Reflector-preserved lithological upscaling

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Slogan

"...from seismic reflectivity to connectivity..."

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Motivation

Equivalent media techniques

- wash out *singularities*
- loose *specular* reflectivity

Because they average *density* and *constitutive* parameters

Wish list

Upscaling techniques that preserve singularities = reflectivity

- link *lithology* to *reflectivity* (e.g. volume fraction shale in sand/shale mixtures)
- provide information on the *connectivity*
- without *oversampling*

Two strategies

1. Replace *linear windowed* equivalent medium averaging by equivalent medium averaging based on *nonlinear approximations* (e.g nonlinear approximations with wavelets based on recent developments in applied Harmonic analysis).
2. Use (rock) physical arguments based on the existence of *critical phenomena* in *statistical mechanics* (e.g., phase transitions in percolation theory)

[Schoenberg '88, '89, '92']

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Equivalent-medium (EM) approaches

Wave-equation driven (homogenization)

- anisotropy
- difference (harmonic) averages for density
- static behavior of waves, i.e., the centroid

Mixture-model driven (binary mixtures)

- HS bounds
- Voigt-Reuss

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[FJH and Bernabe, '04]
[Bernabe and FJH, '04]
[Maysami and FJI '08]

Our approach

Include *connectivity* in models for the *effective* properties of bi-compositional mixtures **<=> SWITCH**

- sand-shale, gas-hydrate, opal/opal CT
- upper-mantle mineralogy

Studied two cases:

- elastic properties upper mantle
- fluid-flow properties synthetic rock

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[FJH and Bernabe, '04]
[Bernabe and FJH, '04]

Approach cont'd

Develop an upscaling methodology based on

- bi-compositional (sand/shale) mixtures
- two litho phases (LP/HP), namely *weak* and *strong*
- assume volume fraction (p) increases *linearly* with depth

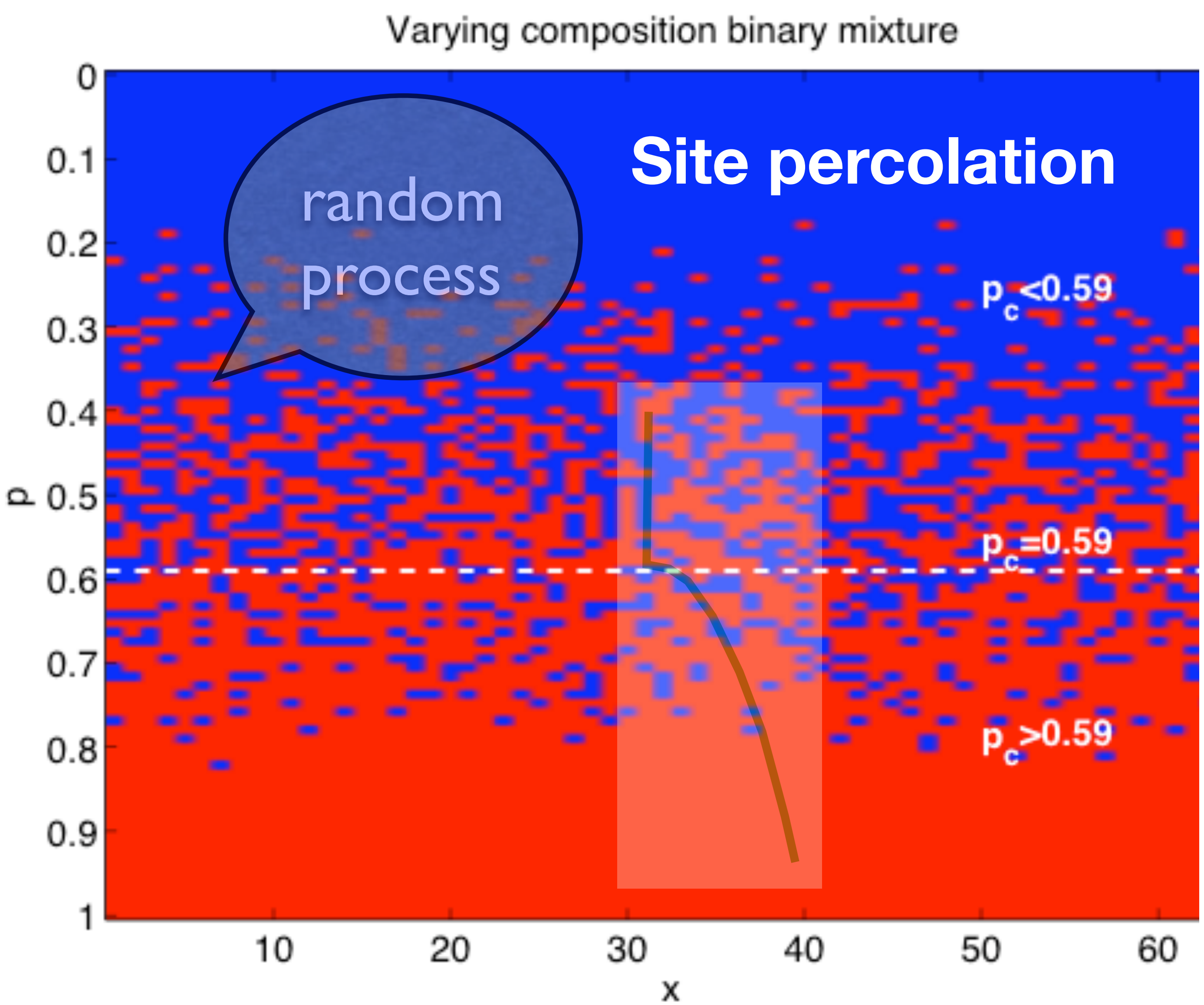
Approach cont'd

Model predictions:

- *volumetric* properties vary smoothly as a function of the volume fractions
- *transport* properties may not...

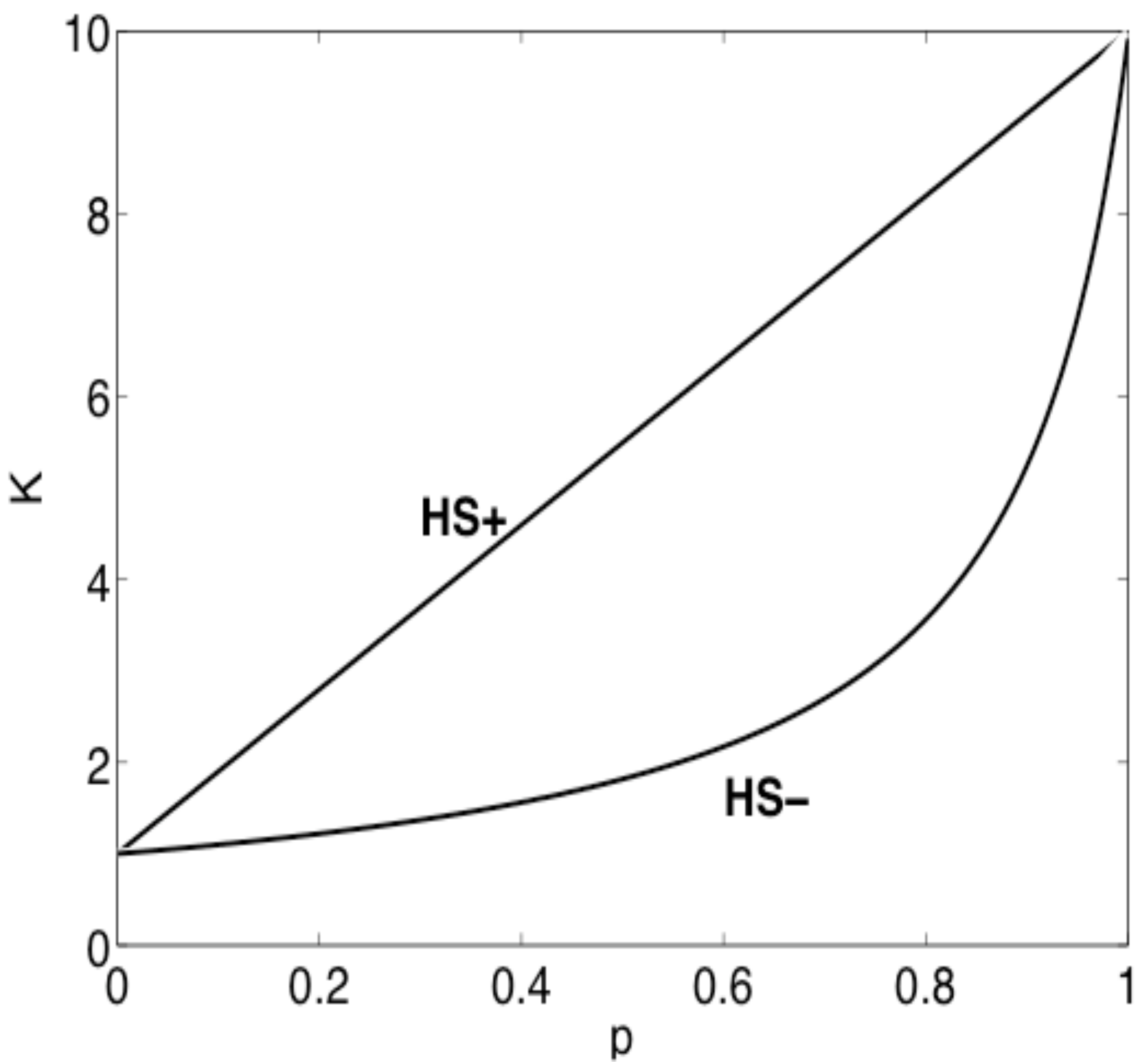
Site-percolation model [FJH and Bernabe, '04]

