# Bayesian ground-roll separation by curvelet-domain sparsity promotion

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**SUMMARY**

The removal of coherent noise generated by surface waves in land based seismic is a prerequisite to imaging the subsurface. These surface waves, termed as ground roll, overlay important reflector information in both the t-x and f-k domains. Standard techniques of ground-roll removal commonly alter reflector information. We propose the use of the curvelet domain as a sparsifying transform in which to preform signal-separation techniques that preserves reflector information while increasing ground-roll removal. We look at how this method preforms on synthetic data for which we can build quantitative results and a real field data set.

## INTRODUCTION

The removal of coherent noise, such as ground-roll, in seismic signals generally comes in two parts, prediction and separation. Developments in the prediction of ground roll have been numerous and varied. Methods utilizing the Fourier (Yilmaz, 2001; Galbraith and Wiggins, 1968; Coruh and Costain, 1983; Embree et al., 1963; Fail and Grau, 1963) or Radon (Russel et al., 1990; Trad et al., 2003) domains, one and two-dimensional wavelets (Deighan and Watts, 1997; Zhang and Ulrych, 2003) and various other techniques (Londono et al., 2005; Karsli and Bayrak, 2004) have all been developed to address the challenges of prediction. The second step, separation, has generally been overlooked. Adaptive subtraction techniques applied to multiples (Verschuur et al., 1992; Guitton and Verschuur, 2004) and other methods exist but have not been developed with the same vigor as the prediction schemes. If a prediction scheme can perfectly predict the coherent noise components, they can be separated with a simple subtraction but in the event that they are not perfect, this may cause problems. Alteration of reflector information and incomplete noise removal (McMechan and Sun, 1991; Liu, 1999; Karsli and Bayrak, 2004) are common and we propose the use of a Bayesian separation algorithm (Saab et al., 2007; Wang et al., 2007) to improve the overall separation of coherent noise and reflector information.

It has been shown that the Bayesian separation method can separate primary and multiple information effectively (Saab et al., 2007; Wang et al., 2007). The Bayesian separation algorithm takes advantage of the sparse representation of seismic data in the curvelet domain by applying an iterative series of thresholding operations to solve a sparsity promoting $\ell_1$ minimization problem. This method utilizes a set of parameters to help characterise the signal separation by incorporating information such as prediction fidelity and expected sparsity of the to-be-separated results. These parameters allow flexibility that is needed to adapt to many different separation problems as ground roll and reflector information both exhibit large variation between different data sets and problems.

In this abstract, we look at this signal separation method in the context of ground-roll removal. We will examine two sets of data to understand the effects, limitations and benefits of using this separation scheme on seismic data. The first example involves a set of synthetic data for which accurate signal-to-noise ratio measurements can be made. The use of this synthetic data allows us to control the fidelity of the predictions and gives us the ability to test a range of erroneous surface wave predictions and analyse the separation results. A real field data set will also be used to show how this method can be utilized to improve real denoising schemes that already exist simply by the addition of a signal separation algorithm. We show that by using a signal separation method, more coherent noise can be removed and, more importantly, more reflector information can be preserved.

## BAYESIAN SEPARATION METHOD

Introduced by Saab et al. (2007); Wang et al. (2007), the Bayesian separation method is in the same group of sparse signal separation methods as the block-coordinate relaxation method (Herrmann et al., 2007). The block-coordinate relaxation method was shown to be effective in certain situations for coherent noise removal for ground roll (Yarham et al., 2006) and multiples (Herrmann et al., 2007) but care needs to be taken with regards to the thresholding function which can tends to be too aggressive. This over thresholding can cause the signal separation may fail or curvelet-like artifacts may be generated that are undesirable. This second generation method works around these problem by introducing control parameters in the minimization of the function

$$
\begin{aligned}
f(\mathbf{x}_1,\mathbf{x}_2) \quad = \quad & \lambda_1||\mathbf{x}_1||_{1,\mathbf{w}_1}+\lambda_2||\mathbf{x}_2||_{1,\mathbf{w}_2}+\frac{||\mathbf{A}\mathbf{x}_2-\mathbf{b}_2||_2^2}{\sigma_2^2} \\
+ \quad & \frac{||\mathbf{A}(\mathbf{x}_1+\mathbf{x}_2)-(\mathbf{b}_1+\mathbf{b}_2)||_2^2}{\sigma^2}. \qquad (1)
\end{aligned}
$$

