Curvelet-domain matched filtering

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Thanks to Gilles Hennenfent & Eric Verschuur

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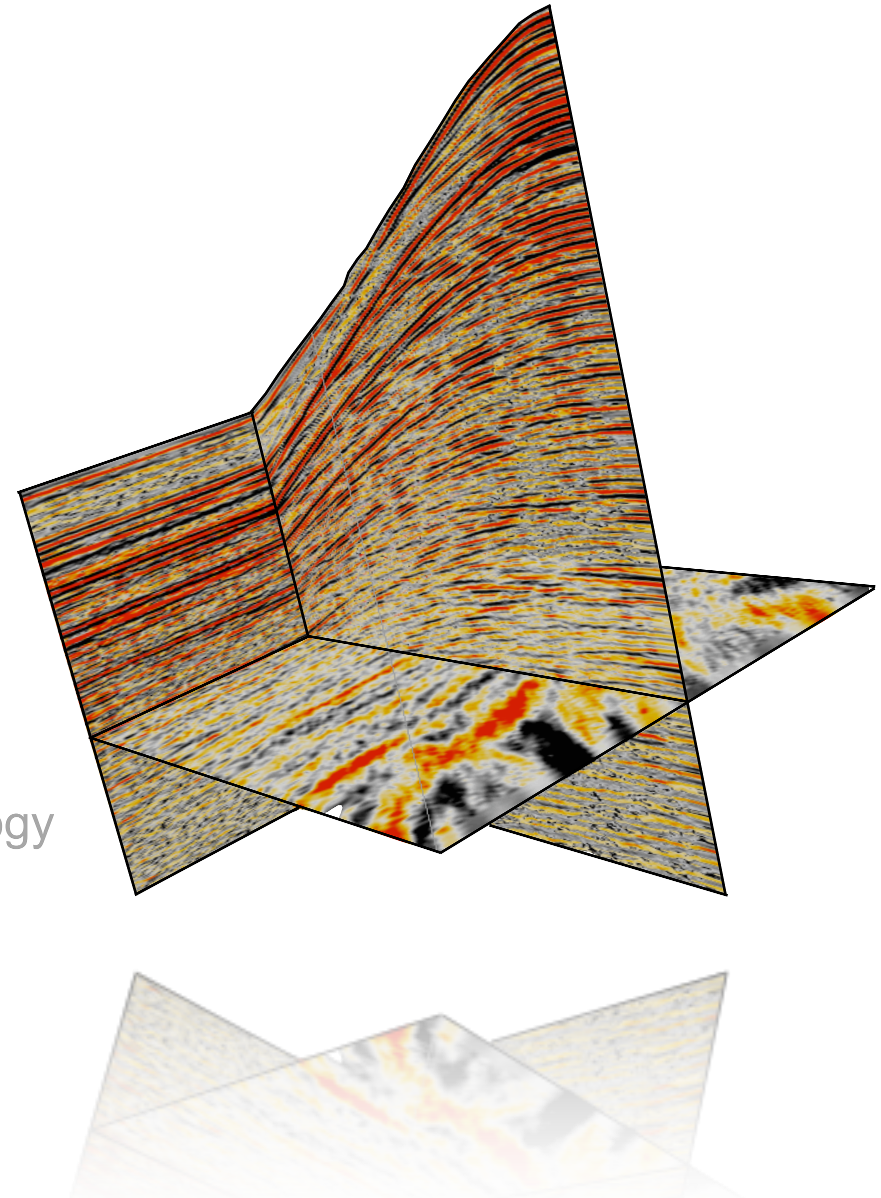

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Motivation

- *Transform-domain* matched-filtering forms the basis of
  - *adaptive* subtraction during surface-related multiple elimination [Verschuur ‘97]
  - *idem* during surface-wave removal with interferometry [Vasconcelos ’08, Wapenaar ‘08]
  - scaling during migration “preconditioning” based on migrated-remigrated image matching [Symes ’08,F.J.H. et. al, ‘08]

- *Fourier-based* matching
  - accounts for amplitude-spectra mismatches & global kinematic errors
  - fails for errors that vary spatially & as function of the local dip

- *Spatial & windowed Fourier* matching
  - run risk of over fitting (loss of primary energy)

- *Curvelet-domain* matching in phase space
  - corrects for *amplitude* errors that vary *smoothly* as a function of position & dip

History

- *Fourier*-based matched filtering was built on the premise that

$$\mathbf{m}_{\text{true}} \approx \mathbf{F}\mathbf{m}_{\text{predicted}} \text{ with } \mathbf{F} = \mathcal{F}^H \text{diag}\left(\hat{\mathbf{f}}\right)\mathcal{F}$$

- Estimated during a *global* least-squares estimation procedure

$$\hat{\mathbf{f}} = \arg\min_{\hat{\mathbf{g}}} \frac{1}{2}\|\hat{\mathbf{d}} - \hat{\mathbf{g}}\hat{\mathbf{m}}_{\text{predicted}}\|_2^2 + \lambda\|\mathbf{L}_{\mathcal{F}}\hat{\mathbf{g}}\|_2^2$$

  - $\mathbf{L}_{\mathcal{F}}$ *Fourier-space* sharpening operator that promotes smoothness
  - for each offset separately

- Estimated primaries:

$$\tilde{\mathbf{p}} = \mathbf{d} - \mathbf{F}\mathbf{m}_{\text{predicted}}$$

History cont'd

- Recently proposed alternatives [F.J.H '04-'08, Neelamani '08]

- Use **global** *Fourier-domain* matching to correct the multiples, i.e.,

$$\tilde{\mathbf{m}}_{\mathrm{matched}} = \mathbf{F}\mathbf{m}_{\mathrm{predicted}}$$

- Use these matched predictions as a *template* for a curvelet-domain separation by thresholding [F.J.H '04], i.e.,

$$\tilde{\mathbf{p}} = \mathbf{C}^H \mathbf{S}_{\mathbf{w}}\left(\mathbf{C}\mathbf{d}\right) \quad \text{with} \quad \mathbf{w} \propto \left|\mathbf{C}\tilde{\mathbf{m}}_{\mathrm{matched}}\right|$$

$$\mathbf{S}_w\left(x\right) = \mathrm{sign}(x)\max\left(0, |x| - w\right)$$

  - element-by-element separation
  - assumes correct amplitudes
  - assumes no kinematic errors

History cont'd

- Soft thresholding special case of weighted one-norm optimization [F.J.H et.al '07]

$$\begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{d}\|_2 + \|\mathbf{x}\|_{1,\mathbf{w}} \\ \tilde{\mathbf{p}} = \mathbf{S}^H \tilde{\mathbf{x}} \end{cases}$$

- and a special case of Neelamani's ['08] elementwise approach

$$\min_{a_\mu,\, \phi_\mu} \left|\{\mathbf{C}_c \mathbf{d}\}_\mu - a_\mu \exp(j\phi_\mu)\{\mathbf{C}_c \tilde{\mathbf{m}}_{\mathrm{matched}}\}_\mu\right|$$

$$\text{subject to} \quad a_\mu^{min} \leq a_\mu \leq a_\mu^{max}, \quad \phi_\mu^{min} \leq \phi_\mu \leq \phi_\mu^{max}, \quad \text{for} \quad \mu \in \mathcal{M}$$

for $\phi_\mu^{\min\,/\,\max} = 0$

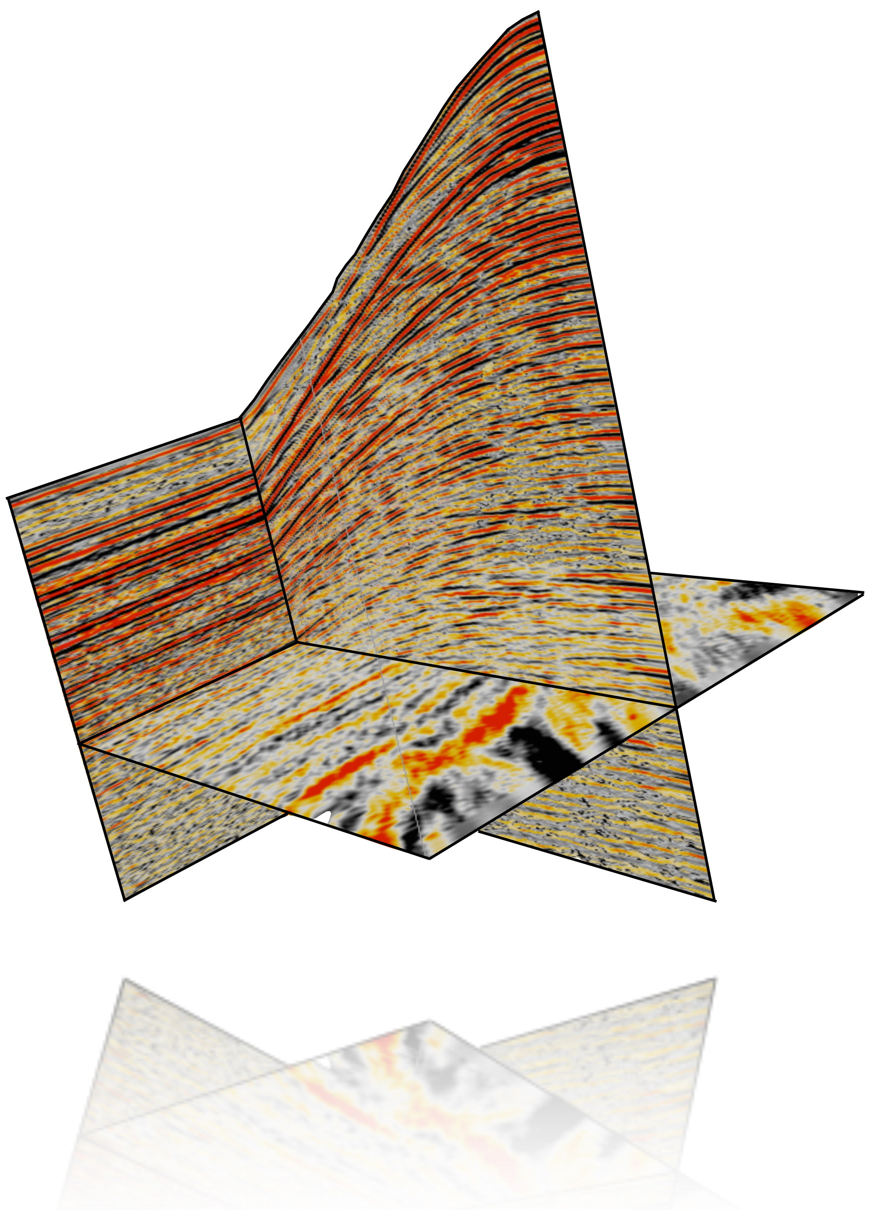
Our focus

- Adaptively correct for *amplitude* errors
  - vary *smoothly* as a function of position and dip
  - e.g., spherical divergence, incomplete data & other surface-related effects...

- Model driven
  - assume a *forward* model between *predicted* and *true* multiples
  - introduce a *regularized* inversion method based on this *forward* model

- Exploit curvelets
  - their *phase-space* partitioning
  - their ability to approximate zero-order *Pseudodifferential* operators [F.J.H et.al. ‘08]
  - their *sparsity* [Wang et. al. ‘08] (during the non-adaptive Bayesian separation)

The curvelet transform



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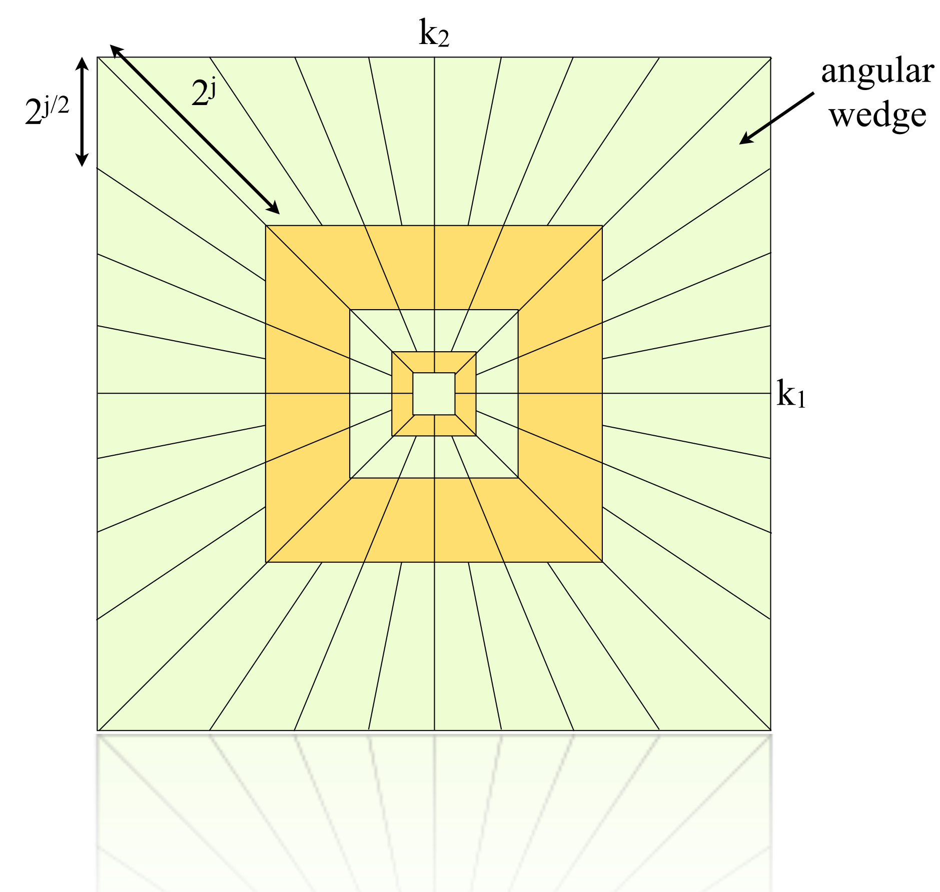
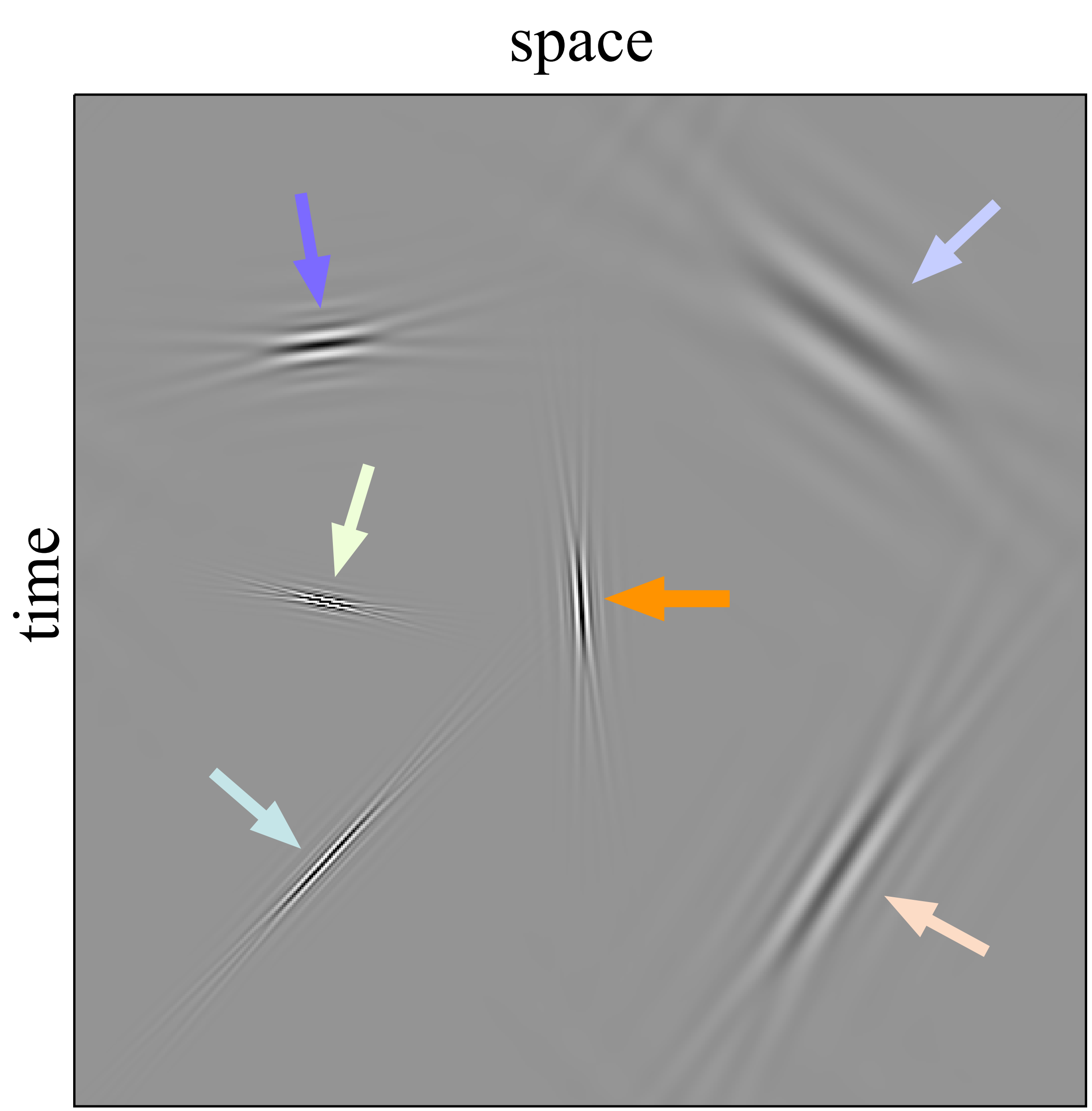
Representations for seismic data

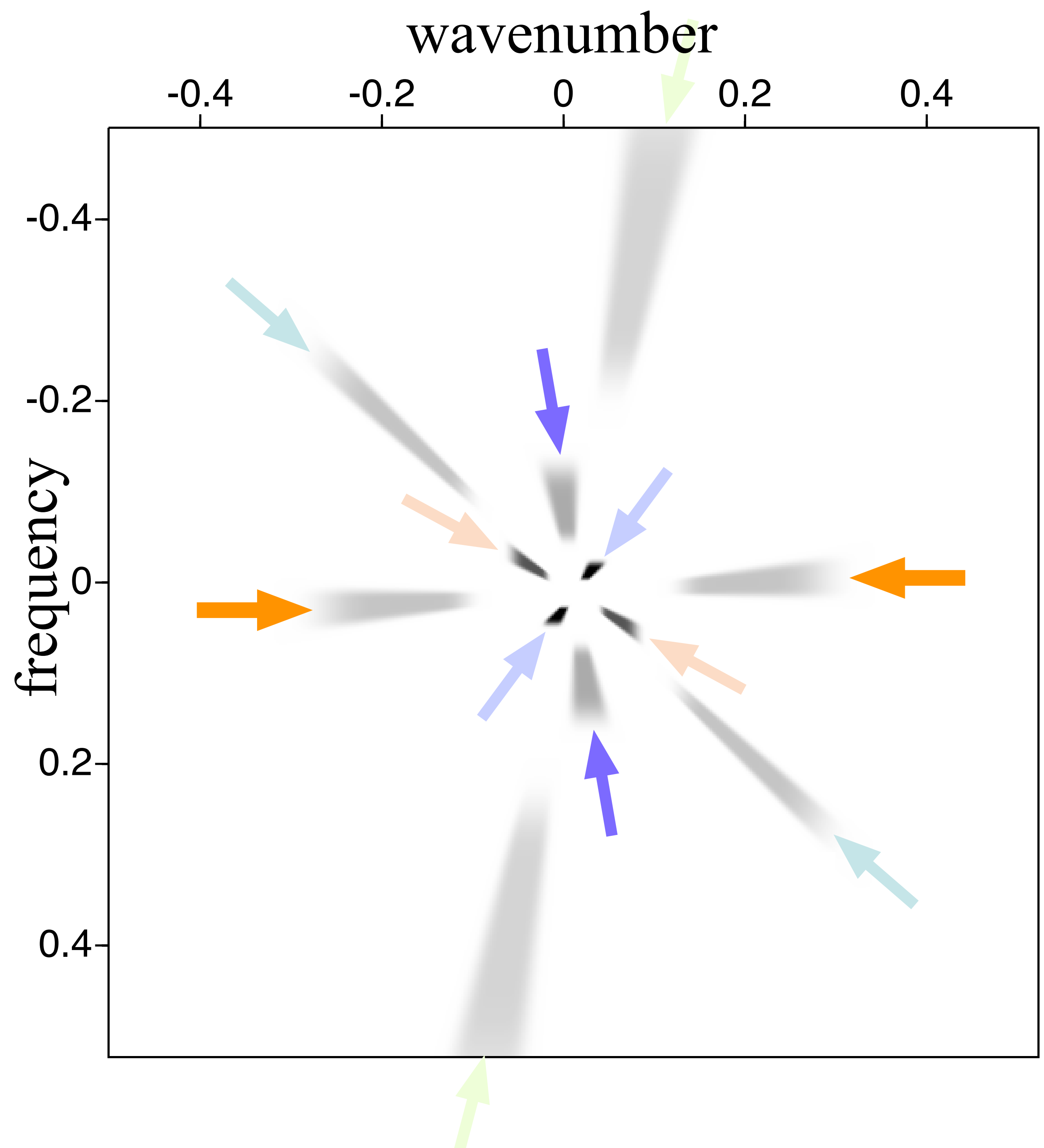
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

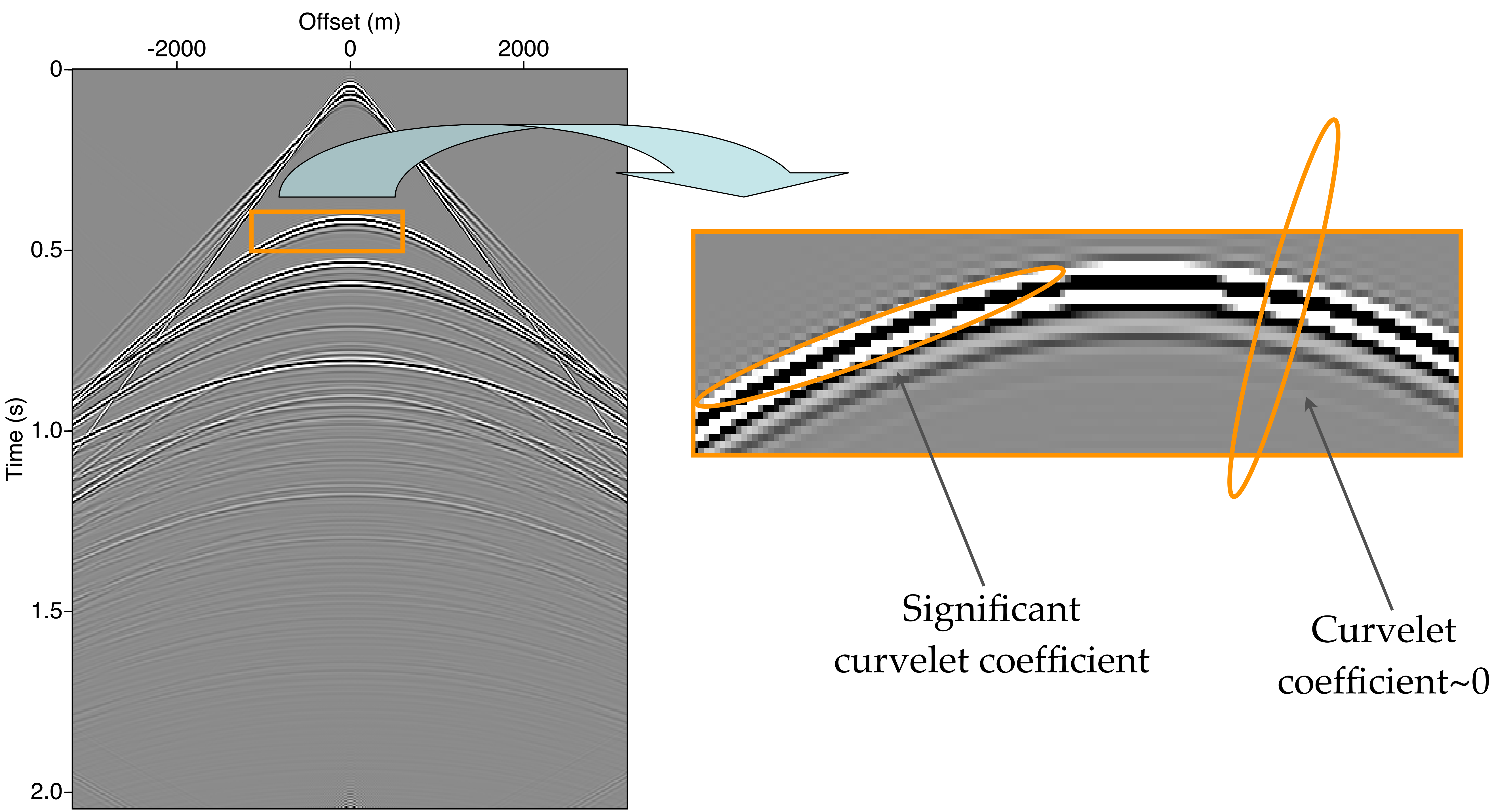
- curvelet transform
  - **multiscale**: tiling of the FK domain into dyadic coronae
  - **multidirectional**: coronae sub-partitioned into angular wedges, # of angles doubles every other scale
  - **anisotropic**: parabolic scaling principle
  - **local**



