Seismic noise: the good the bad and the ugly

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ABSTRACT

In this paper, we present a nonlinear curvelet-based sparsity-promoting formulation for three problems related to seismic noise, namely the 'good', corresponding to noise generated by random sampling; the 'bad', corresponding to coherent noise for which (inaccurate) predictions exist and the 'ugly' for which no predictions exist. We will show that the compressive capabilities of curvelets on seismic data and images can be used to tackle these three categories of noise-related problems.

Introduction

In this paper, we present recent developments of the application of the curvelet transform (see e.g. Candes et al., 2006; Hennenfent and Herrmann, 2006b) to problems that involve different types of noise in seismic data. Our approach banks on two fundamental properties of curvelets, namely the

- **detection of wave-fronts** without requiring *prior* information on the dips or on the velocity model;
- **invariance** of curvelets under the action of wave propagation.

These two properties make this transform suitable for a robust formulation of problems, such as seismic data regularization (Hennenfent and Herrmann, 2006a; Herrmann and Hennenfent, 2007), primary-multiple separation (Herrmann et al., 2006a), ground-roll removal (Yarham et al., 2006) and stable migration-amplitude recovery (Herrmann et al., 2006b). All these methods exploit sparsity in the curvelet domain that is a direct consequence of the above two properties and corresponds to a rapid decay for the magnitude-sorted curvelet coefficients. This sparsity allows for a separation of 'noise' and 'signal' underlying all these problems (see e.g. Hennenfent and Herrmann, 2006b; Herrmann et al., 2006a).

Curvelets: As can be observed from Fig. 1, curvelets are localized functions that oscillate in one direction and that are smooth in the other directions. They are

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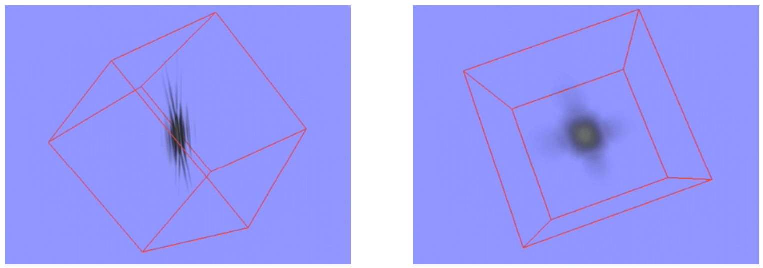
multiscale and multi-directional and because of their anisotropic shape (they obey the so-called parabolic scaling relationship, yielding a width $\propto 2^{j/2}$ and a length $\propto 2^j$ with j the scale), curvelets are optimal for detecting wavefronts. This explains their high compression rates for seismic data and images as reported in the literature (Candes et al., 2006; Hennenfent and Herrmann, 2006c; Herrmann et al., 2006a,b).



(a) (b)

Figure 1: Example of a 3-D curvelet. Notice the oscillations in one direction and the smoothness in the other two directions.

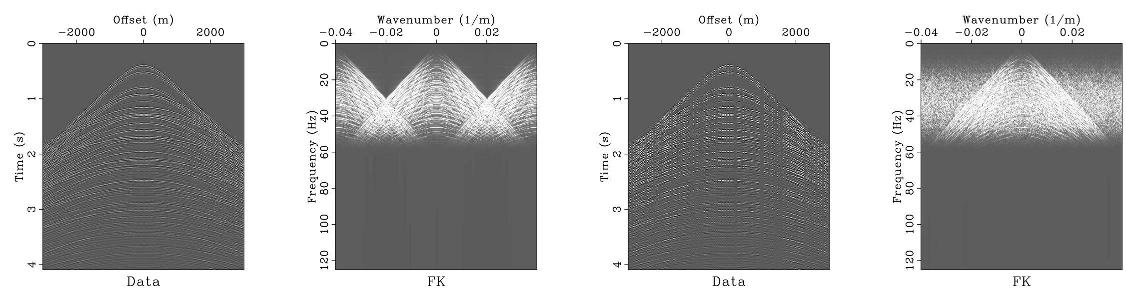
Sparsity promoting inversion: High compression rates for signal representations are a prerequisite for the robust formulation of stable signal recovery problems and other inverse problems. These compression rates allow for a nonlinear sparsity promoting solution. As such sparsity-promoting norm-one penalty functionals are not new to the geosciences (see for instance the seminal work of Claerbout and Muir (1973), followed by many others), where sparsity is promoted on the model. What is different in the current surge of interest in sparsity-promoting inversion, known as 'compressed sensing' (Candes et al., 2005; Donoho et al., 2006), is (i) the existence of sparsity promoting transforms such as the curvelet transform; (ii) the deep theoretical understanding on what the conditions are for a successful solution. This work can be seen as the application of these recent ideas to the seismic situation and involves the solution of the following norm-one nonlinear program,