LP Sand



HP Shale

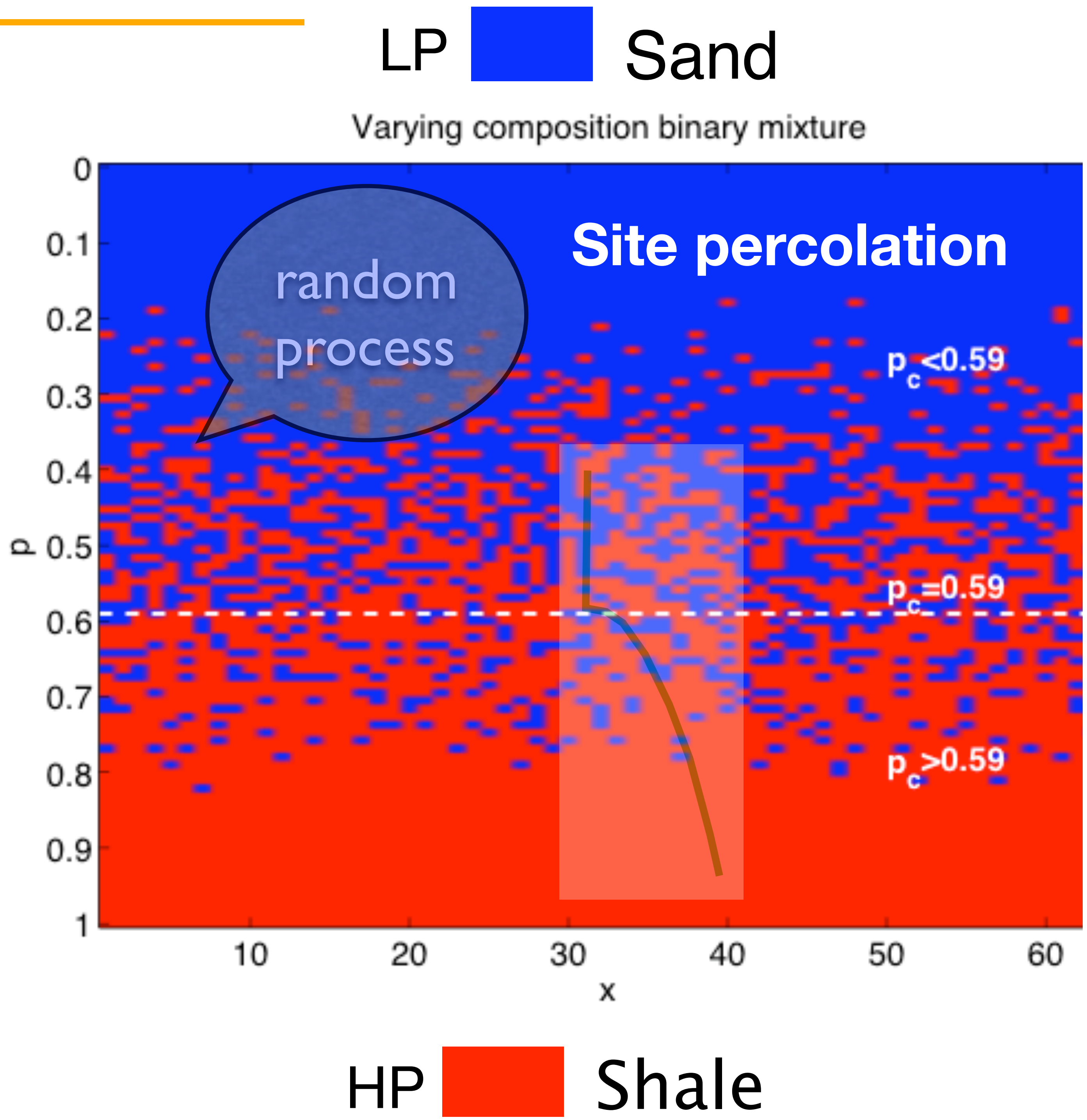
elastic properties

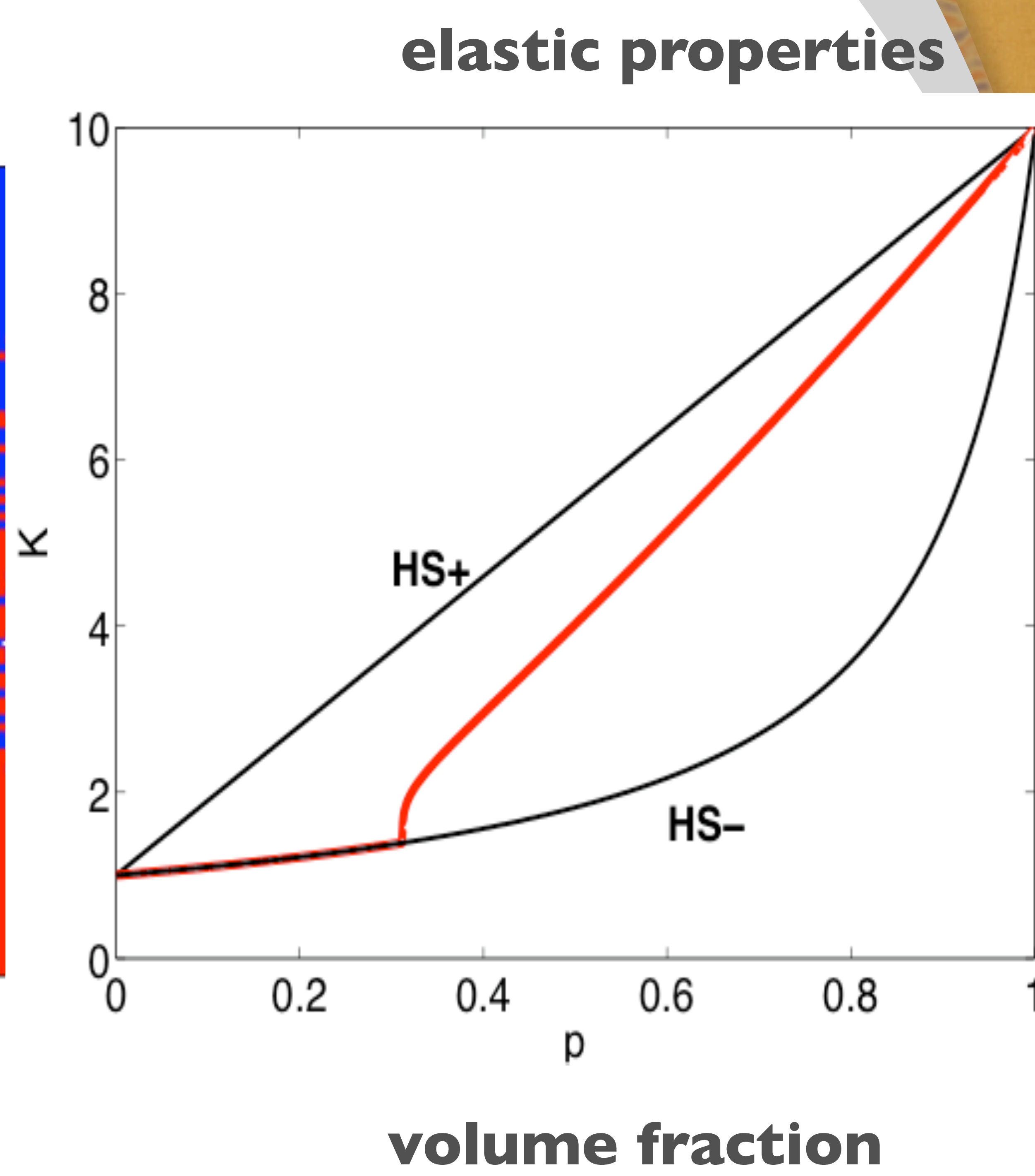


volume fraction

Site-percolation model

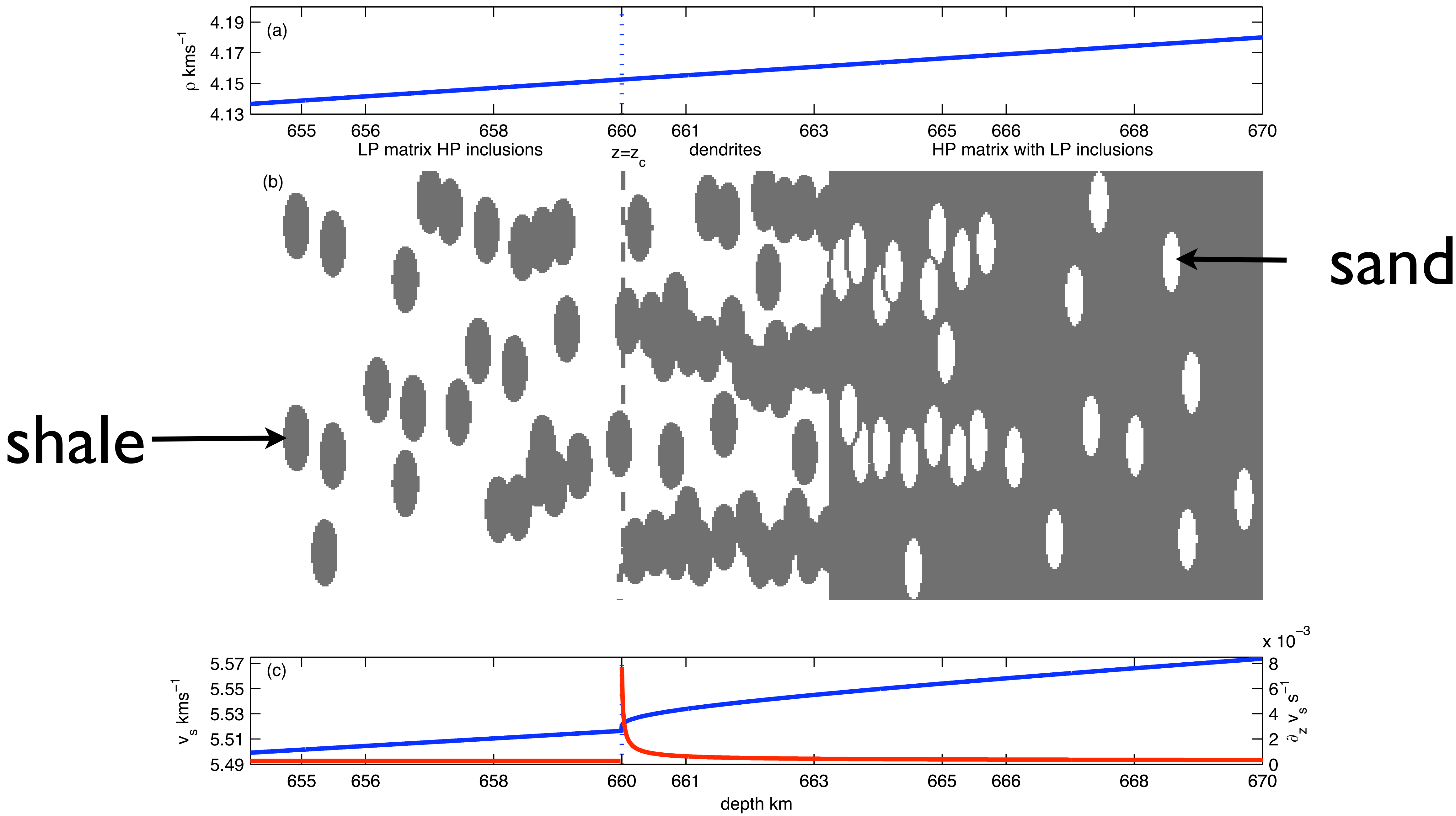
[FJH and Bernabe, '04]





[FJH and Bernabe, '04]

Percolation model



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[Knight '05]

Our approach

Incorporate *geometry* in description binary mixtures.

Distinguish between

- volumetric properties (density & porosity) - do not depend on *geometry/connectivity*
- transport properties (permeability, stiffness, wavespeed) - depend on *geometry/connectivity*

Regimes

For $p < p_c$,

- *weak* mixture with *random disconnected strong* inclusions
- with increasing depth more *strong* inclusions are deposited
- at a *critical* volume fraction (depth), a ***connected*** cluster of *strong* HP is formed

Regimes

For $p = p_c$,

- an infinite cluster of *connected* **strong** (HP) material is formed

Regimes

For $p > p_c$,

- not all HP inclusions are part of the *infinite* HP cluster
- *isolated* HP clusters are *embedded* in the remaining LP to form a *mixture* M
- volume fraction that belongs to HP *infinite* cluster

$$\mathrm{p}^* = p\left(\frac{p-p_c}{1-p_c}\right)^{\beta} \text{ for } \beta > 0$$

Switch

Strength of material proportional to cluster size, i.e.,

$$p^* = \begin{cases} 0 & \text{if} \quad p < p_c \\ p\left(\frac{p-p_c}{1-p_c}\right)^\beta & \text{if} \quad p \geq p_c. \end{cases}$$

Generates a fractional-order singularity at the critical volume fraction.

Universality scale exponents

Scale exponents of Percolation depend on

- dimension & type - e.g. Bond vs Site Percolation
- statistical properties of the mixing
- if *isotropic*, then the scale exponents are *universal*

We use Site Percolation in 3-D yielding $\beta = 0.41$

Regimes

Volume fraction of mixed material is $\mathrm{q}^* = 1 - p^*$.

To model the *mixture*, we need the volume fractions of its LP/HP parts

$$\mathrm{q}_M = (1-p)/\left((1-p) + (p - p^*)\right) \text{ and } p_M = (1 - q_M),$$

yielding

$$p_M = 1 - \frac{q}{1 - p\left(\frac{p-p_c}{1-p_c}\right)^{\beta}}.$$

Ledbetter et al. (1984); Gai et al. (1984); Deptuck et al. (1985); Turosov et al. (1986); Marion and Nur (1989); Favier et al. (1997); Novikov et al. (2001),Stauffer and Aharony (1994),Herrmann and Bernab´e, 2004a; Bernab´e et al., 2004)

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Percolation

Well known that *binary* mixtures are strong when *strong* material is *connected* and *weak* otherwise.

Onsets of *connectivity* yield transitions similar to *phase transitions* predicted by *Statistical Mechanics*

- e.g. the onset of magnetization below Cury temperature

Takes *connectivity* into account...

[Hashin and Shtrikman (1962)]

Bounds

Both the HP and LP phases are elastically isotropic and HP inclusions are spherical so the HP/LP mixture is locally (statistically) isotropic.

Model materials with *isolated randomly* distributed inclusions inside *connected* matrix.

Use *upper* bound when *strong* component forms the *connected* matrix.

Use *lower* bound otherwise.

[Marion and Nur (1989); Favier et al. (1997); Novikov et al. (2001); Saidi et al. (2003)]

Bounds

Bulk modulus ***above*** *critical* depth

$$K = K_{LP}\left(1 + \frac{p(K_{HP} - K_{LP})}{q(K_{HP} - K_{LP})a_{LP} + K_{LP}}\right)$$

with $p = q - 1$ and

$$\mathrm{a}_{LP} = 3K_{LP}/(3K_{LP} + 4G_{LP})$$

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[Marion and Nur (1989); Favier et al. (1997); Novikov et al. (2001)]
[Saidi et al. (2003), Herrmann and Bernabe '04; Bernabe and Herrmann, '04]

Bounds

Bulk modulus ***below*** *critical* depth

$$K = K_H \left(1 + \frac{q^*(K_M - K_{HP})}{p^*(K_M - K_{HP})a_{DHLP} + K_{HP}} \right),$$

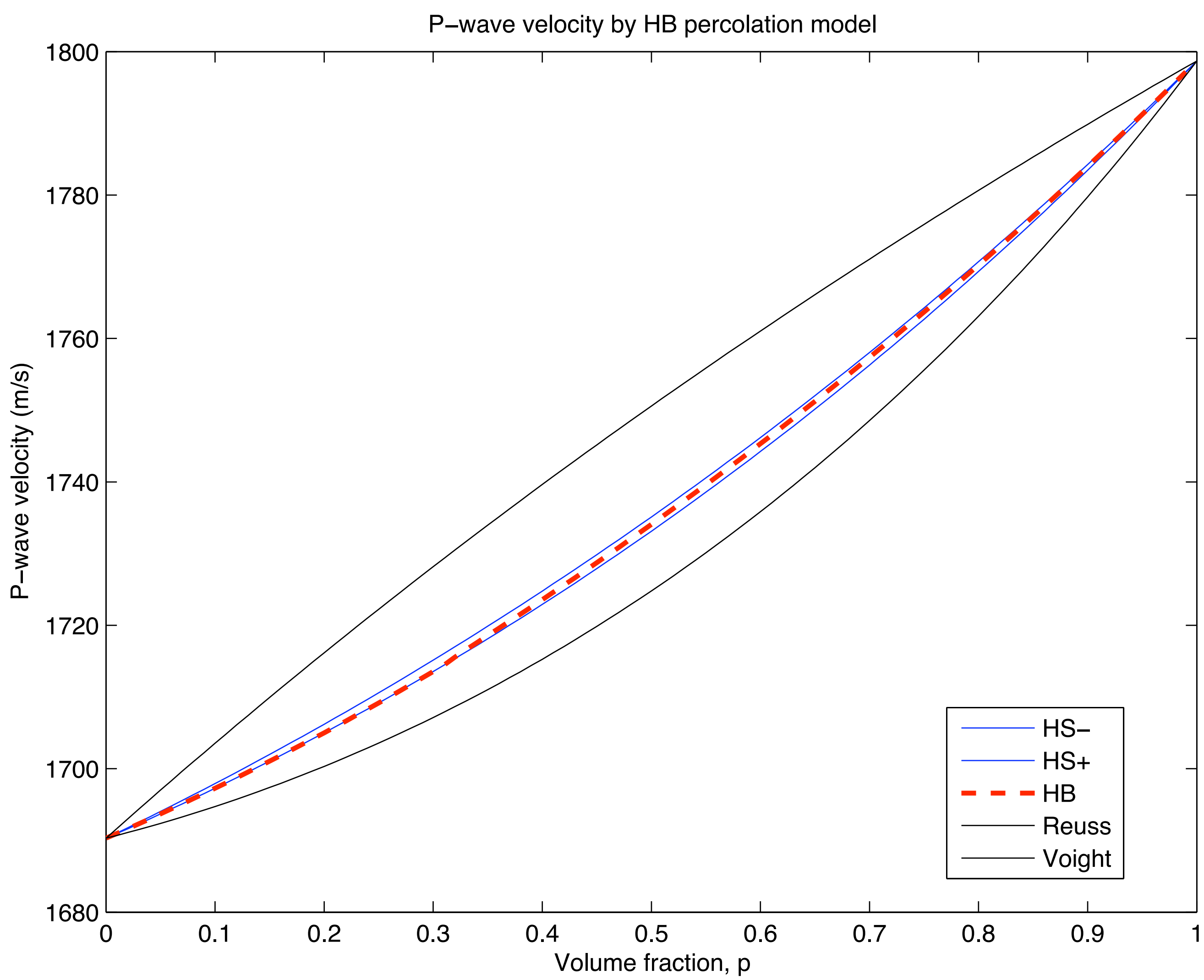
with

$$K_M = K_{LP} \left(1 + \frac{p_M(K_{HP} - K_{LP})}{q_M(K_{HP} - K_{LP})a_{LP} + K_{LP}} \right).$$