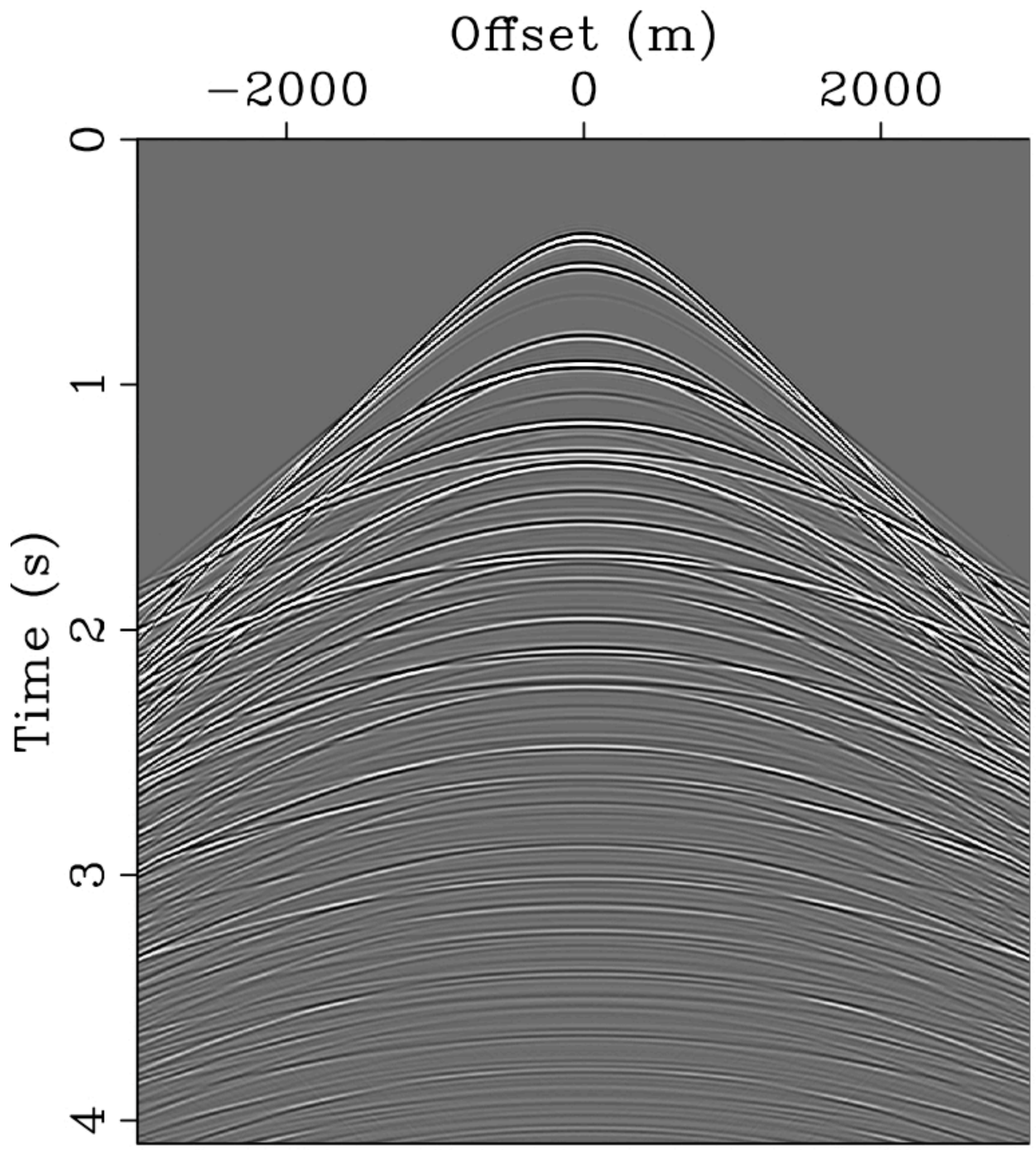
2D discrete curvelets



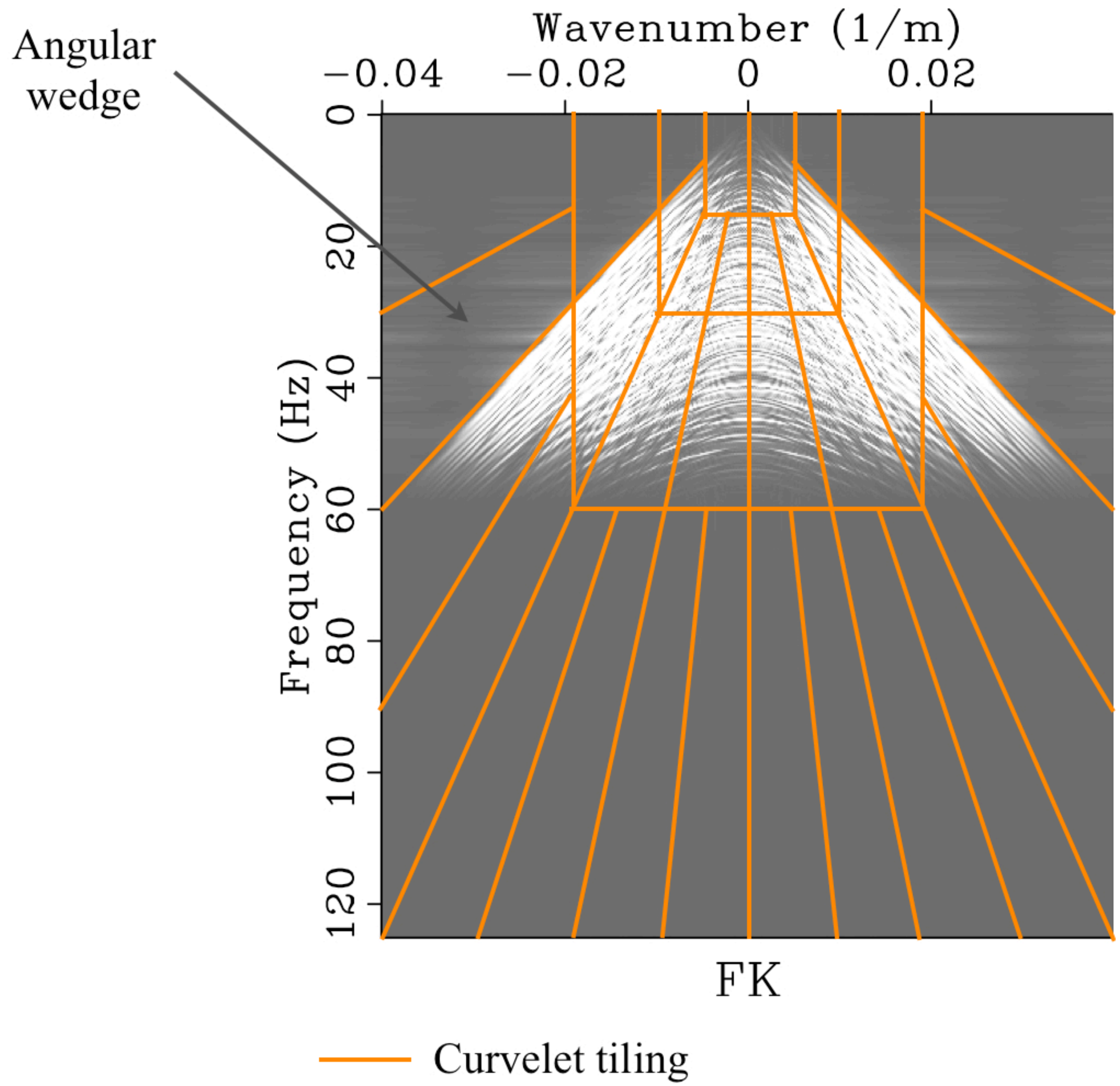



Offset (m)
-2000
0
2000
0
0.5
1.0
1.5
2.0
Time (s)
Significant curvelet coefficient
Curvelet coefficient~0

Curvelet tiling & seismic data



Data



FK

Curvelet tiling superimposed on the FK domain

Curvelet processing workflow (forward transform C)

t-x domain data

f-k transform whole data

f-k domain data

cover the f-k domain with dyadic tiling
to determine appropriate curvelet transform

tiled f-k domain data

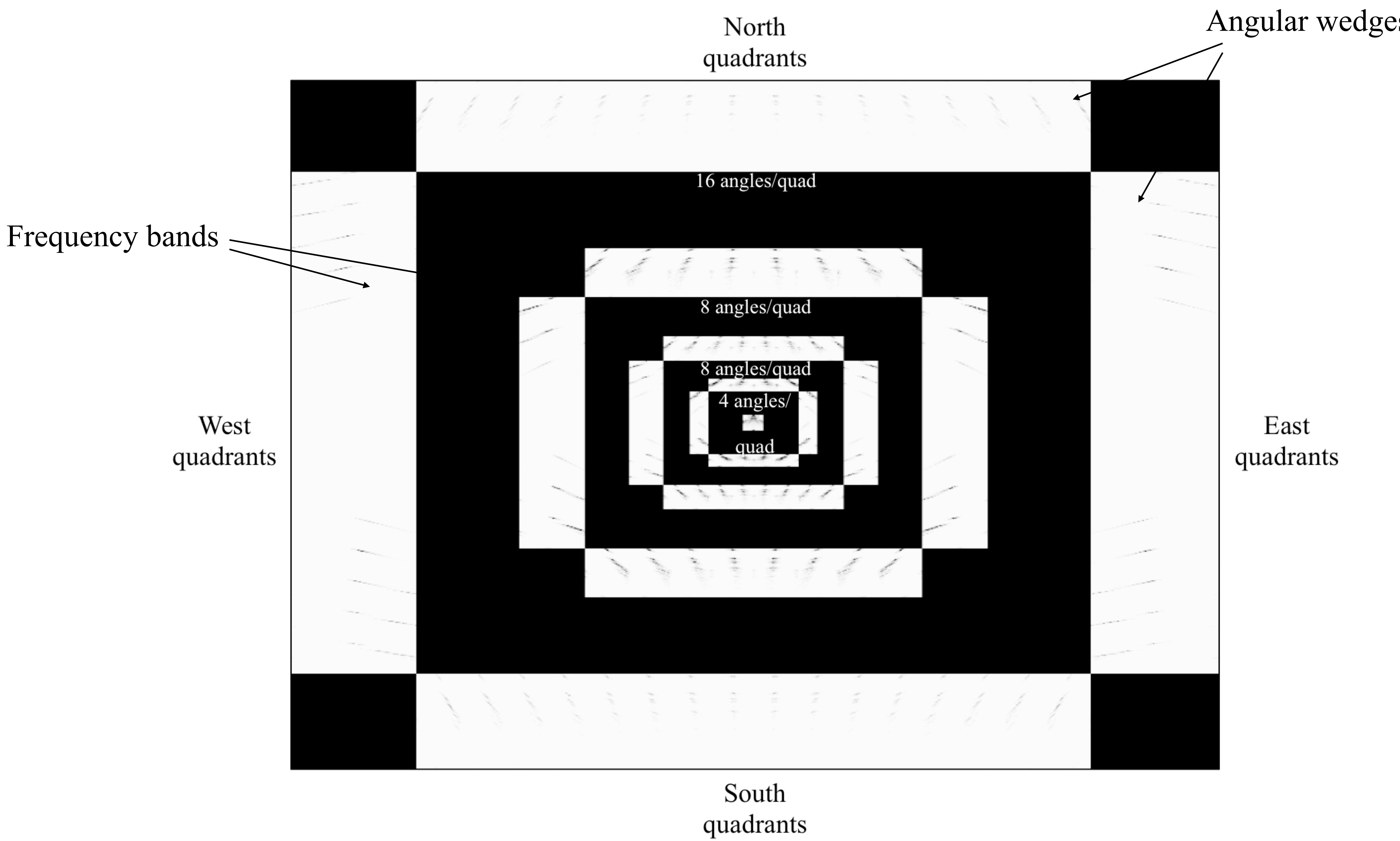
window each tile segment in the f-k domain

Inverse f-k transform each tile segment window

tiled curvelet domain data

(This is the domain in which curvelet operations takes place)

Curvelet tiling & seismic data



Curvelet processing workflow (inverse transform C^T)