$$\mathbf{P}_\epsilon : \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{d}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases} \tag{1}$$

in which $\mathbf{y}$ is the (incomplete) data, $\mathbf{A}$ the synthesis matrix and $\mathbf{S}^T$ the inverse sparsity transform. Both these matrices consist of the inverse curvelet transform matrix, $\mathbf{S}^T$ (the symbol ^{T} denoting the transpose) compounded with other operators depending on the problem. The above constrained optimization problem is solved to an accuracy of ϵ that depends on the noise level. Finally, $\widetilde{\mathbf{d}}$ stands for the recovered vector with the symbol $\widetilde{}$ reserved for optimized quantities.

The 'good': random sampling gives rise to incoherent noise

Sampling of seismic wavefields is based on the assumption that equally-sampled data is good. Indeed, when Nyquist's sampling theorem is met, equidistant sampling allows for a perfect reconstruction of the wavefield. Unfortunately, adequate sampling of steeply dipping events, such as ground roll, are often unfeasible in practice. In case of *regular* subsampling, this leads to the well-known phenomonon of *aliasing* as illustrated in Fig. 2. This alliasing leads to a difficult to predict and separate coherent 'noise' in the Fourier domain. Random subsampling, on the other hand, leads to a noisy spectrum for the same number of samples (see Fig. 2(d)). This is an example of 'good' noise that can easily be separated. Denoising in that case corresponds to seismic data regularization and boils down to solving $\mathbf{P}_\epsilon$ with $\mathbf{A} := \mathbf{R}\mathbf{C}^T$, $\mathbf{S} := \mathbf{C}$ and $\mathbf{y} = \mathbf{R}\mathbf{d}$ for the incomplete data. This formulation corresponds the formulation for curvelet recovery by sparsity-promoting inversion (CRSI), which has successfully been applied to the recovery of incomplete seismic data (see e.g. Hennenfent and Herrmann, 2006a). In this formulation, $\mathbf{R}$ is the restriction matrix, selecting the rows from the curvelet transform matrix that correspond to active traces. As opposed to other recovery methods, such as sparse Fourier recovery and plane wave destruction, curvelet-based methods have the advantage of working in situations where there are conflicting dips without stationarity assumptions. The method exploits the high-dimensional continuity of wavefronts and as Fig. 3 demonstrates, recovery results improve when using the 3-D curvelet transform compared to the 2-D transform. This can be explained because the 3-D curvelets capture more of the signal's energy and hence are better able to separate the coherent seismic energy from the incoherent and hence 'good' noise related to random subsampling.



(a) (b) (c) (d)

Figure 2: Fourier spectra for incomplete data. **(a)** Regularly missing data leads to a strongly aliased spectrum **(b)** as opposed to **(c)** data missing according an uniform distribution that gives rise to the noisy Fourier spectrum **(d)**. Observe that the Fourier spectrum for the random subsampled data looks noisy while the regular undersampled data displays the well-known and harmful periodic imprint of aliasing.

