Multiple prediction from incomplete data with the focused curvelet transform

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joint work with Deli Wang and Gilles Hennenfent.

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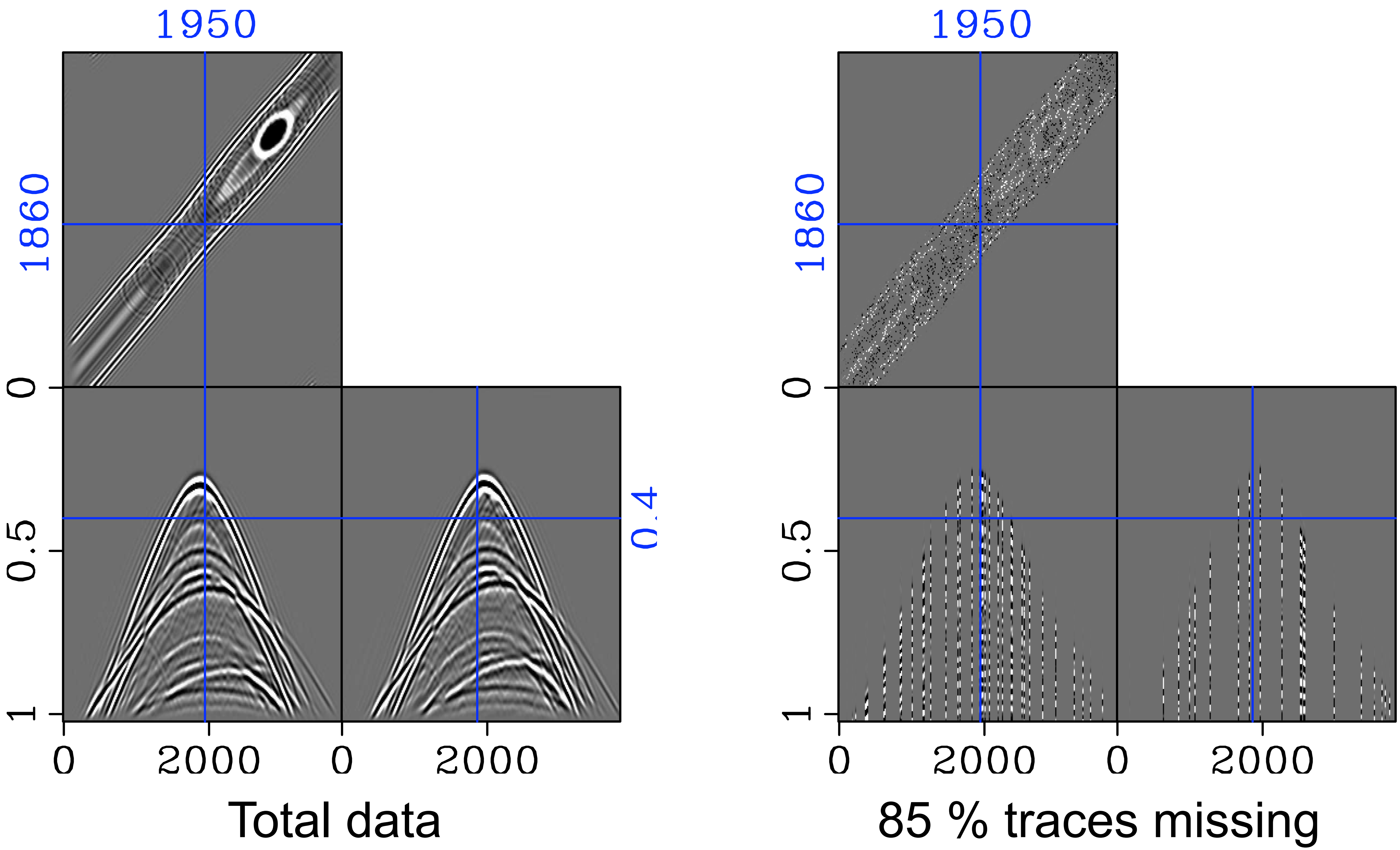
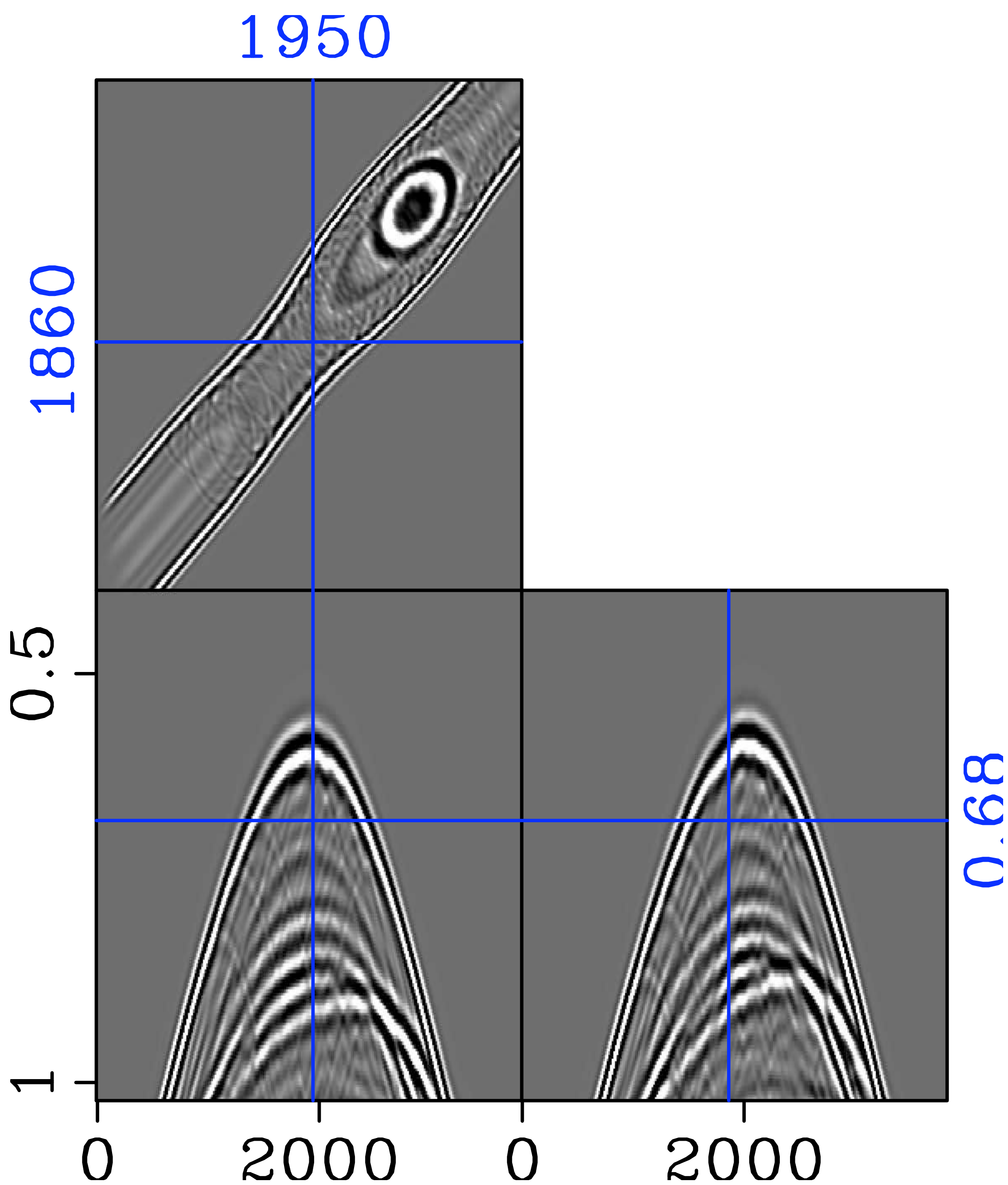
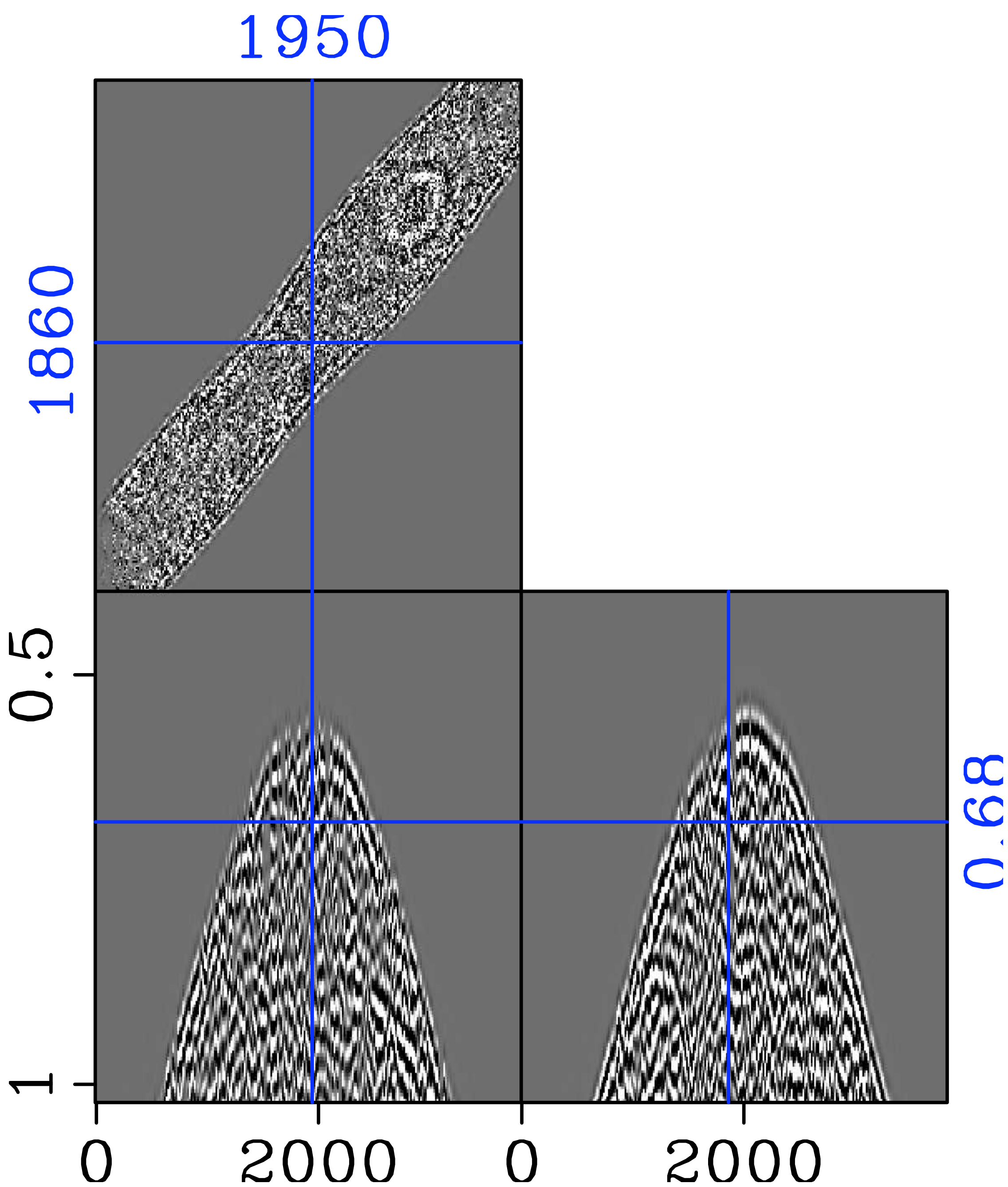
The problem


