Bayesian ground-roll separation by curvelet-domain sparsity promotion

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SEG 78th Annual Meeting
November 10th, 2008

Seismic Laboratory for Imaging and Modeling

Introduction

- Motivation
- Sparsity
  - Wavelet-like transforms and decay rates
  - Curvelets
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  - Results
- Conclusions

Motivation

- Improve ground-roll separation
  - *Preserve* reflector information
  - *Increase* ground-roll removal
- Find a *sparsifying* domain for the ground-roll *separation* problem
- Adapt tools designed for *primary-multiple* to separation to *ground-roll* removal
- Examine *feasibility* of methods on synthetic data
- Improve denoising flows by introducing a signal separation method

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Sparsity
Wavelet-like transforms – Physical domain

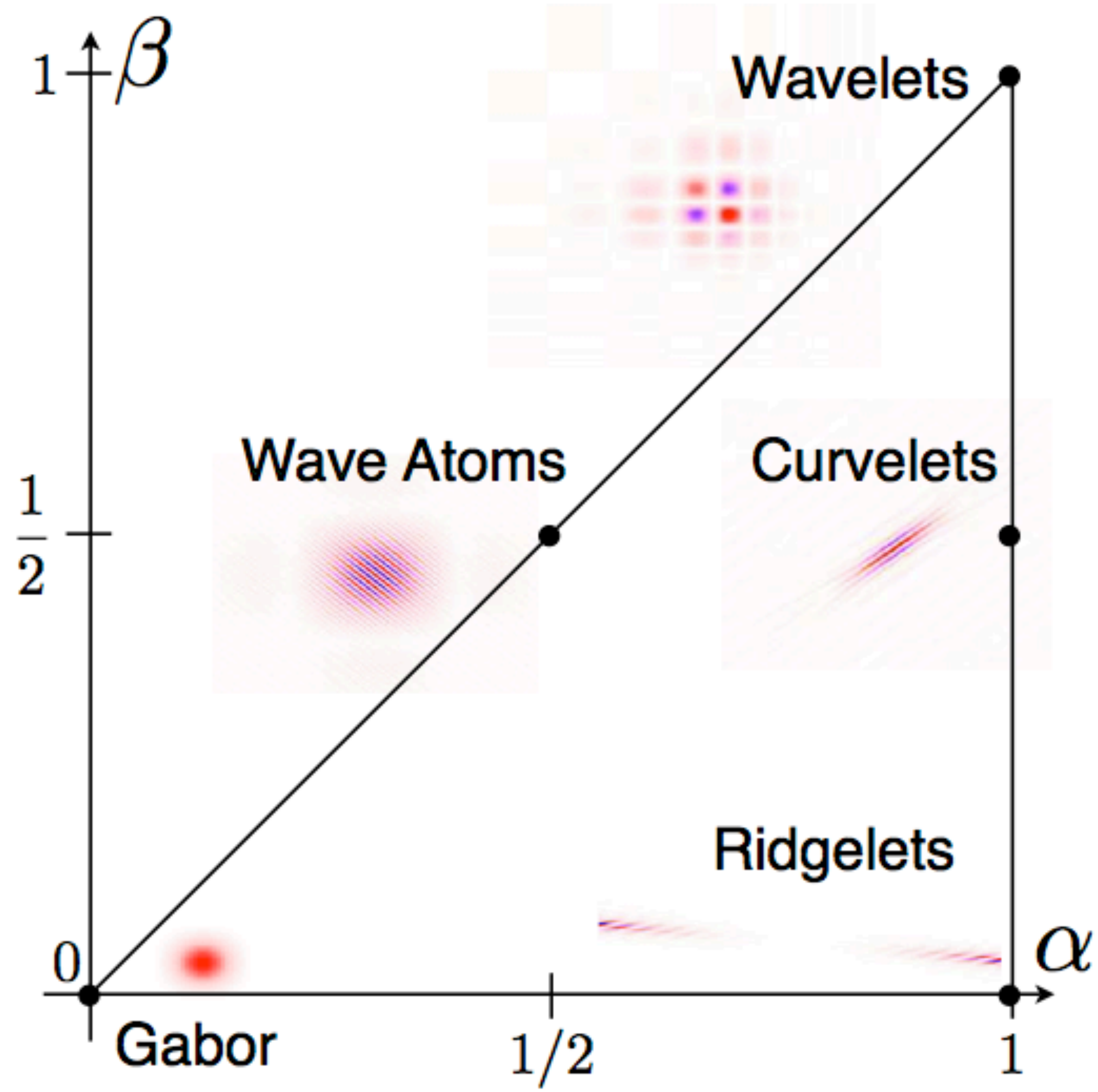
D_j –Dilation Matrix

$$D_j = \begin{bmatrix} 2^{j\alpha} & 0 \\ 0 & 2^{j\beta} \end{bmatrix}$$

j — Scale index

α — Multiscale parameter

β — Directionality parameter



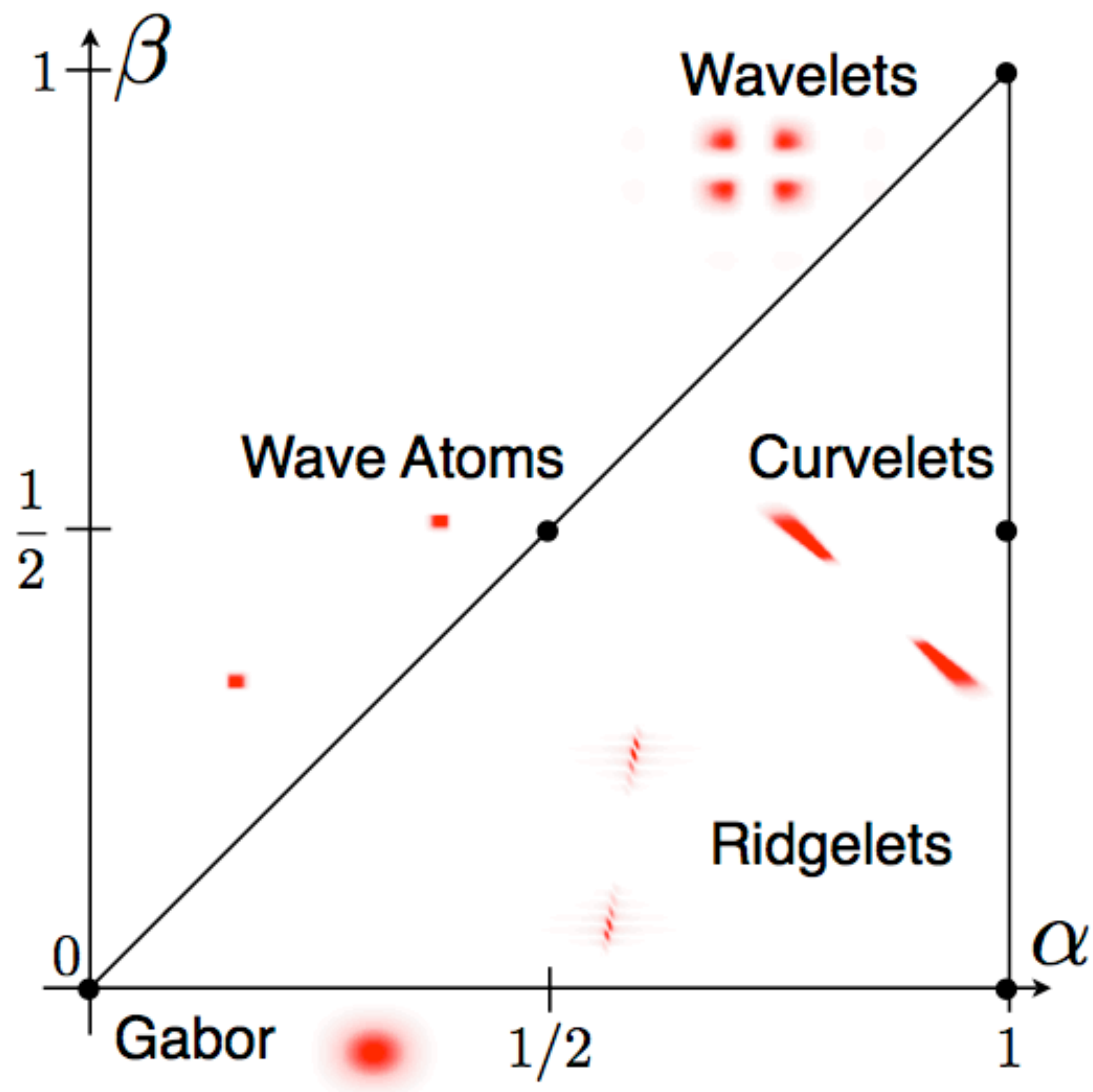
Adapted from Demanet and Ying (06)

