

Deconvolution with curvelet-domain sparsity

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Outline

- Motivation
- Brief tutorial on “*Curvelets*”
- Brief introduction to “*Sparsity*”
- Curvelet based Deconvolution algorithm
- Results
- Conclusions
- Challenges & Future work
- Acknowledgements
- References

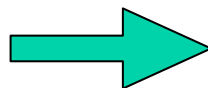
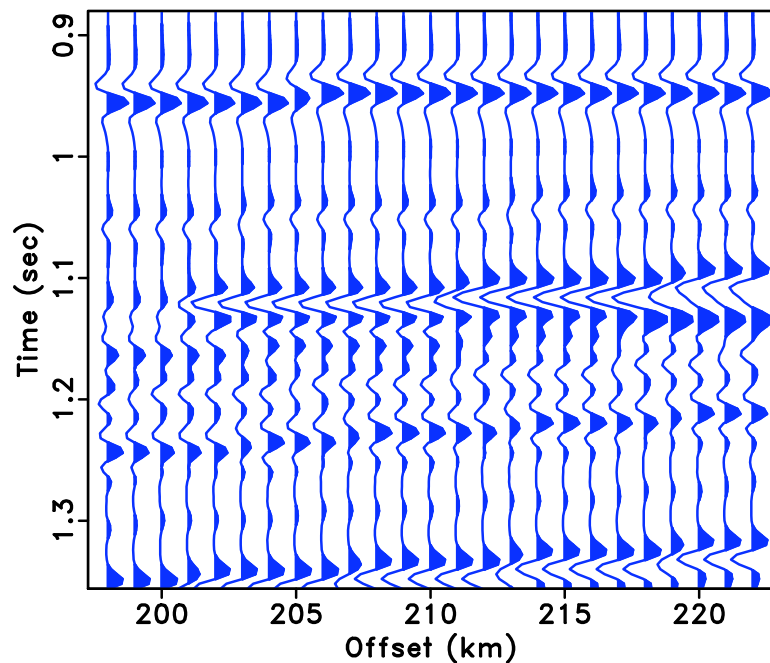
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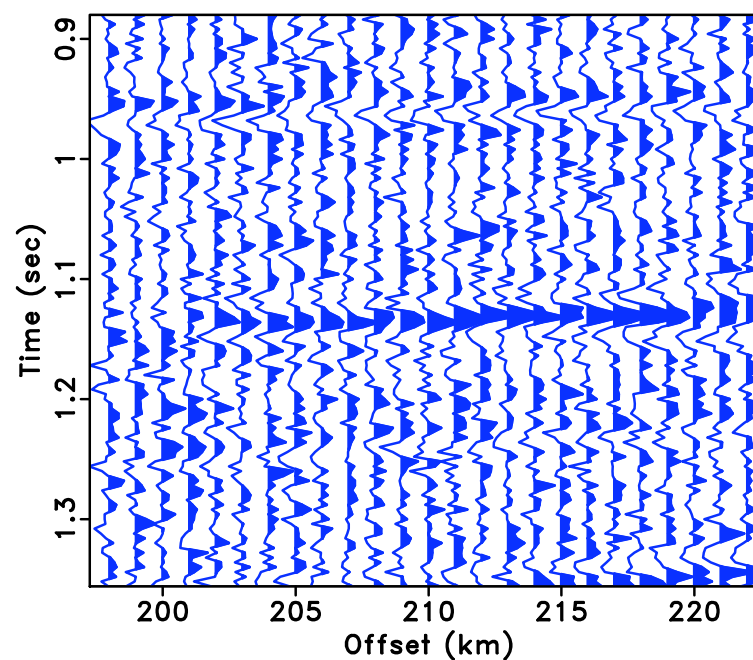
Motivation

- Assumption of spiky reflectivity is too limited to describe seismic reflectivity [Herrmann and Bernabe, 2004], [Herrmann, 2005], [Maysami and Herrmann, 2008]
- There is an inherent continuity along reflectors of a seismic image [Hennenfent et al., 2005]
- Because of the multi-dimensional structure the reflectivity is sparse in curvelet-domain [Hennenfent et al., 2005]

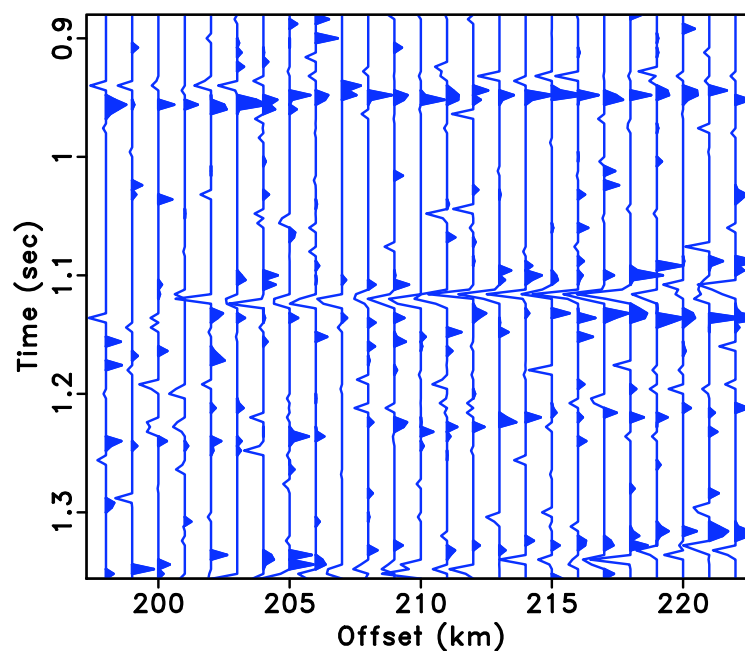
Non-spiky model



Data



Sparse spike deconvolution

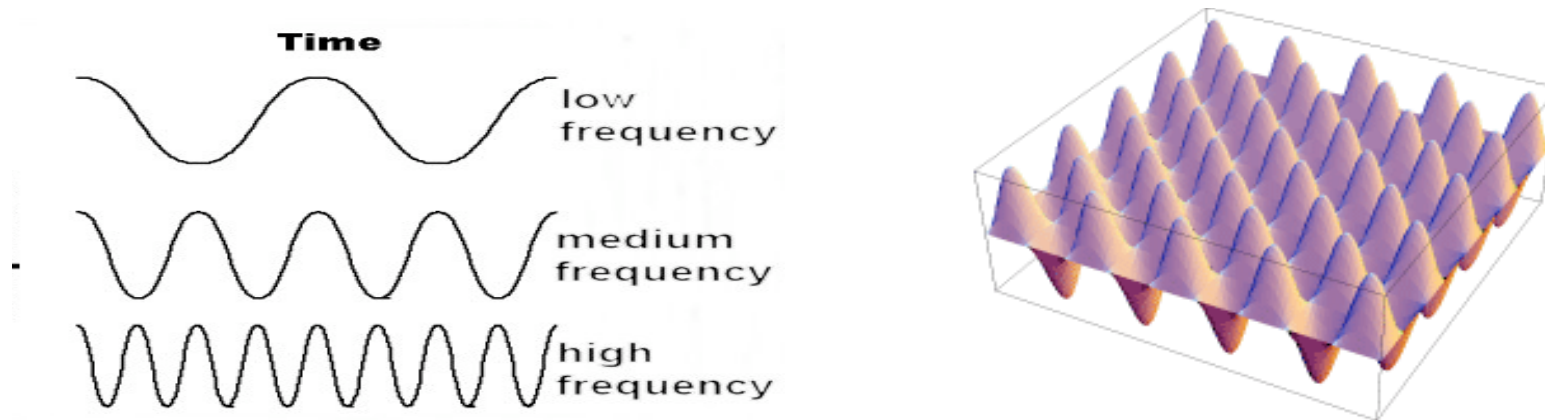


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Lets start from very basic
concepts of trasforms!!

Fourier Transform



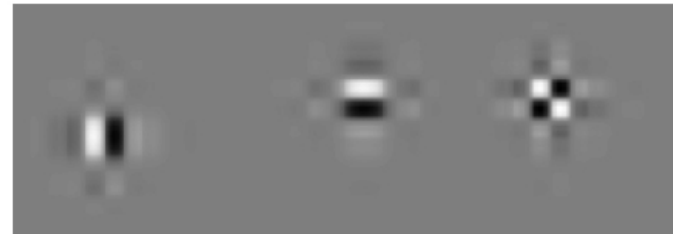
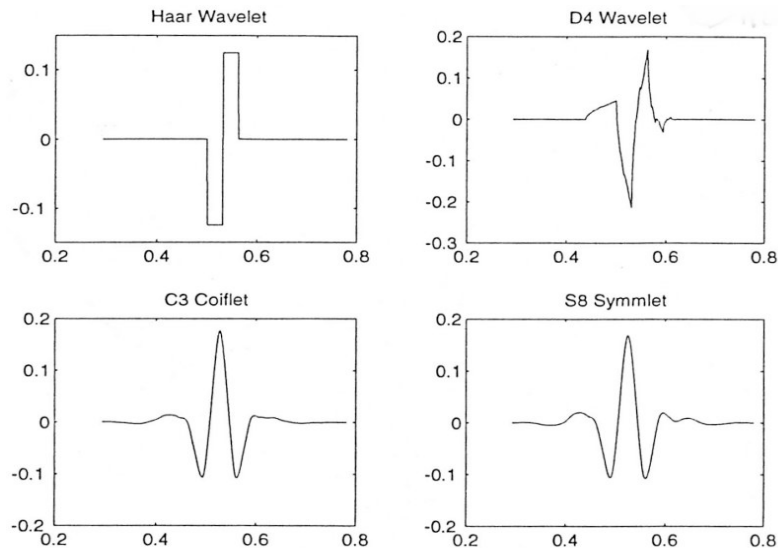
Decomposes into sinusoids / monochromatic plane waves
(global)

$$d = \sum_i \lambda_i e^{-j\omega t}$$

$$d = \sum_i \lambda_i \psi(\text{frequency, phase})$$

$$d = F^* x$$

Wavelet transform

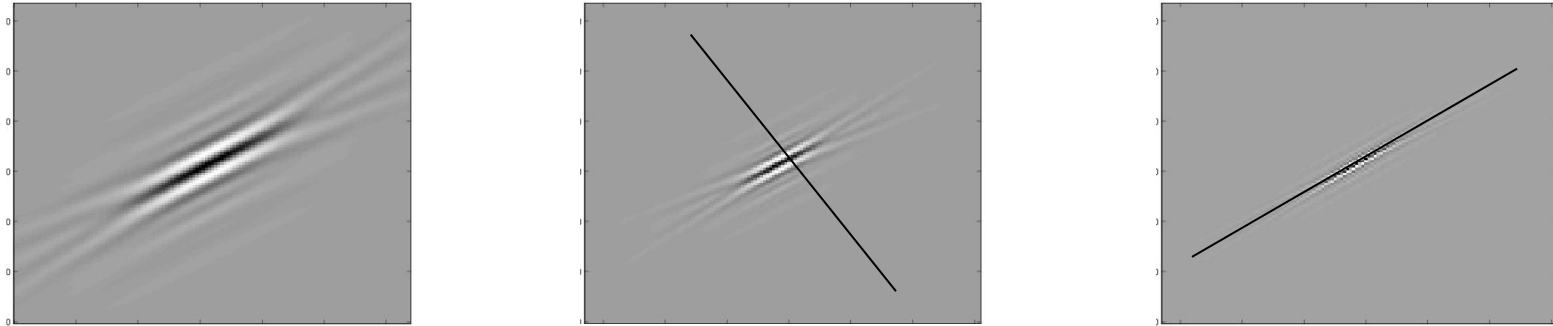


Decomposes into wavelets (local)

$$d = \sum_i \lambda_i \psi(\text{position}, \text{frequency} - \text{bandwidth})$$

$$d = W^T x$$

Curvelet transform



[Figures adapted from www.curvelet.org]