The values $\mathbf{x}_1$ and $\mathbf{x}_2$ represent the estimated reflector and ground roll to be generated by the separation algorithm from the total data $\mathbf{b}$. The predictions for the reflector ($\mathbf{b}_1$) and ground roll ($\mathbf{b}_2$) such that $\mathbf{b}=\mathbf{b}_1+\mathbf{b}_2$ are given. This method makes use of the curvelet synthesis matrix $\mathbf{A}$ and weights applied to the $\ell_1$ minimization to promote sparsity and facilitate the signal separation. The weighting vectors are defined as $\mathbf{w}_1=\max\{|\mathbf{A}^T\mathbf{b}_2|,\varepsilon\}$ and $\mathbf{w}_2=\max\{|\mathbf{A}^T\mathbf{b}_1|,\varepsilon\}$ to incorporate the predictions.

The introduced control parameters ($\lambda_1$, $\lambda_2$ and $\eta$) allow input of prior information pertaining to the expected sparsity of the reflectors and ground roll estimates through $\lambda_1$ and $\lambda_2$, respectively. The parameter $\eta$ represents the fidelity of the two predictions such that reducing $\eta$ reduces the effect of the reflector prediction while increasing it has the opposite effect. As we will see, the parameters of the Bayesian separation are relatively insensitive to small changes making this method stable.

The minimization of Equation 1 is accomplished by applying a soft thresholding operator $T_u = \text{sign}(\tilde{\mathbf{x}}) \cdot \max(0, |\tilde{\mathbf{x}}| - |\mathbf{u}|)$ iteratively. This method of solving an $\ell_1$ minimization follows from the work of Elad et al. (2005) and is similar to the previous block-coordinate relaxation method. From the initial starting conditions, the $n^{th}$ iteration computes

$$\begin{aligned}
\mathbf{x}_1^{n+1} &= T_{\frac{\lambda_1 \mathbf{w}_1}{2\eta}}[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n + \mathbf{x}_1^n] \\
\mathbf{x}_2^{n+1} &= T_{\frac{\lambda_2 \mathbf{w}_2}{2(1+\eta)}}[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{x}_2^n \\
&+ \frac{\eta}{1+\eta}(\mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n)]. \qquad (2)
\end{aligned}$$

With strictly positive weights, it can be shown that this algorithm converges to the minimizer of $f(x_1, x_2)$. The separation algorithm is described in further detail in Wang et al. (2008).

## NUMERICAL RESULTS

Two sets of data are used to examine the effectiveness of our separation method. The first set of data is a synthetic where we have control over both the reflector and the ground roll information. This allows the signal-to-noise ratios (SNR) to be calculated and compared for various parameter settings and predictions. The predictions used in the separation algorithm are controlled and purposely varied to examine how the separation method preforms under different errors. Next, we switch to a real data set to highlight the ability of the Bayesian separation method to improve the separation of reflector and ground roll information.

### Synthetic testing

To generate the synthetic data, we use a full-elastic finite-difference code (Graves, 1996). The synthetic data is generated as two separate signals, consisting of ground roll and reflectors, and then combined to form the total data set. This gives us a ground truth for the reflector information used in calculating the SNR, and allows for the generation of the surface wave on a finer grid making it more accurate. The reflector dataset is generated on a ten meter grid with a horizontal layer at 500m, followed by two dipping reflectors centered at 1500 and 2650m depth above a half space. The ground roll is generated on a 1m grid with linear increase in compressional velocity, shear wave velocity, and density from the surface to 25m depth over a half space. Within the layers, a one percent normally distributed random perturbation is added to all three parameters varying the model in two dimensions. Before combination, all signals are convolved with a Ricker source wavelet with a center frequency of 20 Hz. This provides a dataset with an initial SNR of -1.67 dB with respect to ground-roll free data.