Similar *expressions* hold for *shear* modulus G.

$$\begin{aligned} \rho_1 &= 1700 \\ \rho_2 &= 1400 \\ K_1 &= 3.5e9 \\ K_2 &= 2.8e9 \\ G_1 &= 1.5e9 \\ G_2 &= 0.9e9 \\ p_c &= 0.3116 \\ \beta &= 0.41 \end{aligned}$$

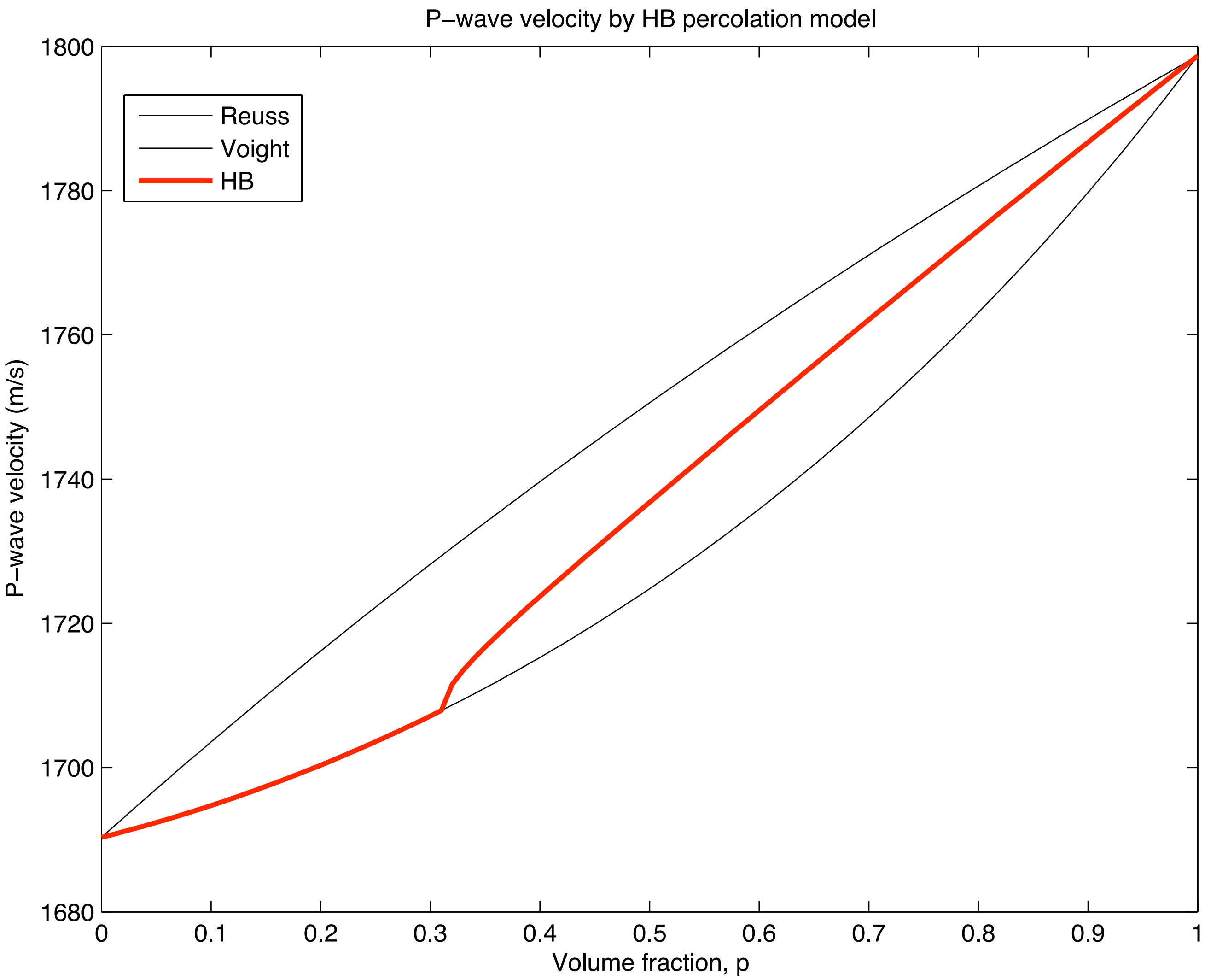
Equivalent medium



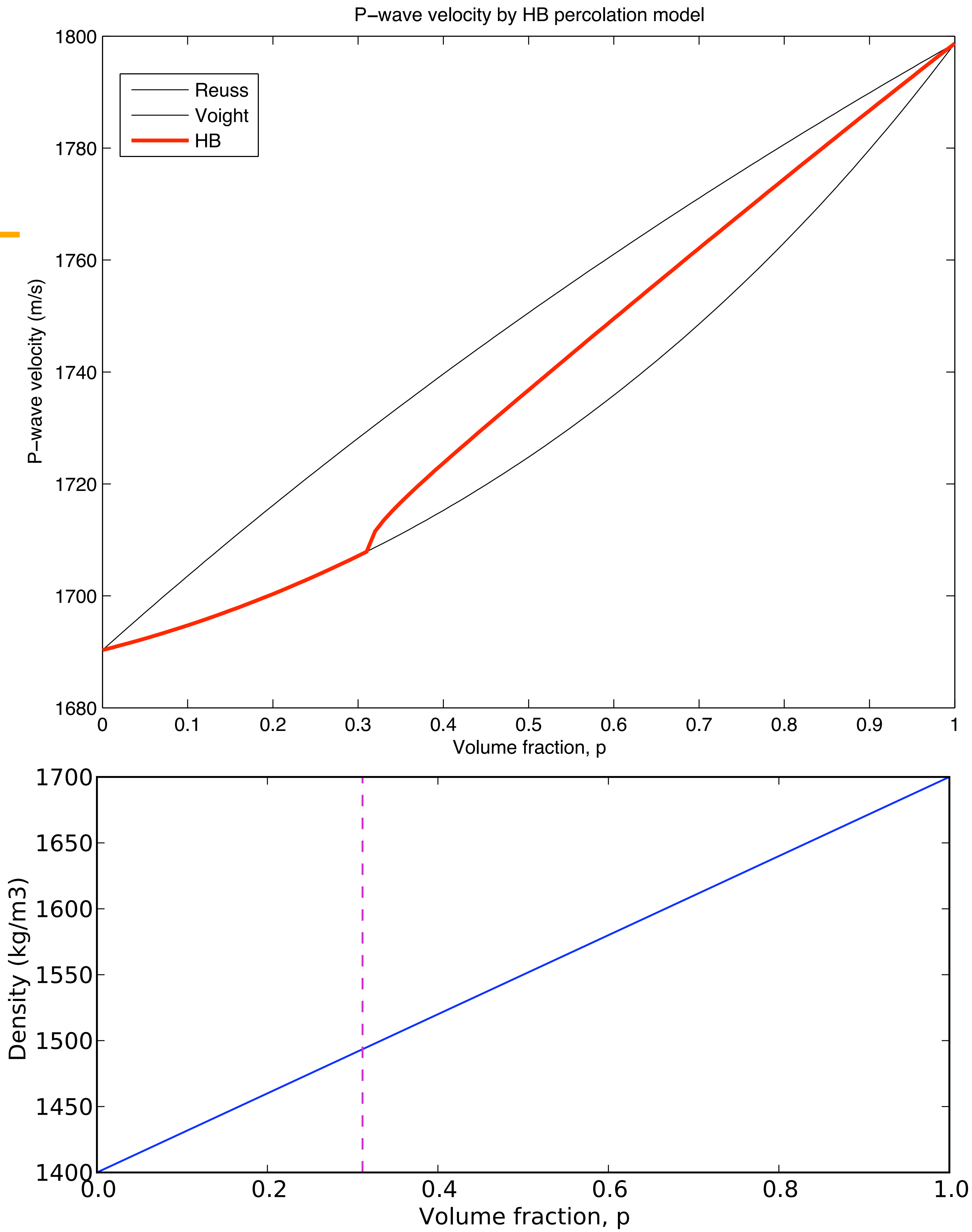
- Profile is smooth
- HS bounds are narrow
- Reuss-Voigt are wide

$$\rho_1 = 1700$$
$$\rho_2 = 1400$$
$$K_1 = 3.5e9$$
$$K_2 = 2.8e9$$
$$G_1 = 1.5e9$$
$$G_2 = 0.9e9$$
$$p_c = 0.3116$$
$$\beta = 0.41$$

Percolation model



- ***Below*** p_c, HP is *disconnected*, use ***lower*** *bound*
- ***Above*** p_c, HP is connected, switch to ***upper*** *bound* with appropriate volume fractions
- Switching leads to ***singularity*** at $p = p_c$
- Use Reuss-Voigt



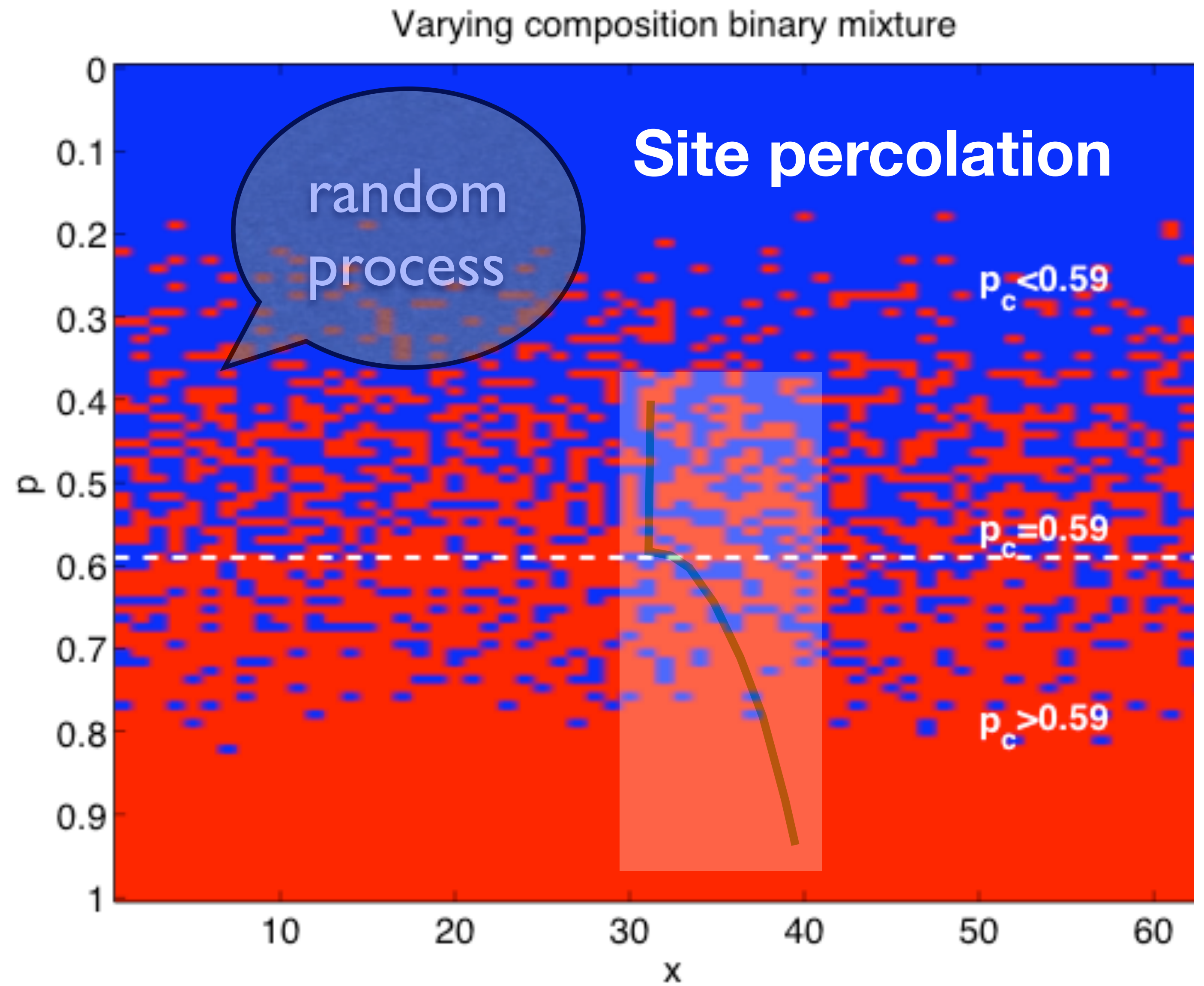
Density varies ***smoothly***

Velocity does ***not***

Singularity generates ***specular*** *reflectivity*

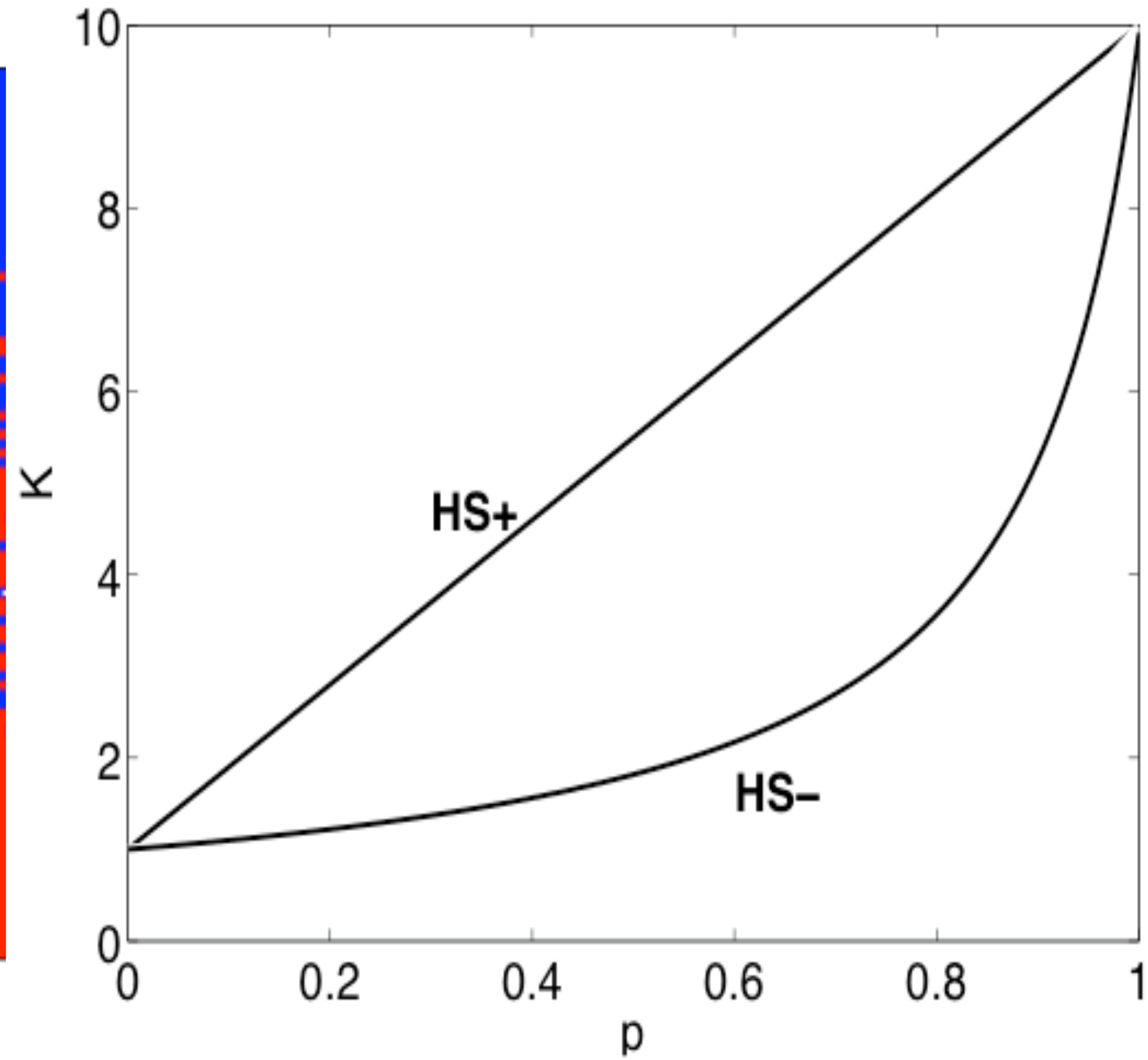
Site-percolation model [FJH and Bernabe, '04]

