# Recent results in curvelet-based primary-multiple separation: application to real data

Deli Wang[1,2], Rayan Saab[3], Ozgur Yilmaz[4], Felix J. Herrmann[2]

*1.College of Geoexploration Science and Technology, Jilin University,Changchun,China*
*2.Seismic Laboratory for Imaging and Modeling*
*Department of Earth & Ocean Sciences*
*The University of British Columbia at Vancouver*
*3.The Department of Electrical and Computer Engineering,The University of British Columbia*
*4.Department of Mathematics,The University of British Columbia*

# Contents

# Introduction

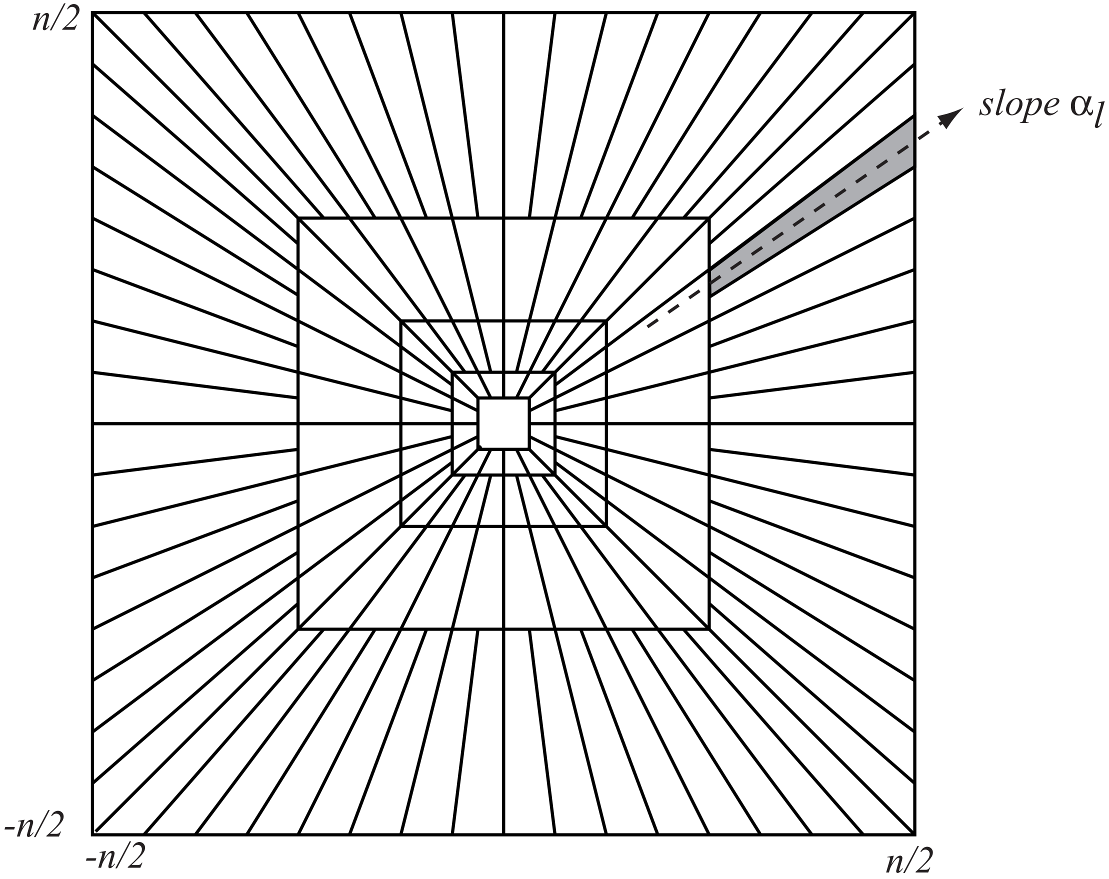
Problems with WE-based multiple elimination

- imperfect multiple predictions
- failure of direct subtraction after matched filtering

Exploit the ability of curvelets to

- sparsify the to-be-separated signal components
- separation based on the curvelet parameterization
  - location
  - dip
  - scale

# Introduction




**Discrete frequency tiling**

**One curvelet**

*Ying et al,2005*

# Contents

- Introduction
- Curvelet-based rimary-multiple separation
- Examples
- Discussion and Conclusion
- Acknowledgments

# Curvelet-based separation

Forward model

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n}$$

Soft thresholding

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T S_w(\mathbf{C}\mathbf{s})$$

where

$$S_w(x) := \operatorname{sgn}(x) \cdot \max(0, |x| - w)$$

and $w := |\mathbf{C}\breve{\mathbf{s}}_2|$

- predictions may contain moderate
  - amplitude, phase
  - and sign errors

*Herrmann et al,2006*

# Curvelet-based separation

## Nonlinear optimization from a Bayesian perspective

Forward model

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \qquad \text{(total data)}$$

$$\mathbf{s}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{n}_2 \qquad \text{(multiples)}$$

$$\mathbf{s}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{n} - \mathbf{n}_2 \qquad \text{(primaries)}$$

where

$\mathbf{x}_1$ curvelet coefficients primaries

$\mathbf{x}_2$ curvelet coefficients multiples

$\mathbf{A}_{1,2}$ inverse curvelet transforms

# Curvelet-based separation

Separate by solving the nonlinear problem

$$\mathbf{P_w}: \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda_1 \|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2 \|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\breve{\mathbf{s}}_2 - \mathbf{A}_2\mathbf{x}_2\|_2^2 + \mu \|\breve{\mathbf{s}}_1 + \breve{\mathbf{s}}_2 - \mathbf{A}_1\mathbf{x}_1 - \mathbf{A}_2\mathbf{x}_2\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}_1\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}_2\tilde{\mathbf{x}}_2. \end{cases}$$

where

- $\breve{\mathbf{s}}_{1,2}$ predicted primaries (1) and multiples (2)
- $\mathbf{A}_{1,2}$ inverse discrete curvelet transforms
- $\lambda_{1,2}$ and $\mu$ are control parameters

Can be solved by iterative soft thresholding.

*(For a detailed description please refer to Rayan Saab et al.,2007 and his later presentation this section)*

# Contents

# Examples

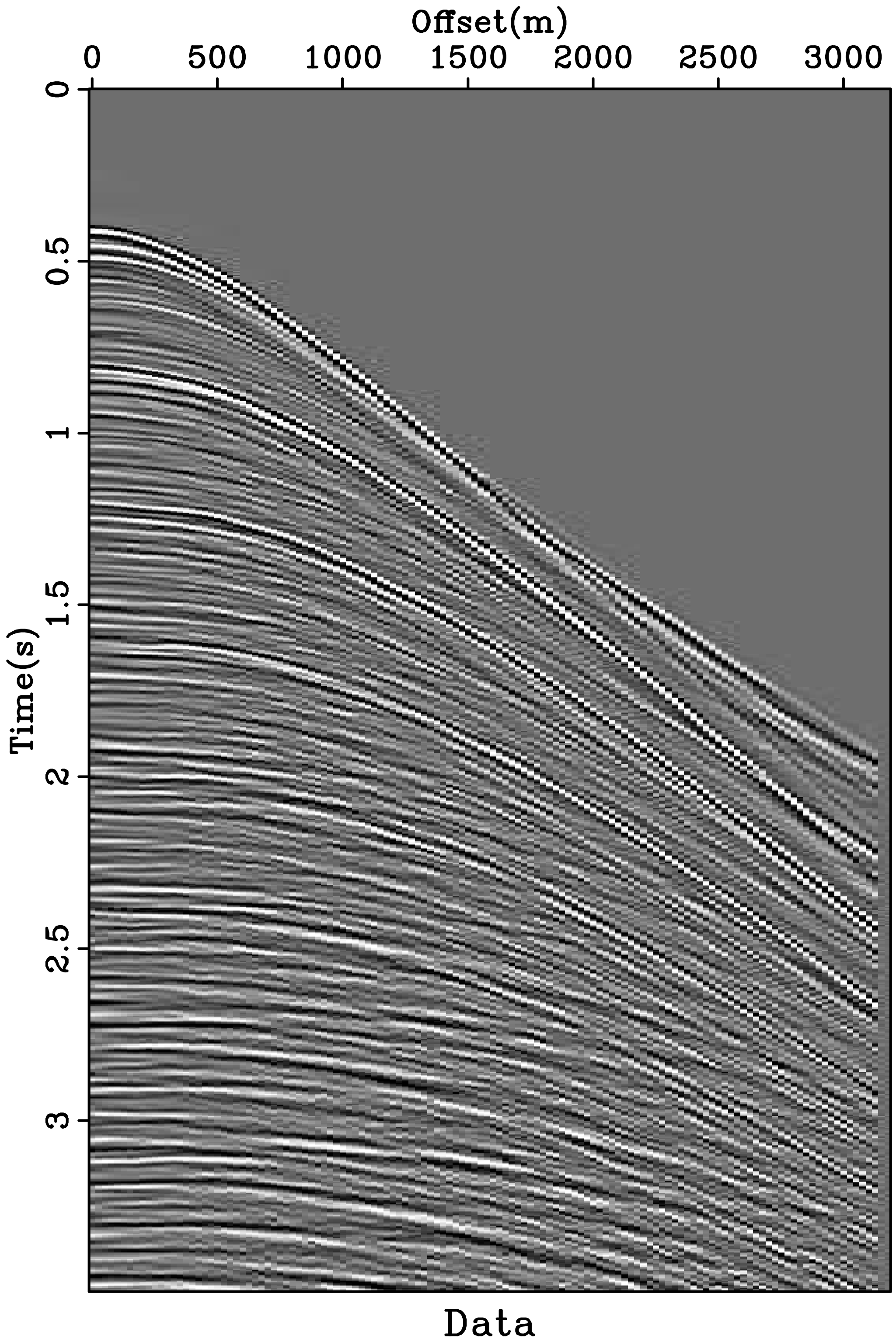
Example 1

Saga data:

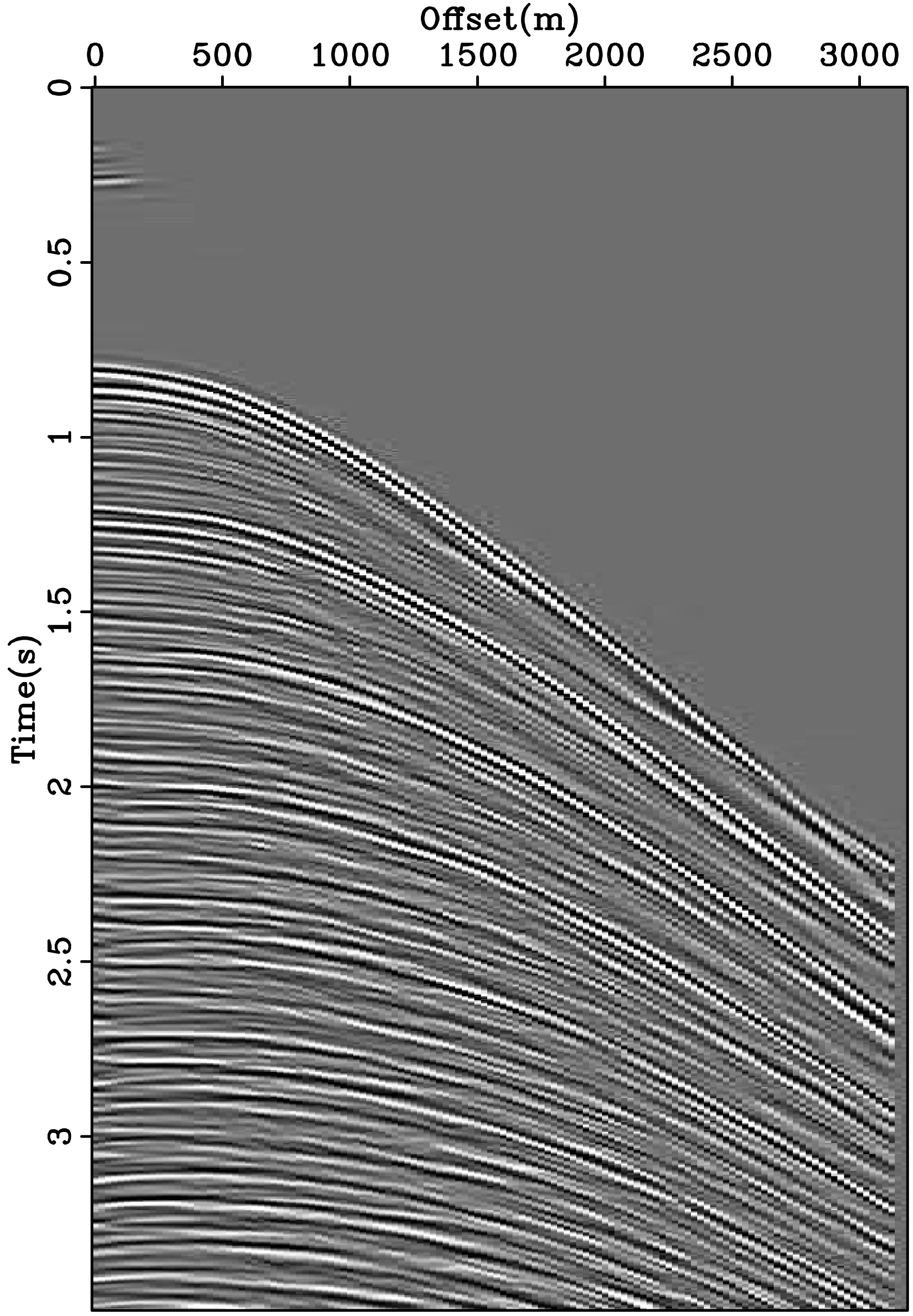
128 shots
128 traces/shot
1024 samples/trace

The original data contains many strong surface-related multiples

# Example 1

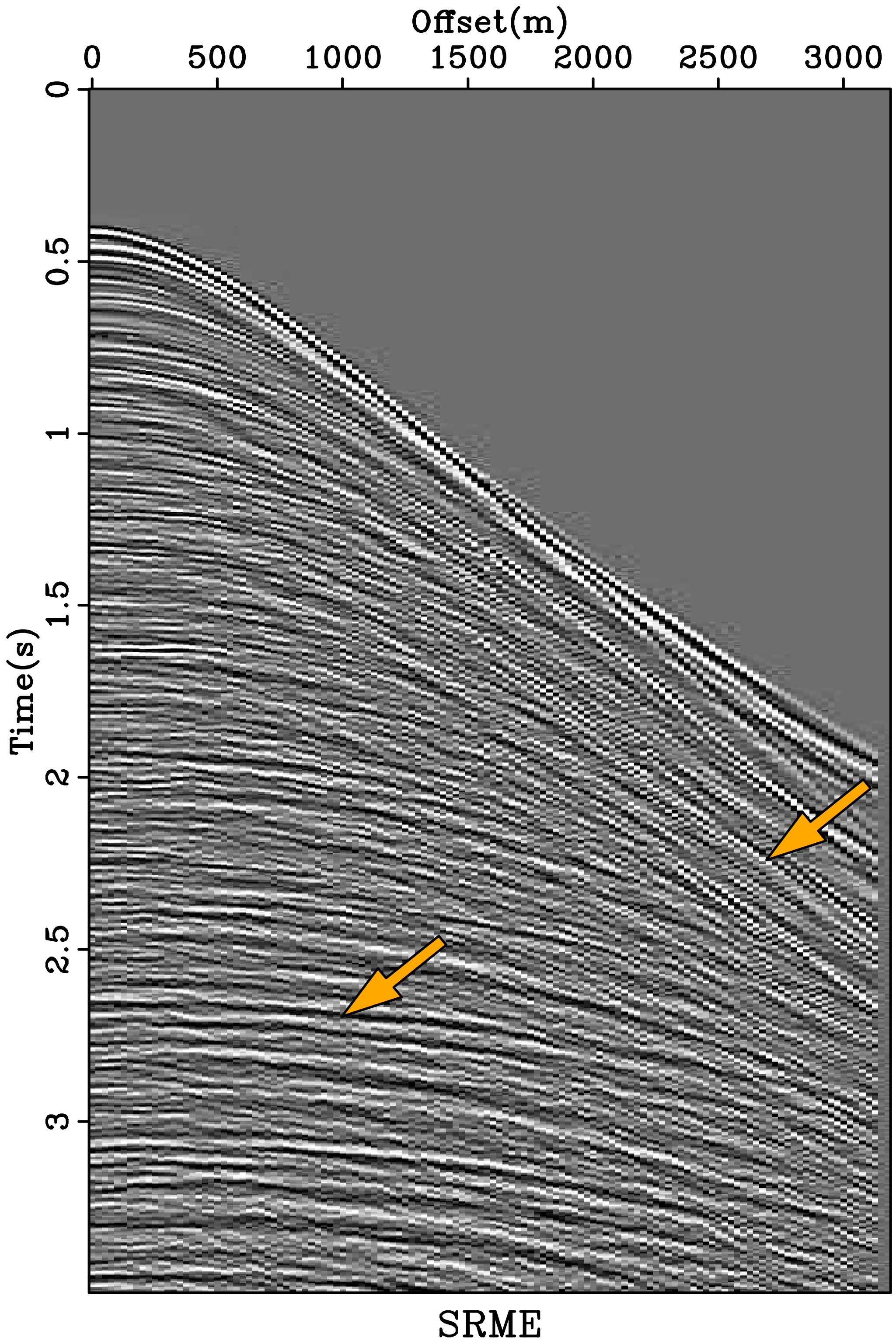

Offset(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
Data

# Example 1



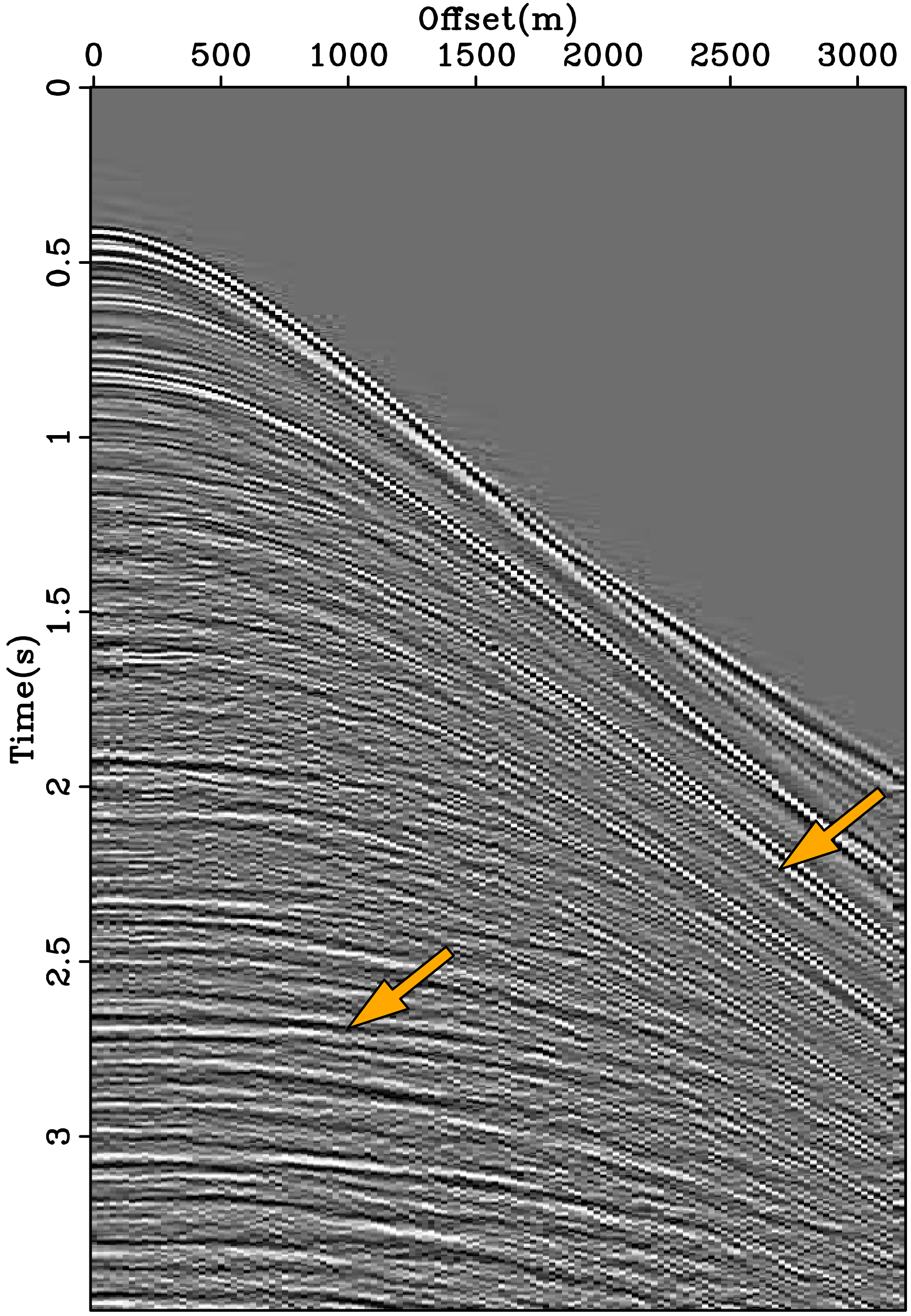
Multiples

# Example 1



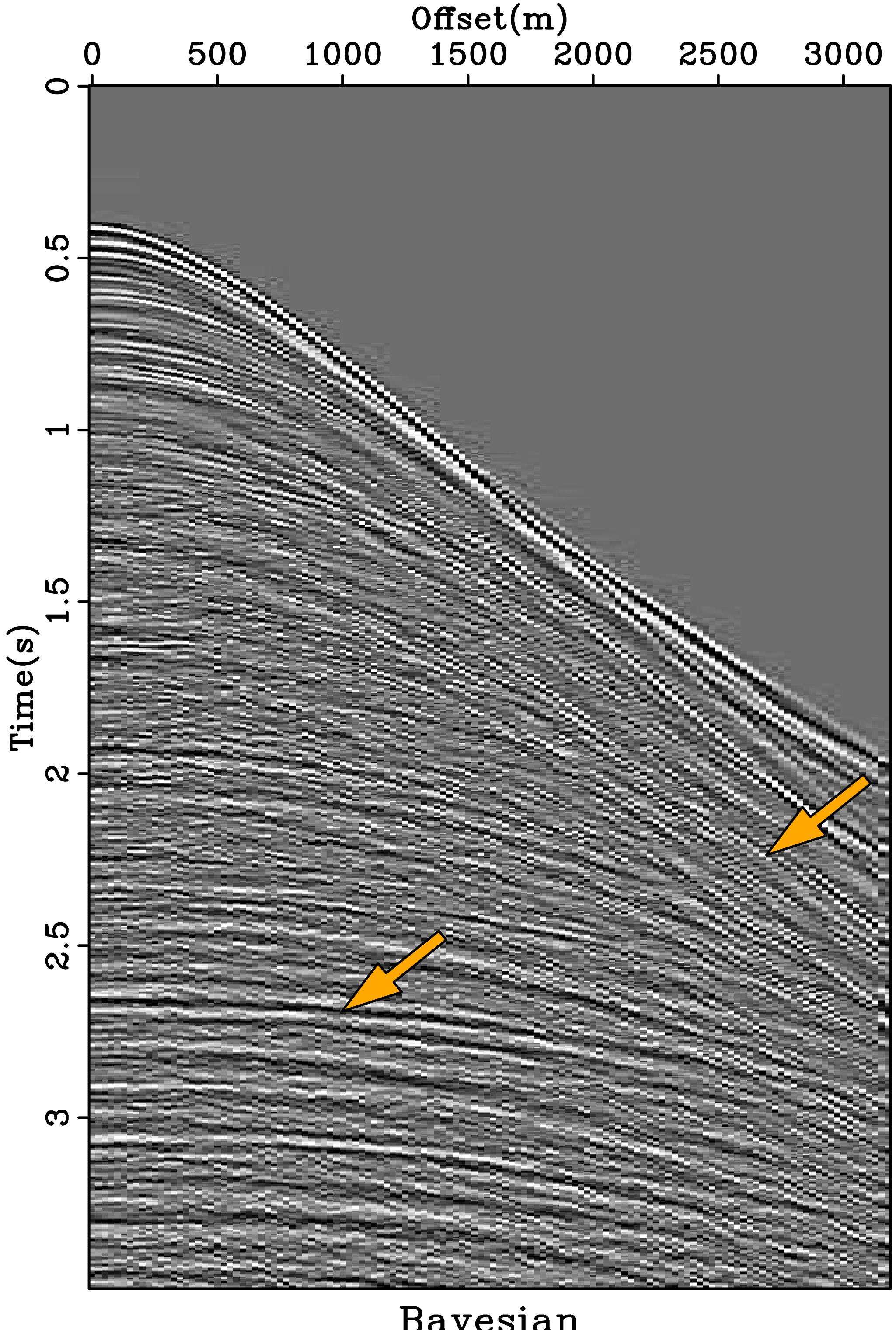
Offset(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
SRME