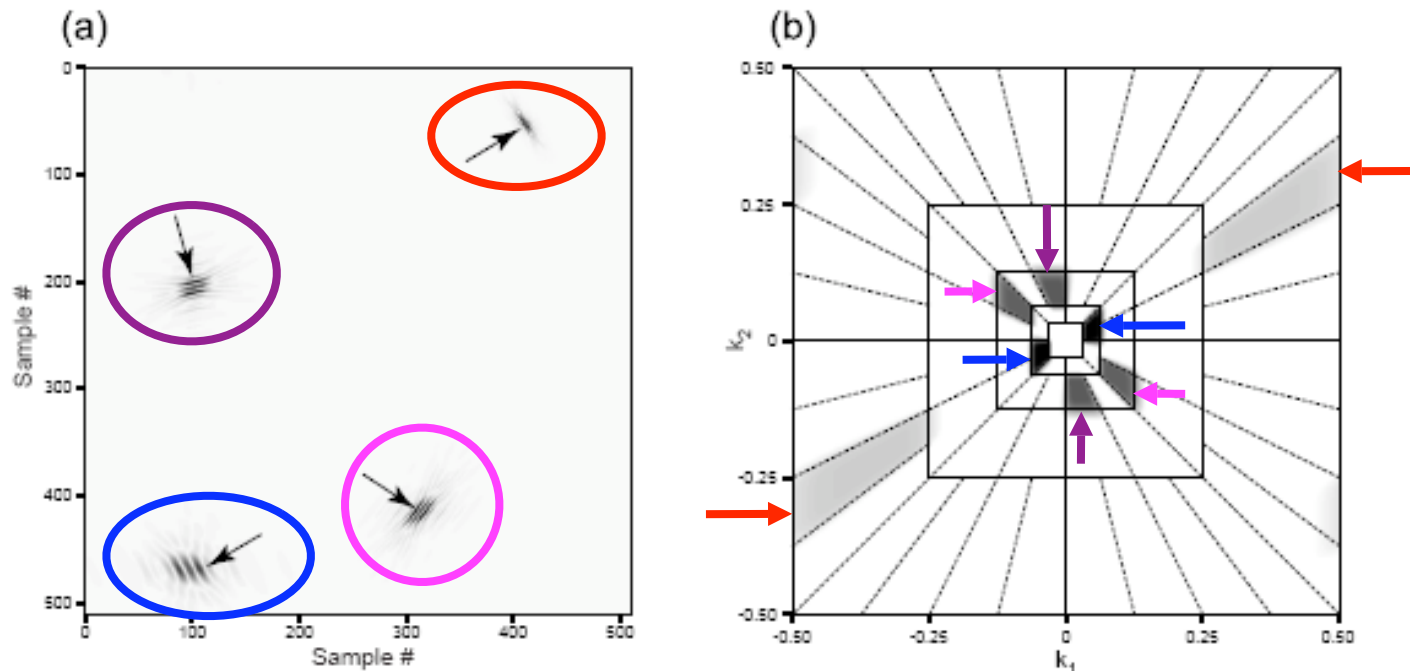
Decomposes into localized plane waves (local)

$$d = \sum_i \lambda_i \psi(\text{position}, \text{frequency}, \text{angle})$$

$$d = C^T x$$

Curvelets are oscillatory in one direction and smooth in other perpendicular direction.

A few curvelets



Spatial

F-K

[Figure adapted from Herrmann et al., 2008]

Properties:

Multi-scale: tiling of the FK domain into dyadic coronae

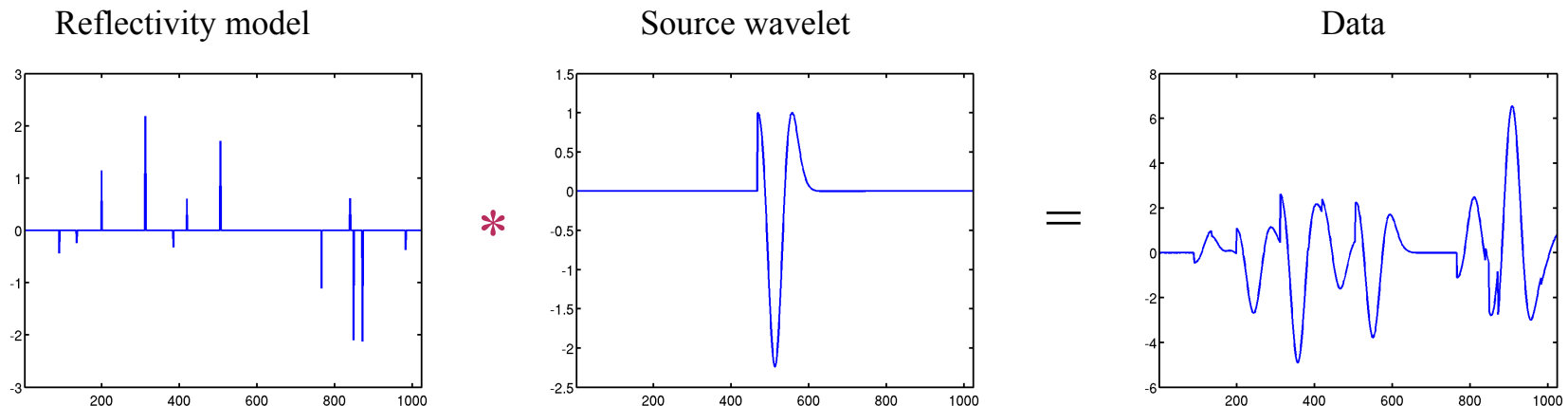
Multi-directional: coronae sub-partitioned into angular wedges, # of angle doubles every other scale

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The concept of sparsity

The idea started with sparse spike deconvolution !!

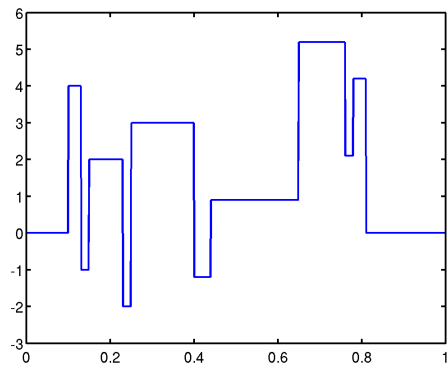


The reflectivity is sparse in time domain.

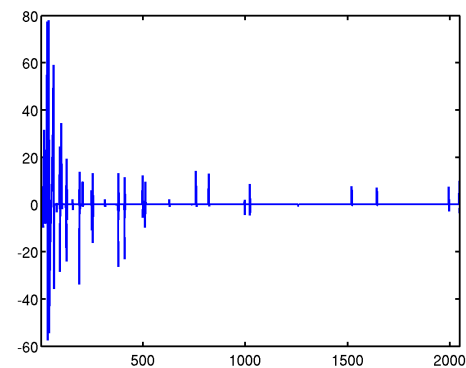
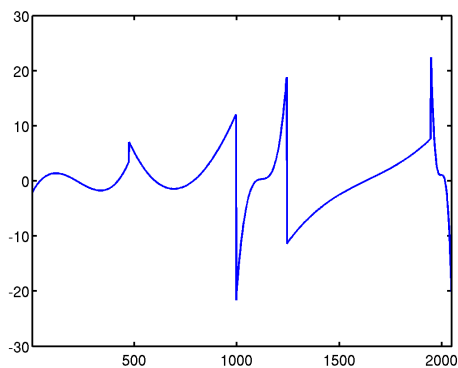
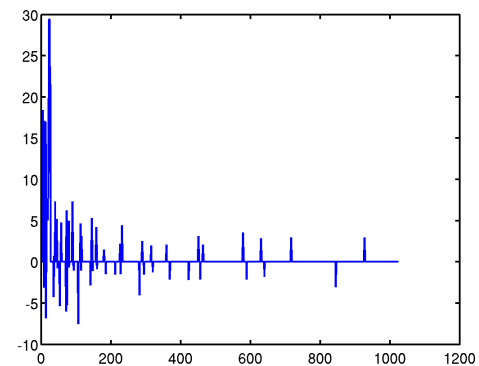
Thus we enforce sparsity of reflectivity during deconvolution.

Wavelet sparsity

Model

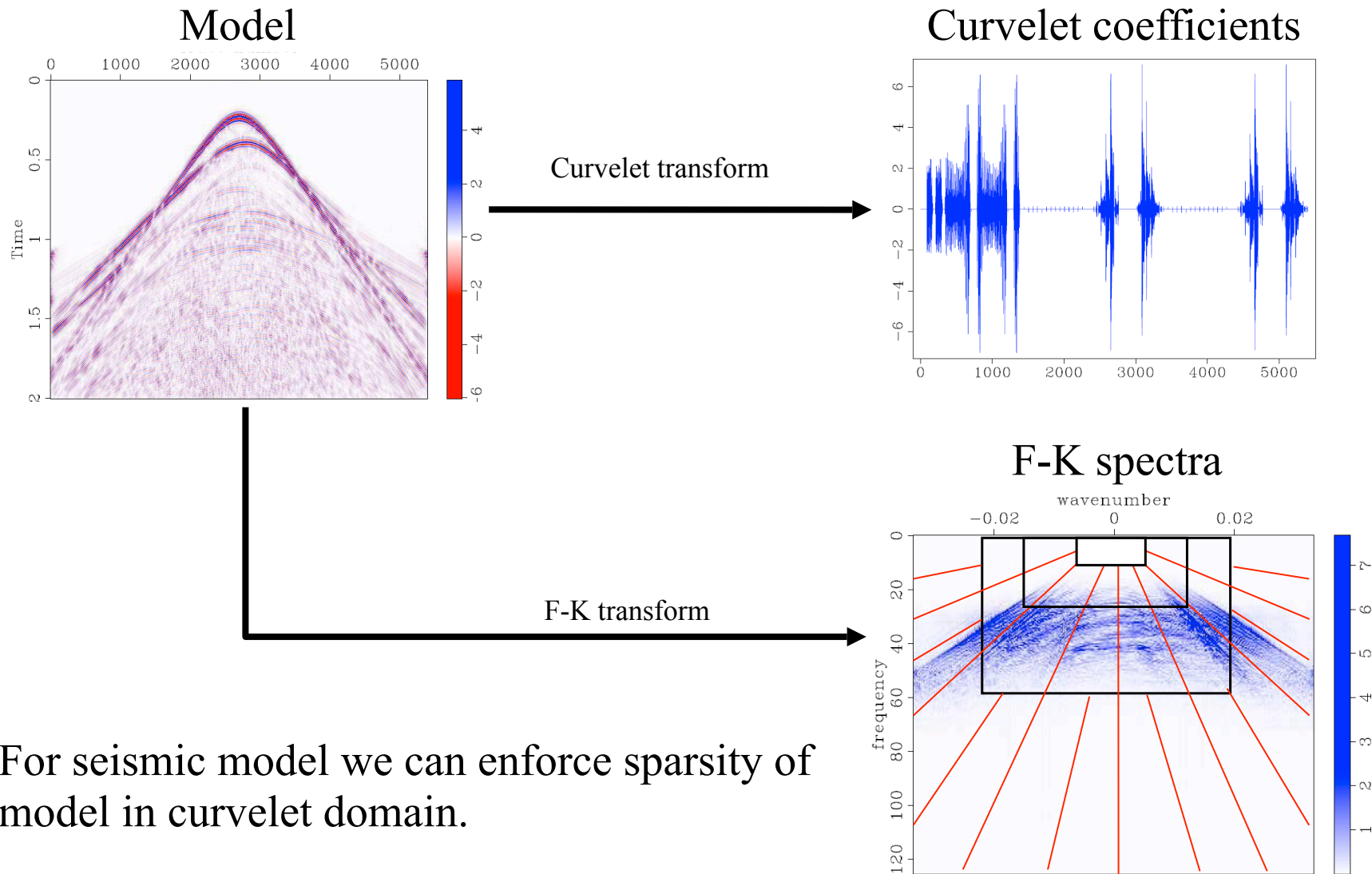


Wavelet coefficients



For these models we can enforce the sparsity of model in wavelet domain.