Sparsity

Wavelet-like transforms - Fourier domain

D_j —Dilation Matrix

$$D_j = \begin{bmatrix} 2^{j\alpha} & 0 \\ 0 & 2^{j\beta} \end{bmatrix}$$



j — Scale index

α — Multiscale parameter

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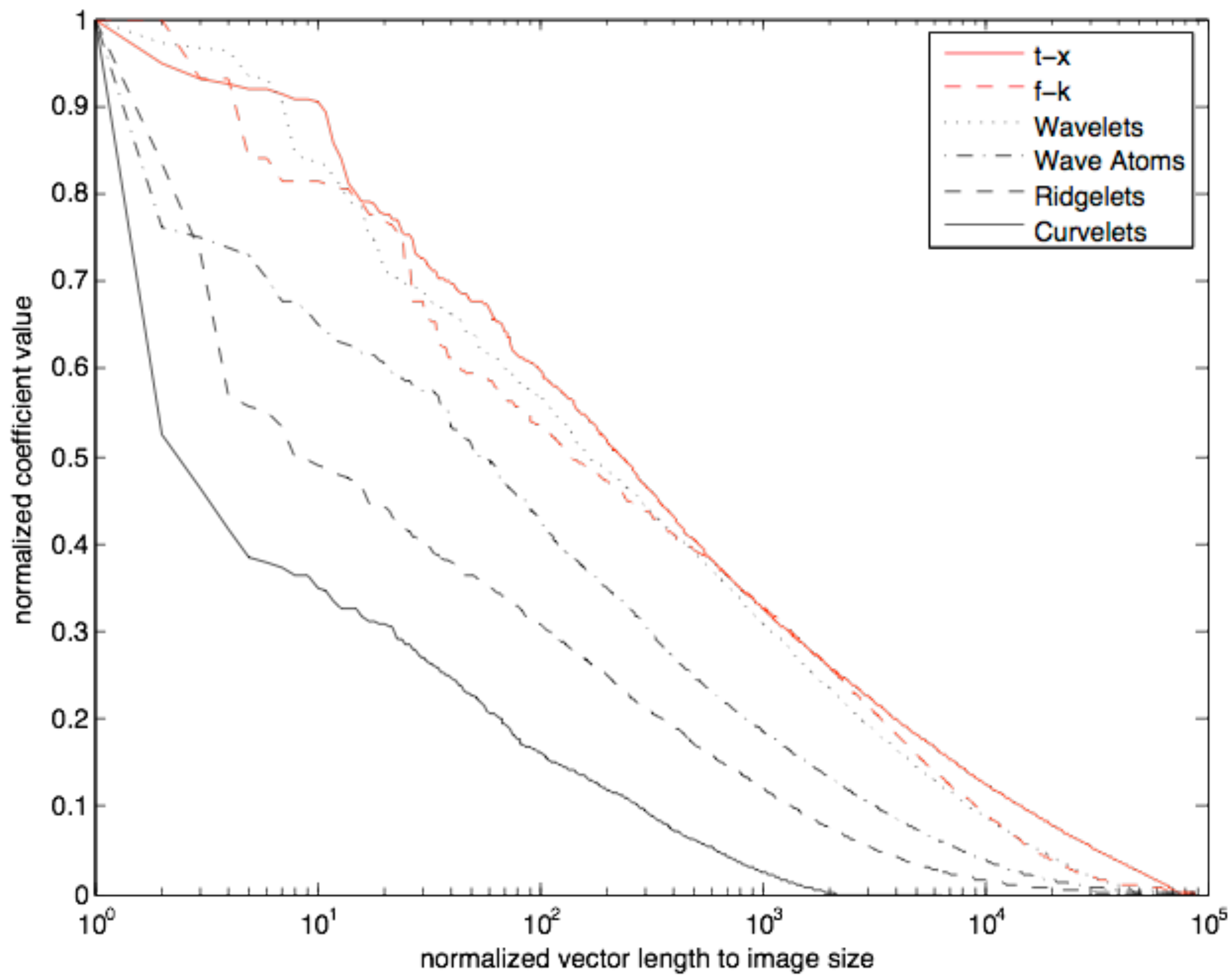
Adapted from Demanet and Ying (06)

Sparsity

Coefficient decay rates - real data

$\mathbf{A}$ Sparsity promoting transform

Decay rates for the "Oz25" dataset



Curvelets are the most sparse domain for seismic data. More energy in fewer coefficients.

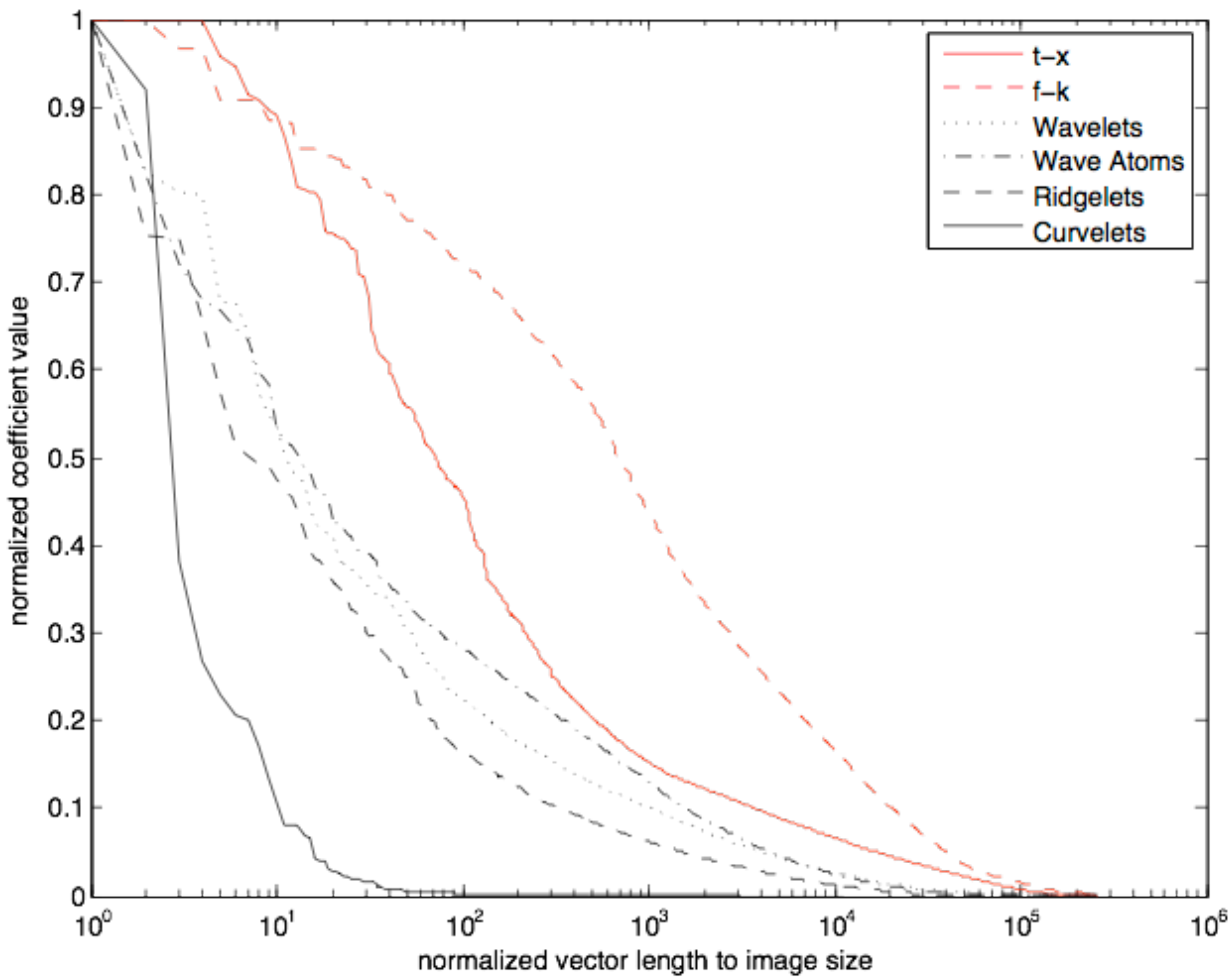
How do we take advantage of this?

One-norm minimizations solved through iterative thresholding.

Sparsity
Coefficient decay rates – synthetic data

$\mathbf{A}$ Sparsity promoting transform

Decay rates for synthetic dataset



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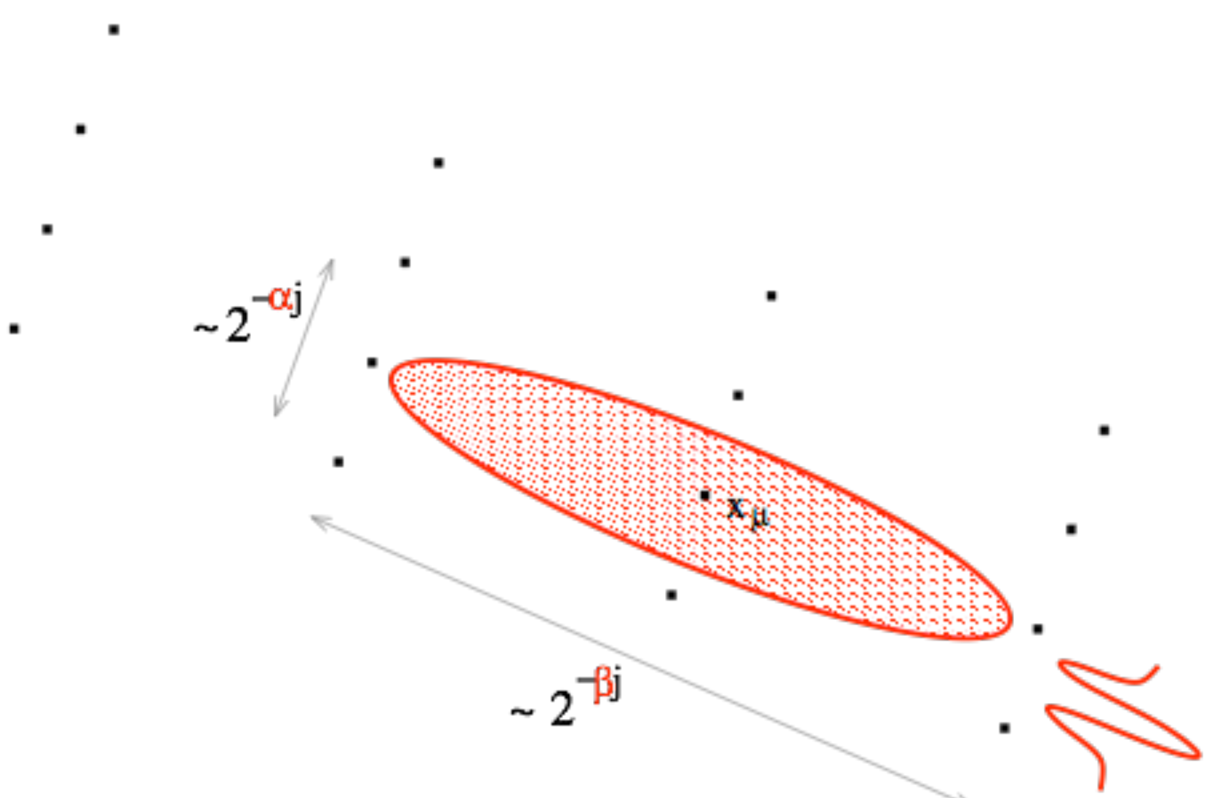
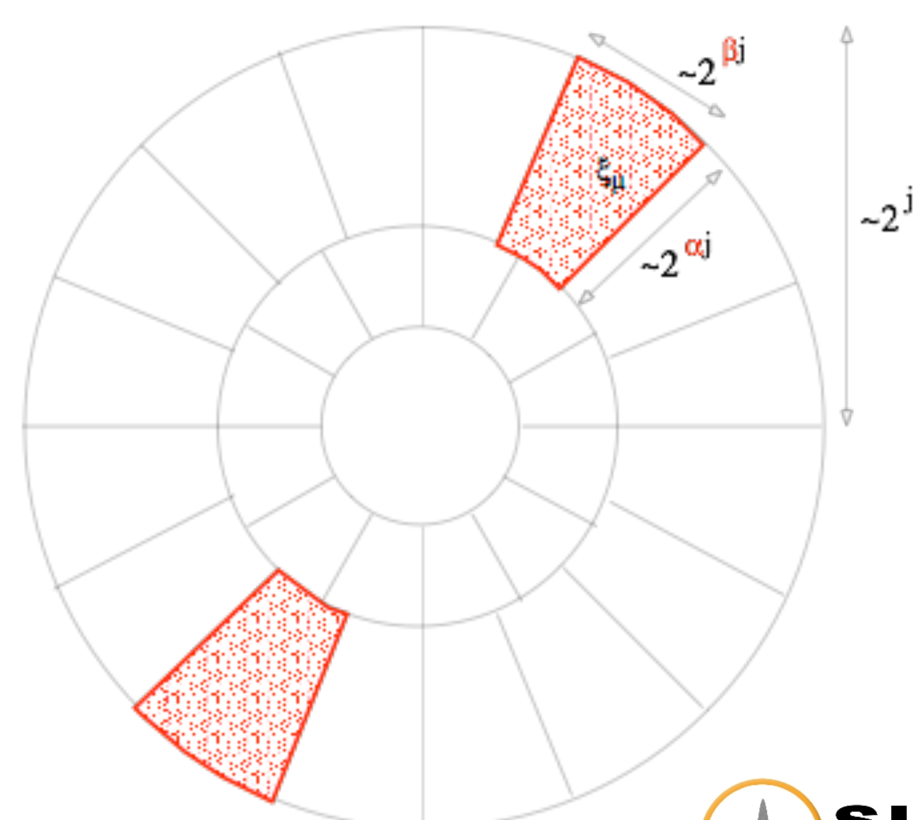
Sparsity