# Example 1

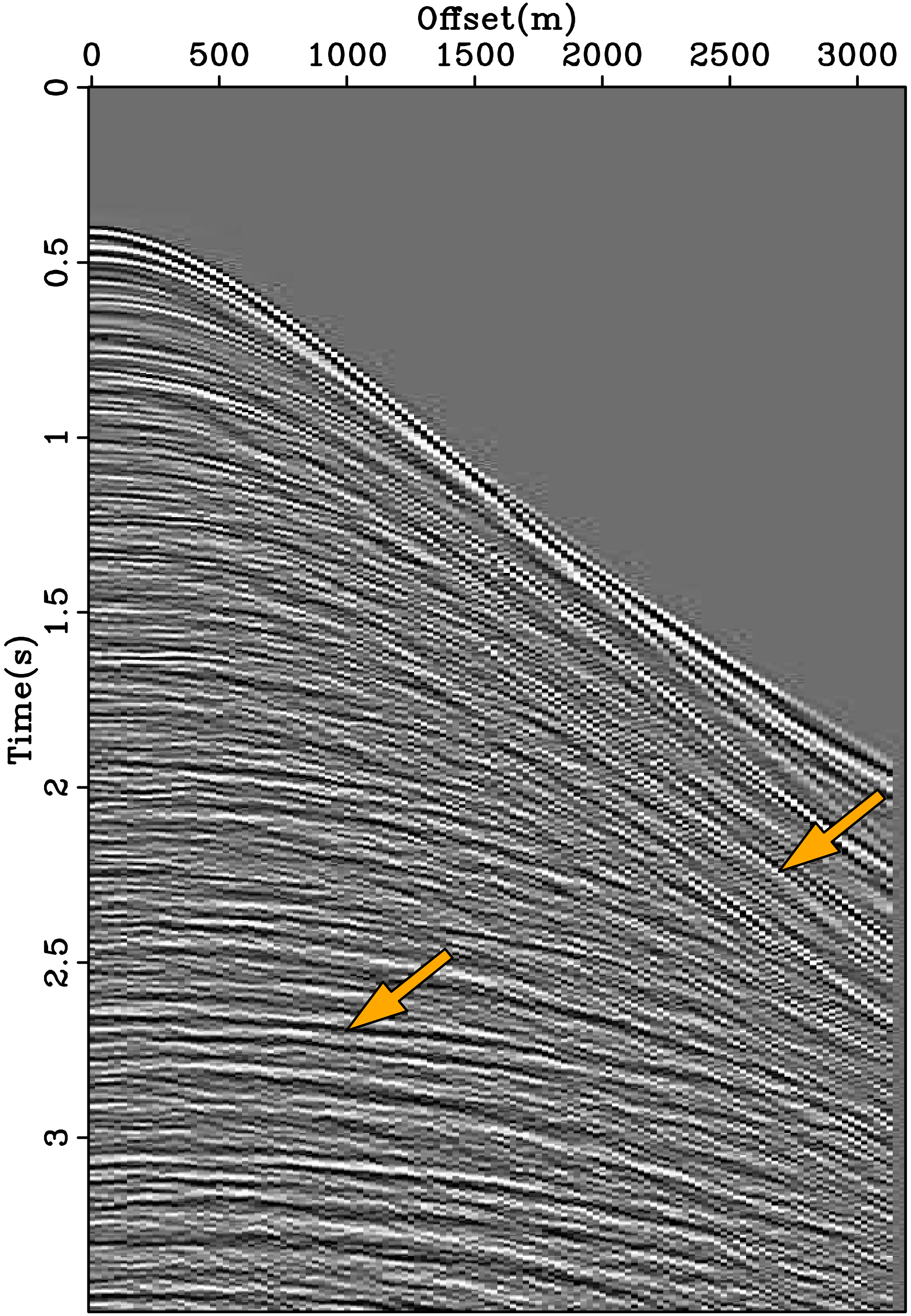



Single threshold

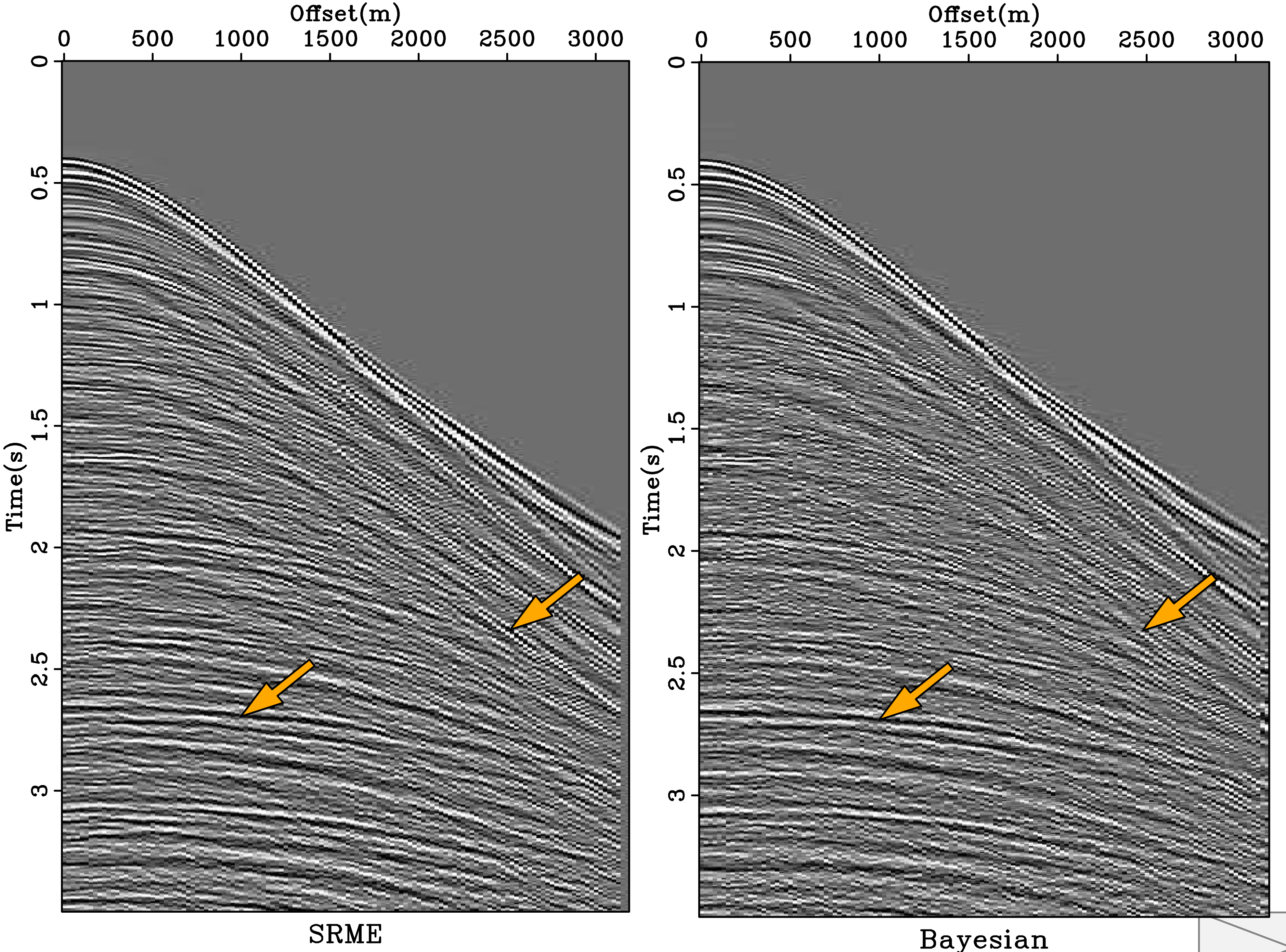
# Example 1


Offset(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
Bayesian

# Example 1

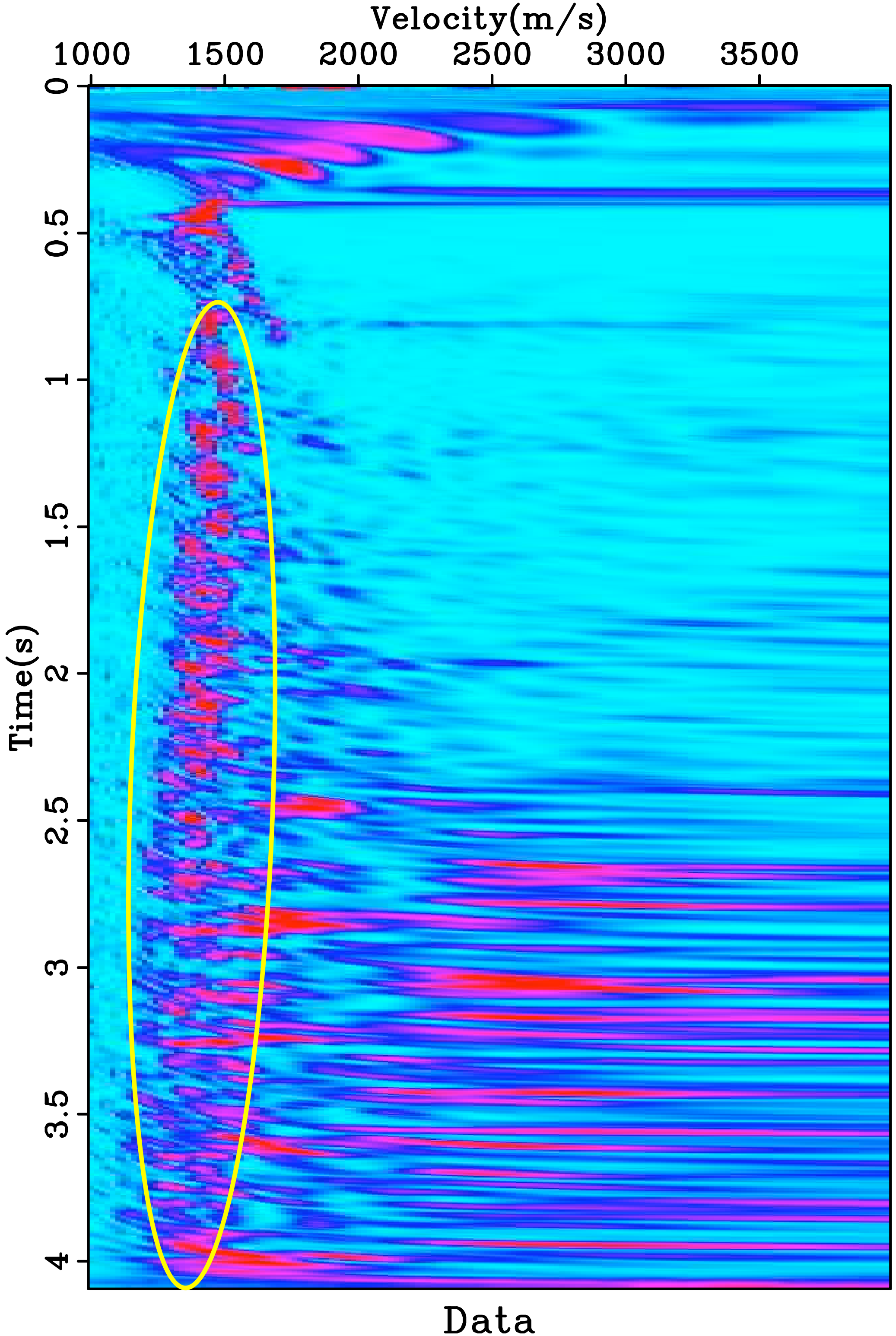


# Example 1



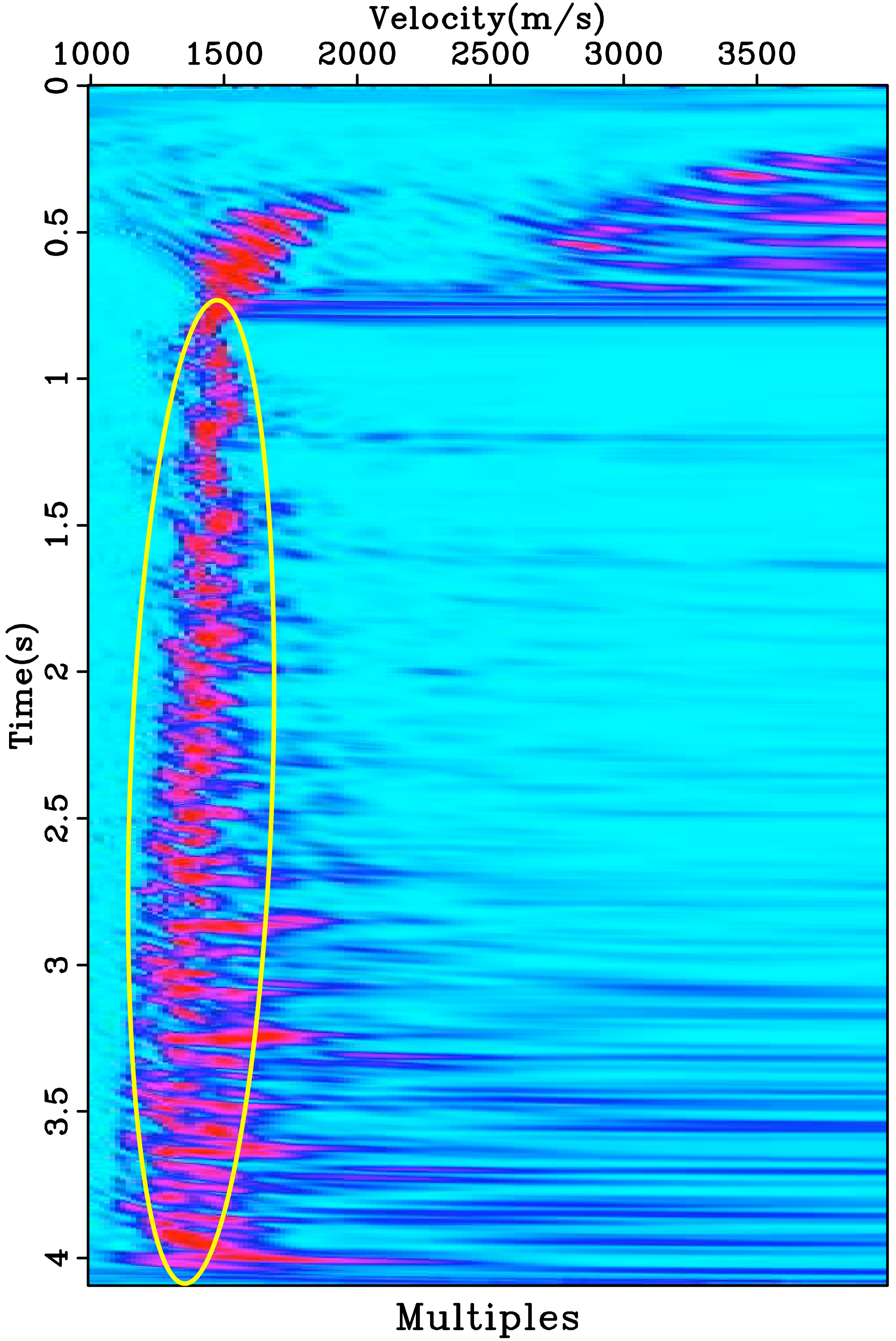
Offset(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
SRME
Offset(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
Bayesian

