Robust Seismic Image Amplitude Recovery Using Curvelets

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Seismic Laboratory for Imaging and Modeling

References

- **Curvelets and its invariance,**
  - Curvelet and FIOs, Candes and Demanent (2002)
  - Hardy space for FIOs, Smith (1998)
- **Migration amplitude recovery**
  - Optimal scaling for RTM, Symes (2007)
  - Illumination based migration, Rickett and Claerbout (2000)
  - Hessian Approximation based, Mulder (2003)
  - True amplitude migration, Zhang (2003)
  - Least square migration, Kuhl and Sacchi (2001)
- **Continuity promoting and anisotropic diffusion,**
  - Regularization for denoising, Scherzer (2003)
  - In seismic imaging, Fehmer (2003)
- **Optimization method,**
  - Soft-thresholding, Donoho (1995)
  - Iterative thresholding, Daubechies (2005)
  - Gradient based optimization, Nocedal (2001)

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Overview

- Definition of problem
- Imaging as an inversion problem
- Curvelets and their properties
- Curvelets and their invariance under the normal operator
- Normal operator approximation
- Curvelets factorization of the normal operator
- Problem reformulation
- Optimization
- Results
- Conclusion

Definition of the problem

- Basic imaging problem

$$\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n}$$

- Desired seismic image characteristics:
  - Broad-band
    - Sharp, high resolution
    - 2D curves/3D sheets
  - Continuity along the reflectors
- Noise in seismic images
  - Random noise
    - Instruments distortion
    - Ambient
  - Imaging operator imperfections

Imaging as an inverse problem

- Following inversion problem is introduced

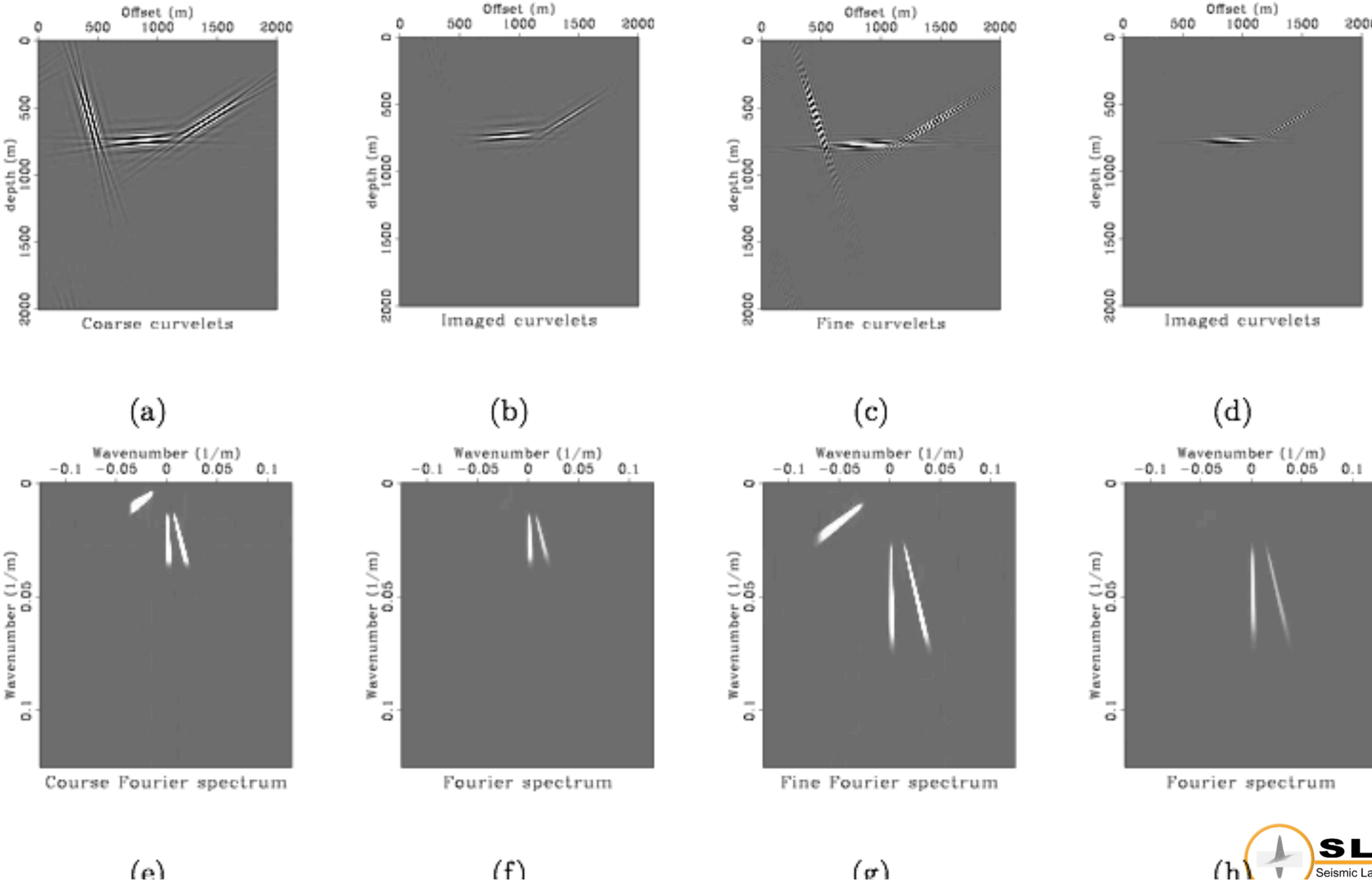
$$\min_{\mathbf{m}} J(\mathbf{m}) \text{ subject to } |\mathbf{d} - \mathbf{K}\mathbf{m}|_2^2 \leq \epsilon$$