Curvelet sparsity



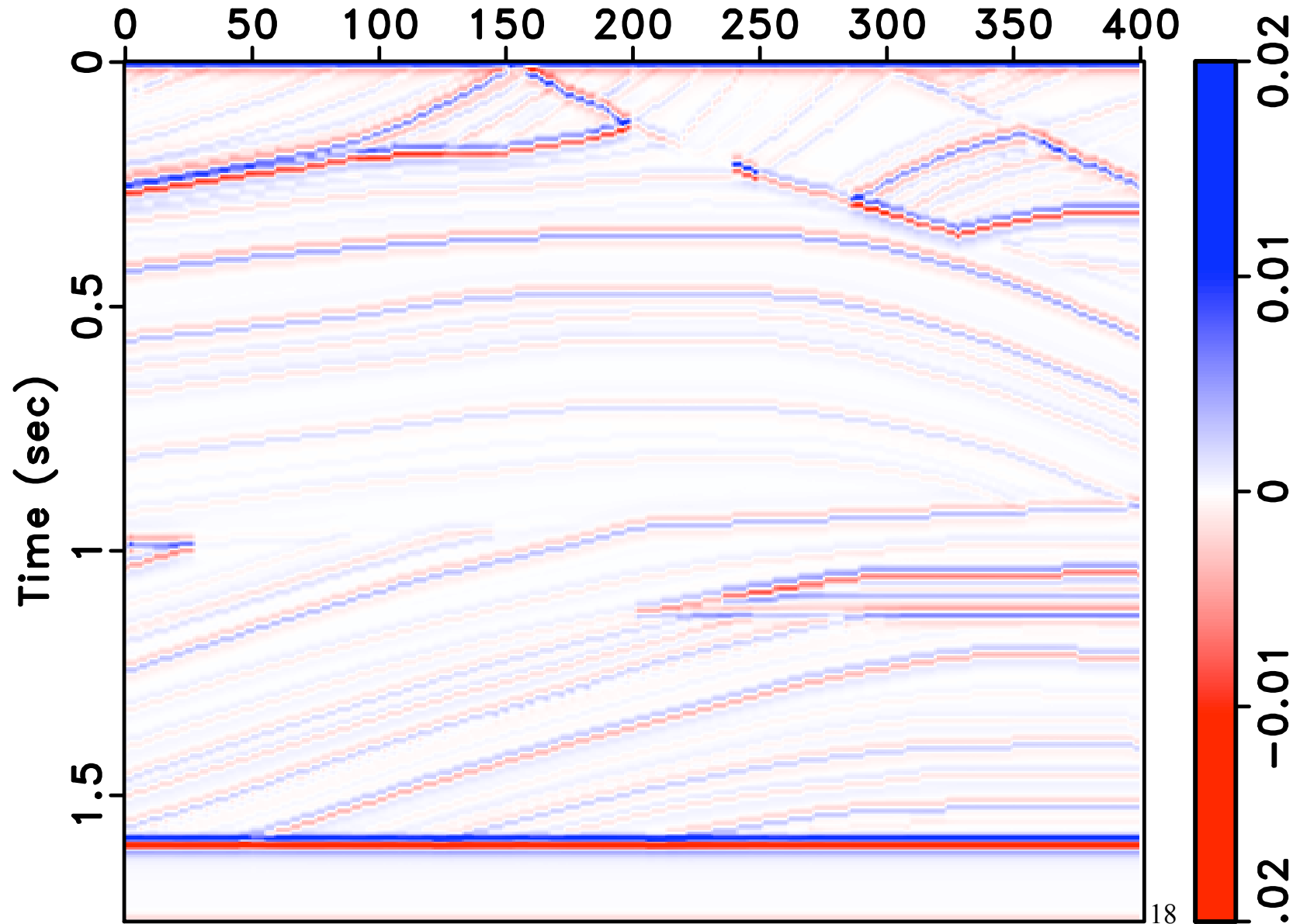
For seismic model we can enforce sparsity of model in curvelet domain.

Sparsifying Transforms

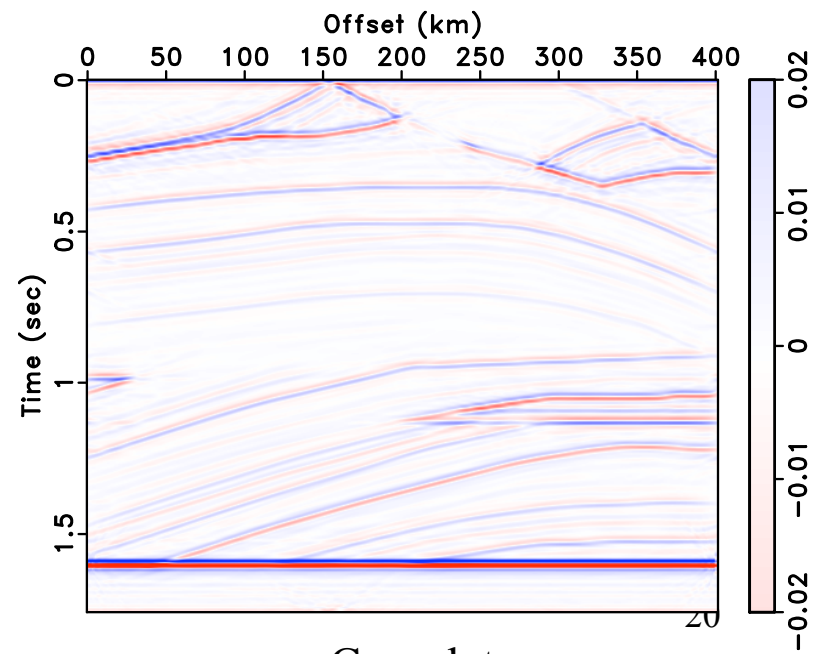
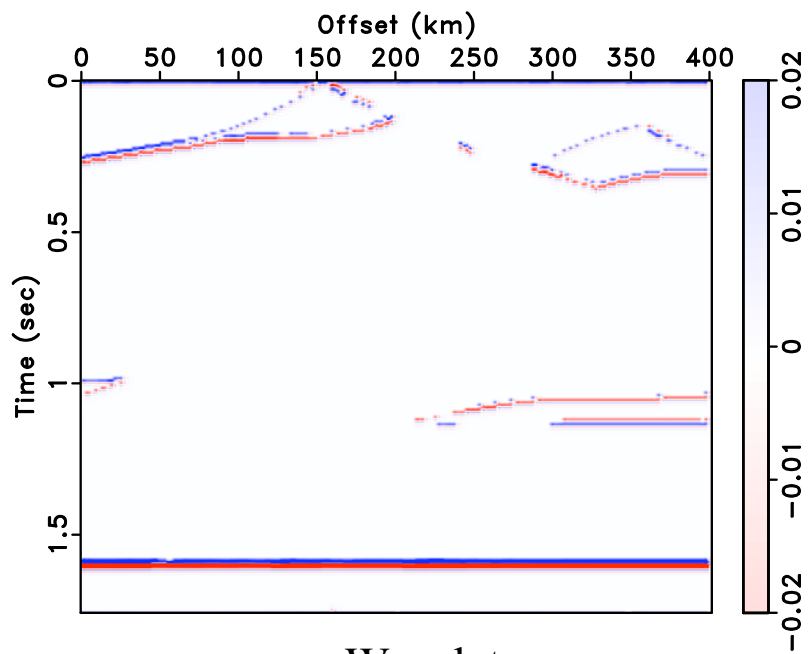
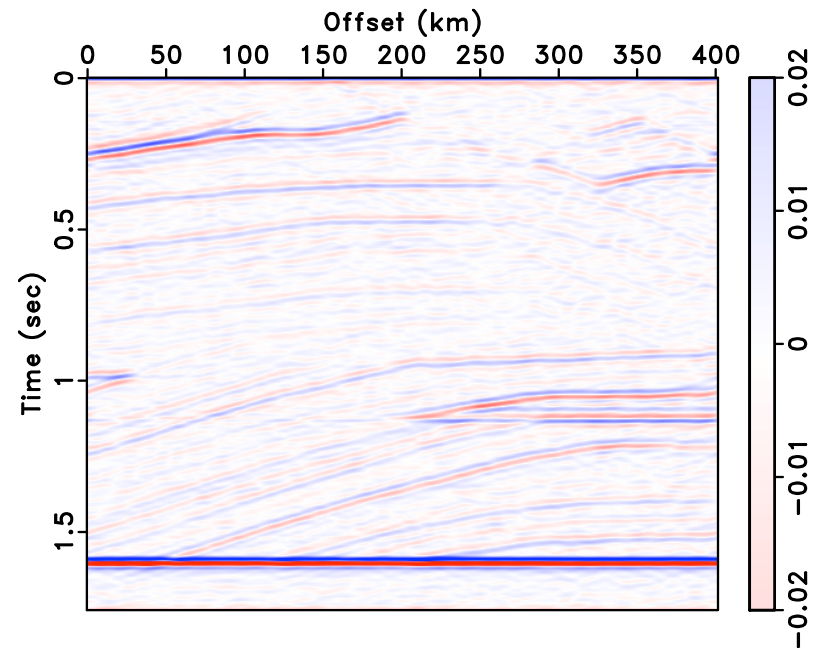
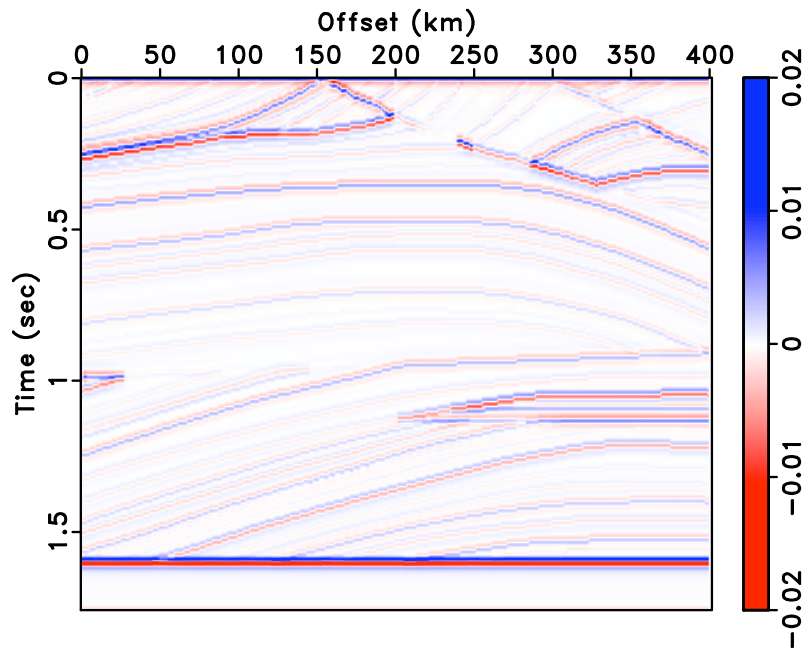
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	curve-like events (2D singularities)

[courtesy G. Hennenfent]

Reflectivity Model



Now we reconstruct
Reflectivity Model with 1% largest
amplitude sorted coefficients in different
transform domains



Seismic reflectivity is sparse in curvelet domain !!!!