# Example 1

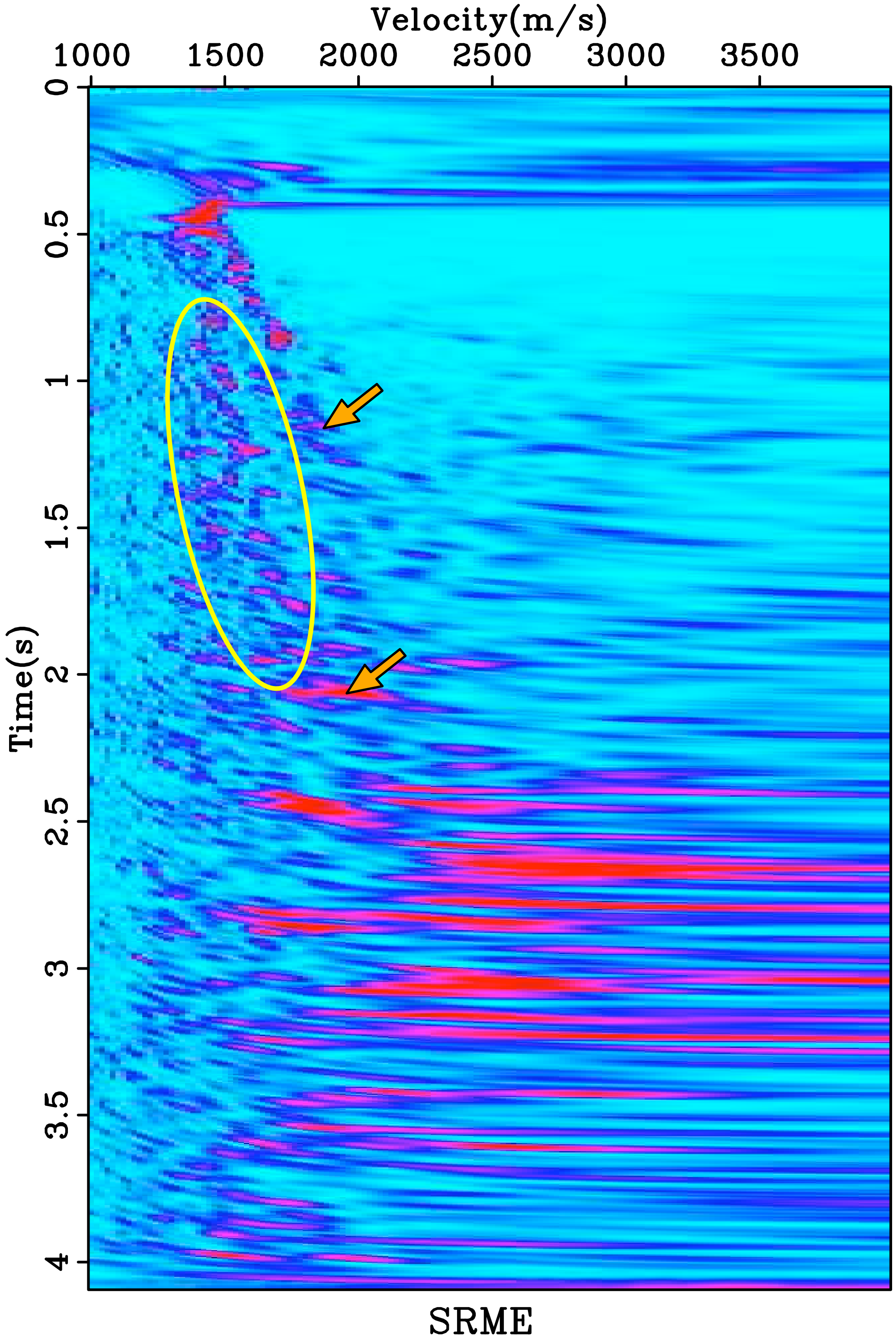


Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Data

# Example 1

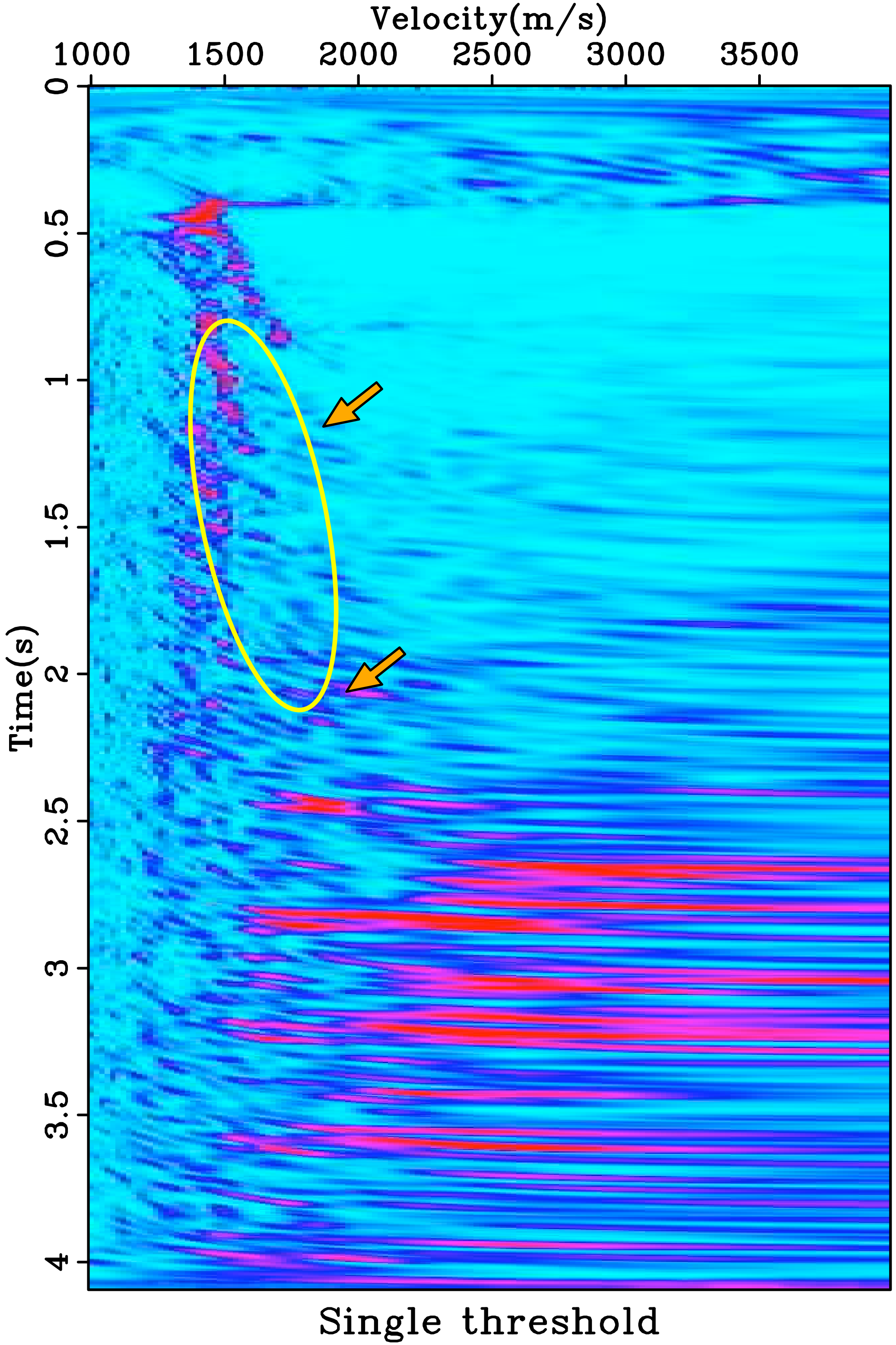

Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Multiples

# Example 1



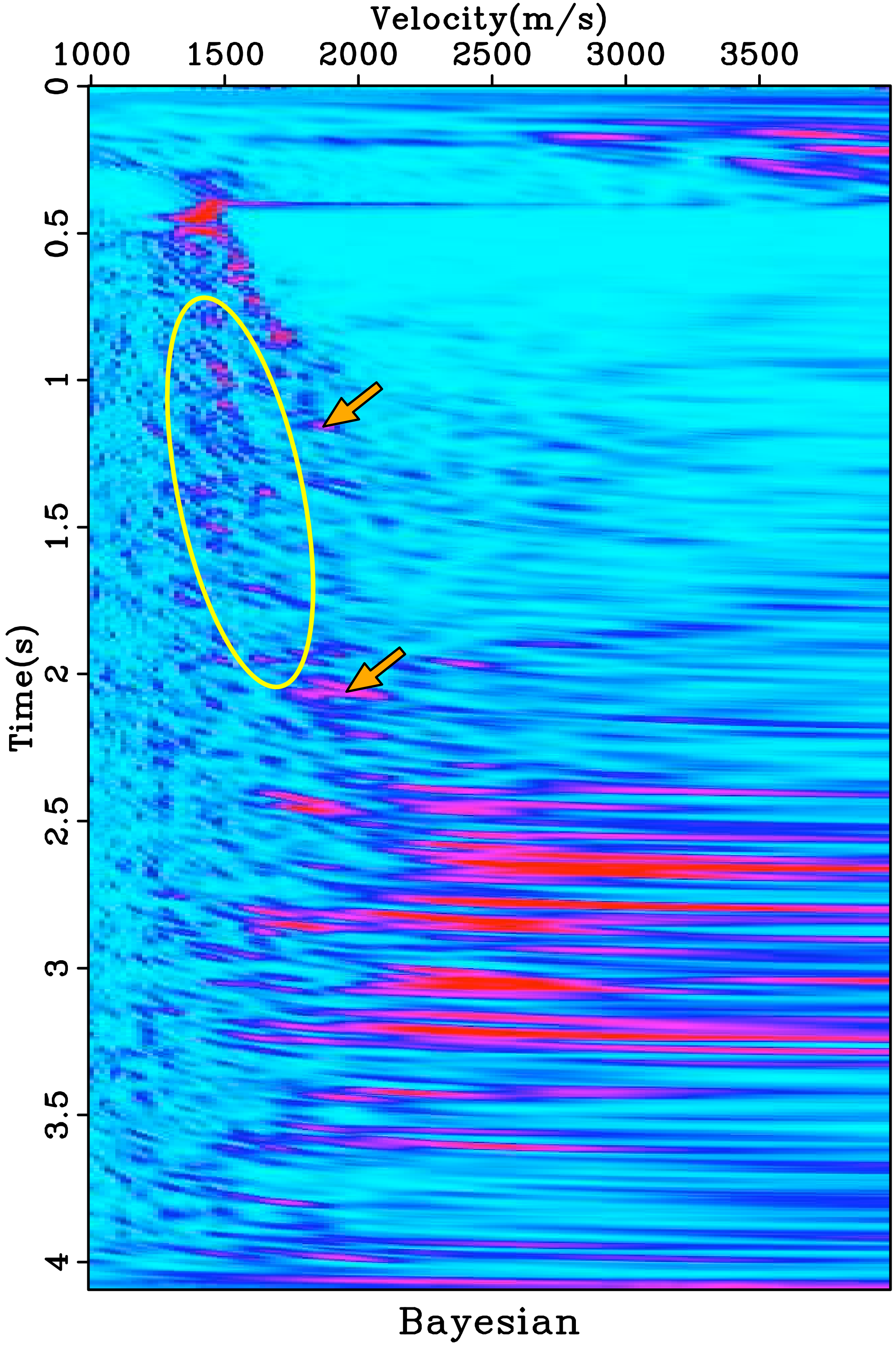
Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
SRME

# Example 1



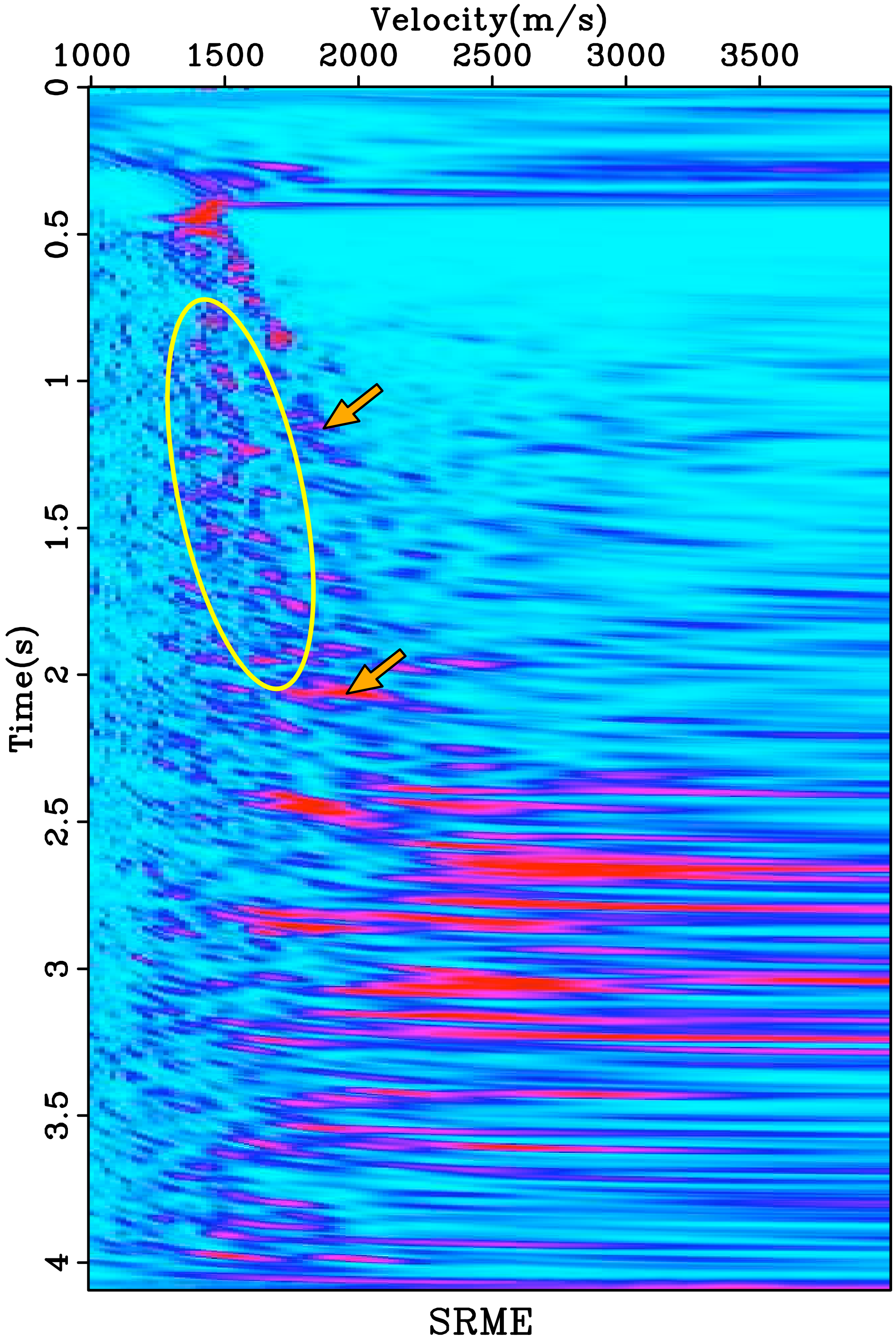
Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Single threshold

# Example 1



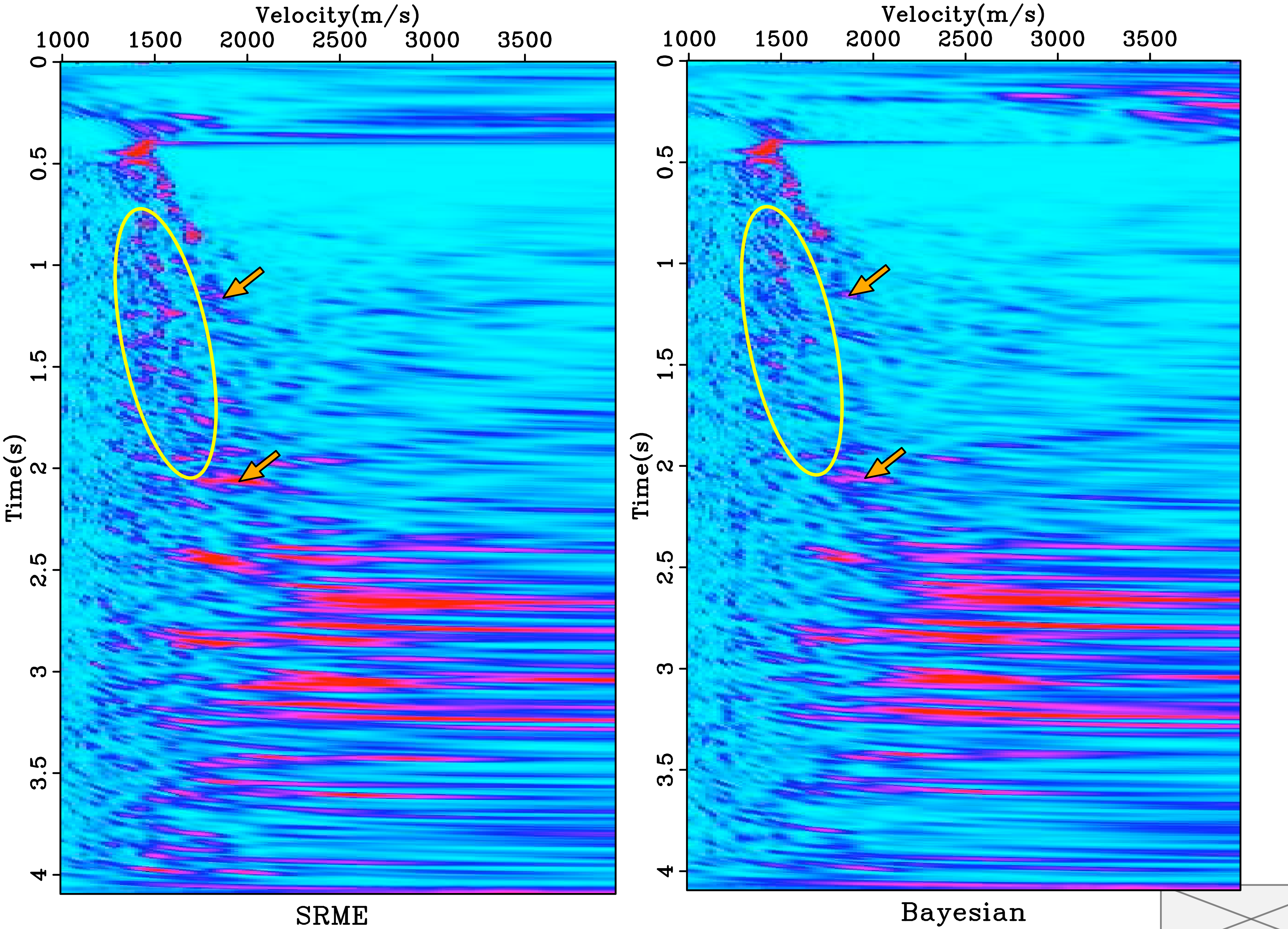
Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Bayesian

# Example 1



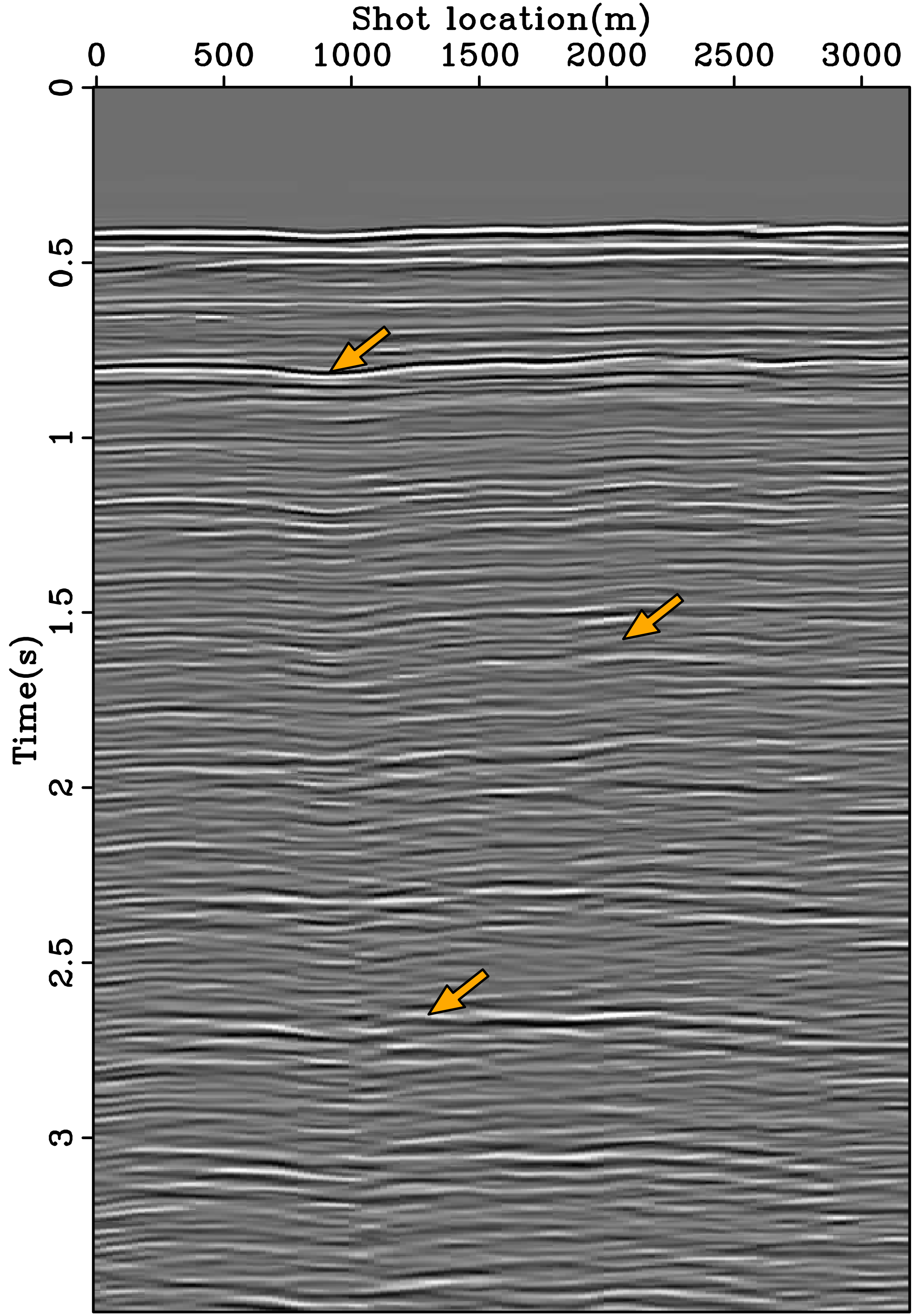
Velocity(m/s)
1000
1500
2000
2500
3000
3500
0
0.5
1
1.5
2
2.5
3
3.5
4
Time(s)
SRME

# Example 1



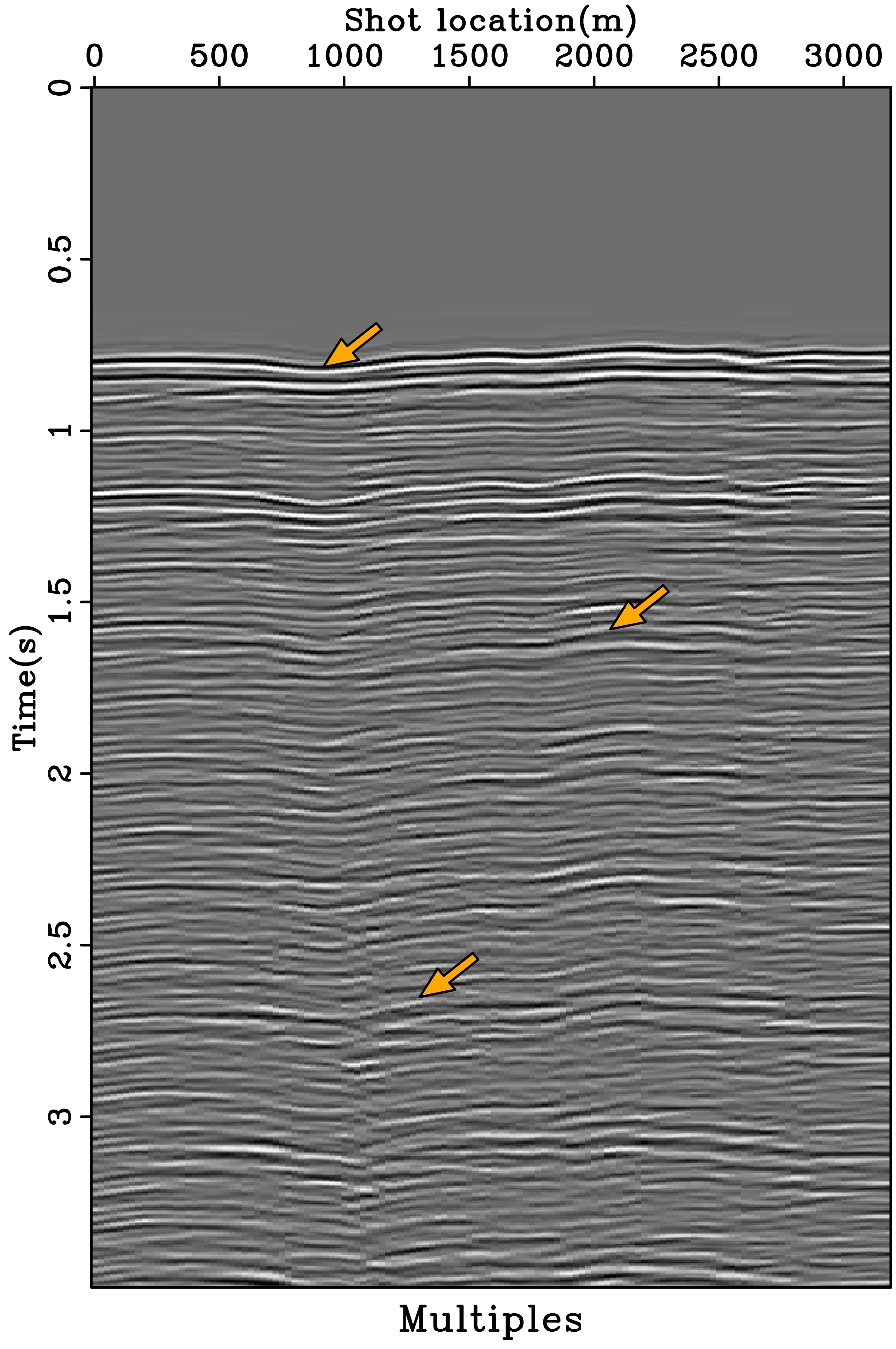
Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
SRME
Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Bayesian