Curvelets (Candès, E., Demanet, L., Donoho, D., and Ying, L., 2005)

- *Compact* support in the f-k domain, *rapid decay* in t-x
- $\alpha^2 \sim \beta$
- Tight frame
- Slightly less than 8 times redundant in 2D
- Fast to compute, built on Fourier transforms $O(N \cdot log N)$





Figures from Demanet and Ying (06)

Signal separation methods

Block-coordinate relaxation (BCR)

- Introduced by Starck, Elad, Donoho 2004.
- Modified for seismic data processing using curvelets to separate multiples by Herrmann, Boeniger and Verschuur, 2007.
- Used for ground-roll removal by Yarham, Boeniger and Herrmann, 2006.

Bayesian formulation

- Developed by Saab, Wang, Yilmaz and Herrmann, 2007 to separate signals in sparse domains.
- Designed for multiple and ground-roll separation.
- Addresses instability of the BCR method.

Curvelet-based Bayesian separation

Forward model: [Saab et. al '07, Wang et.al '07, '08]

$$\mathbf{b} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \qquad \text{(total data)}$$

$$\mathbf{b}_1 = \mathbf{A}\mathbf{x}_1 + \mathbf{n}_1 \qquad \text{(predicted primaries)}$$

$$\mathbf{b}_2 = \mathbf{A}\mathbf{x}_2 + \mathbf{n}_2 \qquad \text{(predicted multiples)}$$

where

$\mathbf{x}_1$ curvelet coefficients of *primaries*

$\mathbf{x}_2$ curvelet coefficients of *multiples*

$\mathbf{A}$ inverse curvelet transform

Curvelet-based Bayesian separation

Estimate $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ by solving

$$\begin{aligned}
&\arg\max_{\mathbf{x_1},\mathbf{x}_2} P(\mathbf{x}_1,\mathbf{x}_2|\mathbf{b}_1,\mathbf{b}_2) \\
&\quad = \arg\max_{\mathbf{x_1},\mathbf{x}_2} \; P(\mathbf{x}_1,\mathbf{x}_2)P(\mathbf{n})P(\mathbf{n}_2) \\
&\quad = \arg\max_{\mathbf{x_1},\mathbf{x}_2} \; \exp\left(-\alpha_1\|\mathbf{x}_1\|_{1,\mathbf{w}_1} - \alpha_2\|\mathbf{x}_2\|_{1,\mathbf{w}_2} - \frac{\|\mathbf{A}\mathbf{x}_2-\mathbf{b}_2\|_2^2}{\sigma_2^2} - \frac{\|\mathbf{A}(\mathbf{x}_1+\mathbf{x}_2)-(\mathbf{b}_1+\mathbf{b}_2)\|_2^2}{\sigma^2}\right) \\
&\quad = \arg\max_{\mathbf{x_1},\mathbf{x}_2} \; -\left(\alpha_1\|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \alpha_2\|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \frac{\|\mathbf{A}\mathbf{x}_2-\mathbf{b}_2\|_2^2}{\sigma_2^2} + \frac{\|\mathbf{A}(\mathbf{x}_1+\mathbf{x}_2)-(\mathbf{b}_1+\mathbf{b}_2)\|_2^2}{\sigma^2}\right)
\end{aligned}$$

yielding estimates for

$$\tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \text{ (primaries)}$$

$$\tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2 \text{ (multiples)}$$

Curvelet-based Bayesian separation

Involves the solution of the following nonlinear problem:

$$\mathbf{P_w}: \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda_1 \|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2 \|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\underline{\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2}\|_2^2 + \eta \|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

where

$\mathbf{b}_2$ predicted multiples

$\mathbf{A}$ inverse discrete curvelet transforms

$\tilde{\mathbf{s}}_{1,2}$ estimated primaries(1)and multiples(2)

$\lambda_{1,2}$ and η are control parameters

Can be solved by iterative soft thresholding.

Curvelet-based Bayesian separation

Given initial estimates of $\mathbf{x}_1^0$ and $\mathbf{x}_2^0$, the n^{th} iteration of the algorithm proceeds as follows

$$\begin{aligned}\mathbf{x}_1^{n+1} &= \mathbf{T}_{\frac{\boldsymbol{\lambda}_1 \mathbf{w}_1}{2\eta}}\left[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n + \mathbf{x}_1^n\right]\\ \mathbf{x}_2^{n+1} &= \mathbf{T}_{\frac{\boldsymbol{\lambda}_2 \mathbf{w}_2}{2(1+\eta)}}\left[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{x}_2^n + \frac{\eta}{\eta+1}(\mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n)\right]\end{aligned}$$

where $\mathbf{T}_{\mathbf{u}} : \mathbb{R}^{|\mathcal{M}|} \mapsto \mathbb{R}^{|\mathcal{M}|}$ is the elementwise soft-thresholding operator, i.e.,

$$T_{u_\mu}(v_\mu) := \frac{v_\mu}{|v_\mu|} \cdot \max(0, |v_\mu| - |u_\mu|)$$

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Curvelet-based Bayesian separation

Parametrization:

η	Prediction confidence parameter
λ_1	Expected reflector sparsity
λ_2	Expected surface wave sparsity

Limiting case:

$\eta \rightarrow \infty$ Total lack of confidence => block-relaxation

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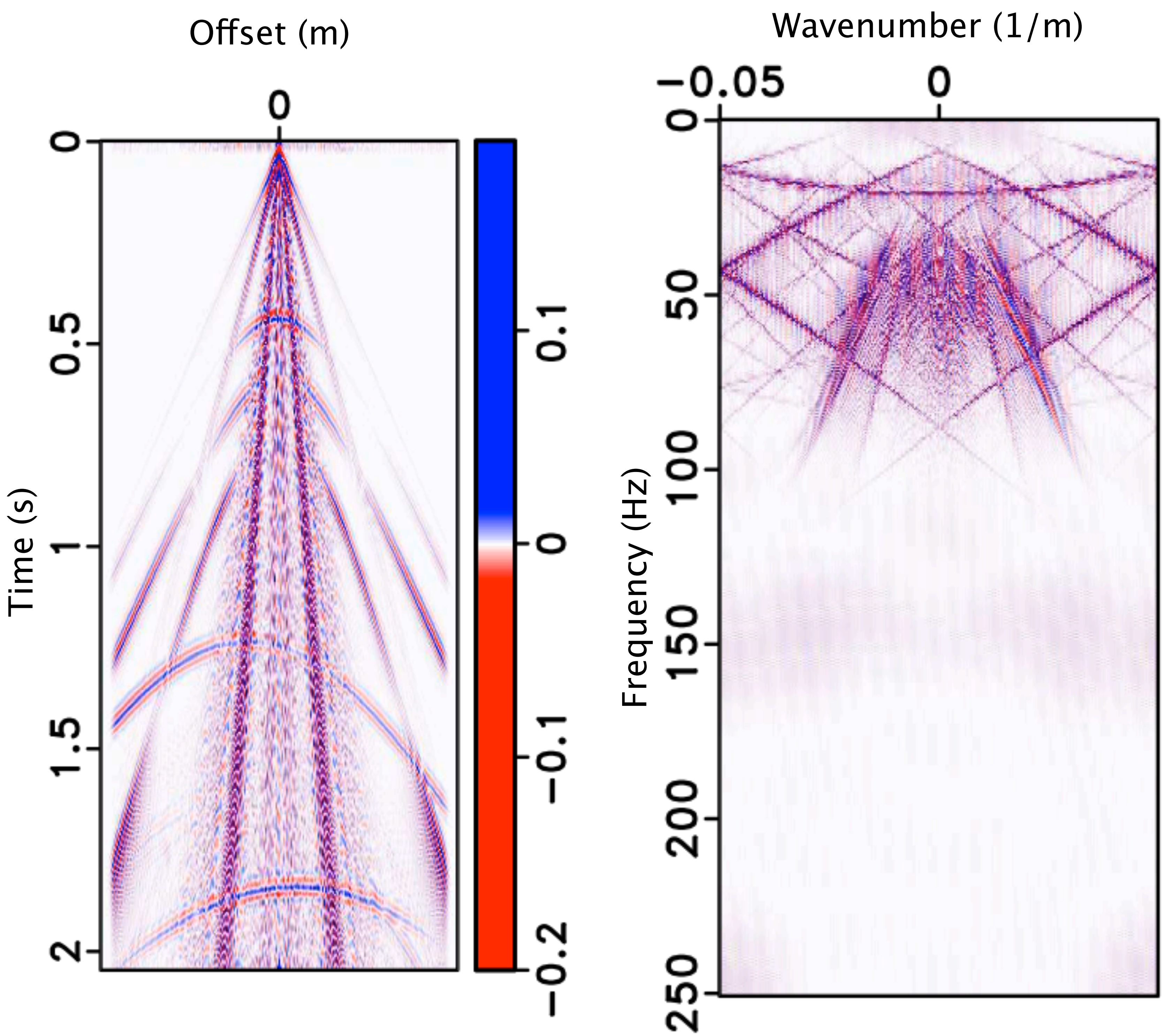
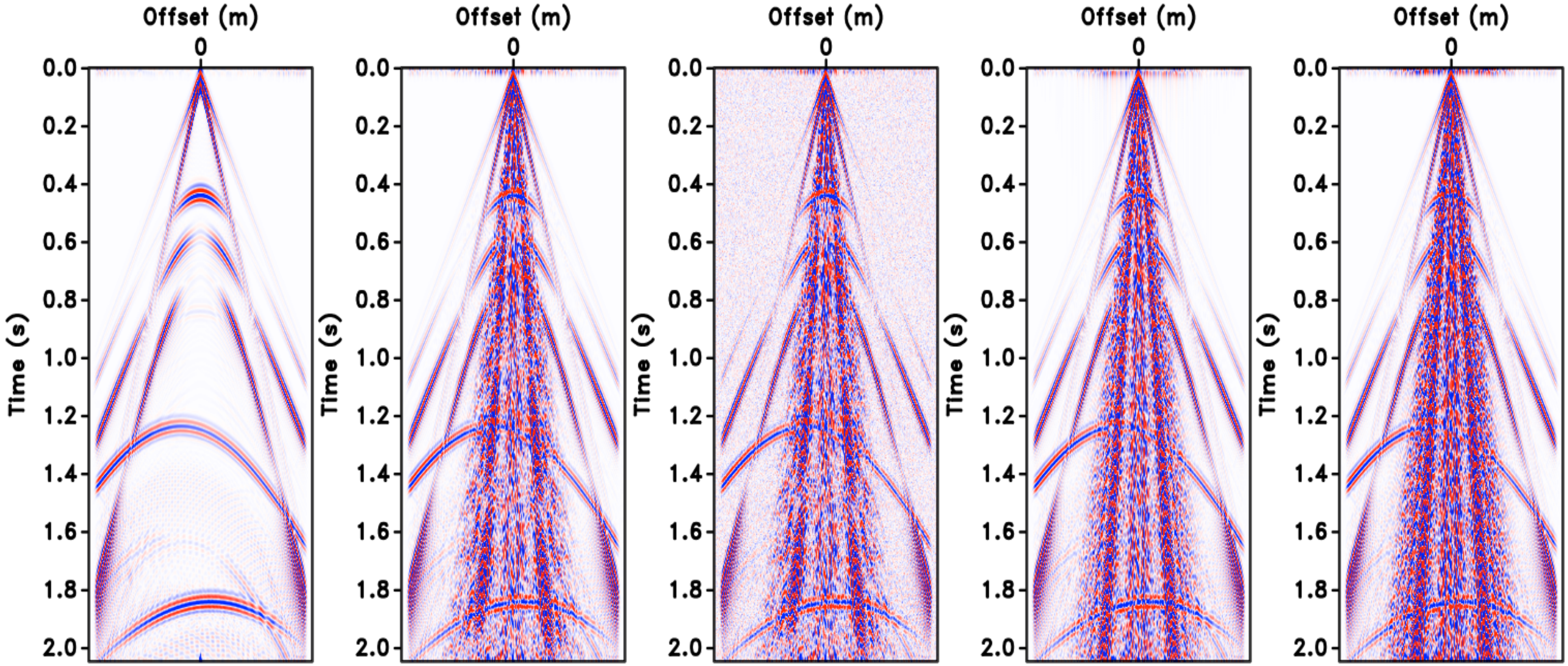
Synthetic-data example
Data - Elastic Finite Difference



- Reflectors
  - 10m grid
  - 3 Reflectors
    - 500 (m) flat
    - 1500 (m) dipping right
    - 2650 (m) dipping left
- Surface Wave
  - 1m Grid
  - 25m surface layer
    - Linear increase in parameters

Synthetic-data example

Benchmark tests: direct subtraction



Exact model

5% model error

5% model error with noise

Hilbert transform

Exact with 180 phase shift

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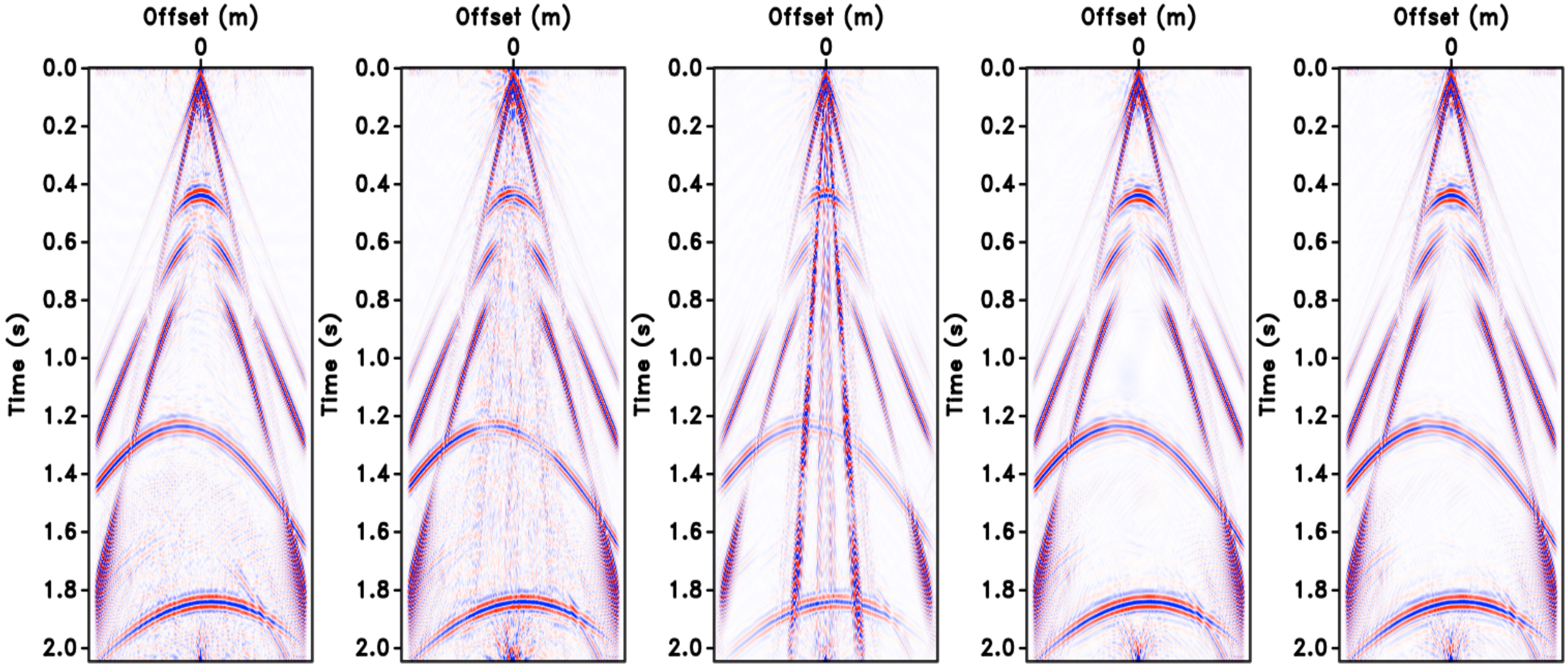
Synthetic-data example