- J(m) is the norm or penalty function
- This norm has to
  - explore the continuity along the reflectors
  - explore the sparsity of image in the curvelet domain
  - reduce the artifacts from the image
  - enhance the reflectors
  - remove the noise from the seismic image

Curvelets and their properties

- Curvelets:
  - are multiscale and multi-directional
  - sparsely present seismic images
  - are *invariant* under the action of idealize normal operator
  - are constructed as tight frames
  - transformation is fast
  - are reliably used for denoising in image processing applications

Examples (three curvelets)



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Normal operator approximation

- Approximation with curvelet eigenvalue-like decomposition:

$$C^T D C r \approx K^T K r$$

- Diagonal matrix is smooth in the curvelet domain
- Computationally cheap, requires only "one" evaluation of the normal operator

Diagonal approximation

- Estimation with smoothness constraint

$$\mathrm{C^T D C r} \approx \mathrm{K^T K r}$$

$$\begin{pmatrix} \mathrm{C^T diag(v)} \\ \mathrm{L} \end{pmatrix} \mathrm{d} = \begin{pmatrix} \mathrm{K^T K r} \\ 0 \end{pmatrix}$$

$$\mathbf{L} = [\mathbf{D_x}^T\ \mathbf{D_y}^T\ \mathbf{D}_{\boldsymbol{\theta}}^T]^T \qquad \mathbf{v} = \mathbf{Cr}$$

- Solve using LSQR method
- Explore smoothness in curvelet's phase space

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Problem reformulation

- Forming the normal equation,

$$\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n} \Rightarrow \mathbf{K}^T\mathbf{d} = \mathbf{K}^T\mathbf{K}\mathbf{m} + \mathbf{K}^T\mathbf{n}$$
$$\Rightarrow \mathbf{y} \approx \mathbf{A}\mathbf{A}^T\mathbf{m} + \mathbf{e}$$
$$= \mathbf{A}\mathbf{x}_0 + \mathbf{e}$$

- with

$$\mathbf{y} = \mathbf{K}^T\mathbf{d}, \qquad A = C^T\sqrt{D}$$

- A is *scaled* inverse curvelet transform

Recovery problem formulation

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = (\mathbf{A}^T)^\dagger \tilde{\mathbf{x}}. \end{cases}$$

- with

$$J(\mathbf{x}) = \overbrace{\alpha||\mathbf{x}||_1}^{Sparsity} + \overbrace{\beta||\mathbf{\Gamma}^{1/2}(\mathbf{A}^T)^{-1}\mathbf{x}||_2}^{Continuity}$$

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Optimization method

Step 1: update of the Jacobian of $\frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2$:

$$\mathbf{x} \leftarrow \mathbf{x} + \mathbf{A}^T \left(\mathbf{y} - \mathbf{A}\mathbf{x}\right);$$

Step 2: project onto the ℓ_1 ball $S = \{\|\mathbf{x}\|_1 \leq \|\mathbf{x}_0\|_1\}$ by soft thresholding

$$\mathbf{x} \leftarrow T_\lambda(\mathbf{x});$$

Step 3: project onto the anisotropic diffusion ball $C = \{\mathbf{x} : \ J(\mathbf{x}) \leq J(\mathbf{x}_0)\}$ by

$$\mathbf{x} \leftarrow \mathbf{x} - \beta \boldsymbol{\nabla}_{\mathbf{x}} J_c(\mathbf{x})$$

with

$$\boldsymbol{\nabla}_{\mathbf{x}} J_c(\mathbf{x}) = 2\mathbf{A}^\dagger \boldsymbol{\nabla} \cdot \left(\boldsymbol{\Lambda} \boldsymbol{\nabla} \left(\left(\mathbf{A}^T\right)^\dagger \mathbf{x}\right)\right).$$