# Example 1

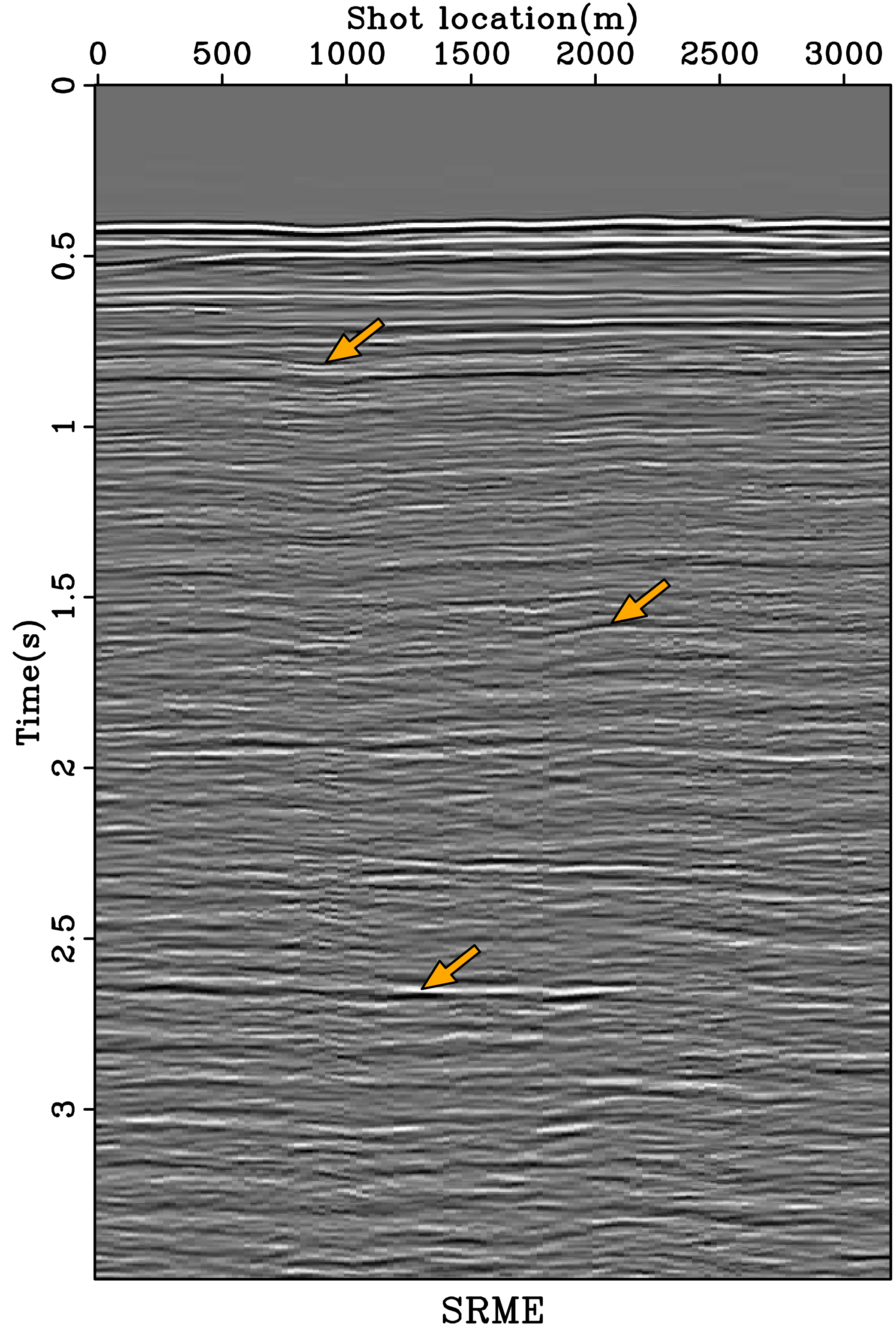



Data

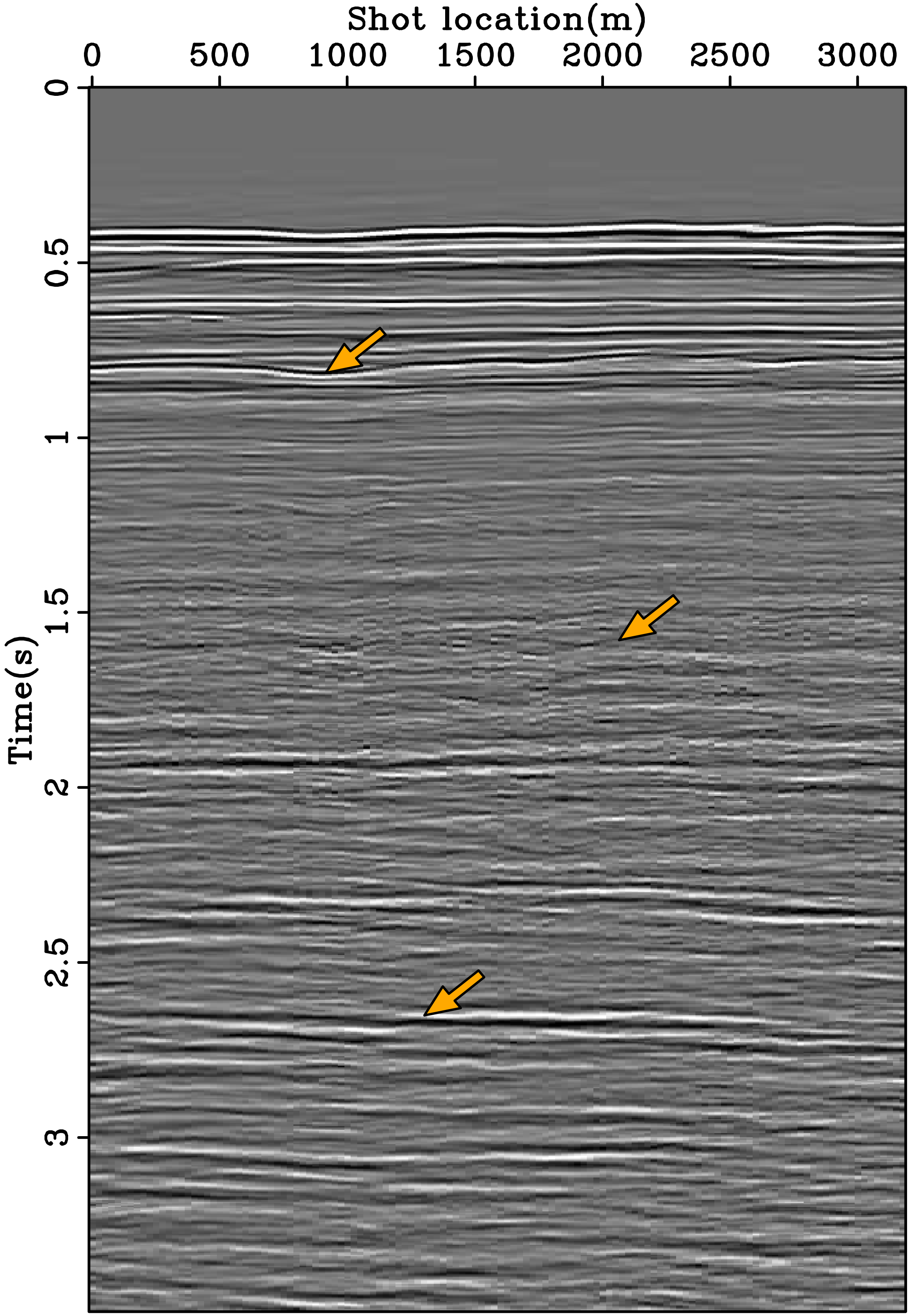
# Example 1


Shot location(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
Multiples

# Example 1

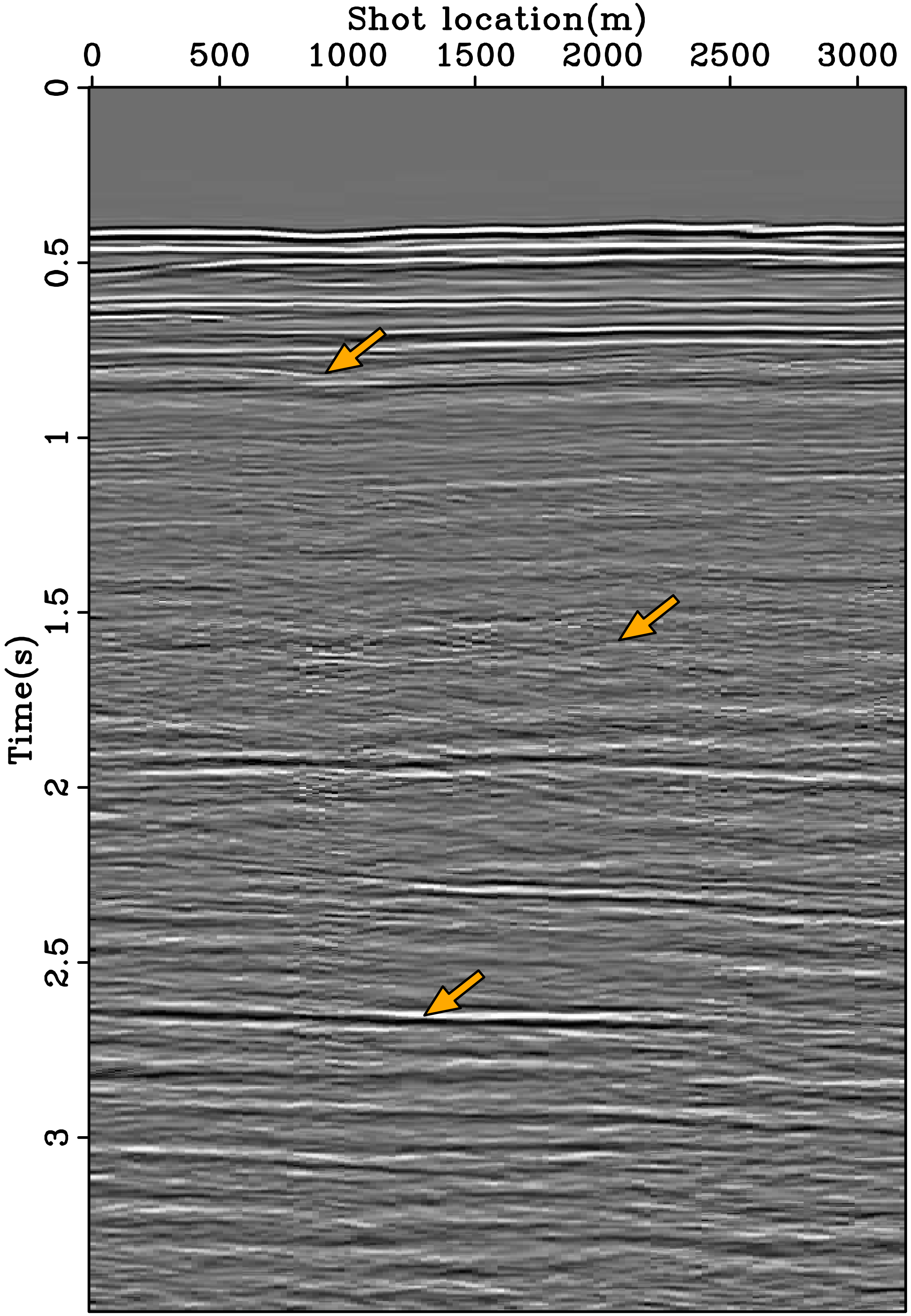

Shot location(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
SRME

# Example 1



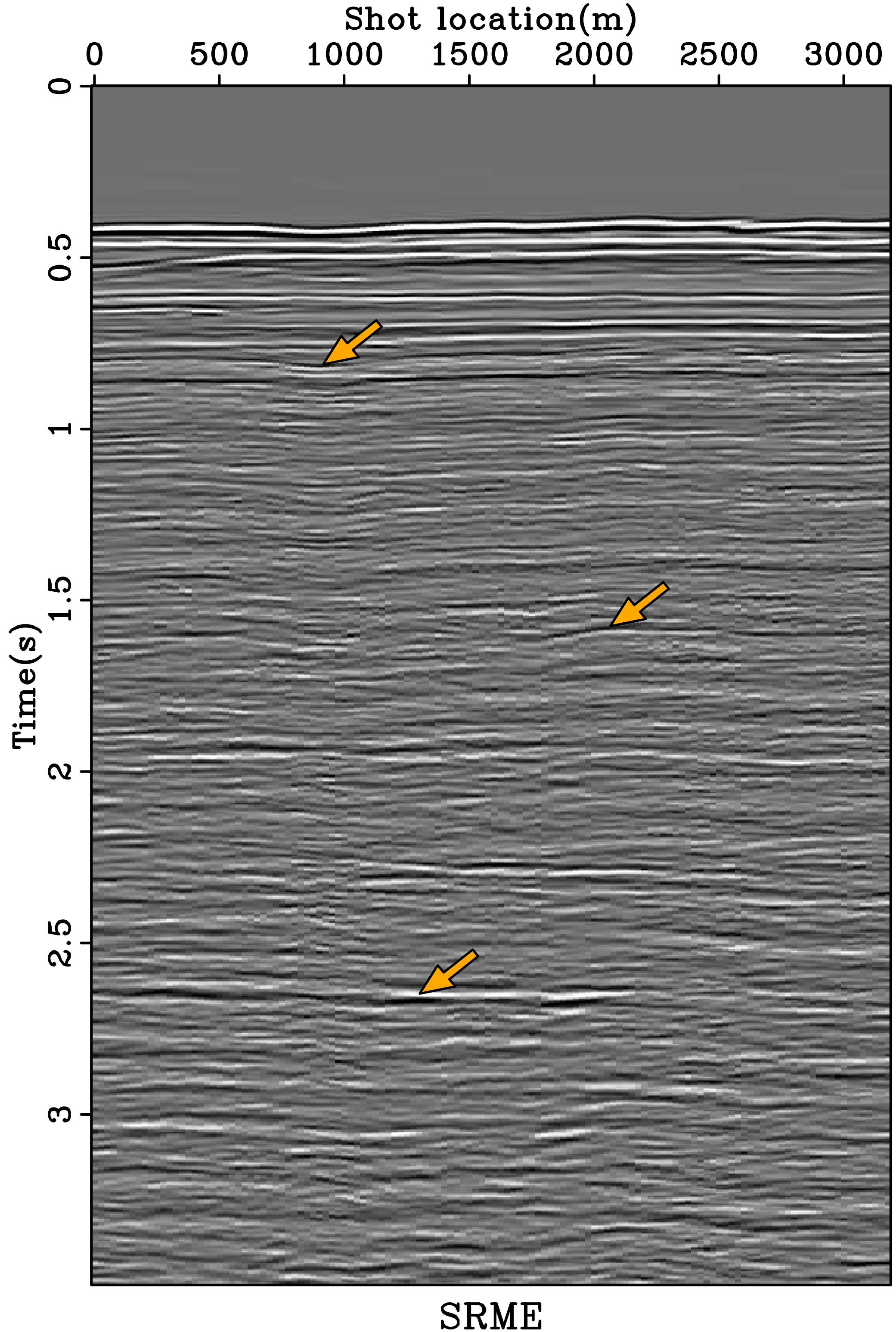

Single threshold

# Example 1

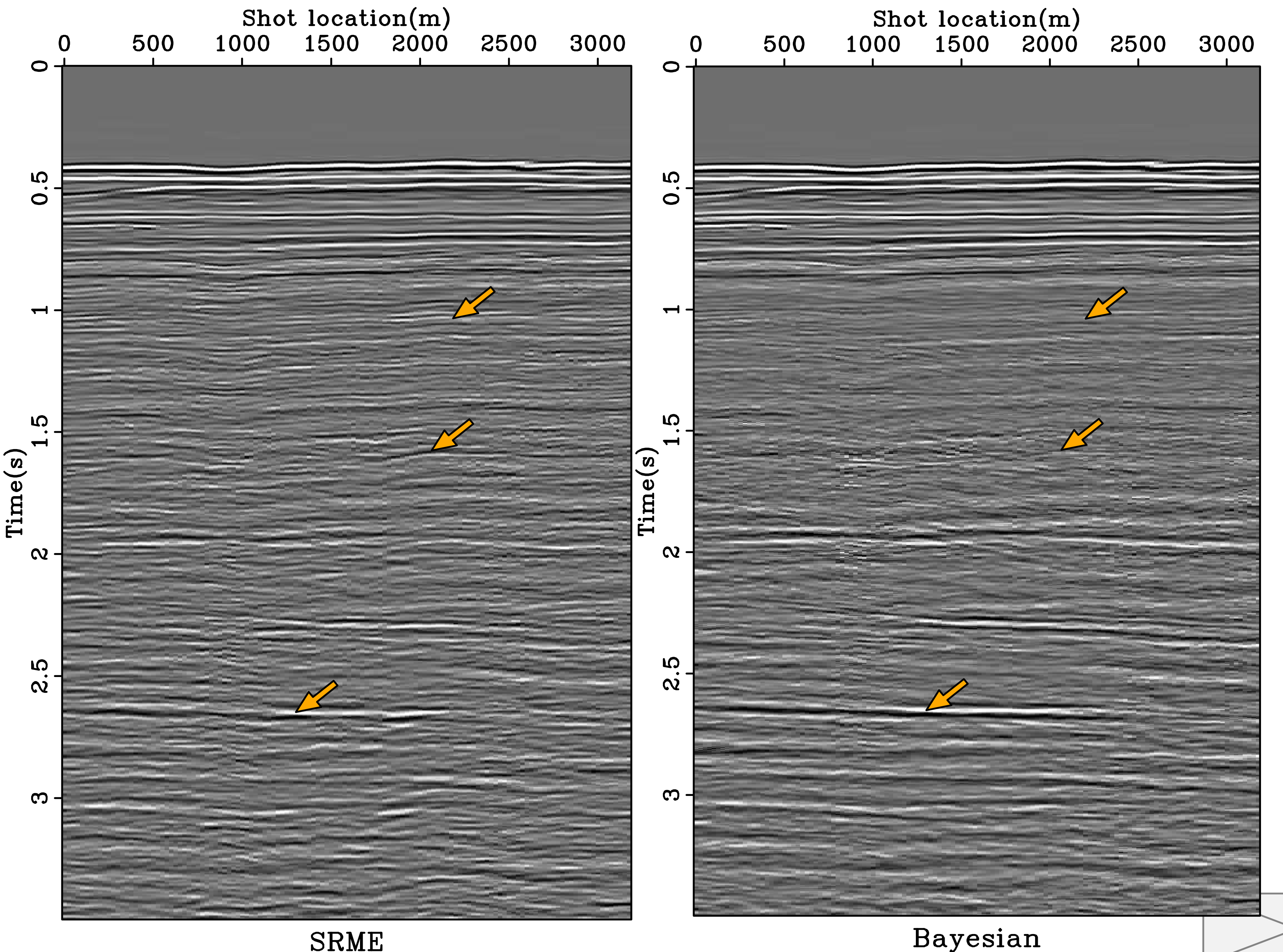



Bayesian

# Example 1


Shot location(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
SRME

# Example 1



Shot location(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
SRME
Shot location(m)
0
500
1000
1500
2000
2500
3000
Time(s)
0
0.5
1
1.5
2
2.5
3
Bayesian

# Examples

Example 2

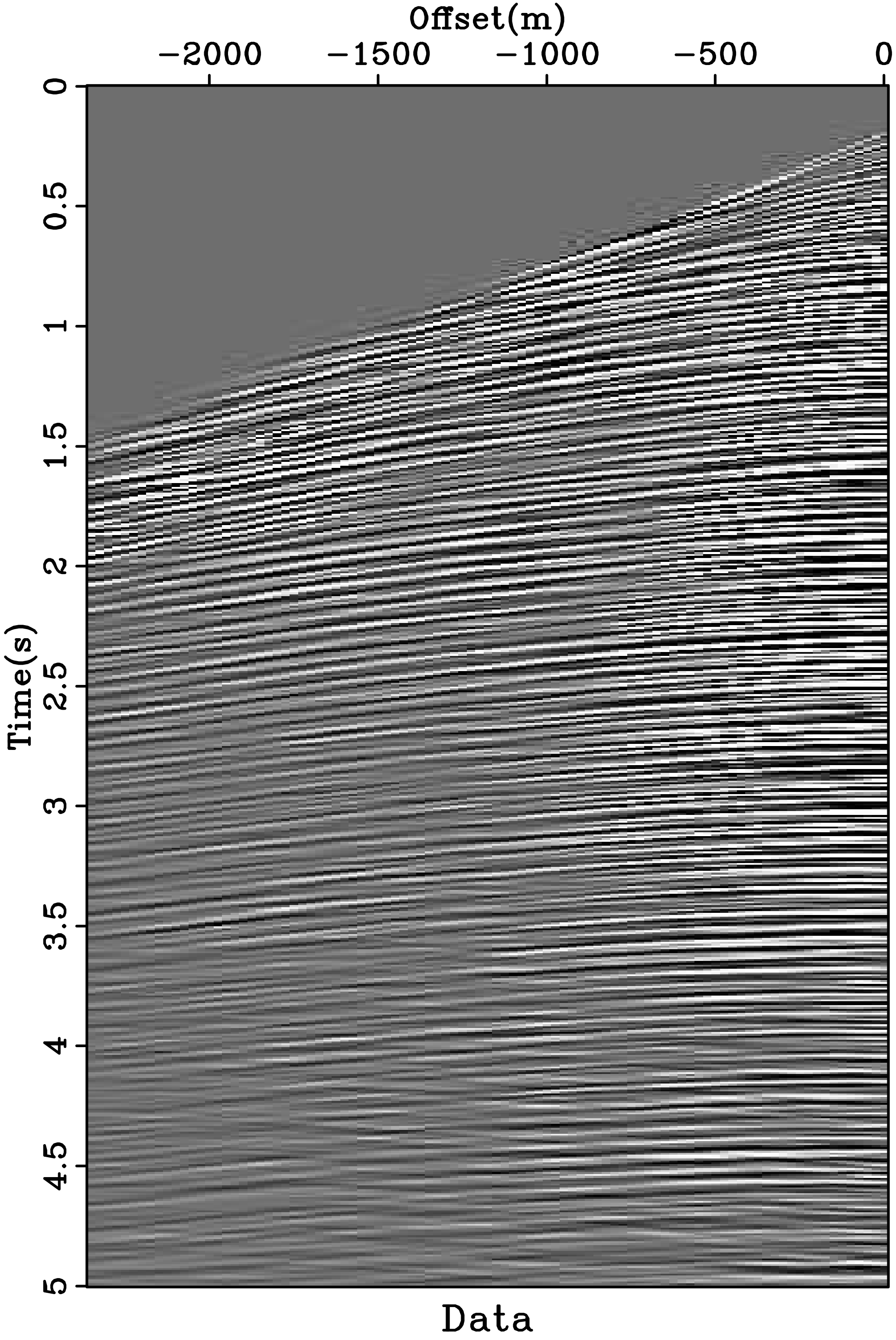
Gulf of Suez data:

340 shots

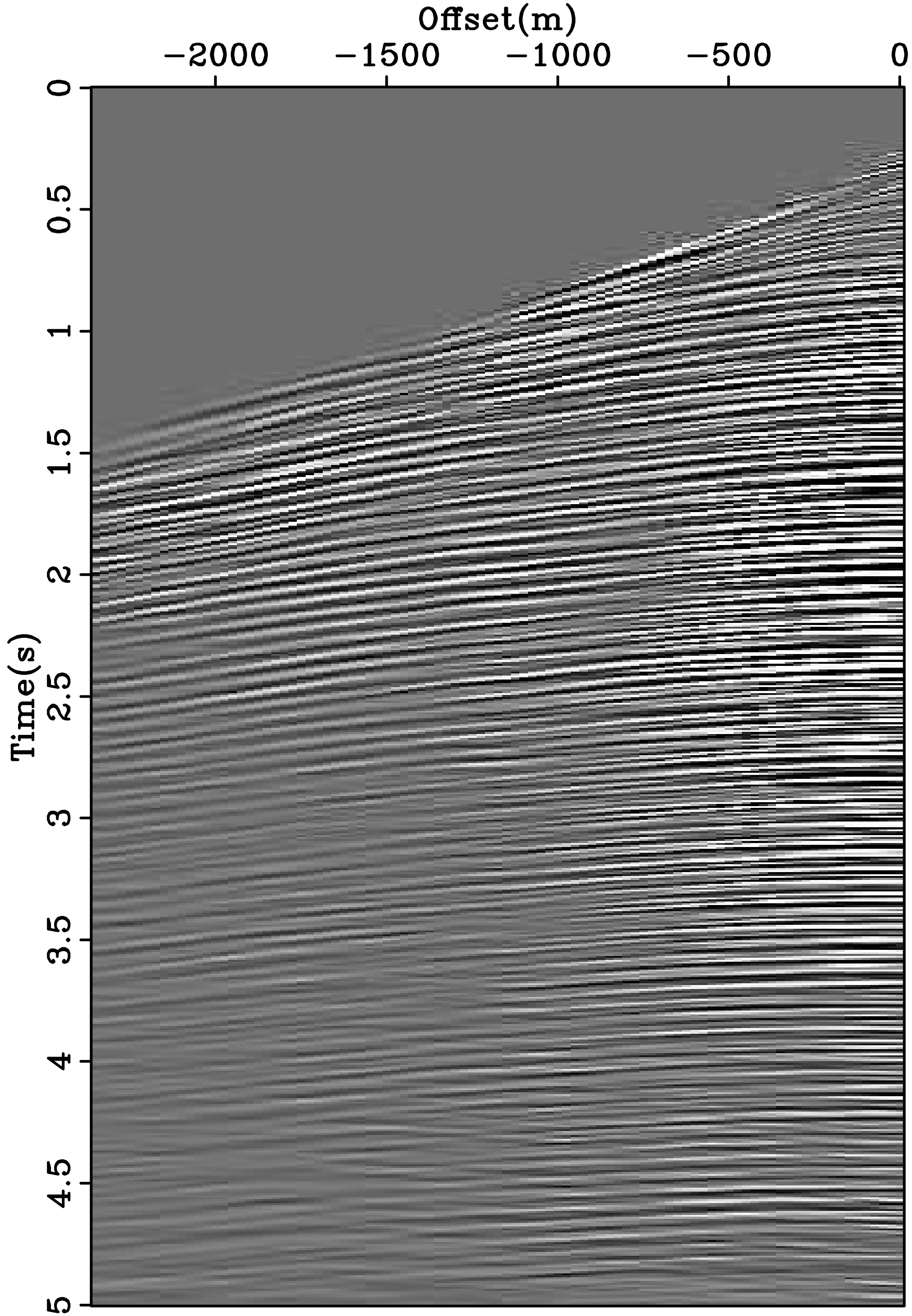
95 traces/shot
626 samples/trace

The original  data contains many short period multiples and surface-related multiples