Four different synthetic examples are used to test the separation schemes. The first test uses the exact ground roll as a prediction to our separation scheme. The second test simulates inaccurate modeling by using a surface wave prediction that contains modeling errors from the exact surface wave. This erroneous prediction is generated with 5% uniformly distributed random error applied to the surface layers as well as the start and end values of the linearly increasing parameters. The third test uses a noisy dataset and prediction consisting of the erroneous modeled ground roll with added noise. The added white noise exhibits a normal distribution and a maximum variation of 5% of the maximum of the signal and is applied to both the dataset and the prediction. The last test involves the exact ground roll to which we apply a Hilbert transform ($\pi/2$ degrees phase rotation) in the time direction. Table 1 shows the signal-to-noise ratios of the experiments utilizing a straight subtraction, the Bayesian separation method and a block-coordinate relaxation scheme previously used in ground-roll separation (Yarham et al., 2006).

| Noise prediction | Subtraction | Bayesian separation | BCR |
|---|---|---|---|
| 1) Exact noise | 147.96 | 20.58 | 13.80 |
| 2) 5% model error | -4.38 | 9.58 | 9.41 |
| 3) Noisy test #2 | -4.52 | 9.09 | 3.53 |
| 4) Hilbert transform | -4.67 | 14.93 | 13.33 |

Table 1: Signal-to-noise ratios for Bayesian and block-coordinate relaxation (BCR) separation methods. The SNR for the synthetic data is -1.67 dB for all tests except for the third test which has added white noise decreasing the SNR to -1.92 dB. These values were calculated with respect to data that has no ground roll or noise present.

While utilizing the synthetic data set, it is also important to look at the sensitivity of the parameters associated with the signal separation. Table 2 shows variations of the chosen separation values ($\lambda_1^*, \lambda_2^*, \eta^*$) and the effect on the signal-to-noise ratios as the Bayesian signal separation method is preformed with the prediction containing 5% model error. Parameters $\lambda_1^*, \lambda_2^*$ and $\eta^*$ are the values used to generate the results in Table 1. These results show stability for variations of the sparsity parameters with $\eta$ as the most sensitive parameter.

| SNR (dB) | $2 \cdot \lambda_1^*, \lambda_2^*$ | $\lambda_1^*, \lambda_2^*$ | $\lambda_1^*, 2 \cdot \lambda_2^*$ |
|---|---|---|---|
| $0.1 \cdot \eta^*$ | 3.33 | 4.46 | 4.46 |
| $\frac{1}{2} \cdot \eta^*$ | 5.83 | 8.88 | 9.00 |
| $\eta^*$ | 6.93 | 9.59 | 9.02 |
| $2 \cdot \eta^*$ | 6.48 | 1.27 | 2.78 |
| $10 \cdot \eta^*$ | -2.56 | -3.38 | -3.18 |

Table 2: Sensitivity of the parameters associated with the Bayesian signal separation scheme. The parameters $\lambda_1^*$, $\lambda_2^*$ and $\eta^*$ refer to 10, 2 and 3.5 respectively. These parameters are the same used to for the results of the second test in Table 1. These values are estimated by the expected sparsity of the estimated signal and the fidelity of the ground roll prediction. Small adjustments were then made by parameter search to select the most effective values for the synthetic data.

### Real field data

This data was shot in the foothills of the Rocky Mountains and contains ground roll and reflector information showing complex structure. This real data, shown in Figure 1(a), set faces common challenges such as unknown true reflector and ground roll signals, possible aliased surface waves and large amplitude variations. There is a strong ground roll signature for this set of geophones as the shot location is midway along the line.

This section of data is a single string of geophones of a larger three-dimensional dataset.

A prediction for the Bayesian separation method has been provided. This consists of a dataset with the coherent and incoherent noise removed by conventional methods, has been provided. This denoised set of reflectors will be used as a reflector prediction for the signal separation. The goal of our separation algorithm is to improve the ground roll and reflector predictions (seen in Figure 1(b) and 1(c)) by reducing the amount of ground roll in the reflector prediction and moving the incorrectly identified reflector information in the ground roll prediction back to the reflector estimation.

The parameters of the Bayesian solver need to be estimated to generate an effective separation. The sparsity of the expected estimates will generally lead to the ground roll estimate being more sparse than the reflectors estimate. The reflector information is present in the majority of the signal and is complex, containing large and small scale structures. The ground roll occupies a region defined by the low velocity of the surface wave. There is an extra noise component as part of the ground roll prediction consisting of the removal of dead traces and other nose removal processes that were applied to generate the reflector estimate but this is not expected to contribute much to the overall sparsity of the signal. For these reasons, we set $\lambda_1 = 1$ and $\lambda_2 = 3$. The parameter $\eta$ is set to 3 in this case as the reflector prediction was constructed to exclude most of the surface wave. We attempt to recover events that were inadvertently removed through the original noise removal processes . We will also simultaneously reduce the noise in the reflector signal. As the reflector prediction scheme was similarly applied to all the slices of this data set, these parameters can generally be applied to all of the two-dimensional slices.