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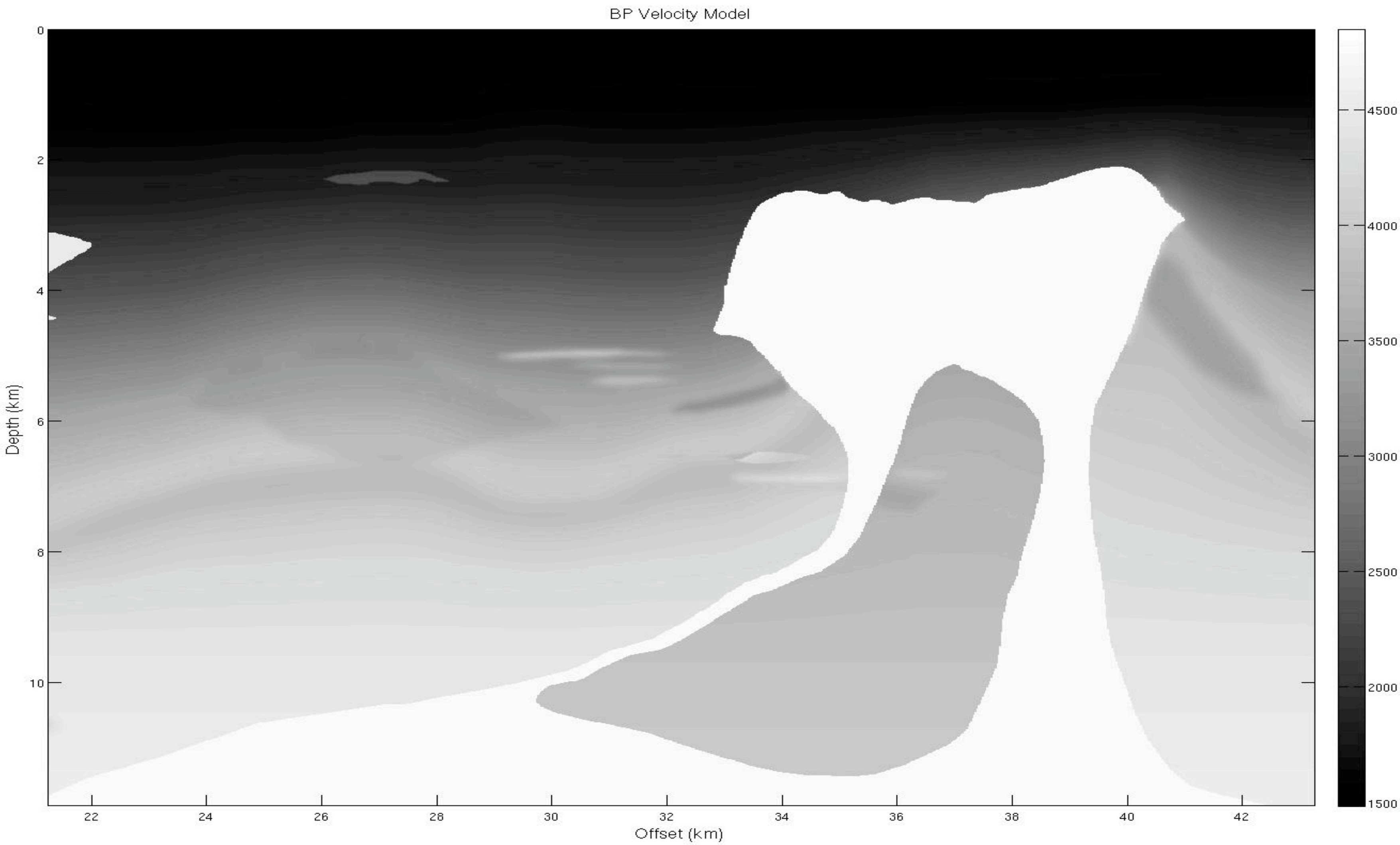