# Example 2

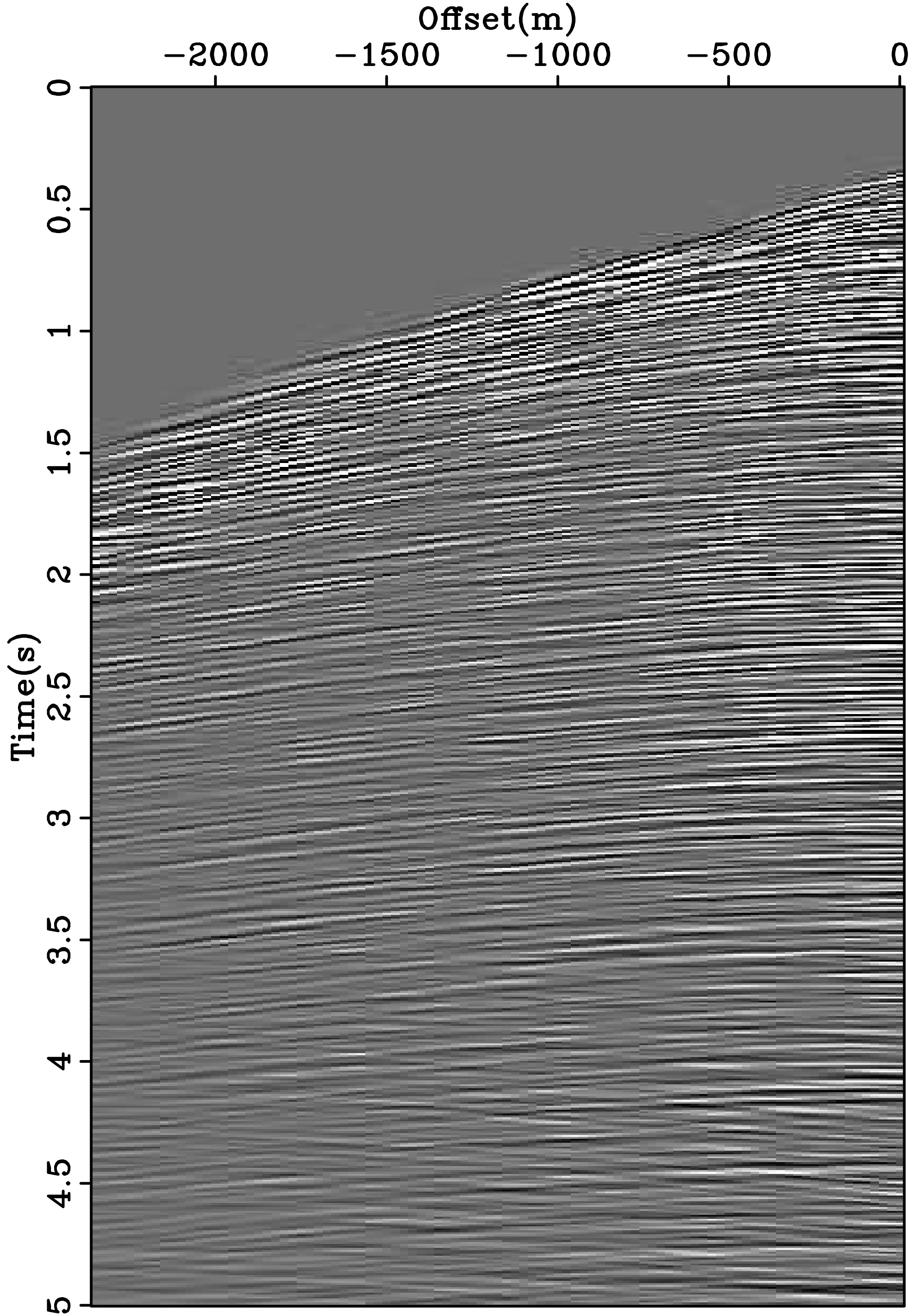


# Example 2



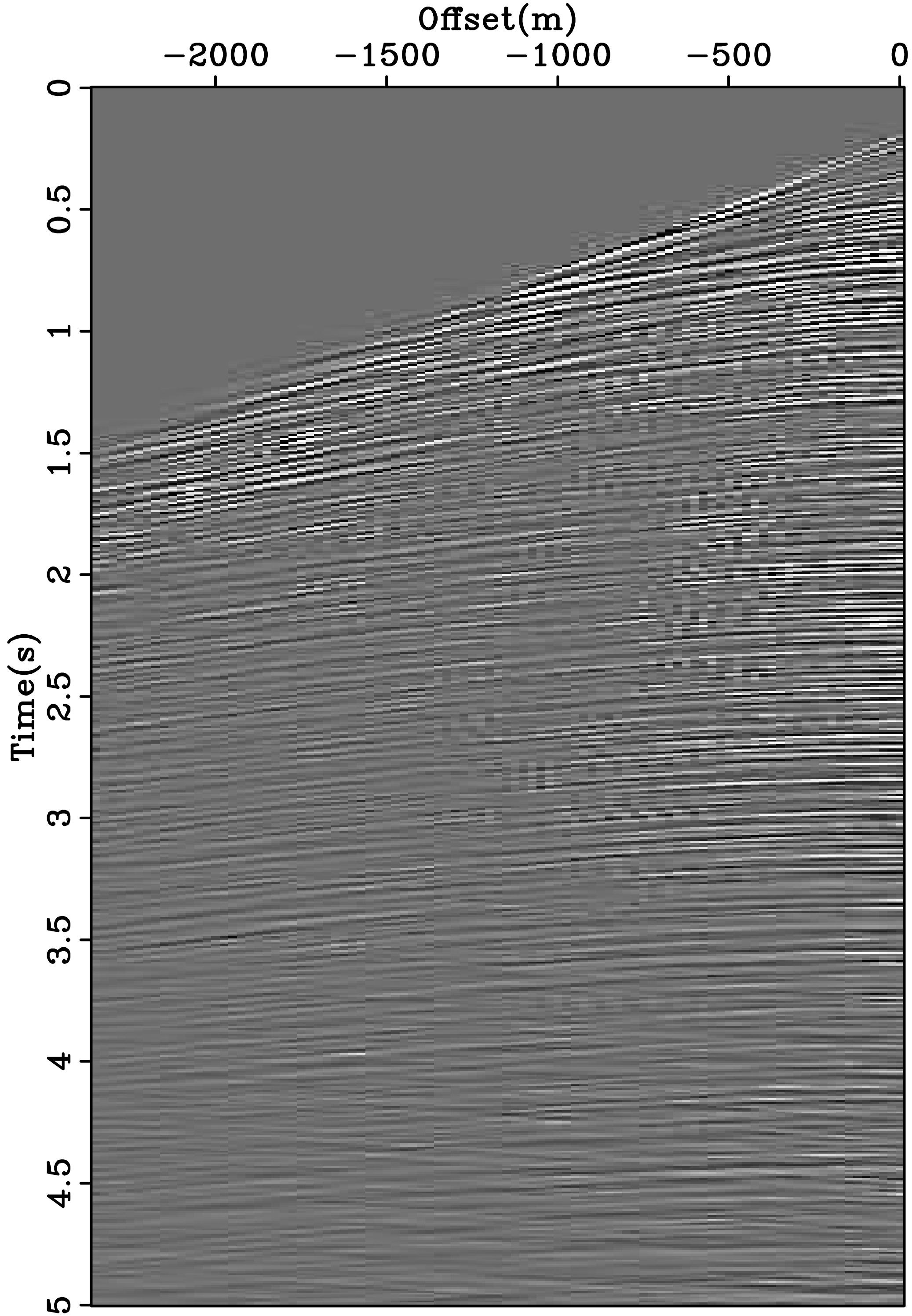

Multiples

# Example 2



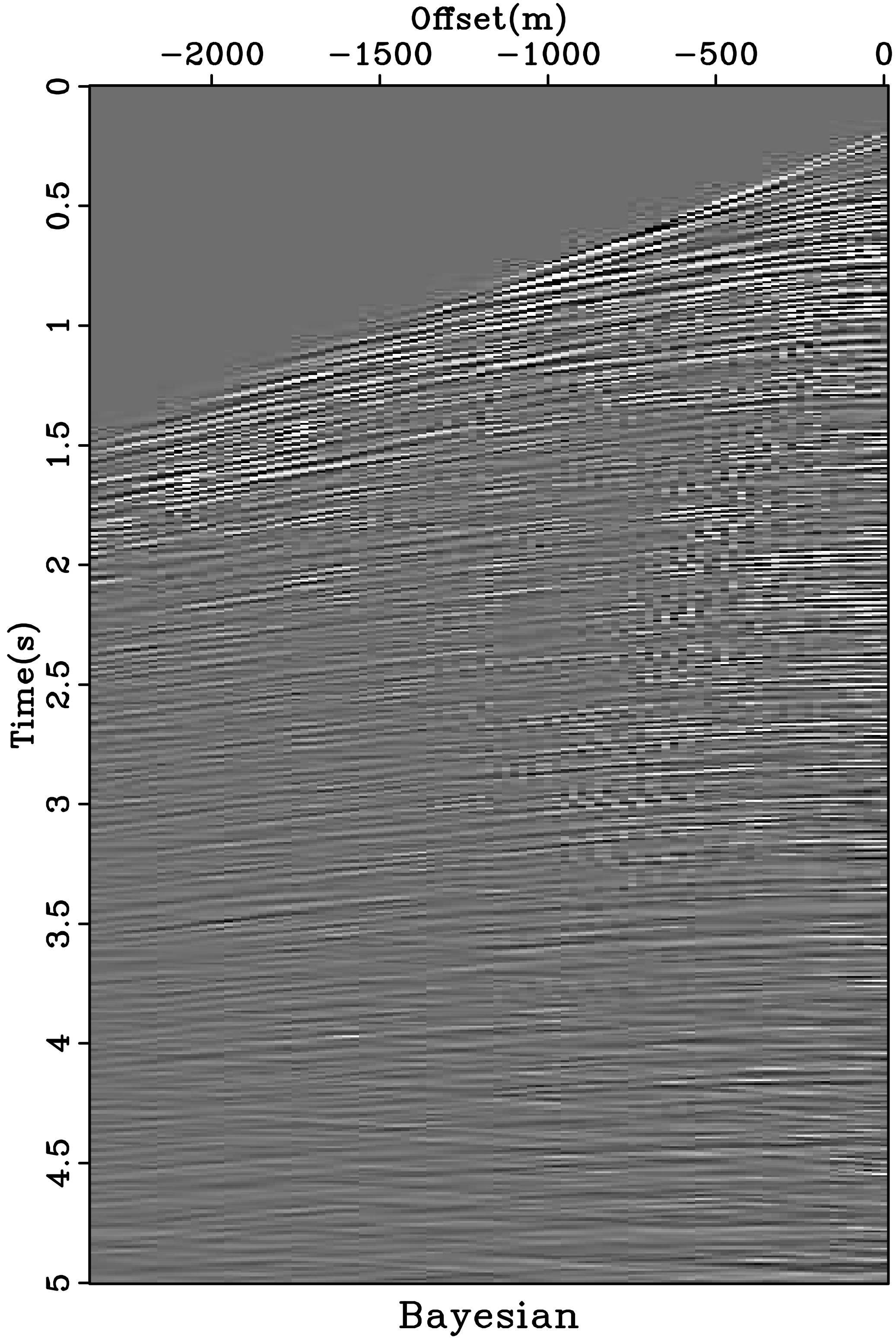

SRME

# Example 2



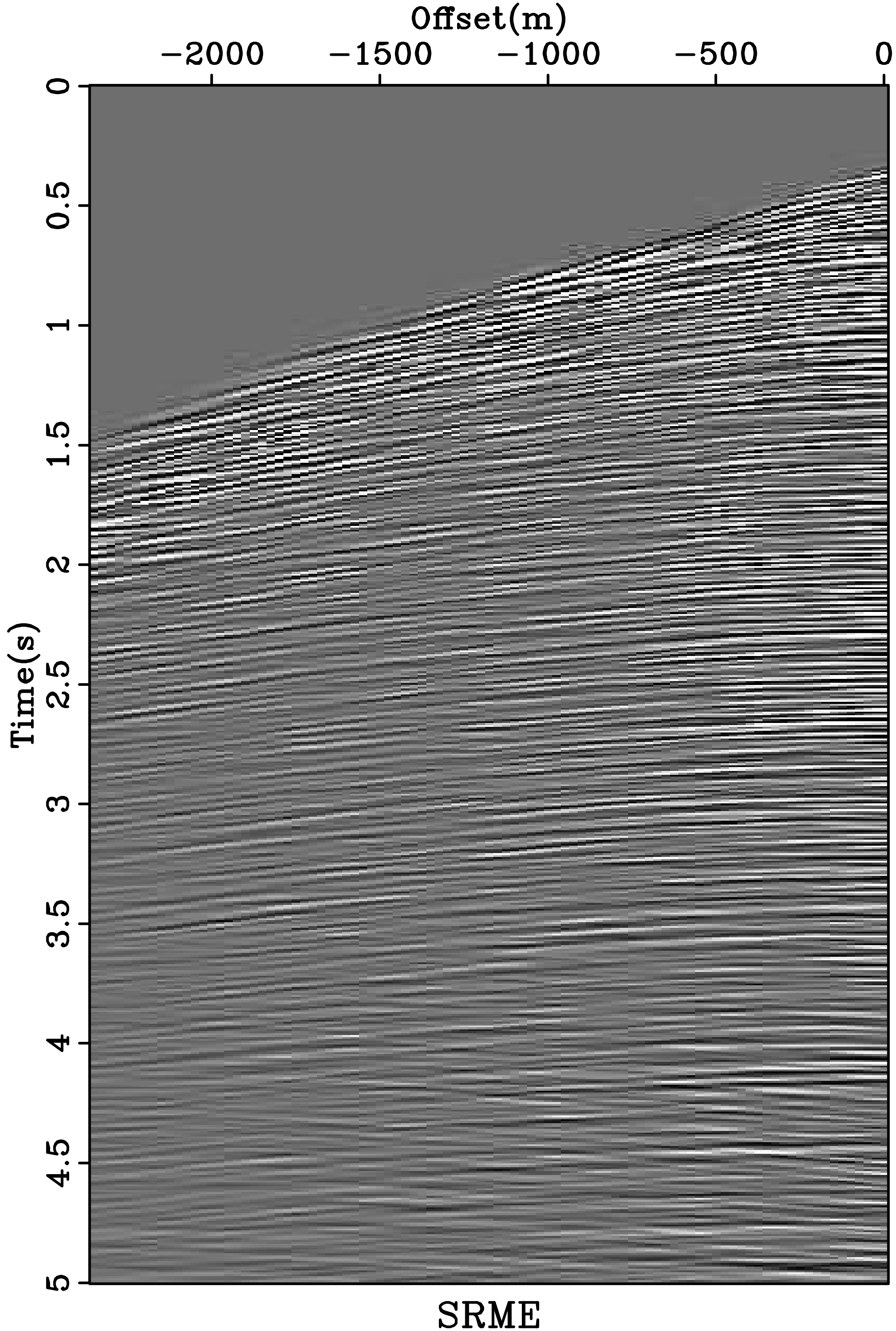

Single threshold

# Example 2

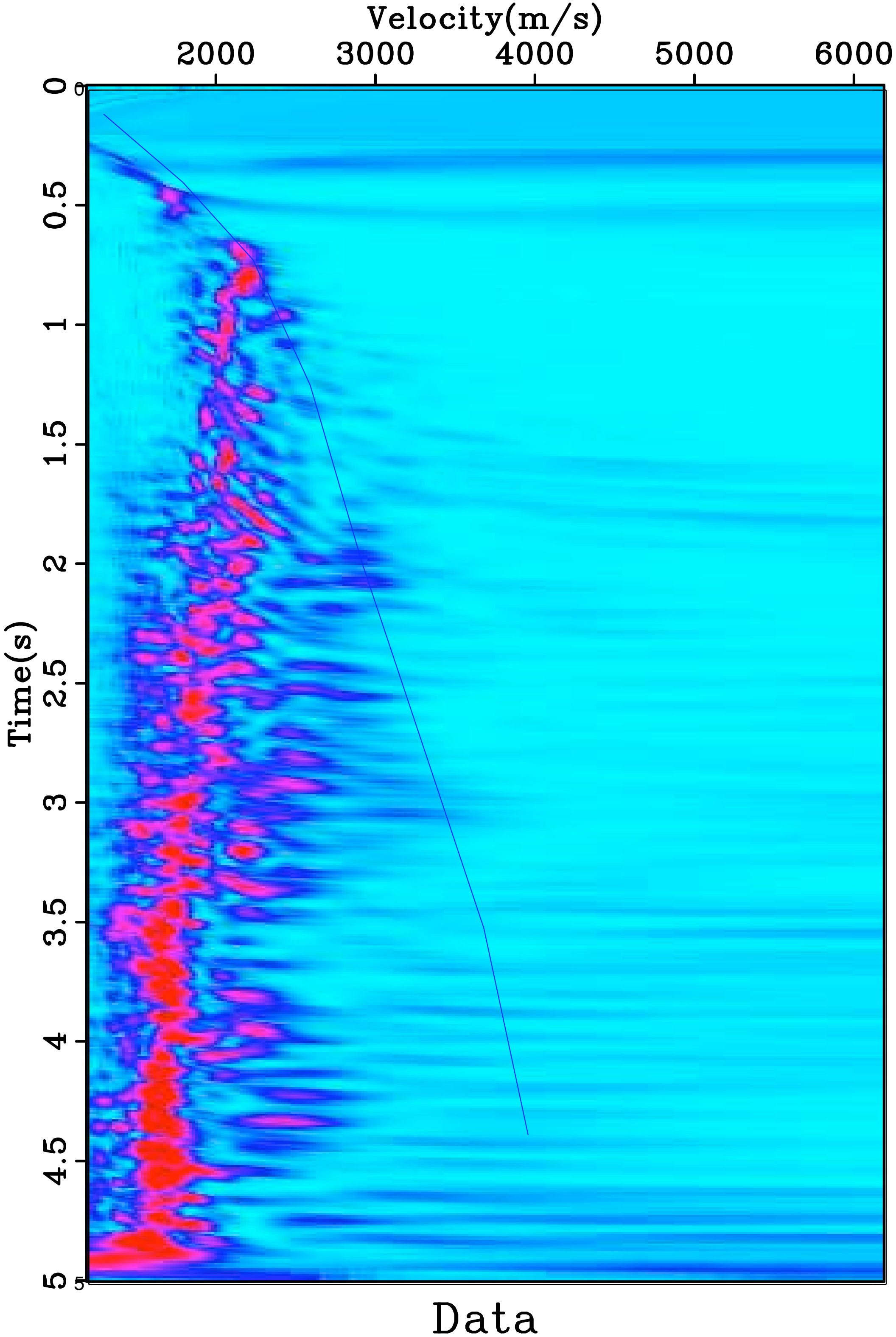


Offset(m)
−2000
−1500
−1000
−500
0
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Bayesian

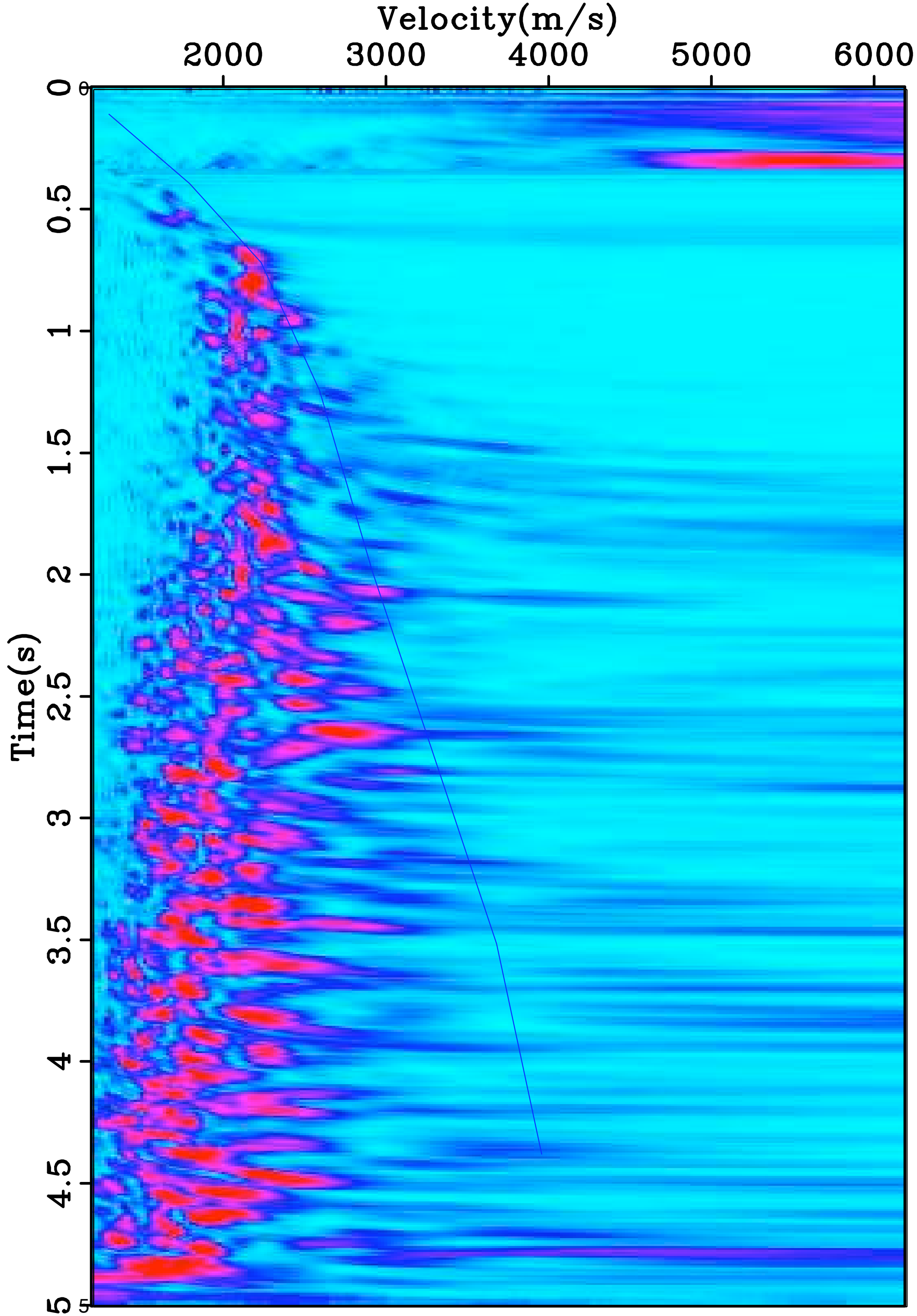
# Example 2



# Example 2

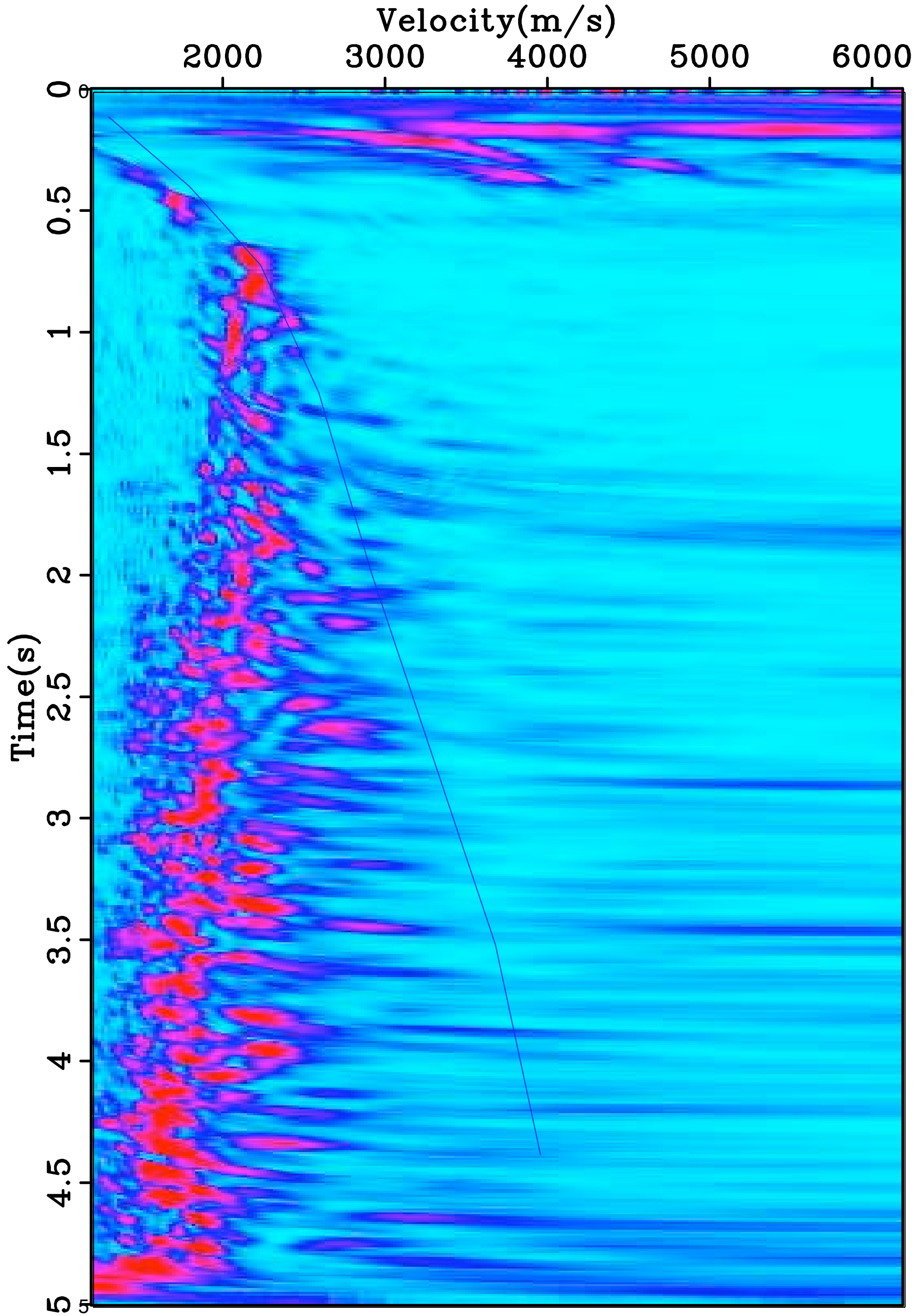

Velocity(m/s)
2000
3000
4000
5000
6000
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Data

