SEG HOUSTON 2009

# Compressive simultaneous full-waveform simulation

Tim T.Y. Lin* *jointly with Felix J. Herrmann and Yogi A. Erlangga*

SLIM
University of British Columbia

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*timtylin@gmail.com

# Wave-equation methods

noun

SYNONYM: very slow methods

## why?

modeling costs

# revolutions

**Acquision:** Slip-sweep, HFVS, ISS

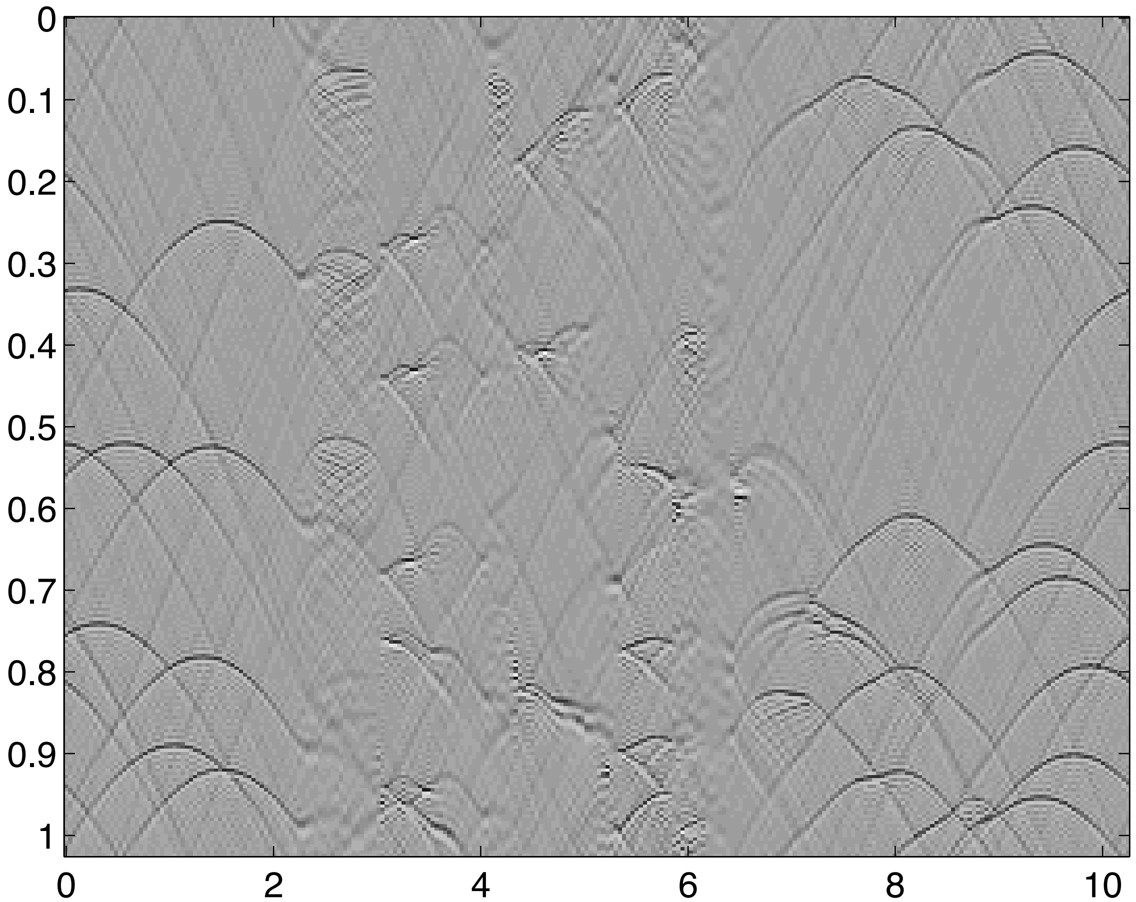
**Modeling:** ?

**Imaging:** Check-pointing, Frequency domain inversion

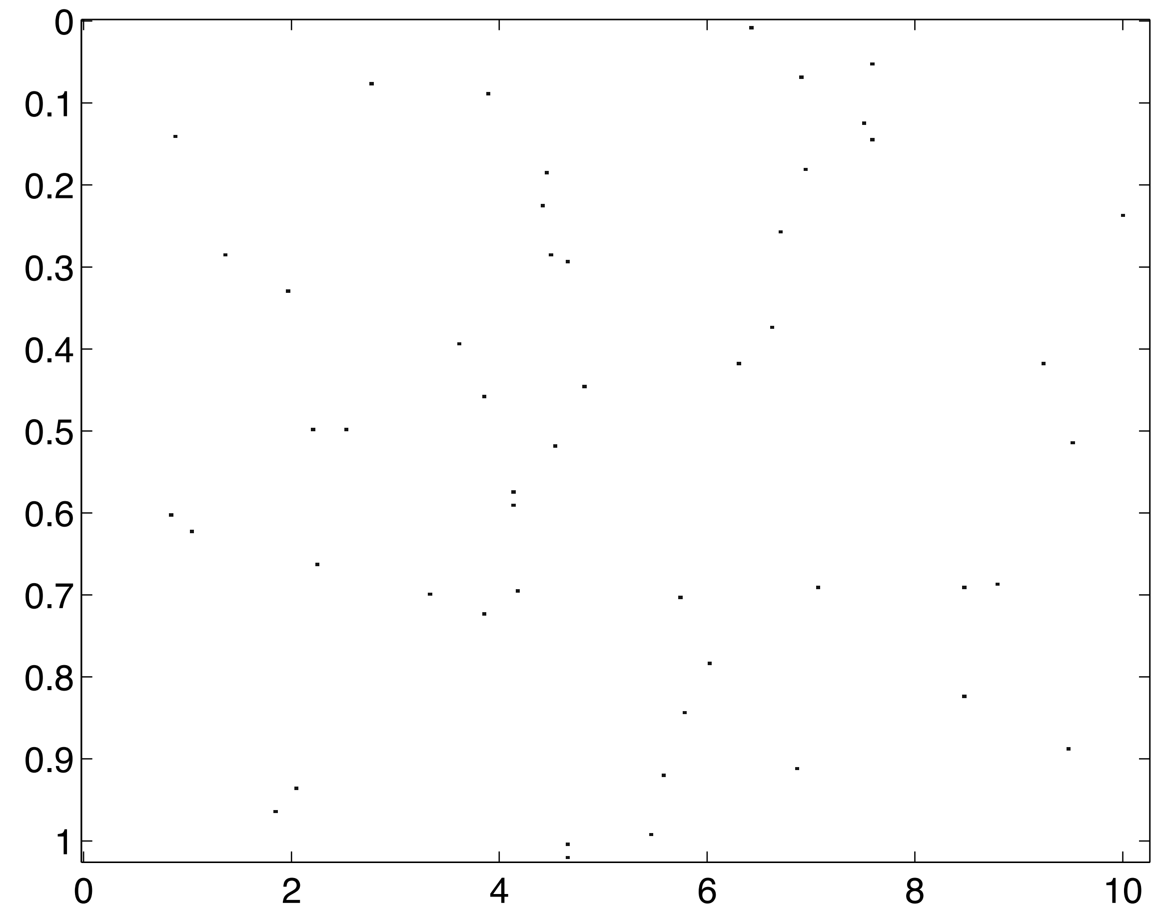
# holy grail

Can we use simultaneously acquired data directly in seismic imaging?

# from SEG 2007




propagated 1.5km down




recovered though L1 inverson

Restricted One-Way Wavefield extrapolator to 1% of original eigenvalues of Helmholtz operator

(Lin & Herrmann 07, Demanet & Peyre 08)

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This talk is concerned with frequency-domain wave equation seismic modeling:

$$\mathcal{H}(\omega)u(\omega, x_s; x) := \left(-\nabla^2 + \frac{\omega^2}{c^2(x)}\right) u(\omega, x_s; x) = b(\omega, x_s)$$

*Invert Helmholtz operator H on b to obtain frequency-domain wavefield u(w)*

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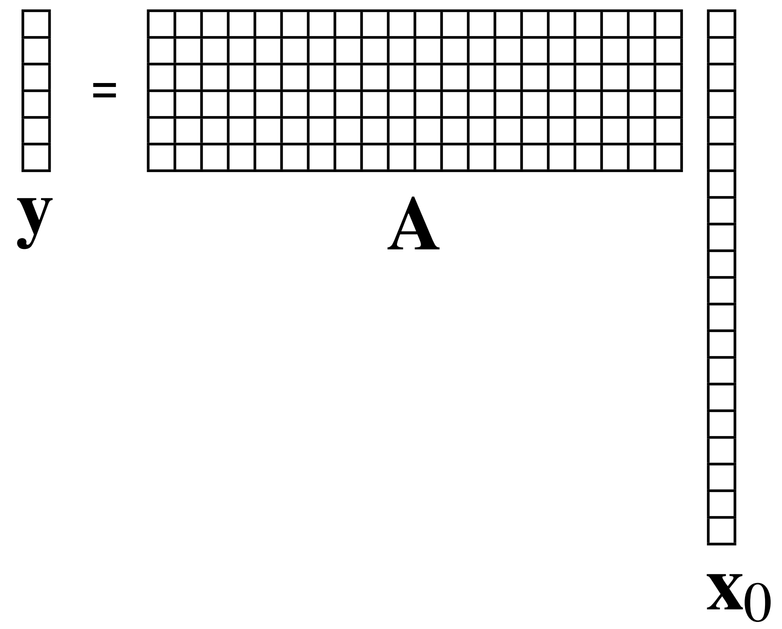
## Single Shot

$$\mathbf{u}(\omega) := \begin{bmatrix} \mathbf{u}(\omega_1) \\ \vdots \\ \mathbf{u}(\omega_{n_f}) \end{bmatrix} = \begin{bmatrix} \mathbf{H}^{-1}(\omega_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}^{-1}(\omega_{n_f}) \end{bmatrix} \begin{bmatrix} \mathbf{b}(\omega_1) \\ \vdots \\ \mathbf{b}(\omega_{n_f}) \end{bmatrix}$$

## Multiple Shots

$$\begin{bmatrix} \mathbf{u}_1(\omega) & \ldots & \mathbf{u}_{ns}(\omega) \end{bmatrix} = \mathbf{H}^{-1} \begin{bmatrix} \mathbf{b}_1(\omega) & \ldots & \mathbf{b}_{ns}(\omega) \end{bmatrix}$$

# Compressed sensing




Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

(Candes, Romberg, Tao, 2006; Wakin, Baraniuk, Laska, 2006, Lustig, Donoho, Pauly, 2006)

# Compressed sensing

**conditions:**

- **A** obeys the *restricted isometry principle*
- $\mathbf{x}_0$ is *sufficiently sparse*