We will enforce this sparsity in our algorithm.

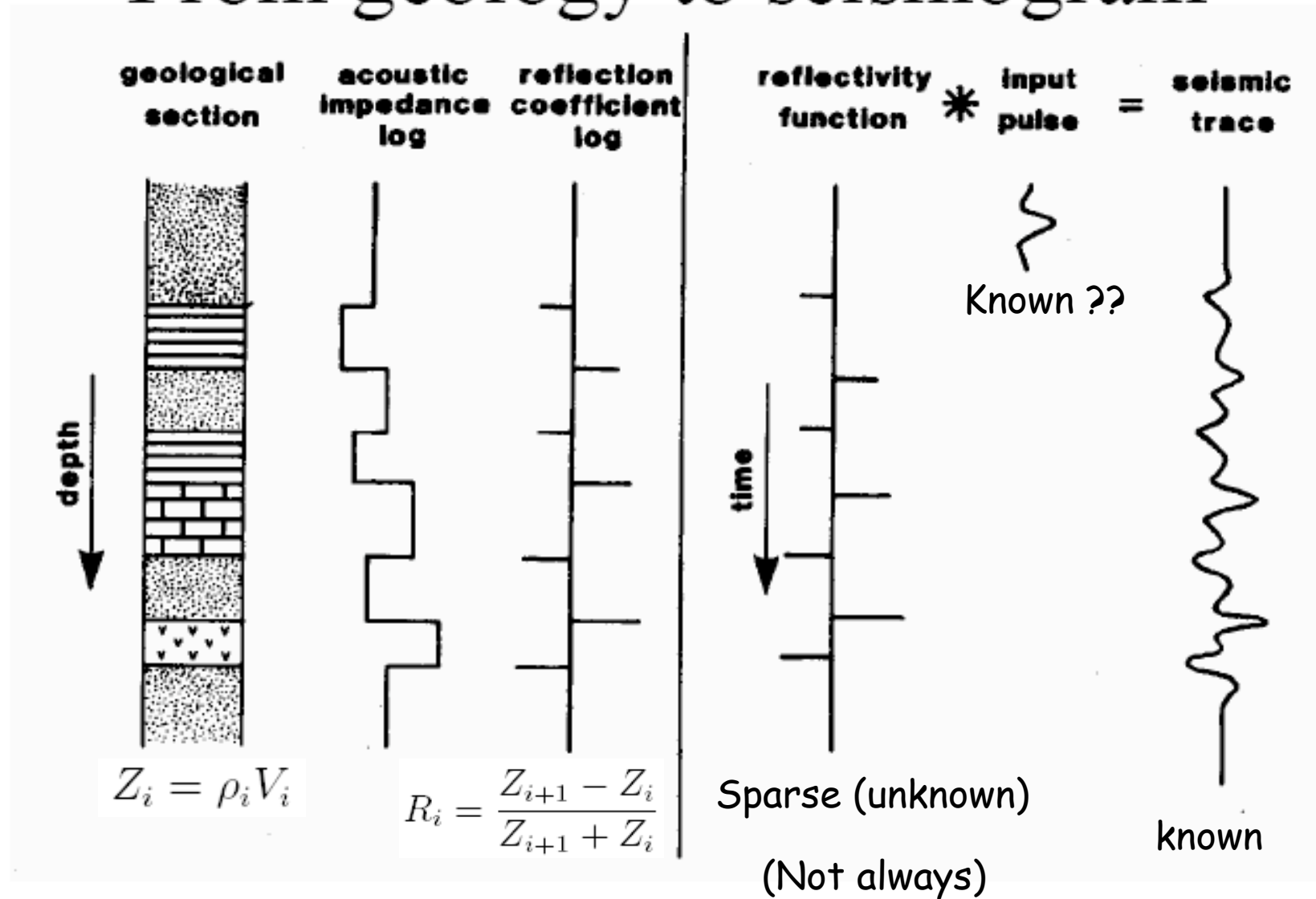
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Traditional methods

- Least squares deconvolution
- Frequency domain Weiner method
- Sparse spike deconvolution

From geology to seismogram



The Forward Problem

$$\mathbf{y} = \mathbf{A}\mathbf{m} + \mathbf{n}$$

$\mathbf{y} \rightarrow$ Noisy Signal

$\mathbf{m} \rightarrow$ True signal

$\mathbf{n} \rightarrow$ Noise

$\mathbf{A} \rightarrow$ Convolution Operator

The Inversion Approach

Enforce the sparsity in the curvelet domain (get me curvelet like reflectivity).

$$\begin{aligned} \min_x \quad & \|\mathbf{x}\|_1 \\ \text{s.t.} \quad & \|\mathbf{y} - \mathbf{A}\mathbf{C}^T \mathbf{x}\|_2 \leq \epsilon \\ \hat{\mathbf{m}} = \mathbf{C}^T \hat{\mathbf{x}} \end{aligned}$$

Obey data consistency (estimated data-actual data \leq noise)

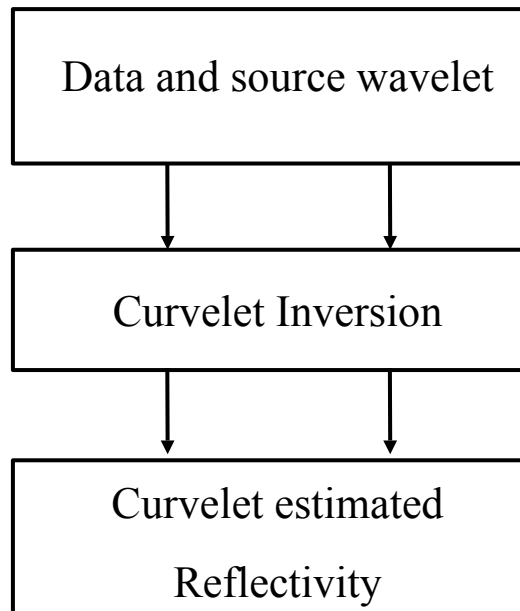
$\mathbf{C}^T \rightarrow$ Curvelet Synthesis Operator

$\mathbf{A} \rightarrow$ Convolution Operator

$$\epsilon^2 \approx \sigma^2 [N + \sqrt{2N}] \quad (\chi^2\text{-misfit})$$

Inversion approach

Using the source wavelet and data we try to find a reflectivity model which is:



(a) sparse in curvelet domain.

(b) obeys data consistency.

Data & Model

- Marmousi model is half differentiated in the frequency domain to obtain non-spiky reflectivity.
- Reflectivity is convolved with Ricker wavelet (central frequency ~30 Hz)
- Noise (standard deviation=.05) is added.

Our approach

- Inversion Operators are formed in SPARCO [Van den Berg et. al]
- The solution is found by SPGL₁ which requires less number of matrix-vector multiplication [van den Berg and Friedlander, 2008; Hennenfent et al., 2008]
- The noise level was estimated by Chi-square misfit criteria [Candes et al., 2005]
- The results were compared with Sparse spike deconvolution (C= I)

$$\min_m \|m\|_1 \quad \text{s.t.} \quad \|y - Am\|_2 \leq \varepsilon$$

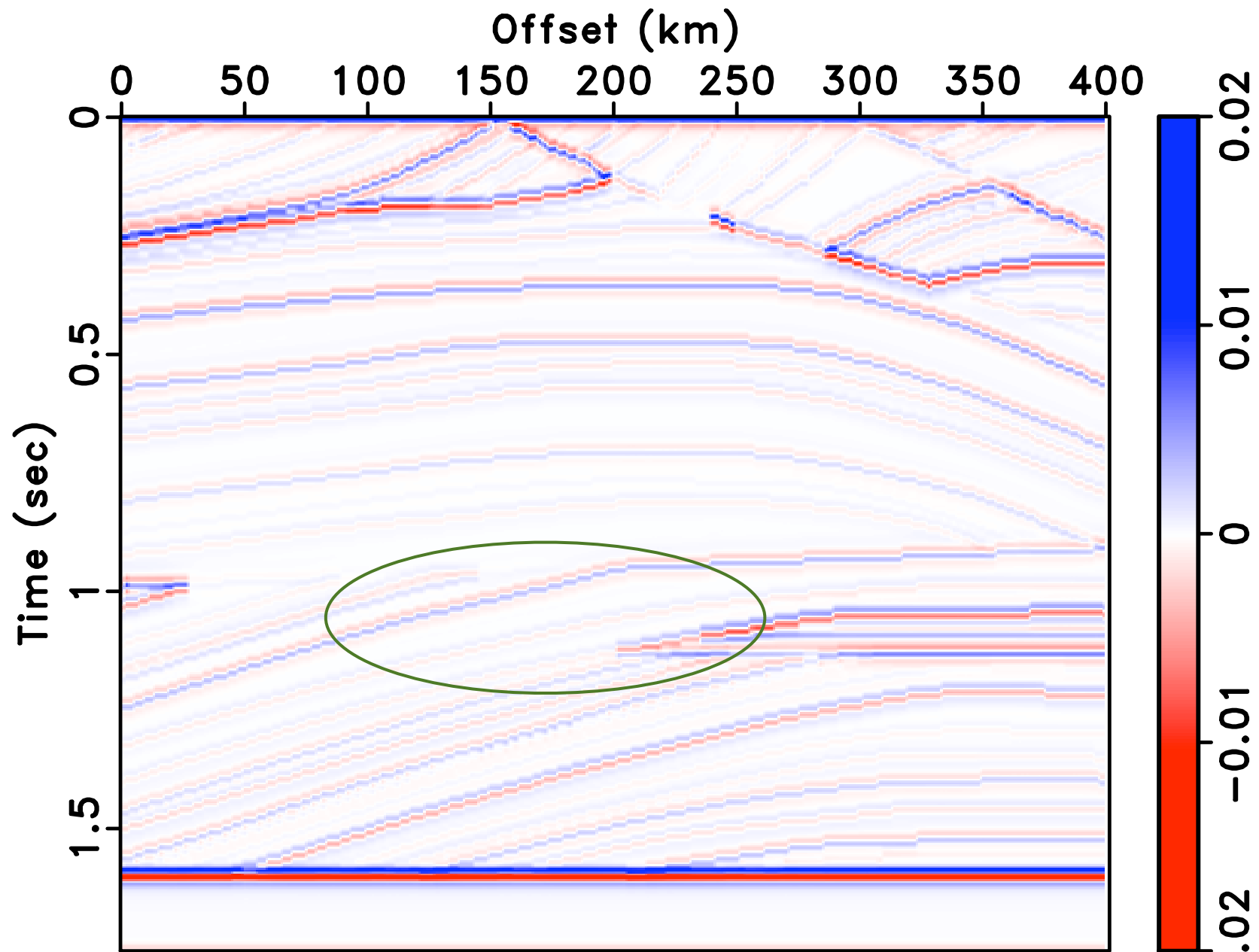
[Taylor et al., 1979; Oldenburg et al., 1981]