1950
1860
0
0.5
1
0
2000
0
2000
0.4
Total data
1950
1860
0
0.5
1
0
2000
0
2000
0.4
85 % traces missing

The problem cont'd

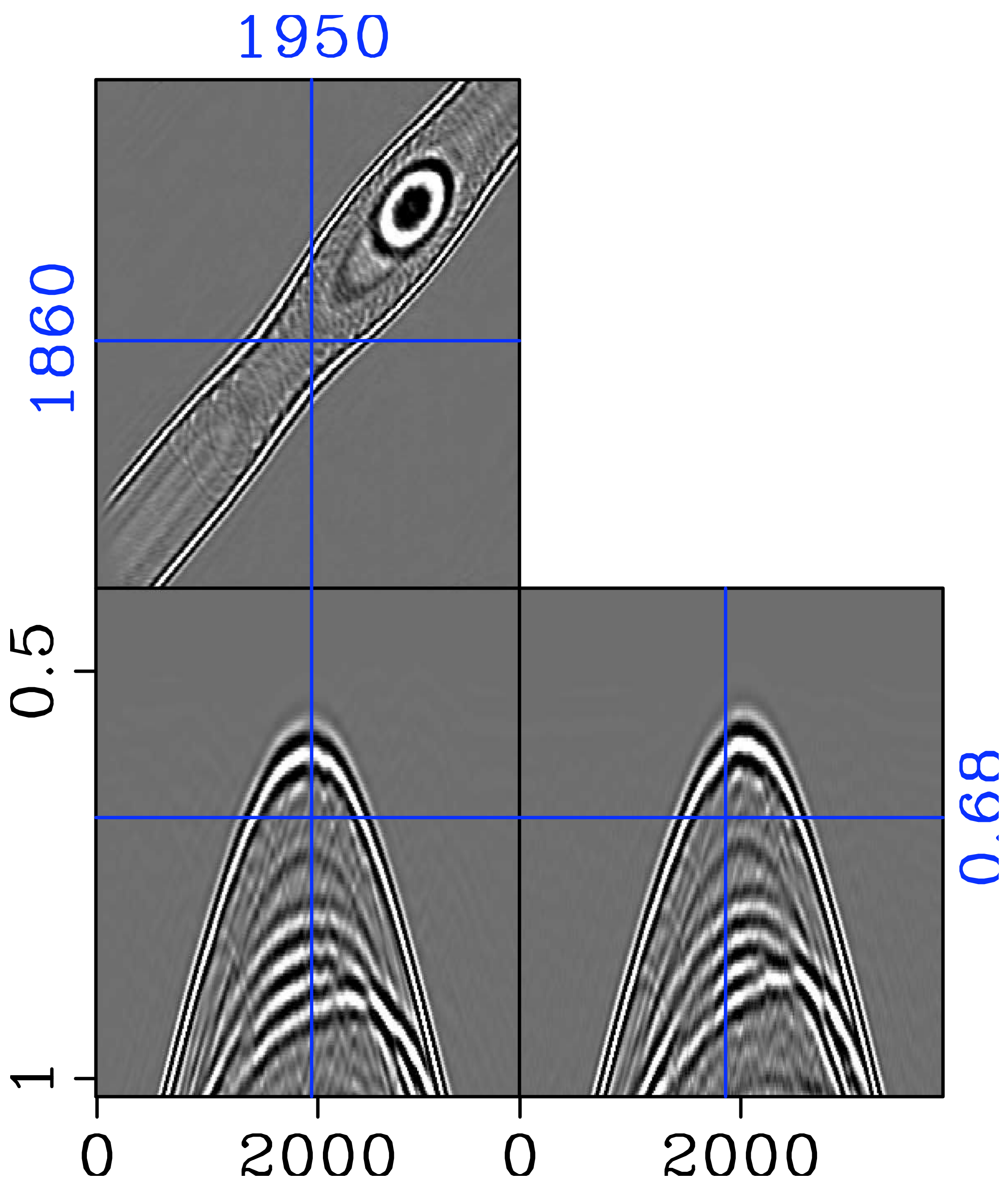


SRME from complete data

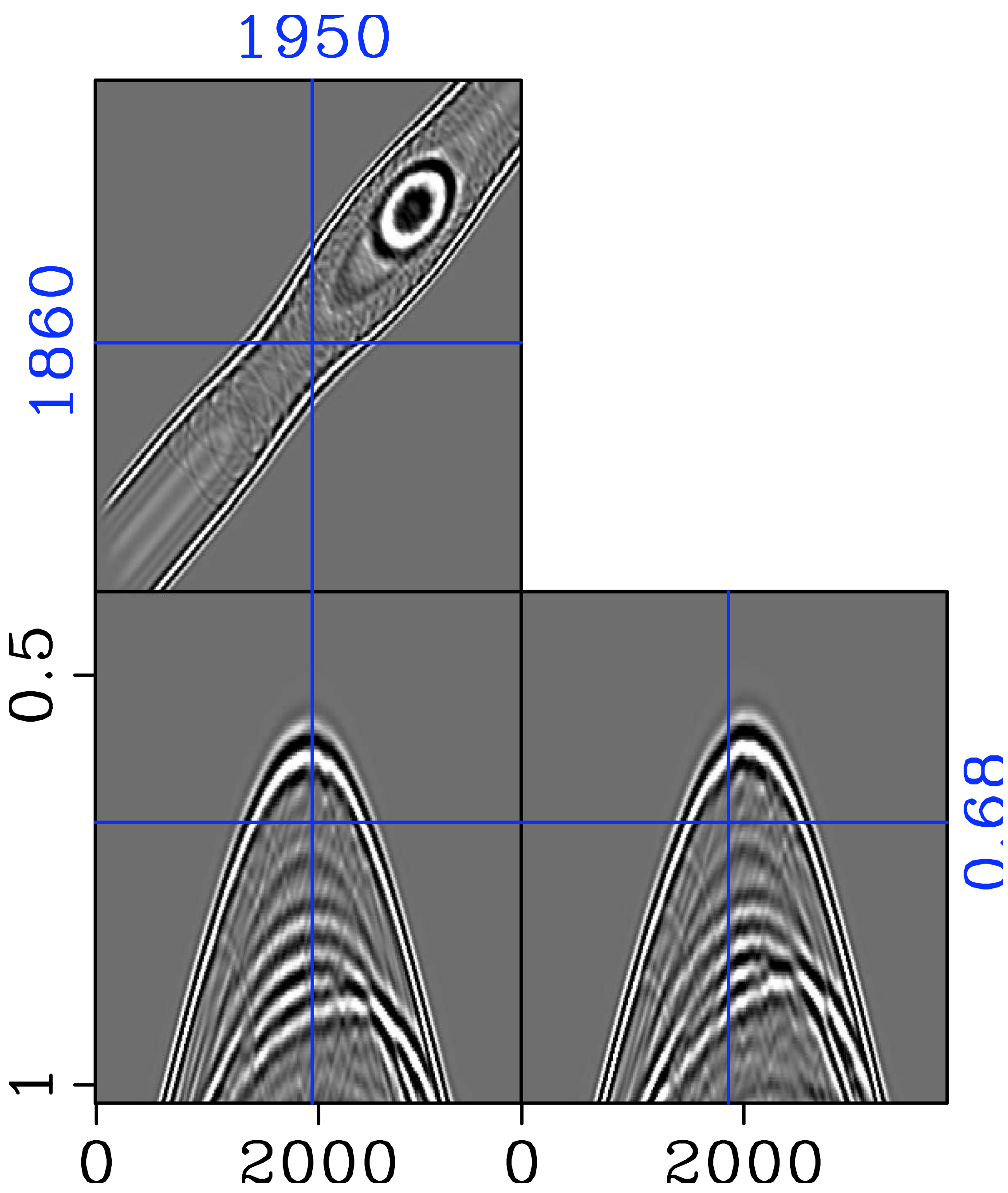


SRME from missing data

Our solution



SRME from recovered data



SRME from original data

Motivation

Data-driven (SRME) multiple prediction requires *fully* sampled data.

The Focal transform (Berkhout & Verschuur '06) allows for

- mapping of multiples => primaries
- incorporation of *prior* information in the recovery

Present a curvelet-based scheme for sparsity-promoting

- recovery of missing data
- prediction of primaries from multiples
- data inverse ...