**procedure:**

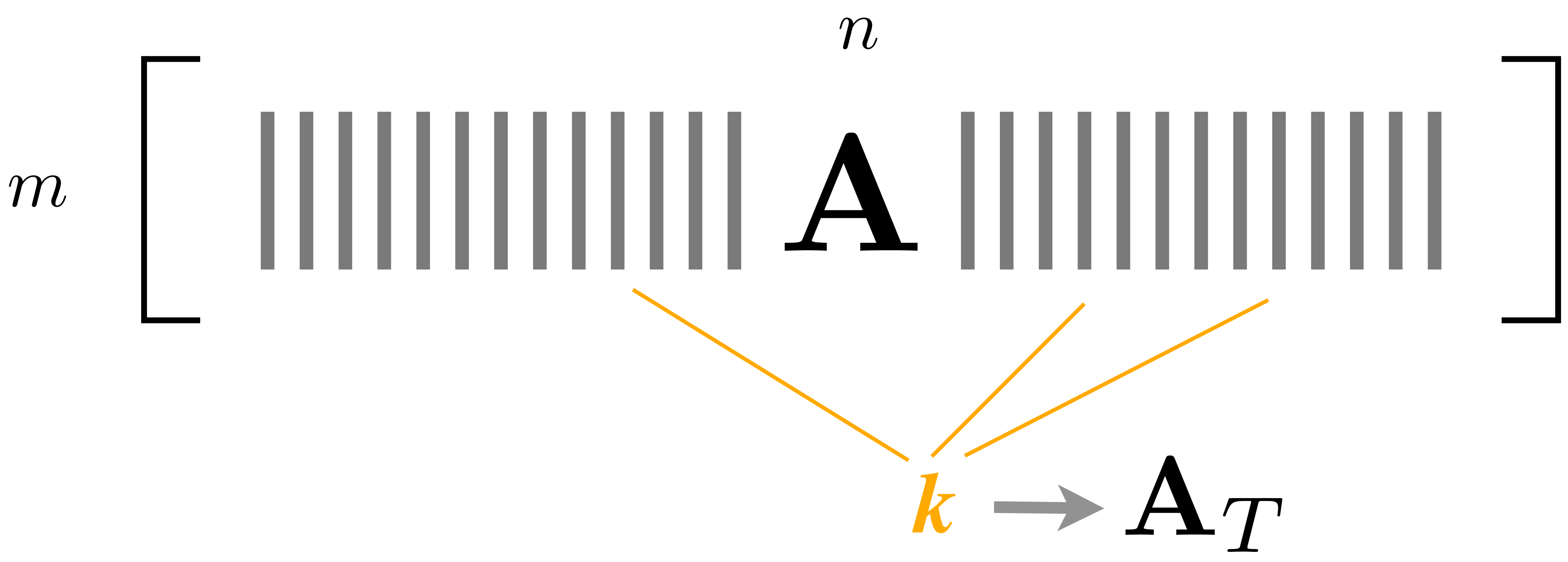
$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

**performance:**

- *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

## RIP for $k \leq m \ll n$

$$(1-\delta_k)\|\mathbf{x}_T\|_{\ell 2} \leq \|\mathbf{A}_T\mathbf{x}\|_{\ell 2} \leq (1+\delta_k)\|\mathbf{x}_T\|_{\ell 2}$$



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## RIP for $k \leq m \ll n$

$\mathbf{A}_T$ how close is it to an orthonormal basis?

(if close enough, then if $\mathrm{NNZ}(\mathbf{x}) \leq k/2$ ,
$\mathbf{S}^\dagger \mathbf{x}_{\mathrm{ml}} = \mathbf{G}$ with overwhelming probability)

# Compressed sensing

**Some popular choices for A in literature**

- Restricted random gaussian projections
- Restricted random signs projections
- Fourier transform with randomly missing frequencies

$$\mathbf{y}(\omega) = \mathbf{RMu}(t)$$

# Commutativity of RM

- Is there equivalence between
  - CS sampling of ***full*** solution for separate single-source experiments
  - Solution of ***reduced*** *system* after CS sampling the collective single-shot source wavefield

$$
\mathbb{P}_1 : \begin{cases} \mathbf{B} = \mathbf{D}^* \underbrace{\mathbf{s}}_{\text{single shots}} \\ \mathbf{HU} = \mathbf{B} \\ \mathbf{y} = \mathbf{RMDU} := \mathbf{RMd} \end{cases} \iff \mathbb{P}_2 : \begin{cases} \underline{\mathbf{B}} = \underline{\mathbf{D}}^* \underline{\mathbf{s}} = \underline{\mathbf{D}}^* \underbrace{\mathbf{RMs}}_{\text{simul. shots}} \\ \underline{\mathbf{HU}} = \underline{\mathbf{B}} \\ \underline{\mathbf{y}} = \underline{\mathbf{DU}}. \end{cases}
$$

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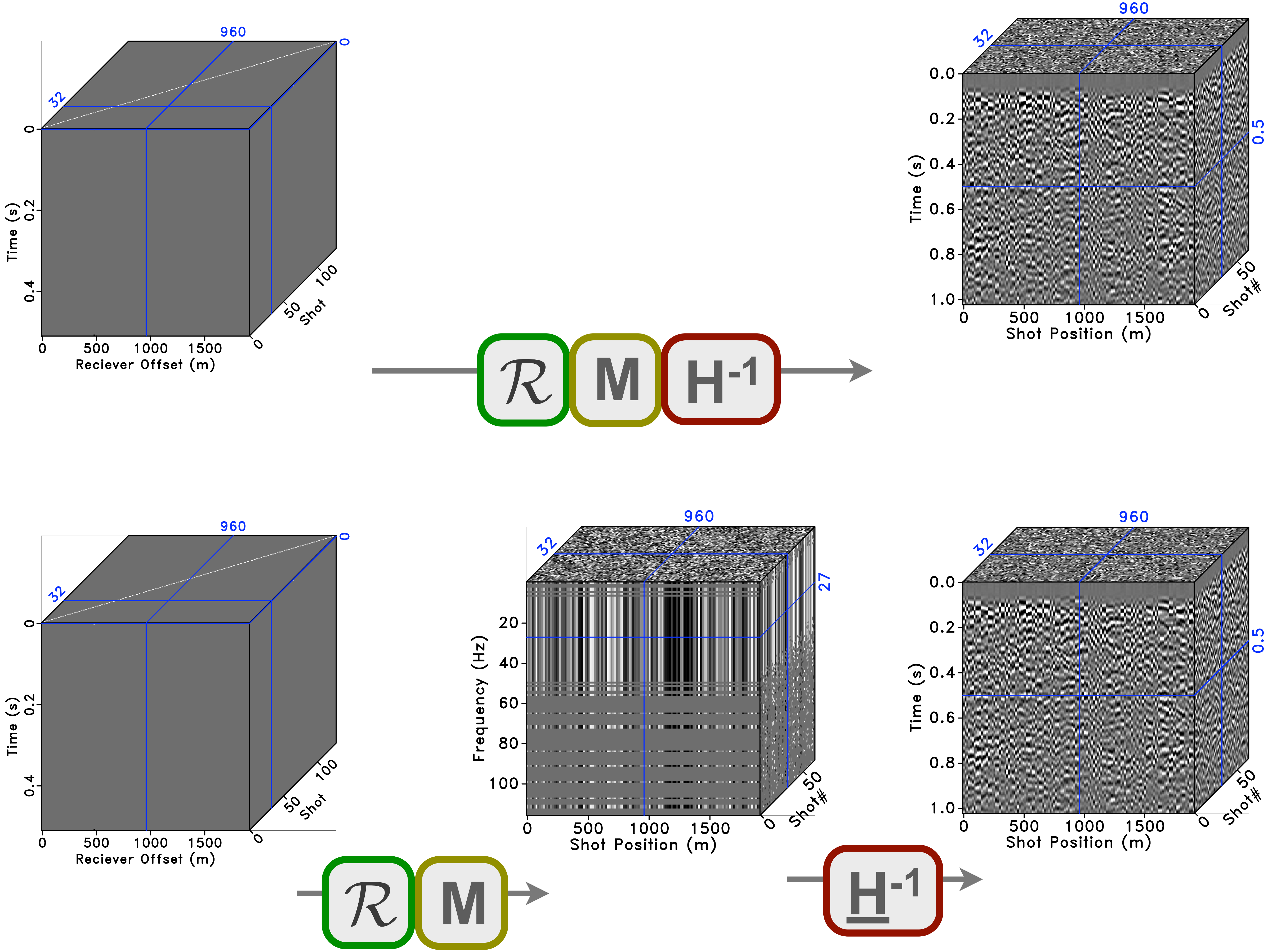
**Single Shot**

$$\mathbf{u}(\omega) := \begin{bmatrix} \mathbf{u}(\omega_1) \\ \vdots \\ \mathbf{u}(\omega_{n_f}) \end{bmatrix} = \begin{bmatrix} \mathbf{H}^{-1}(\omega_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}^{-1}(\omega_{n_f}) \end{bmatrix} \begin{bmatrix} \mathbf{b}(\omega_1) \\ \vdots \\ \mathbf{b}(\omega_{n_f}) \end{bmatrix}$$

**Multiple Shots**

$$\begin{bmatrix} \mathbf{u}_1(\omega) & \ldots & \mathbf{u}_{ns}(\omega) \end{bmatrix} = \mathbf{H}^{-1} \begin{bmatrix} \mathbf{b}_1(\omega) & \ldots & \mathbf{b}_{ns}(\omega) \end{bmatrix}$$



Time (s)
Reciever Offset (m)
Shot
960
32
Time (s)
Shot Position (m)
Shot#
0.5
$\mathcal{R}$
M
H-1
Frequency (Hz)
27

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# Defining RM

- natural restriction in Fourier (**F**) with *importance* sampling in the temporal direction
- CS encoding matrix (**N**) along shots => simultaneous sources
- assures *incoherence* with sparsifying transform

For each **simultaneous** shot, define different restrictions

$$\mathbf{RM} = \begin{bmatrix} \mathbf{R}_1^{\Sigma} \otimes \mathbf{R}_1^{\Omega} \\ \vdots \\ \mathbf{R}_{n_{s'}}^{\Sigma} \otimes \mathbf{R}_{n_{s'}}^{\Omega} \end{bmatrix} \otimes (\mathbf{N} \otimes \mathbf{F})$$

yielding the reduced simulated data

# Defining RM

**CS with Random Convolution (Romberg '08)**

$$\mathbf{RM} = \overbrace{\begin{bmatrix} \mathbf{R}_1^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_1^{\Omega} \\ \vdots \\ \mathbf{R}_{n_{s'}}^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_{n_{s'}}^{\Omega} \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left( \mathbf{F}_2^* \, \mathrm{diag}\left( e^{\hat{i}\boldsymbol{\theta}} \right) \otimes \mathbf{I} \right)}^{\text{random phase encoder}} \mathbf{F}_3,$$

$$\theta_w = \mathrm{Uniform}([0, 2\pi])$$

# question

How do I really *know* if I lost anything?