tiled curvelet domain data

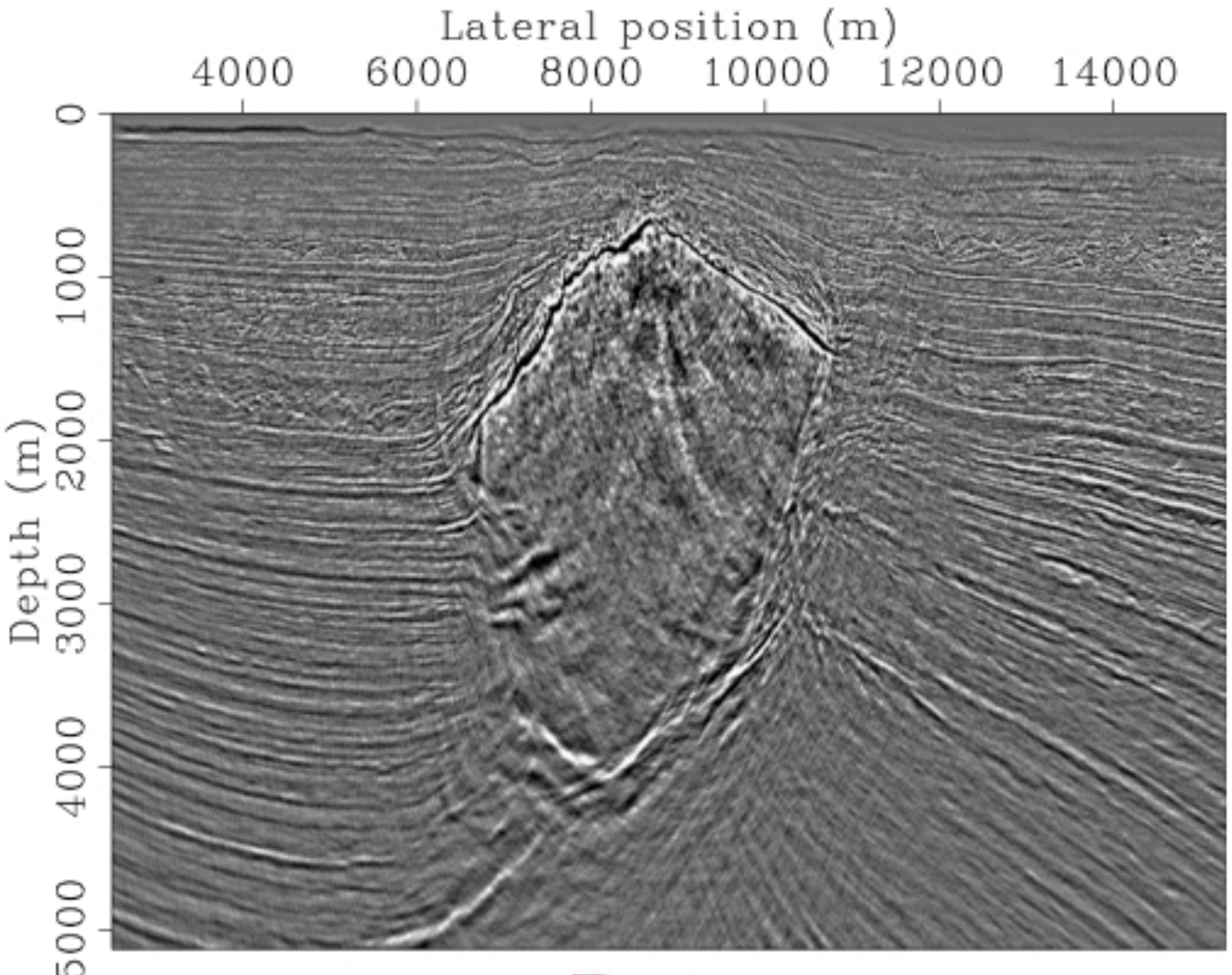
↓ f-k transform each curvelet tile

tiled f-k domain data

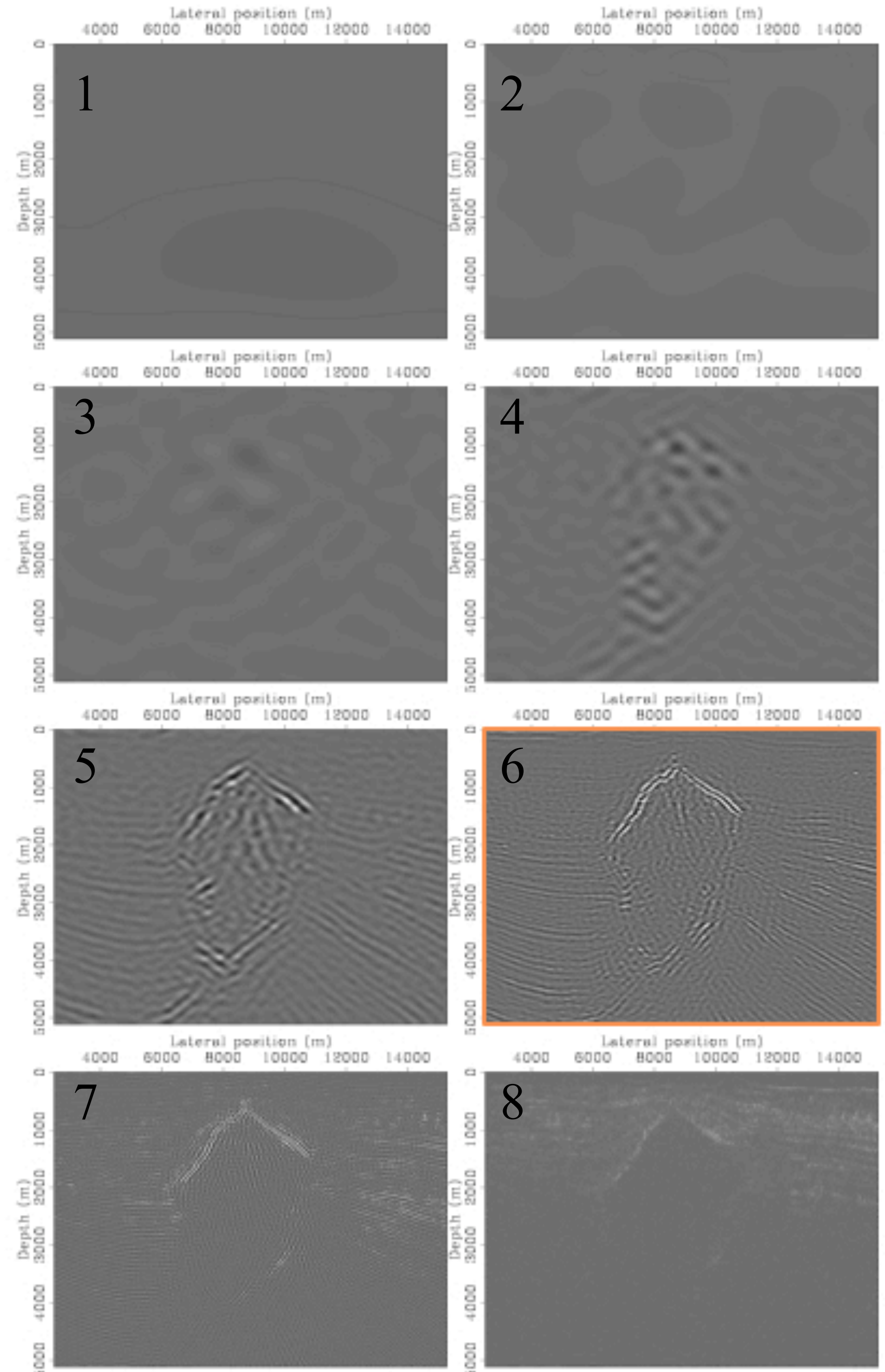
↓ Inverse f-k transform whole data

t-x domain data

Real data frequency bands example

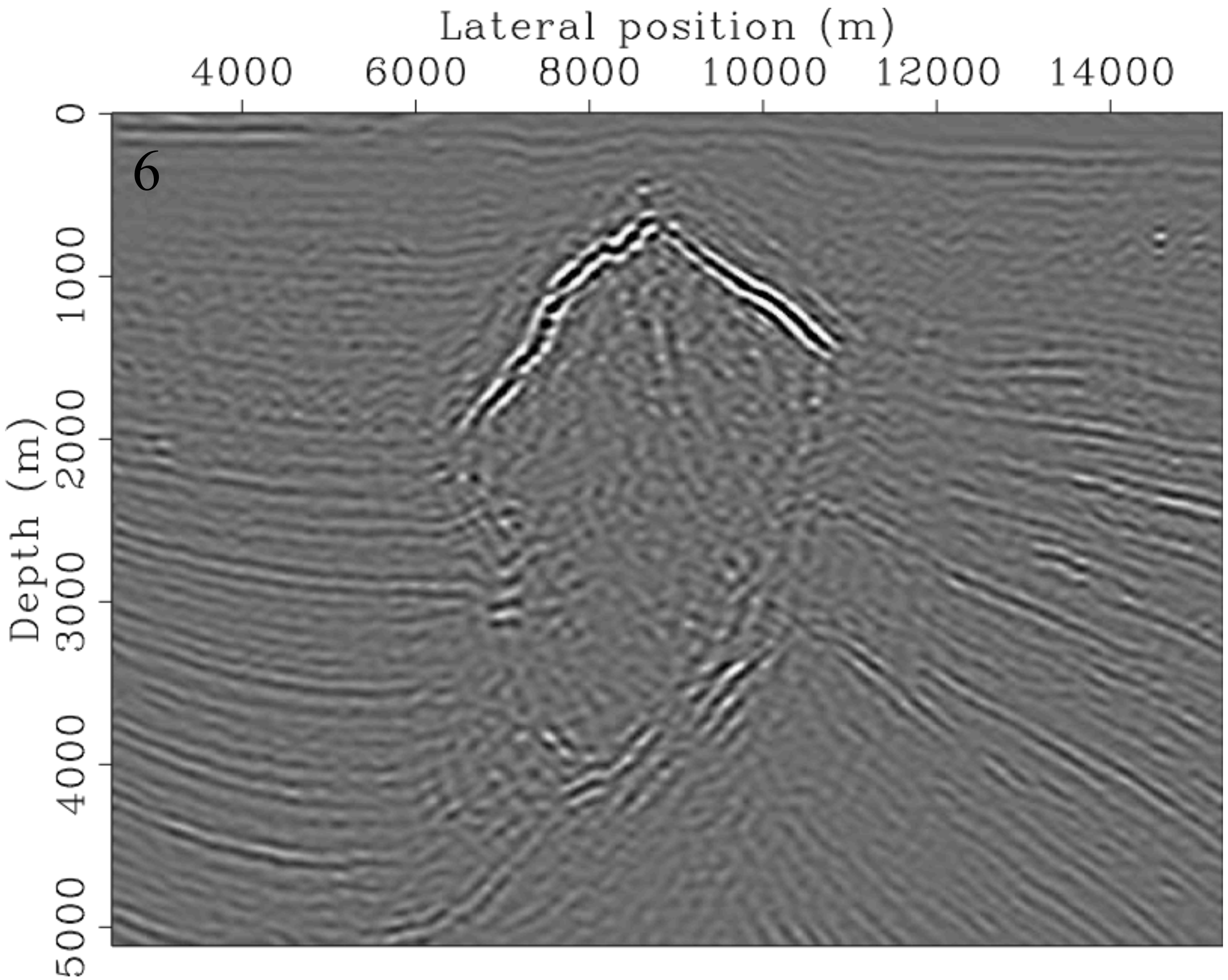


Data

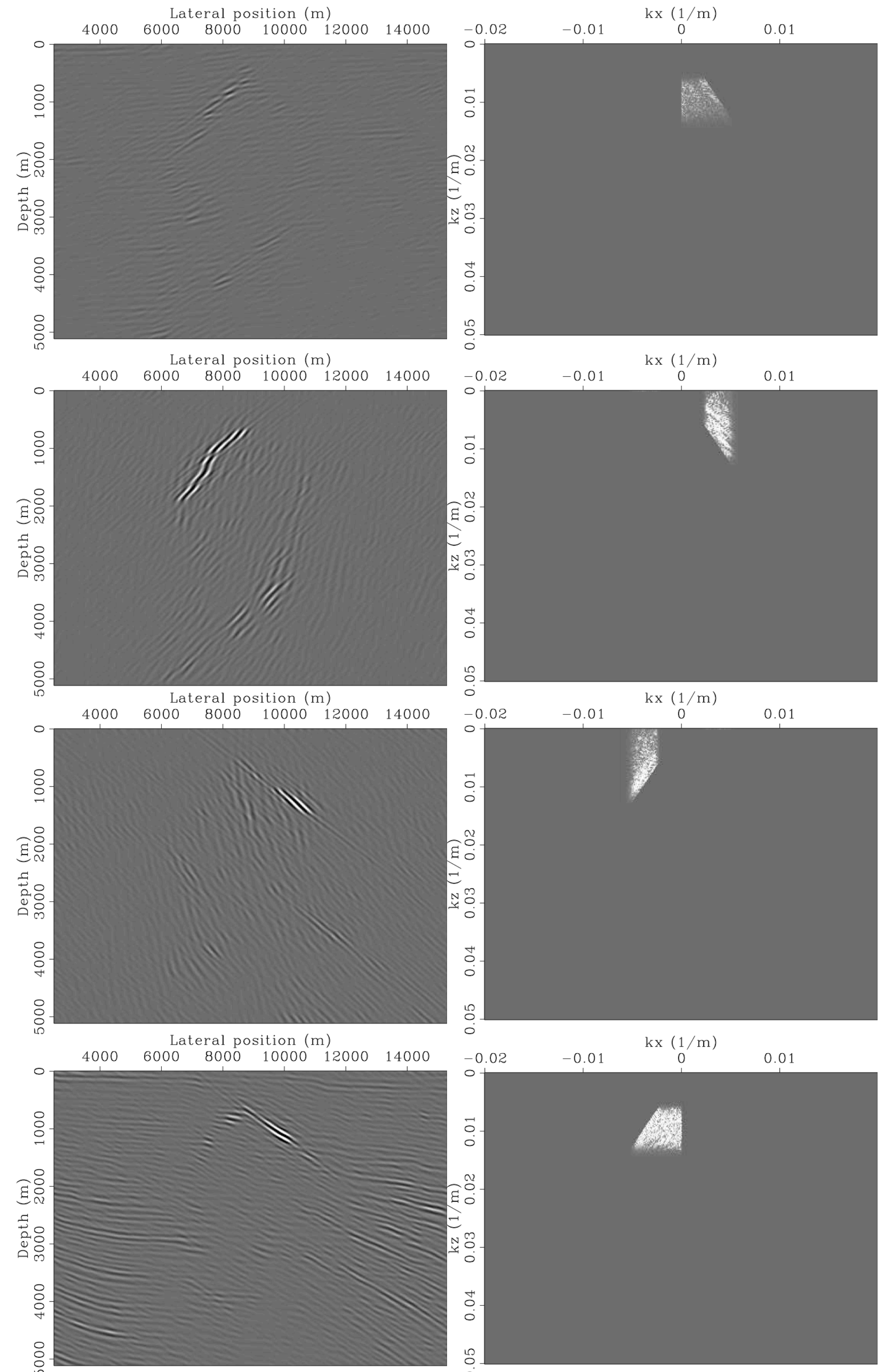


Decomposition in frequency bands

Single frequency band angular wedges

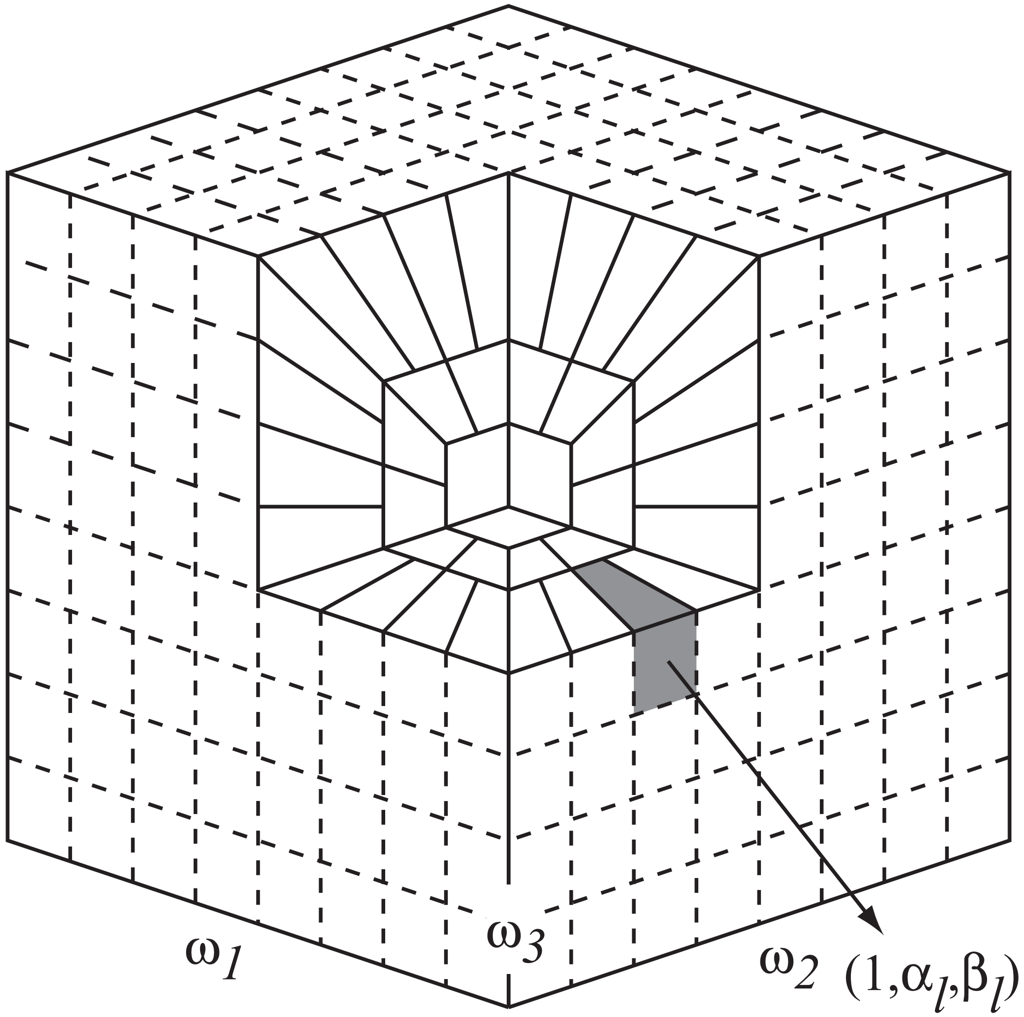


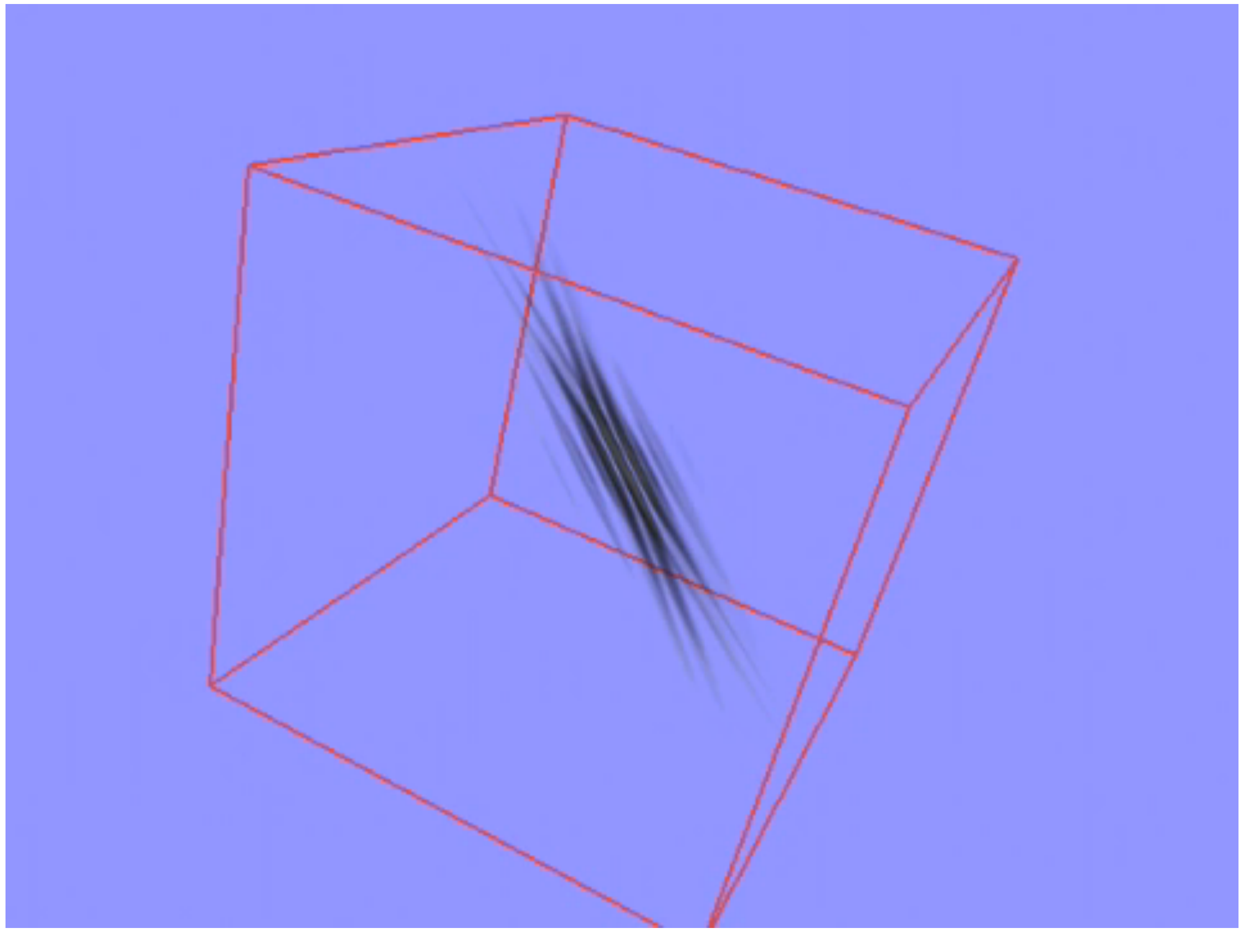
6th scale image



Decomposition in angular wedges

3D discrete curvelets

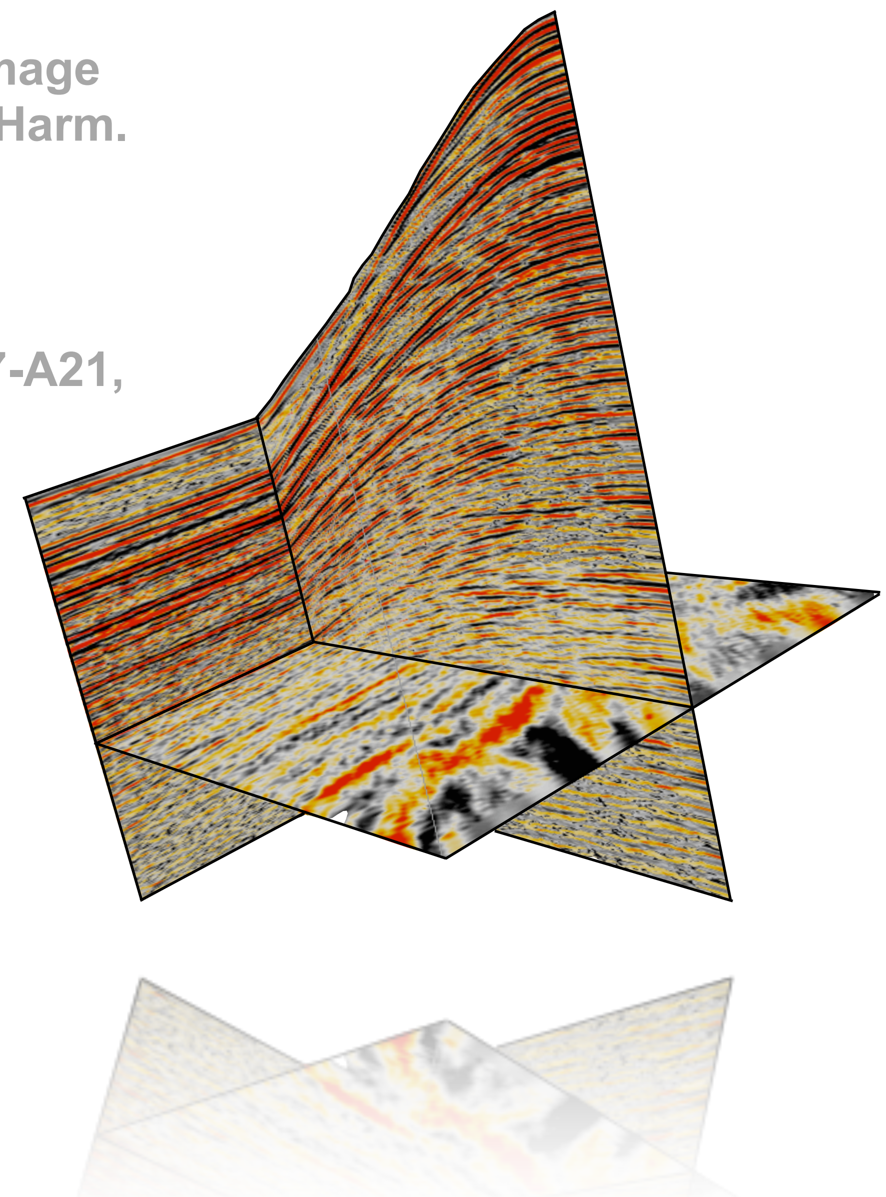




Curvelet-domain matched filter

Herrmann, F. J., Moghaddam, P. and Stolk, C. Sparsity- and continuity-promoting seismic image recovery with curvelet frames. App. & Comp. Harm. Analys., Vol. 24, No. 2, pp. 150-173, 2008.

Herrmann, F. J., Wang, D and Verschuur, D. J. Adaptive curvelet-domain primary-multiple separation. Geophysics, Vol 73, No. 3, pp. A17-A21, 2008.



The forward model

- Our curvelet-domain matched filtering is built on the premise that

$$\mathbf{m}_{\mathrm{true}} \approx \mathbf{B}\mathbf{m}^0 \text{ with } \mathbf{B} = \mathcal{F}^H b(x,k)\mathcal{F}$$

$$\mathbf{m}^0 = \tilde{\mathbf{m}}_{\mathrm{matched}}, \text{ and } \mathbf{B} \text{ a zero-order } \Psi\mathrm{DO}.$$

- We assume that
  - there are NO kinematic and phase errors
  - *global* conservative Fourier-domain matching removed the “wavelet” => *zero-order*
  - corrections by the symbol **b** vary *smoothly* as a function of *space* and *angle*
- Approximate the action of **B** by curvelet-domain scaling
  - fast evaluation
  - possibility to estimate from data by nonlinear least-squares matching

Diagonalization [F.J.H et. al '08]

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T \mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' is a constant depending on Ψ.

Lemma 1. *With C' some constant, the following holds*

$$\|(\Psi(x,D) - a(x_\nu, \xi_\nu))\varphi_\nu\|_{L^2(\mathbb{R}^n)} \leq C' 2^{-|\nu|/2}. \tag{14}$$

To approximate Ψ, we define the sequence $\mathbf{u} := (u_\mu)_{\mu \in \mathcal{M}} = a(x_\mu, \xi_\mu)$. Let $\mathbf{D}_\Psi$ be the diagonal matrix with entries given by $\mathbf{u}$. Next we state our result on the approximation of Ψ by $C^T \mathbf{D}_\Psi C$.

Estimation matched filter

- Action of **B** can be approximated

$$\mathbf{m}_{\mathrm{true}} \approx \mathbf{B}\mathbf{m}^0 \text{ with } \mathbf{B} \approx \mathbf{C}^H \mathrm{diag}\left(\mathbf{b}\right)\mathbf{C}, \qquad \{b\}_{\mu\in\mathcal{M}} > 0$$

- Diagonal can be estimated during a *global nonlinear* least-squares estimation procedure [Symes '08, F.J.H et. al. '08]

$$\tilde{\mathbf{z}} = \arg\min_{\mathbf{z}} \frac{1}{2}\|\mathbf{d} - \mathbf{C}^T \mathrm{diag}\left(\mathbf{C}\mathbf{m}^0\right) e^{\mathbf{z}}\|_2^2 + \gamma\|\mathbf{L}_{\mathcal{C}} e^{\mathbf{z}}\|_2^2$$

  - $\mathbf{L}_{\mathcal{C}}$ *curvelet-domain* sharpening operator that promotes smoothness
  - guarantees the solution to be *positive*

$$\tilde{\mathbf{m}}_{\mathrm{matched}} = \mathbf{B}\mathbf{m}_0 \text{ with } \mathbf{b} = e^{\tilde{\mathbf{z}}}$$

Estimation matched filter

Solve the system

$$\begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix} \approx \begin{bmatrix} \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{m}^0\} \\ \gamma \mathbf{L}_{\mathcal{C}} \end{bmatrix} e^{\mathbf{z}}$$

$$\mathbf{y} \approx \mathbf{G}_\gamma e^{\mathbf{z}}$$

Use grandient of $J(\mathbf{z}) = \frac{1}{2}\|\mathbf{y} - \mathbf{G}_\gamma e^{\mathbf{z}}\|_2^2$

$$\mathrm{grad} J(\mathbf{z}) = \mathrm{diag}\{e^{\mathbf{z}}\}\left[\mathbf{G}_\gamma^T \left(\mathbf{G}_\gamma e^{\mathbf{z}} - \mathbf{y}\right)\right]$$

to solve with this system with limited-memory BFGS [Nocedal ‘89]

Phase-space regularization

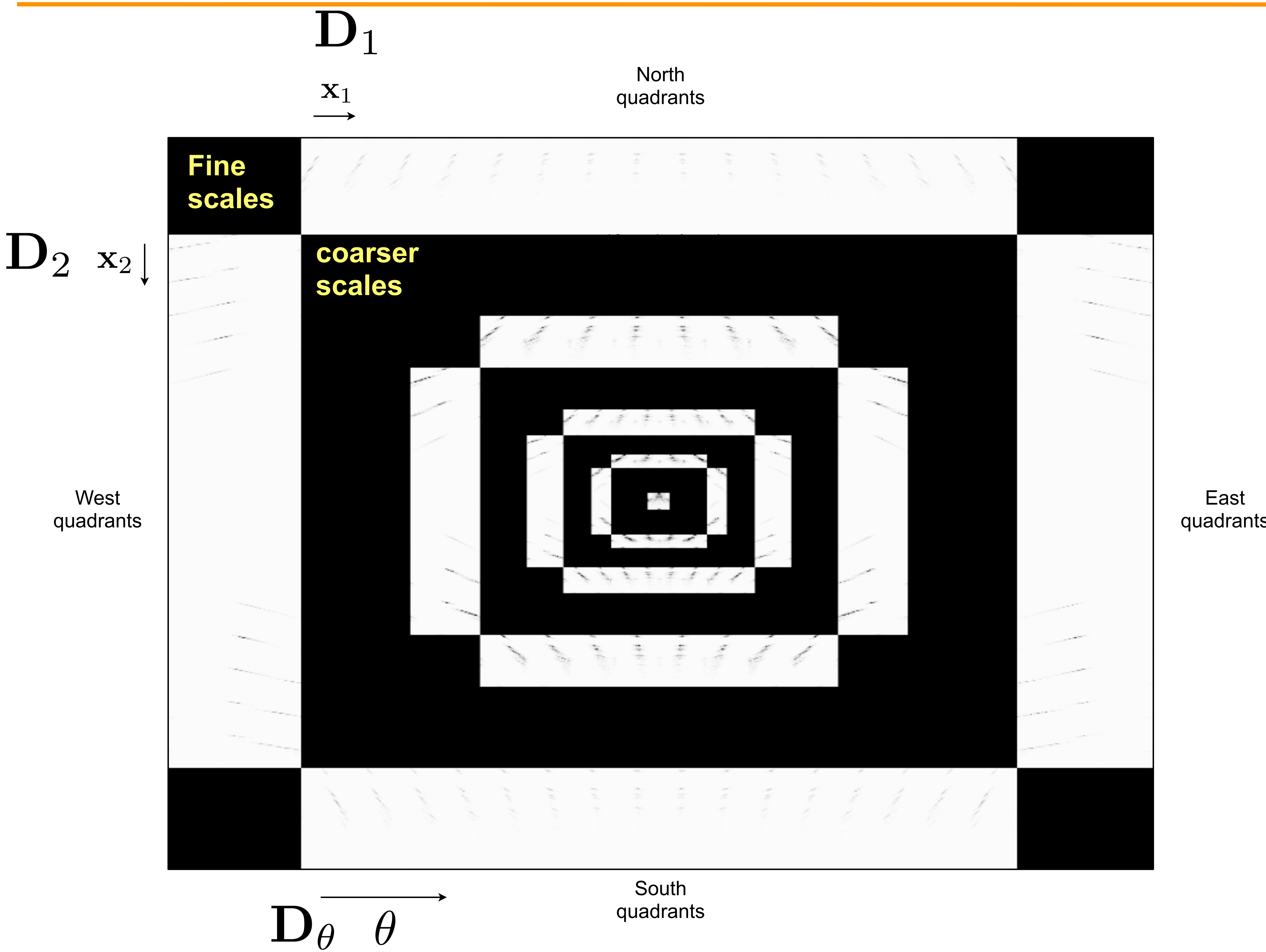
Curvelet-domain *sharpening* operator

$$\mathbf{L}_{\mathcal{C}} = \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \mathbf{D}_\theta^T \end{bmatrix}^T$$

- *First-order differences* in *space* and *angle* directions for each scale
- *Regularization* parameter controls phase-space *smoothness*
- Limit overfitting
- Assure positivity with nonlinear least-squares ...

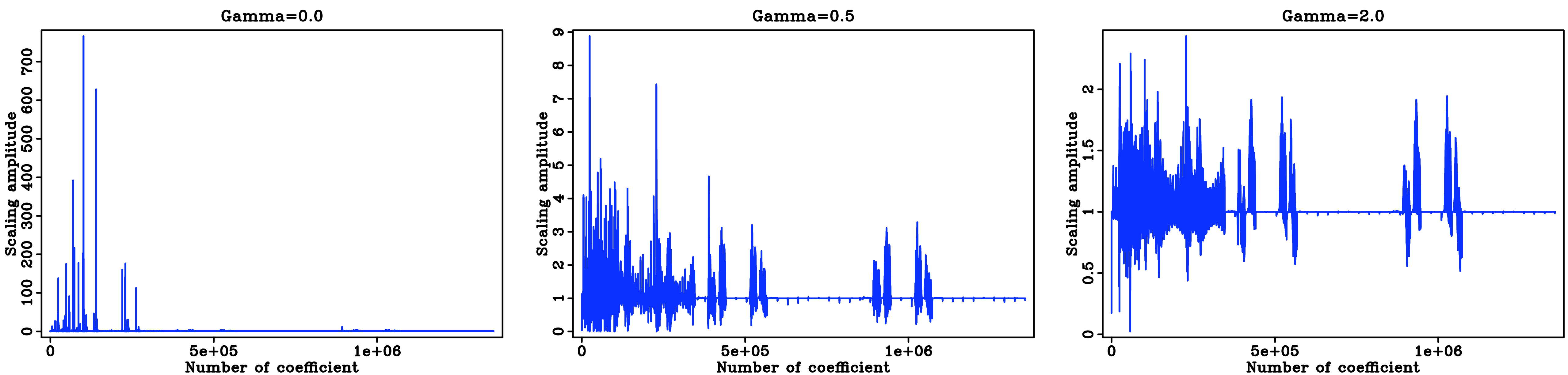
- Matched result is used as input of our Bayesian separation method [Wang et. al. '08]
  - based on sparsity promotion & decorrelation of the signal components
  - offers control on fidelity multiple prediction

Phase-space regularization



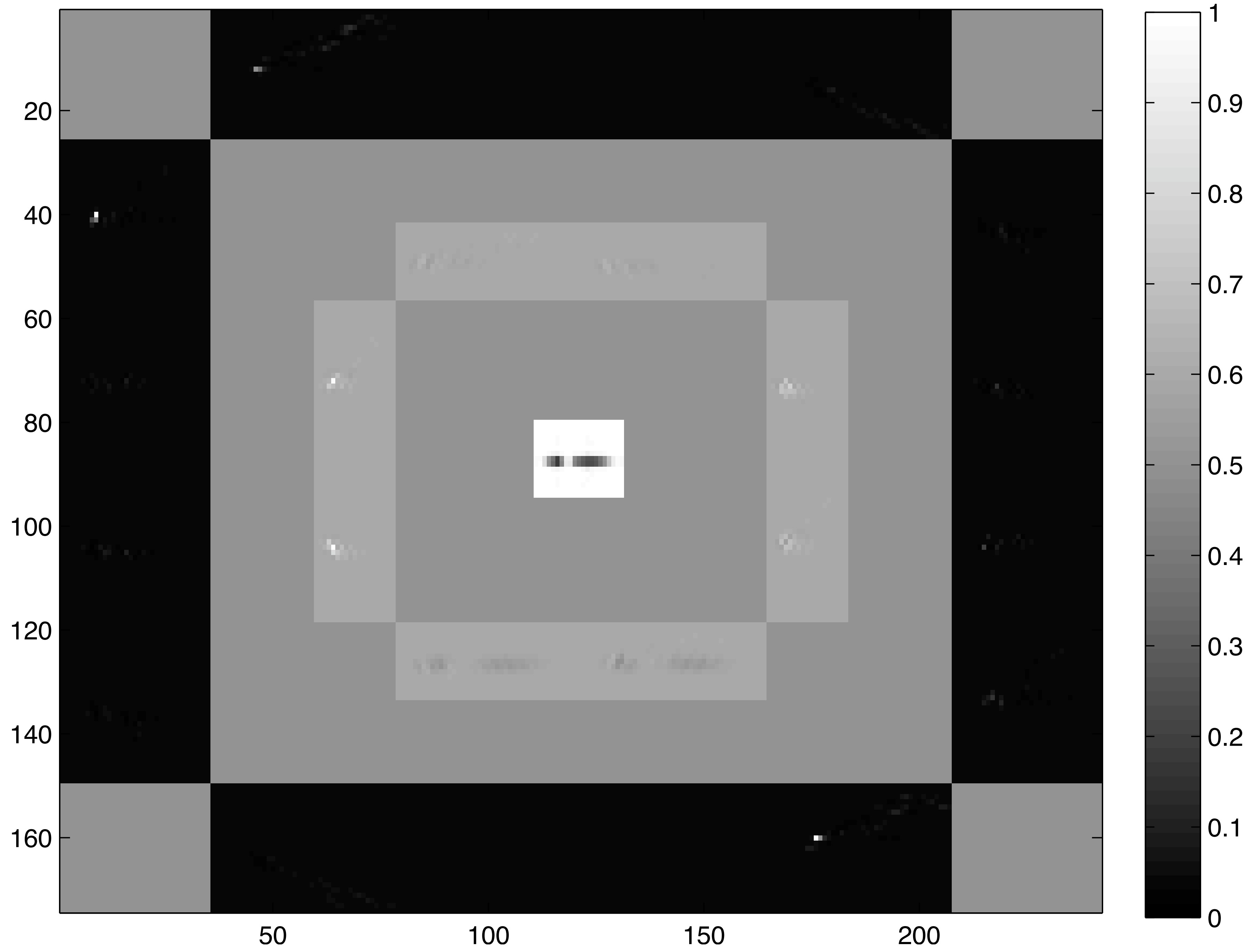
Phase-space regularization

increasing smoothness



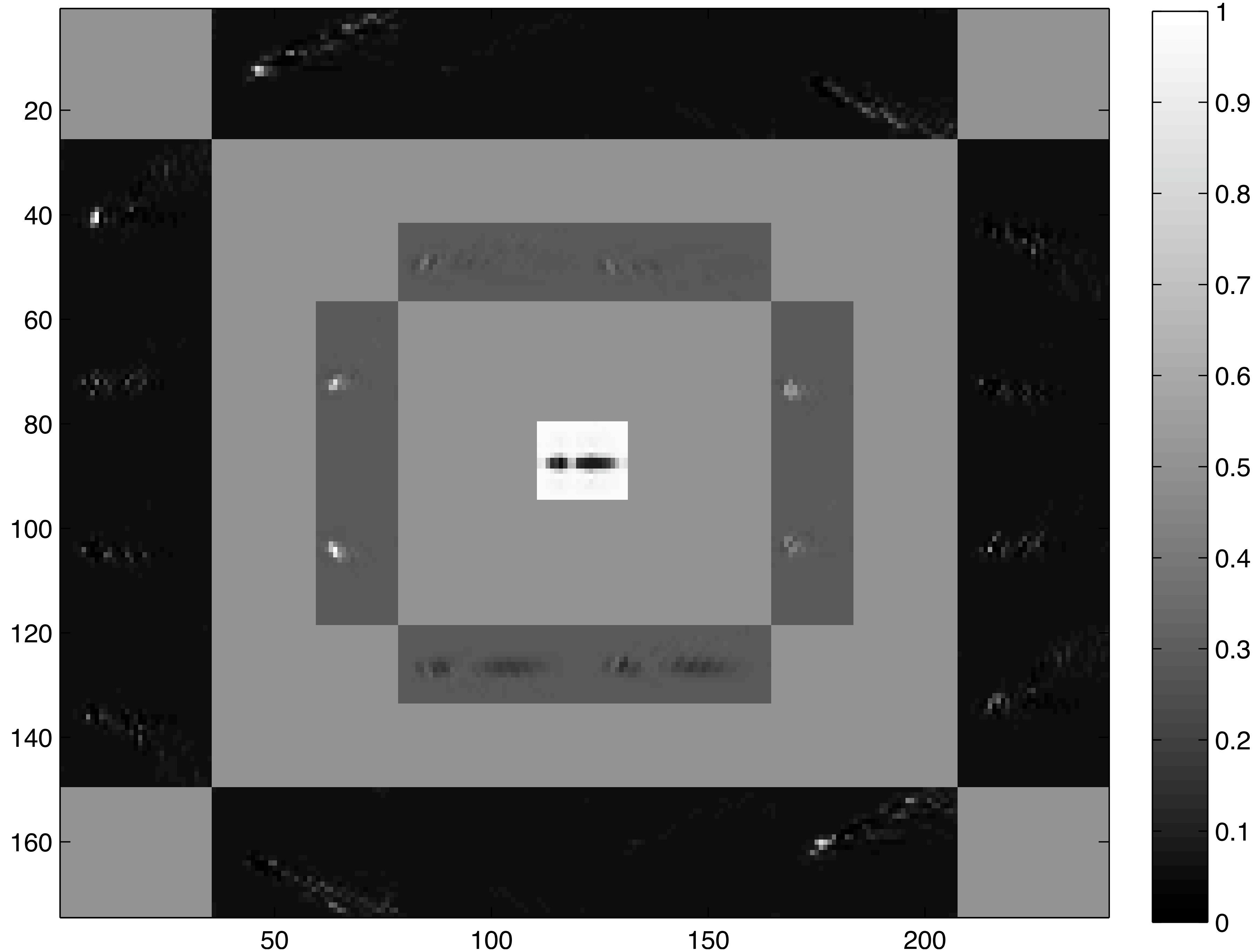
- reduces overfitting
- scaling is positive and reasonable