# Example 2



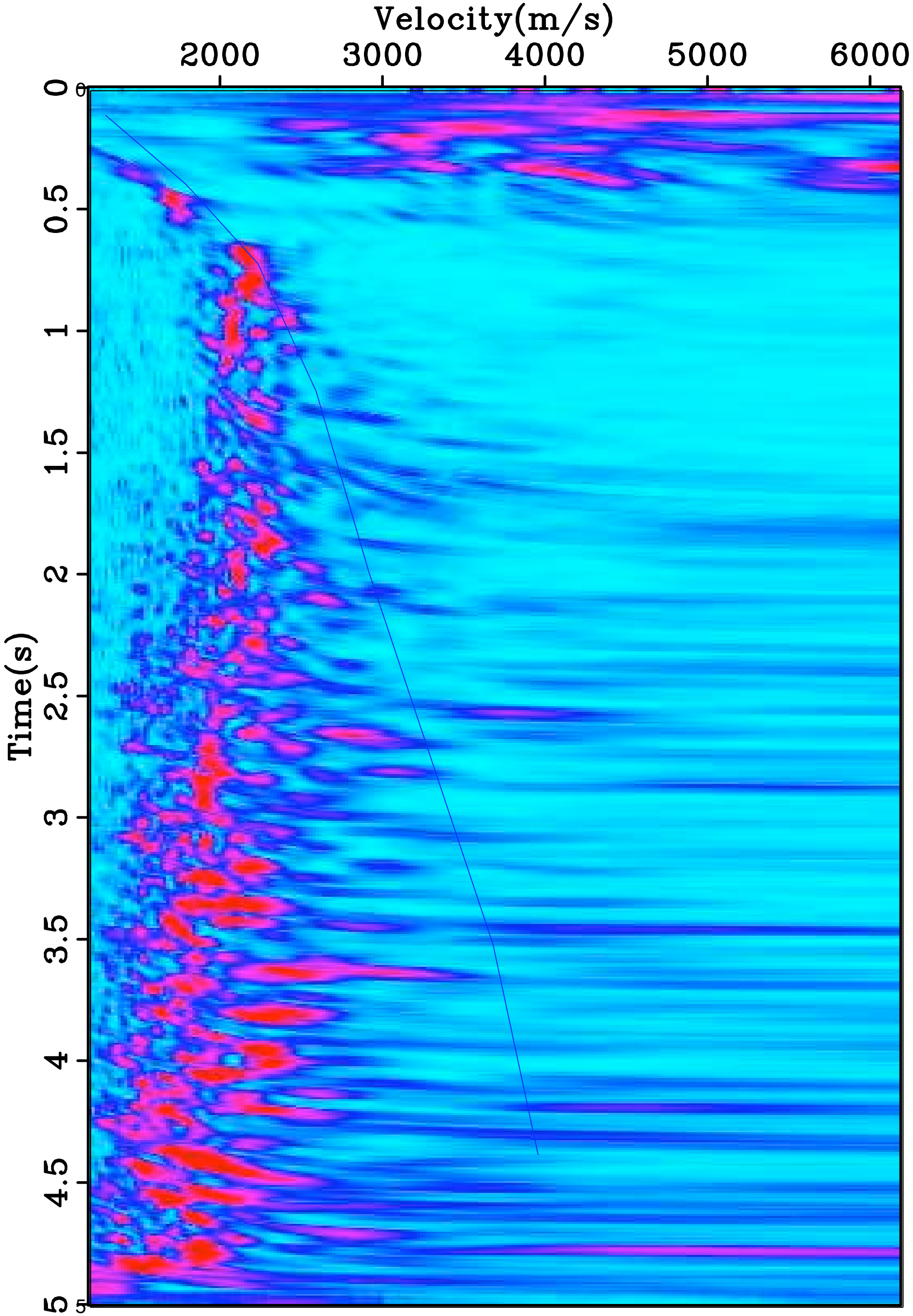

SRME

# Example 2



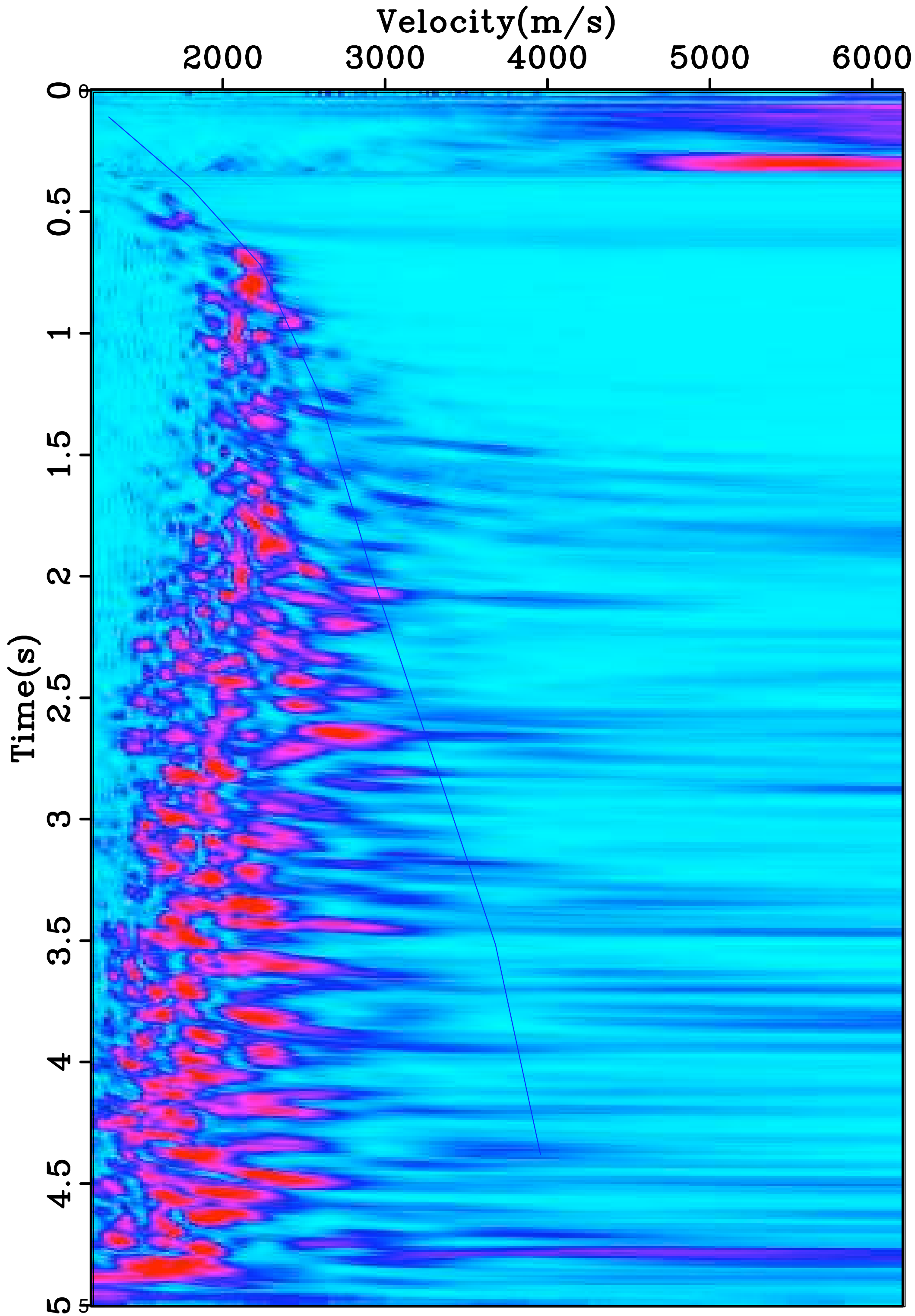

Single threshold

# Example 2



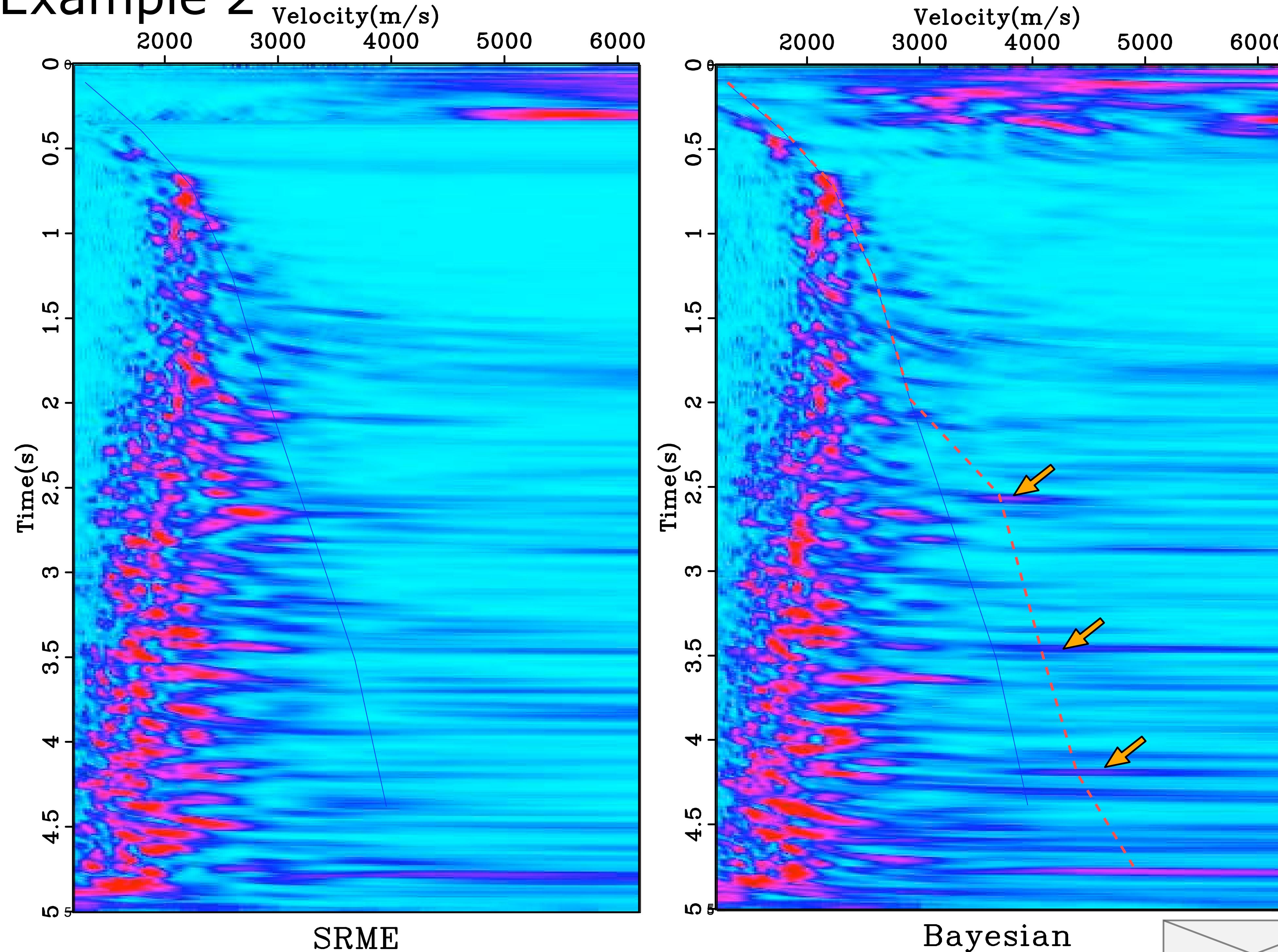

Bayesian

# Example 2



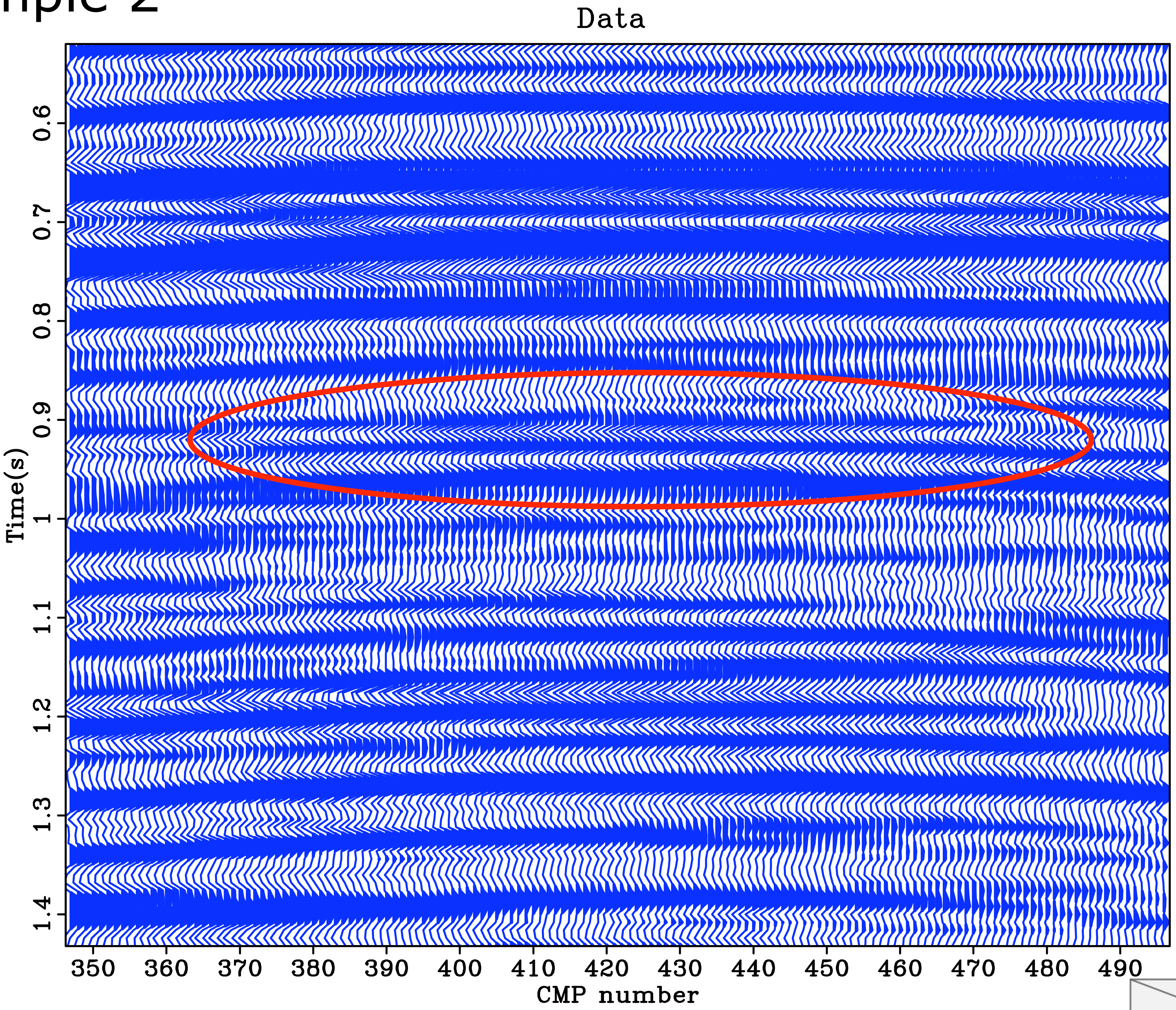
SRME

# Example 2

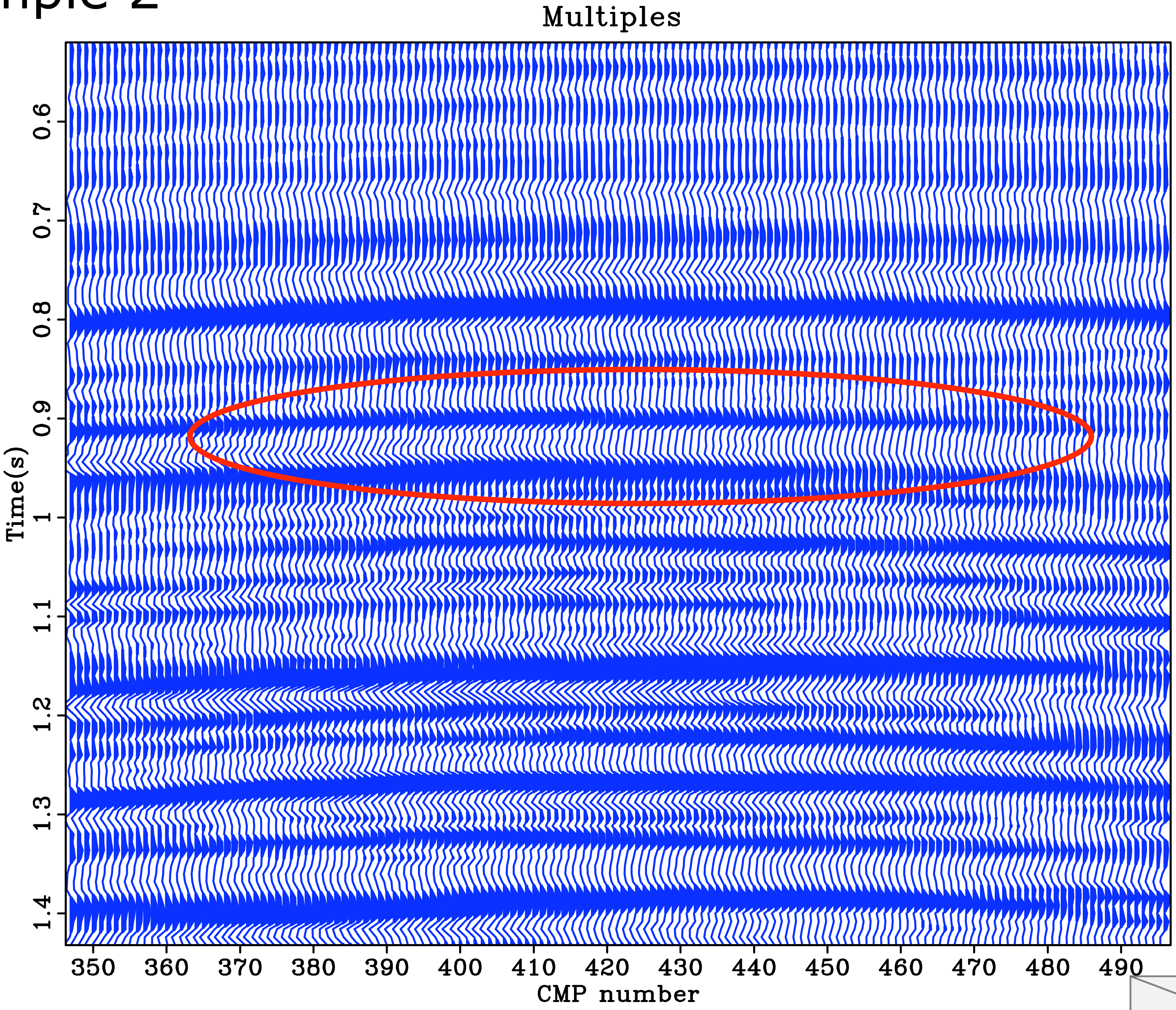


Velocity(m/s)
2000
3000
4000
5000
6000
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
SRME
Velocity(m/s)
2000
3000
4000
5000
6000
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Bayesian