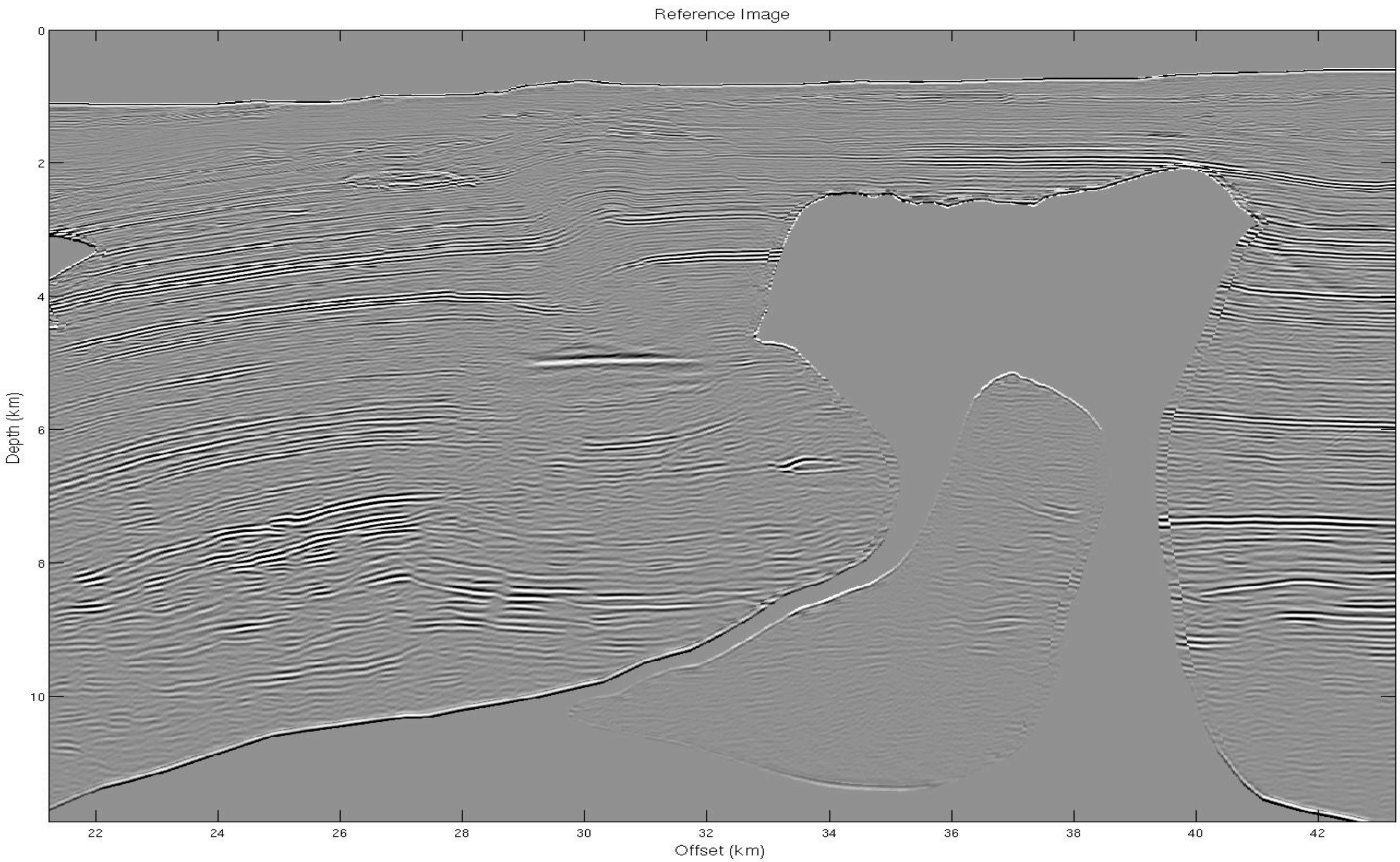
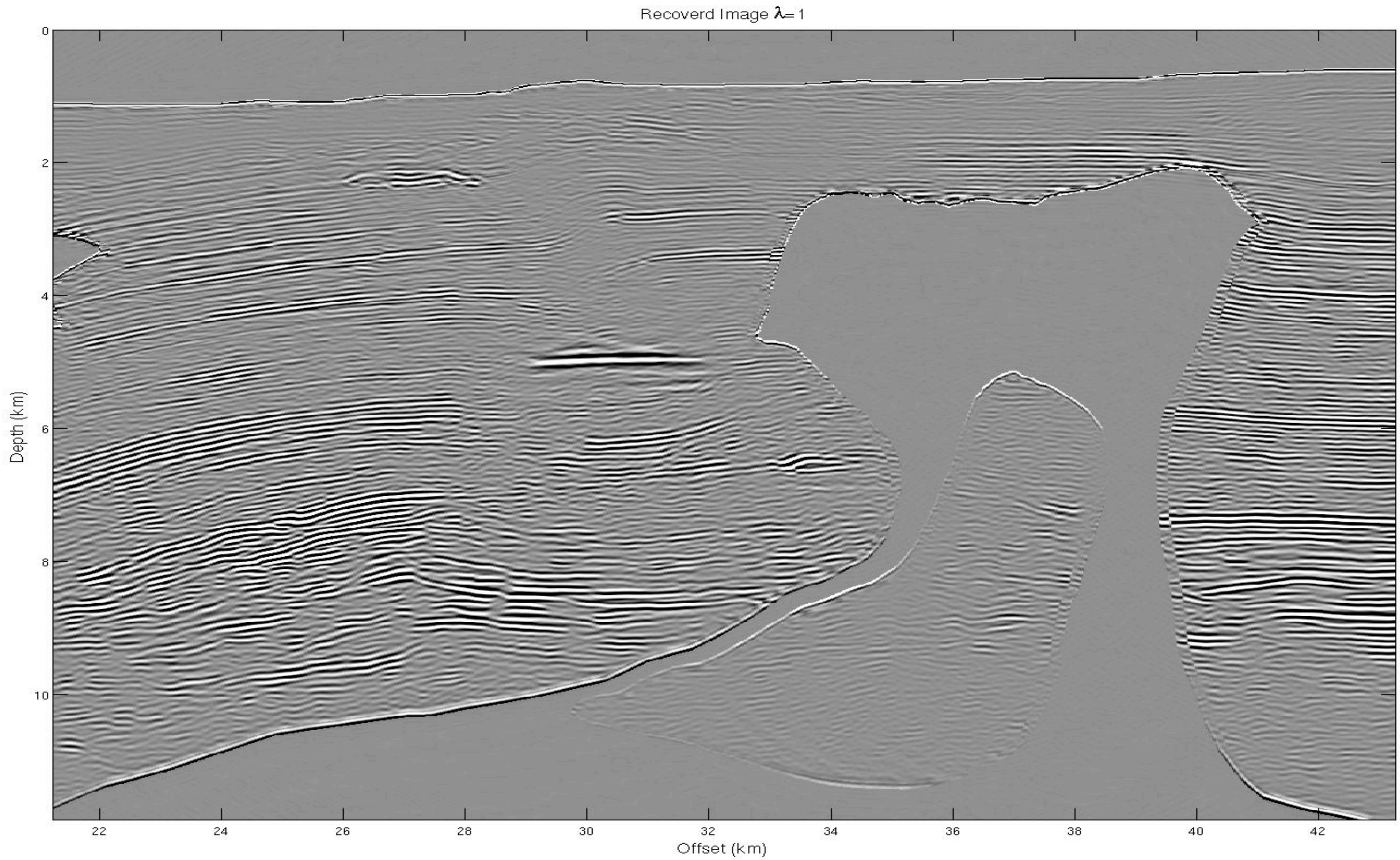
Example: diagonal inversion BP model