Phase-space regularization



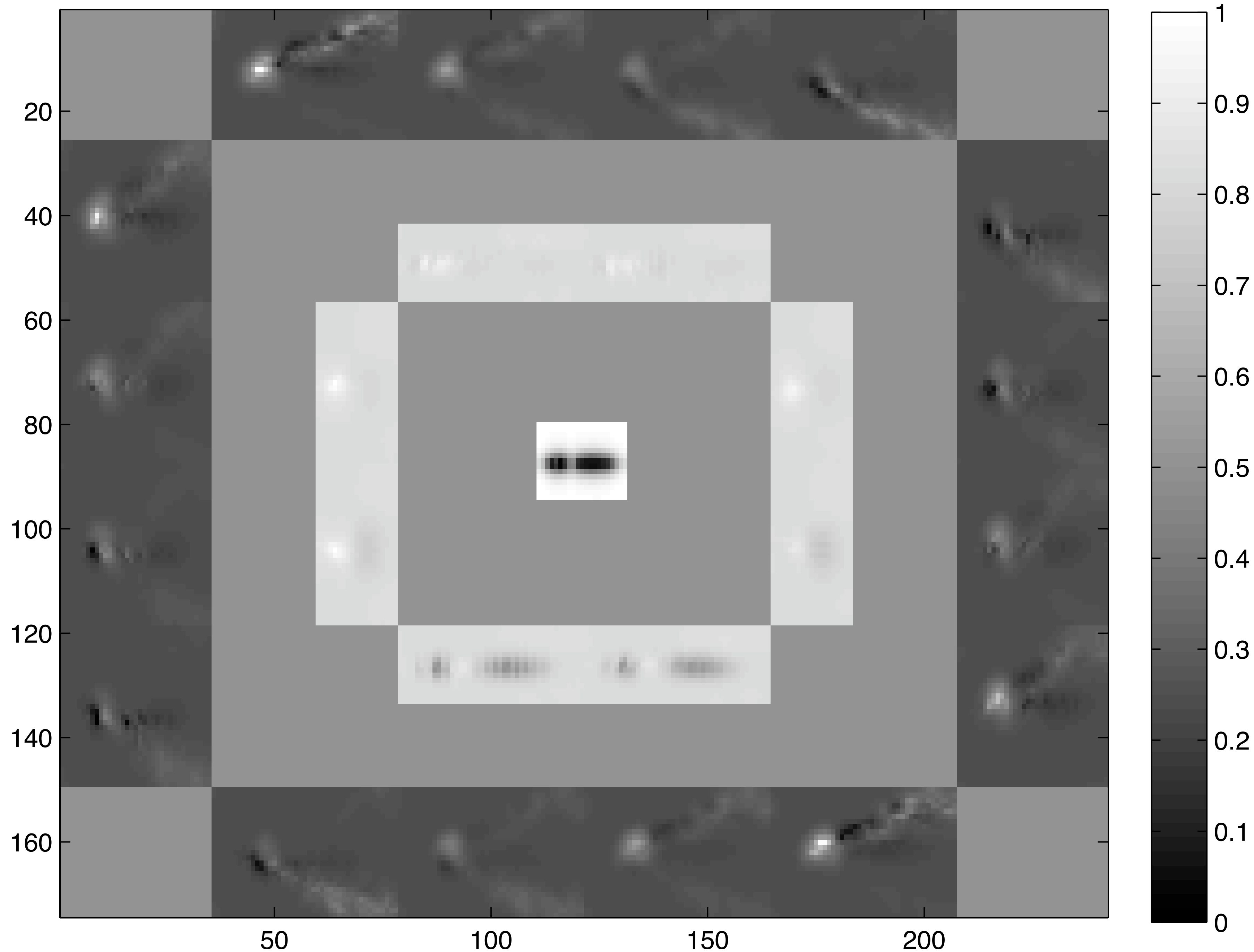
$\gamma = 0.0$

Phase-space regularization



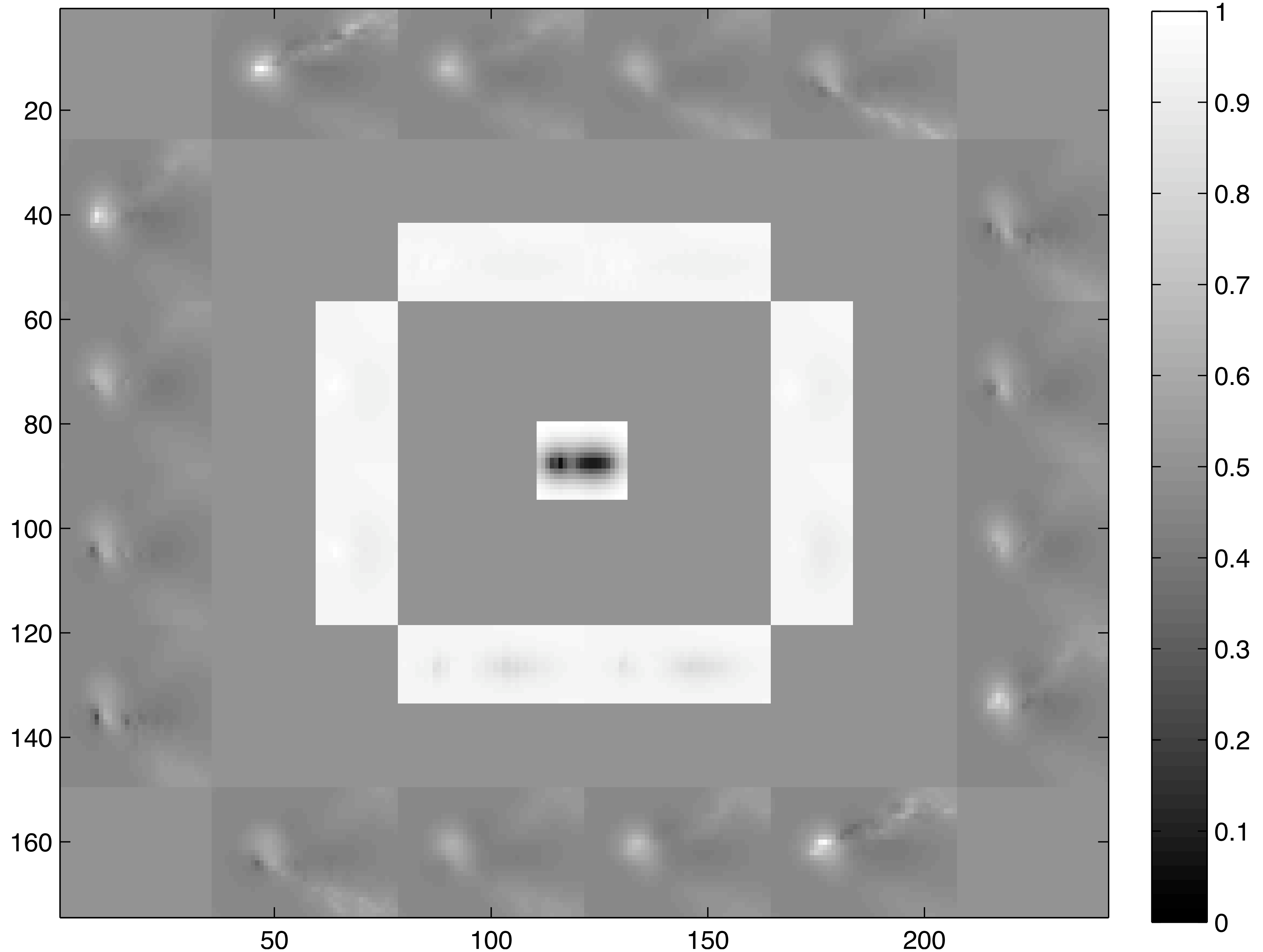
$\gamma = 0.1$

Phase-space regularization



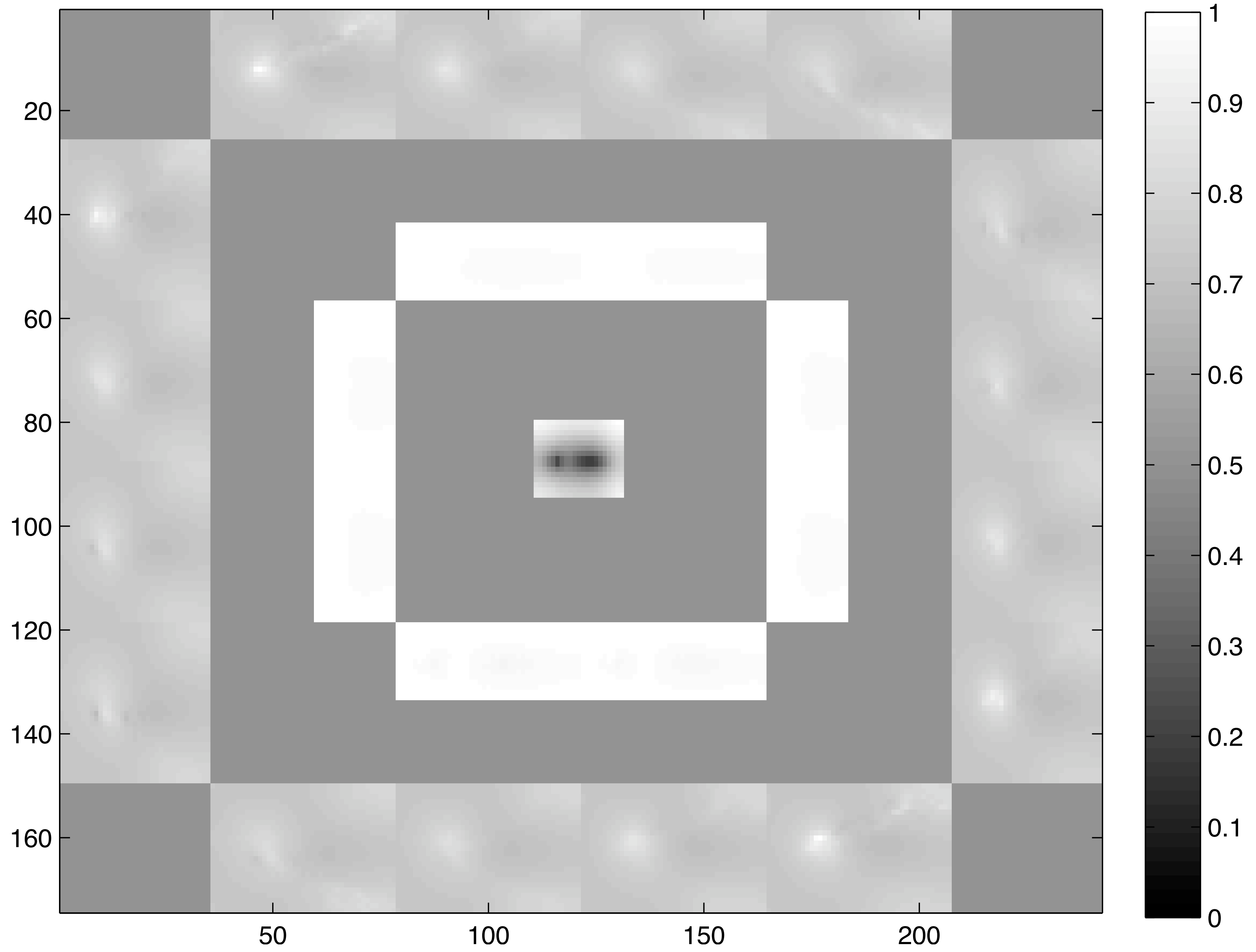
$\gamma = 0.5$

Phase-space regularization



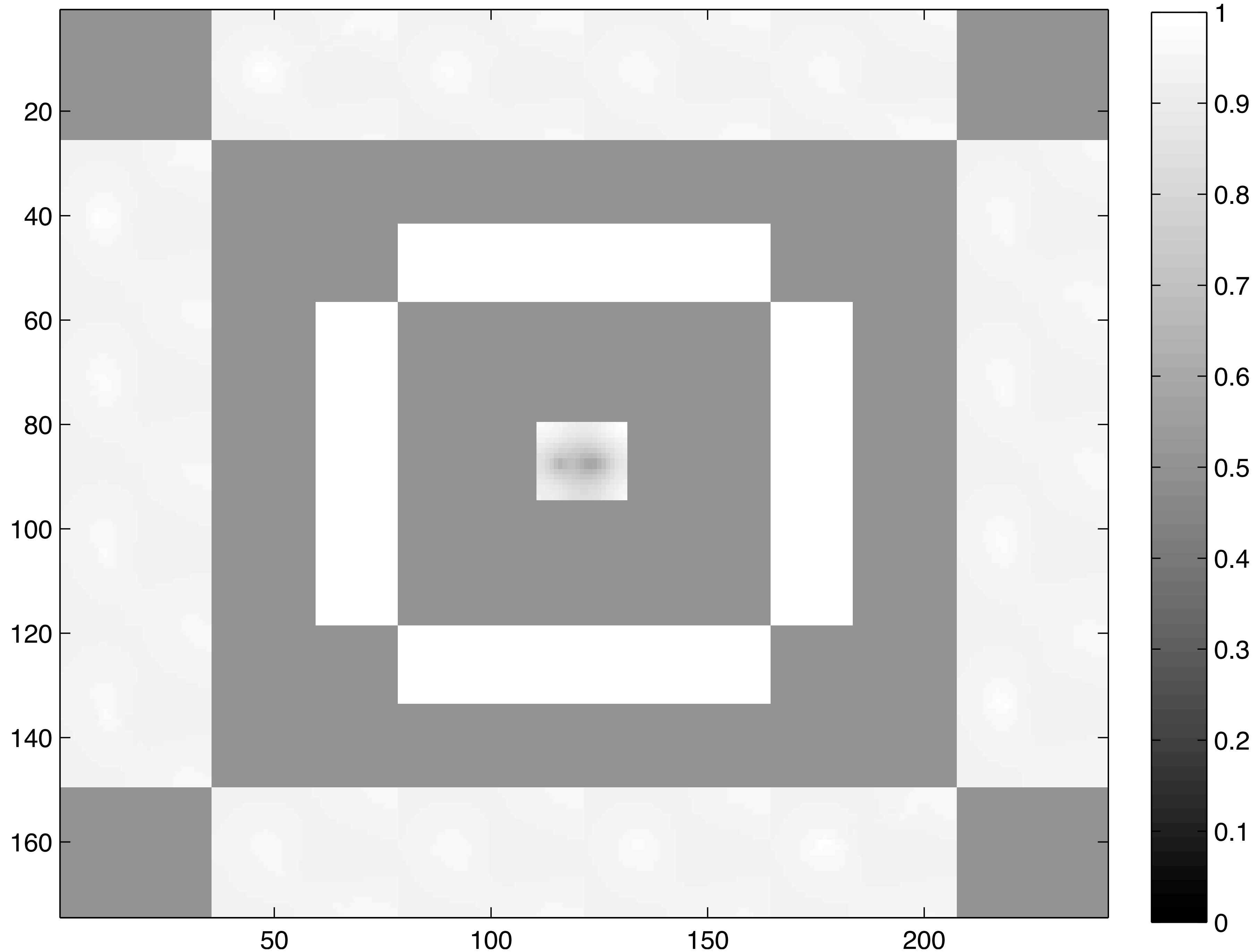
$\gamma = 1.0$

Phase-space regularization



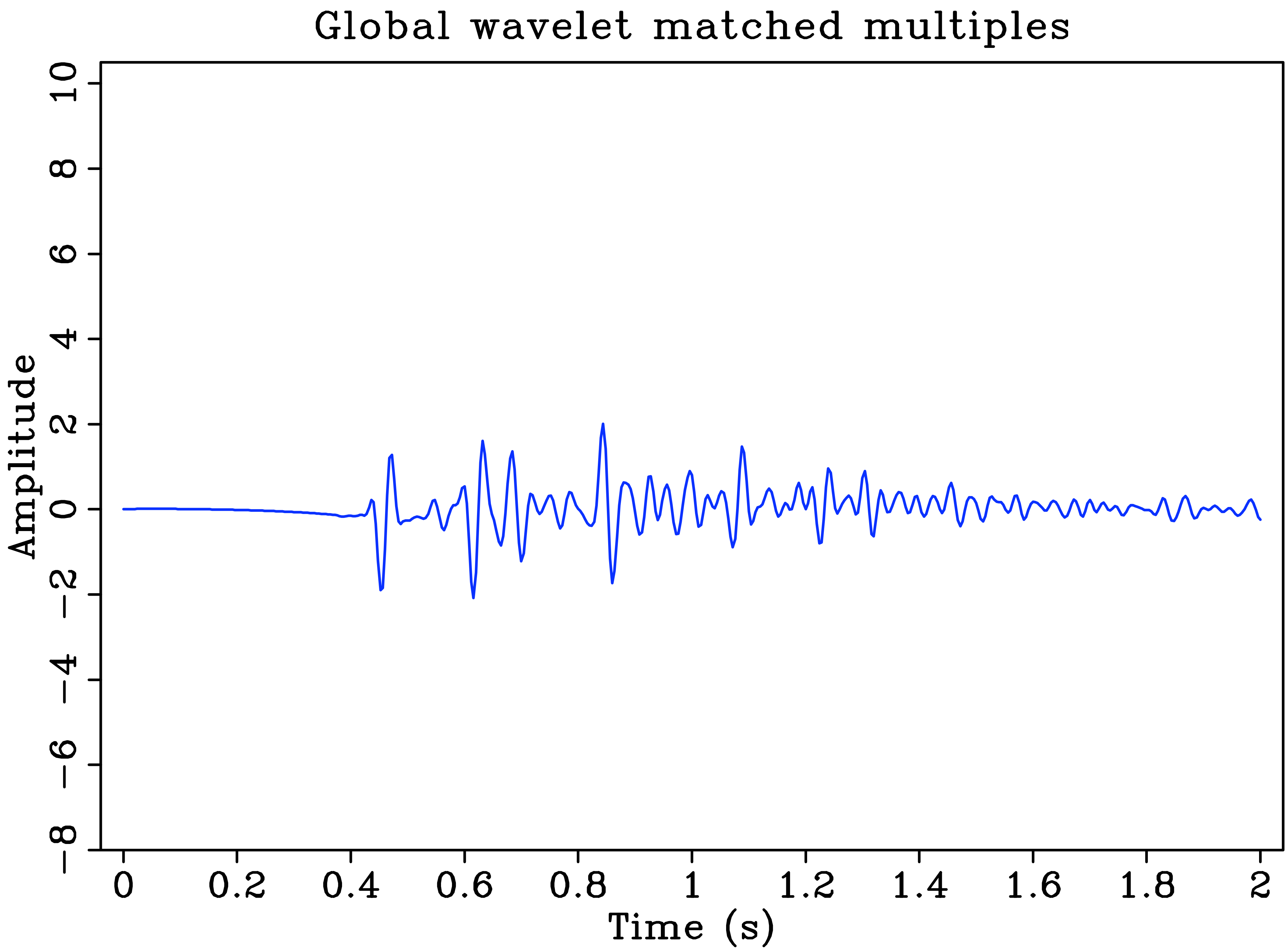
$\gamma = 2.0$

Phase-space regularization



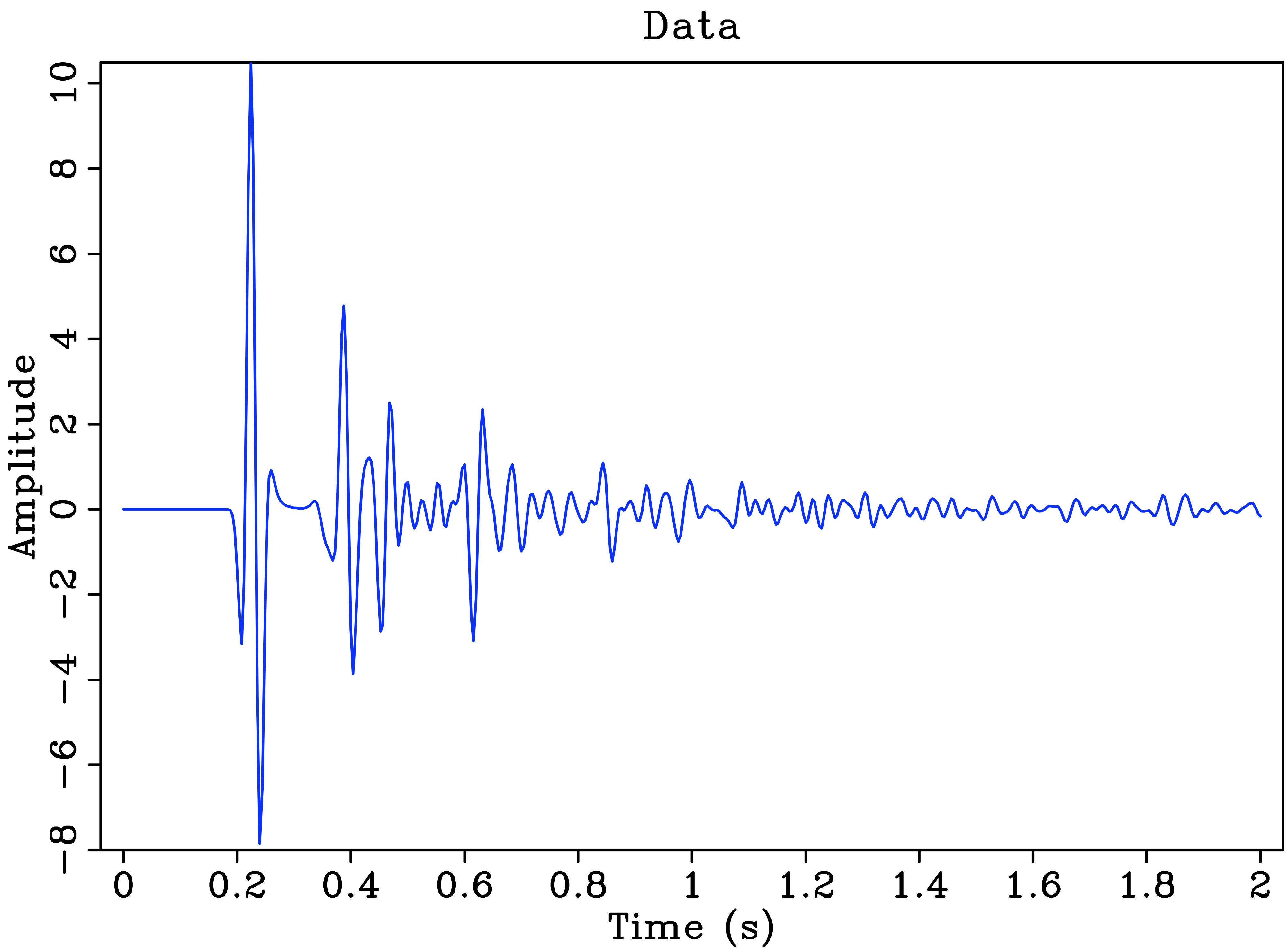
$\gamma = 5.0$

Phase-space regularization



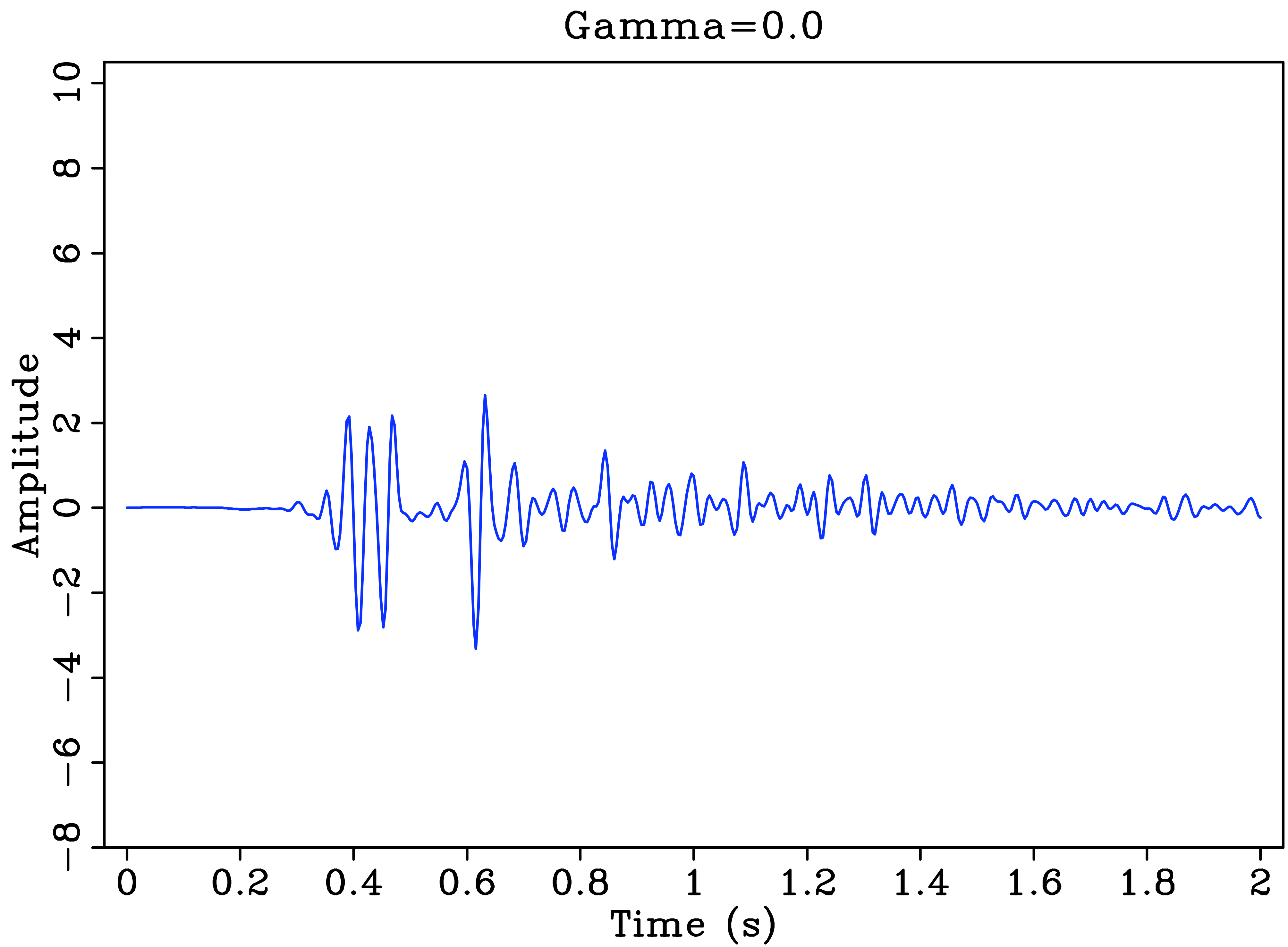
Only global wavelet matching no curvelet matching