# Example 2

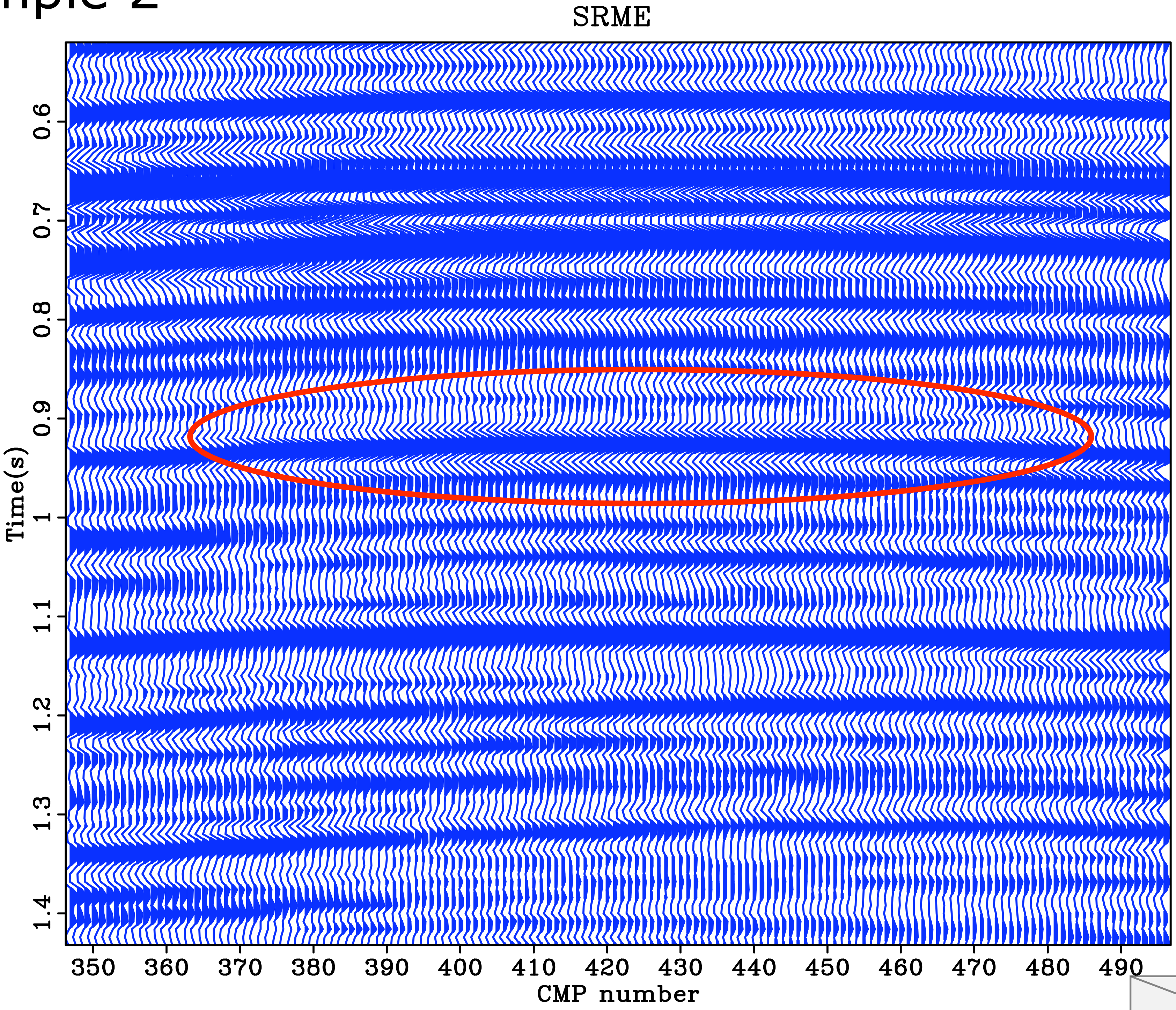

Data
Time(s)
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

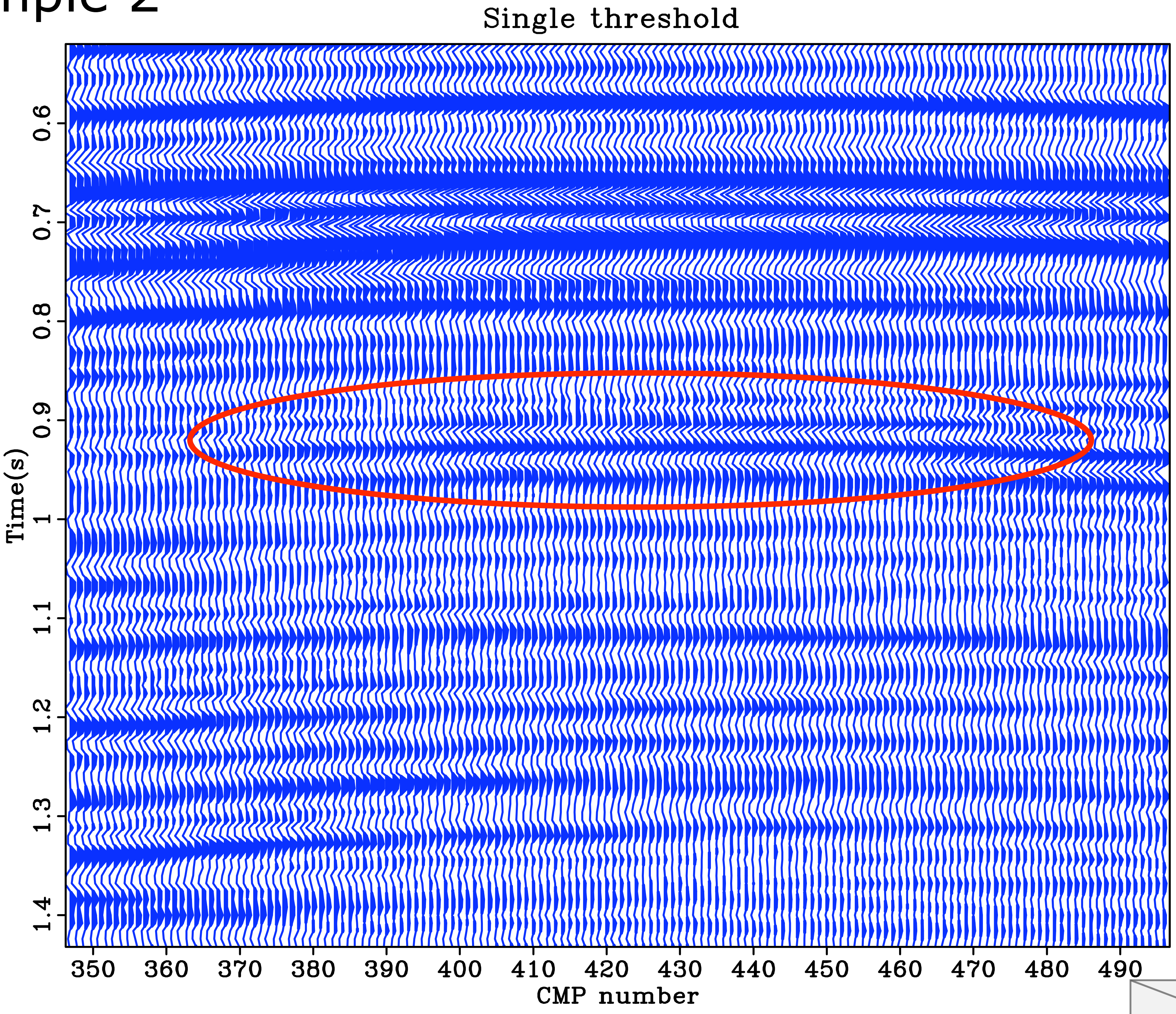


Multiples
Time(s)
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

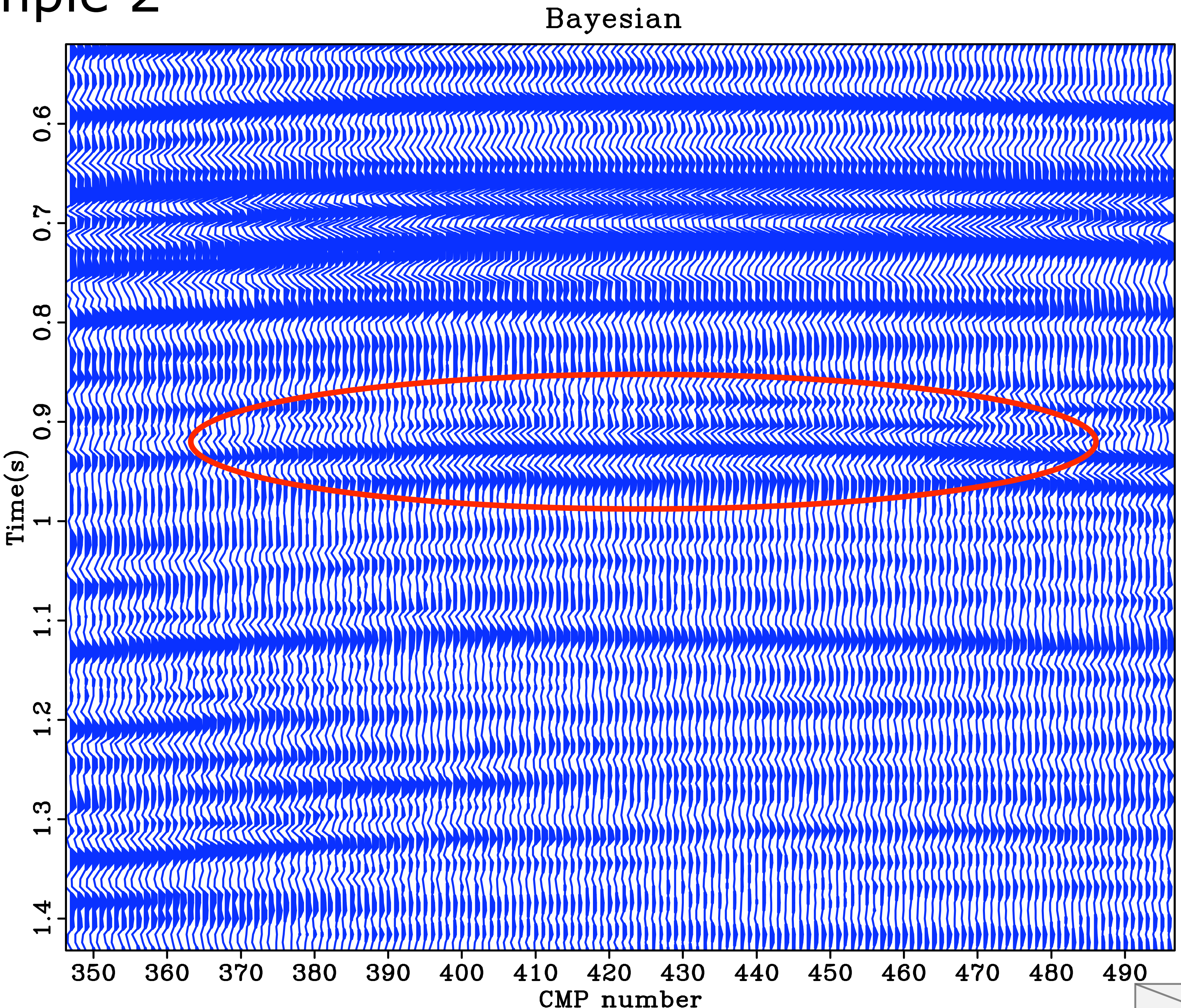

SRME
Time(s)
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

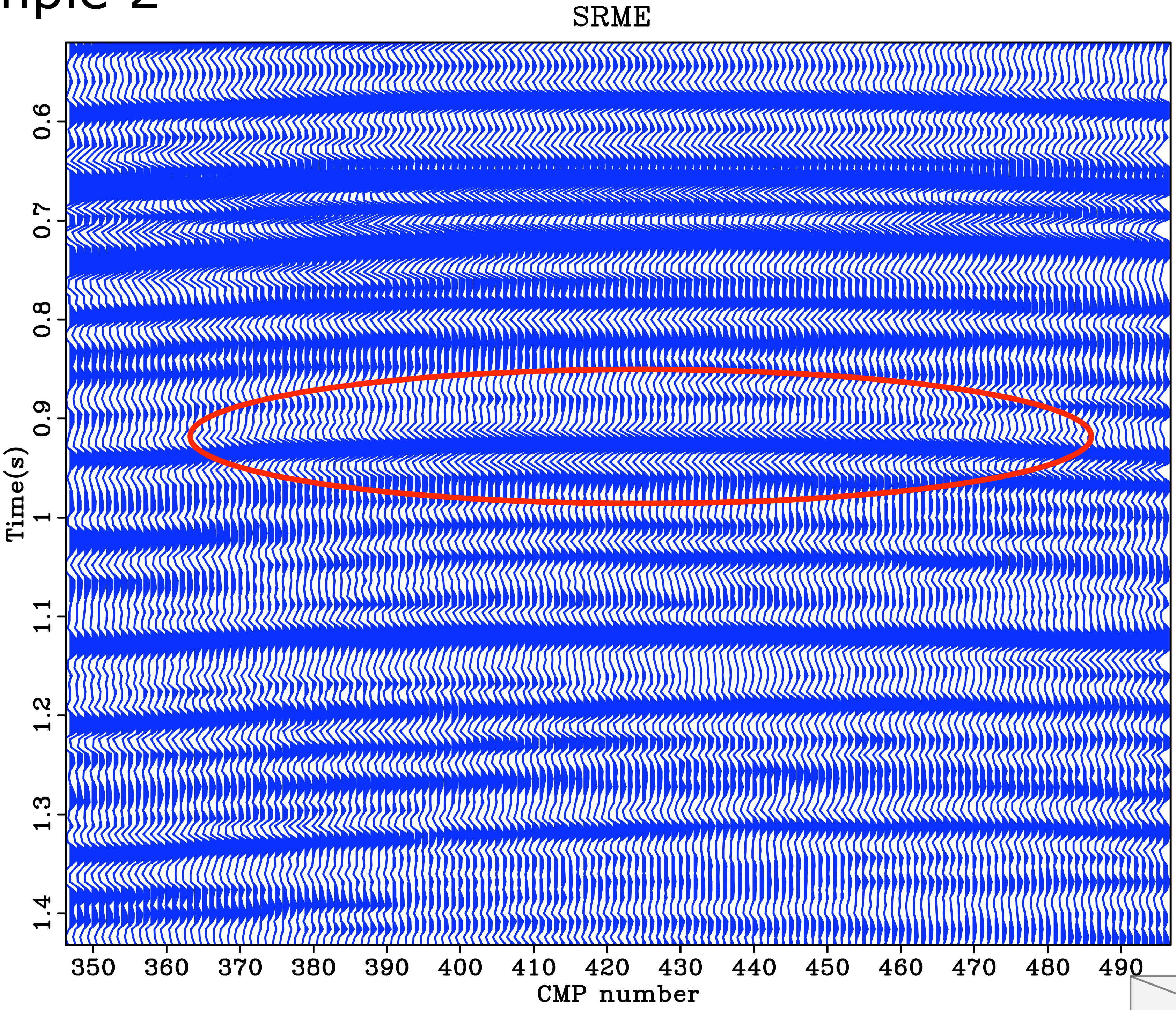


Single threshold
Time(s)
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

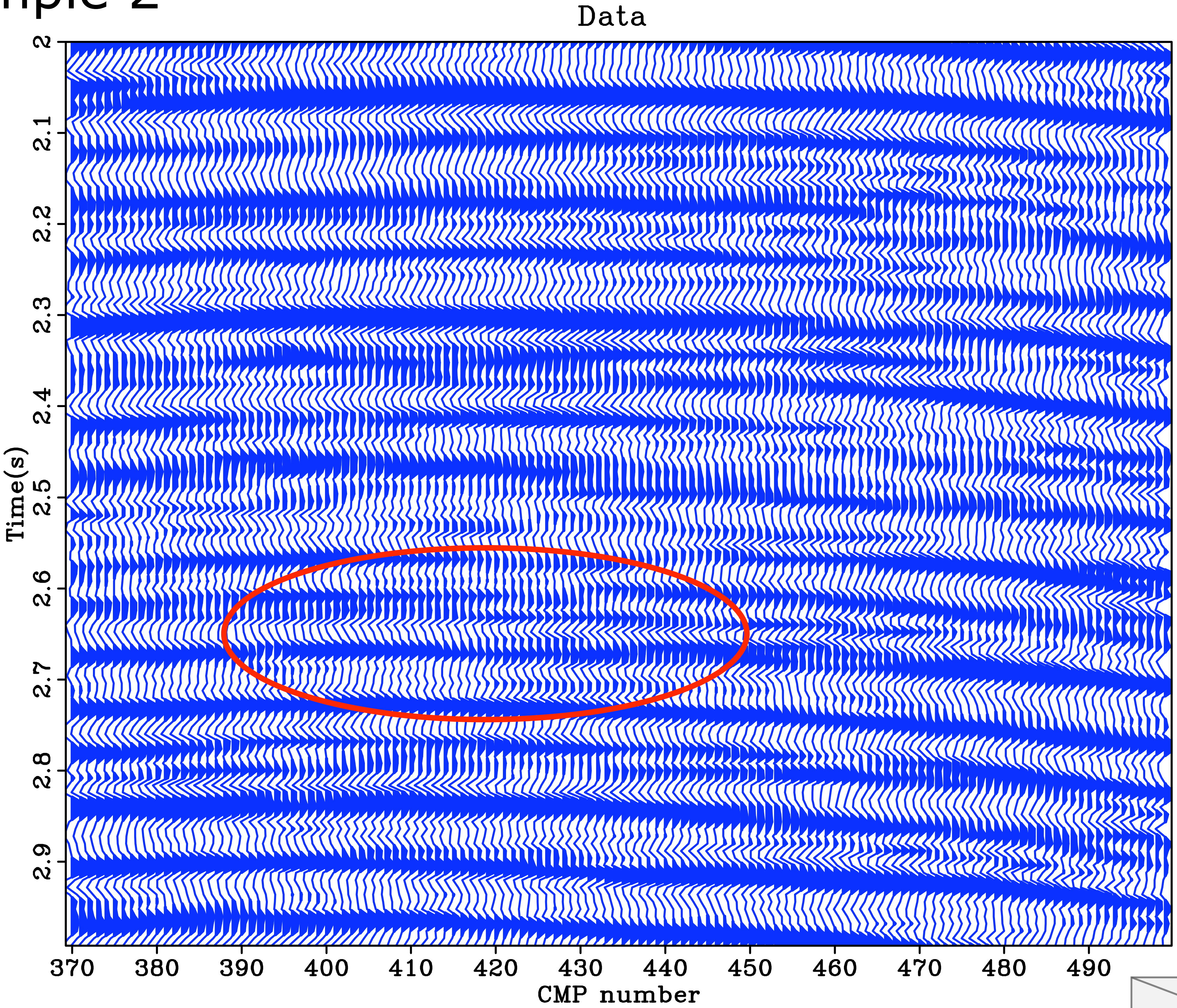
# Example 2


Bayesian
Time(s)
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

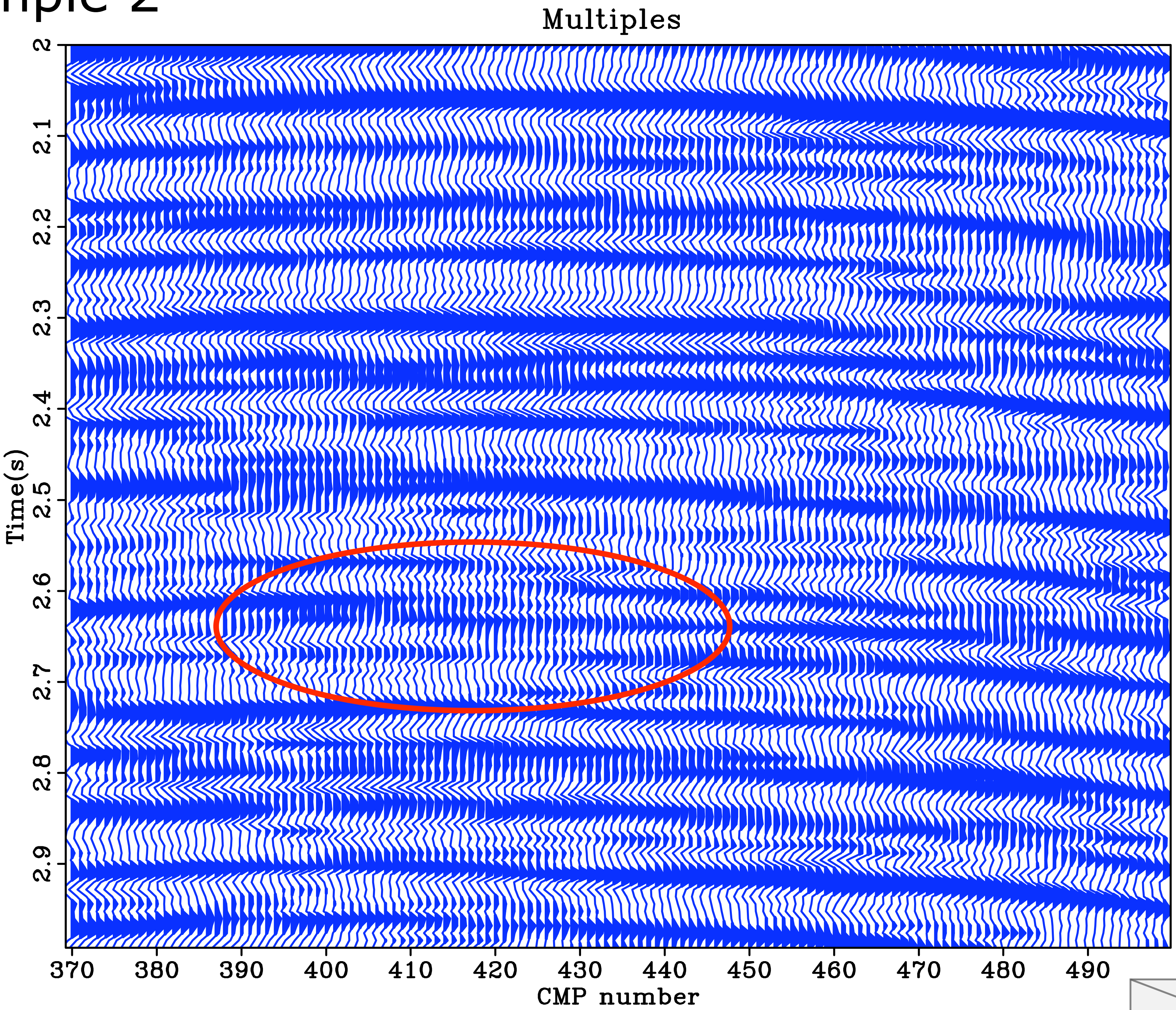
# Example 2


SRME
Time(s)
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

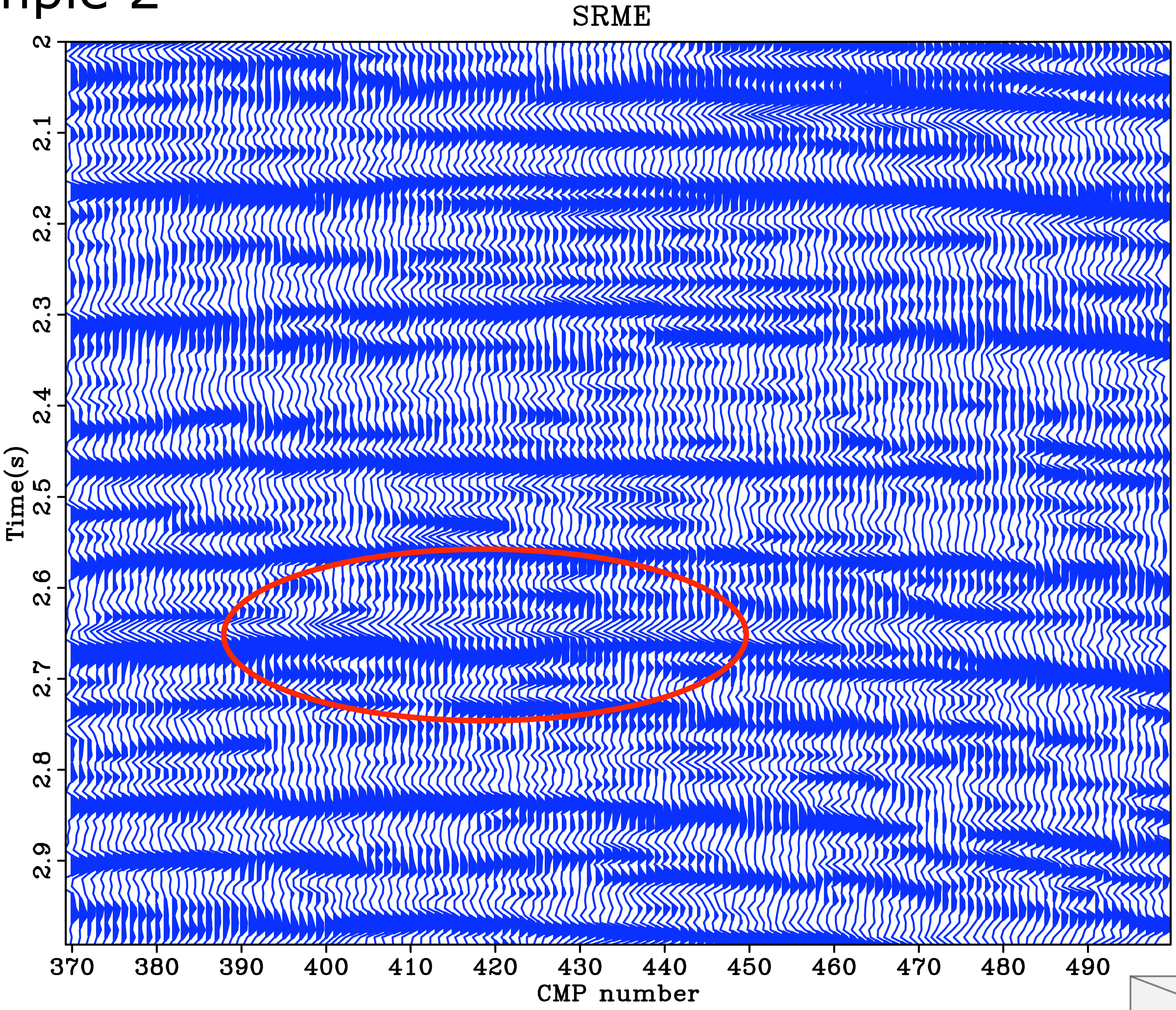

Data
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

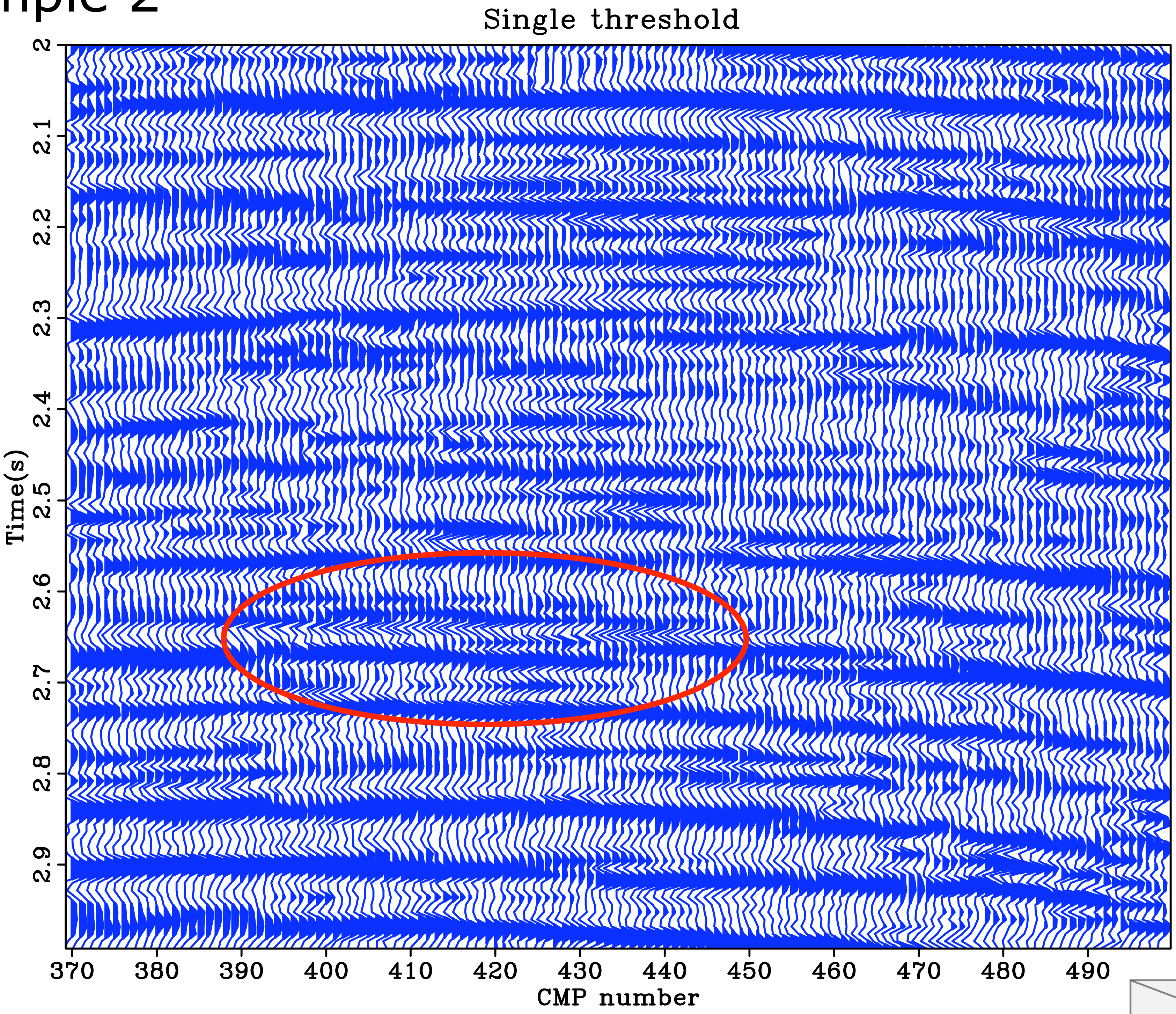


Multiples
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

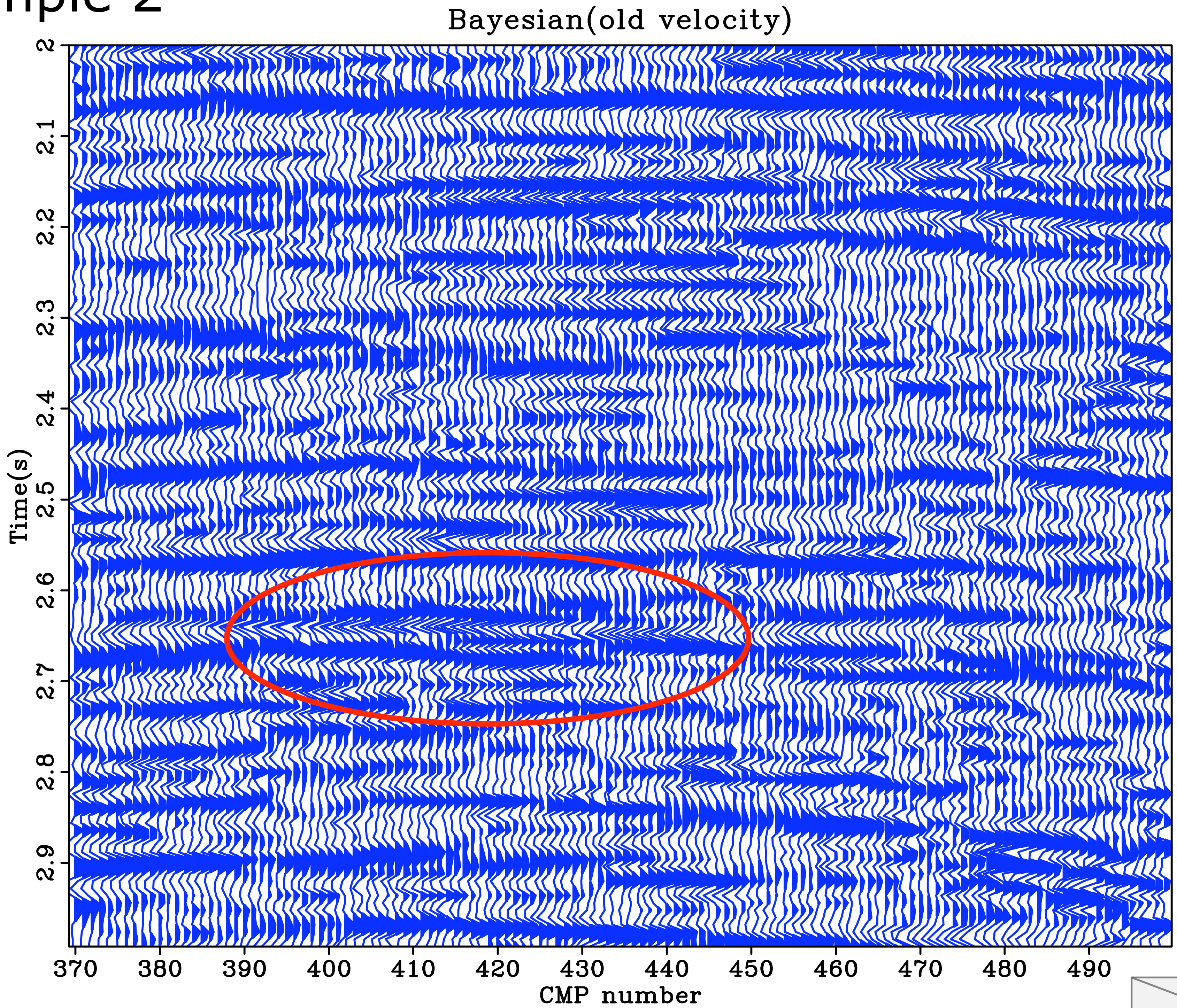
# Example 2


SRME
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

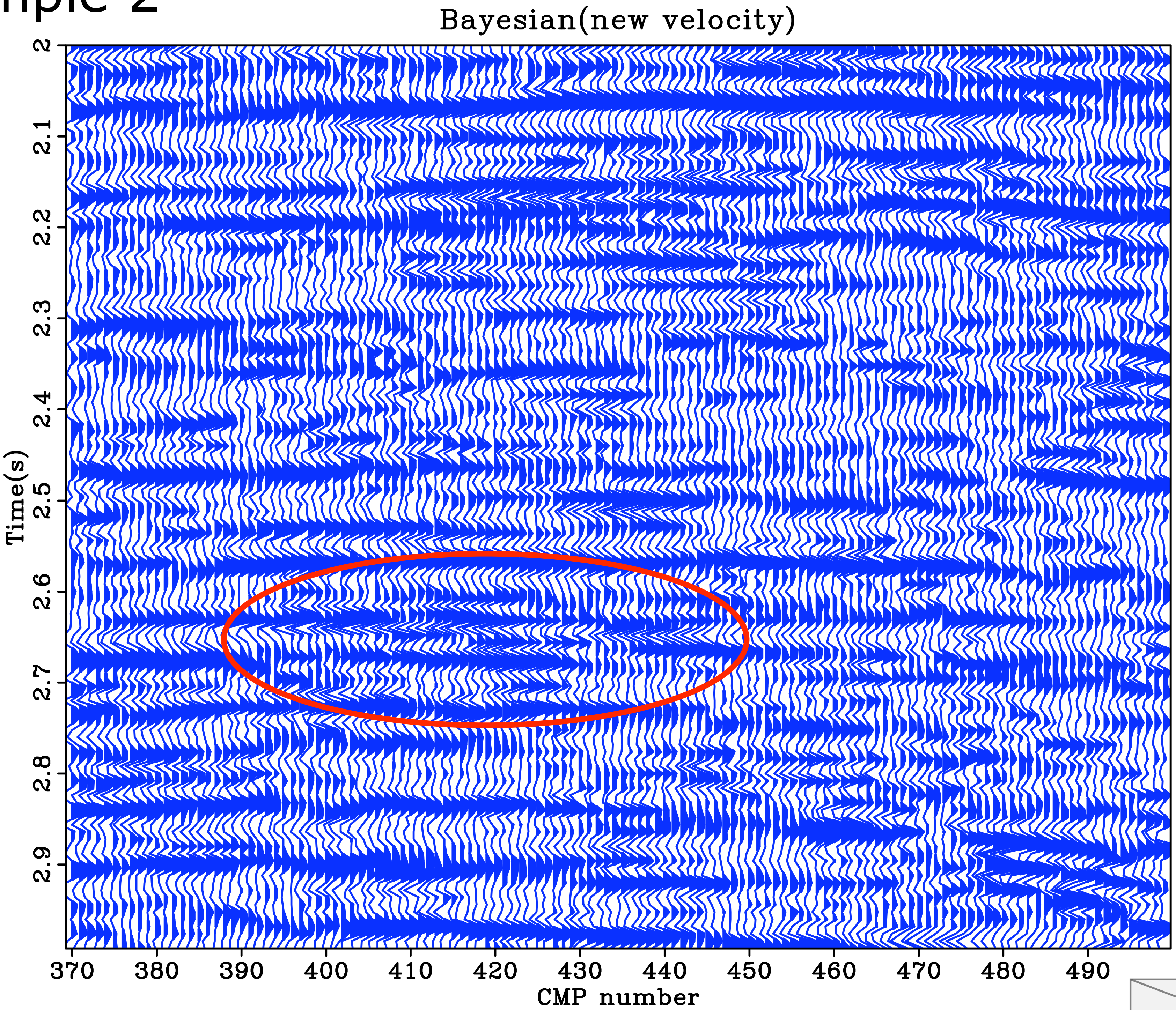
# Example 2


Single threshold
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

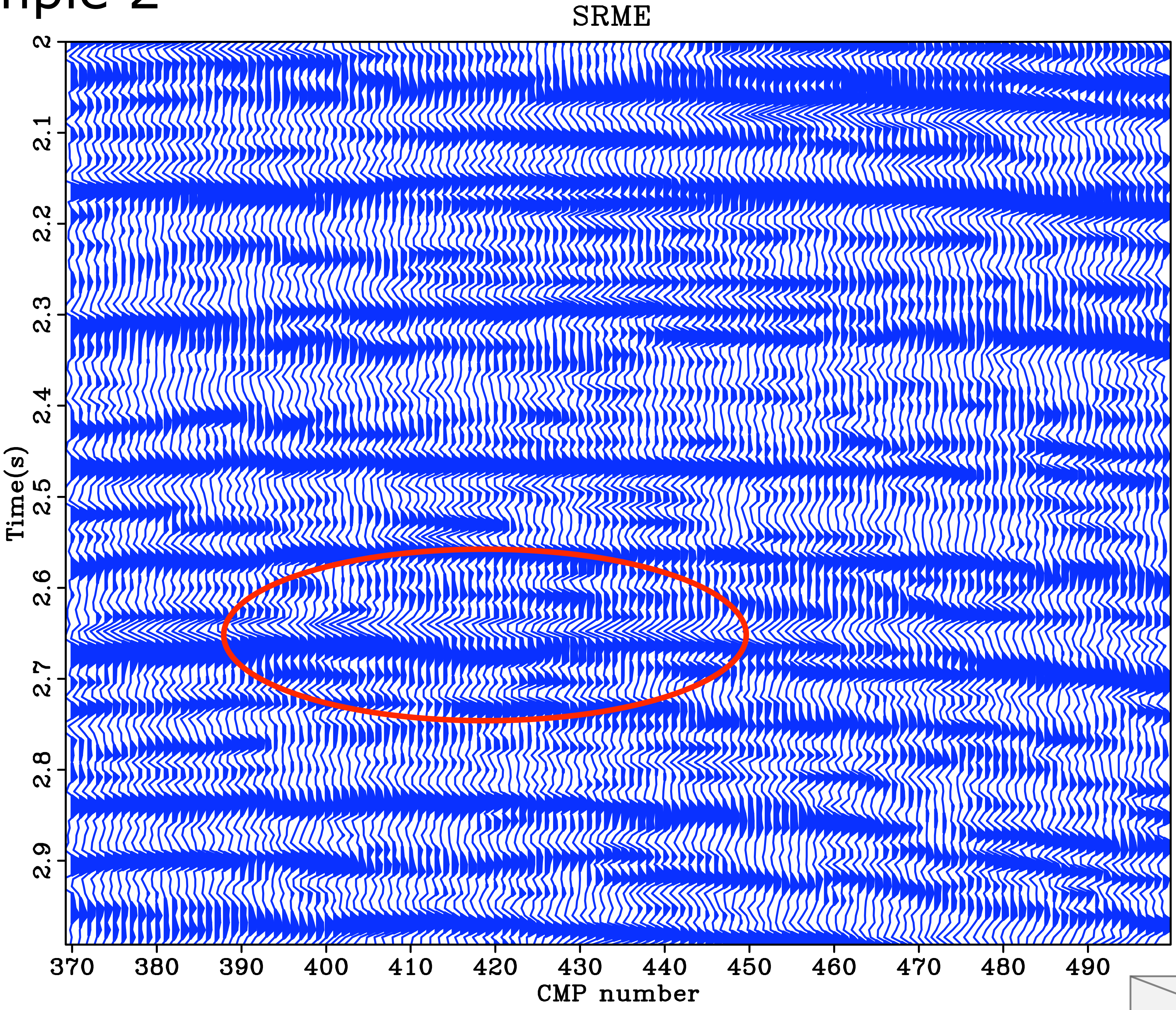

Bayesian(old velocity)
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

# Example 2

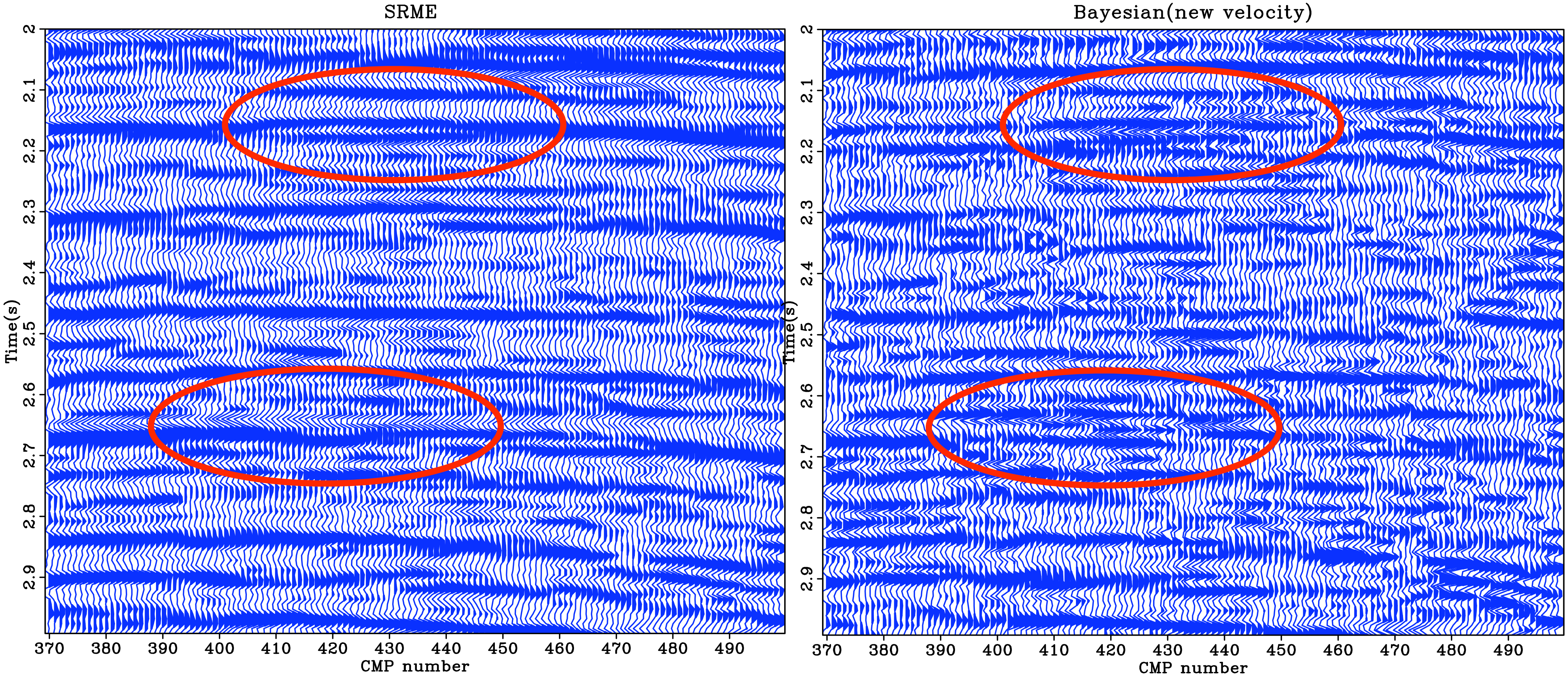


Bayesian(new velocity)
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370
380
390
400
410
420
430
440
450
460
470
480
490
CMP number

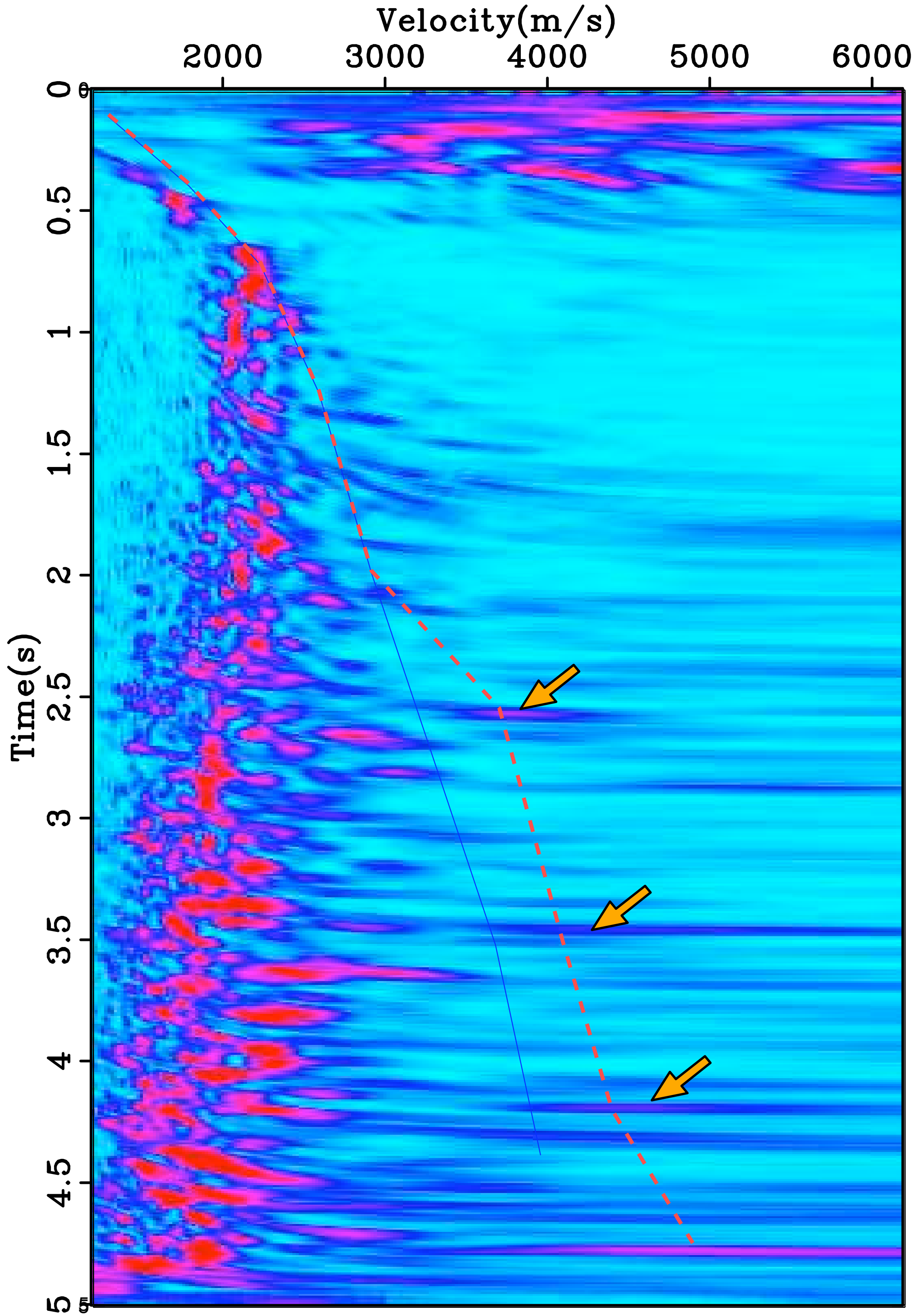
# Example 2


SRME