The calculated reflector and surface wave estimations after five iterations of the Bayesian scheme are shown in Figure 2(a) and 2(b). The difference between the surface wave prediction and estimation is shown in Figure 2(c). From this difference plot, we can see reflector information located in the upper left corner of the image has been moved to the reflector estimate. We can also see much of the residual ground roll energy that was part of the reflector prediction has been moved to the surface wave estimation.

## CONCLUSIONS

The use of the Bayesian signal separation algorithm for seismic coherent noise removal shows improvement over standard subtraction separation and our previous method of block-coordinate relaxation. By taking advantage of the sparse representation of seismic data in the curvelet domain, components of unidentified ground-roll can be removed while restoring erroneously removed reflector information. This technique is relatively fast requiring only a few iterations consisting primarily of Fourier transforms used to generate the curvelet transform. These methods can also be implemented in parallel along the two-dimensional data slices or utilized on fully three-dimensional data with the use of the 3-D curvelet transform.

By utilizing synthetic data, we were able to quantitatively analyse the separation results. This method was able to adapt to various erroneous predictions and generate estimates that preserve the reflector information while separating out the ground roll. The Bayesian signal separation method is most efficient when dealing with phase errors such as with the fourth synthetic data test but is also capable of dealing with modeling errors and noise. This method does not produce the artifacts that were generated with previous method and tends to be faster as it requires less overall iterations.

The previous block-coordinate relaxation method required the use of conservative predictions such that no reflector information was present in the ground roll prediction. With the Bayesian separation algorithm, the lack of a conservative estimate can be compensated by proper setting of the fidelity and sparsity parameters resulting in an improved separation. This is important as the generation of new conservative estimates for datasets can be time consuming and costly so the addition of this flexibility allows this method to be directly implemented on existing problems to improve signal separations.

We have shown how the Bayesian separation method can be utilized to improve previous denoised results by improving the overall signal separation. By taking data that was previously denoised, we have shown that with very simple parameter settings, a improvement can be calculated for both the reflector and the surface wave estimates. This improvement is simple to apply and helps preserve the original reflector amplitudes while improving the overall denoising.