Phase-space regularization



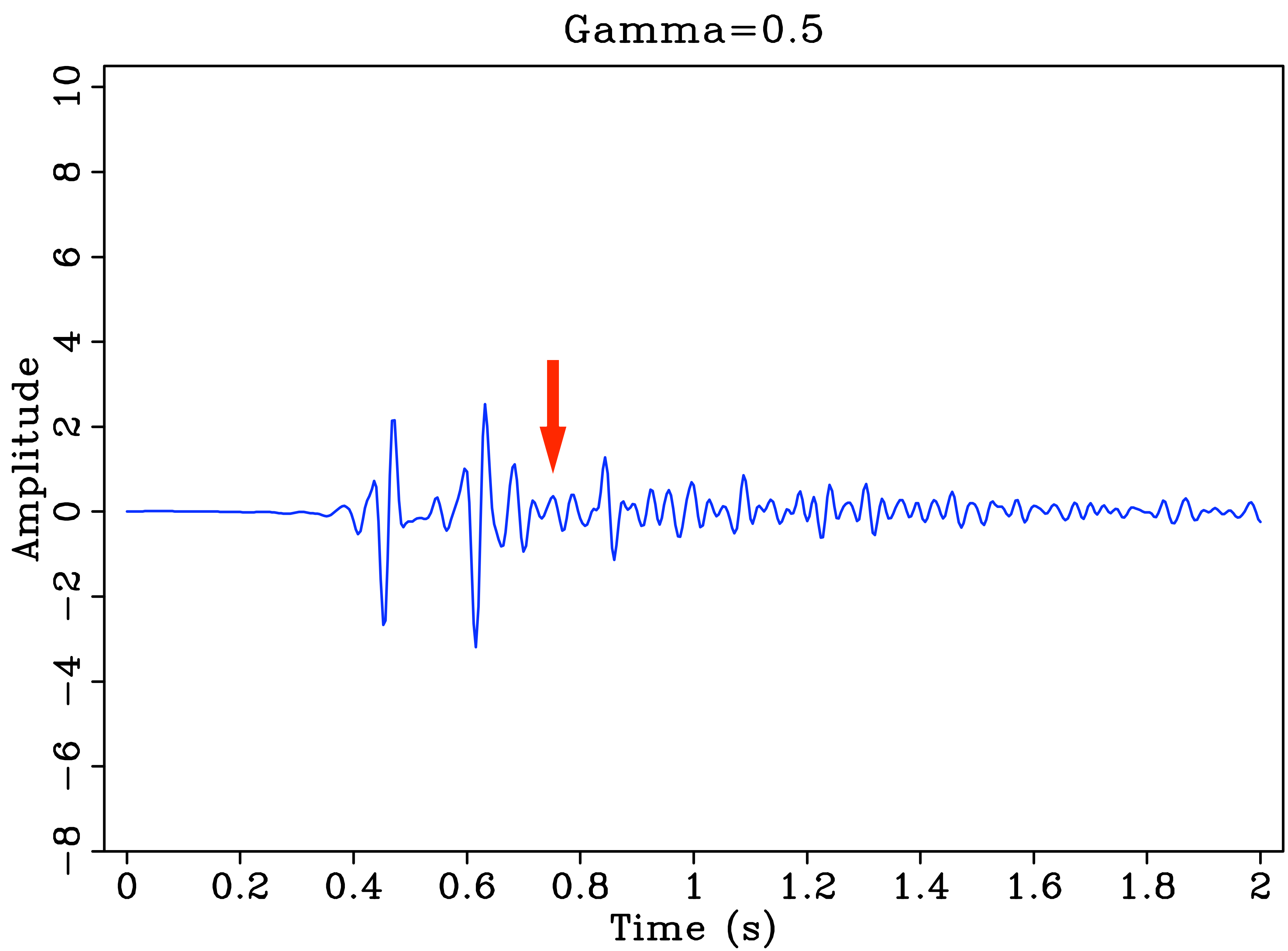
Total data

Phase-space regularization



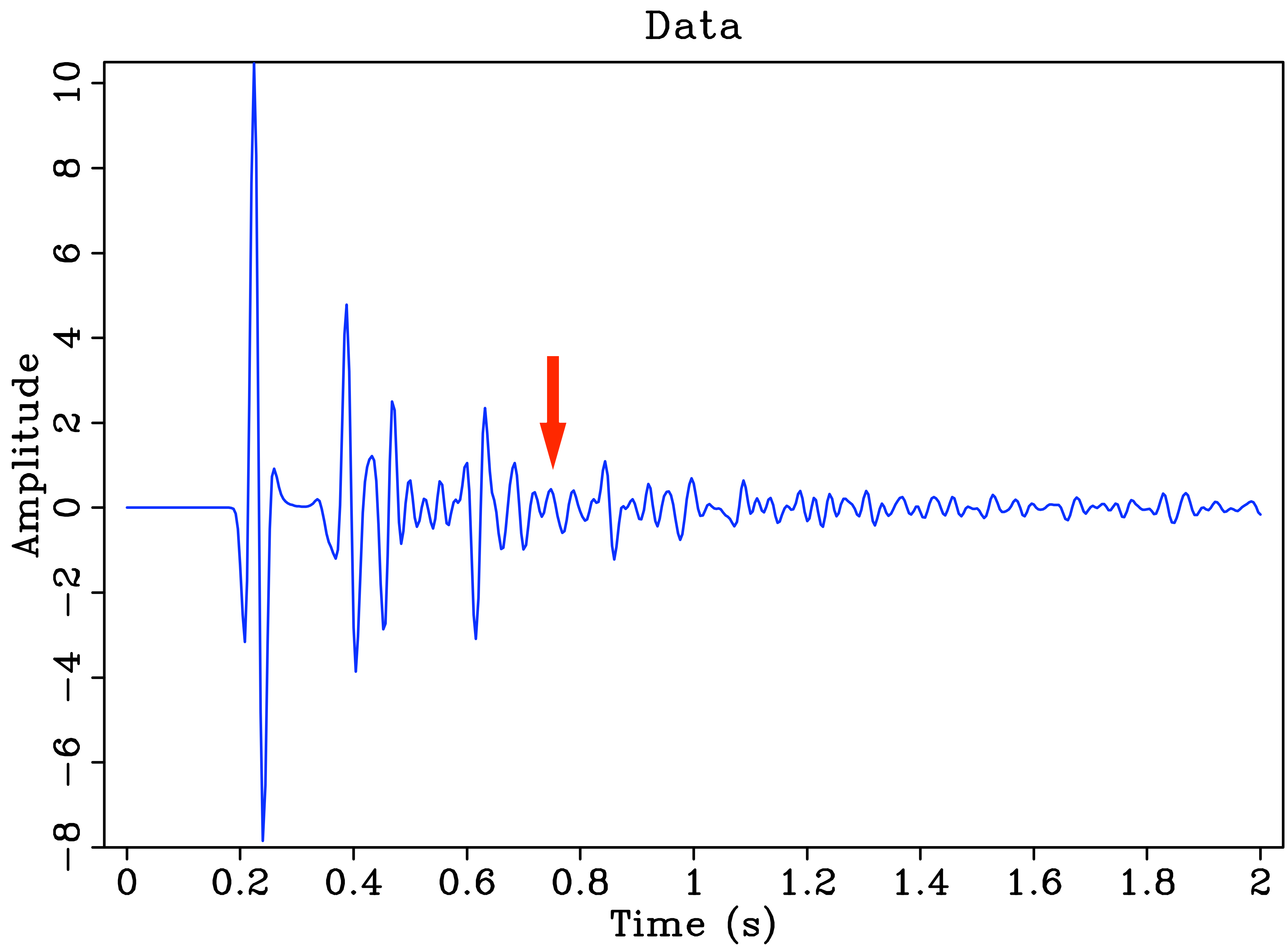
$\gamma = 0.0$

Phase-space regularization



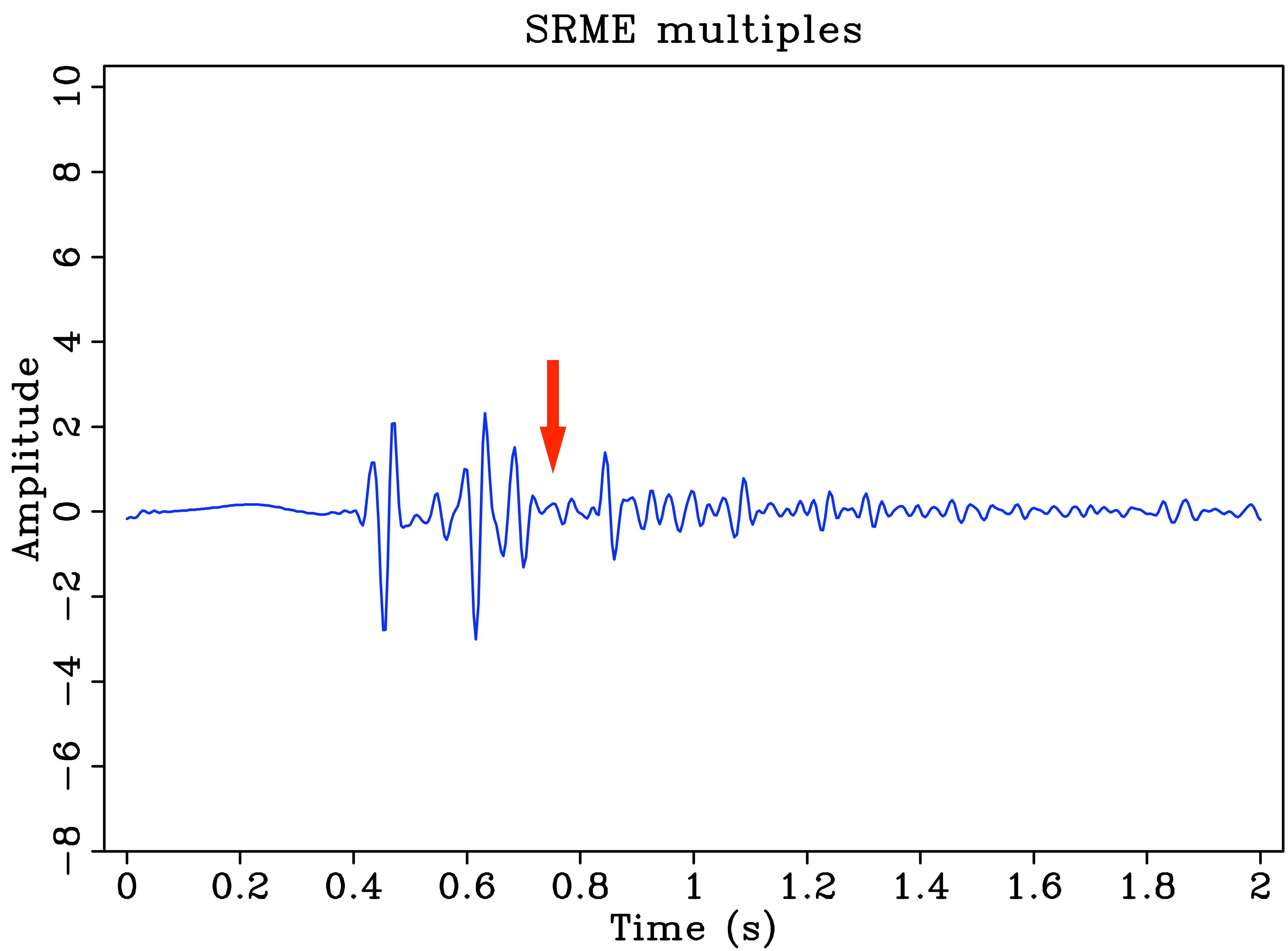
Correct curvelet matching

Phase-space regularization



Total data

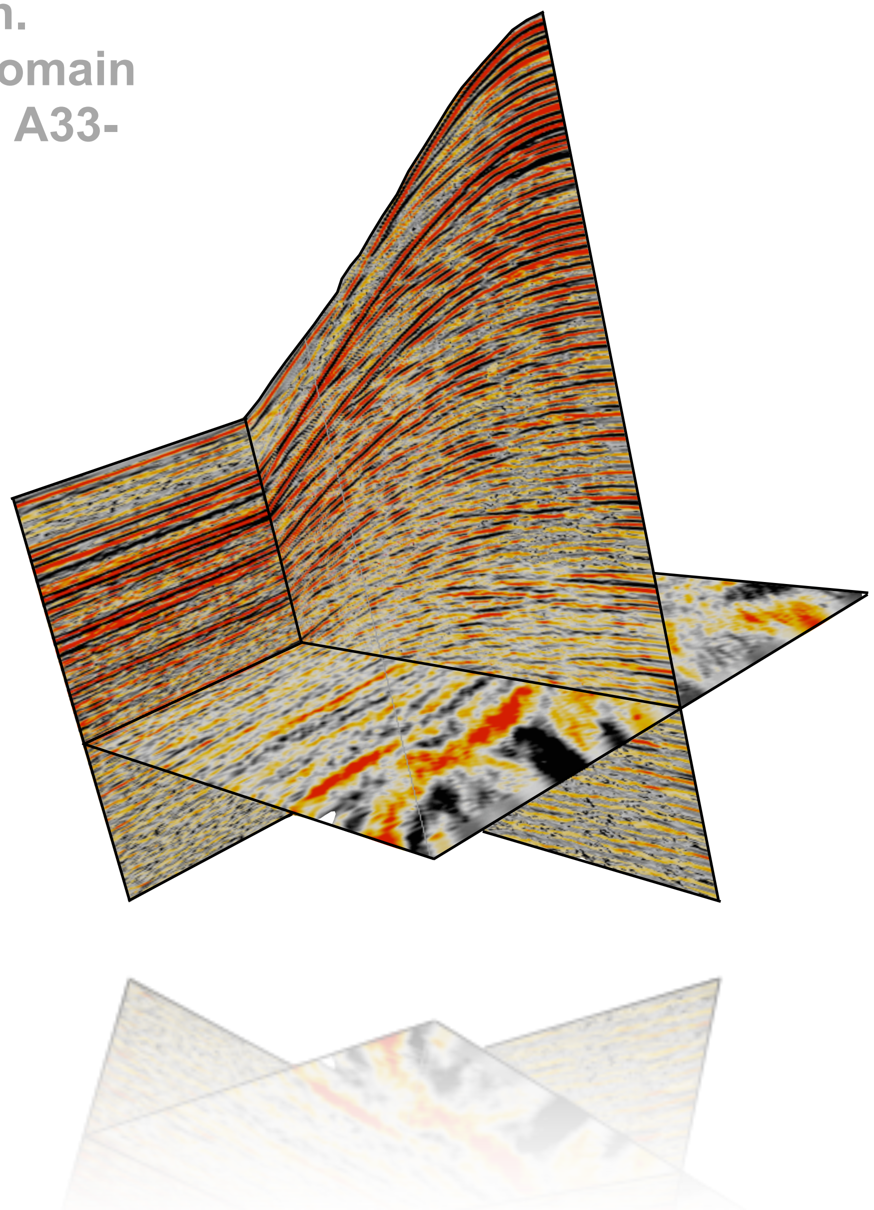
Phase-space regularization



SRME-windowed Fourier-matched multiples

Bayesian separation

D. Wang, R. Saab, O. Yilmaz and F J. Herrmann. Bayesian wavefield separation by transform-domain sparsity promotion. Geophysics, Vol 73, No. 5, A33-A38, 2008.



Curvelet-based Bayesian separation

Forward model: [Saab et. al '07, Wang et.al '07, '08]

$$\mathbf{b} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \quad \text{(total data)}$$

$$\mathbf{b}_1 = \mathbf{A}\mathbf{x}_1 + \mathbf{n}_1 \quad \text{(predicted primaries)}$$

$$\mathbf{b}_2 = \mathbf{A}\mathbf{x}_2 + \mathbf{n}_2 \quad \text{(predicted multiples)}$$

where

$\mathbf{x}_1$ curvelet coefficients of *primaries*

$\mathbf{x}_2$ curvelet coefficients of *multiples*

$\mathbf{A}$ inverse curvelet transform

Curvelet-based Bayesian separation

Involves the solution of the following nonlinear problem:

$$\mathbf{P_w}: \quad \begin{cases} \tilde{\mathbf{x}} \; = \; \arg\min_{\mathbf{x}} \lambda_1 \|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2 \|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\underline{\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2}\|_2^2 + \eta \|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

where

$\mathbf{b}_2$ predicted multiples

$\mathbf{A}$ inverse discrete curvelet transforms

$\tilde{\mathbf{s}}_{1,2}$ estimated primaries(1)and multiples(2)

$\lambda_{1,2}$ and η are control parameters

Can be solved by iterative soft thresholding.

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Curvelet-based Bayesian separation

Given initial estimates of $\mathbf{x}_1^0$ and $\mathbf{x}_2^0$, the n^{th} iteration of the algorithm proceeds as follows

$$\begin{aligned}
\mathbf{x}_1^{n+1} &= \mathbf{T}_{\frac{\boldsymbol{\lambda}_1 \mathbf{w}_1}{2\eta}} \left[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n + \mathbf{x}_1^n\right] \\
\mathbf{x}_2^{n+1} &= \mathbf{T}_{\frac{\boldsymbol{\lambda}_2 \mathbf{w}_2}{2(1+\eta)}} \left[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{x}_2^n + \frac{\eta}{\eta+1}\left(\mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n\right)\right]
\end{aligned}$$

where $\mathbf{T}_{\mathbf{u}} : \mathbb{R}^{|\mathcal{M}|} \mapsto \mathbb{R}^{|\mathcal{M}|}$ is the elementwise soft-thresholding operator, i.e.,

$$T_{u_\mu}(v_\mu) := \frac{v_\mu}{|v_\mu|} \cdot \max(0, |v_\mu| - | u_\mu|)$$

Curvelet-based Bayesian separation

Parametrization:

η	Prediction confidence parameter
λ_1	Expected reflector sparsity
λ_2	Expected surface wave sparsity

Limiting case:

$\eta \rightarrow \infty$ Total lack of confidence => block-relaxation

Workflow

input data

$$\mathbf{m}_{\text{predicted}} = \mathbf{P}\mathbf{p} \text{ (multi-D convolution)}$$

conservative Fourier matching

$$\mathbf{m}_0 = \mathbf{F}\mathbf{m}_{\text{predicted}} \text{ with } \mathbf{F} = \mathcal{F}^H \text{diag}\left(\hat{\mathbf{f}}\right)\mathcal{F}$$

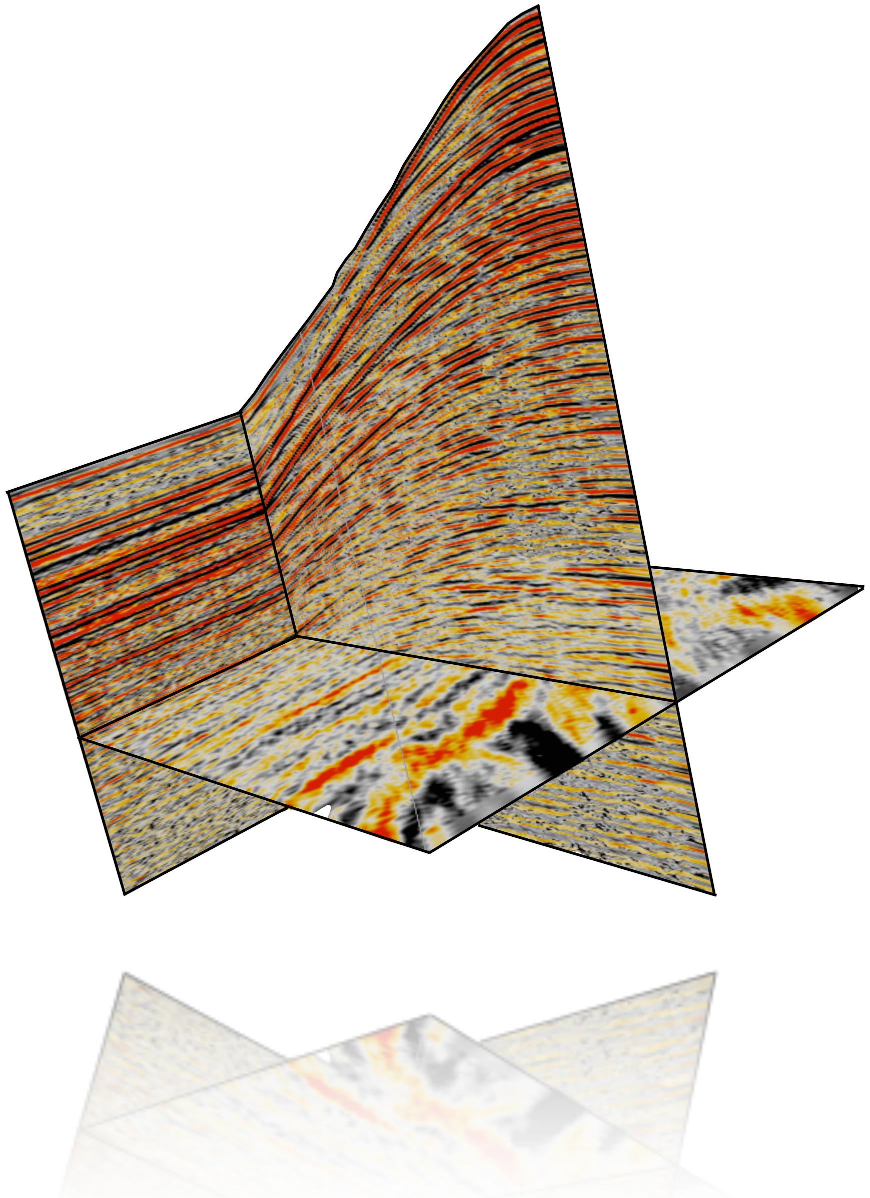
curvelet-domain matching

$$\begin{aligned} \mathbf{b}_2 &= \mathbf{B}\mathbf{m}_0 \\ \text{with } \mathbf{B} &= \mathbf{C}^T \text{diag}\left(e^{\mathbf{z}}\right)\mathbf{C}\mathbf{m}_0 \\ &\approx \mathcal{F}^H b(x,k)\mathcal{F}\mathbf{m}_0 \end{aligned}$$

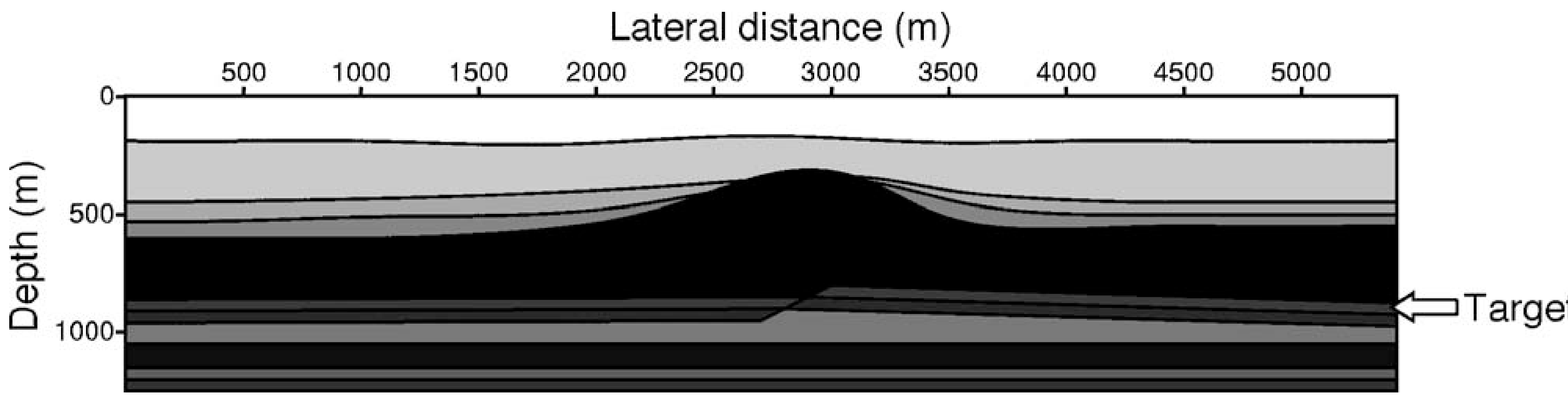
Bayesian separation

$$\mathbf{P}_{\mathbf{w}}: \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda_1\|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2\|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2\|_2^2 + \eta\|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