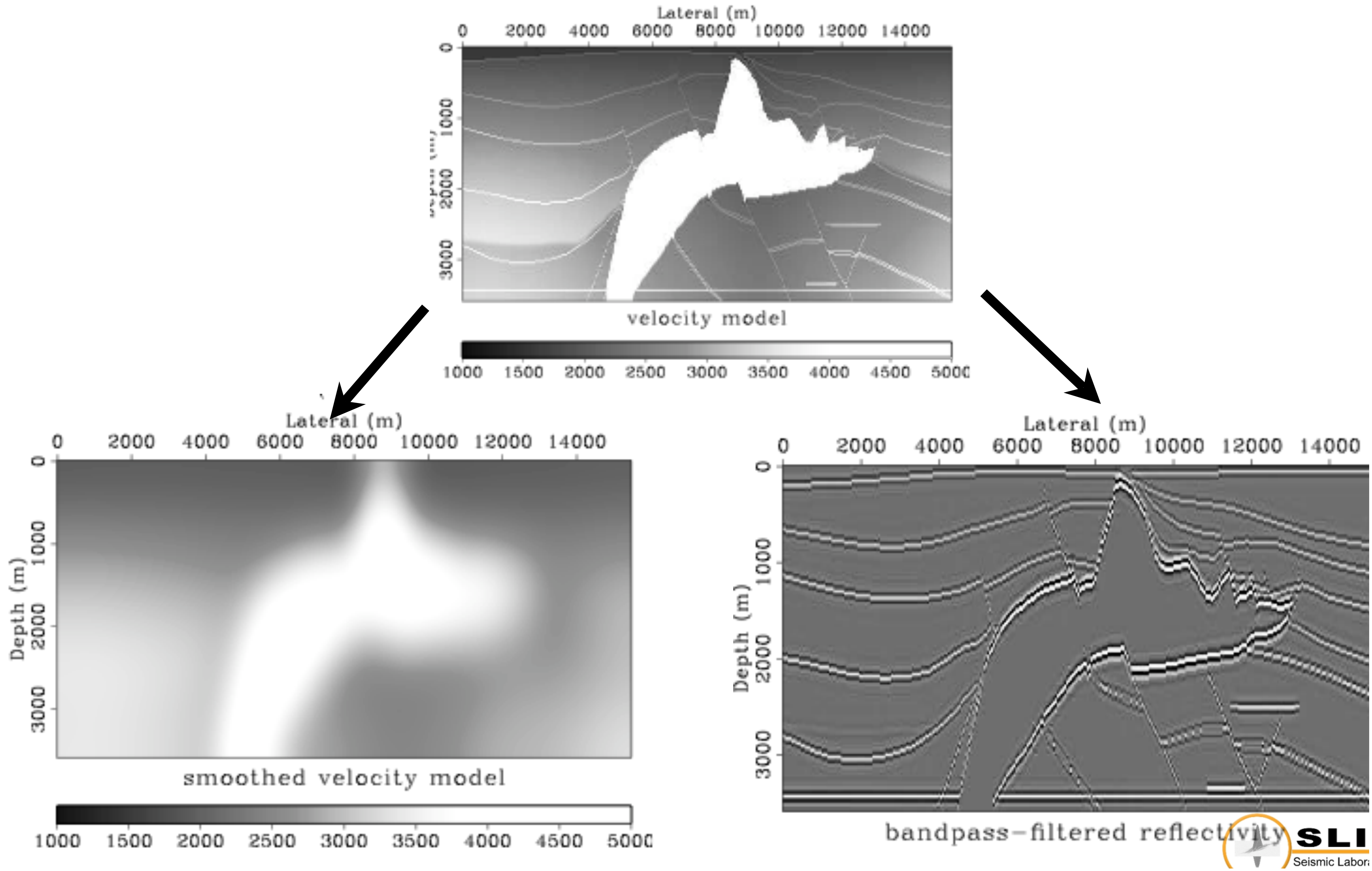
Example: diagonal inversion reference image