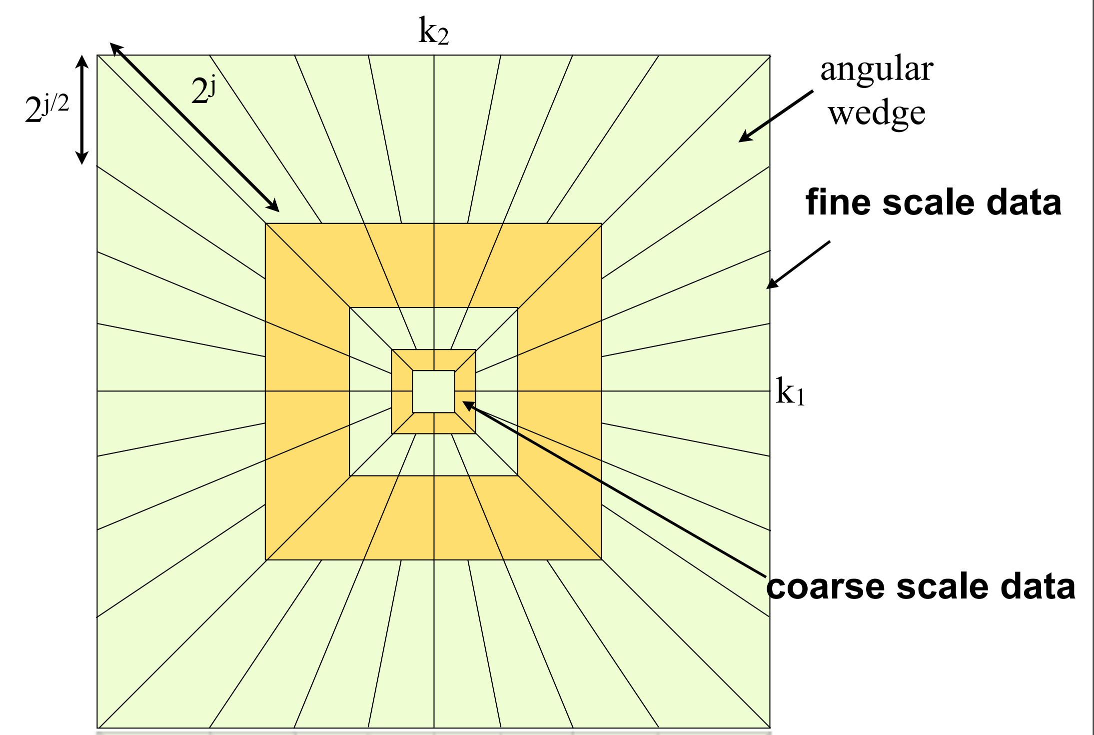
The curvelet transform

Representations for seismic data

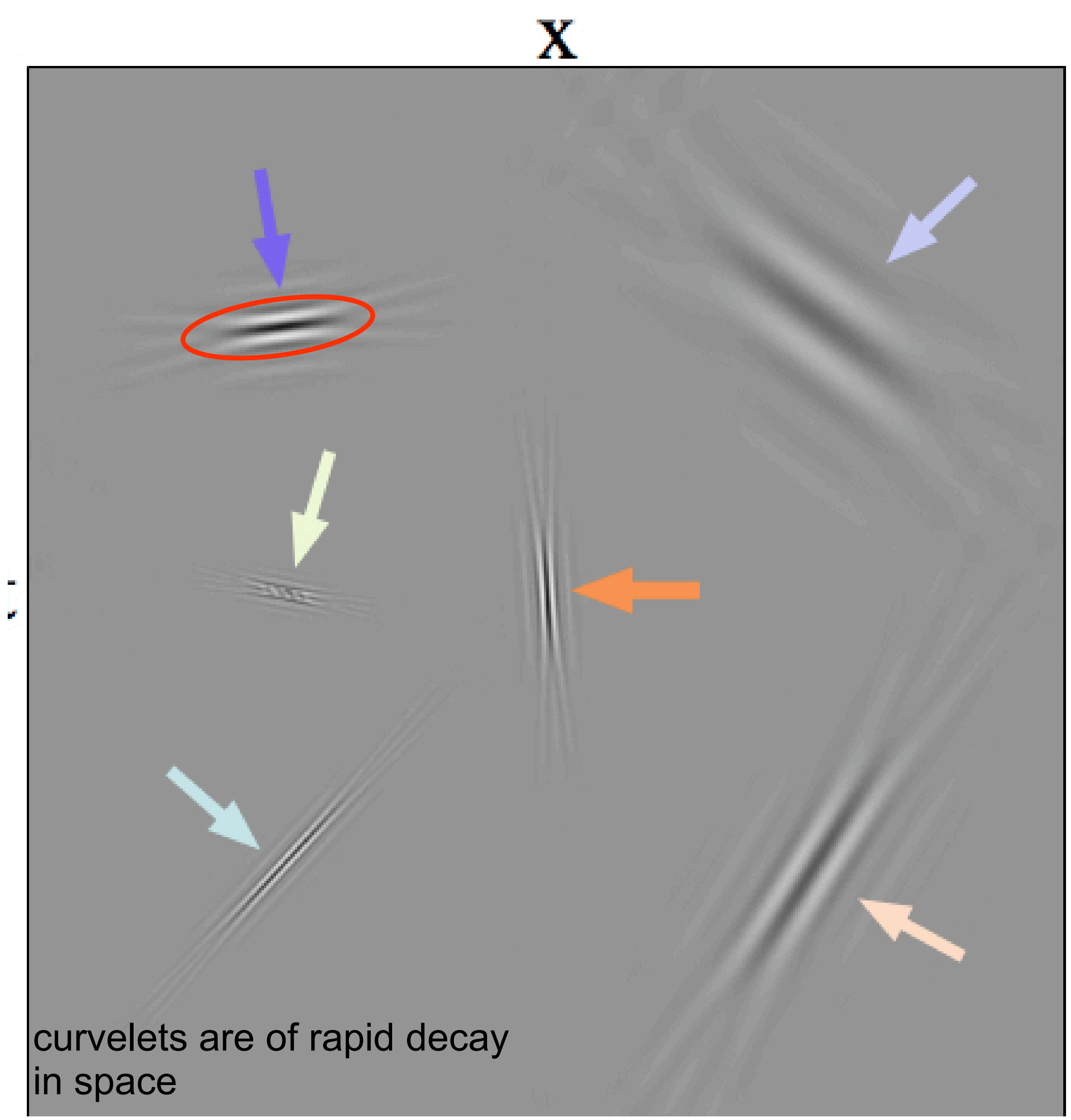
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Properties curvelet transform:

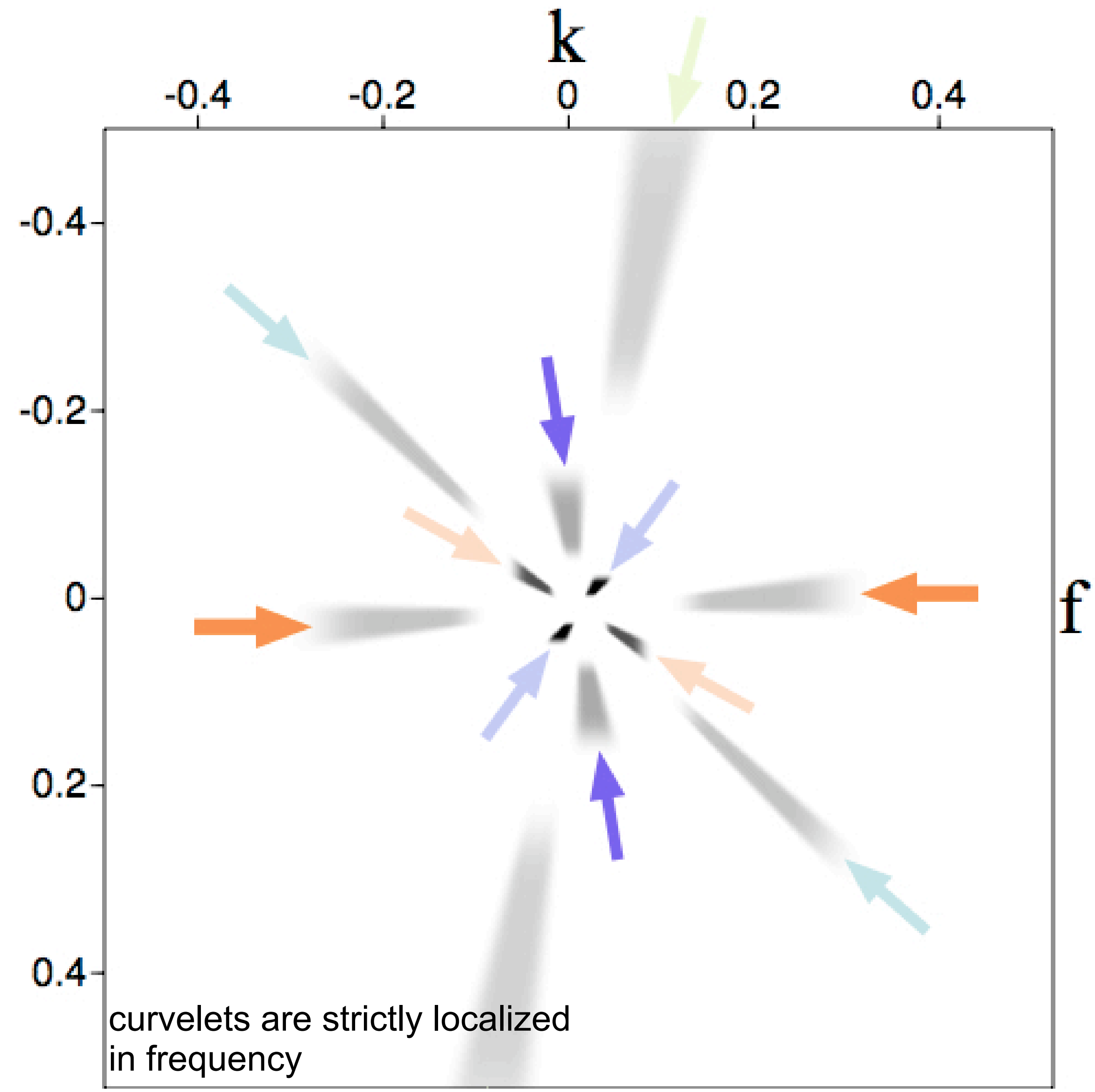
- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy (8 X in 2-D and 24 X in 3-D)**



2-D curvelets



x-t



f-k

Oscillatory in one direction and smooth in the others!