LP Sand



HP Shale

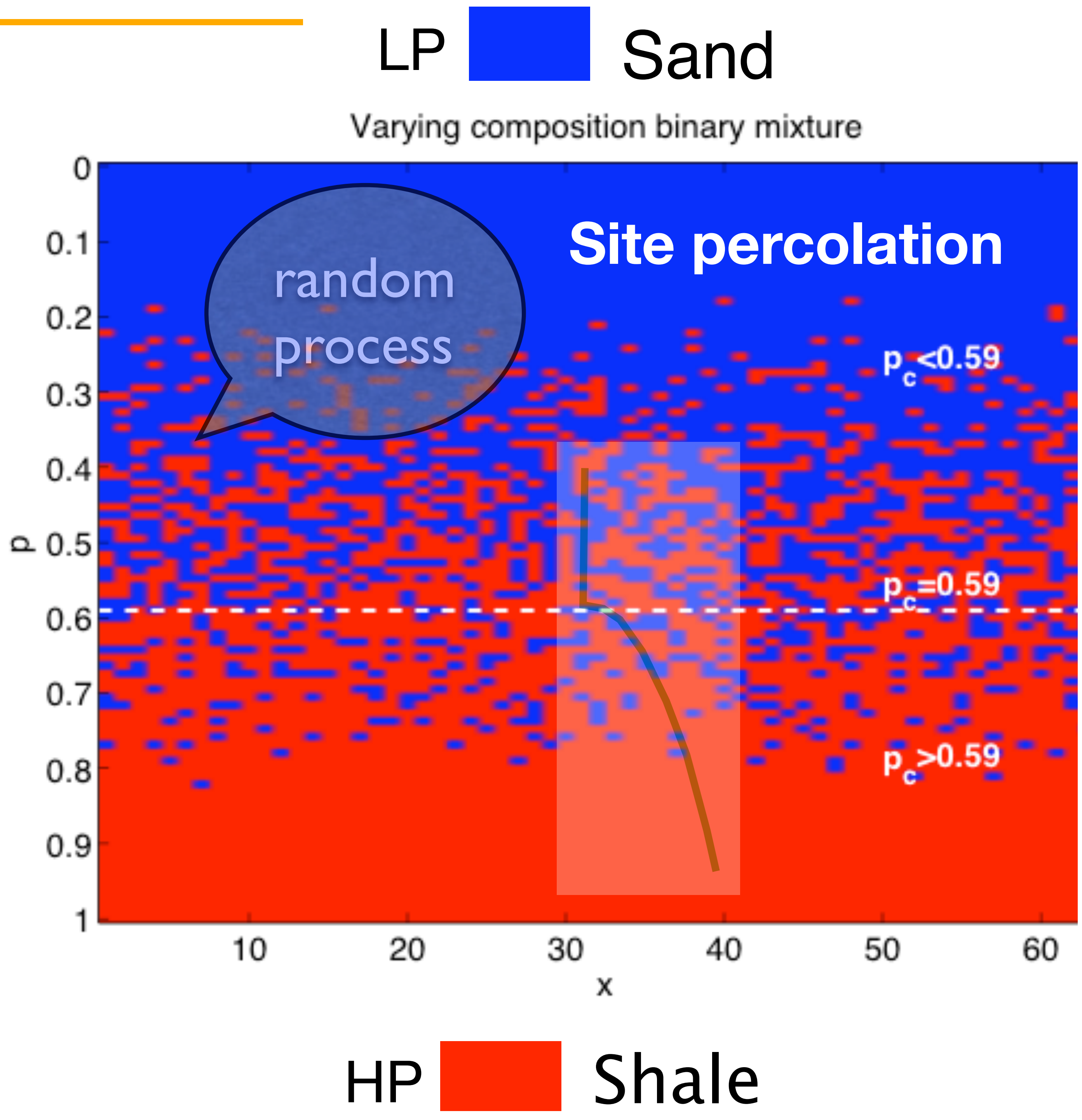
elastic properties

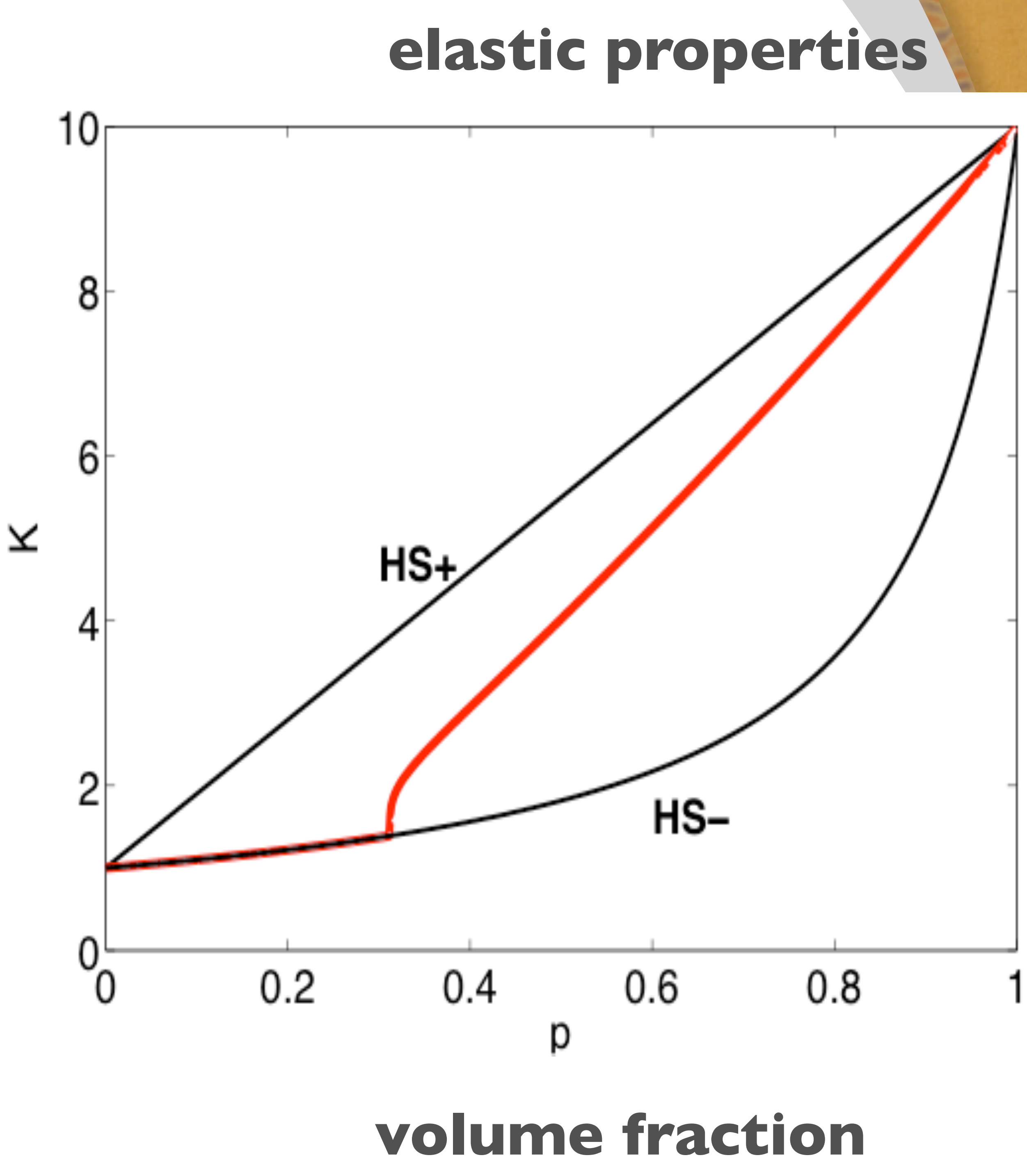


volume fraction

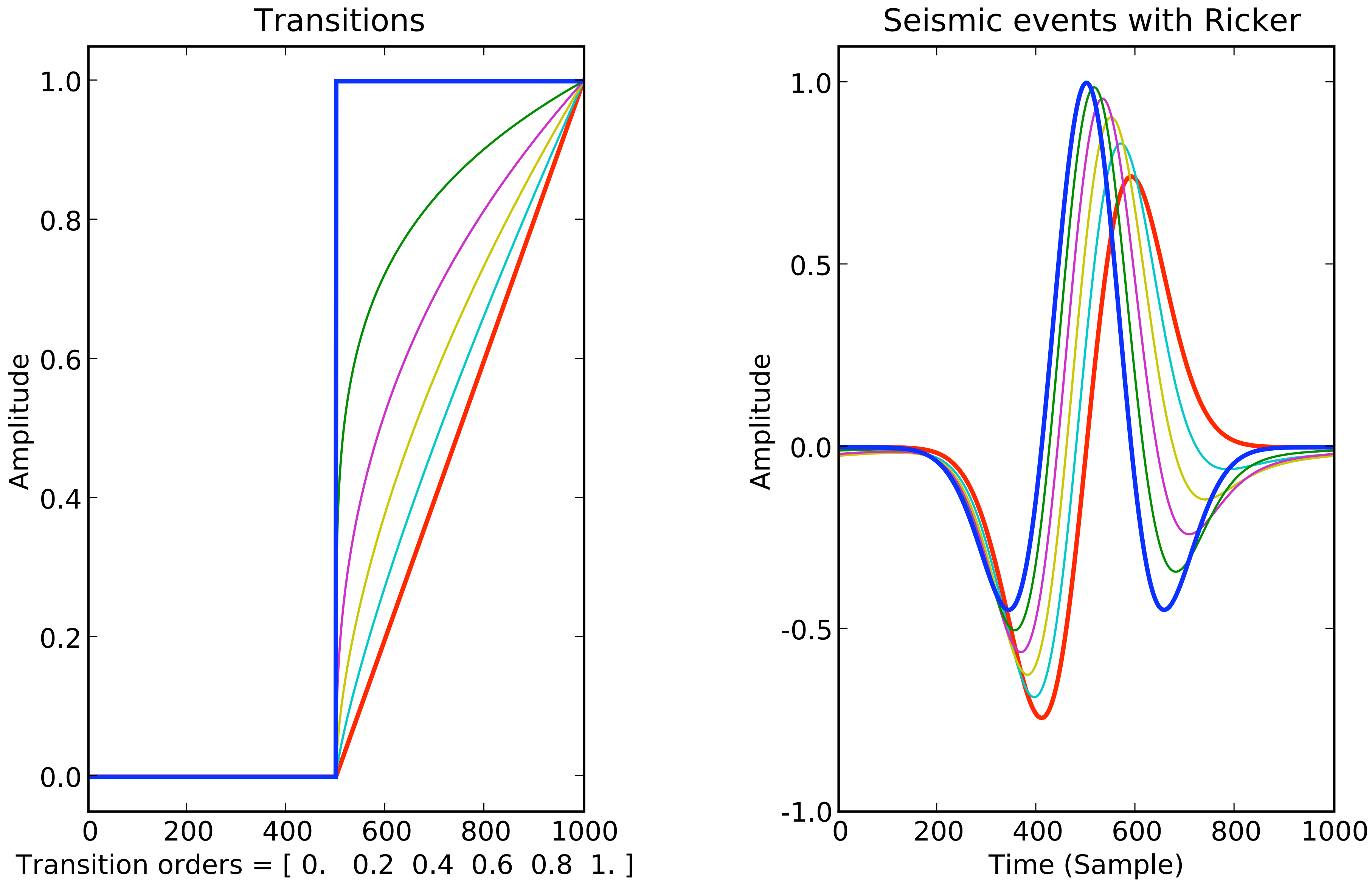
Site-percolation model

[FJH and Bernabe, '04]