Synthetic-data example

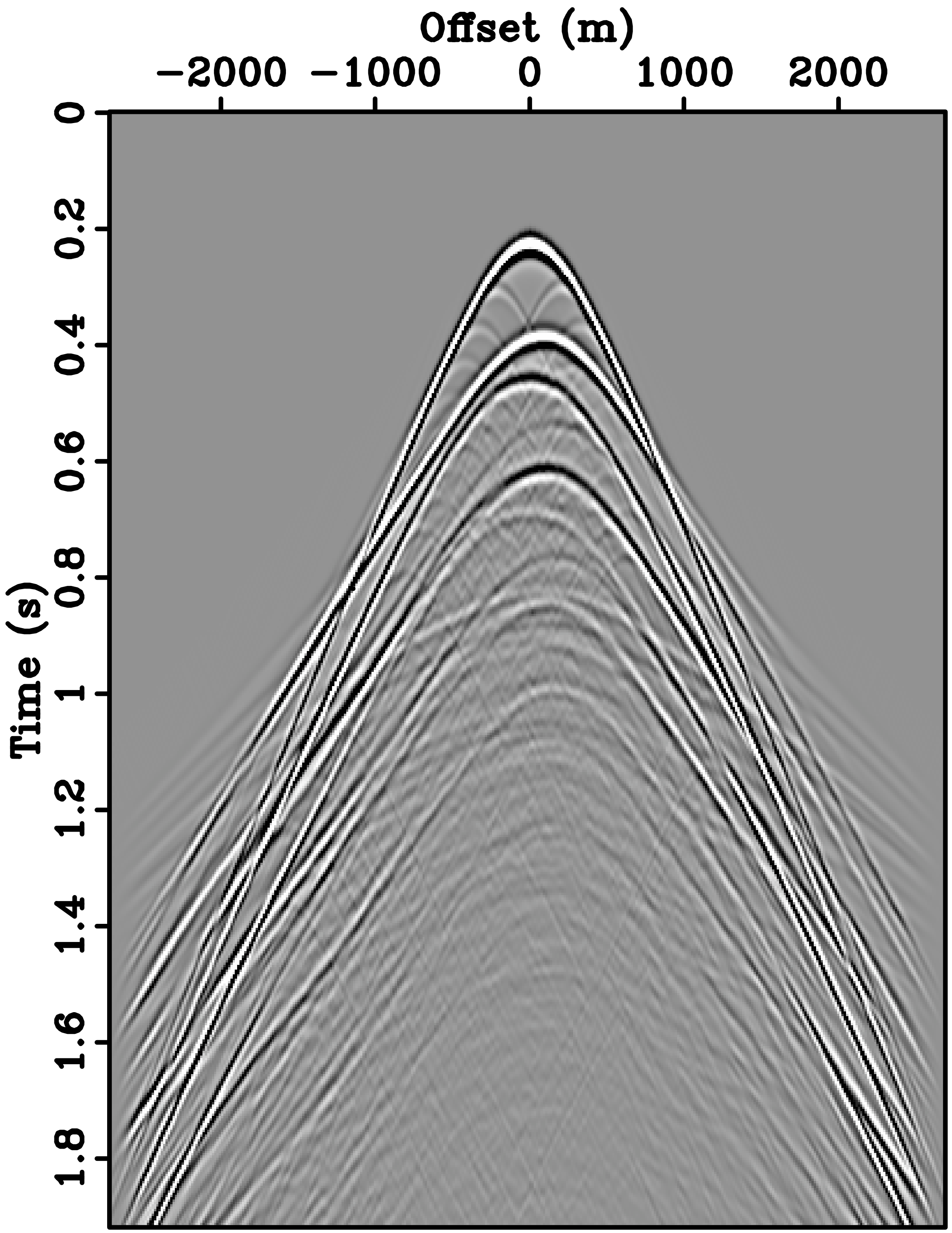


Synthetic-data example

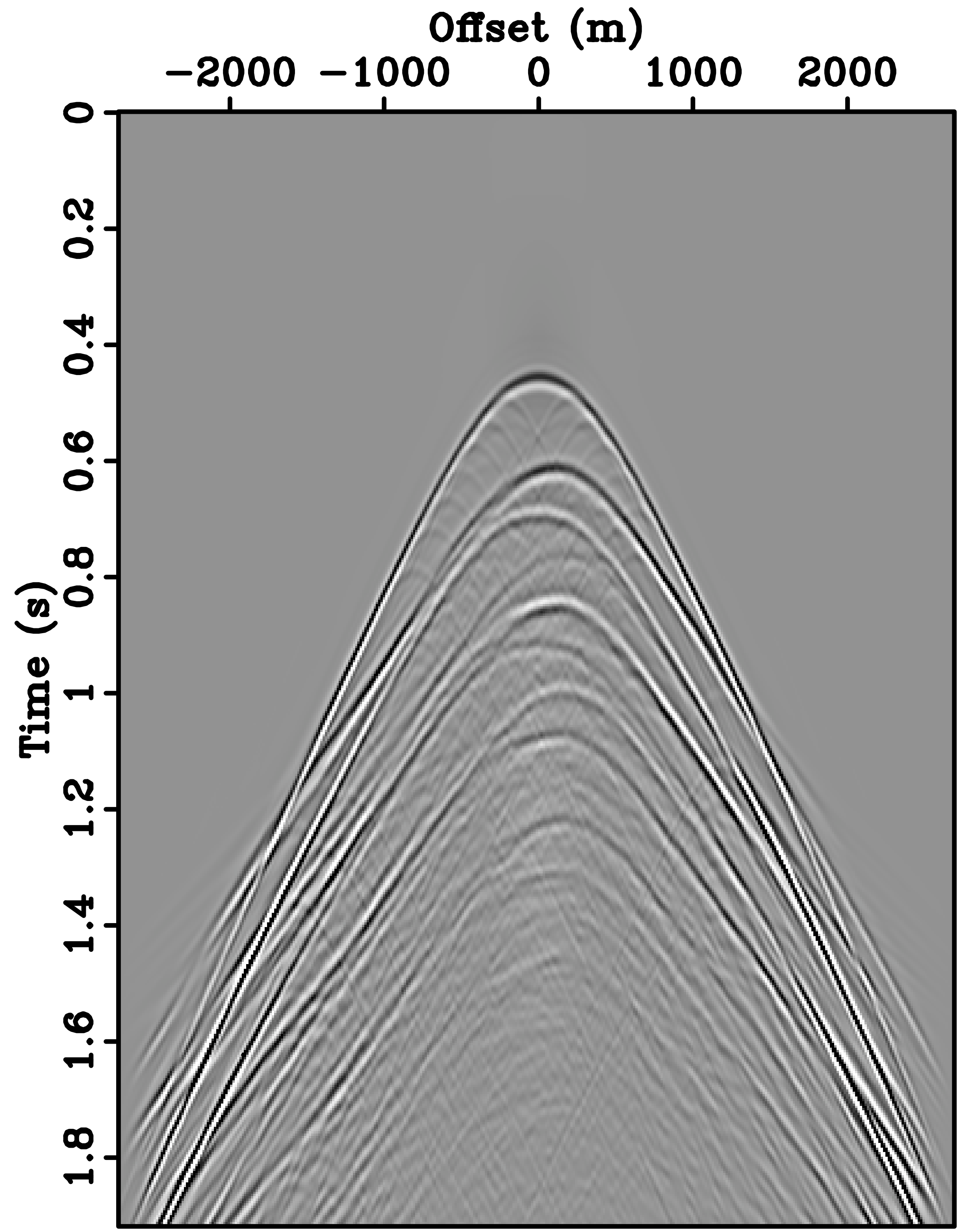


Velocity model used in the synthetic data examples

Synthetic-data example

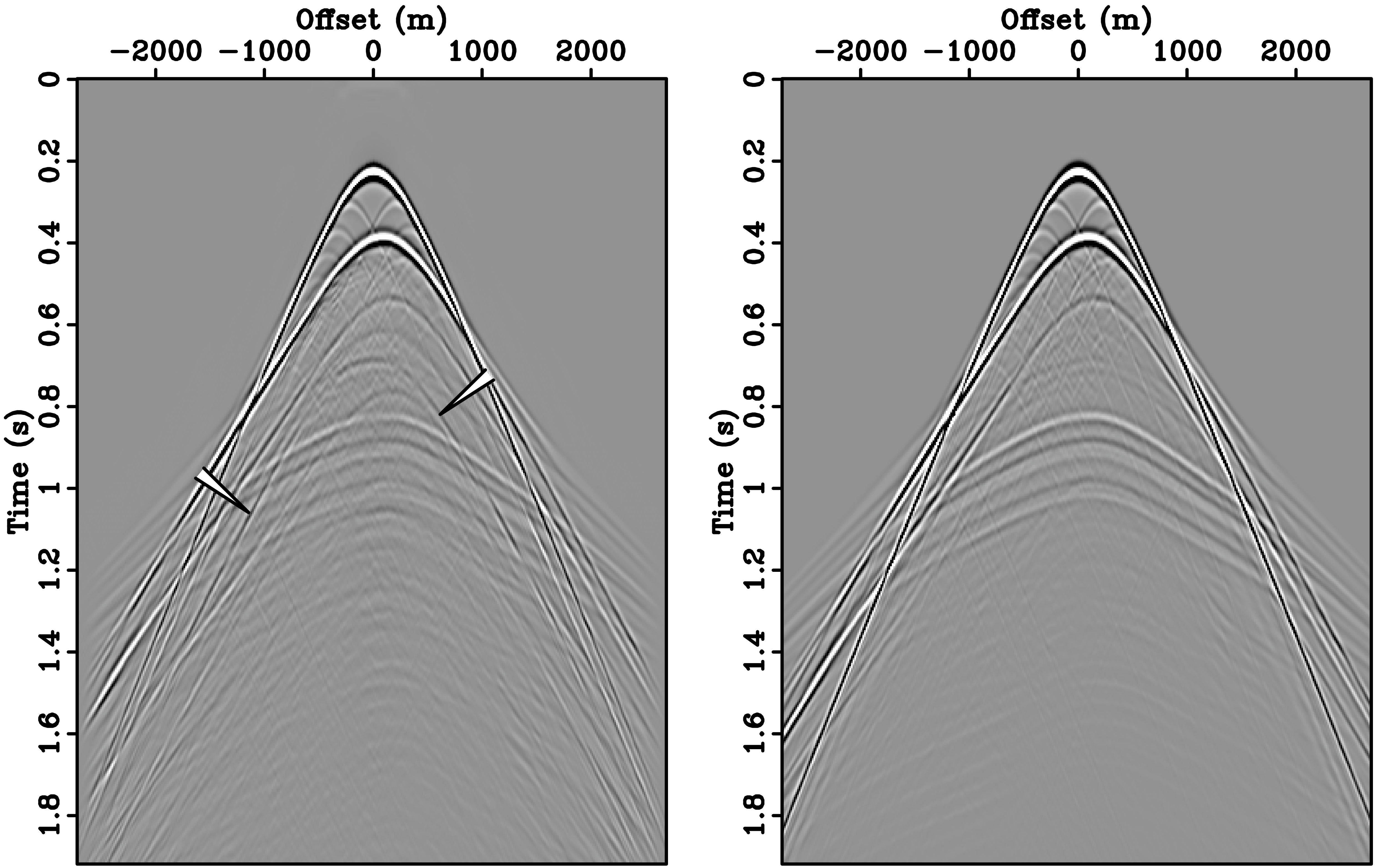


Total data



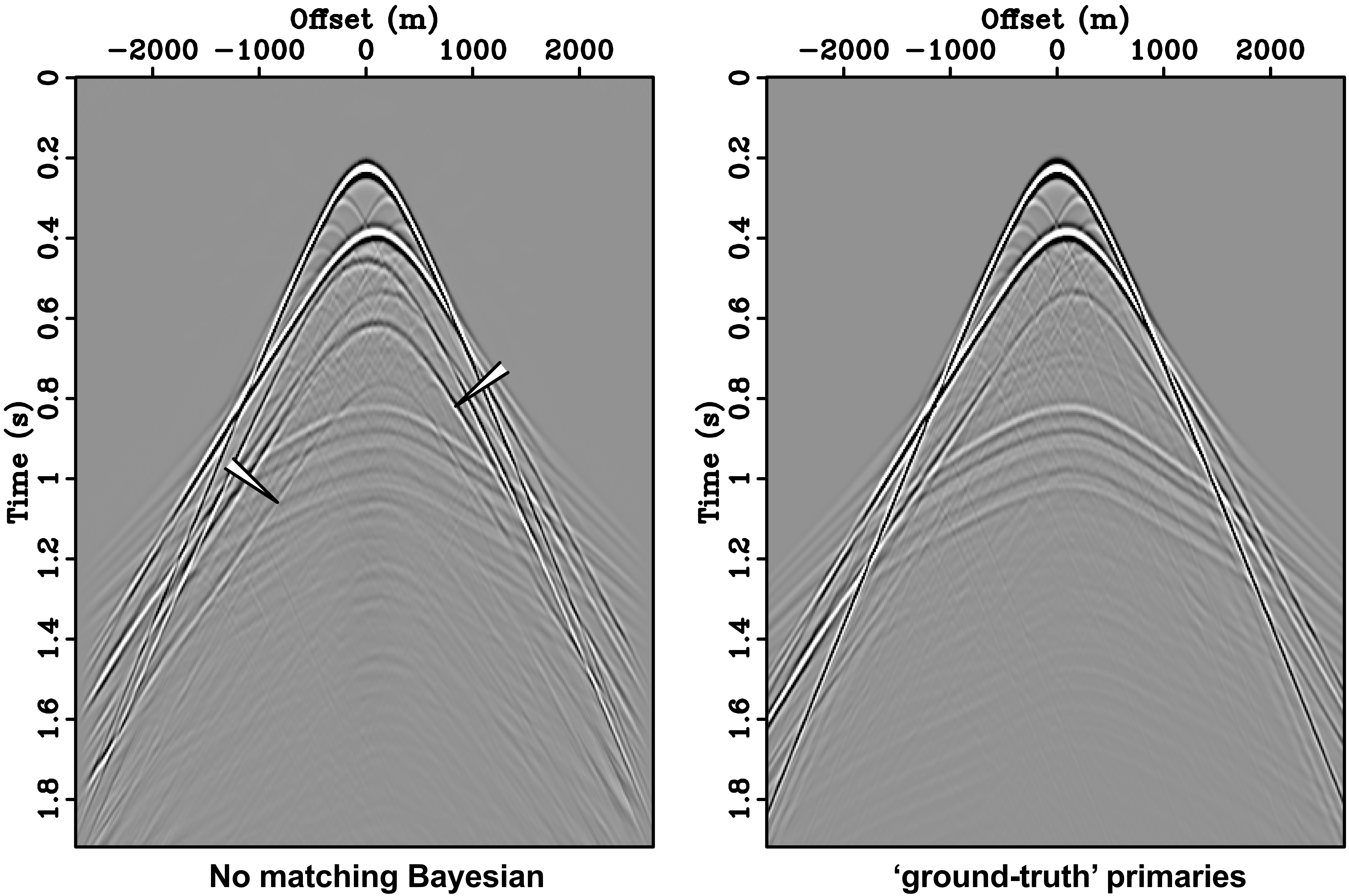
Single-term SRME multiples

Synthetic-data example

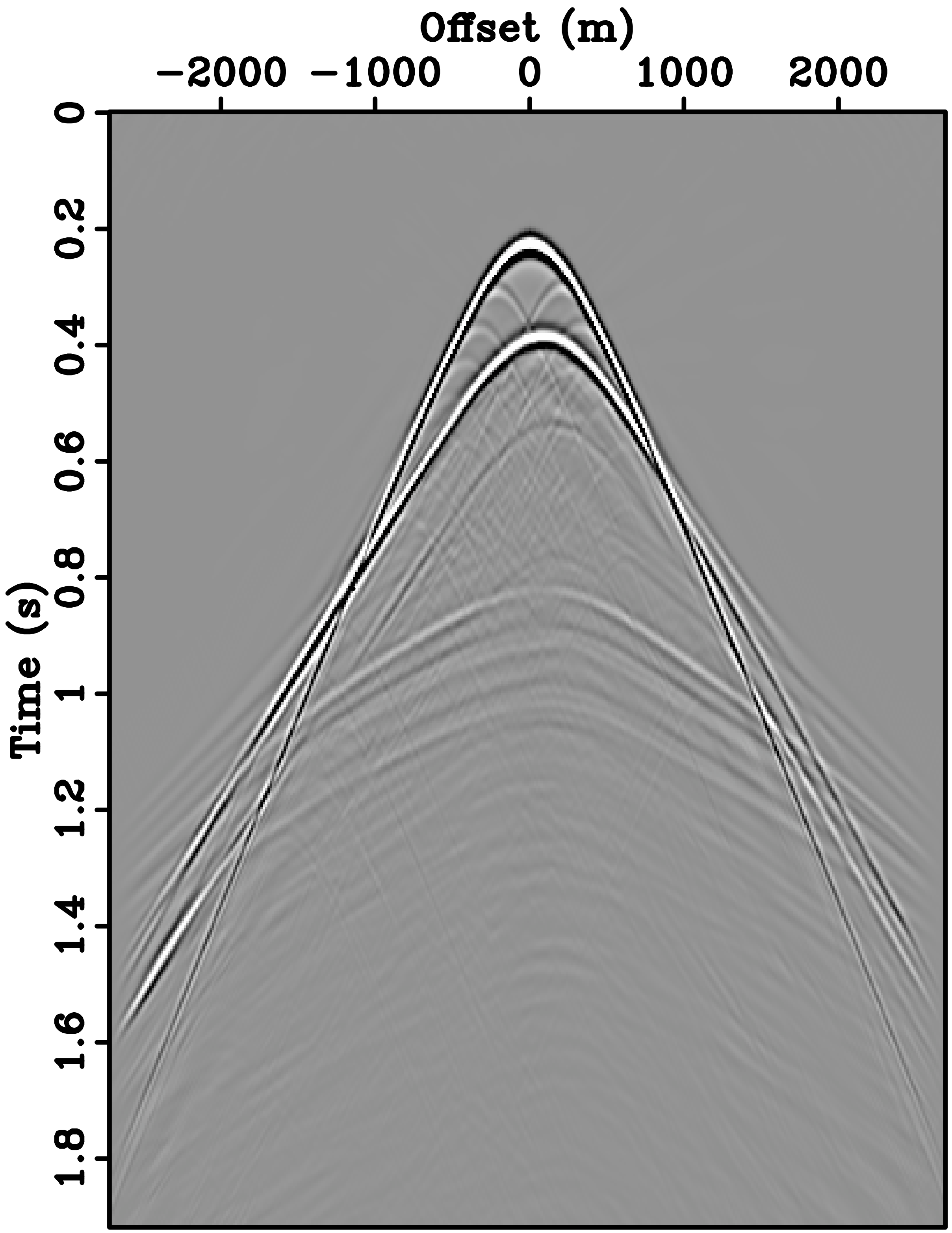


SRME primaries **'ground-truth' primaries**

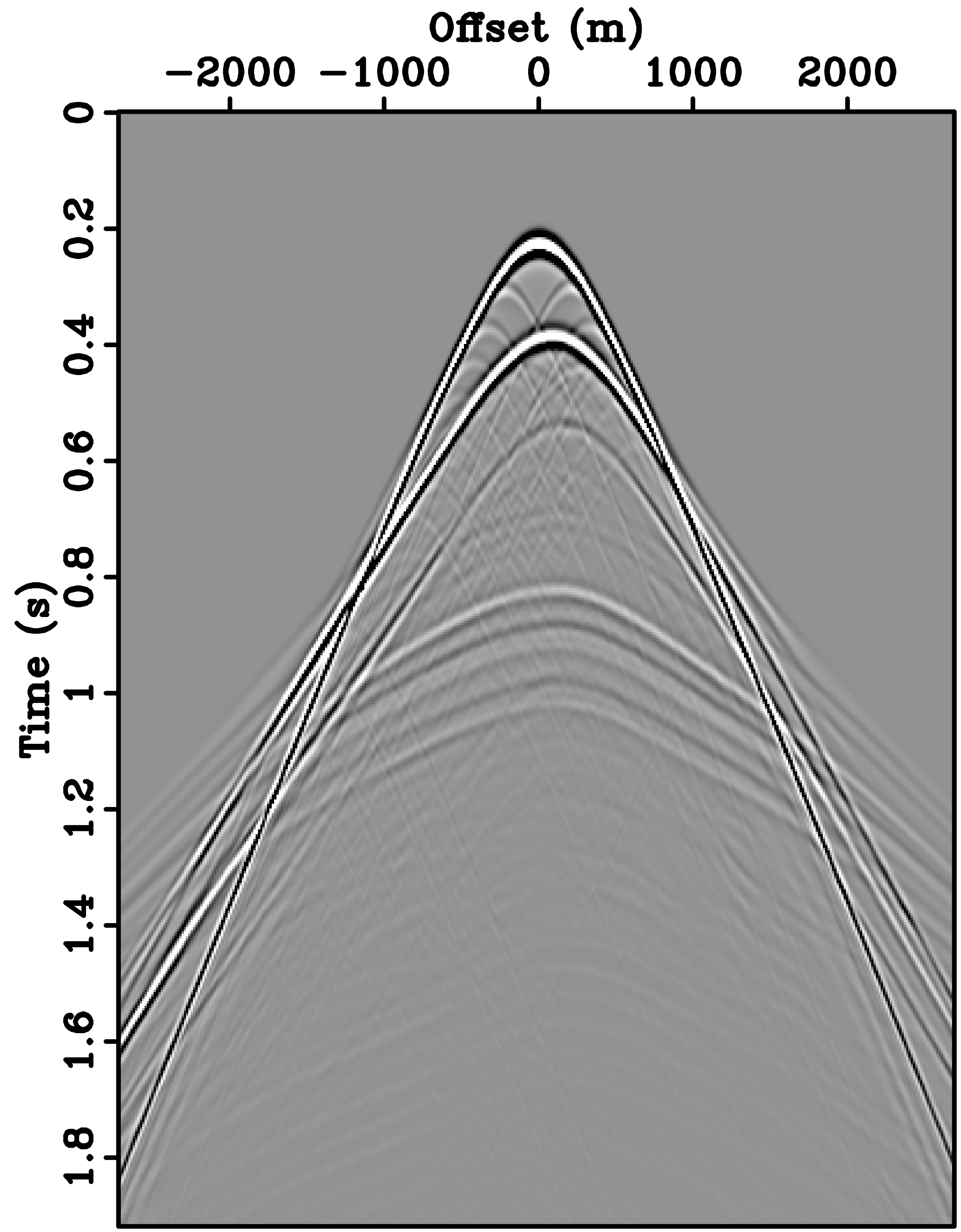
Synthetic-data example



Synthetic-data example

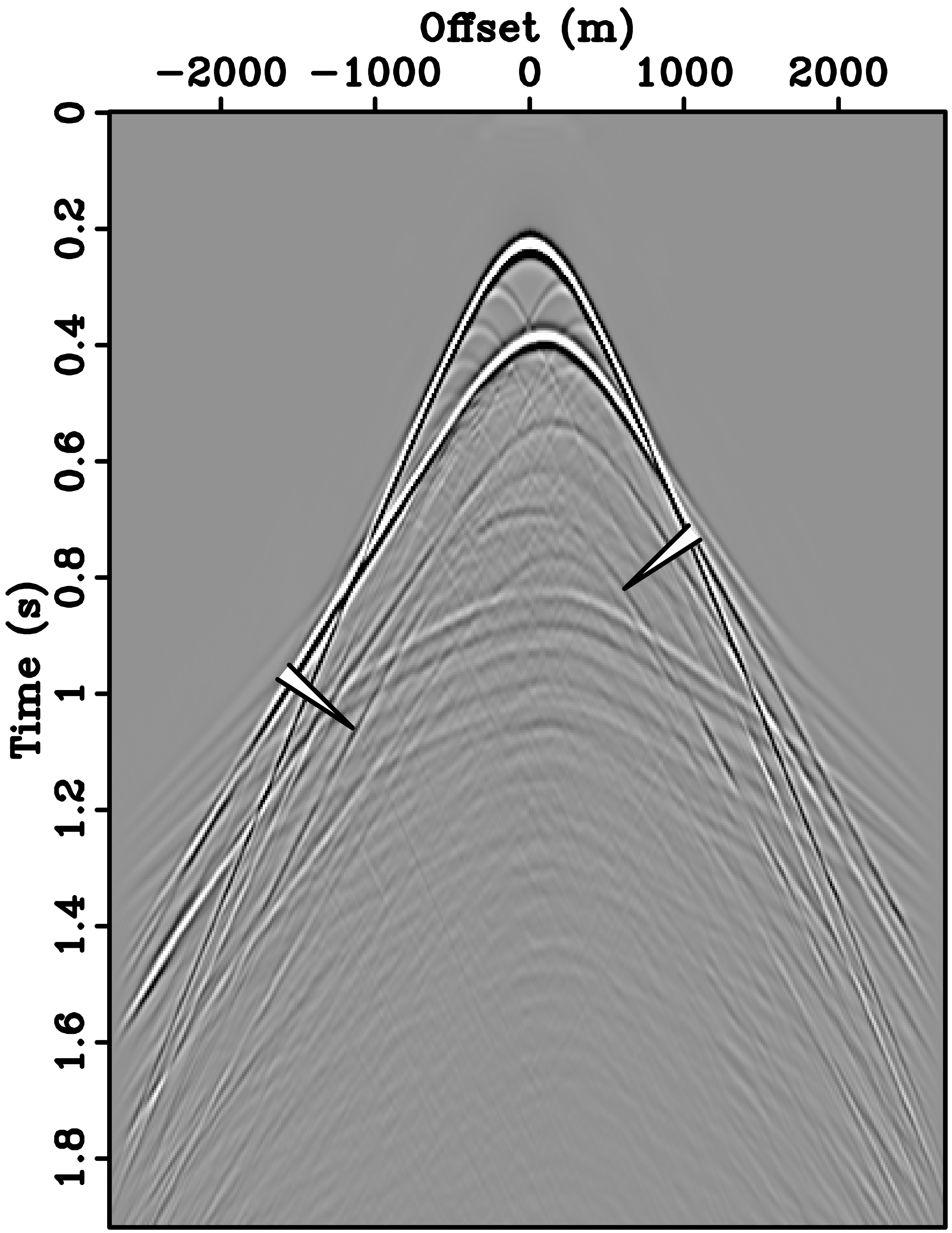


Matched + Bayesian