- SNR is computed by:

$$20 \log_{10} \frac{\| \text{reconstructed model} \|_2}{\| \text{noise} \|_2}$$

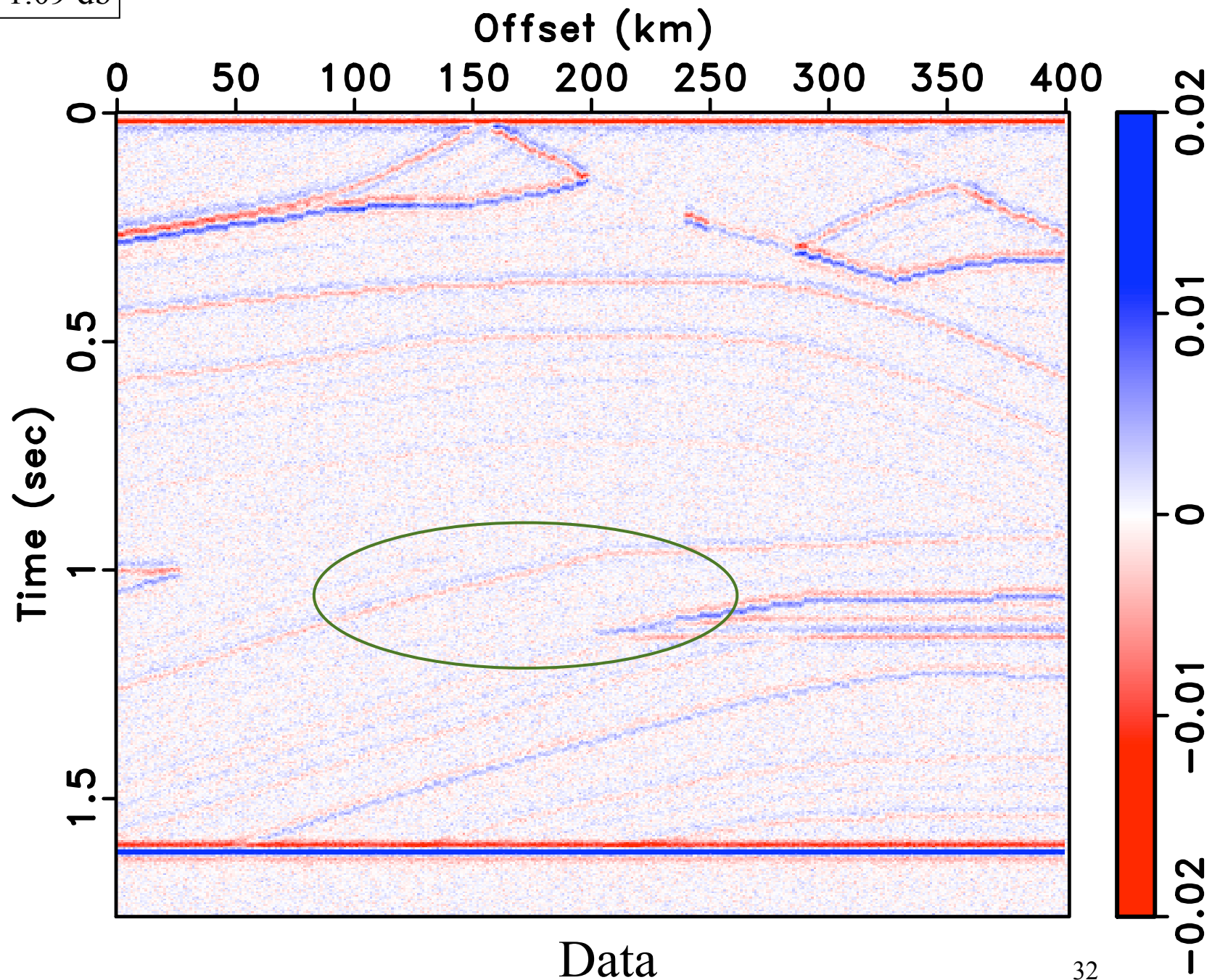
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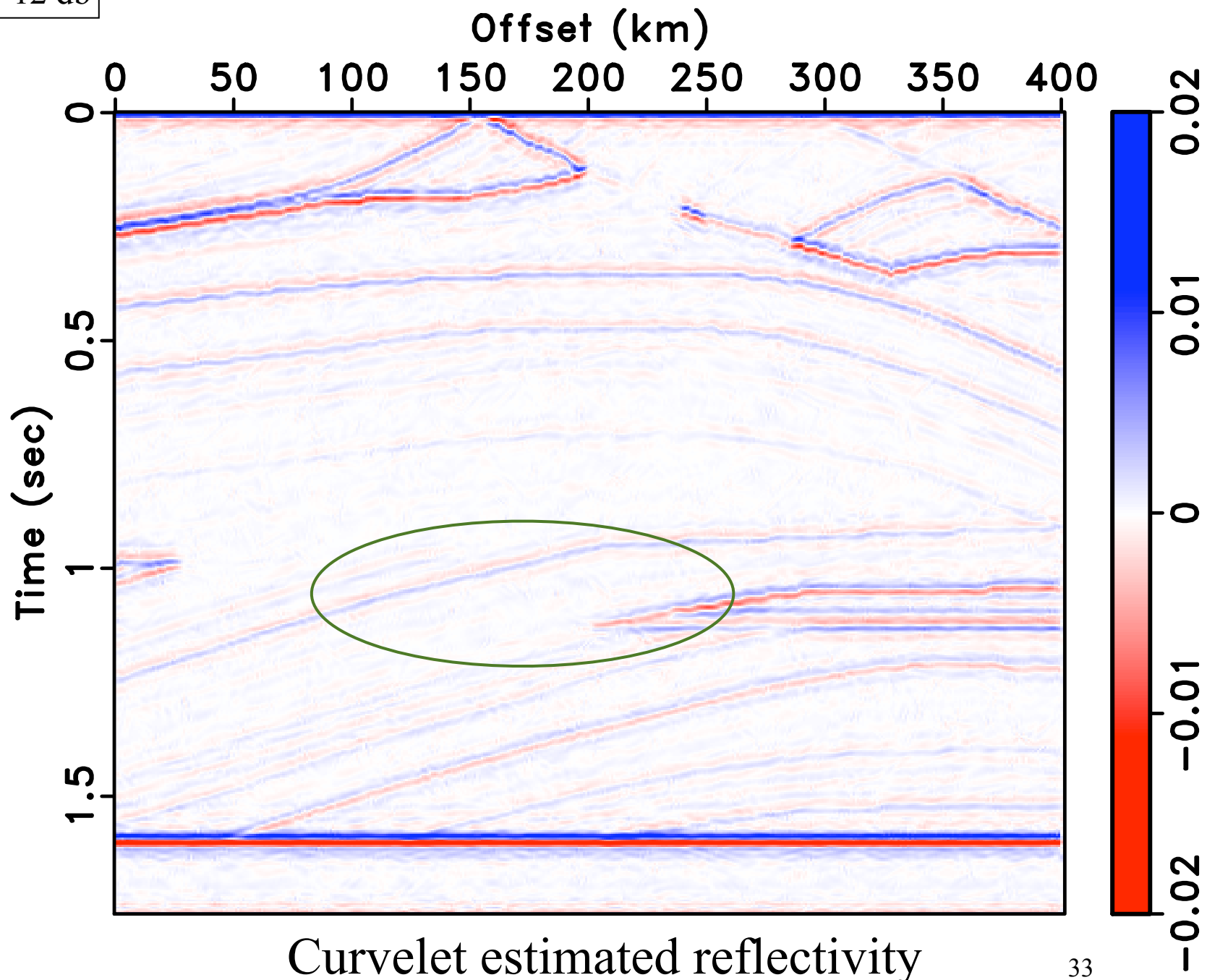