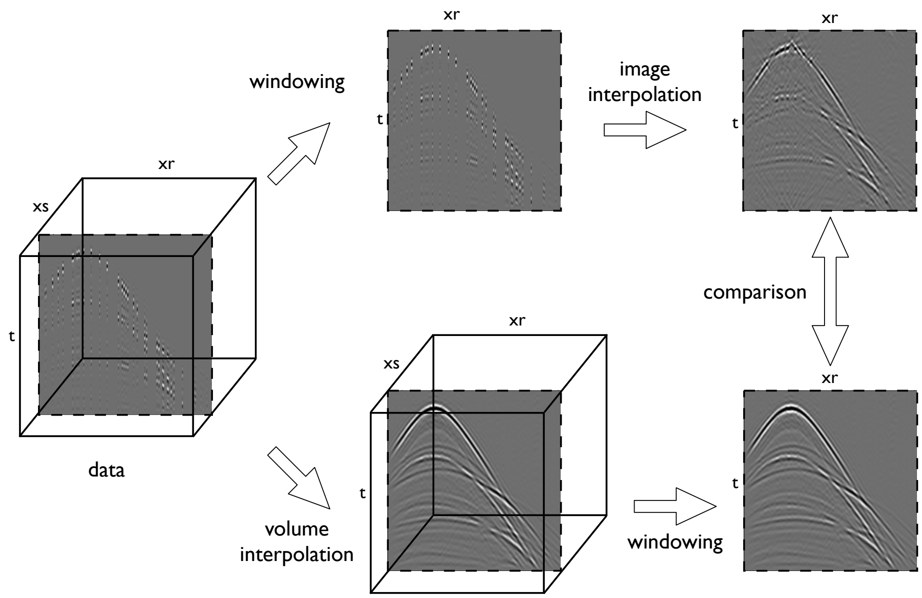


Figure 3: Illustration of sliced versus volumetric interpolation.

The 'bad': coherent signal separation with (inaccurate) predictions

So far, we looked at exploiting the sparsity of curvelets in the data domain for the purpose of recovery from incomplete data. The ability of curvelets to detect wavefronts with conflicting dips, allows for a formulation of a coherent signal separation method that uses inaccurate predictions as weightings. By defining the synthesis matrix as $\mathbf{A} := \begin{bmatrix}\mathbf{C}^T\mathbf{W}_1 & \mathbf{C}^T\mathbf{W}_2\end{bmatrix}$, $\mathbf{x} = \begin{bmatrix}\mathbf{x}_1 & \mathbf{x}_2\end{bmatrix}^T$ and $\mathbf{y} = \mathbf{d} = \mathbf{s}_1 + \mathbf{s}_2$ and by setting the diagonal weighting matrices $\mathbf{W}_{1,2}$ in terms of predictions for two different signal components (e.g., primaries and multiples or reflectivity and ground roll), the solution of $\mathbf{P}_\epsilon$ separates the two signal components (see e.g. Herrmann et al., 2006a; Yarham et al., 2006) even for inaccurate predictions for which least-squares adaptive subtraction fails (see Fig. 4). With this method ground-roll and reflectivity can also succesfully be separated as can be seen from the example plotted in Fig. 5.

The 'ugly': migration amplitude recovery from noisy data

Finally, the precense of unknown sources of clutter in the image space can be a major challenge. For instance, consider the situation where noisy data is migrated, yielding a noisy image, i.e. $\mathbf{y} = \mathbf{K}^T\mathbf{d}$ with $\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n}$. In this expression, $\mathbf{d}$ is the noisy data, $\mathbf{k}$ the demigration operator and $\mathbf{n}$ a Gaussian noise term. The recovery of the reflectivity $\mathbf{m}$ is challenging because the image $\mathbf{y}$ contains a coherent and nonstationary noise term, $\mathbf{K}^T\mathbf{n}$ consisting of migrated noise. To separate this noise term from the imaged reflectivity, we use the following approximate idendity