Example: diagonal inversion recovered image



Example (SEG-AA' Model) linearized Born data



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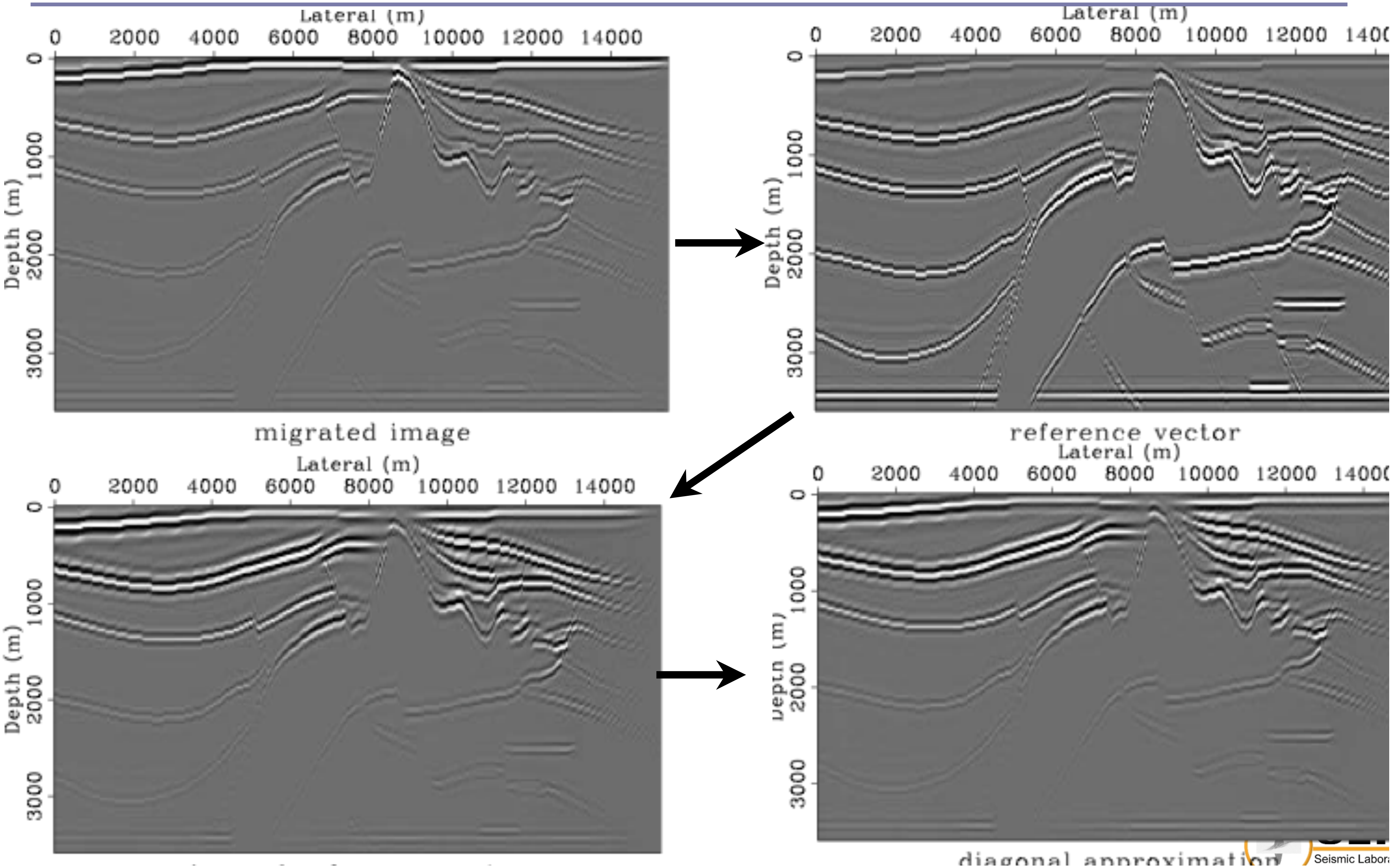