Estimated reflectors

Benchmark tests: block-coordinate separation



Exact model

5% model error

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Hilbert transform

Exact with 180 phase shift

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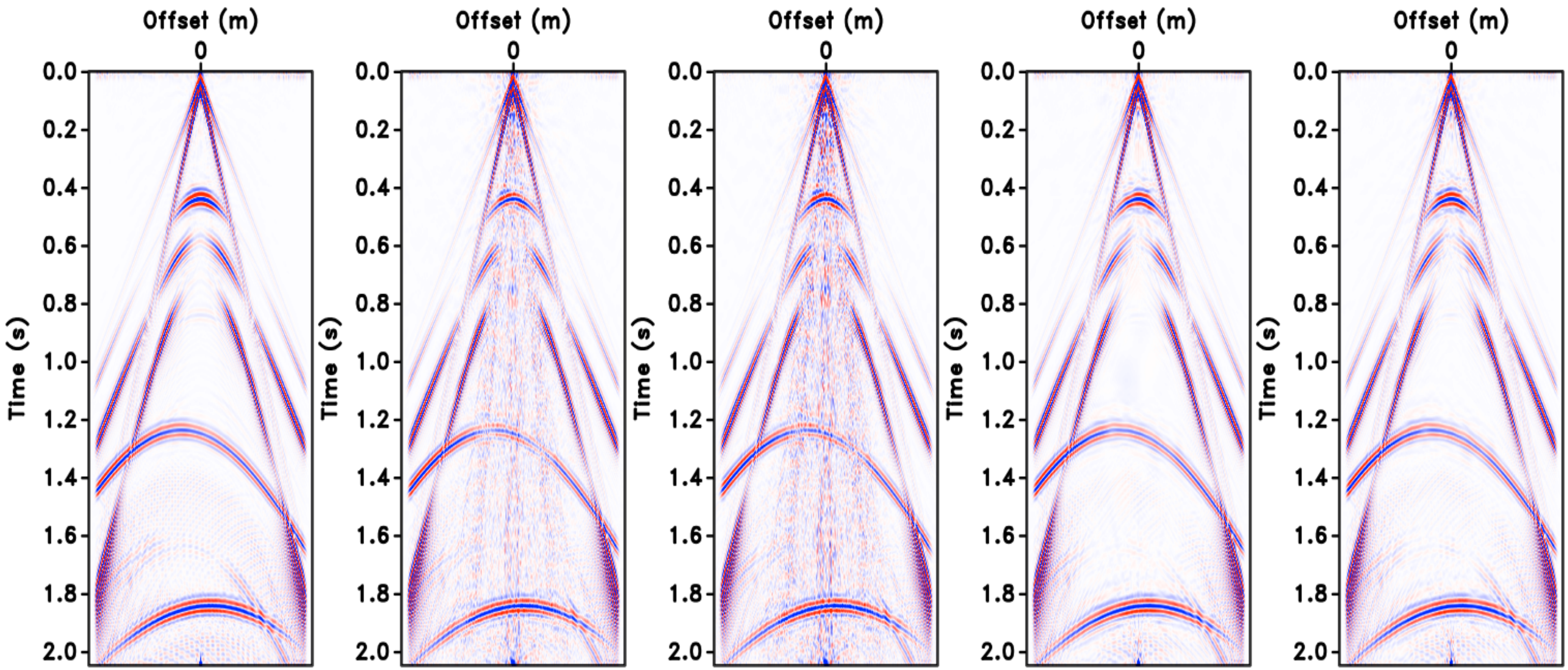
Synthetic-data example

Estimated reflectors

Benchmark rests: Bayesian separation



Exact model

5% model error

5% model error with noise

Hilbert transform

Exact with 180 phase shift

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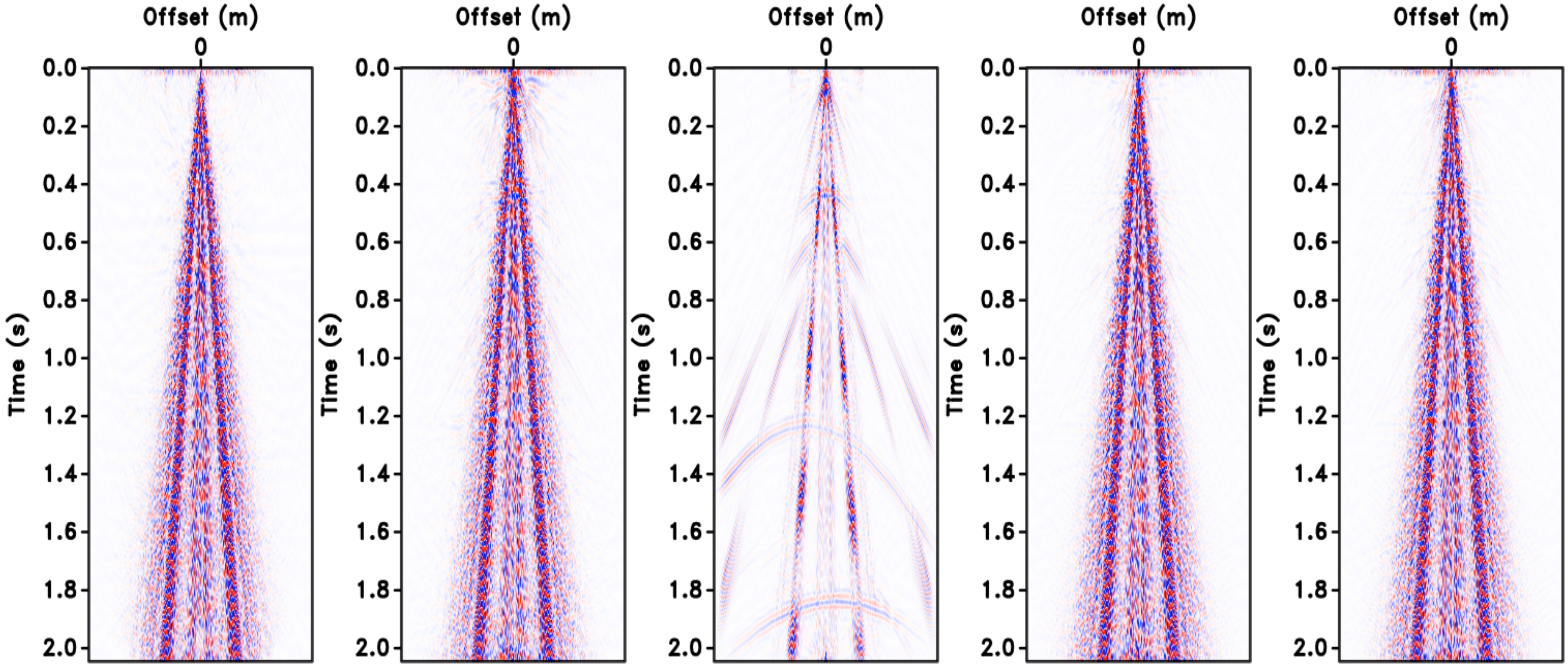
Synthetic-data example

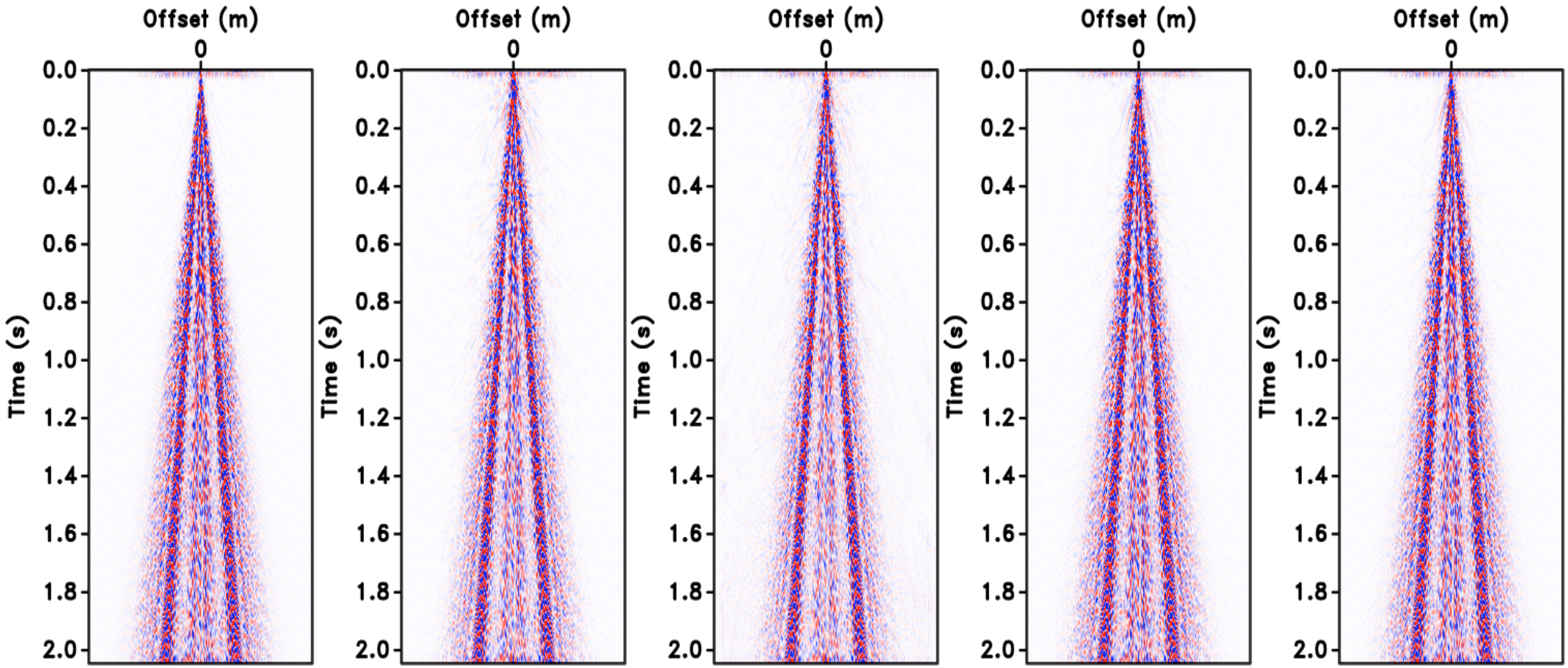
Estimated surface wave
Benchmark tests: block-coordinate separation



Exact model	5% model error	5% model error with noise	Hilbert transform	Exact with 180 phase shift

Synthetic-data example

Estimated surface wave
Benchmark rests: Bayesian separation



Exact model	5% model error	5% model error with noise	Hilbert transform	Exact with 180 phase shift