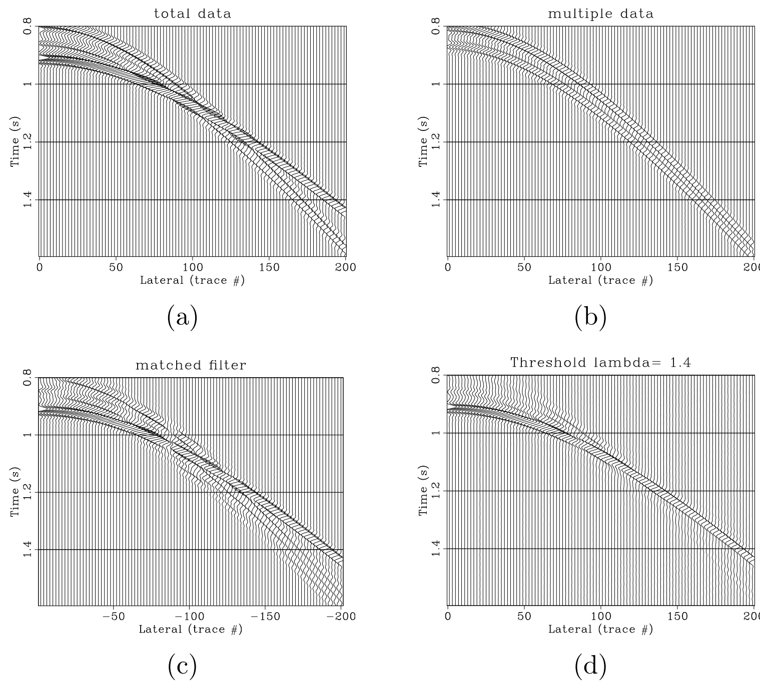


Figure 4: Example of primary-multiple separation through $\mathbf{P}_\epsilon$ for predicted multiples with moveout errors. **(a)** the total data with primaries and multiples. **(b)** the true multiples used for the prediction. **(c)** the result obtained with least-squares adaptive subtraction with localized windows. **(d)** the result obtained with a single curvelet-domain soft thresholding with $\lambda = 1.4$. Notice that least-squares subtractions fails.