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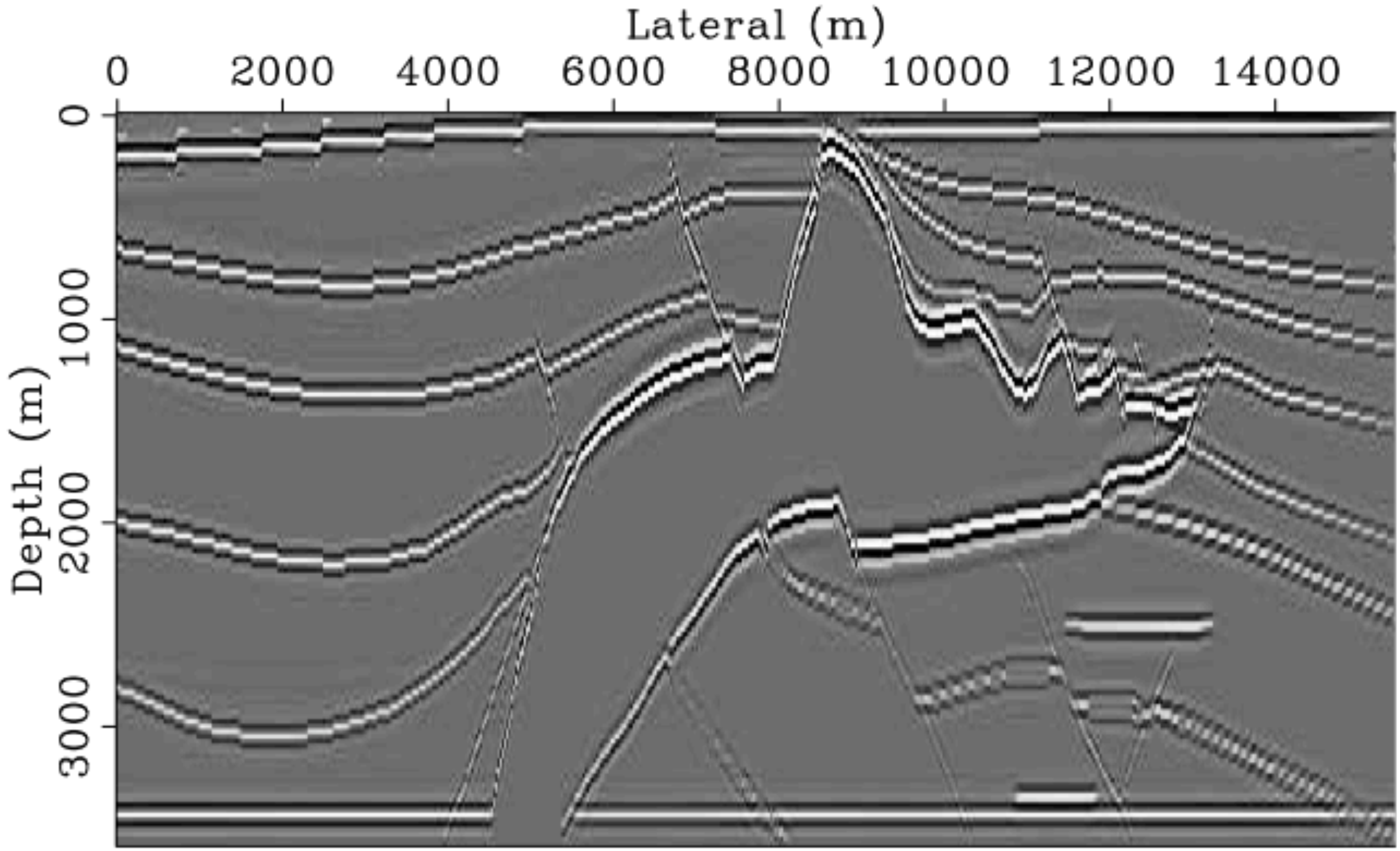
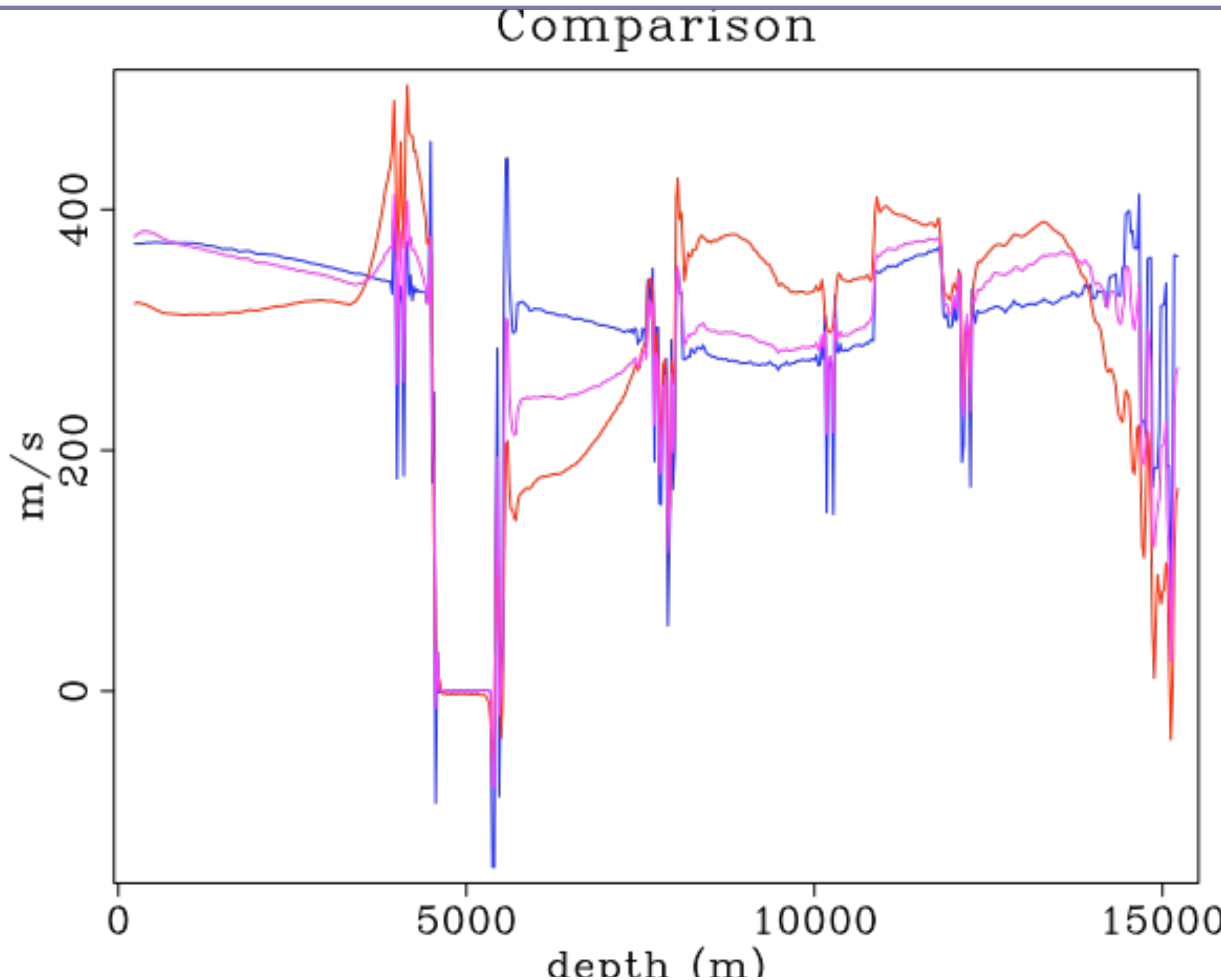
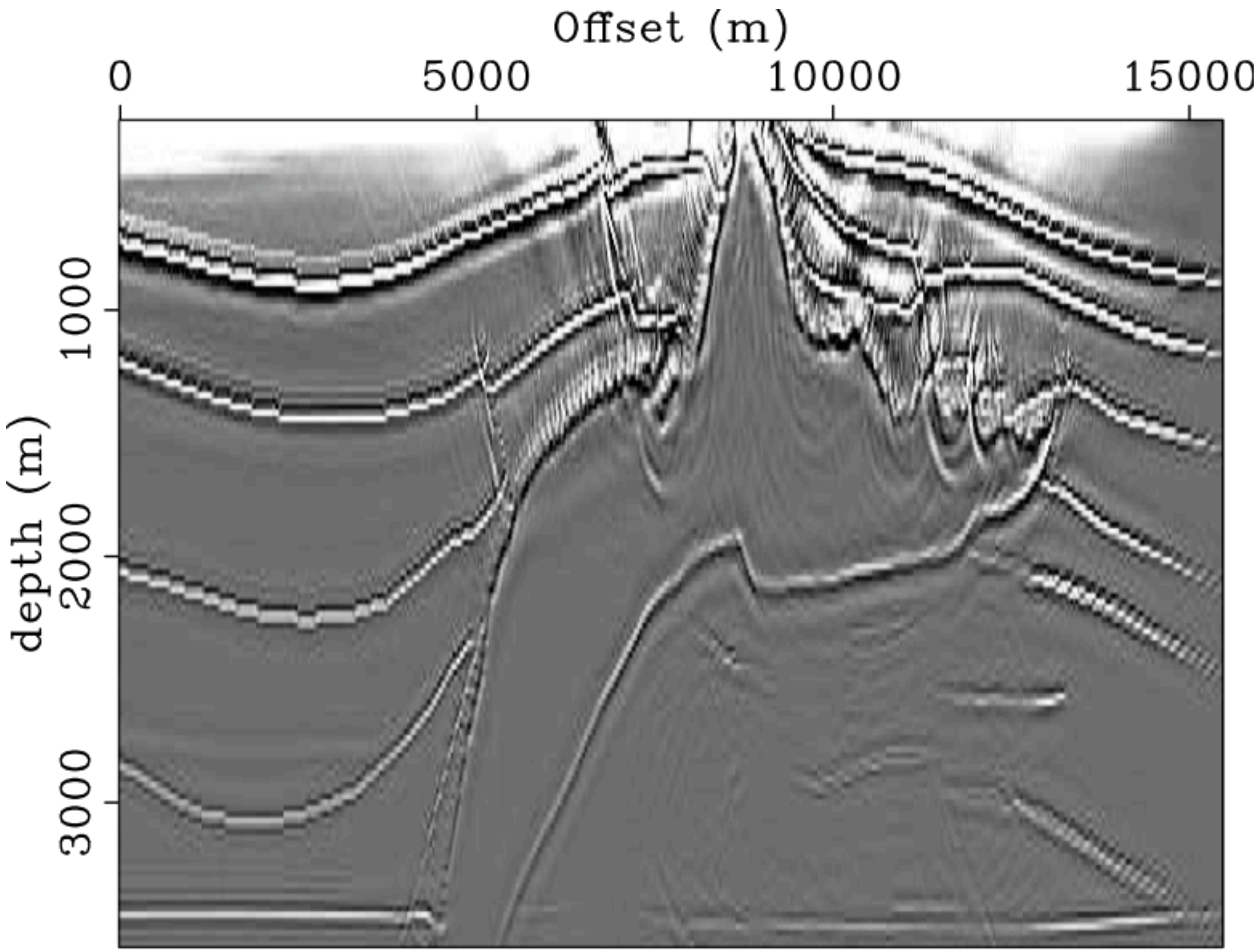