Original reflectivity

SNR ~ -1.09 db

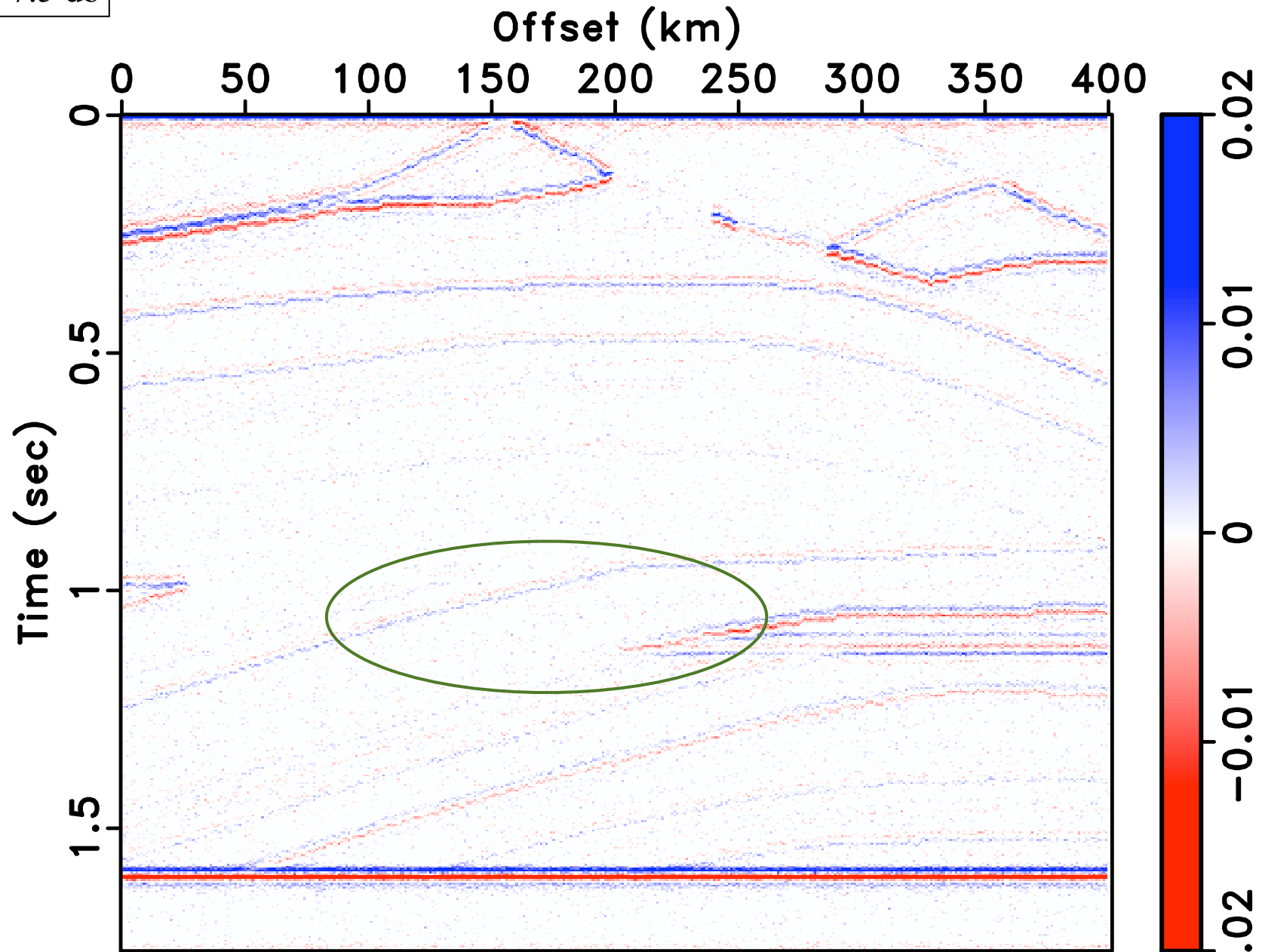


SNR~ 12 db

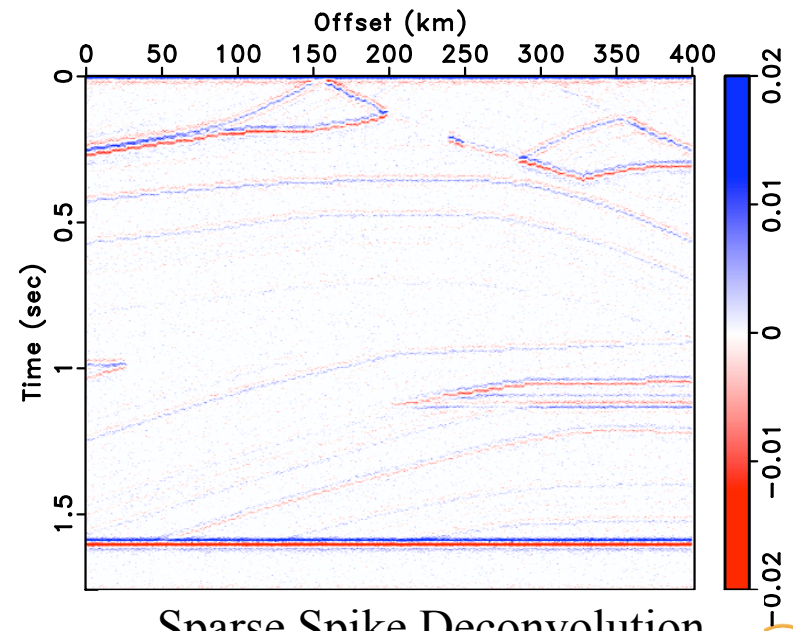
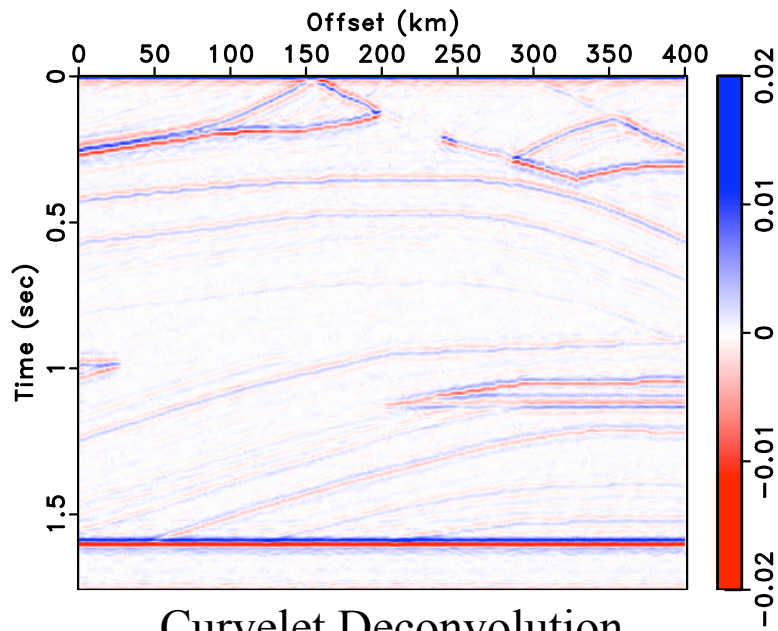
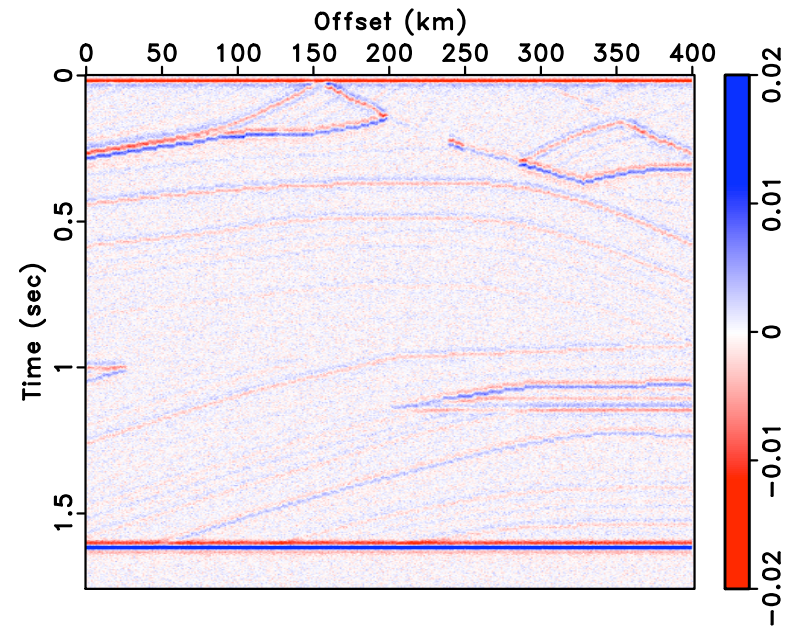
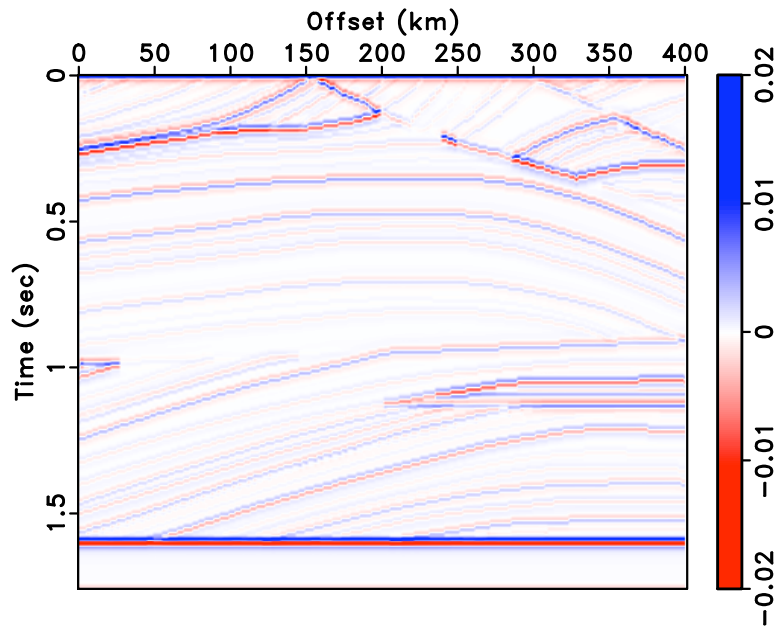