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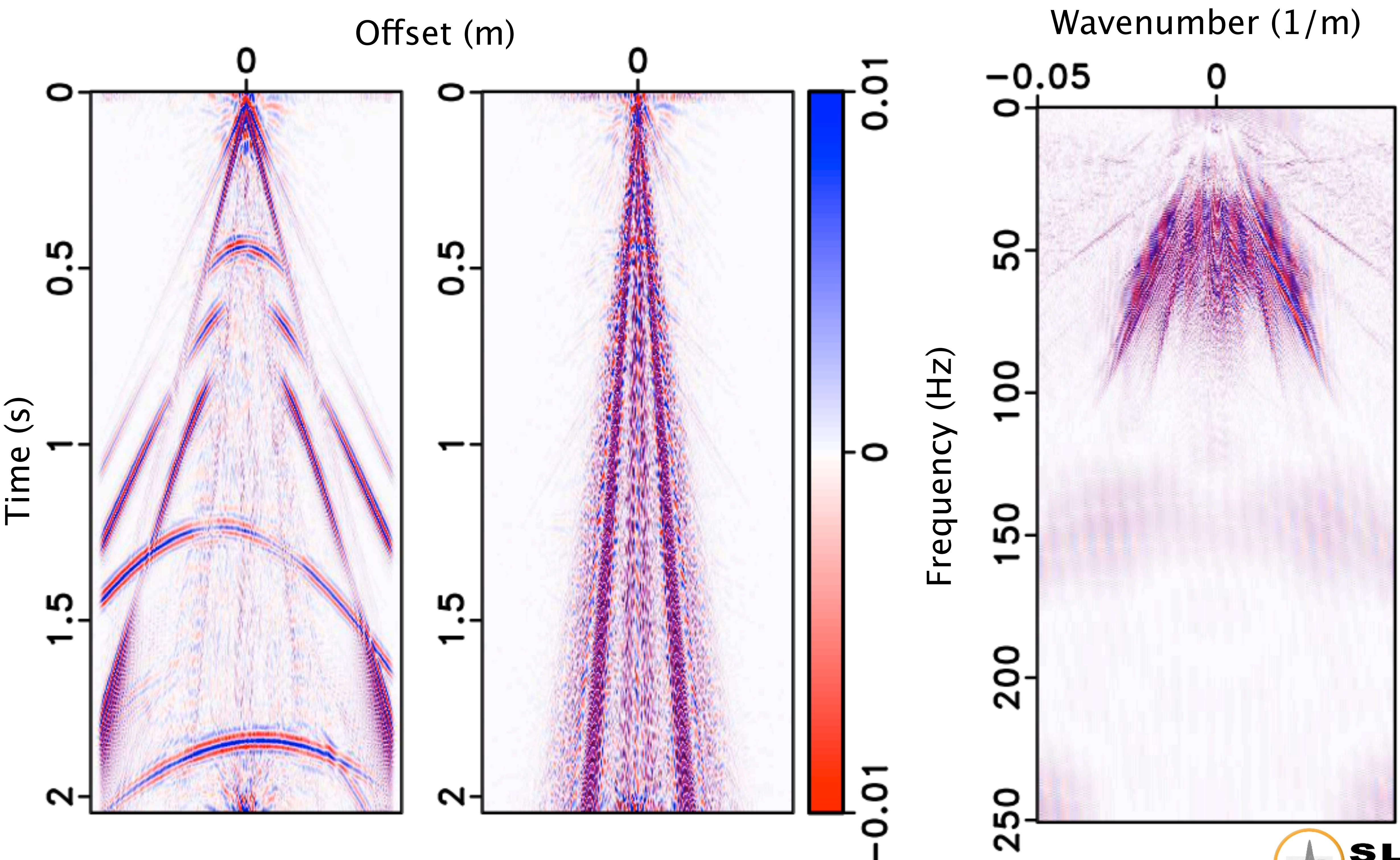
Synthetic-data example
Results: Block-coordinate relaxation



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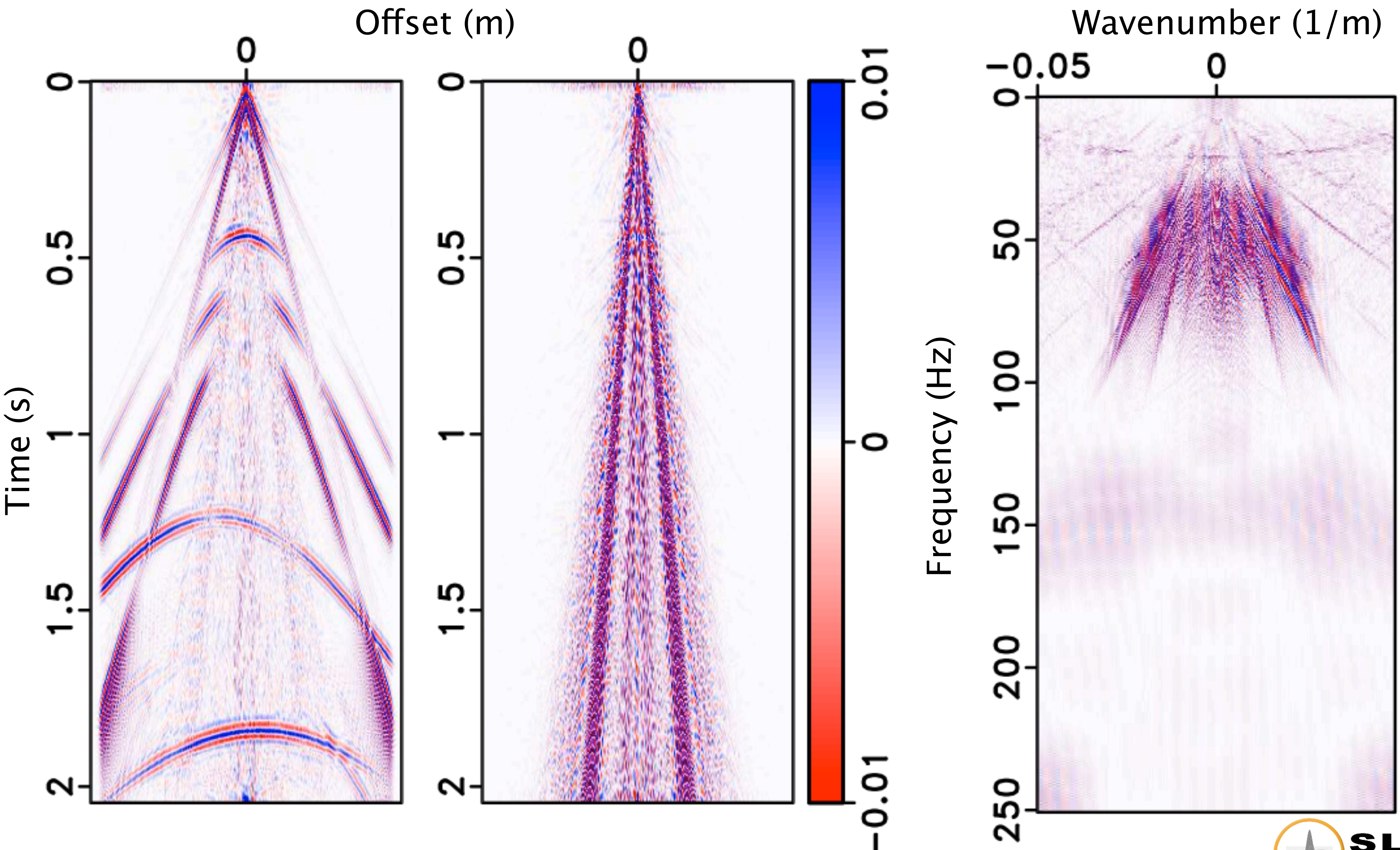
Synthetic-data example
Results: Bayesian formulation



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Synthetic-data example

Signal to Noise Ratio

$$SNR(dB) = 20log_{10}\left(\frac{S_1}{S_2}\right)$$

$$S_1 = RMS(Reflectors)$$

$$S_2 = RMS(Exact\ Surface\ Wave - Estimated\ Surface\ Wave)$$

Reflectors and Surface wave are known for Synthetic Example
Estimated Surface wave is produced by separation method.

Synthetic-data example

Benchmark tests – Results

Noise Prediction	Initial SNR	Subtraction	Bayesian	Block Coordinate Relaxation
Exact Noise	-1.67	147.96	20.58	15.50
5% Model Error	-1.67	-4.38	9.59	9.41
5% Model Error + Noise	-1.92	-4.52	9.09	3.53
Hilbert Transform	-1.67	-4.67	14.93	13.33
Phase Inverse	-1.67	-7.69	14.08	13.10

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Synthetic-data example

Parameter Sensitivity – Bayesian Solver

SNR (dB)	$0.1 \cdot (\lambda_1^*, \lambda_2^*)$	$2 \cdot \lambda_1^*, \lambda_2^*$	λ_1^*, λ_2^*	$\lambda_1^*, 2 \cdot \lambda_2^*$	$10 \cdot (\lambda_1^*, \lambda_2^*)$
$0.1 \cdot \eta^*$	8.35	3.33	4.46	4.46	1.56
$0.5 \cdot \eta^*$	1.86	5.83	8.88	9.00	3.33
η^*	1.79	6.93	9.59	9.02	4.45
$2 \cdot \eta^*$	-3.90	6.48	1.27	2.78	5.97
$10 \cdot \eta^*$	-4.28	-2.56	-3.38	-3.18	8.05

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Synthetic-data example

Parameter Sensitivity – BCR Solver

SNR (dB)	$0.1 \cdot c_1^*$	$0.5 \cdot c_1^*$	c_1^*	$2 \cdot c_1^*$	$10 \cdot c_1^*$
$0.1 \cdot c_2^*$	6.24	0.35	2.30	-1.57	-1.67
$0.5 \cdot c_2^*$	4.09	8.83	0.62	-1.35	-1.66
c_2^*	2.30	5.01	9.41	-0.56	-1.24
$2 \cdot c_2^*$	1.66	2.47	4.02	7.85	8.42
$10 \cdot c_2^*$	0.01	0.01	0.12	0.15	0.21