norm−one and continuity recovered

Example (SEG-AA' model) linearized Born data

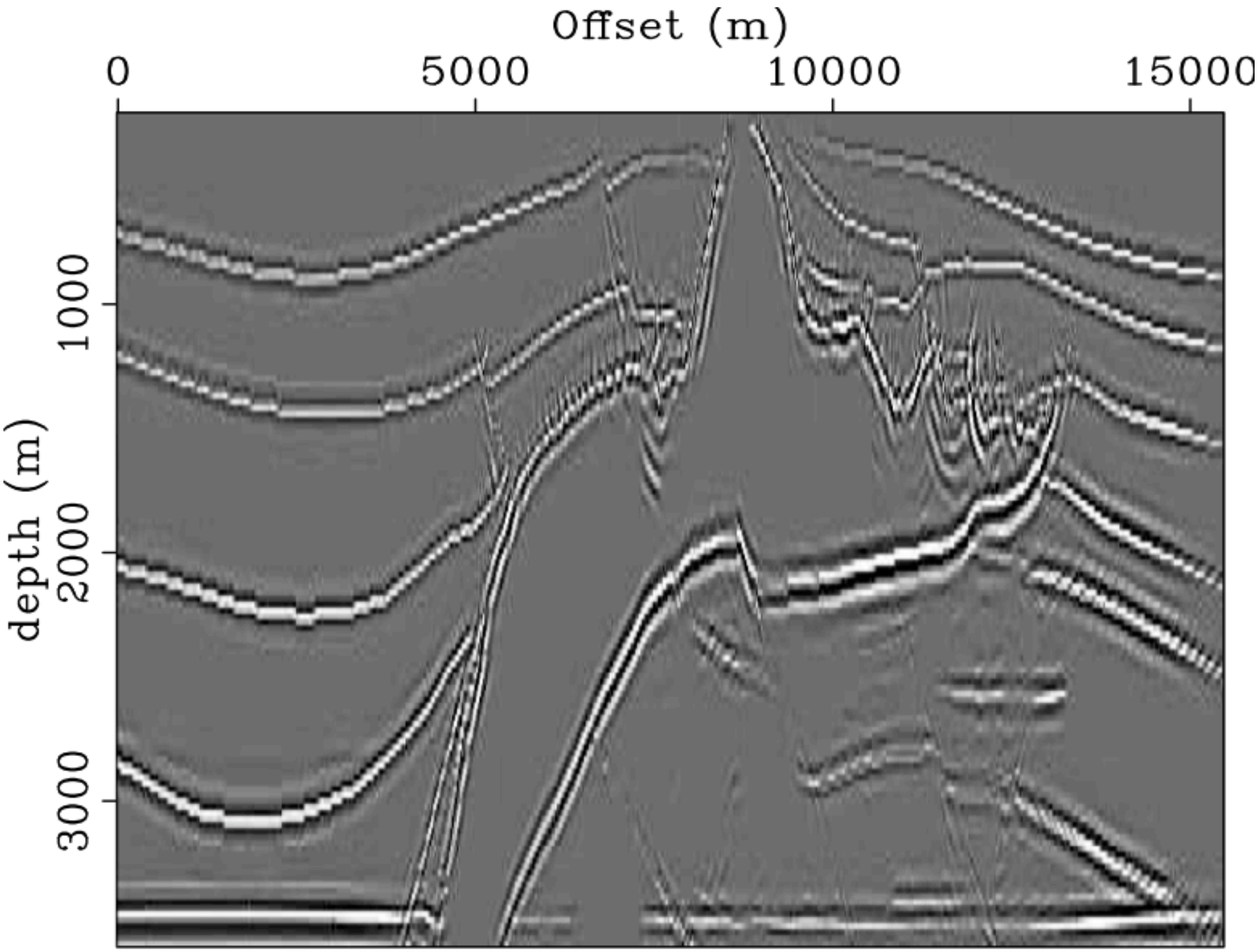


Example (SEG-AA' model) synthetic data



Migrated Image

Example (SEG-AA' model) synthetic data



Amplitude Recovered Image

Conclusion

This work

- … introduces a novel approach to migration amplitude recovery
- … employs an accurate diagonal decomposition of the expensive normal operator
- … employs curvelets as essential elements in both approximation and estimation
- … can be used instead of illumination map or in conjunction with it

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