# Enforcing sparsity

Using Curvelet transform for shot and receiver coordinates

- **Frequency-domain** restrictions perform well under **Wavelet** transform for seismic data (Lin et. al. ’08)
- **Spatial-domain** restrictions perform well under **Curvelet** transform for seismic data (Hennefent et. al. ’07)

**Combine both transforms in the coordinate they are most suited for**

- Wavelet sparsity on temporal-frequency coordinate
- 2D Curvelet sparsity on shot and receiver plane

$$\mathbf{S} = \mathbf{C}_{2d} \otimes \mathbf{W}$$

# Compressed sensing

**conditions:**

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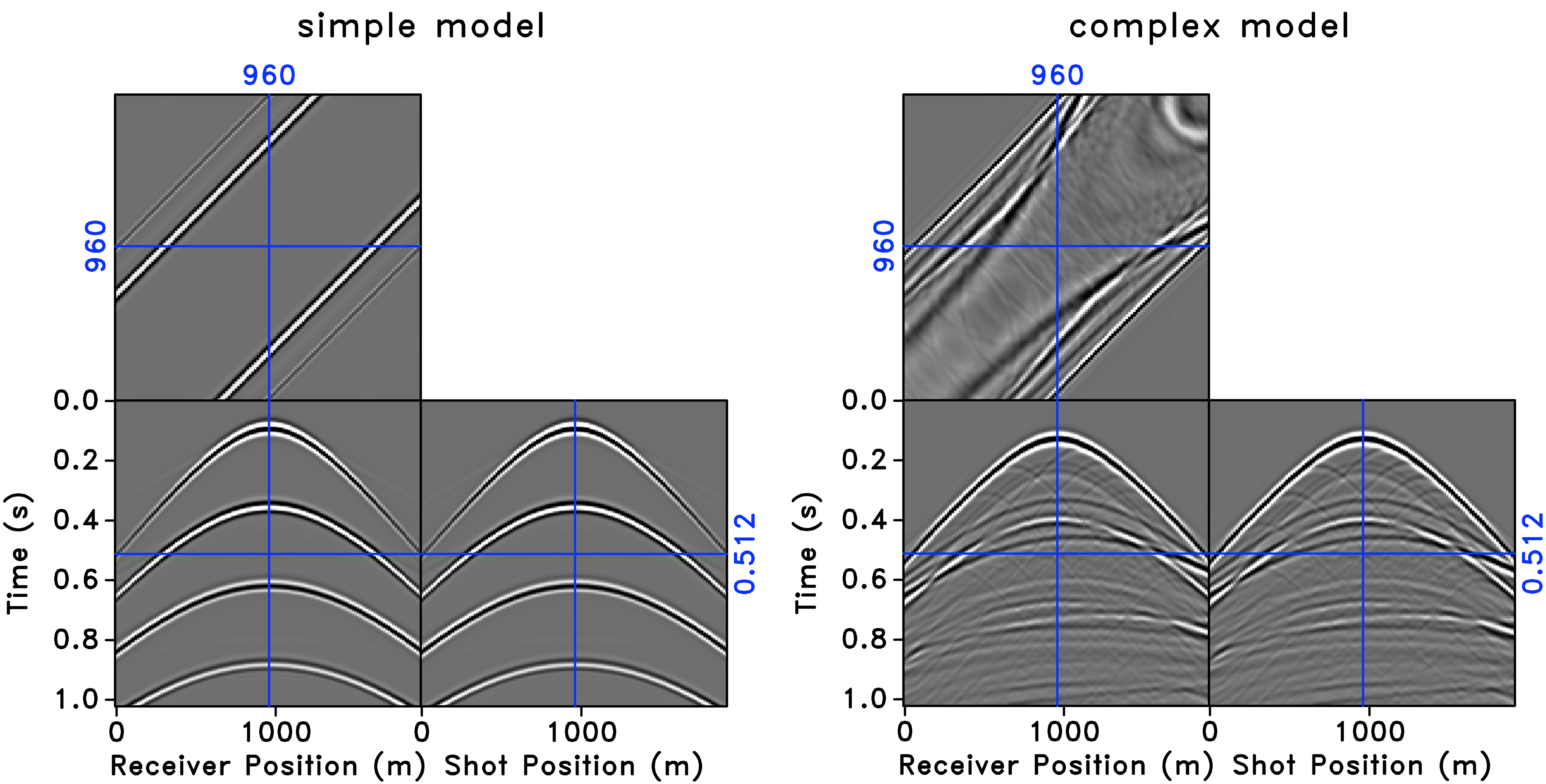
**procedure:**

$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

**performance:**

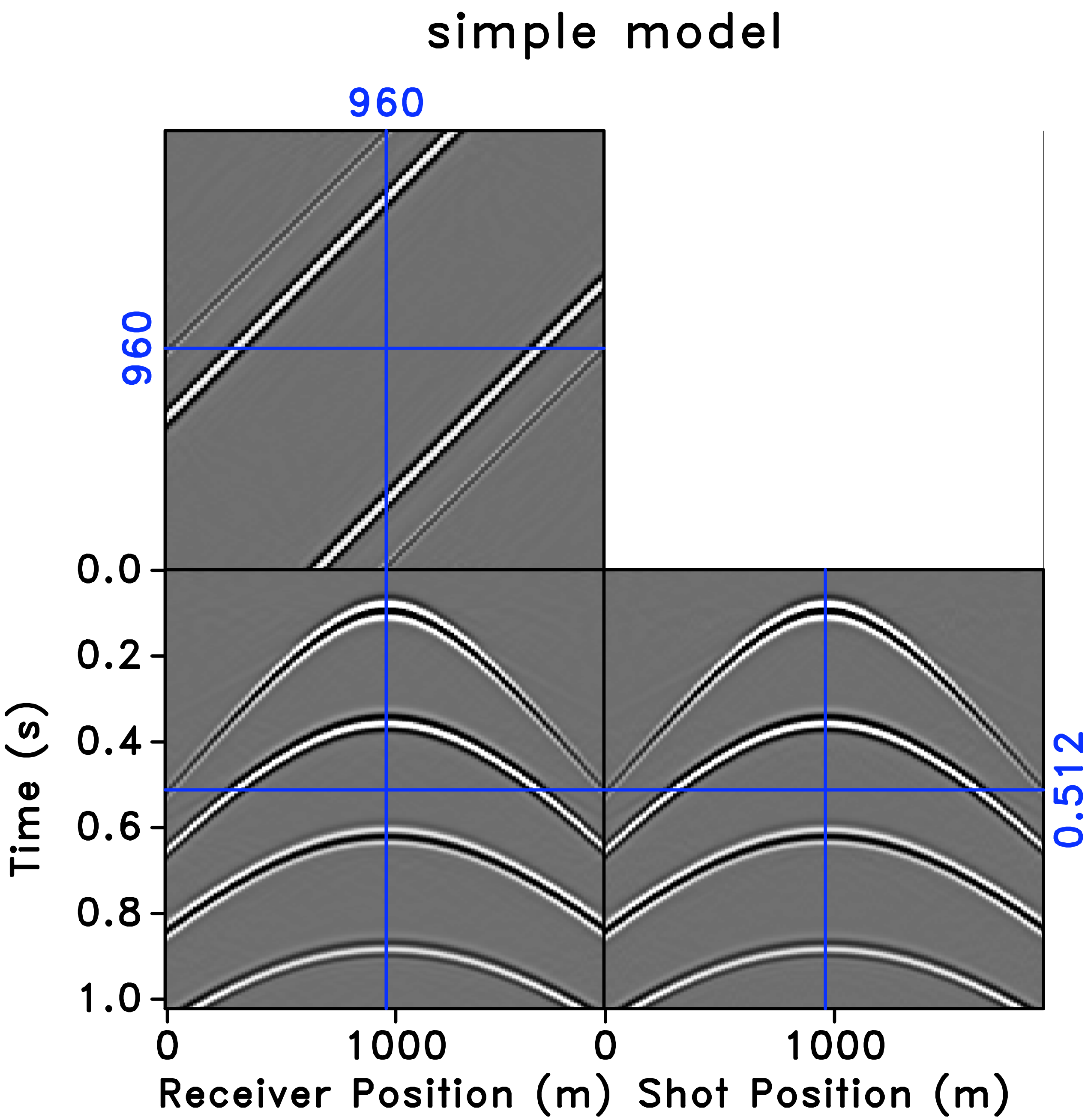
- *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