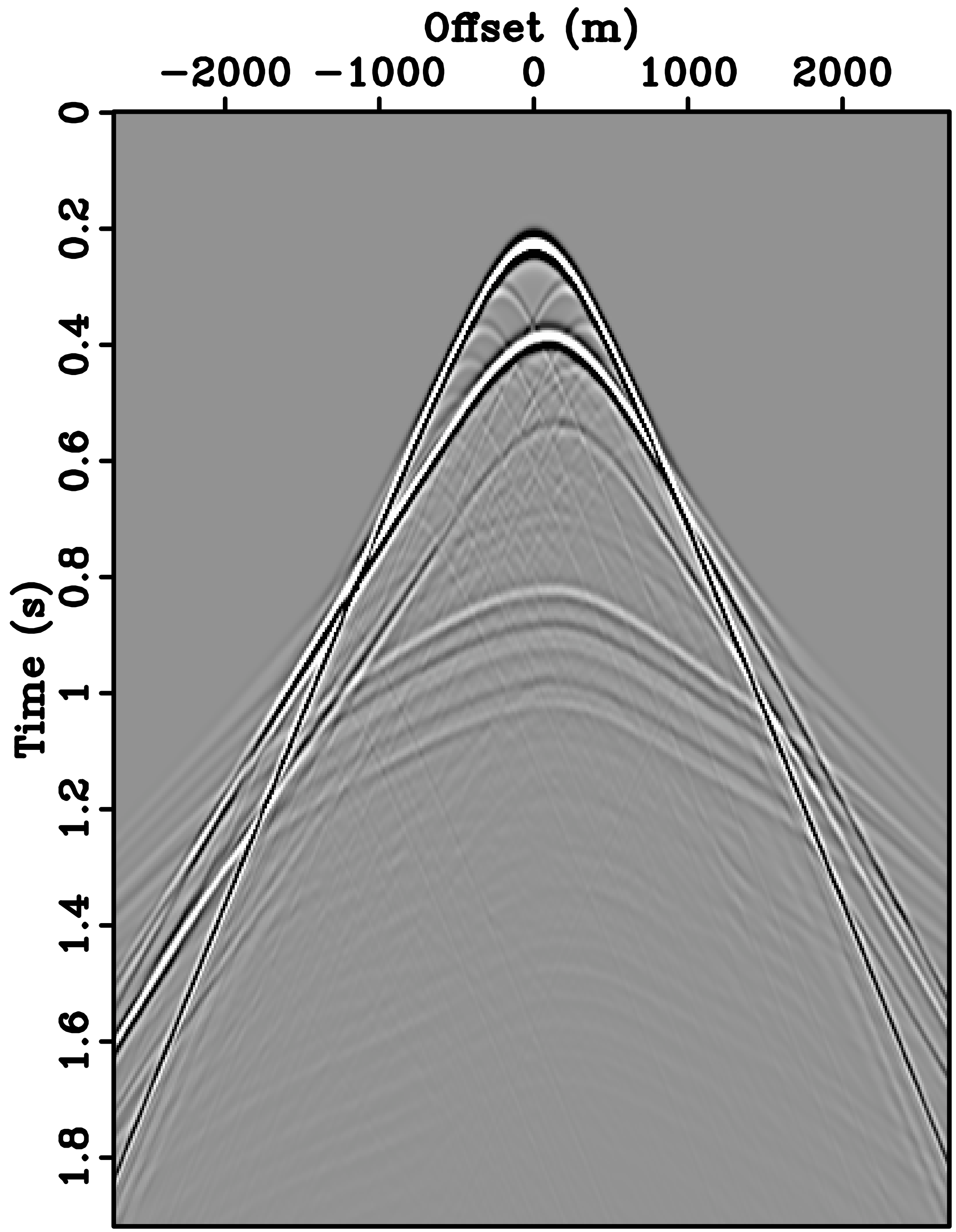


'ground-truth' primaries

Synthetic-data example

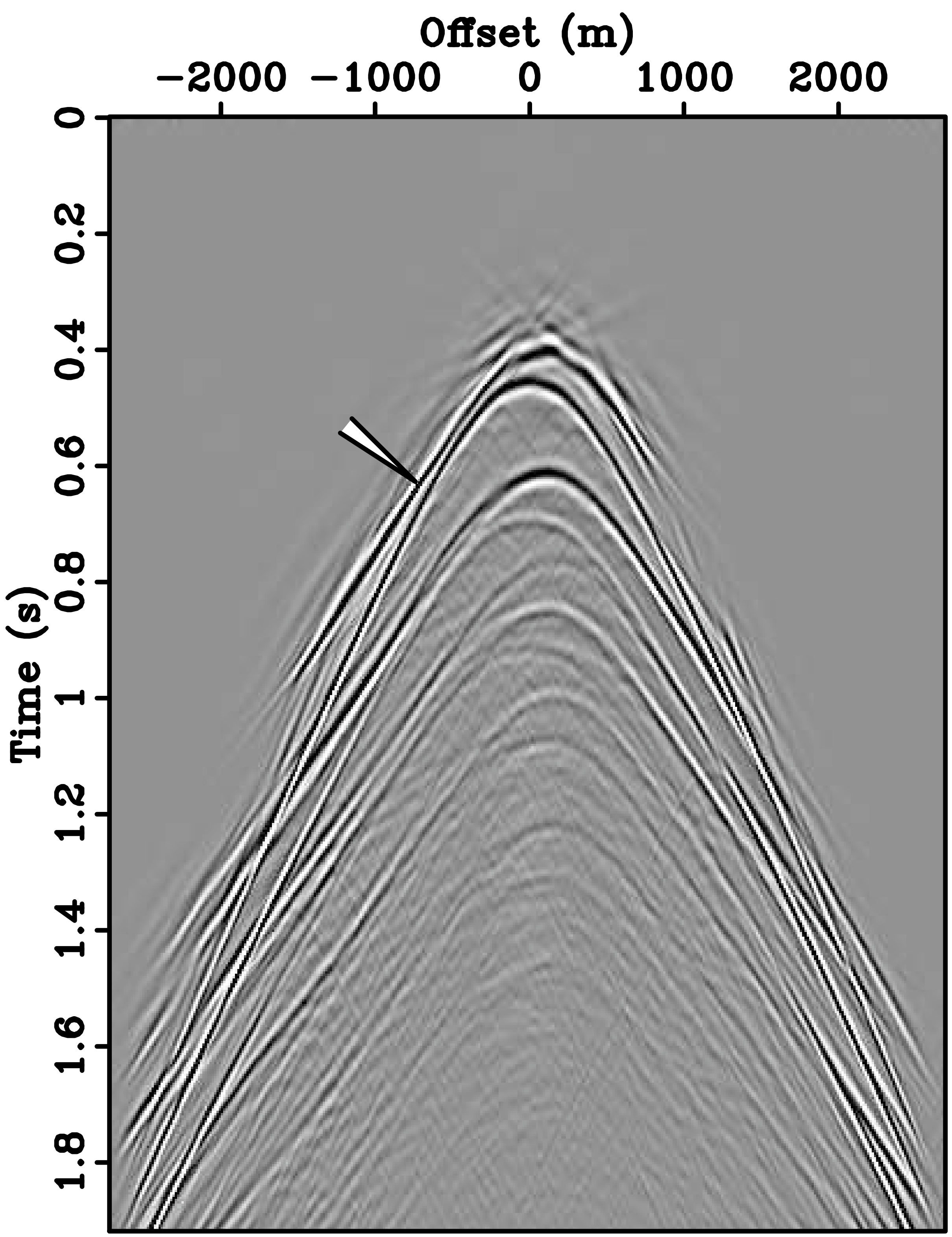


SRME primaries

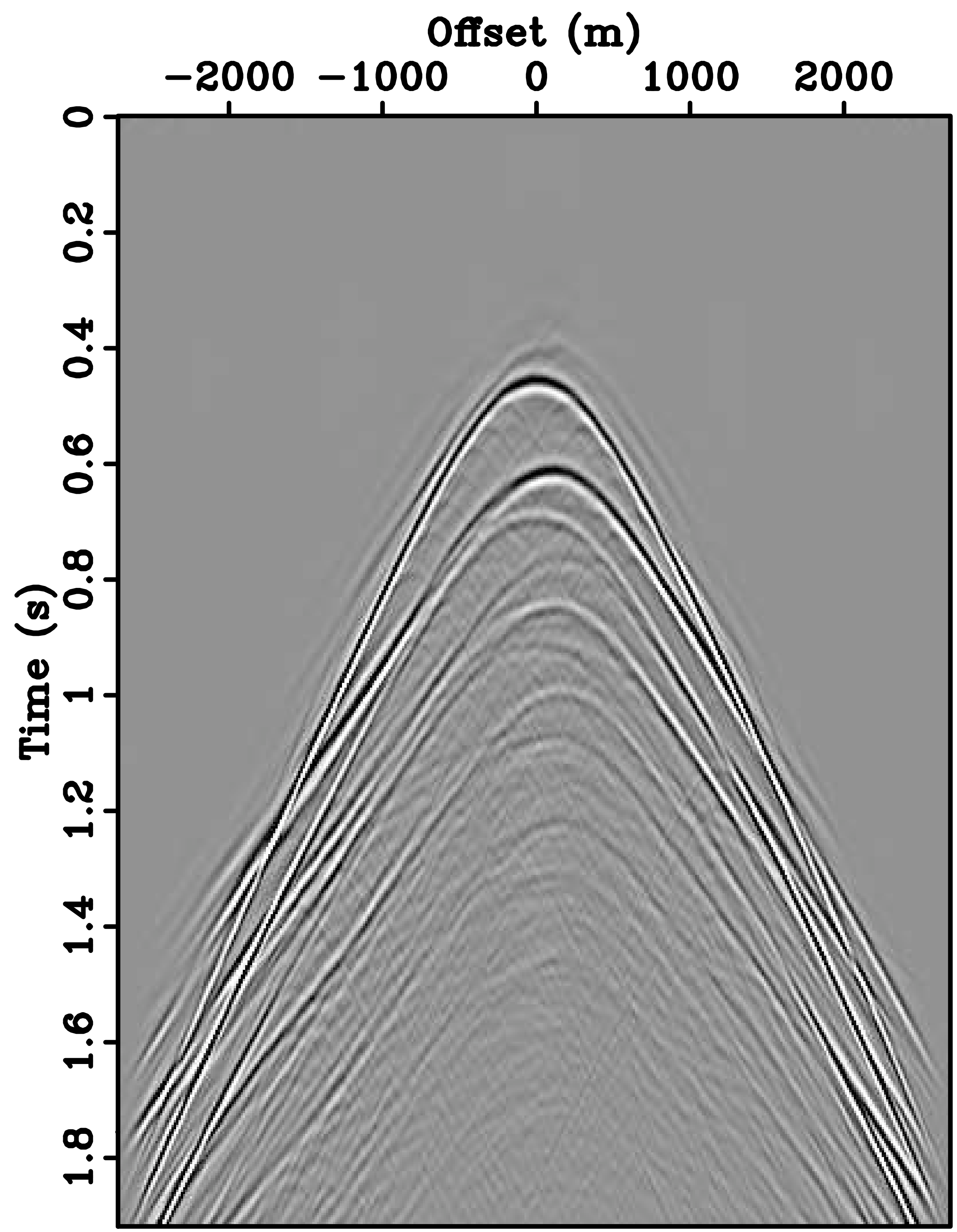


‘ground-truth’ primaries

Synthetic-data example

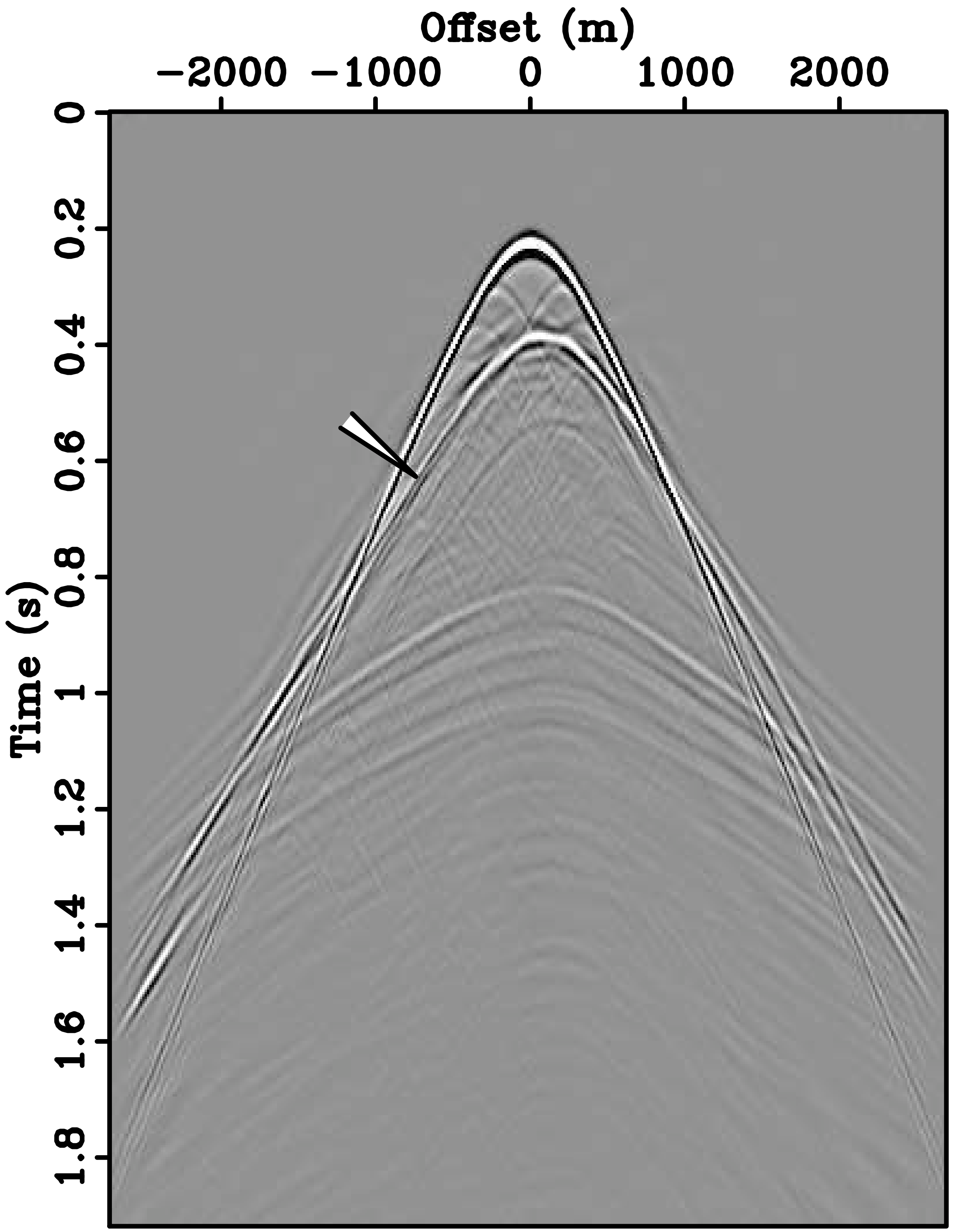


Over matched multiples

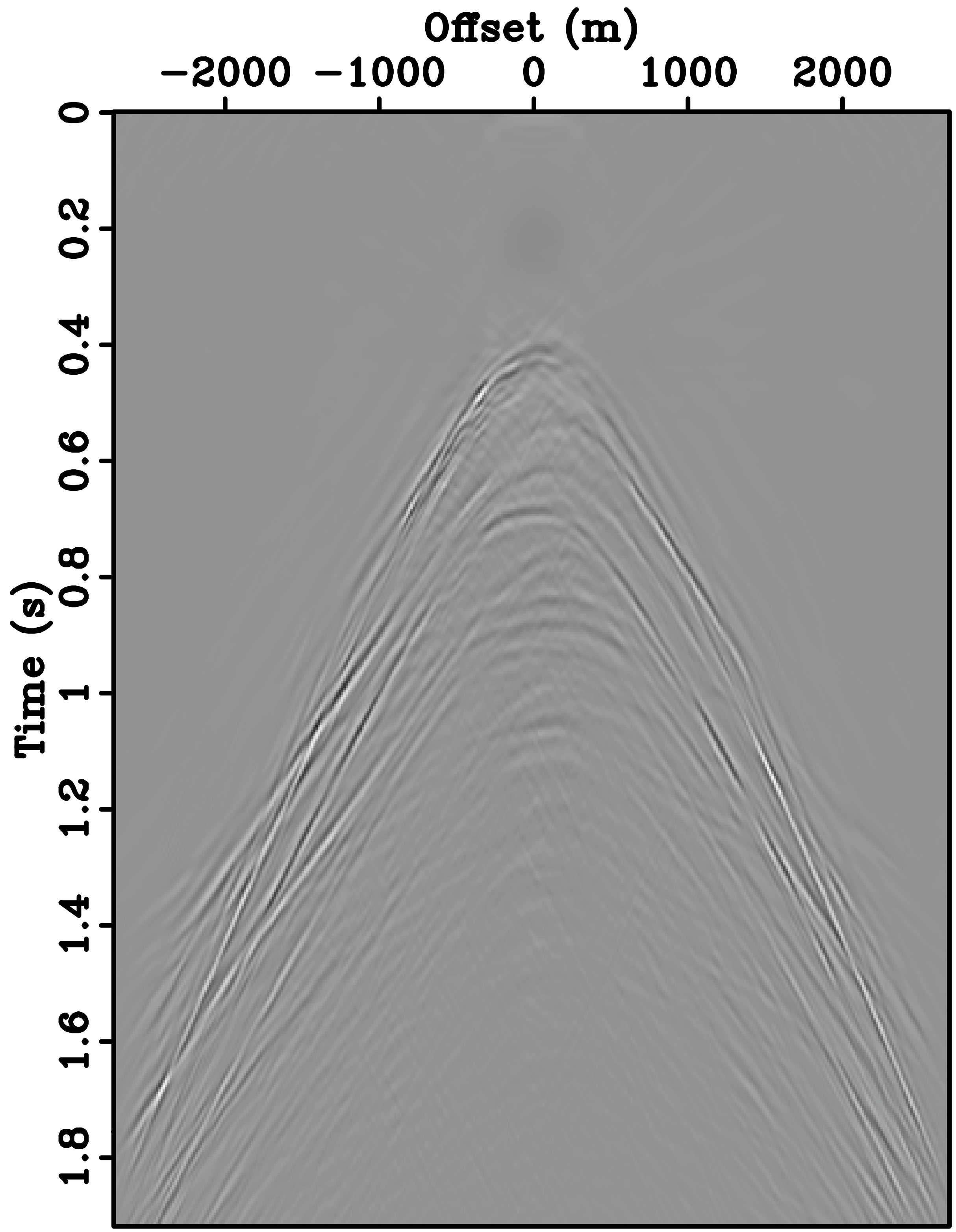


Correctly matched multiples

Synthetic-data example



Estimate for the primaries with over matched multiples



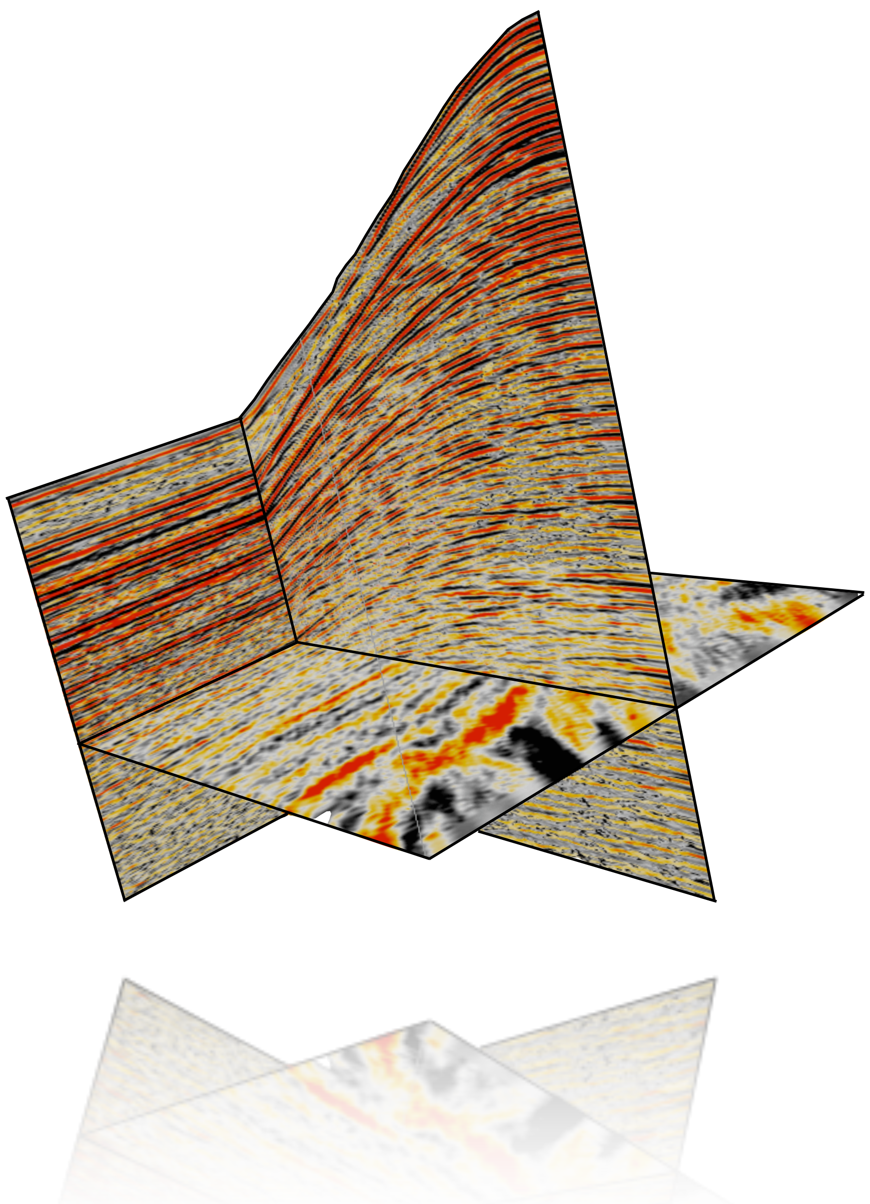
Difference between SRME and curvelet matching

SNRs

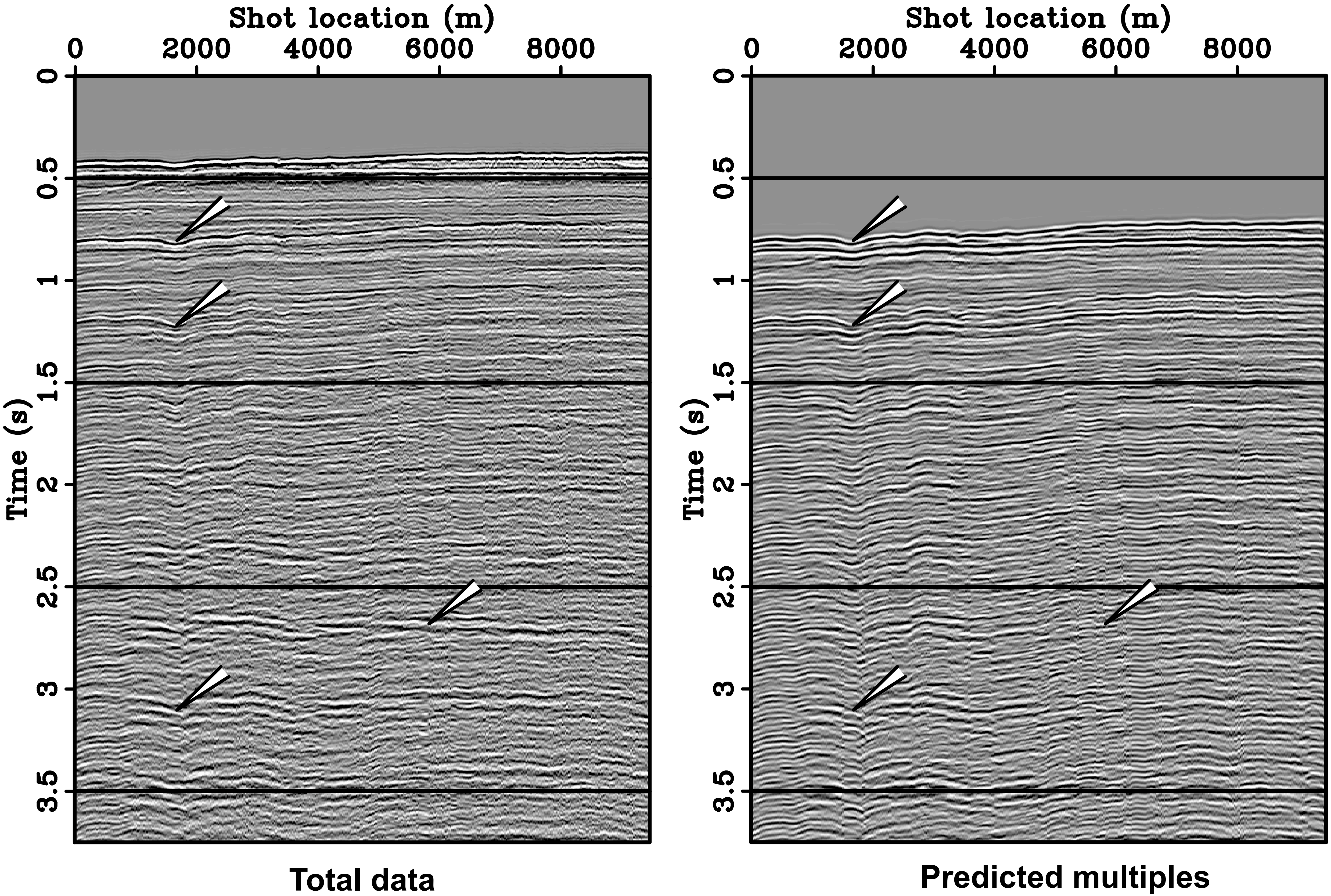
- Comparison with “ground truth”