Obey *parabolic* scaling relation $\mathrm{length} \approx \mathrm{width}^2$

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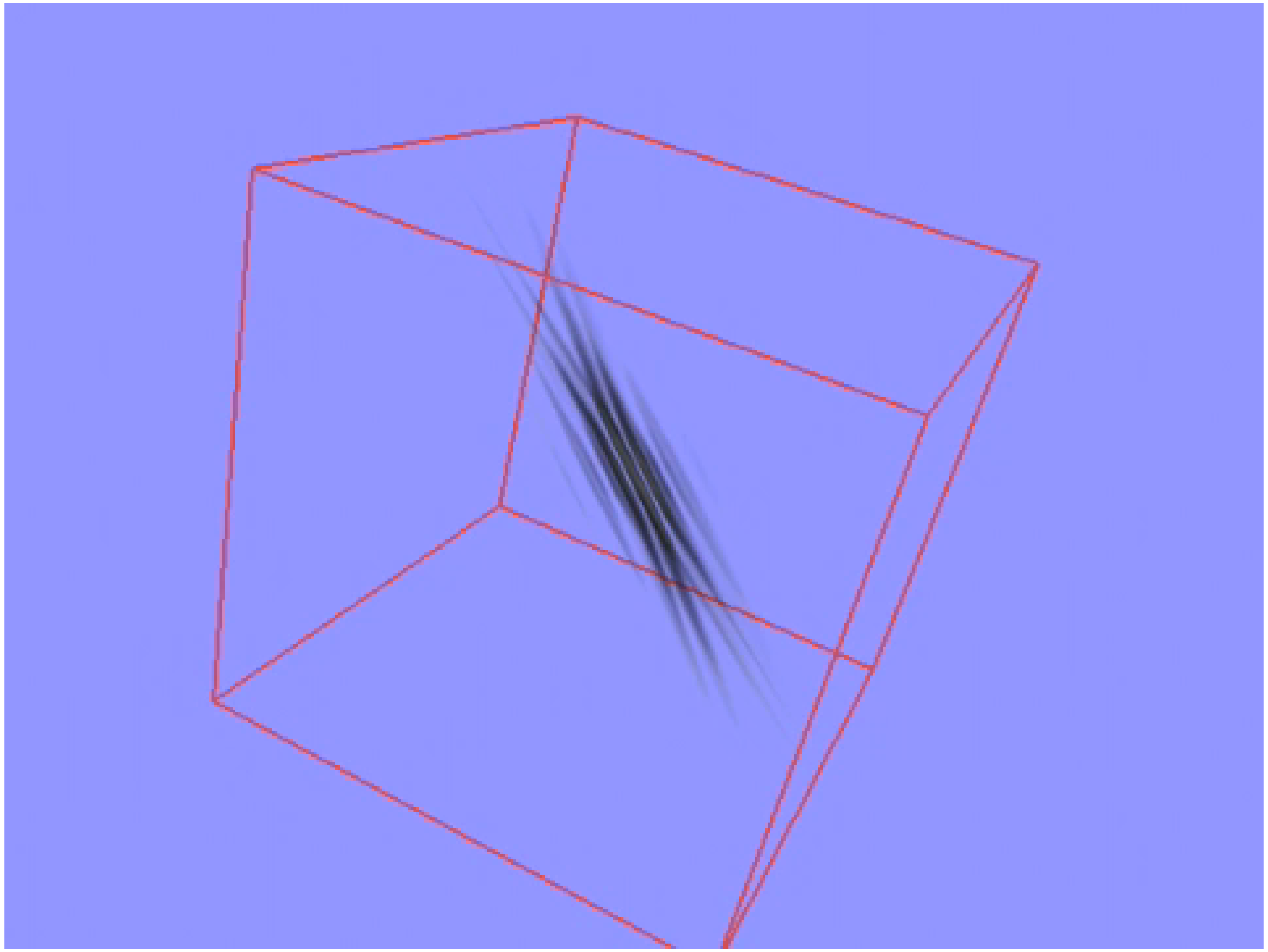
3-D curvelets

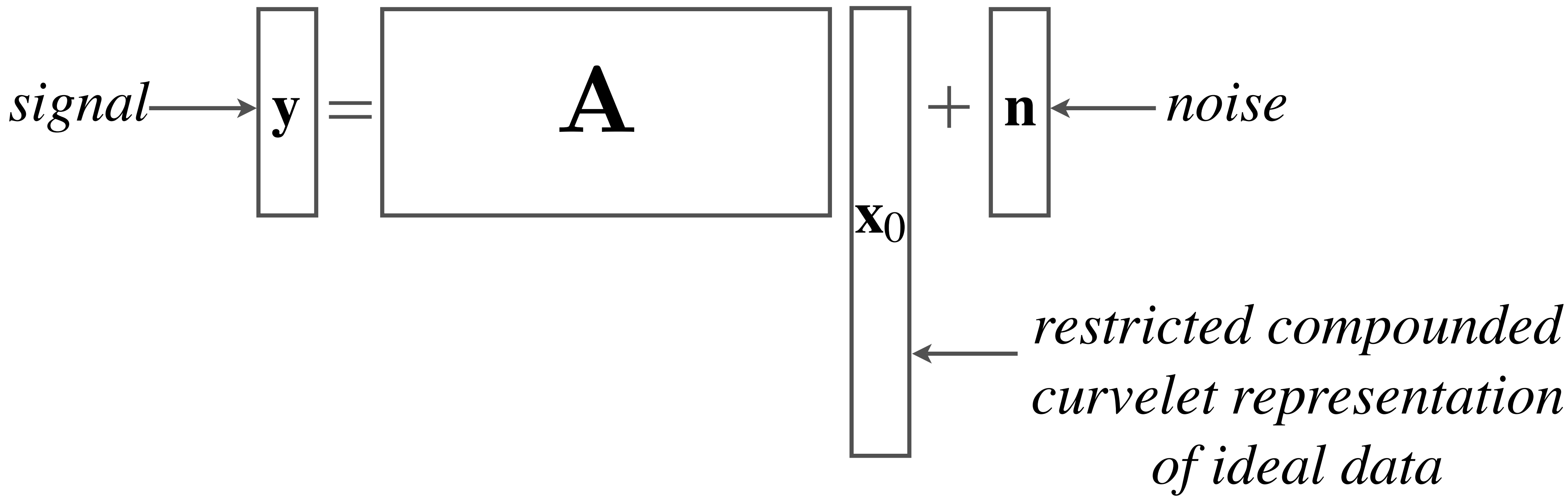


Curvelets are oscillatory in one direction and smooth in the others.