# Green's functions

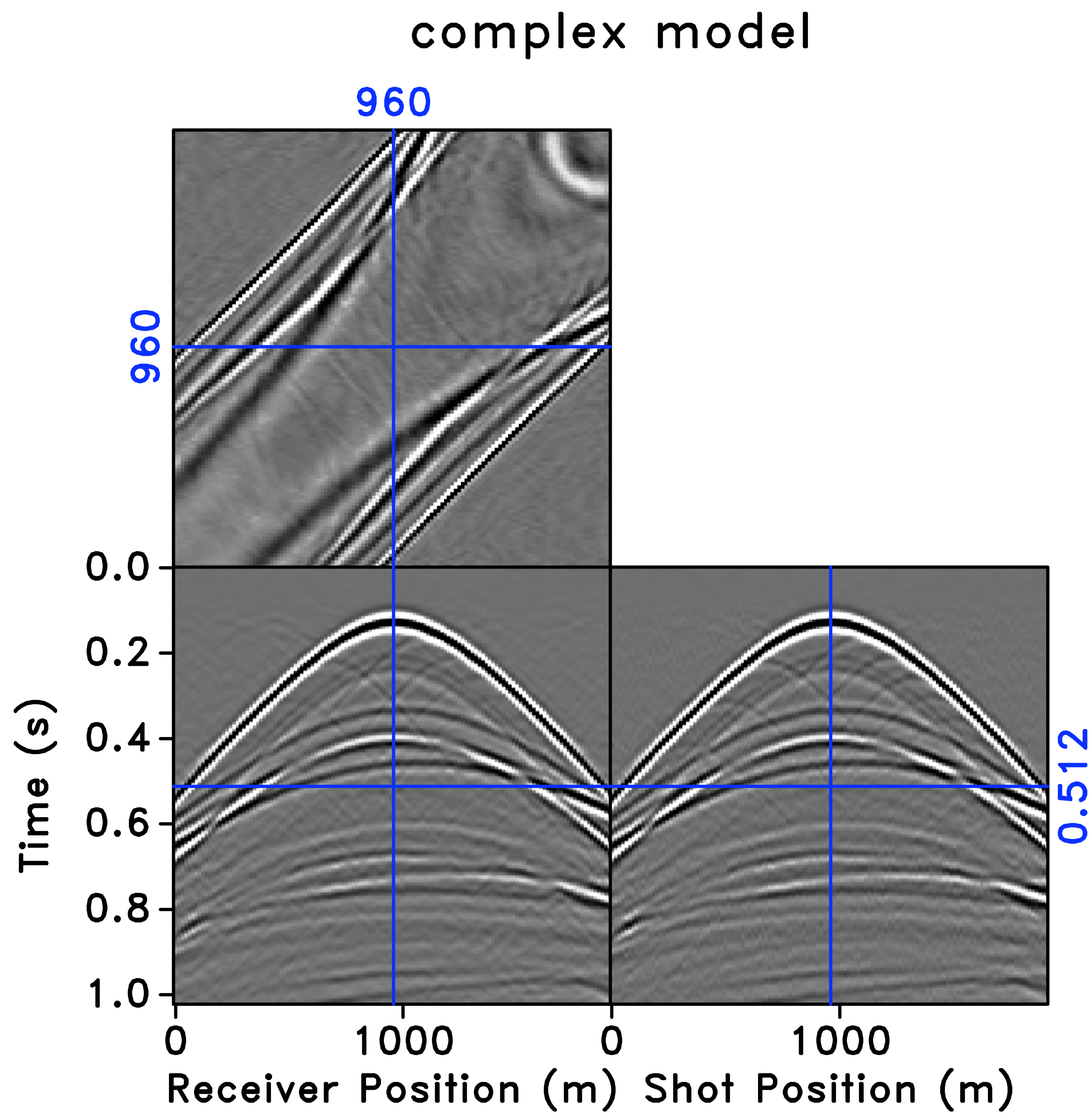



projected to 25% of original data size

# recovered wavefield



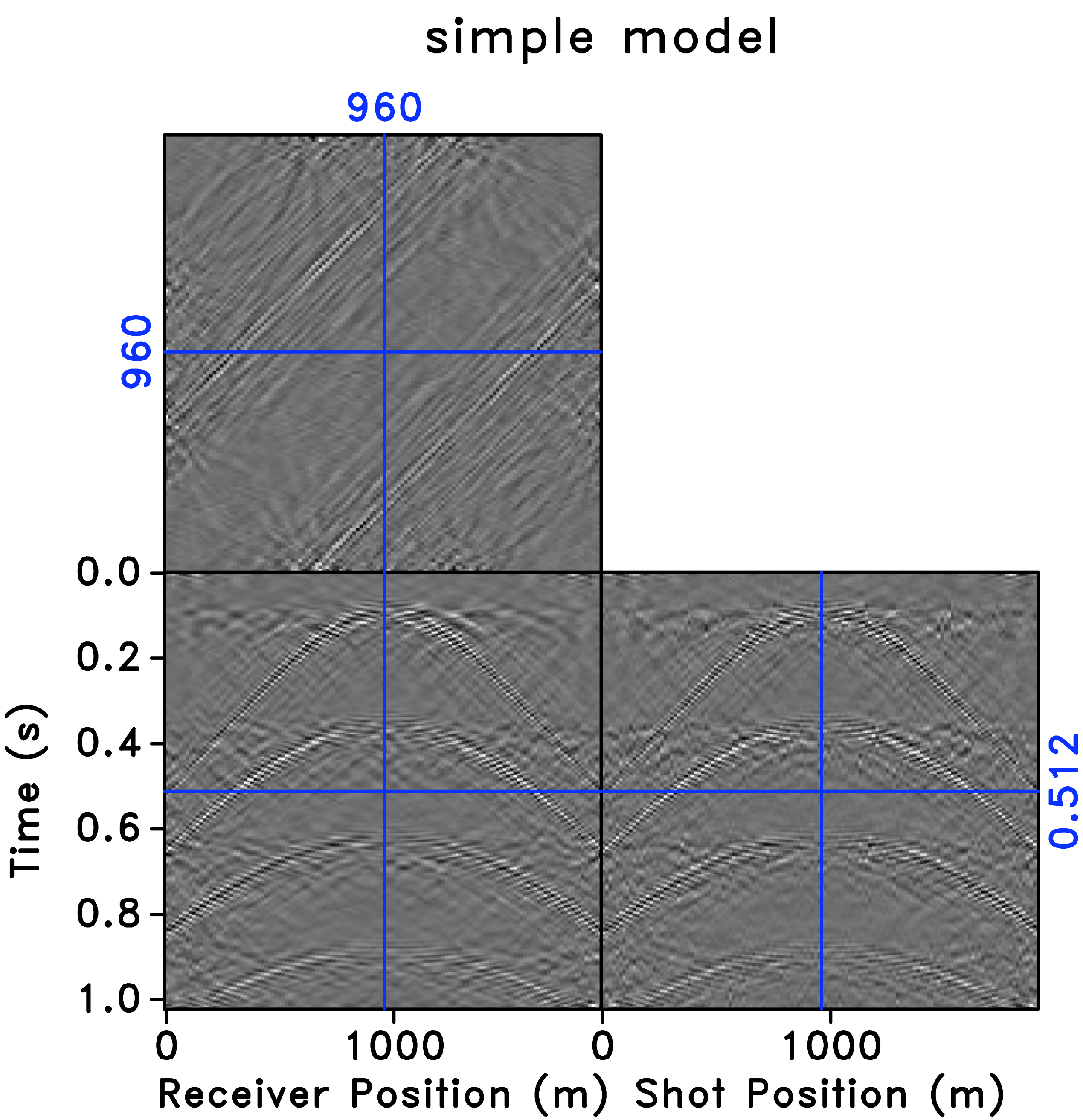

28.1dB



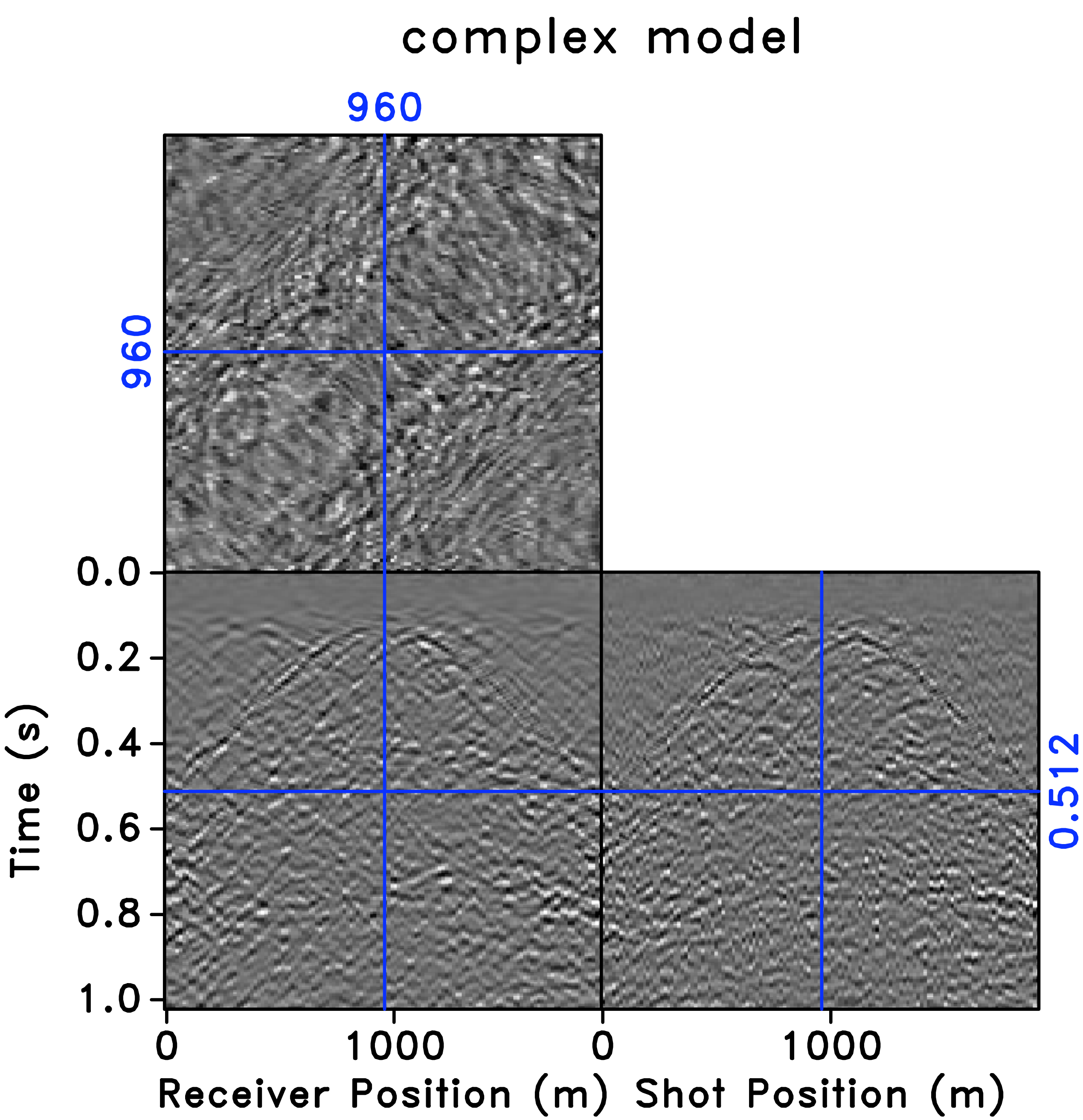

18.2dB

projected gradient (~100x cost of RM and its adjoint)