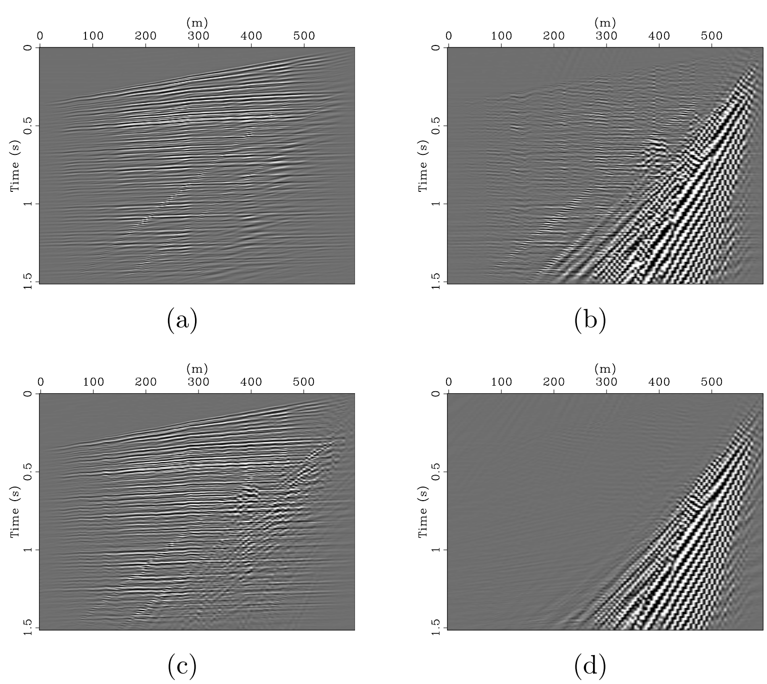


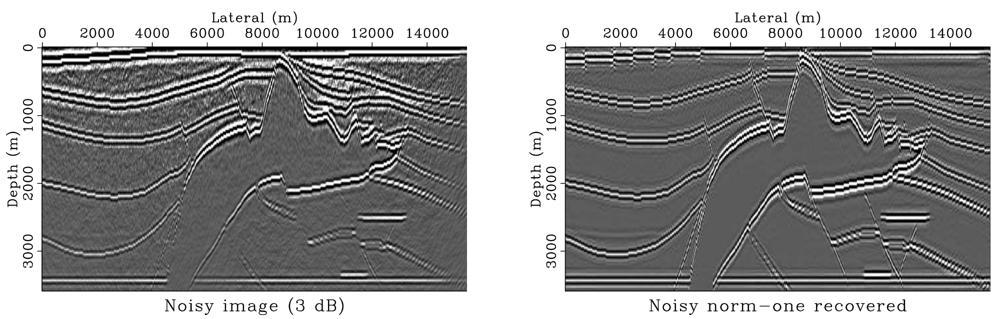
Figure 5: Example of curvelet-domain ground roll removal. **(a)** $f - k$ filtered result. **(b)** the separated ground roll. **(c)** Curvelet-domain separated result obtained by optimization. **(d)** The predicted ground roll. Notice that the predicted ground roll for the curvelet separation is clean and does not contain significant reflection events.

(Herrmann et al., 2006b)

$$\mathbf{A}\mathbf{A}^T\mathbf{r} \simeq \mathbf{\Psi}\mathbf{r} \tag{2}$$

with $\mathbf{r}$ an appropriately chosen reference vector and $\mathbf{\Psi}$ the discrete normal (demigration-migration) operator. The synthesis operator in this case is defined as $\mathbf{A} := \mathbf{C}^T\mathbf{\Gamma}$ with $\mathbf{\Gamma}$ a diagonal weighting matrix. This identity diagonalizes the normal operator and allows for a stable recovery and denoising of the migrated image from 'noisy data' $\mathbf{y}$. After solving for $\mathbf{P}_\epsilon$, the reflectivity is obtained by applying the following synthesis, i.e. by setting $\mathbf{S}^T := \left(\mathbf{A}^T\right)^\dagger$ with † the pseudo inverse.

It can be shown that the diagonal approximation of the normal operator serves two purposes. Firstly, the inversion of the weighted curvelet transform corrects for the amplitudes, as can be observed in Fig. 6(b). Secondly, the diagonal whitens the coloring of the noise term in the image spaced, allowing for succesful denoising by solving $\mathbf{P}_\epsilon$ with ϵ set according the noise level. Results of this procedure on the SEG AA' dataset with a reverse-time migration operator are summarized in Fig. 6, confirm the validity of this approach. The resulting image shows a nice recovery of the amplitudes and removal of most the noise for data with a signal-to-noise ratio of 3 dB.



(a) (b)

Figure 6: Image amplitude recovery for noisy data (SNR 3 dB). **(a)** Noisy image. **(b)** Image after nonlinear recovery from noisy data with $\mathbf{P}_\epsilon$. The clearly visible non stationary noise in **(a)** is mostly removed during the recovery while the amplitudes are also restored.

Discussion

The methodology presented in this paper banks on two favorable properties of curvelets, namely their ability to detect wavefronts and their approximate invariance under wave propagation. These properties allow for a succesful removal of different types of clutter from seismic data. We showed that by compounding the curvelet transform with certain matrices each 'denoising' problem can be cast into one and the same optimization problem. The solution of this optimization problem entails a denoising, where the curvelet coefficients are recovered through a promotion of sparsity. This sparsity allows for a separation of the different 'noisy' signal components. The results show that (i) exploiting the multi-dimensional structure of seismic data with 3-D curvelets leads to a recovery scheme that is able to reconstruct fully sampled data volumes from data with $> 80\,\%$ random traces missing; (ii) the sparsity of curvelet can be used to separate coherent signal components given an (inaccurate) prediction and finally that (iii) the invariance of curvelets under the demigration-migration operator can be used to recover the seismic amplitudes and remove noise in the image spaced by inverting a diagonally weighted curvelet matrix.

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