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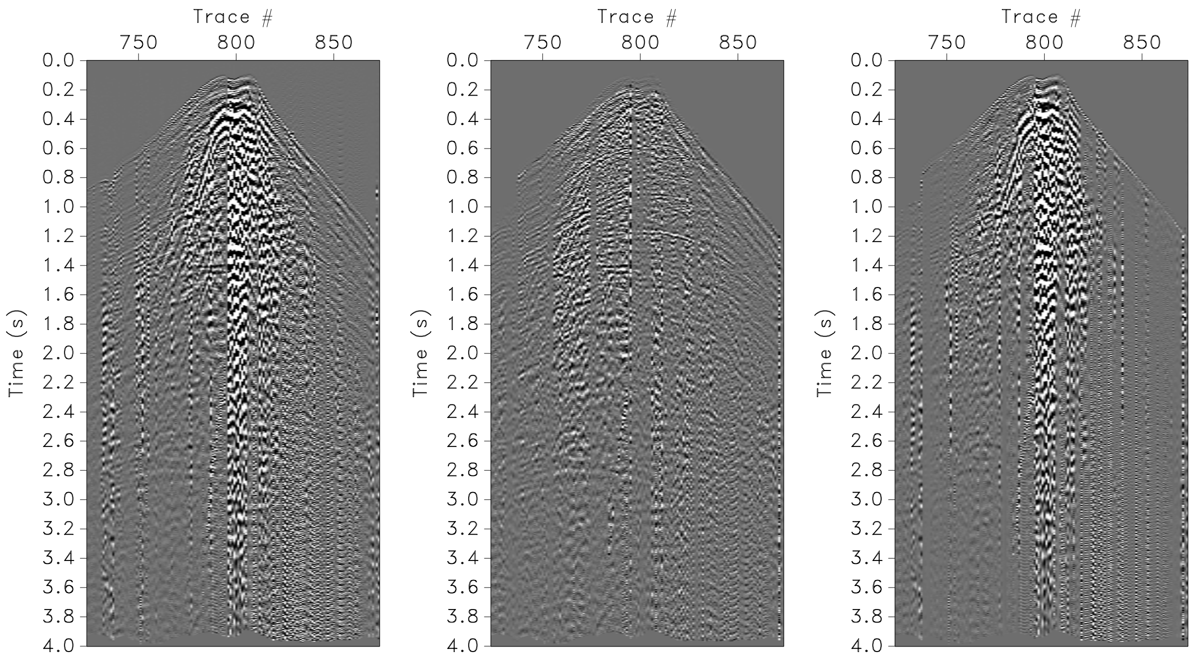
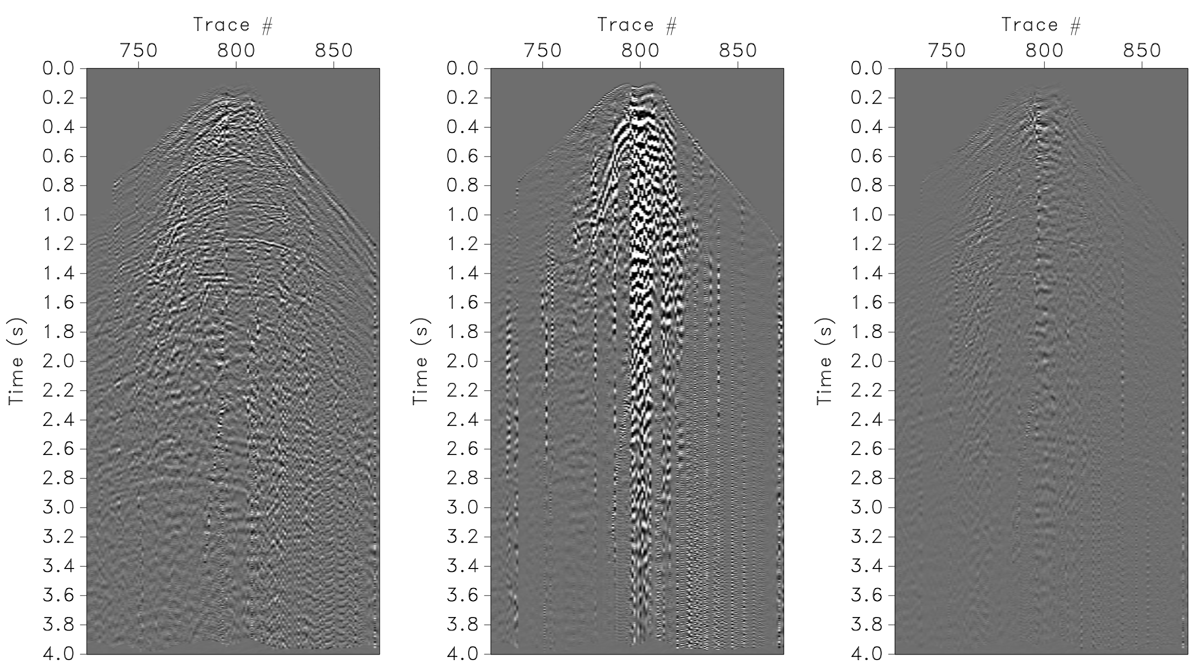



(a) (b) (c)

Figure 1: The real dataset to be improved with the Bayesian separation method (a). This dataset comes from a line that was located near the source thus causing a dominant ground roll signal in the center of the image. The predictions to be used with the Bayesian separation method for the (b) reflectors and (c) surface wave. The reflector prediction is a supplied denoised data set with the surface wave prediction as the difference between the total data and the reflector prediction.




(a) (b) (c)

Figure 2: The Bayesian (a) reflector and (b) surface wave estimates for the real data set after five iterations. These images show improved energy content for both the reflector and surface wave estimates. The difference between the predicted surface wave and the Bayesian estimated surface wave (c). This image shows the energy transferred between predictions through the estimation scheme. As we can see, reflector information, particularly in the top left section of the image and ground roll energy has been transferred back to there appropriate estimates.

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