# 10x difference

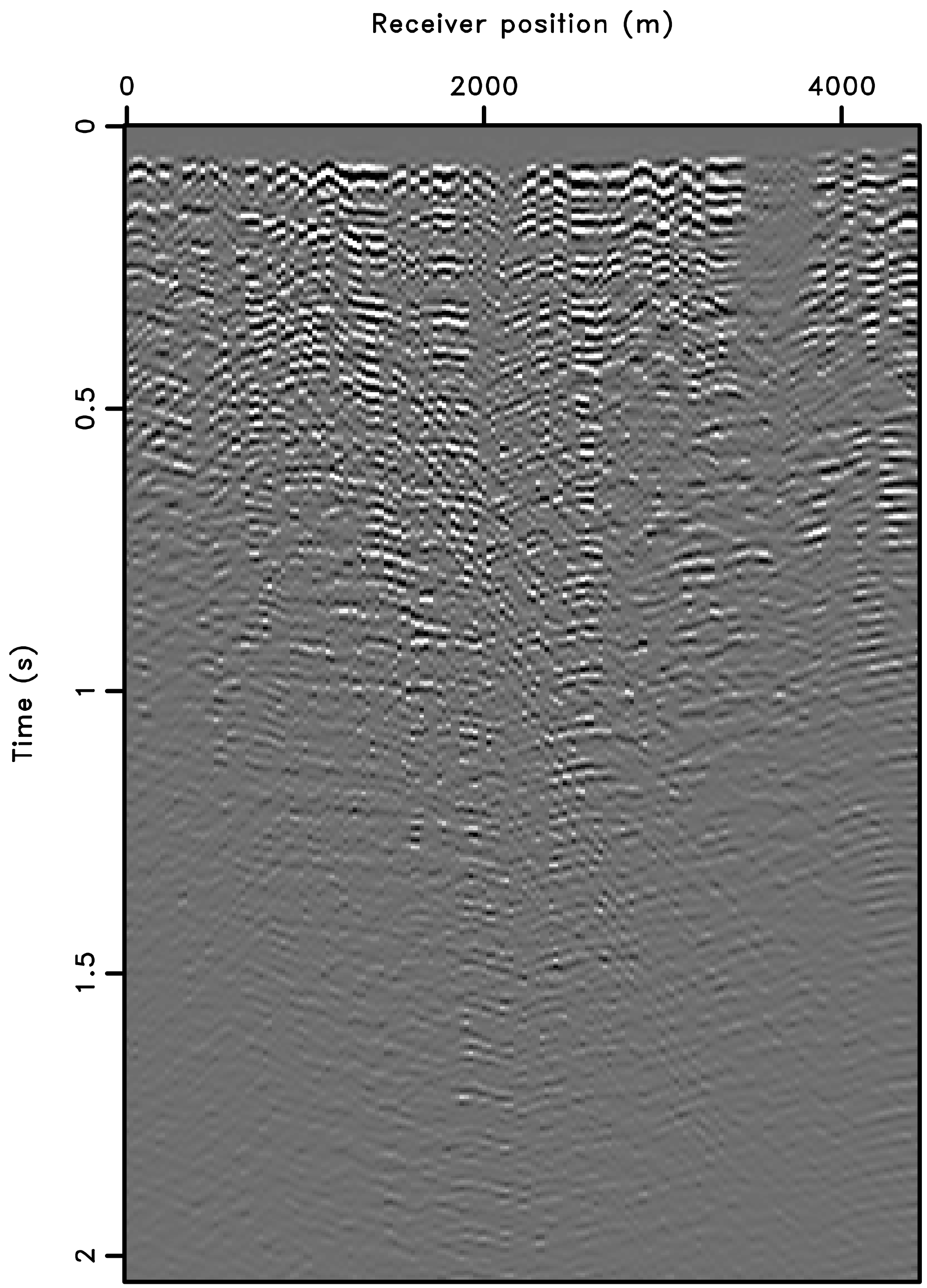



28.1dB

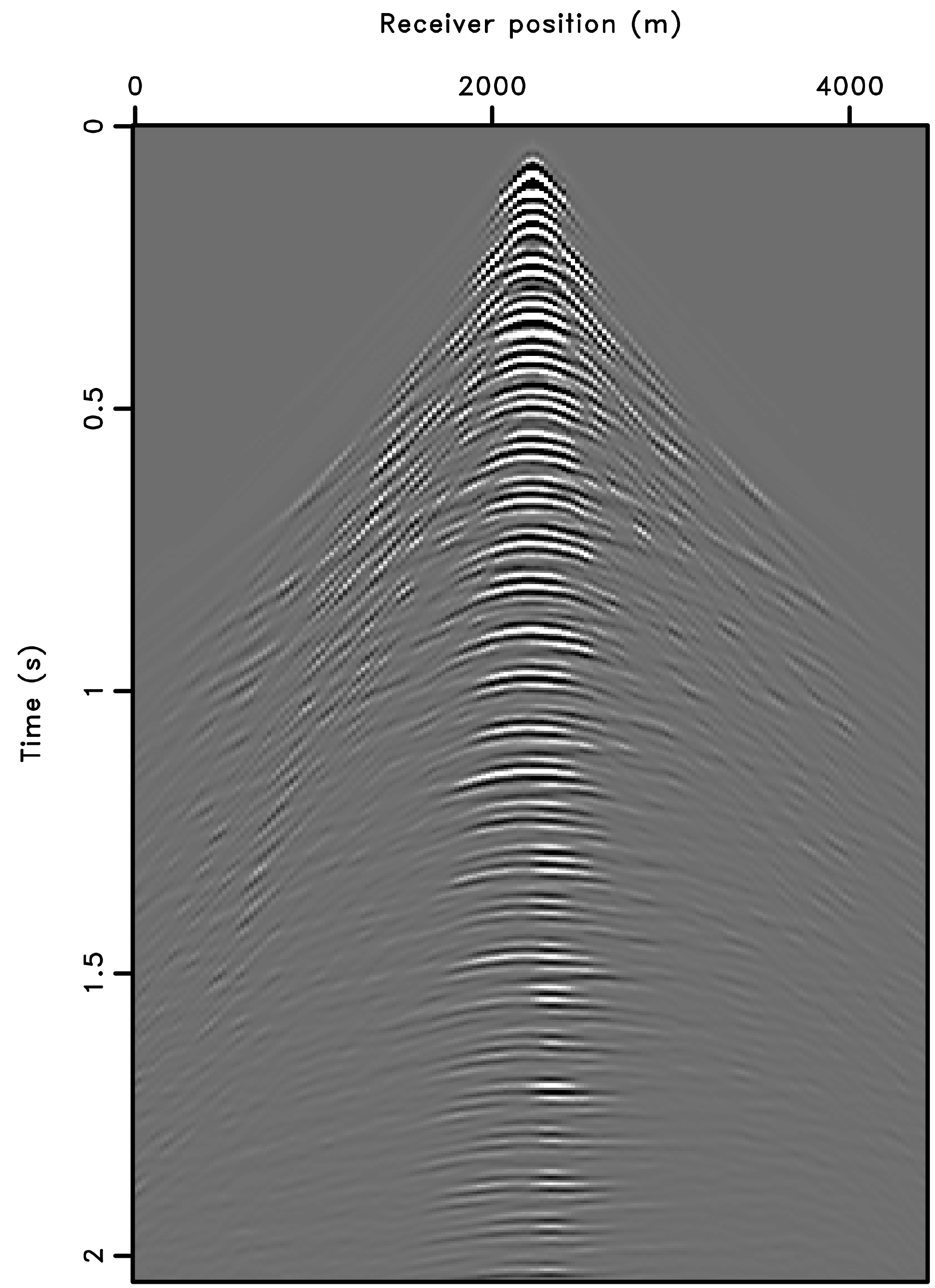



18.2dB

# real marine data

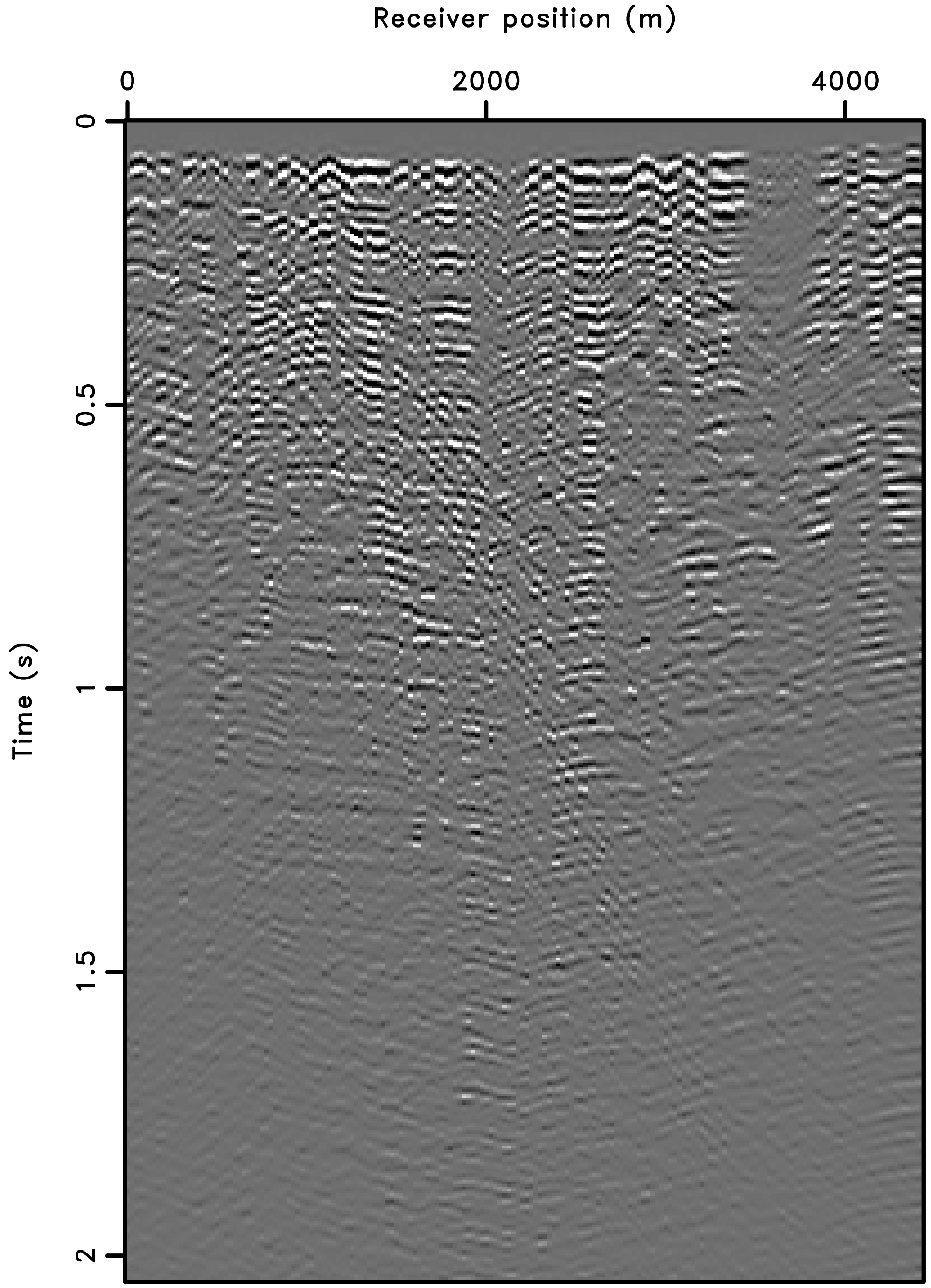



Single simultaneous "simulation"