Synthetic-data example
Summary

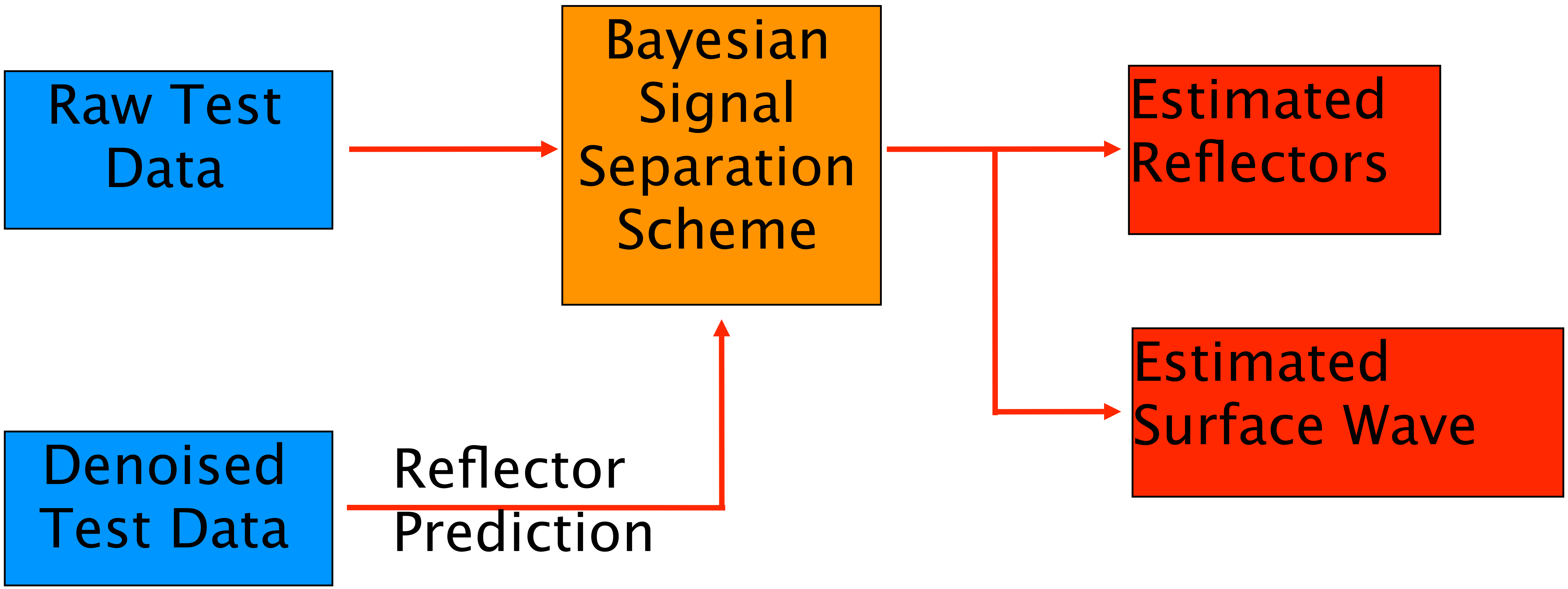
Block-coordinate relaxation (BCR)

- Not as effective as the Bayesian separation method.
- More sensitive to control parameters.
- Works well with phase shifted predictions but does not handle noise well.

Bayesian formulation

- Generates better separations than the block coordinate relaxation method.
- Less sensitive to parameter variations.
- Faster, requiring less iterations.
- Added parameters to enforce stability.

Real-data example
Shell Test Data



η Prediction confidence parameter – Set to 3.0

λ_1 Reflector sparsity parameter – Set to 1.0

λ_2 Surface wave sparsity parameter – Set to 3.0

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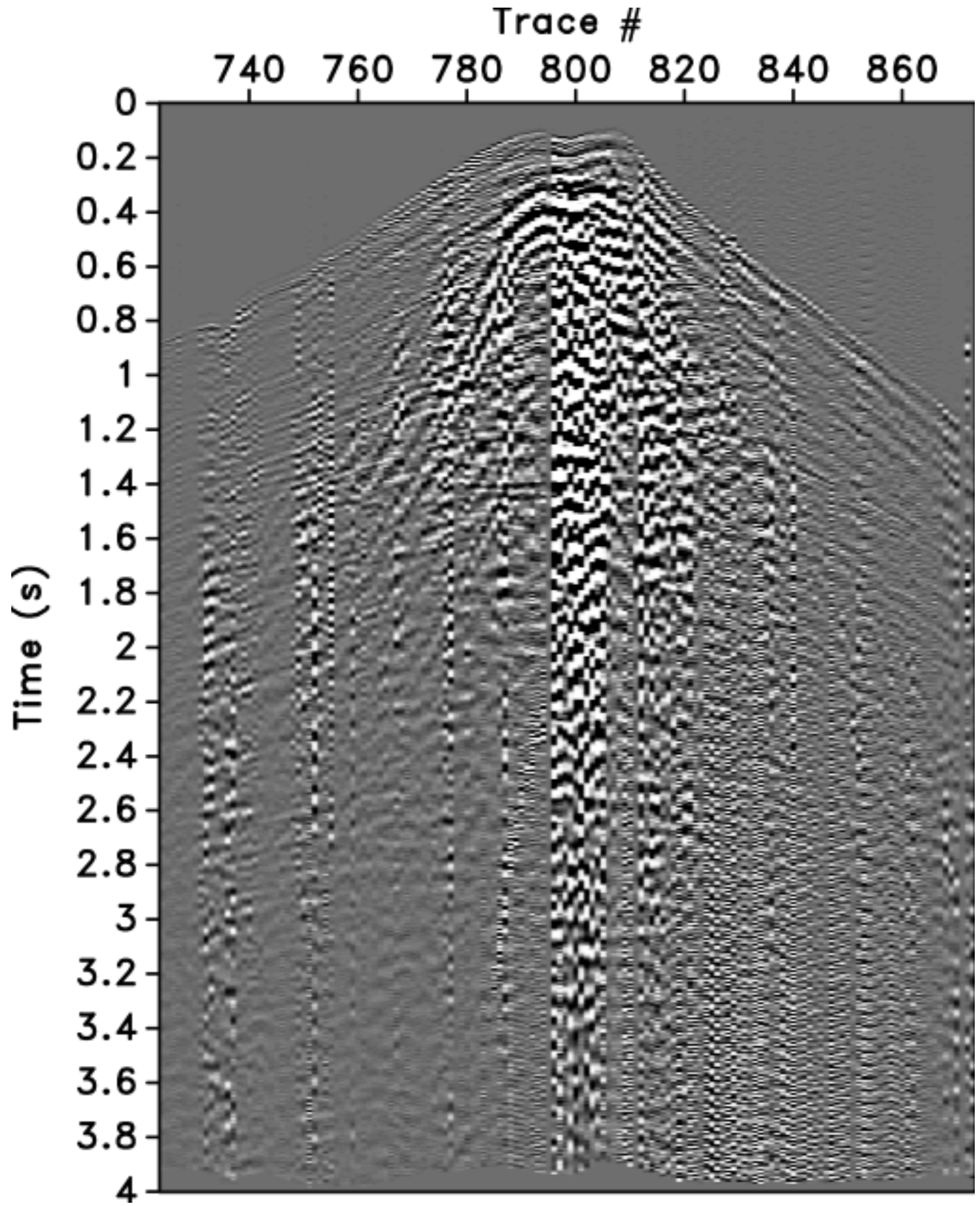
Real-data example
Shell's foothill test data



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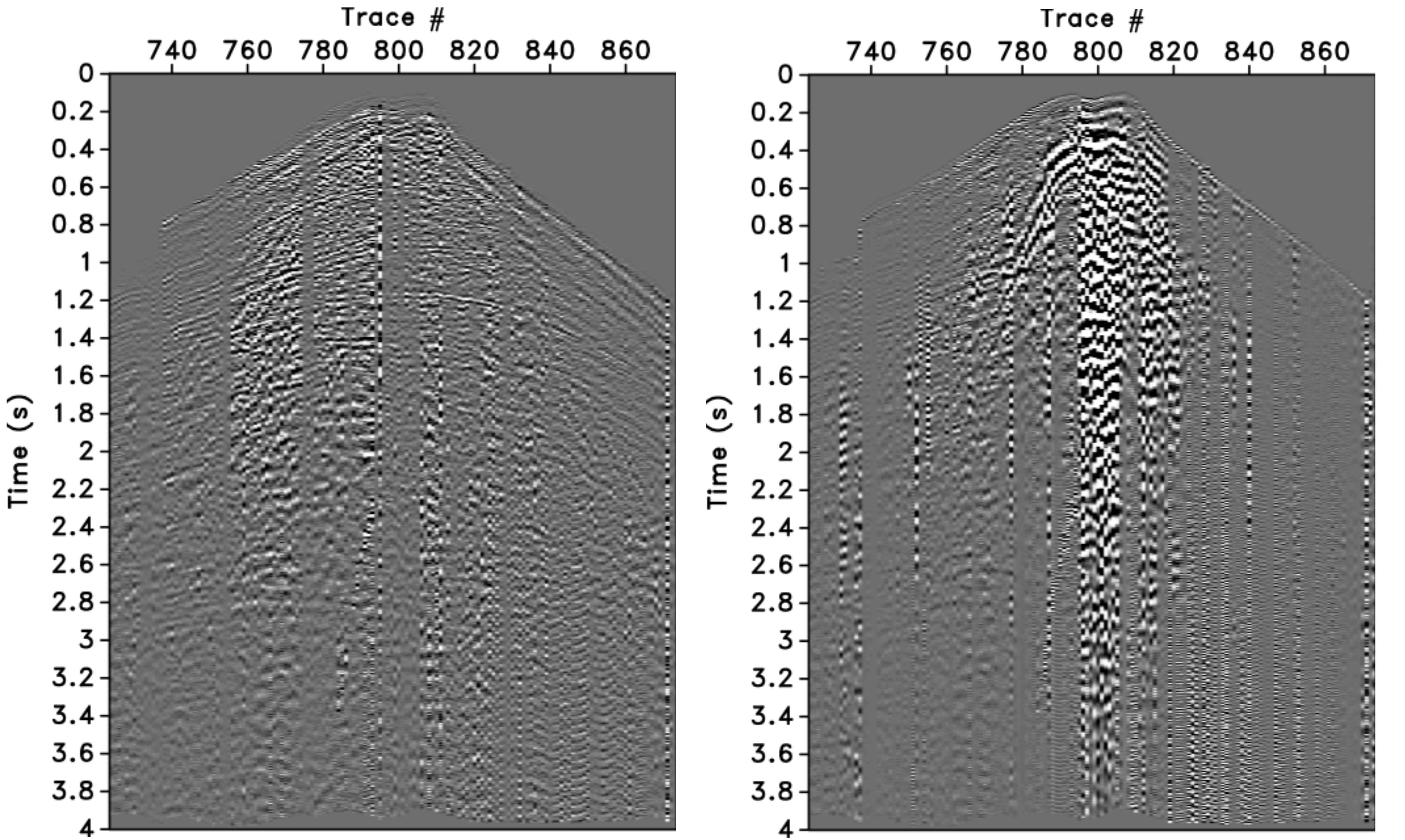
Real-data example
Provided predictions