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370 380 390 400 410 420 430 440 450 460 470 480 490
CMP number

# Example 2


SRME
Bayesian(new velocity)
Time(s)
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
370 380 390 400 410 420 430 440 450 460 470 480 490
CMP number

# Example 2




Bayesian

# Contents

# Discussion and conclusions

- Curvelets represent the ***ideal*** domain for primary-multiple separation
- Curvelet construction allows for a separation based on differences in curvelet attributes and allows for a ***sparsity*** promoting formulation of the primary- multiple separation problem.
- The curvelet's multi-angular parameterization helps the separation, even for ***erroneous*** predictions.
- The ***nonlinear*** optimization algorithm shows a clear improvement in the primary-multiple separation.
- Results application to ***real*** data are encouraging
  - improved velocity panel
  - improved resolution

# Contents

- Introduction
- Curvelet-based primary-multiple separation
- Examples
- Discussion and conclusion
- Acknowledgments

# Acknowledgments

Eric Verschuur, input in primary-multiple separation

The authors of CurveLab, making their codes available.

Sergey Fomel, developed Madagascar

SLIM team: Gilles Hennenfent, Sean Ross Ross, Cody Brown, Henryk Modzelewski et al.

This work was in part financially supported by the NSERC Discovery (22R81254) and CRD Grants DNOISE (334810-05) of F.J.H. and was carried out as part of the SINBAD project with support, secured through ITF, from BG Group, BP, Chevron, ExxonMobil and Shell.