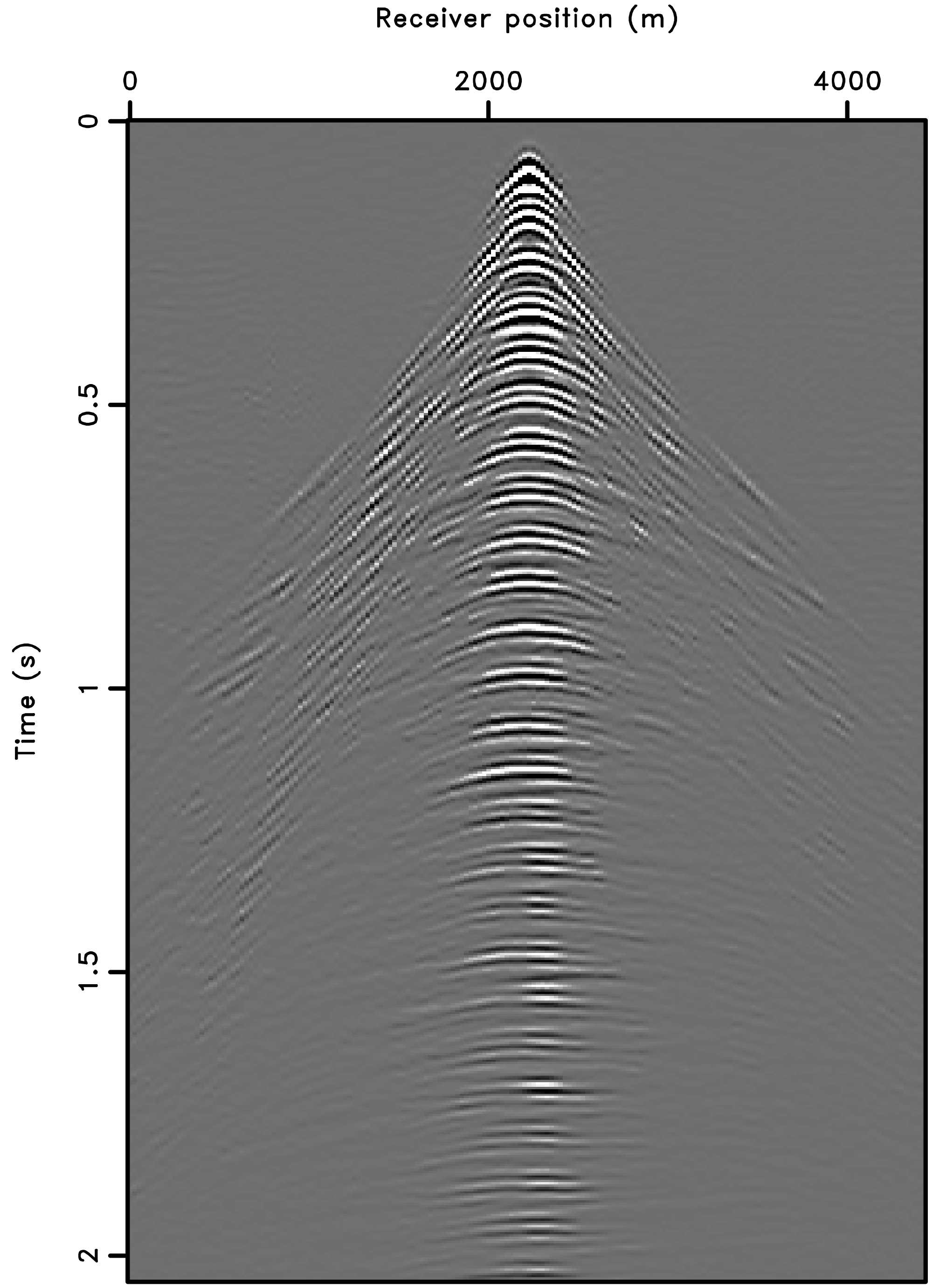



Single shot

# real marine data

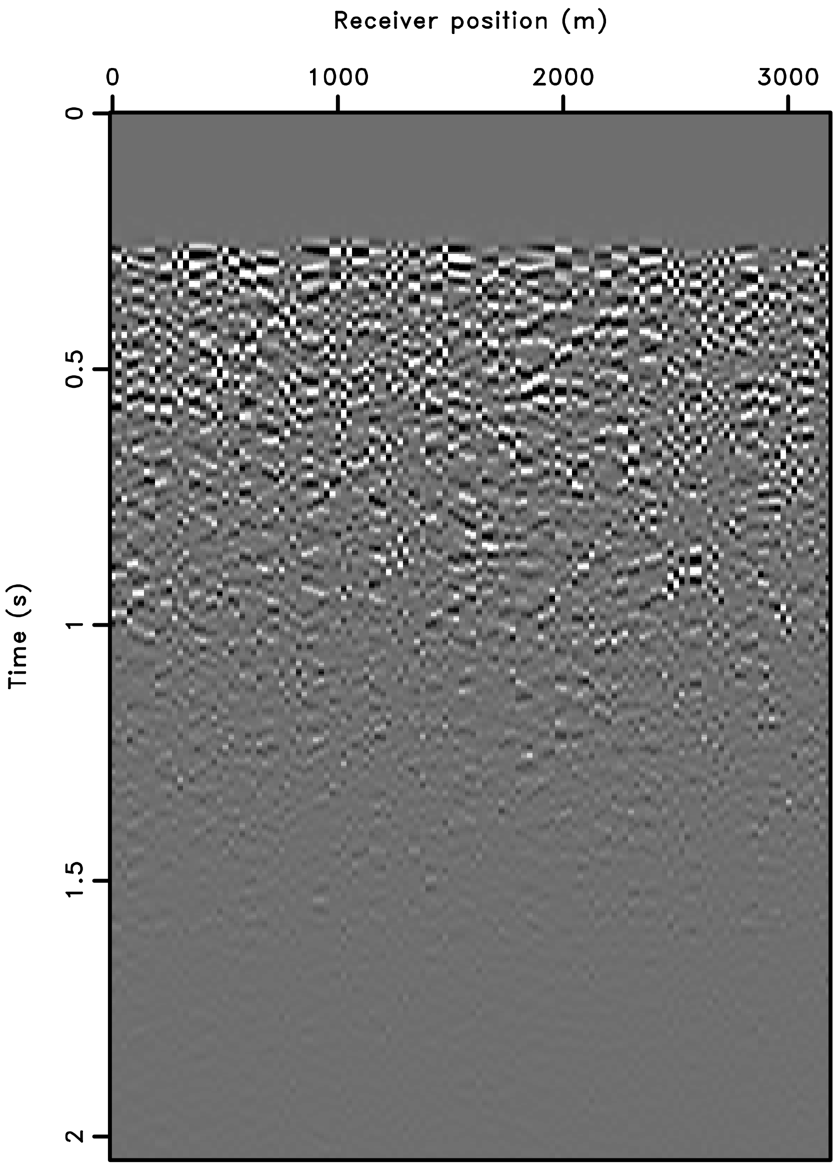



Single simultaneous "simulation"

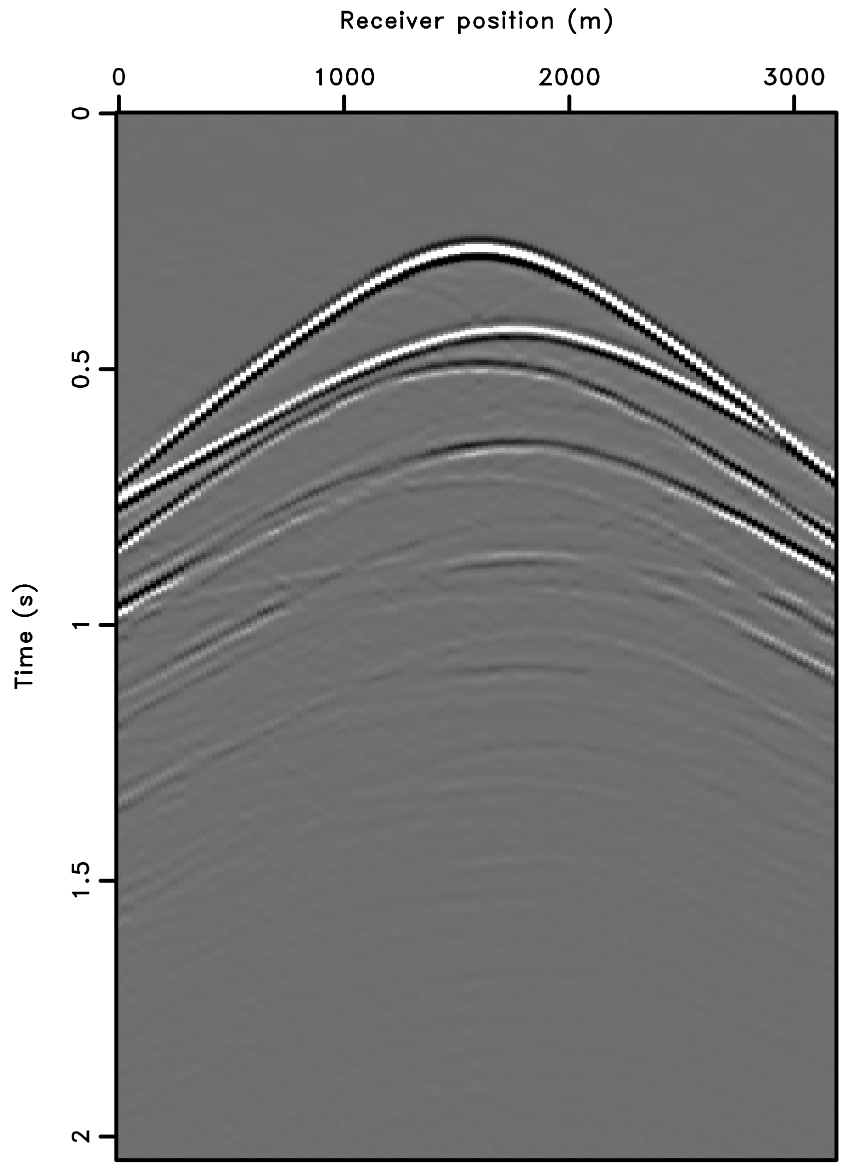



recovered from 25% number of realizations

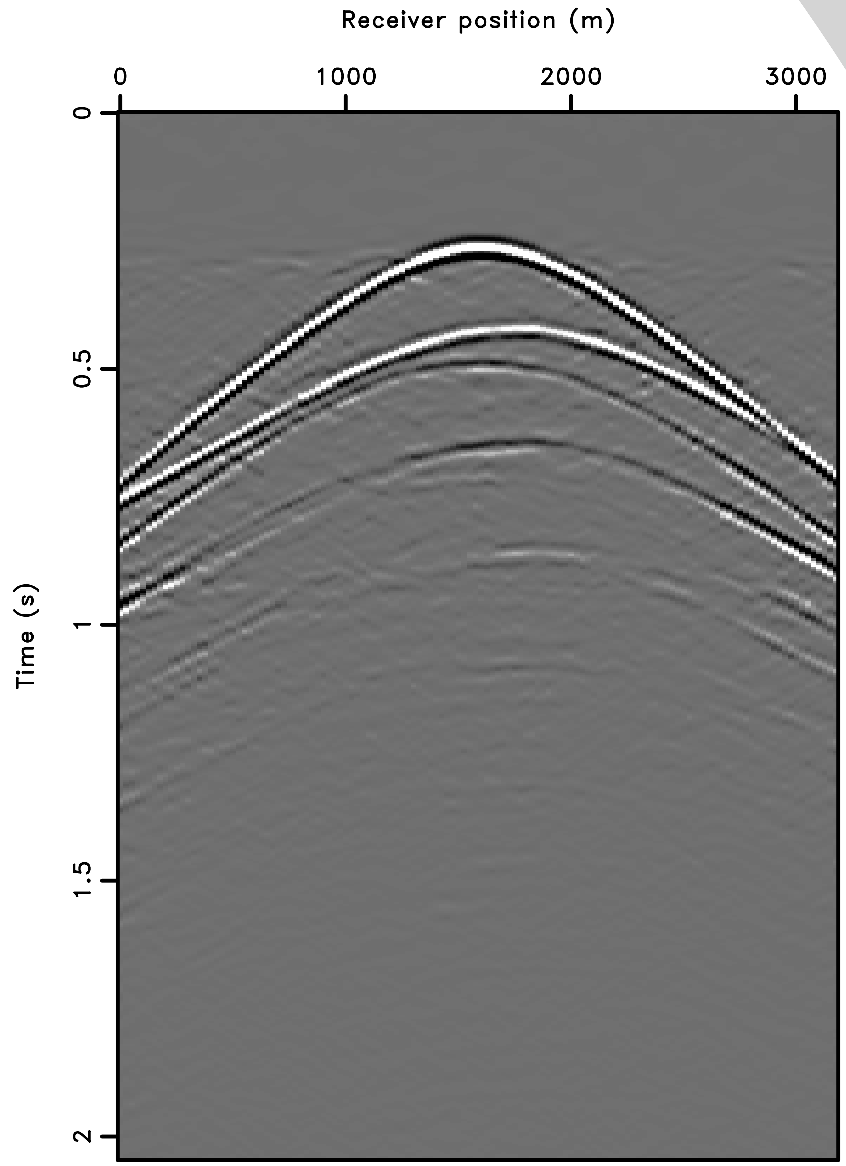
# Reconstruction from different number of realizations of simultaneous simulation (measured in % of number of single-shots)




Single simultaneous simulation




30% number of realizations




20% number of realizations

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Total computed data fraction

# Frequencies / # Shots

| | 0.25 | 0.15 | 0.07 |
|---|---|---|---|
| 2 | 14.3 | 12.1 | 8.6 |
| 1 | 18.2 | 14.5 | 10.2 |
| 0.5 | 22.2 | 16.5 | 10.7 |

$$\mathrm{SNR} = -20 \log \frac{\|\mathbf{d} - \tilde{\mathbf{d}}\|_2}{\|\mathbf{d}\|_2}$$

# key points

Is it *really* a good idea to remove crosstalk before processing?