Singularity analysis



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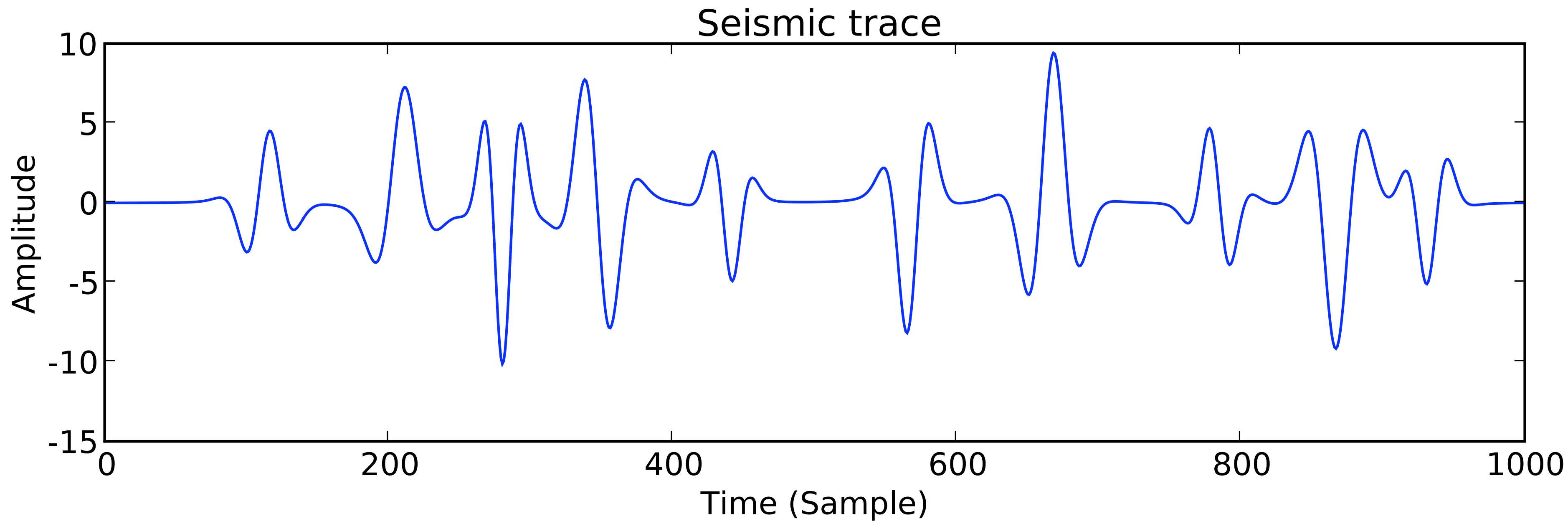
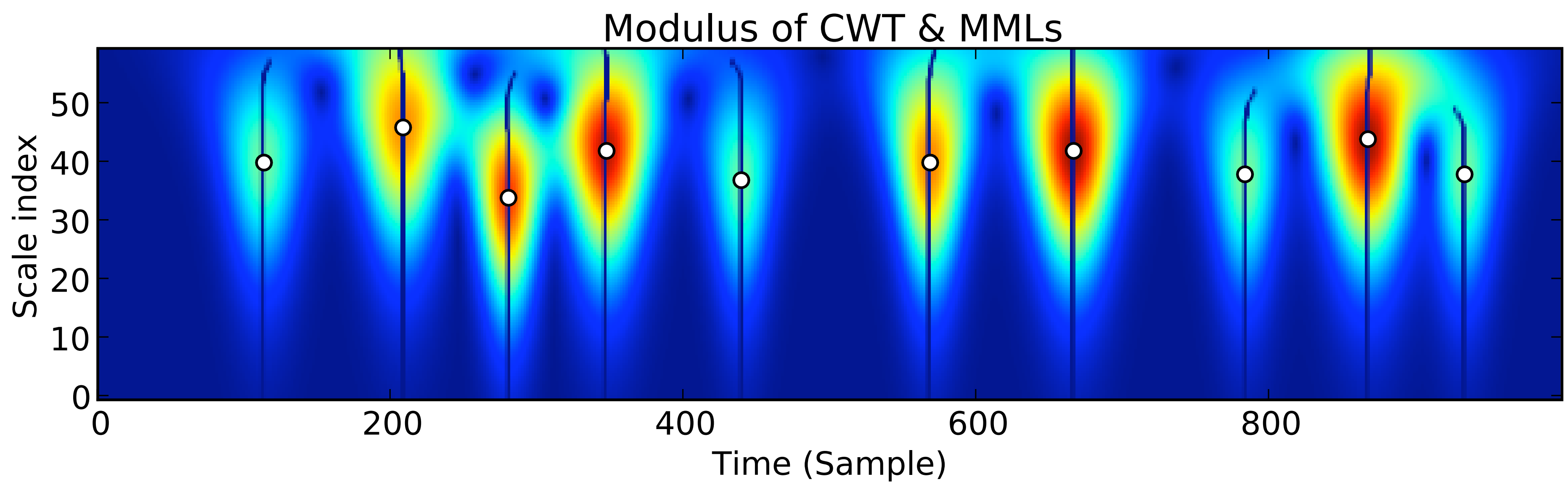
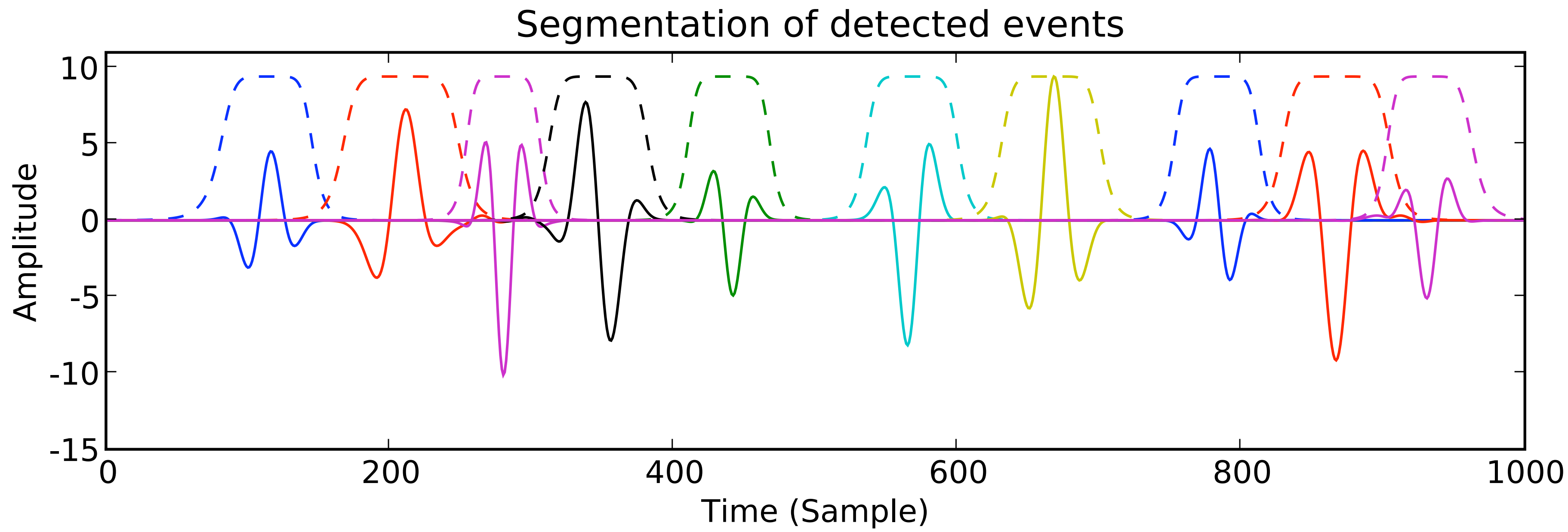
Detection-estimation

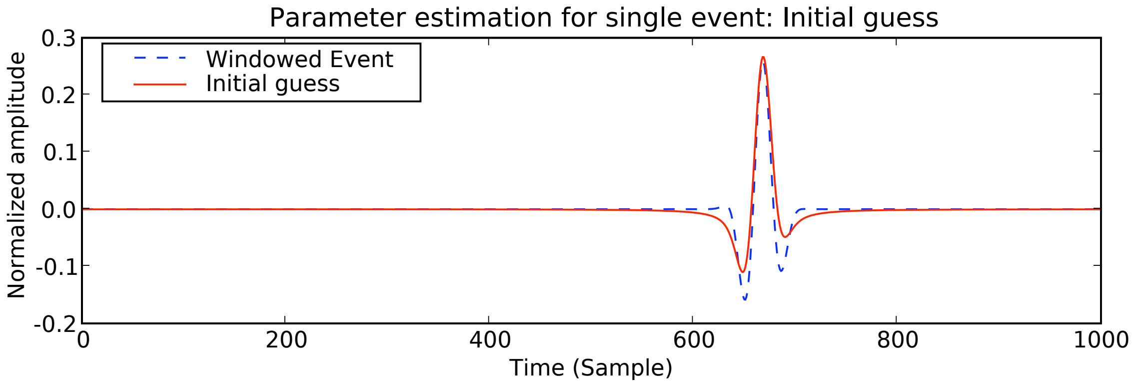


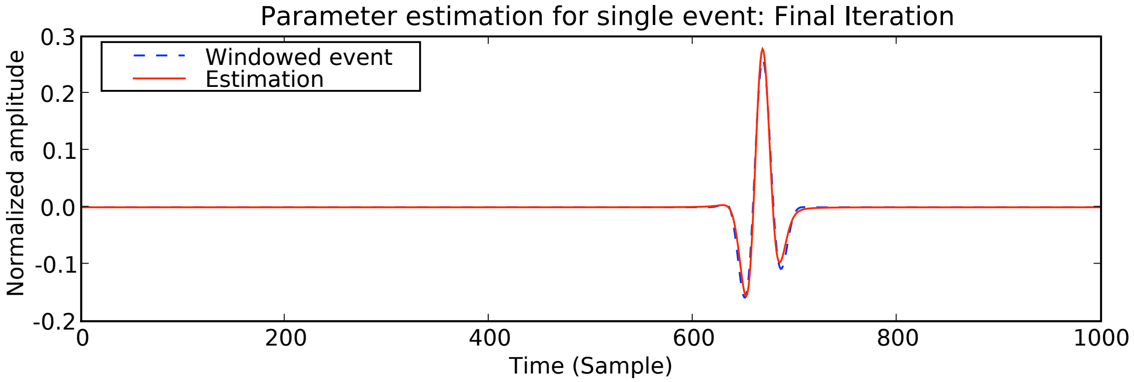
Multiscale detection:



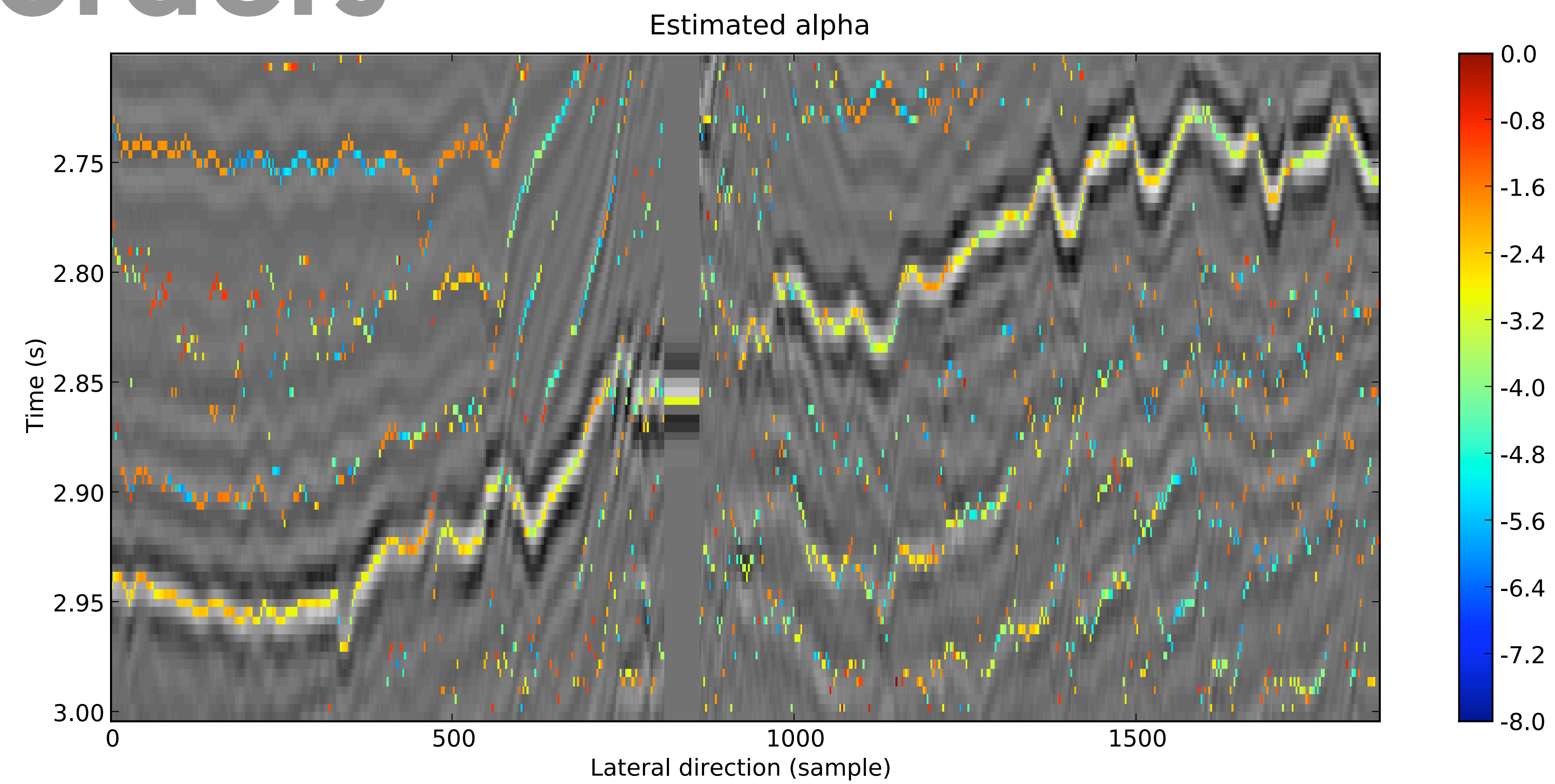
Segmentation & estimation





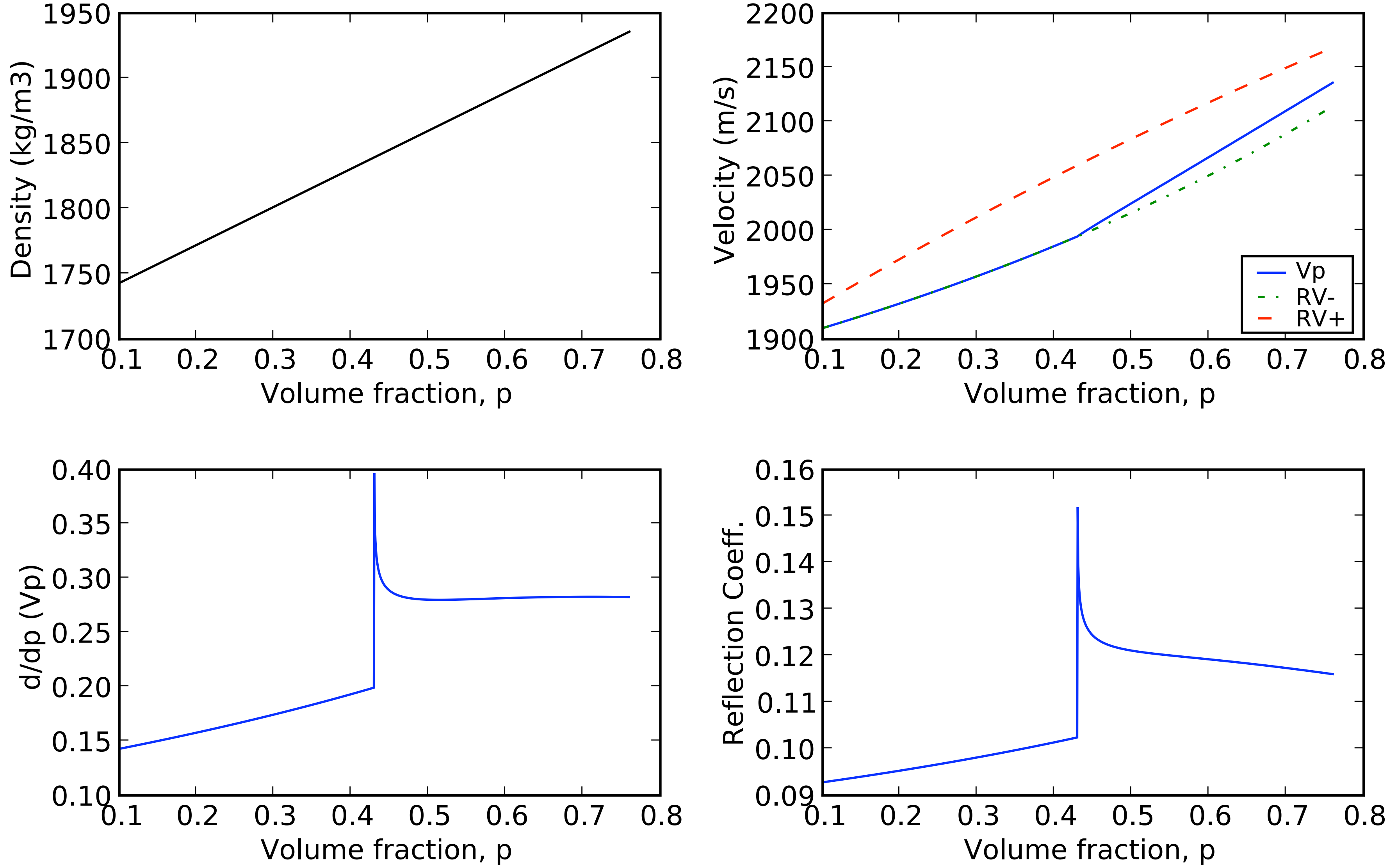


Estimated singularity orders



Used to constrain the scale exponent for well to seismic tie.

Opal-Opal CT transition



[Schoenberg '88, '89, '92']

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Upscaling problem

How can we upscale to preserve reflectivity?

Mike's proposal is to do a moving average equivalent medium averaging-i.e.,

$$c_{v,\sigma}(z) = \sqrt{\frac{1}{\big(\rho * \phi_\sigma\big)(z)\big(\kappa^{-1} * \phi_\sigma\big)(z)}}$$

$$c_{h,\sigma}(z) = \sqrt{\frac{\big(\rho^{-1} * \phi_\sigma\big)(z)}{\big(\kappa^{-1} * \phi_\sigma\big)(z)}}$$

[Foldstad and Schoenberg, '92]

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Upscaling problem

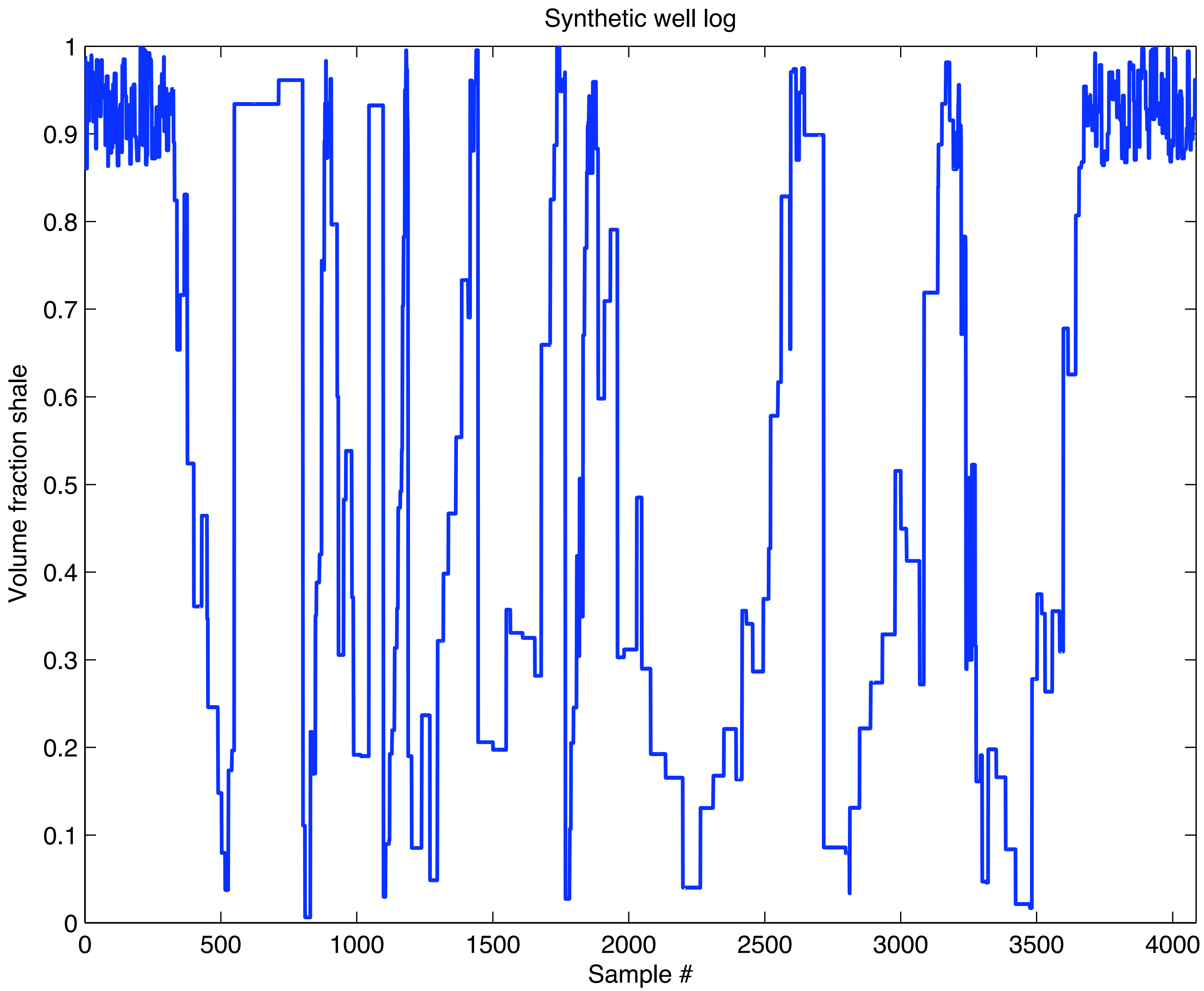
... But in that case we need to "oversample" by a factor of 10 ...

... and this may lead to difficulties during inversion...

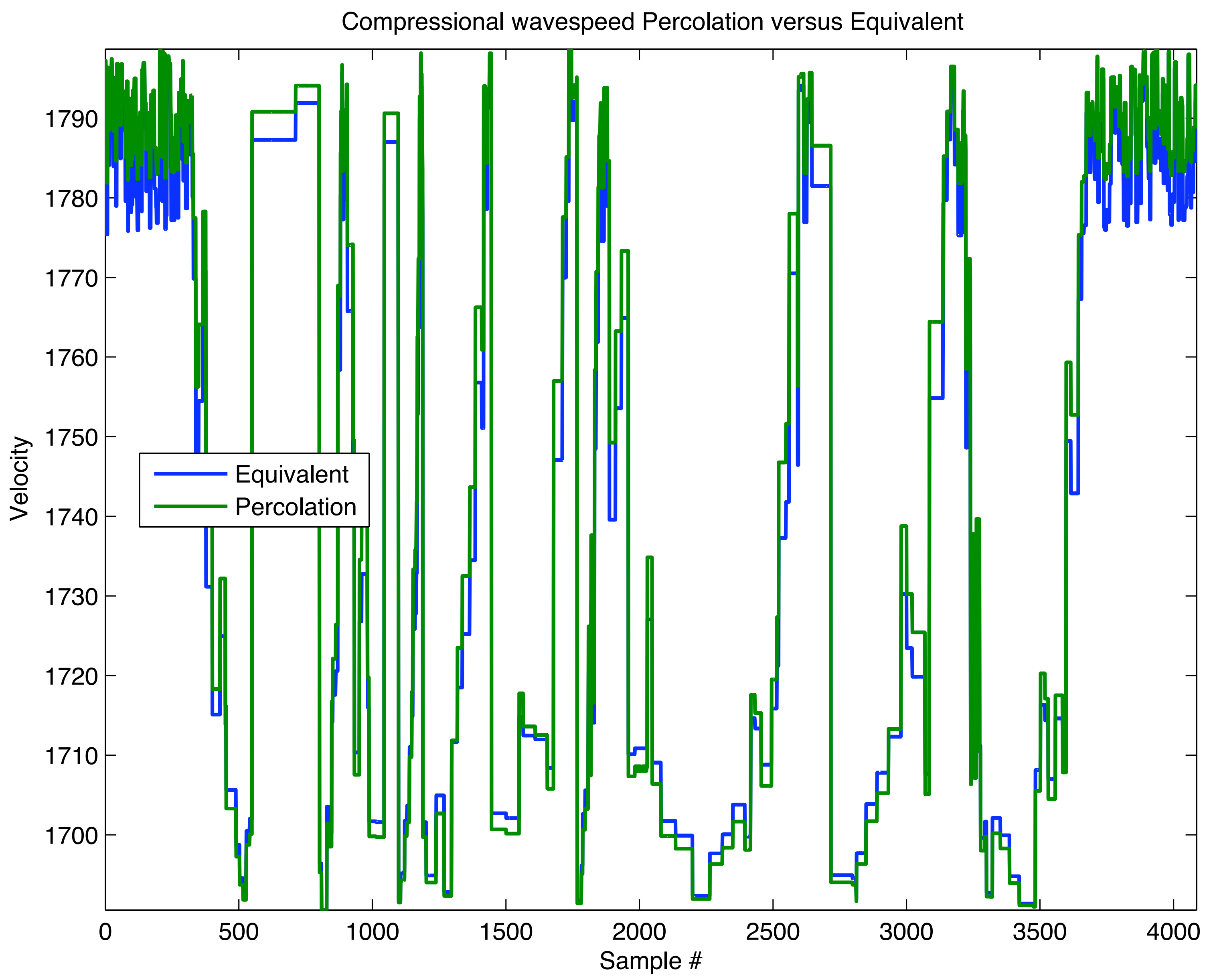
Upscaling problem

We can use our percolation model instead...

Synthetic log

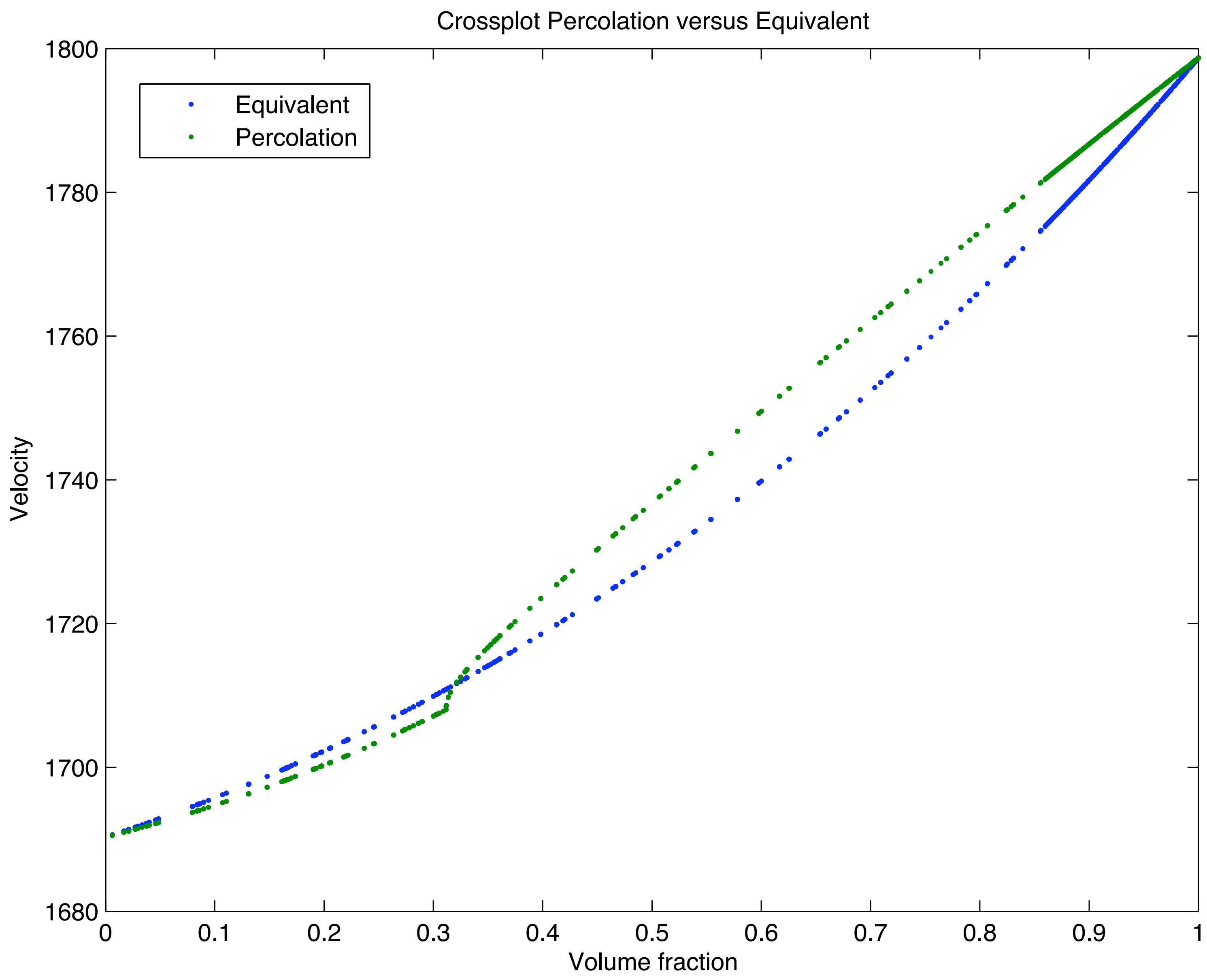


Velocities



Compressional wavespeed Percolation versus Equivalent
Velocity
Sample #
Equivalent
Percolation
1700
1710
1720
1730
1740
1750
1760
1770
1780
1790
0
500
1000
1500
2000
2500
3000
3500
4000