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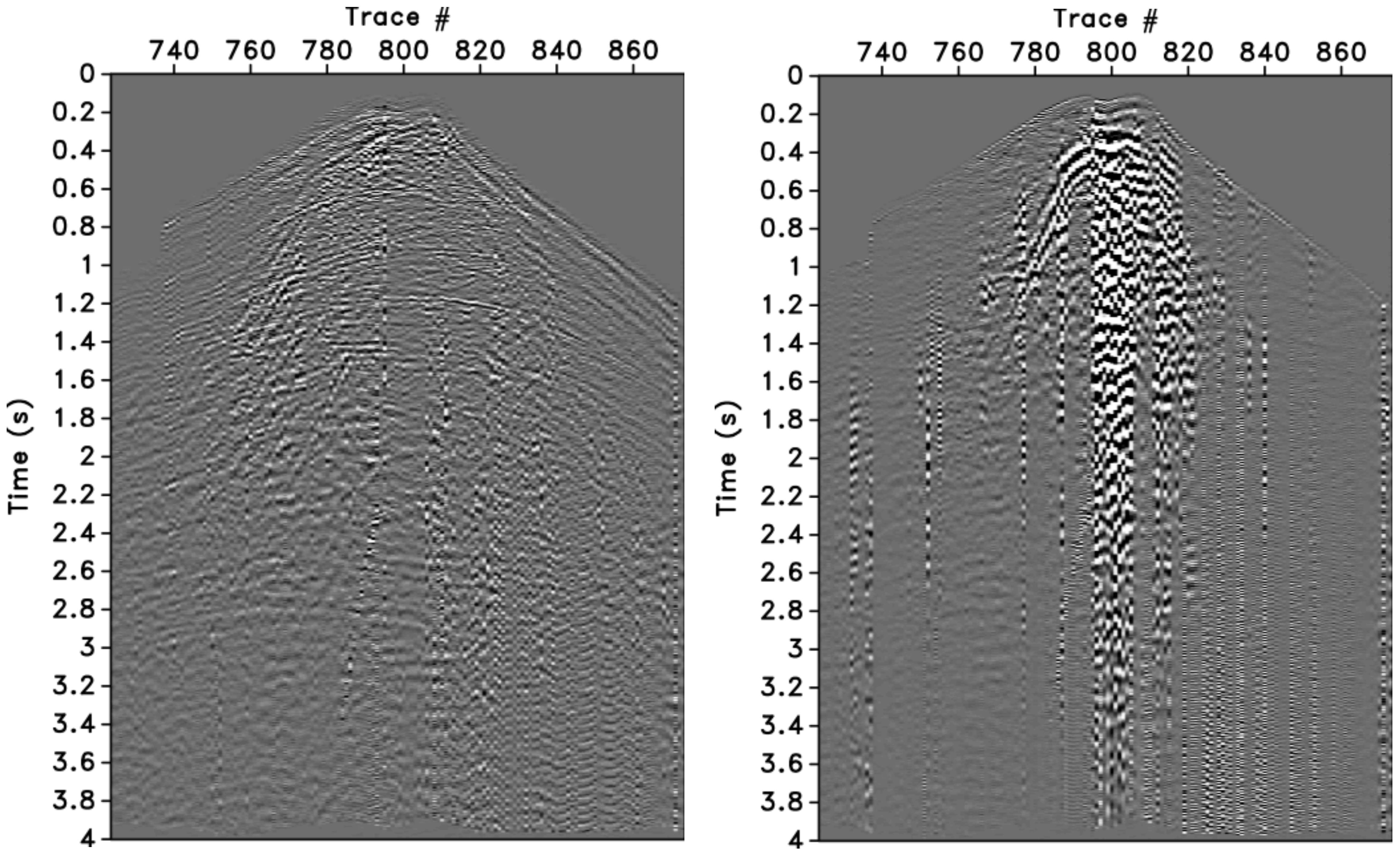
Real-data example

Estimated reflectors and surface wave



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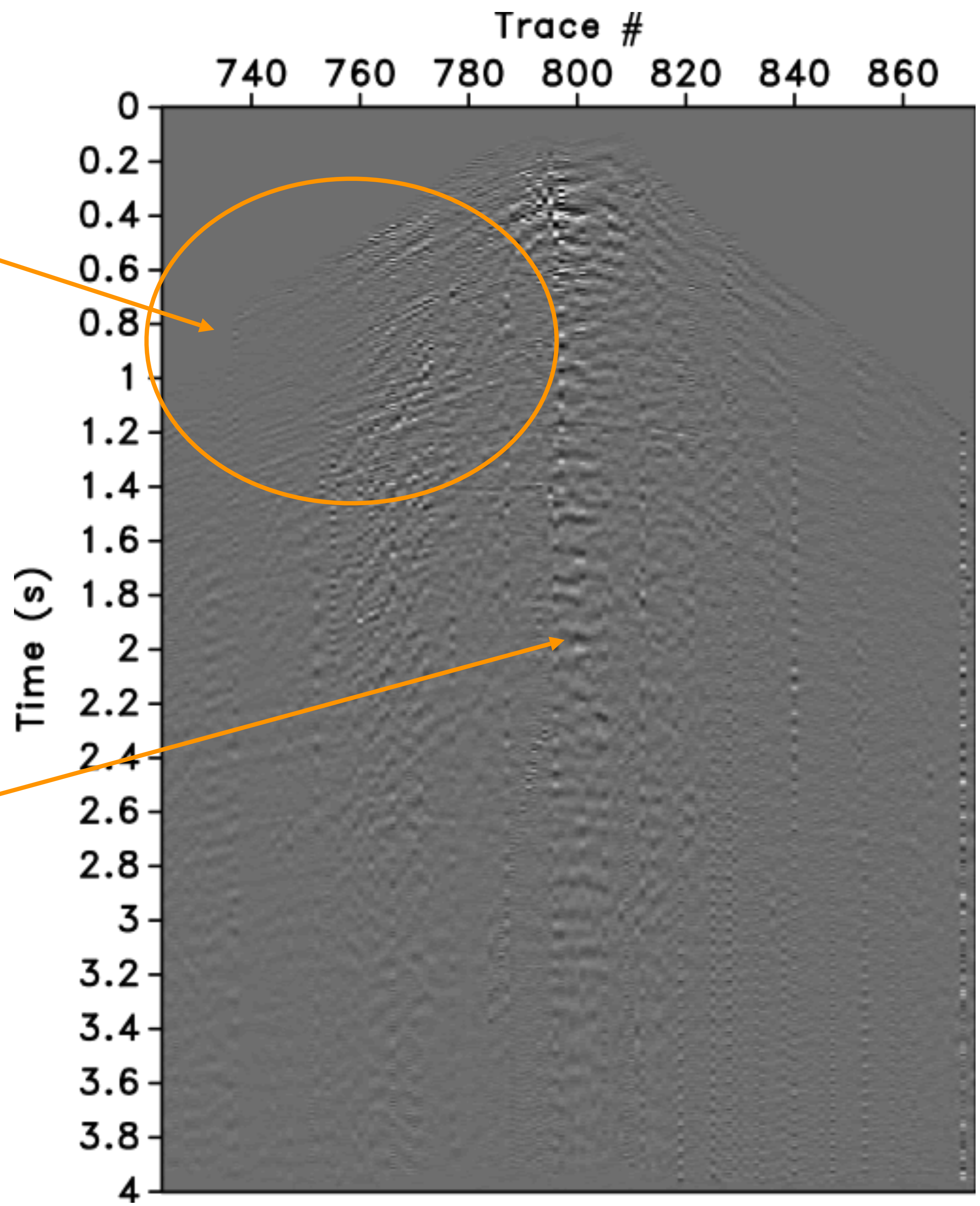
Real-data example
Surface wave difference plot

Recovered reflector information

Increased noise removal



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Conclusions

- Utilized the curvelet domain as our sparsifying transform
  - Sparsest transform of the ones tested
- Shown improvements in ground-roll removal
  - Utilized synthetic data with controllable errors
  - Improved reflector preservation in real seismic data
- Utilized two wavefield separation techniques for ground-roll separation
  - Block coordinate relaxation
    - More sensitive parameters
    - Can degrade small amount of reflector information
  - Bayesian Formulation
    - Less sensitive parameters
    - More control over separation
    - Preserves reflector information

Acknowledgments

David Campden (Shell Canada) for providing the foothill data set

SLIM team members: H. Modzelewski, S. Ross Ross and C. Brown for SLIMpy (slim.eos.ubc.ca/SLIMpy)

E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying for CurveLab (www.curvelet.org)

S. Fomel, P. Sava, and the other developers of Madagascar (rsf.sourceforge.net)

This presentation was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).

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Further reading

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Thank you!

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