If we use non-linear inversions, maybe not necessary

Simultaneous simulation is the key

**Simultaneous data ⟶ Imaging**

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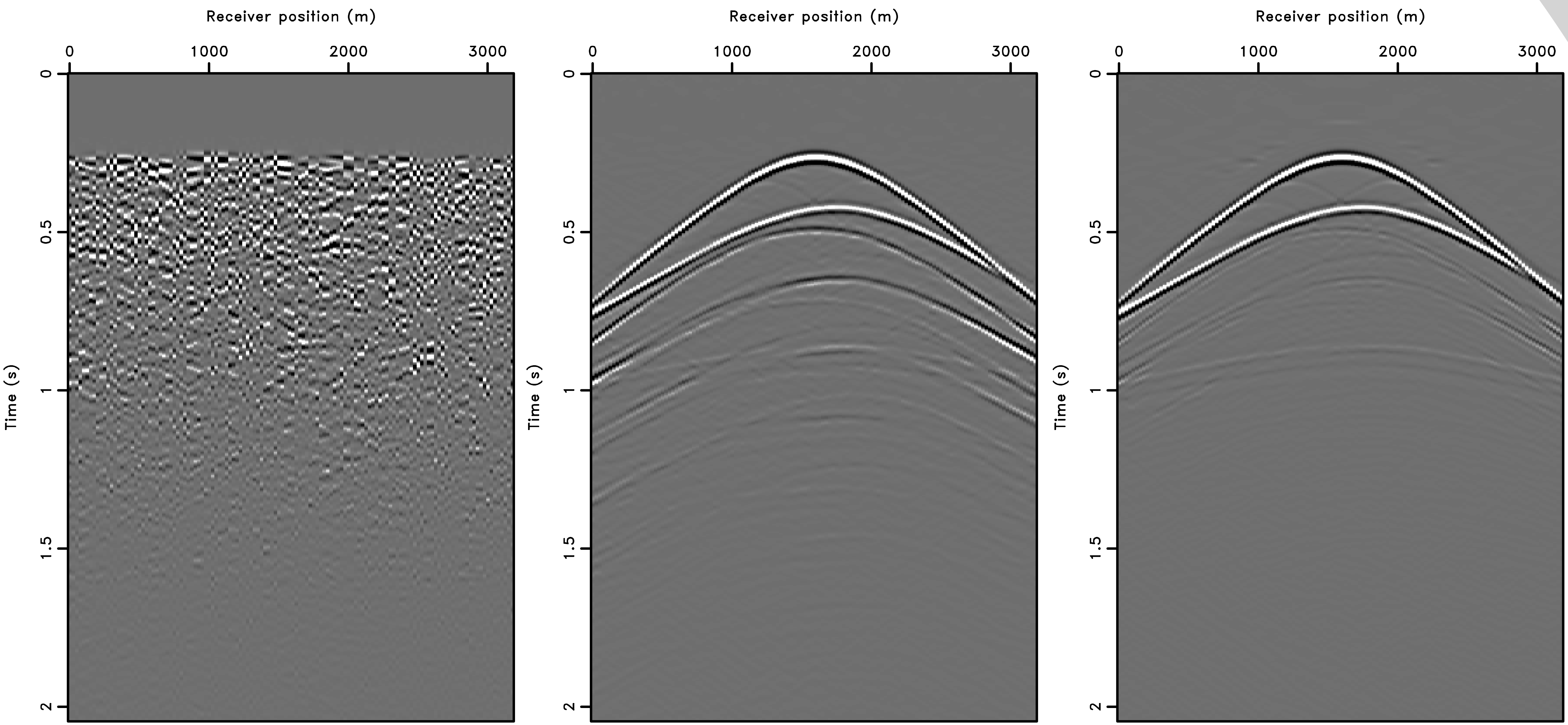
# extension