Cross plots



Crossplot Percolation versus Equivalent
Equivalent
Percolation
Velocity
Volume fraction
1800
1780
1760
1740
1720
1700
1680
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Upscaling dillema

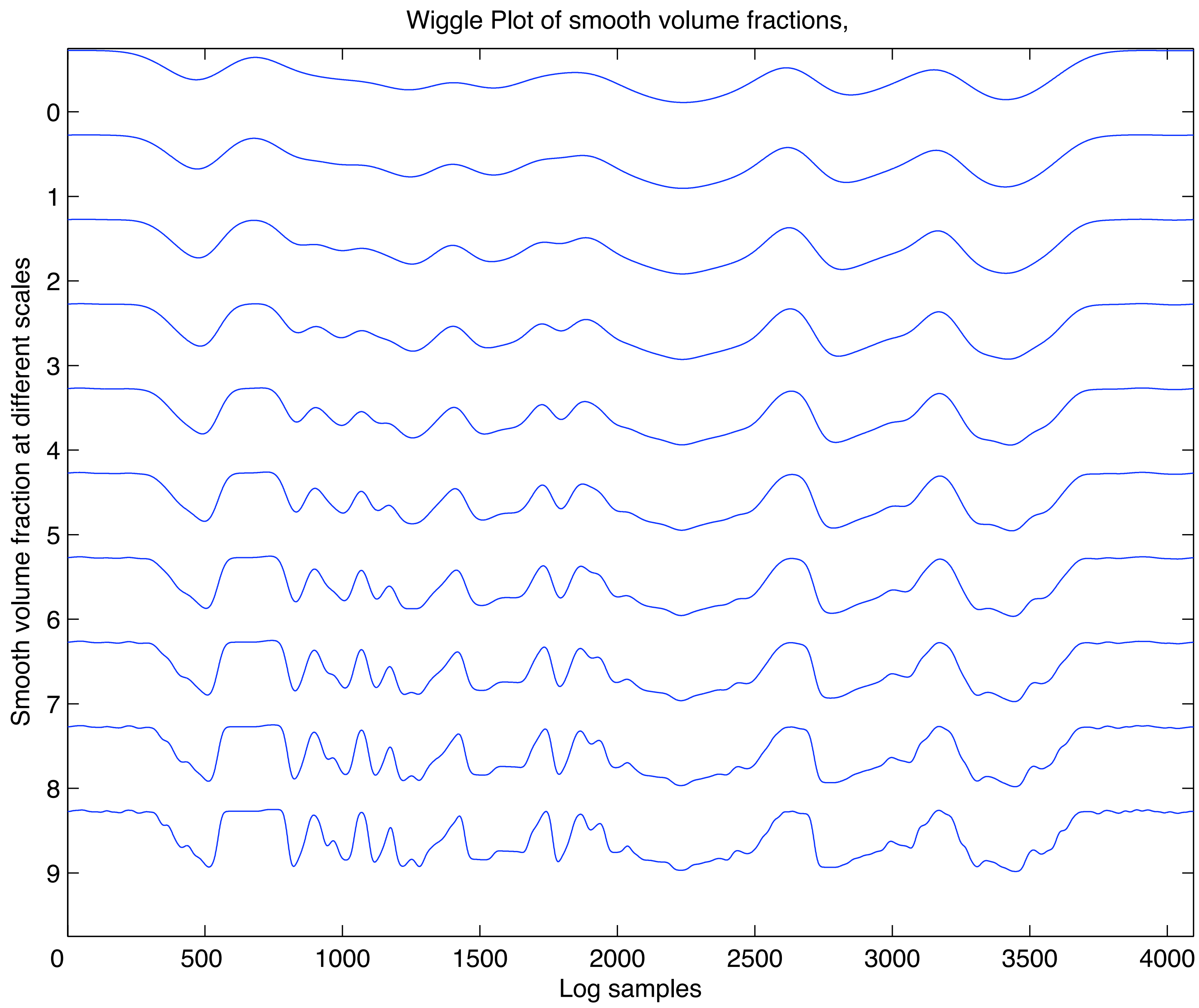
Suppose we are given volume fractions p(z) for shale.

Use the Percolation model to compute fine-grained velocities.

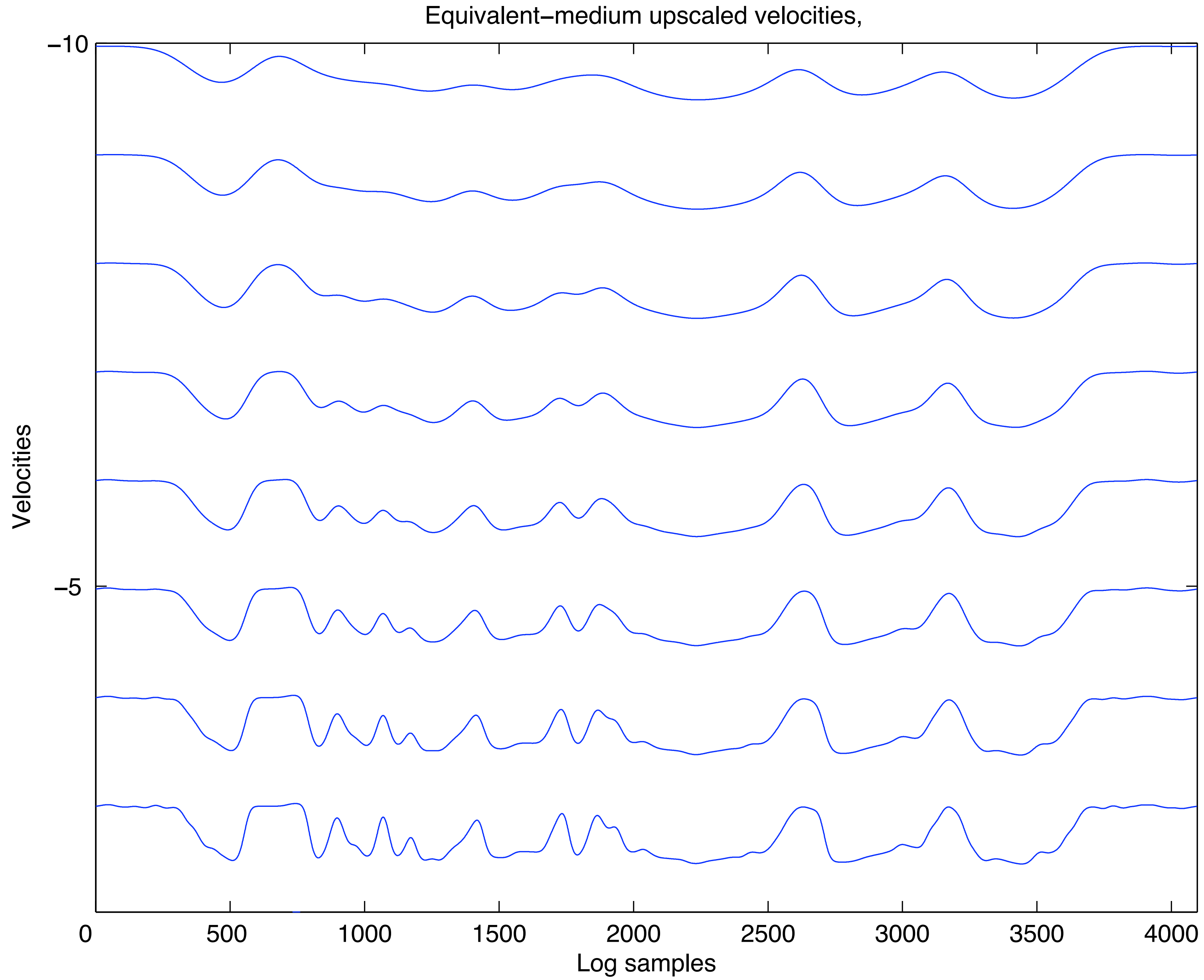
Two options to upscale:

- average *fine-grained* velocities and densities *but* this *smoothes out* the *switch*
- average the *volume fractions because* this *preserves* the *switch*