SRME	9.82
Bayesian separation	7.25
Curvele-domain matching & Bayesuan	11.22

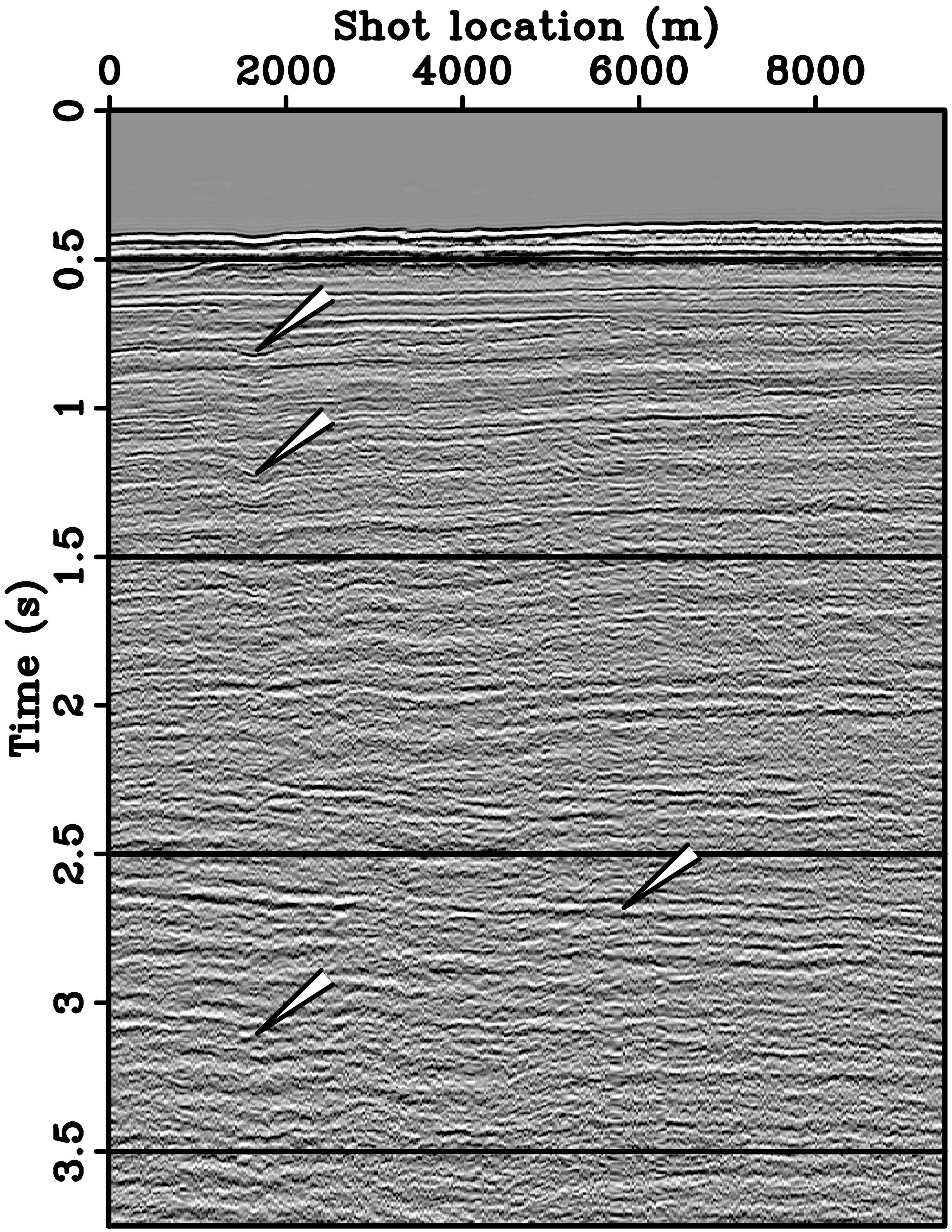
Real-data example



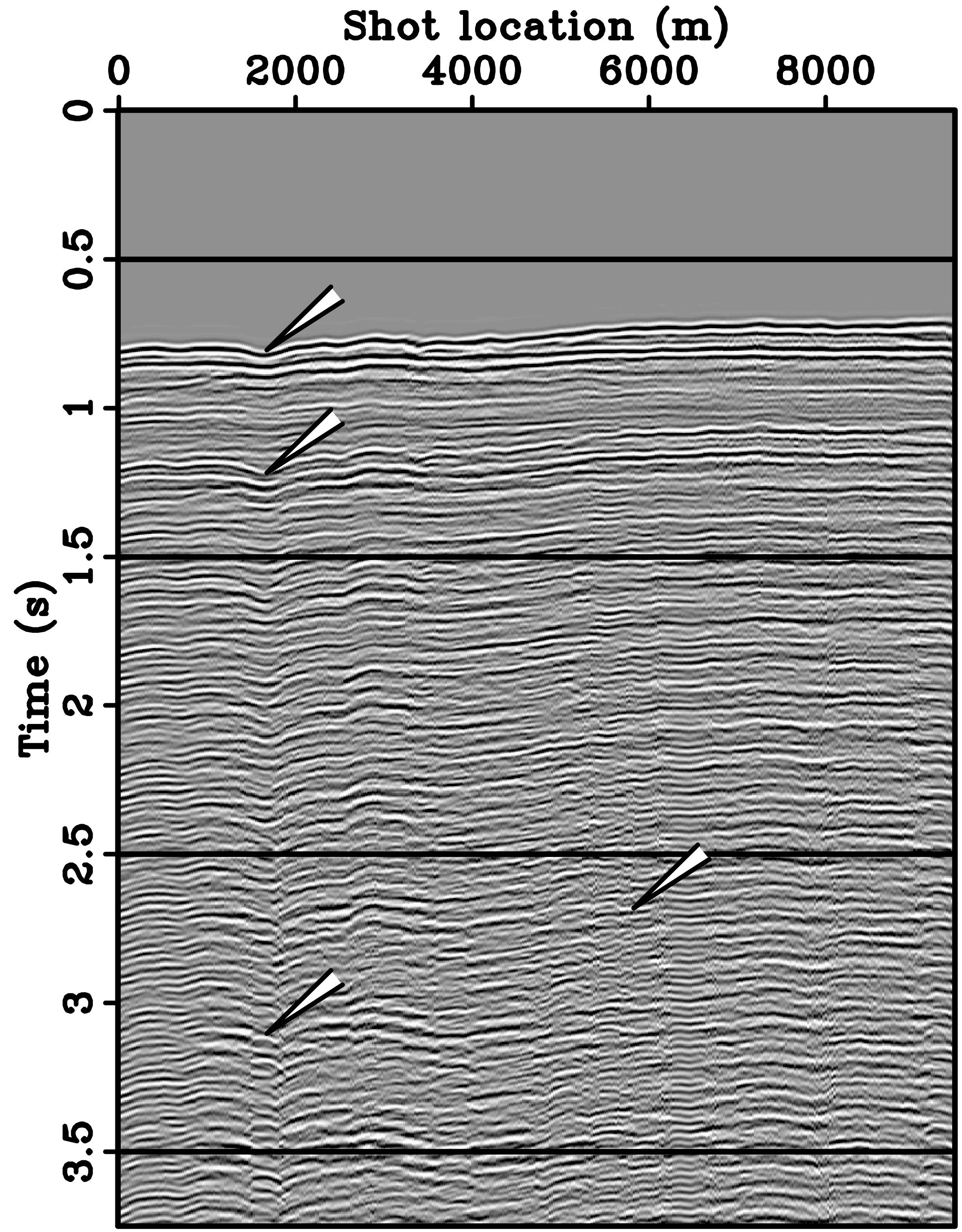
Real-data example



Real-data example

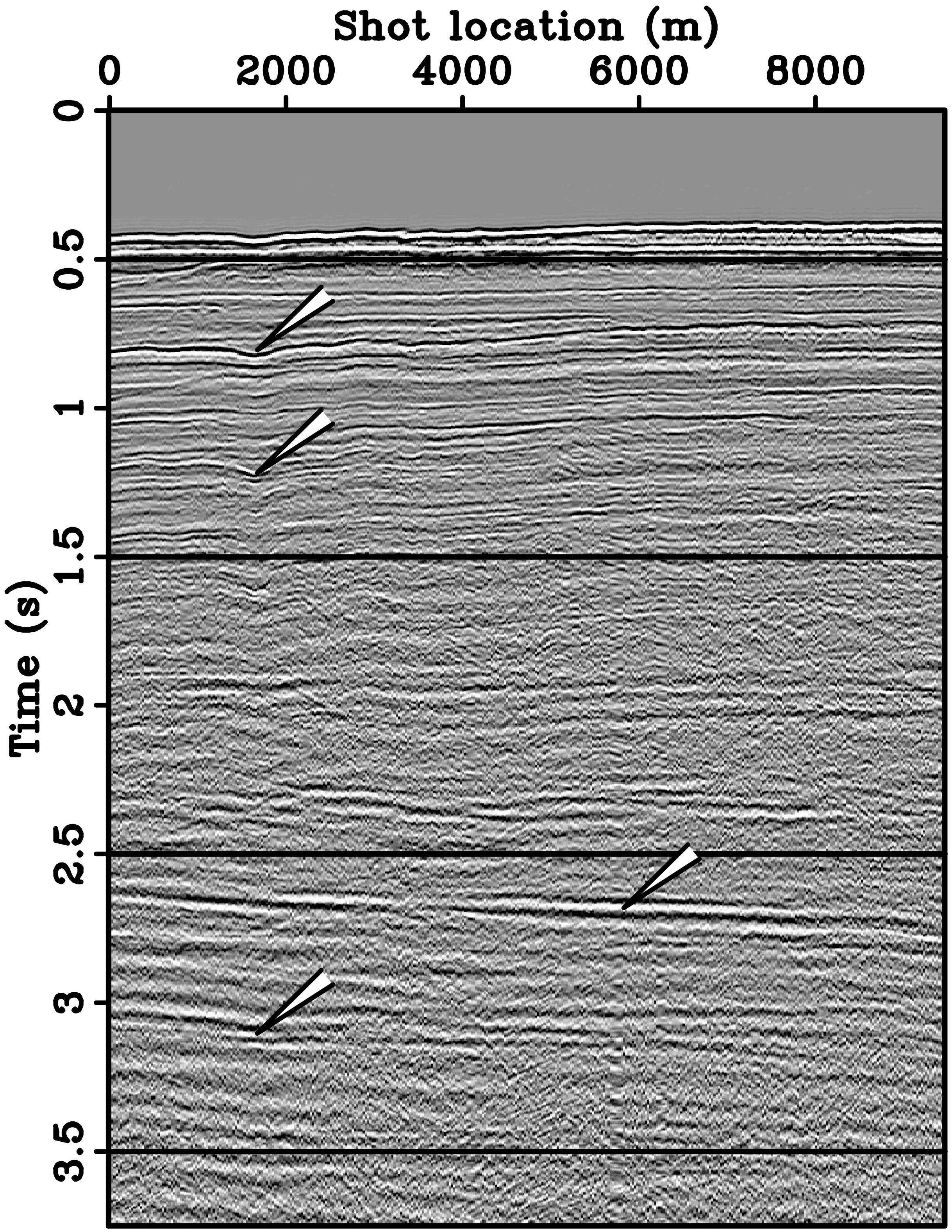


SRME primaries

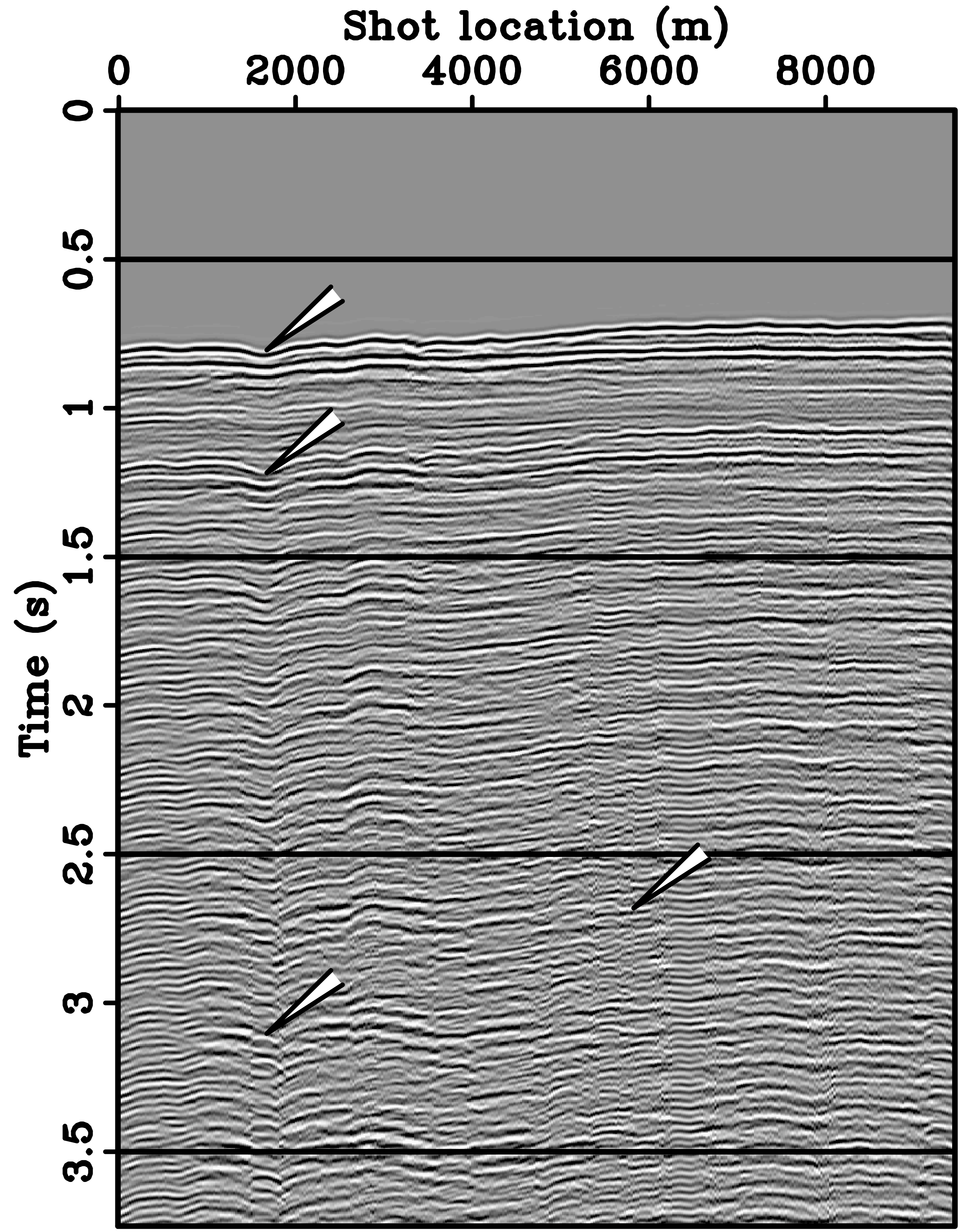


Predicted multiples

Real-data example

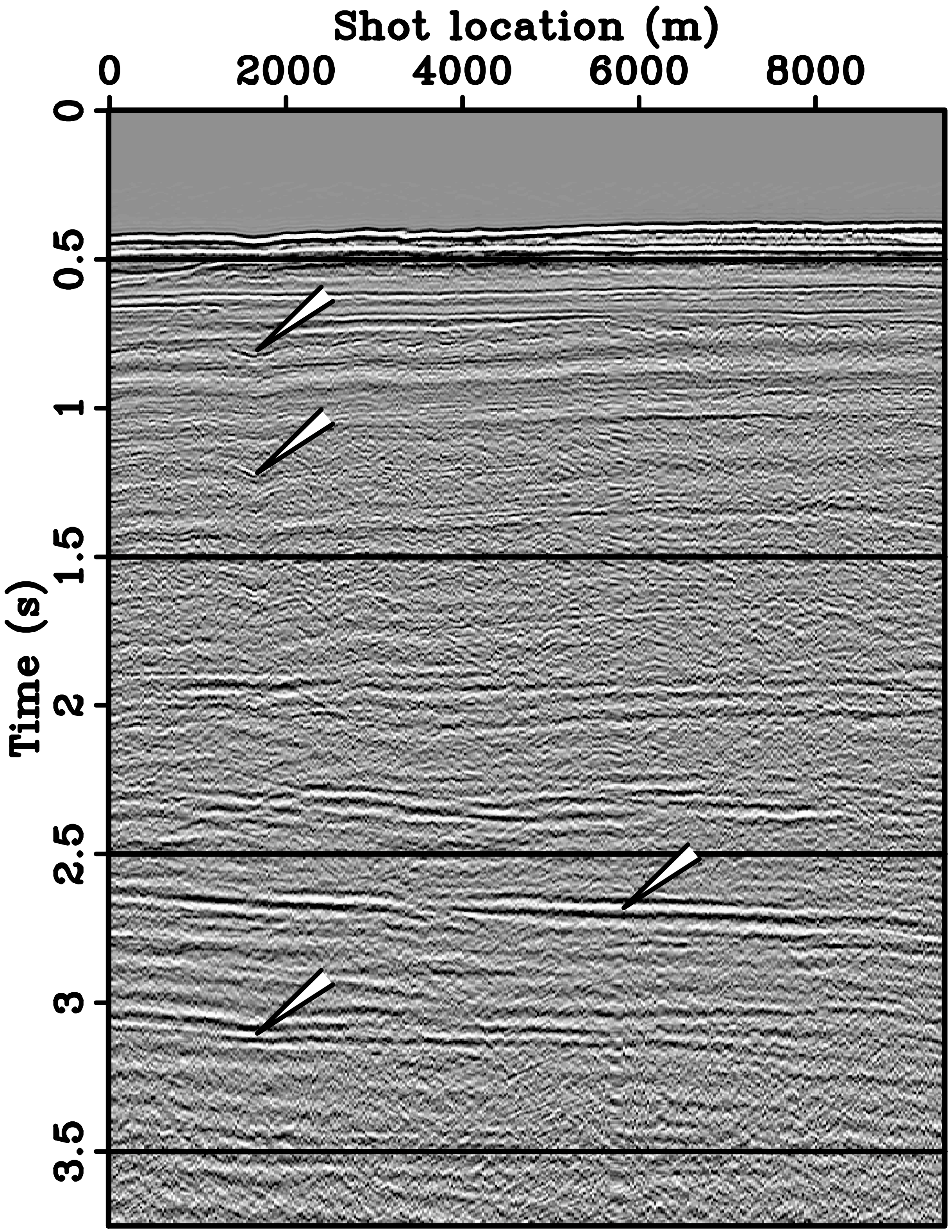


Not scaled Bayesian

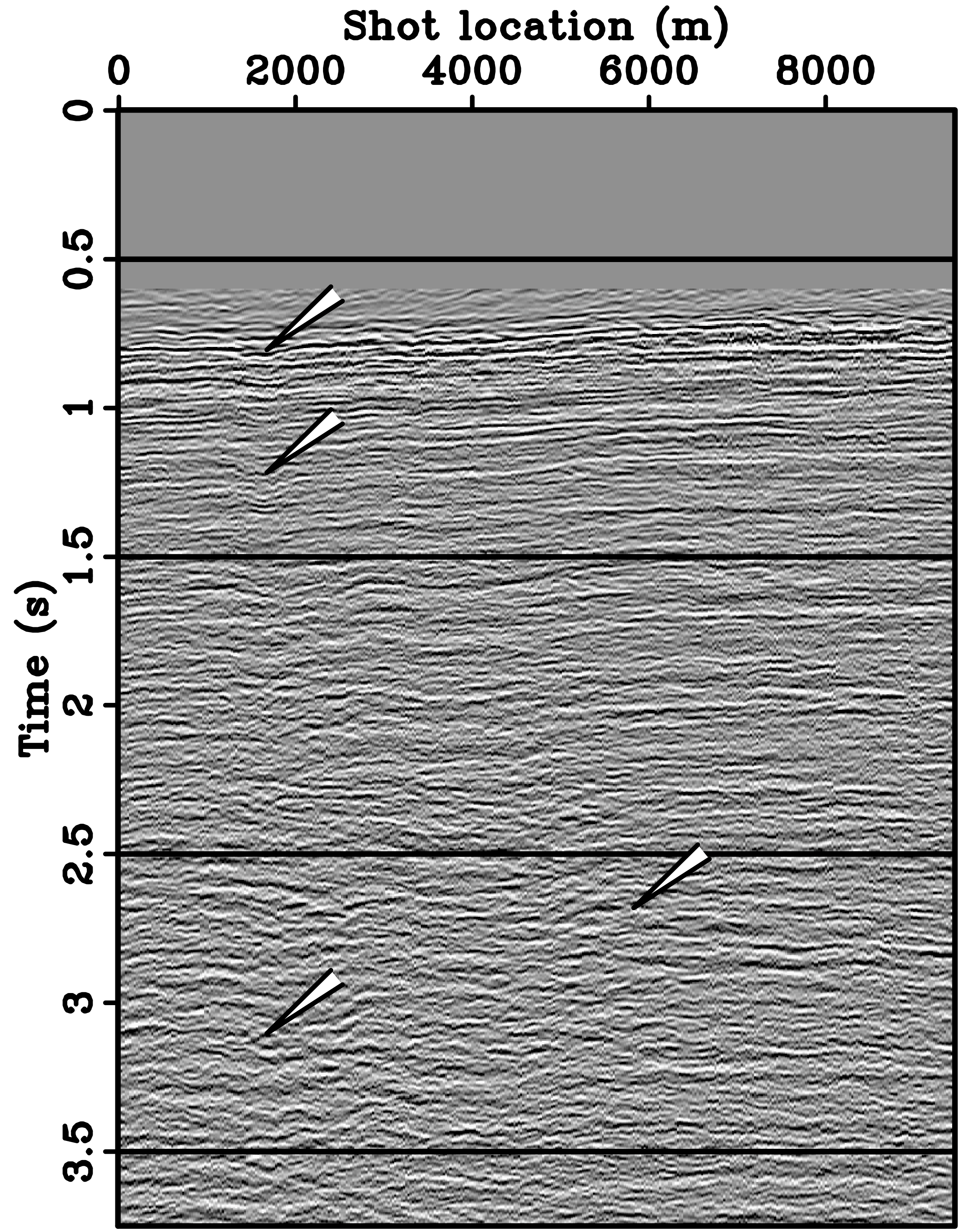


Predicted multiples

Real-data example

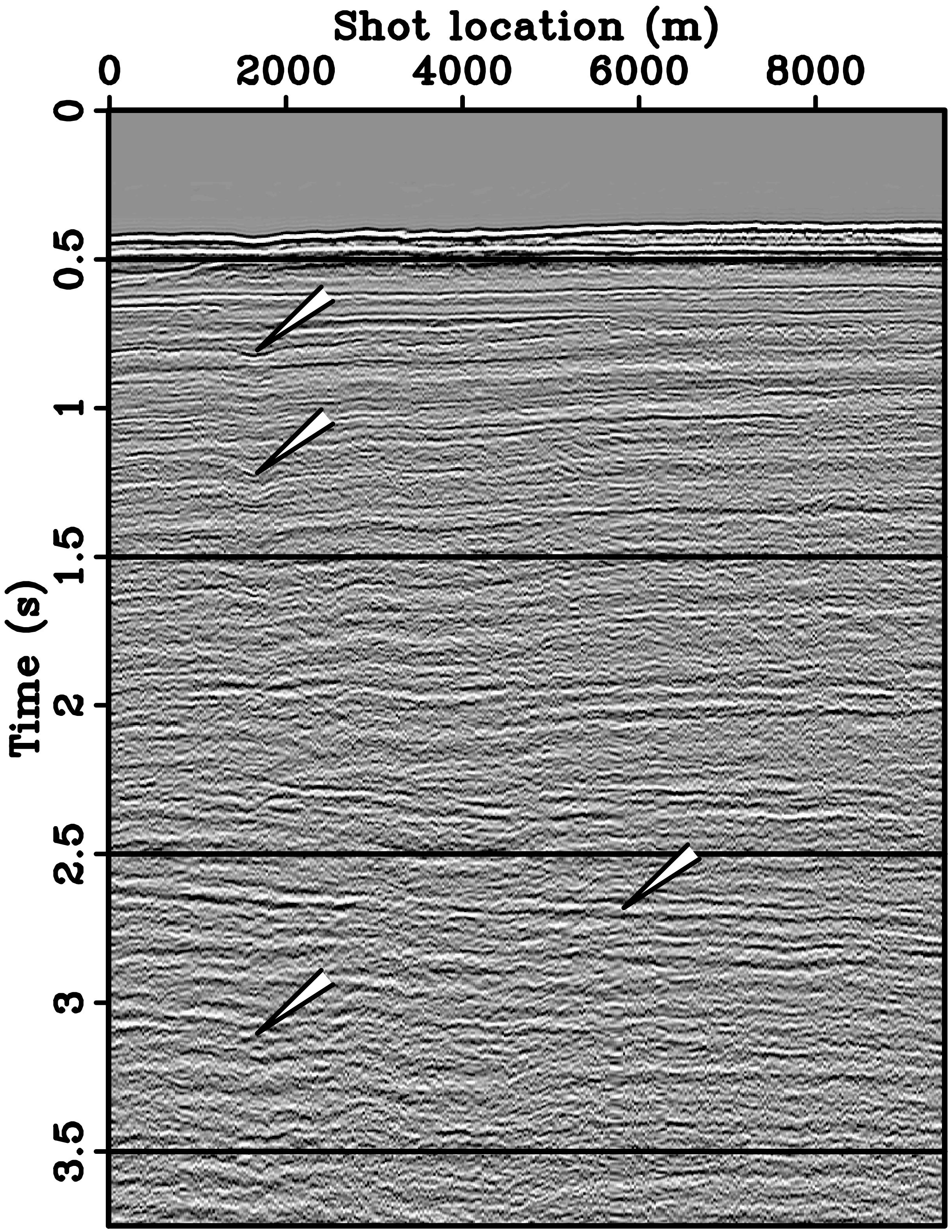


Scaled Bayesian

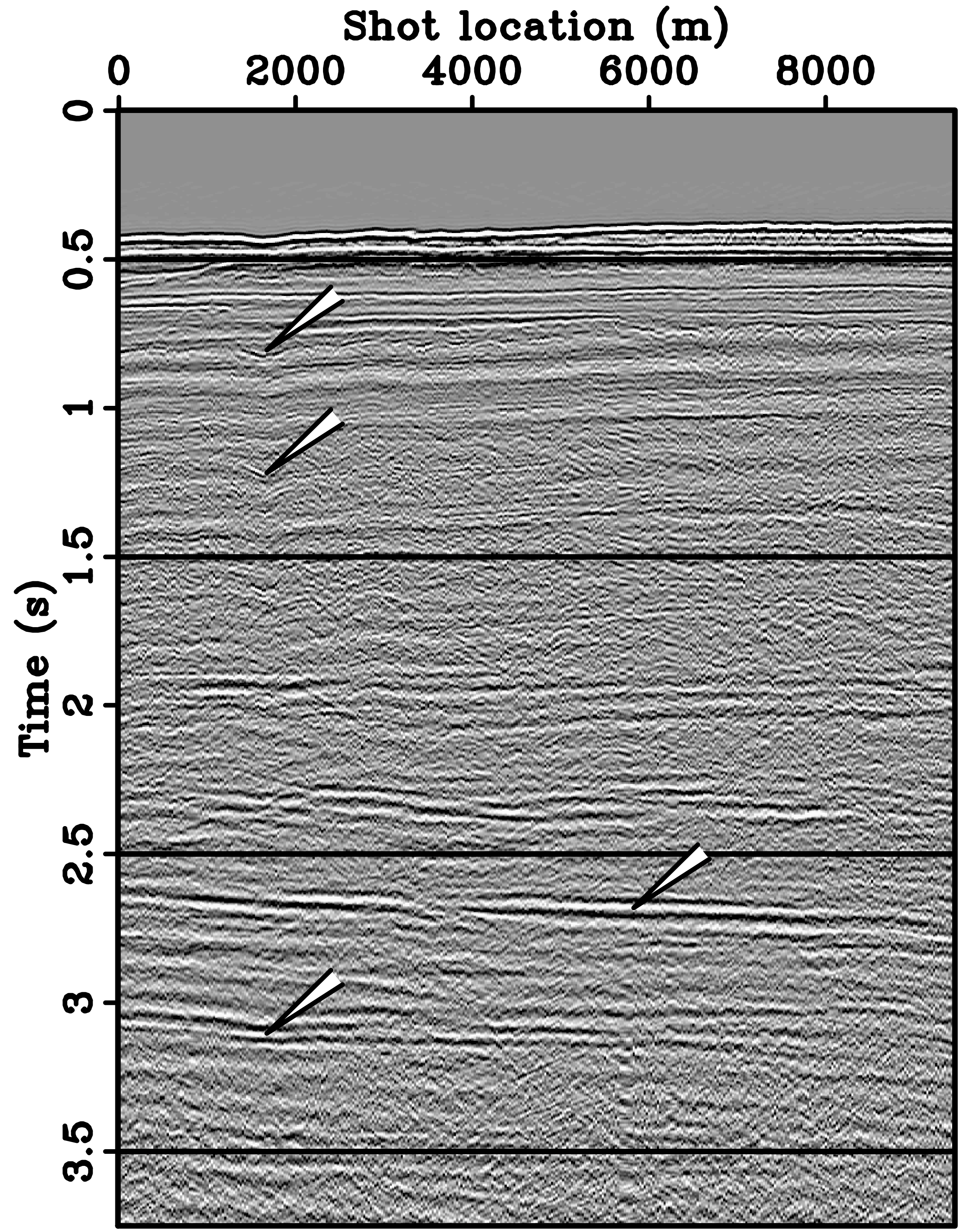


Difference between SRME and scaled Bayesian

Real-data example

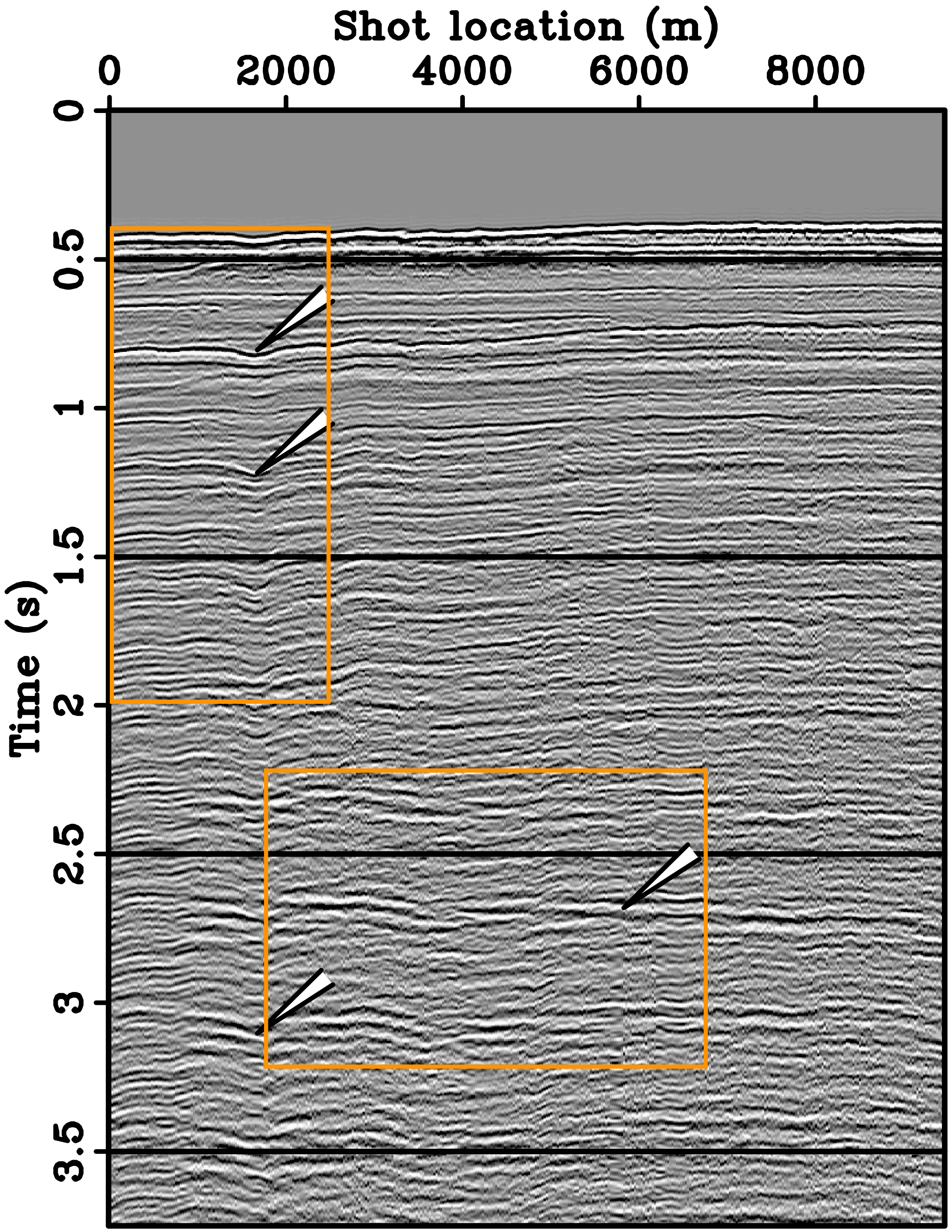


SRME primaries

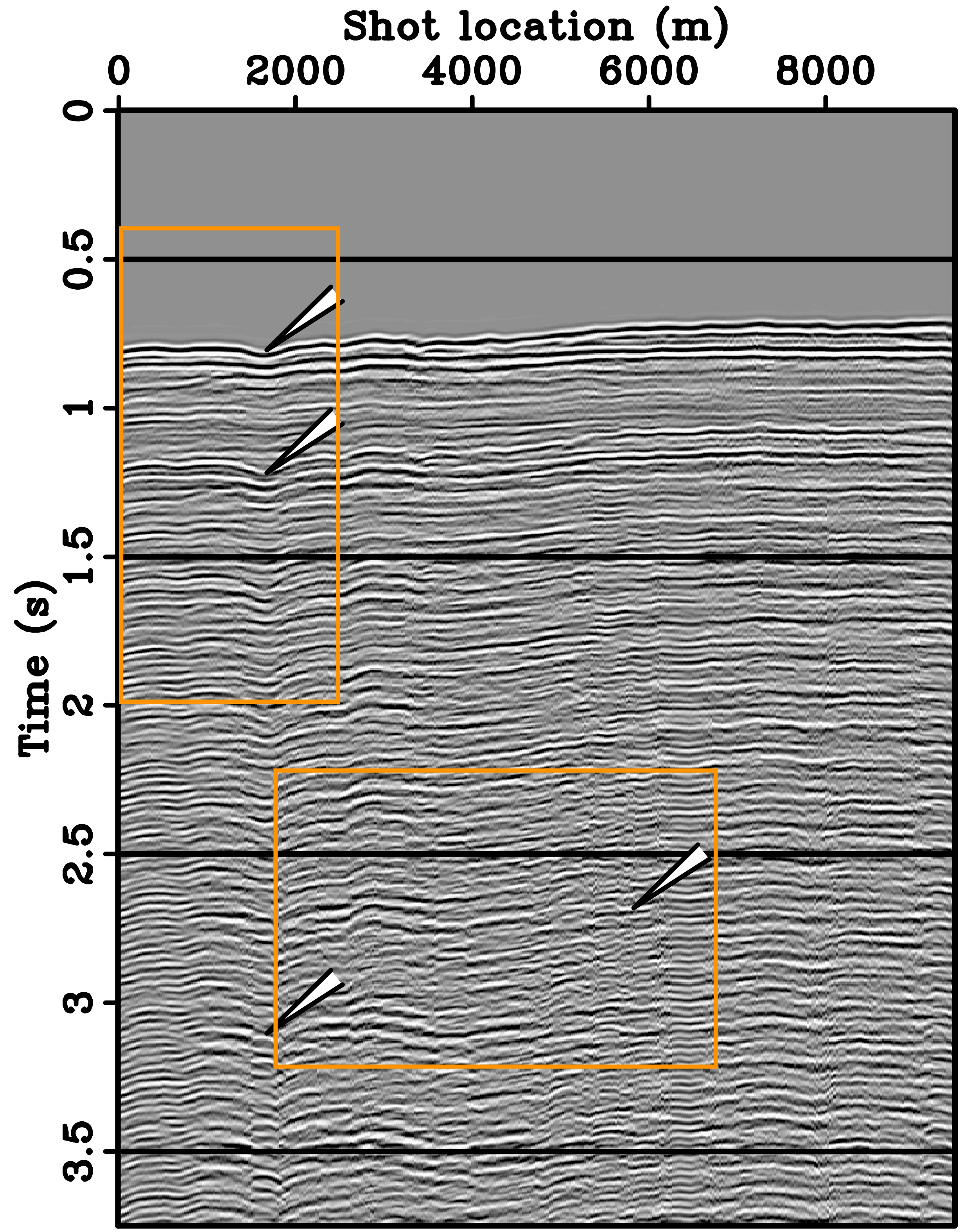


Scaled + Bayesian

Real-data example

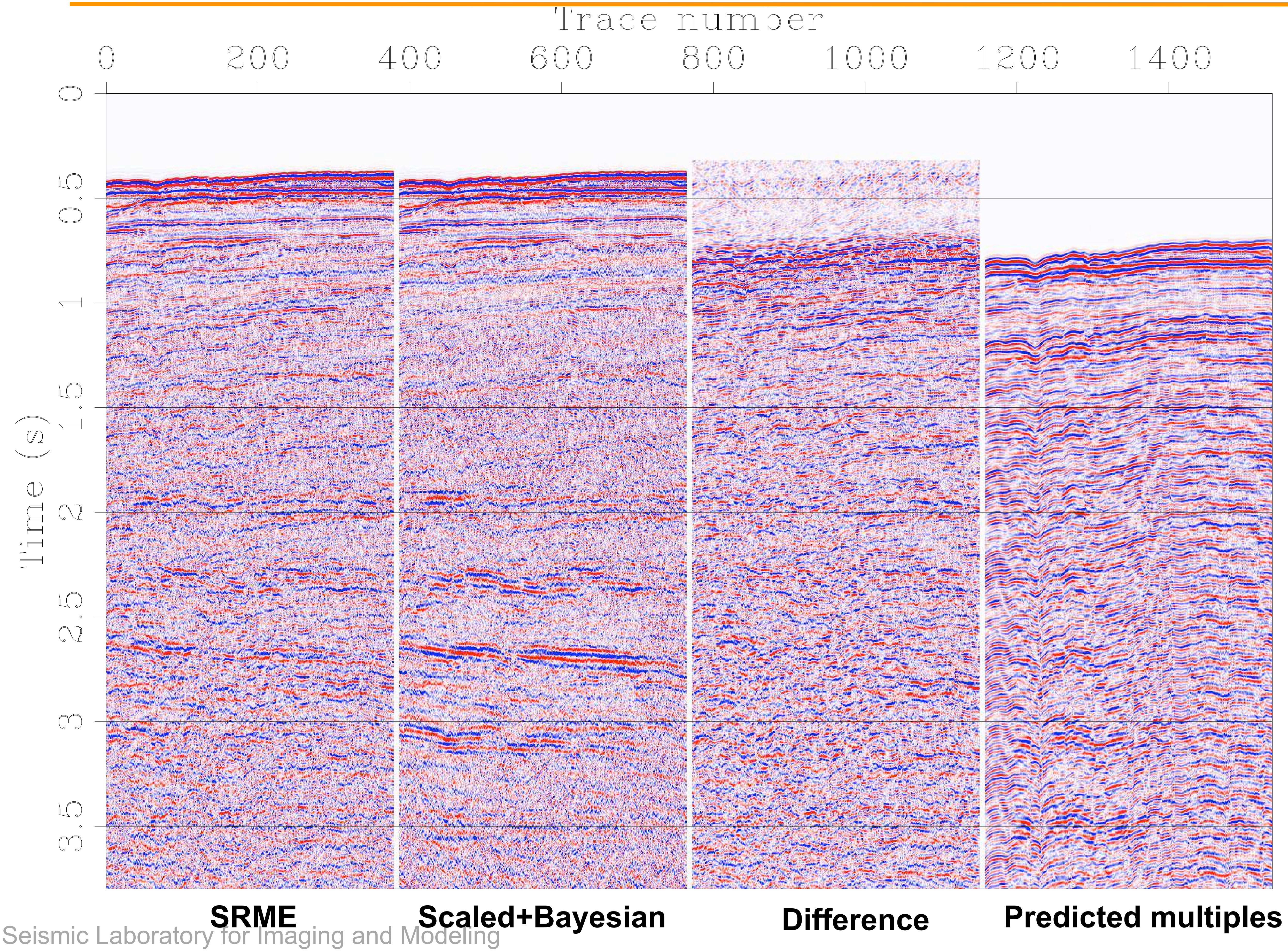


Data



Predicted multiples

Real-data example



Overview approaches

Prediction errors	Kinematic	Amplitude	Phase rotations	Overfitting
Soft thresholding	Minor (curvelet support)	Minor	Minor	Major
Elementwise matched filtering	Moderate	Moderate	Moderate	Minor
Global matching	Minor (curvelet support)	Large (smooth)	Minor	Minor
Bayes	Minor (curvelet support)	Moderate	Minor	Minor
Global matching + Bayes	Minor (curvelet support)	Large (smooth)	Minor	Small

Conclusions

- Adaptive curvelet-domain matched filter significantly improves results
  - reflected in SNR
  - “eye-ball” norm
- Results nearly as good as iterative SRME
- Appropriate for real data for which iterative SRME is often not an option.

- Future plans:
  - more case studies
  - extension to 3-D
  - extension to off-diagonal contributions to “scaling”

Acknowledgments

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Further reading

Herrmann, F. J., Moghaddam, P. and Stolk, C. Sparsity- and continuity-promoting seismic image recovery with curvelet frames. App. & Comp. Harm. Analys., Vol. 24, No. 2, pp. 150-173, 2008.

Herrmann, F. J., Wang, D and Verschuur, D. J. Adaptive curvelet-domain primary-multiple separation. Geophysics, Vol 73, No. 3, pp. A17-A21, 2008.