## Primary estimation

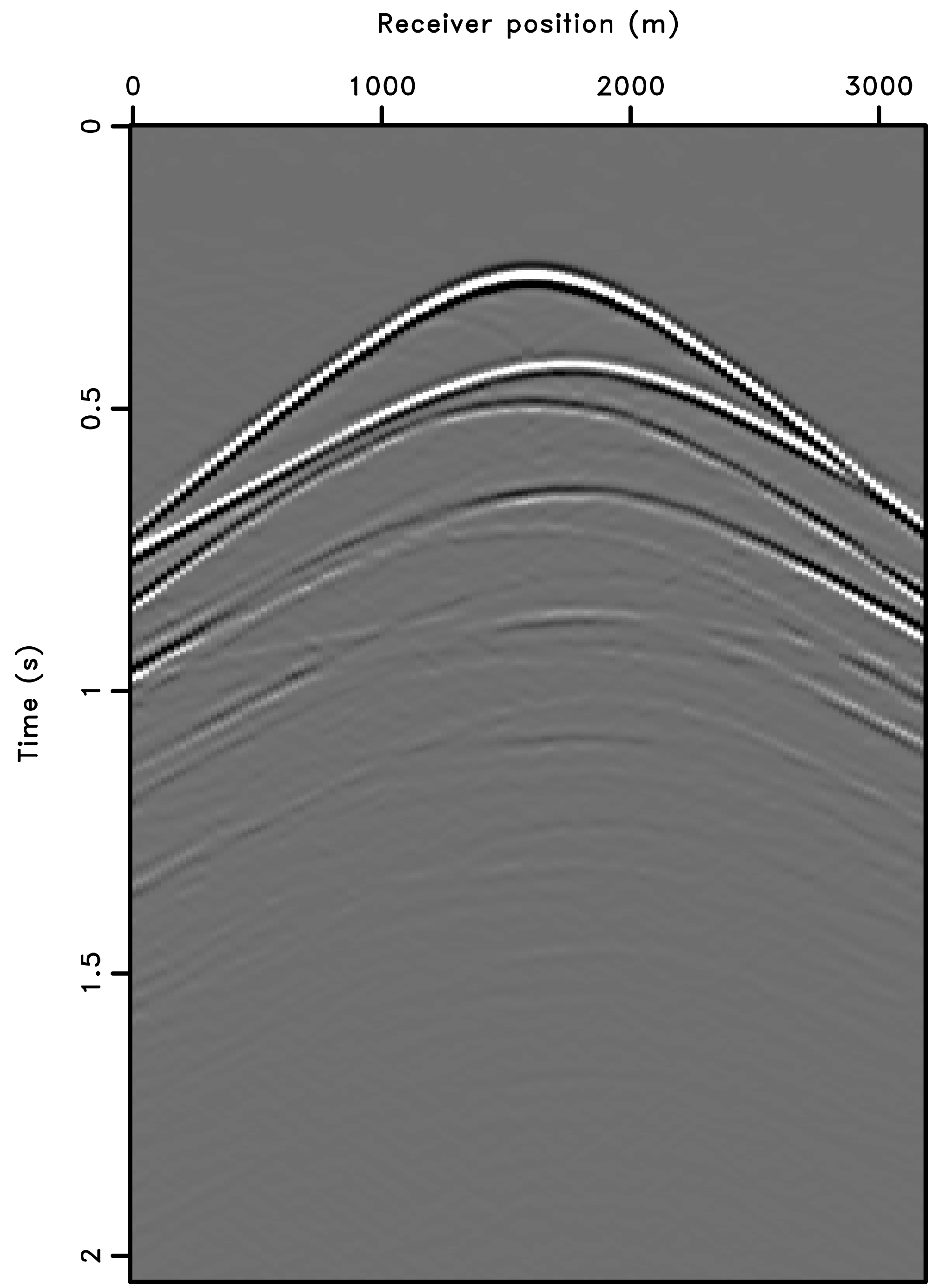
More on this tomorrow from me

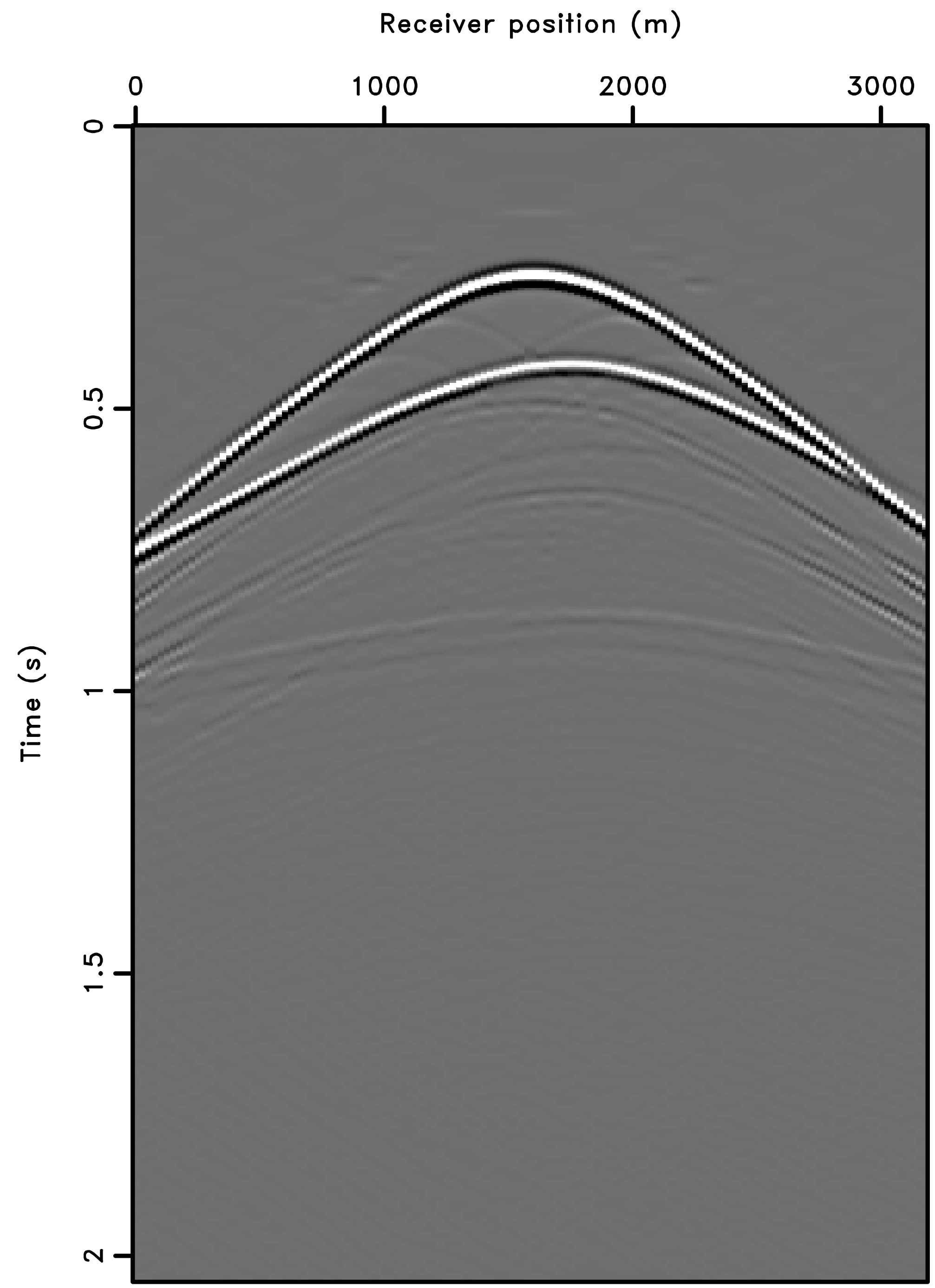
SPMUL 2, 3:10pm

# Primary estimation

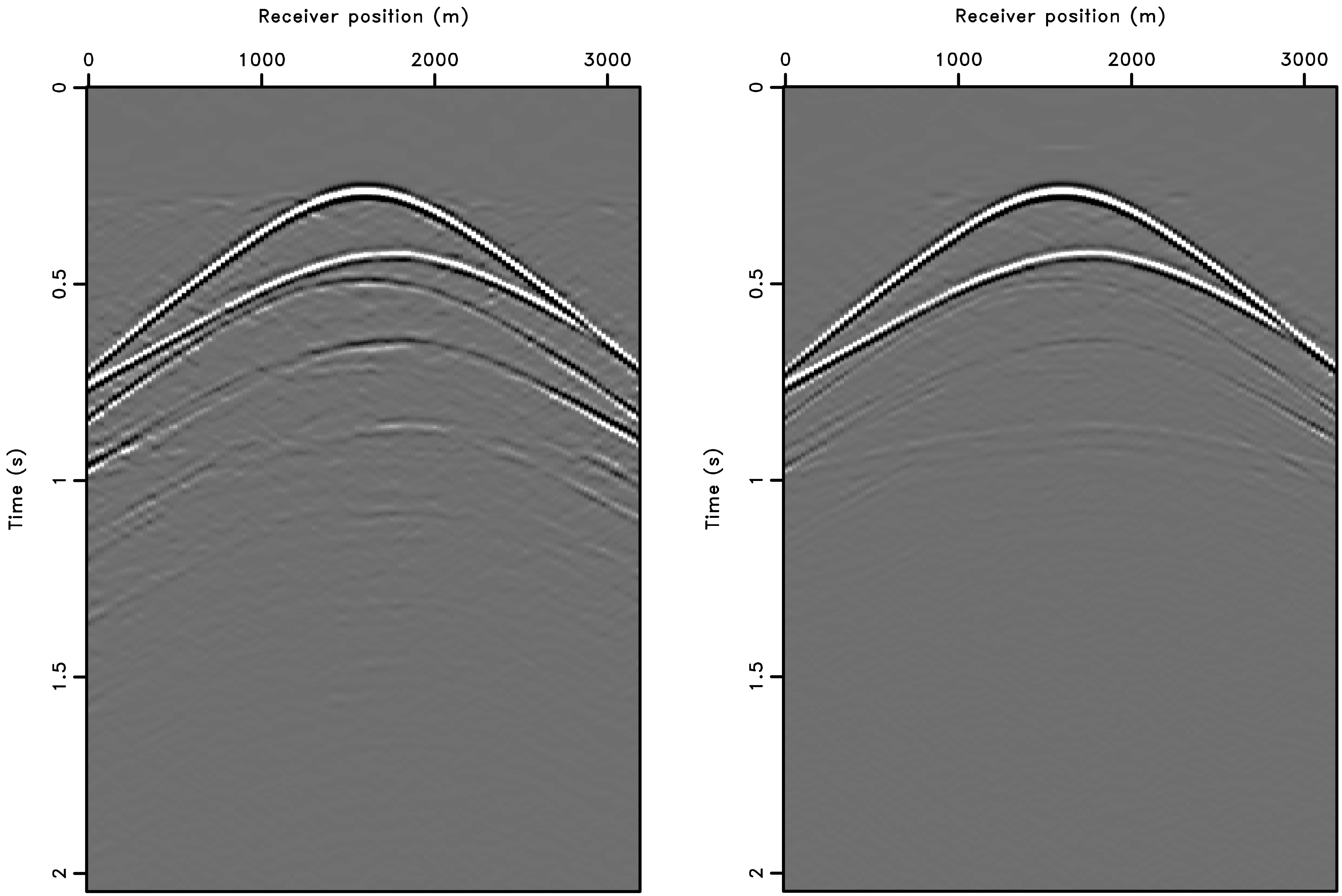


# 50% realizations



# 20% realizations



# extension

## full waveform inversion

$$\min_{\mathbf{U}\in\mathcal{U},\,\mathbf{m}\in\mathcal{M}} \frac{1}{2}\|\mathbf{P}-\mathbf{D}\mathbf{U}\|_2^2 \quad \text{subject to} \quad \mathbf{H}[\mathbf{m}]\mathbf{U}=\mathbf{Q}$$

+ Free surface BC

# extension

$$
\begin{cases}
\mathbf{Q} = \mathbf{D}^* \underbrace{\mathbf{s}}_{\text{single shots}} \\
\mathbf{H}\mathbf{U} = \mathbf{Q} \\
\mathbf{y} = \mathbf{R}\mathbf{M}\mathbf{D}\mathbf{U}
\end{cases}
\iff
\begin{cases}
\underline{\mathbf{Q}} = \underline{\mathbf{D}}^* \underbrace{\mathbf{R}\mathbf{M}\mathbf{s}}_{\text{simul. shots}} \\
\underline{\mathbf{H}\mathbf{U}} = \underline{\mathbf{Q}} \\
\underline{\mathbf{y}} = \underline{\mathbf{D}\mathbf{U}}
\end{cases}
$$

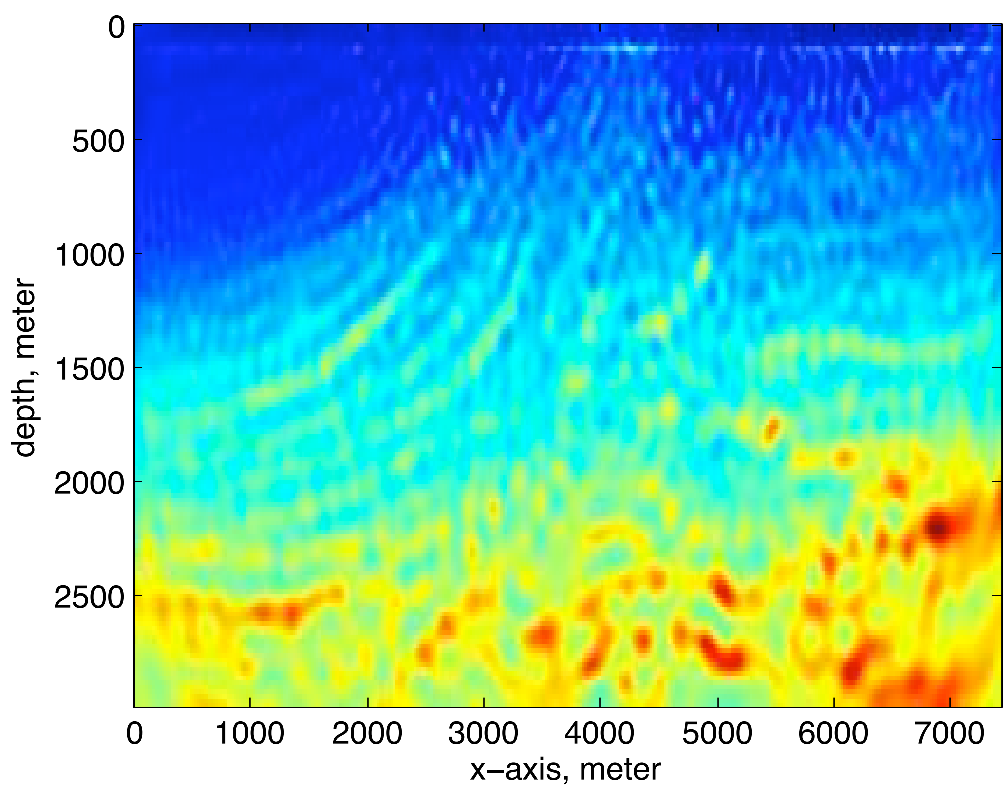
## Reduced system

$$
\min_{\underline{\mathbf{U}} \in \underline{\mathcal{U}},\, \mathbf{m} \in \mathcal{M}} \frac{1}{2} \|\mathbf{y} - \underline{\mathbf{D}\mathbf{U}}\|_2^2 \quad \text{subject to} \quad \underline{\mathbf{H}}[\mathbf{m}]\underline{\mathbf{U}} = \underline{\mathbf{Q}}
$$

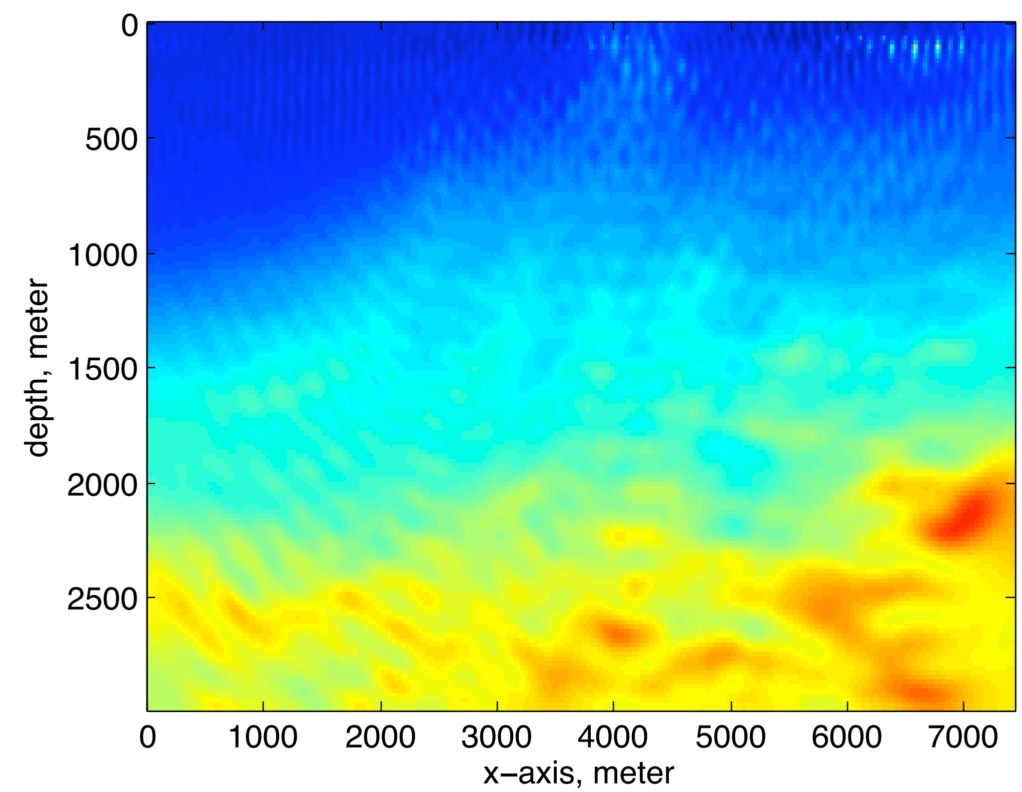
# Gauss-Newton Krylov method

5th iteration

10% data size




Projected using RM




Periodically downsampled

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# extension

## waveform inversion

More on this in the afternoon

SI 3: Herrmann @ 2:45pm

SI 3: Erlangga @ 4:25pm

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# acknowledgements

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