Curvelet estimated reflectivity

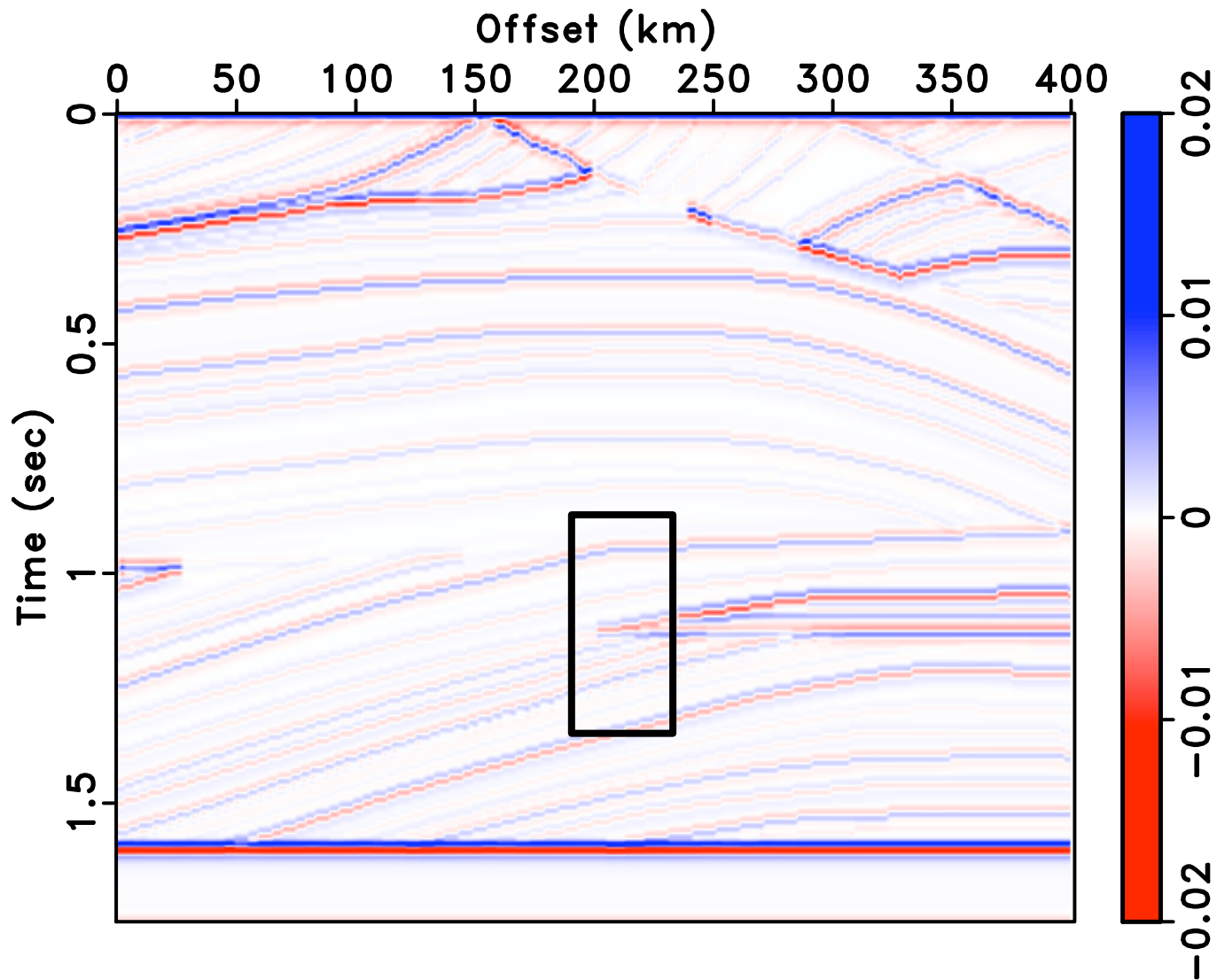
SNR~ 7.5 db



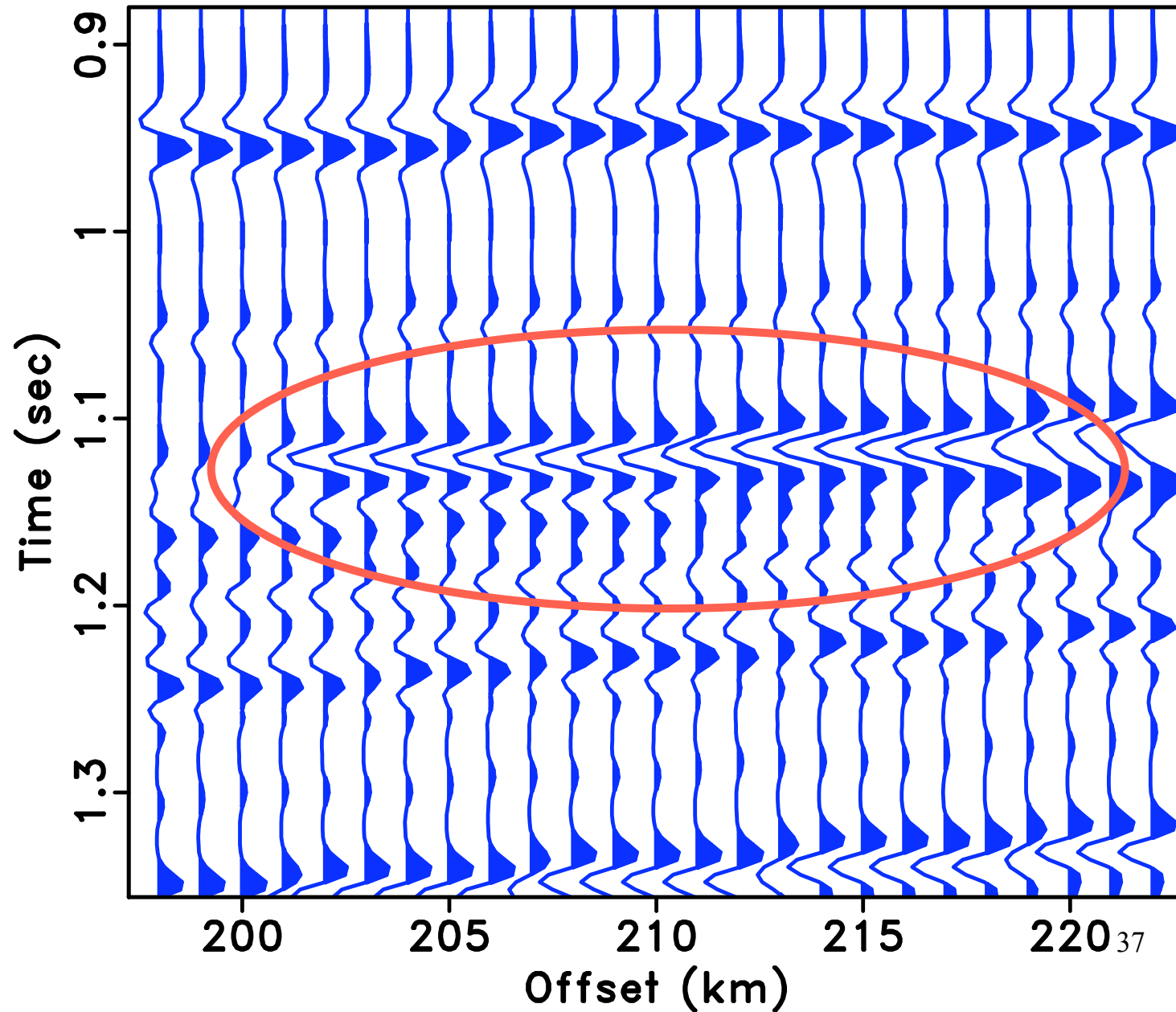
Sparse spike estimated reflectivity



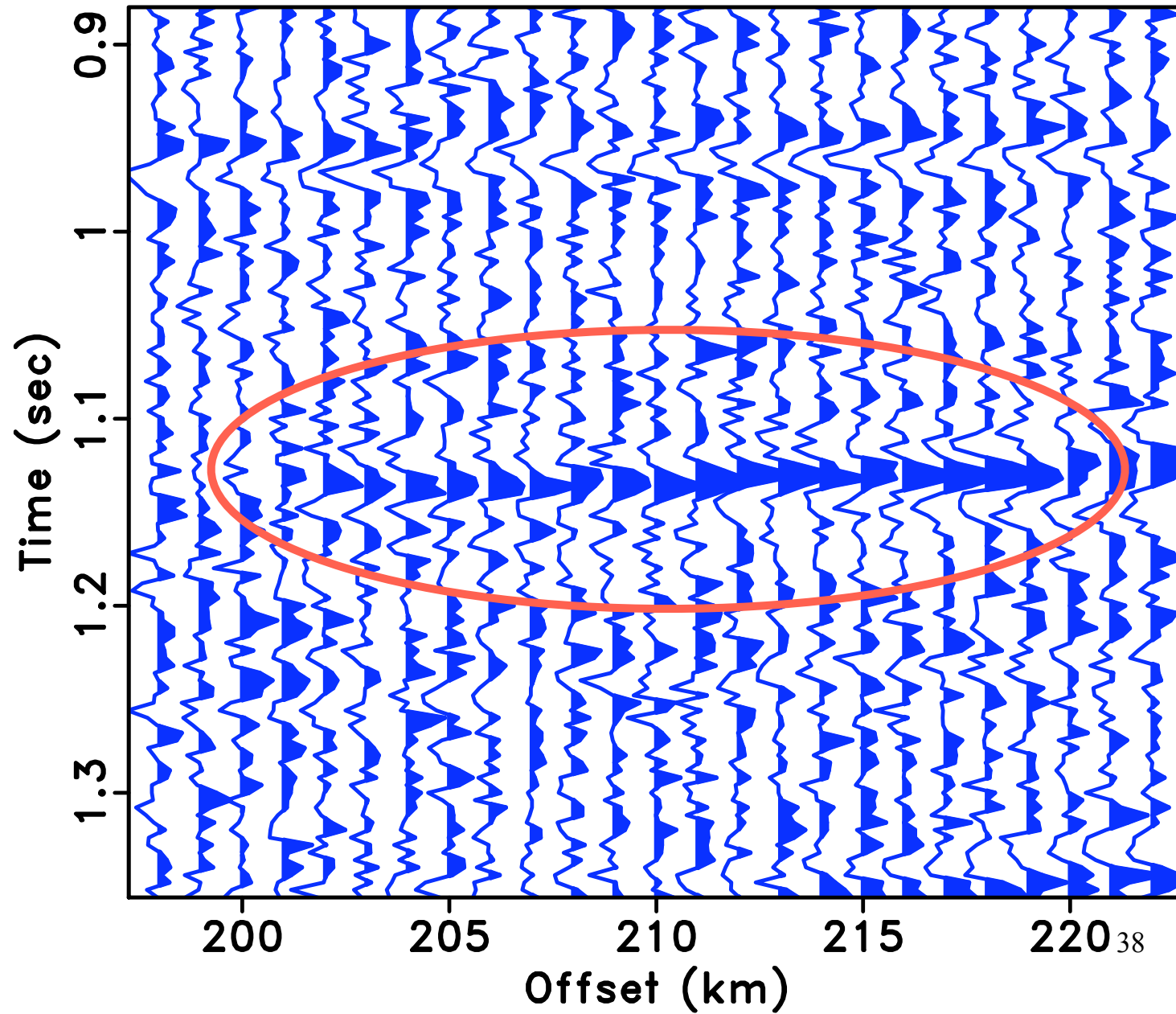
Now we zoom-in the reservoir region



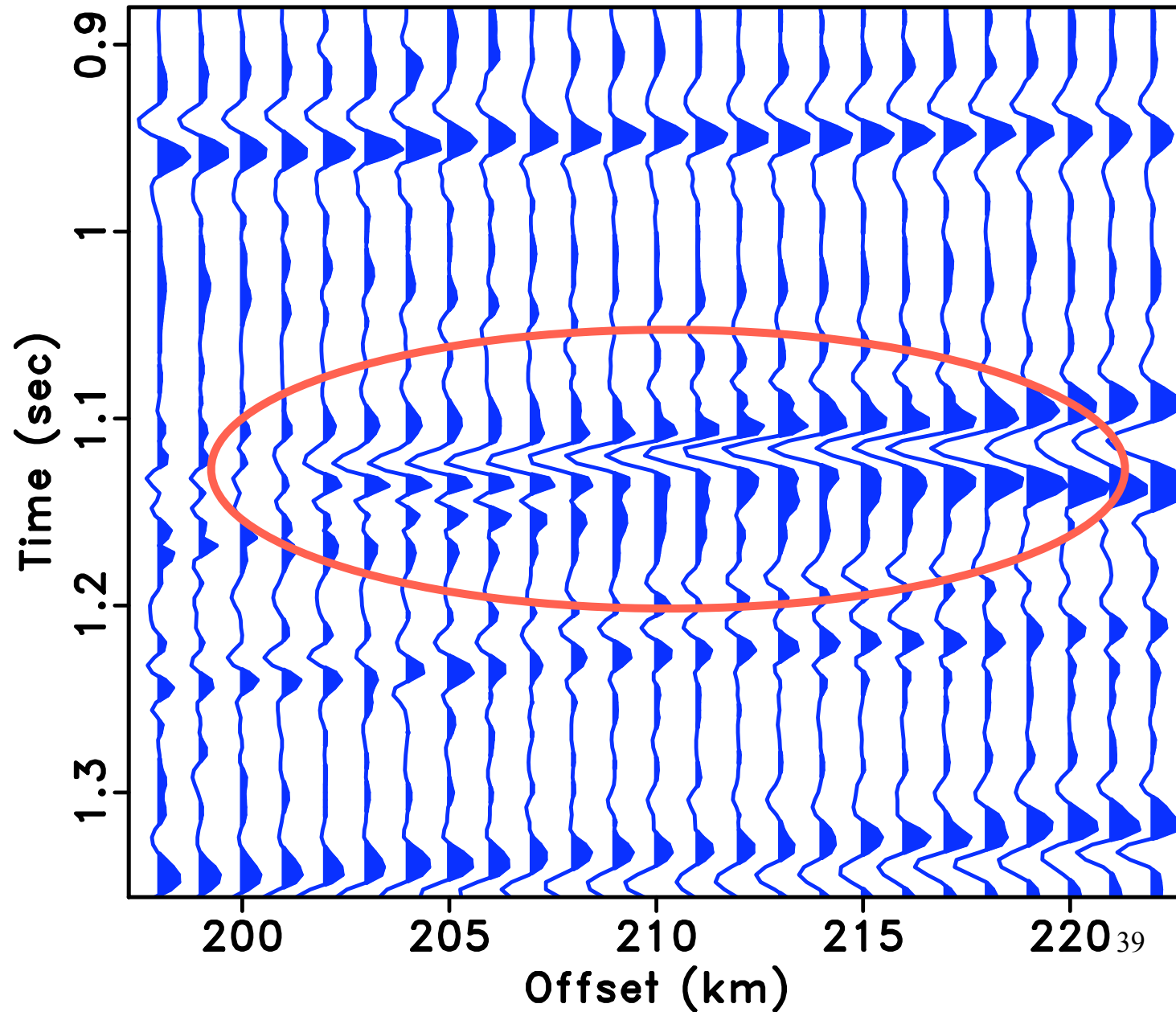
Original Reflectivity



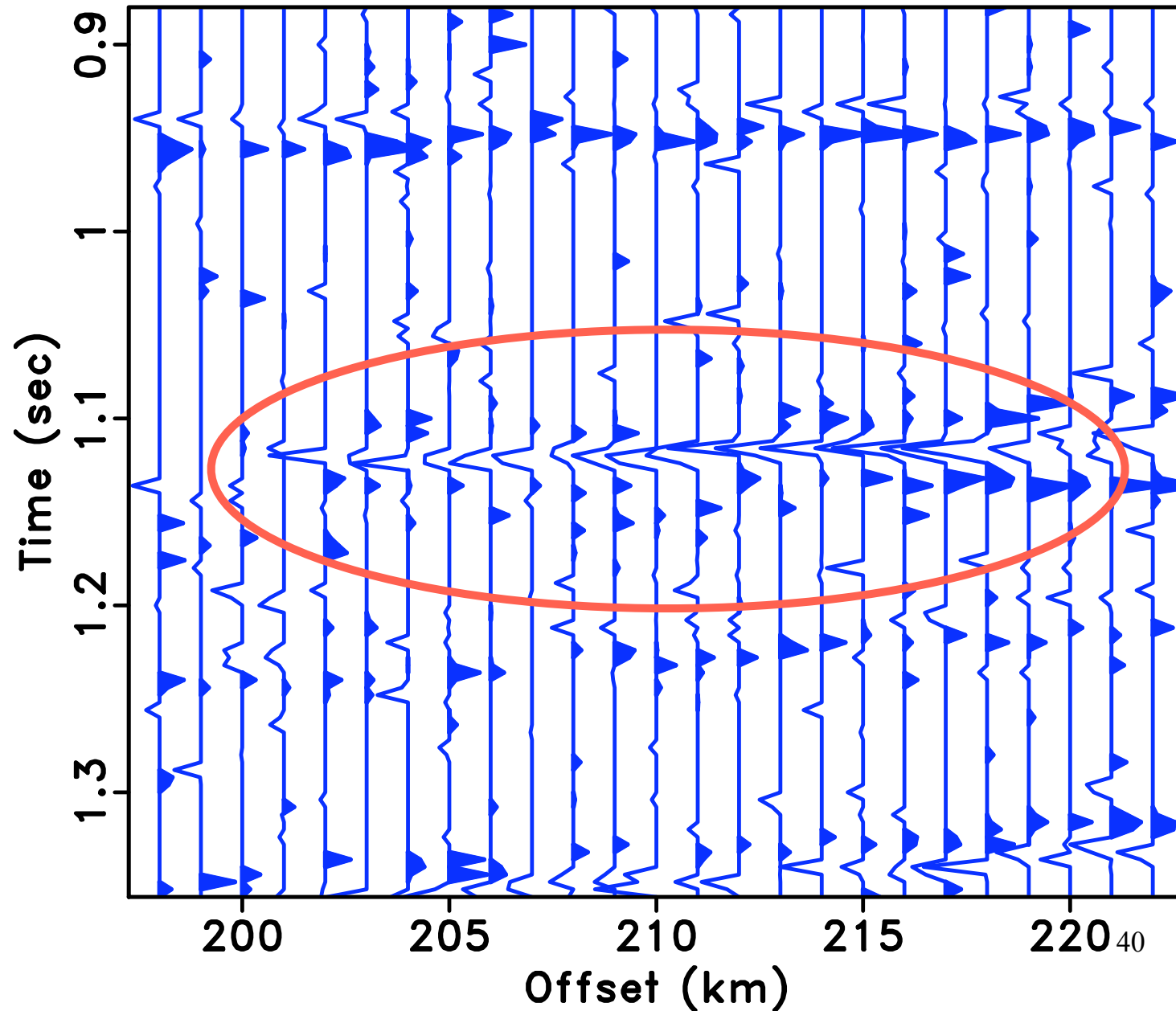
Data



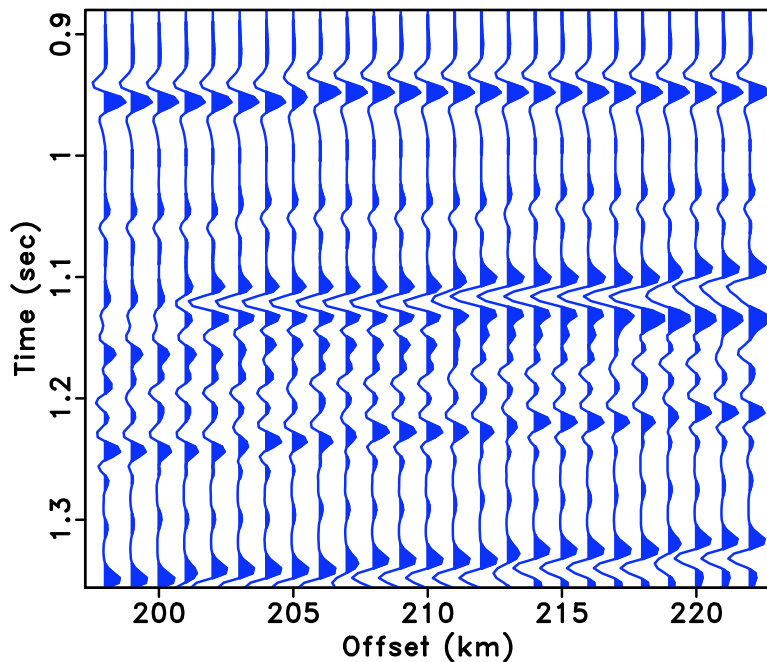
Curvelet Deconvolution



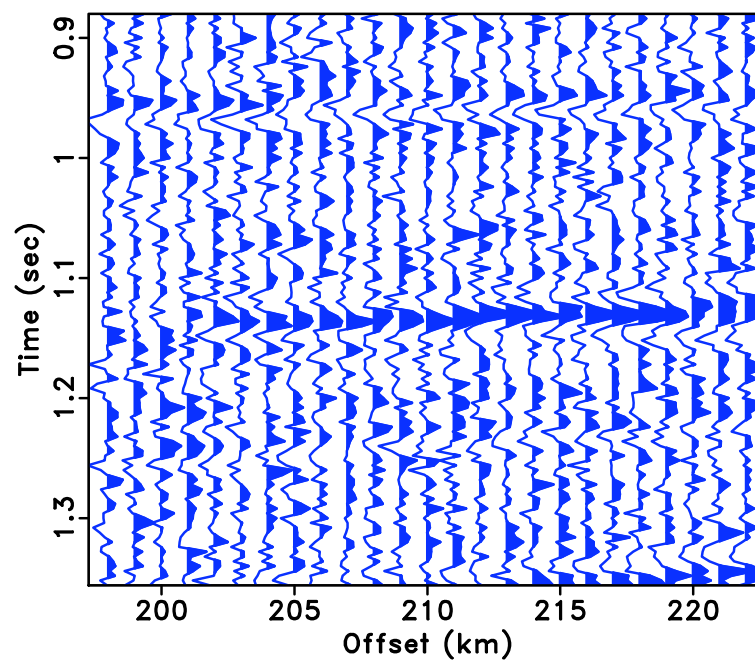
Sparse spike Deconvolution



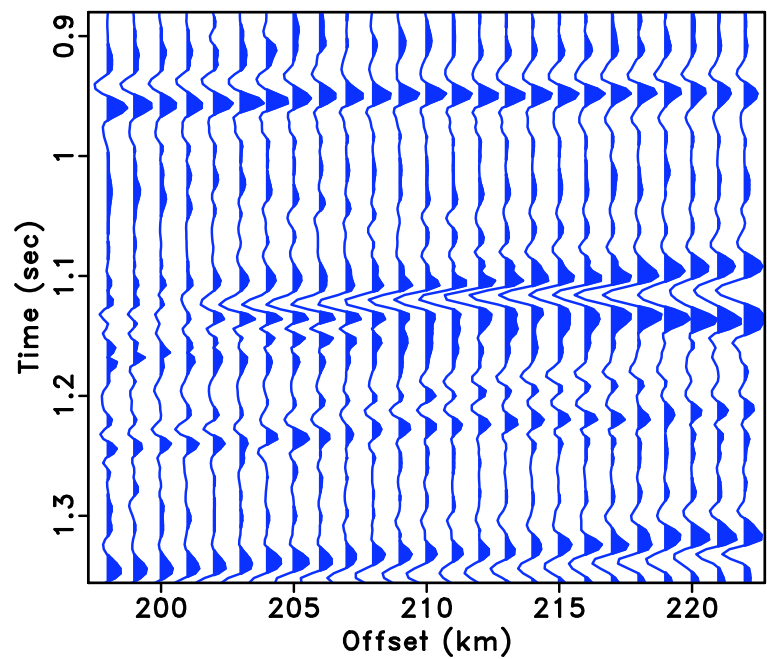
Original Model



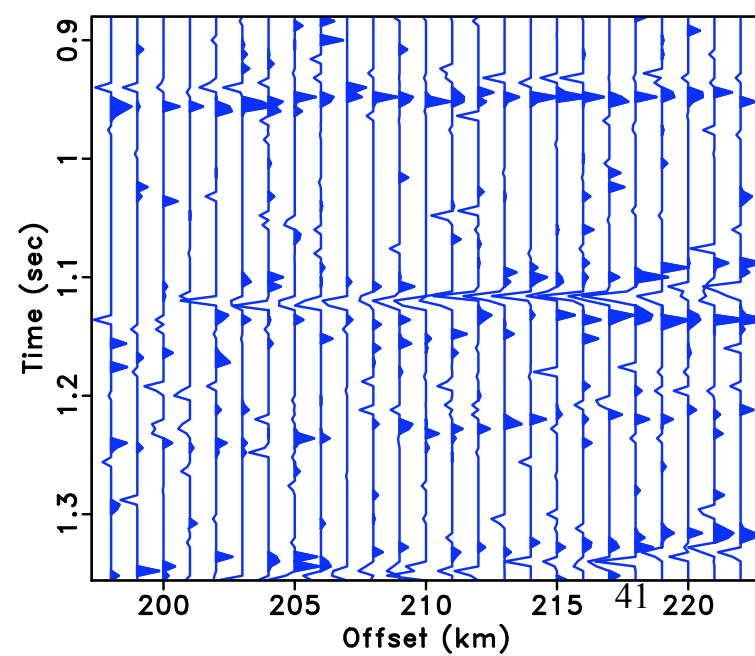
Data



Curvelet Deconvolution



Sparse Spike Deconvolution



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Conclusions

- ◆ We presented a method to deal with Non-spiky reflectivity.
- ◆ The method uses the multi-dimensional structure of earth model (reflectivity) as opposed to trace by trace deconvolution.
- ◆ The use of curvelet domain for seismic signal has few major breakthroughs and the preliminary results for Deconvolution approach looks encouraging.
- ◆ Curvelet based deconvolution approach would be an addition to the tool-kit for seismic data processors which may be advantageous for certain situations.

Challenges & Future work

- ◆ Curvelets are computationally expensive as they are “*redundant*” in nature (8 times in 2D and 24 times in 3D), hence require large memory storage and extra run time thus a parallel version is most favorable.
- ◆ The method can be extended to three dimensions to estimate “*3D reflectivity*” which has more structure.
- ◆ We can solve “*blind deconvolution*” problem and “*wavelet estimation*” for non spiky reflectivity.

Acknowledgements

- ◆ SEG for the Foundation scholarship
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- ◆ S. Fomel, P. Sava, and the other developers of Madagascar (rsf.sourceforge.net)
- ◆ E. van den berg and M. P. Friedlander for SPG/ l_1 (www.cs.ubc.ca/labs/scl/spg11)
- ◆ E. van den Berg, M. P. Friedlander, G. Hennenfent, F. J. Herrmann, R. Saab, and O. Yilmaz for SPARCO (www.cs.ubc.ca/labs/scl/sparco/)

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Questions ??