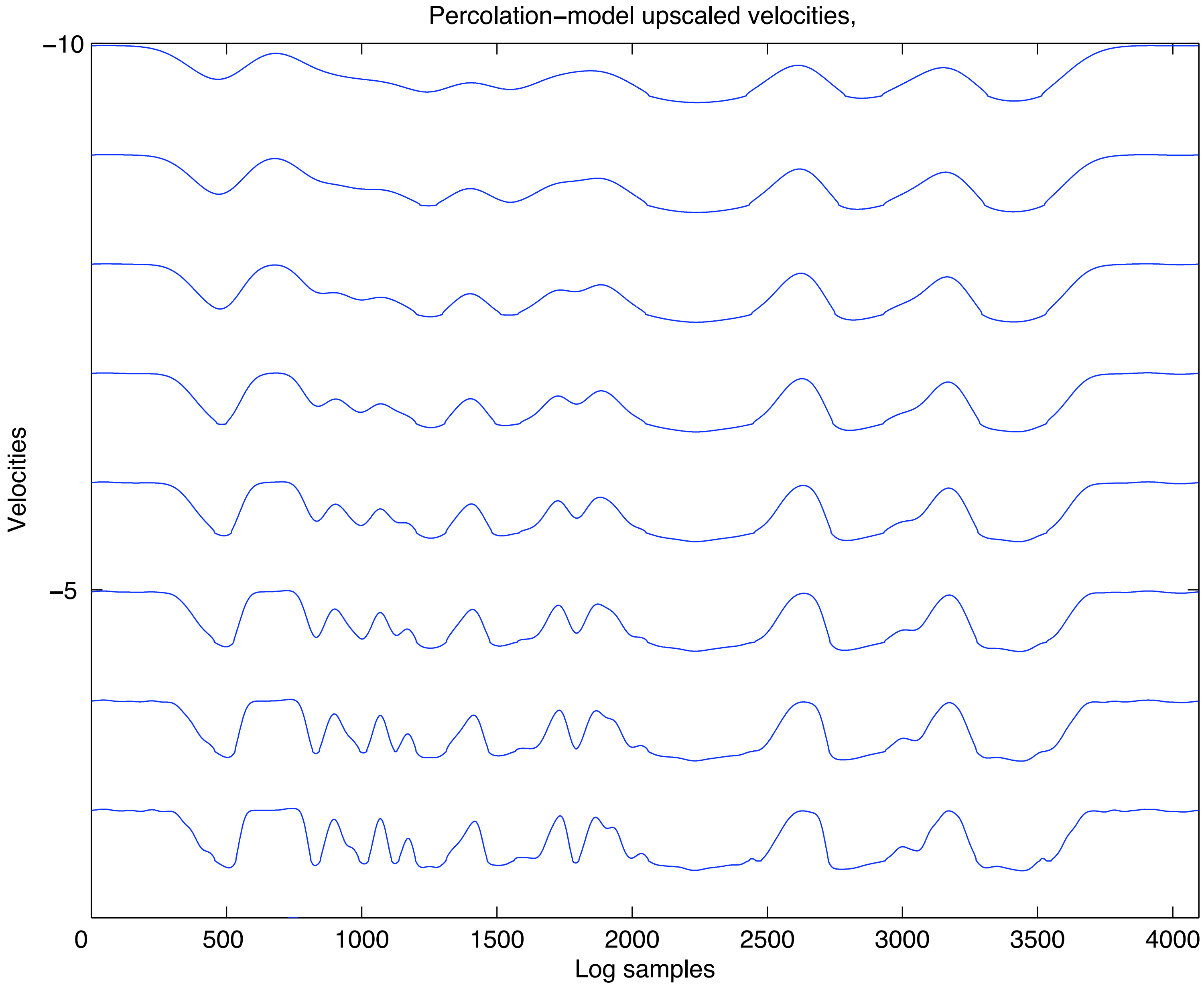
Upscaled lithology



Upscaled EM velocities

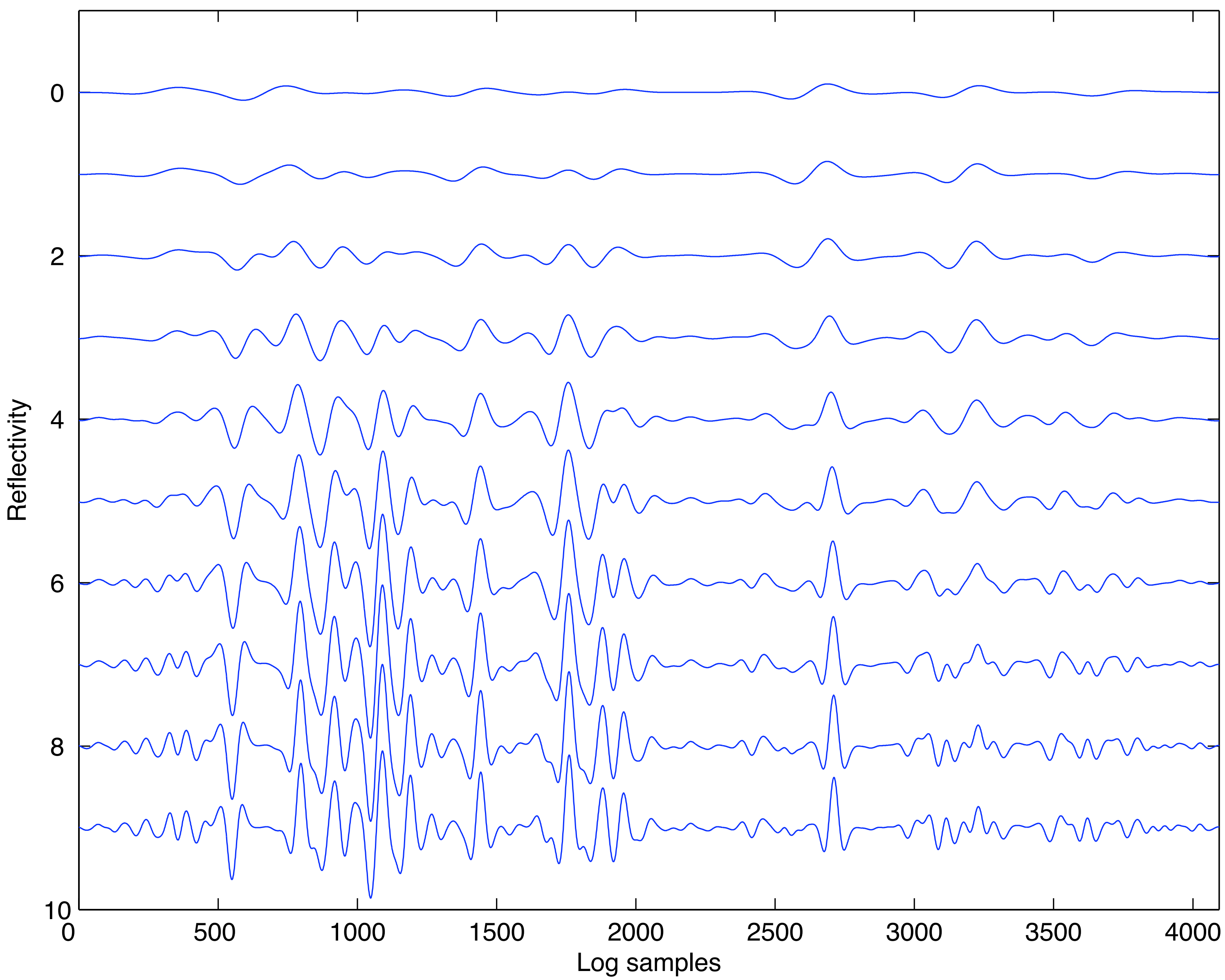


Upscaled Percolation velocities

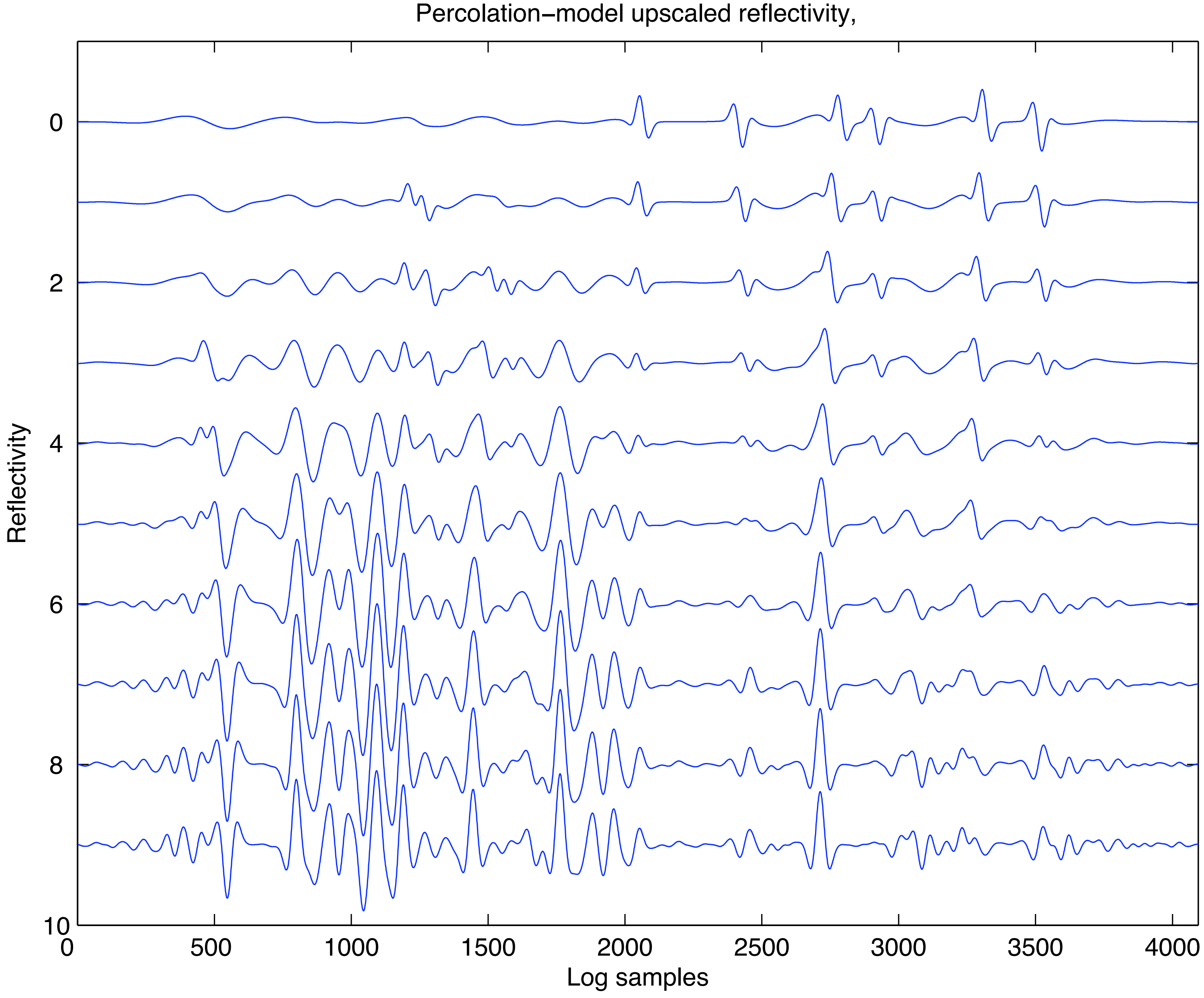


Upscales EM reflectivities

Equivalent−medium upscaled reflectivity,



Upscaled Percolation reflectivities



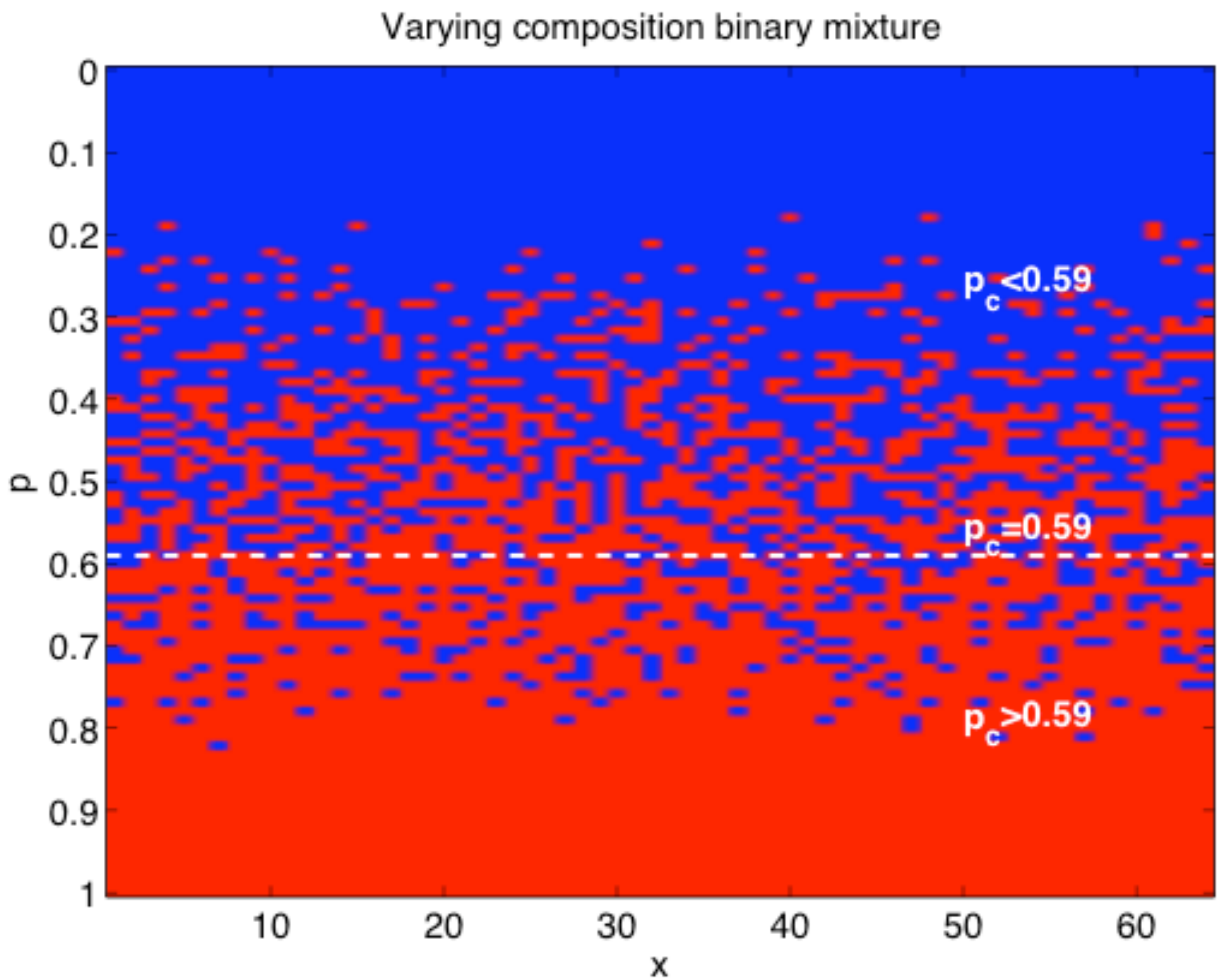
Observations

At *fine* scales, the (zero-order) singularities in the lithology dominate the reflectivity

At *coarse scales*,

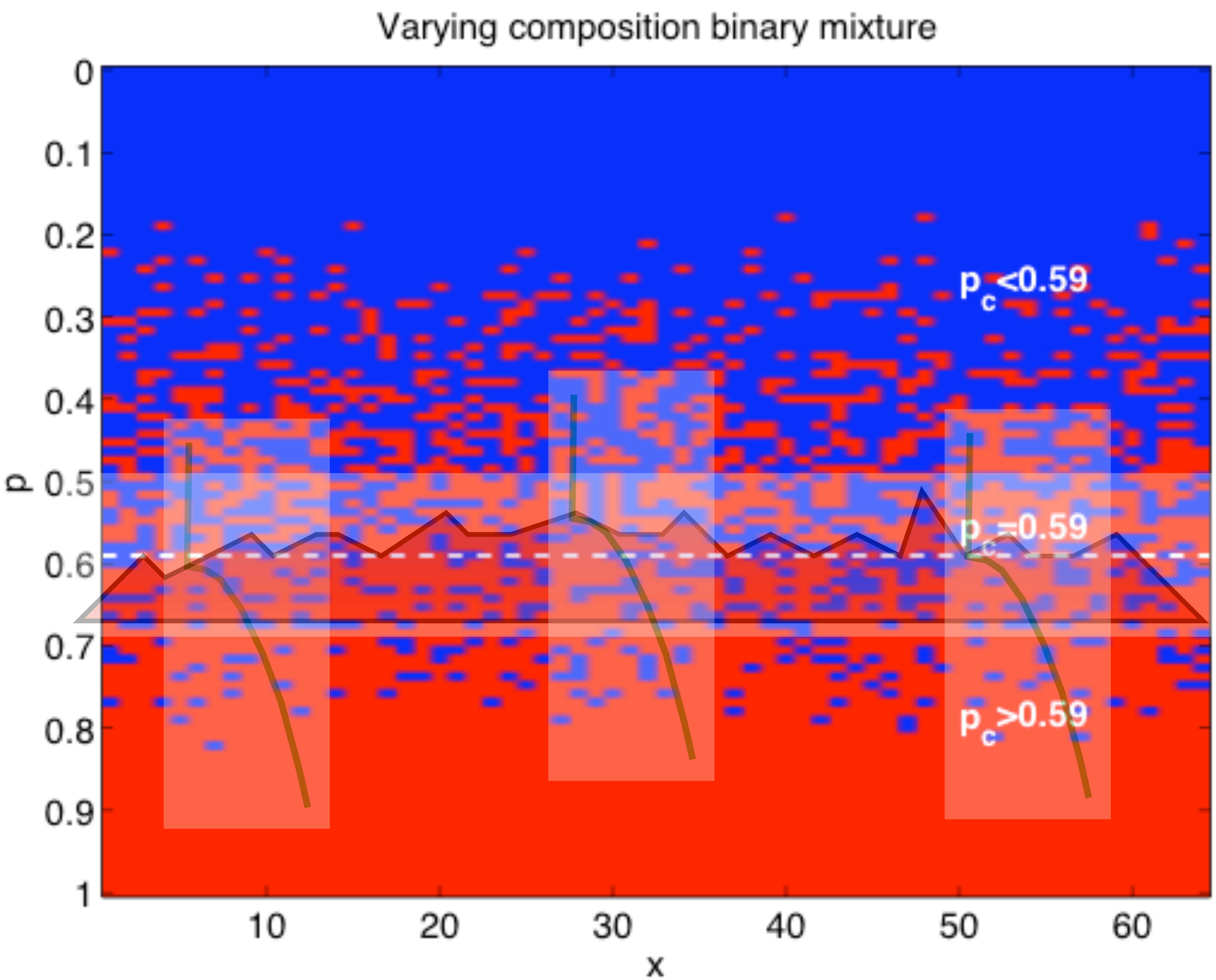
- *EM*-based reflectivity smoothes out
- *Percolation*-based reflectivity is *persistent* and is dominated by the (fractional)-order singularity

Morphology?



Varying composition binary mixture
p
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60
x
p_c<0.59
p_c=0.59
p_c>0.59

Morphology?



Varying composition binary mixture
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0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60
x
p_c<0.59
p_c=0.59
p_c>0.59

Conclusions

Percolation model preserves the singularities

Switch model provided "access" to the fine-structure (connectivity) from macroscopic waves

Rigorous mathematical framework for the "shapes" of these percolation-induced transitions is an open problem...

Acknowledgments

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Thanks to Dave Wilkinson for providing the synthetic log.

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"...a few words"

Thank you for your attention!

more information

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