Curvelet sparsity promotion

Sparsity-promoting program

Solve for $\mathbf{x}_0$



with

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

- exploit ***sparsity*** in the curvelet domain as a ***prior***.
- find the sparsest set of curvelet coefficients that match the data.
- invert an *underdetermined* system.

Focused wavefield reconstruction with curvelets

Focused recovery

Non-data-adaptive Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI) derives from ***curvelet-sparsity*** of seismic data.

Berkhout and Verschuur's ***data-adaptive*** Focal transform derives from ***focusing*** of seismic data by the ***major*** primaries.

Both approaches entail the ***inversion*** of a linear operator.

Combination of the two yields

- *improved* focusing => more *sparsity*
- curvelet *sparsity* => better *focusing*

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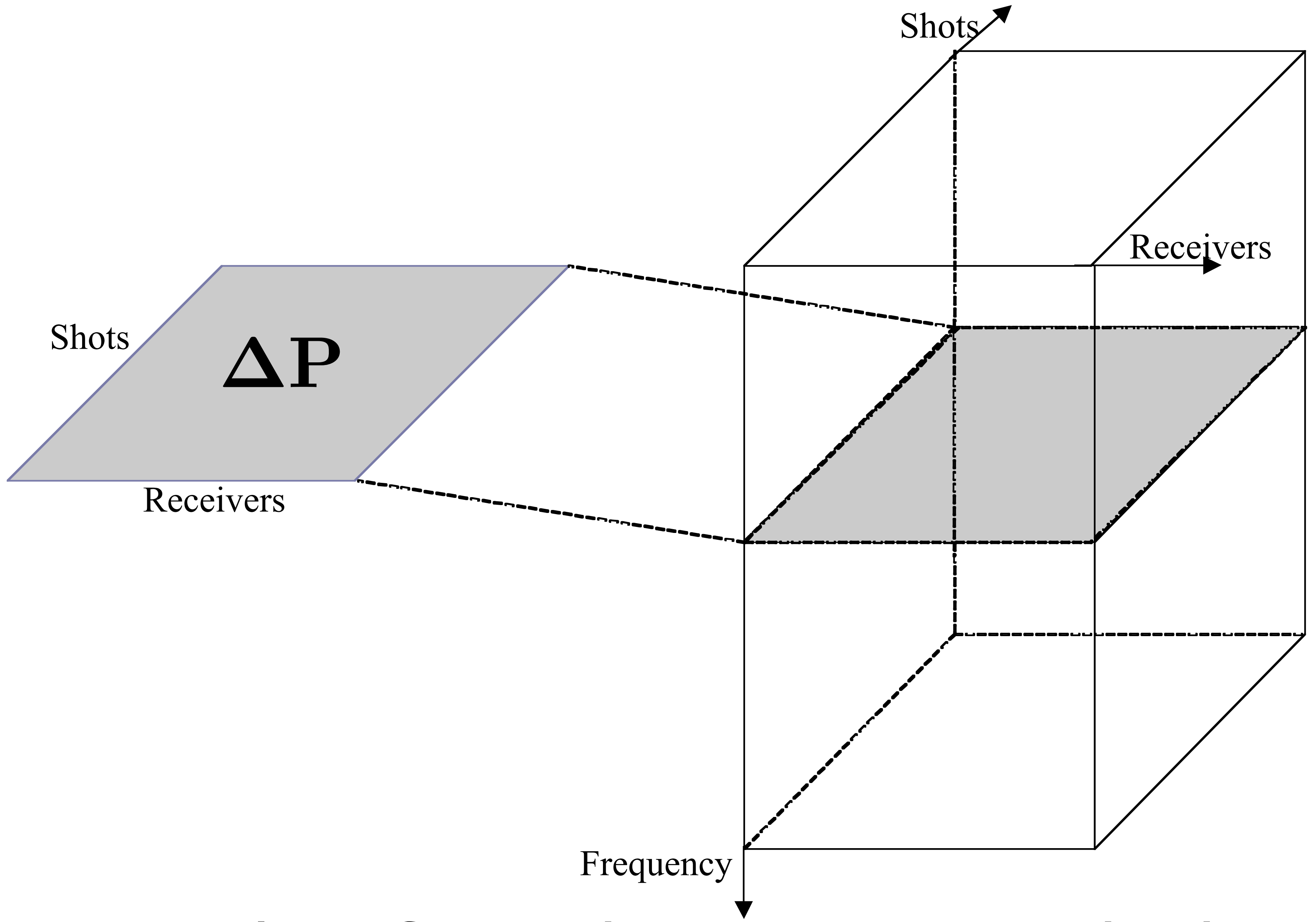
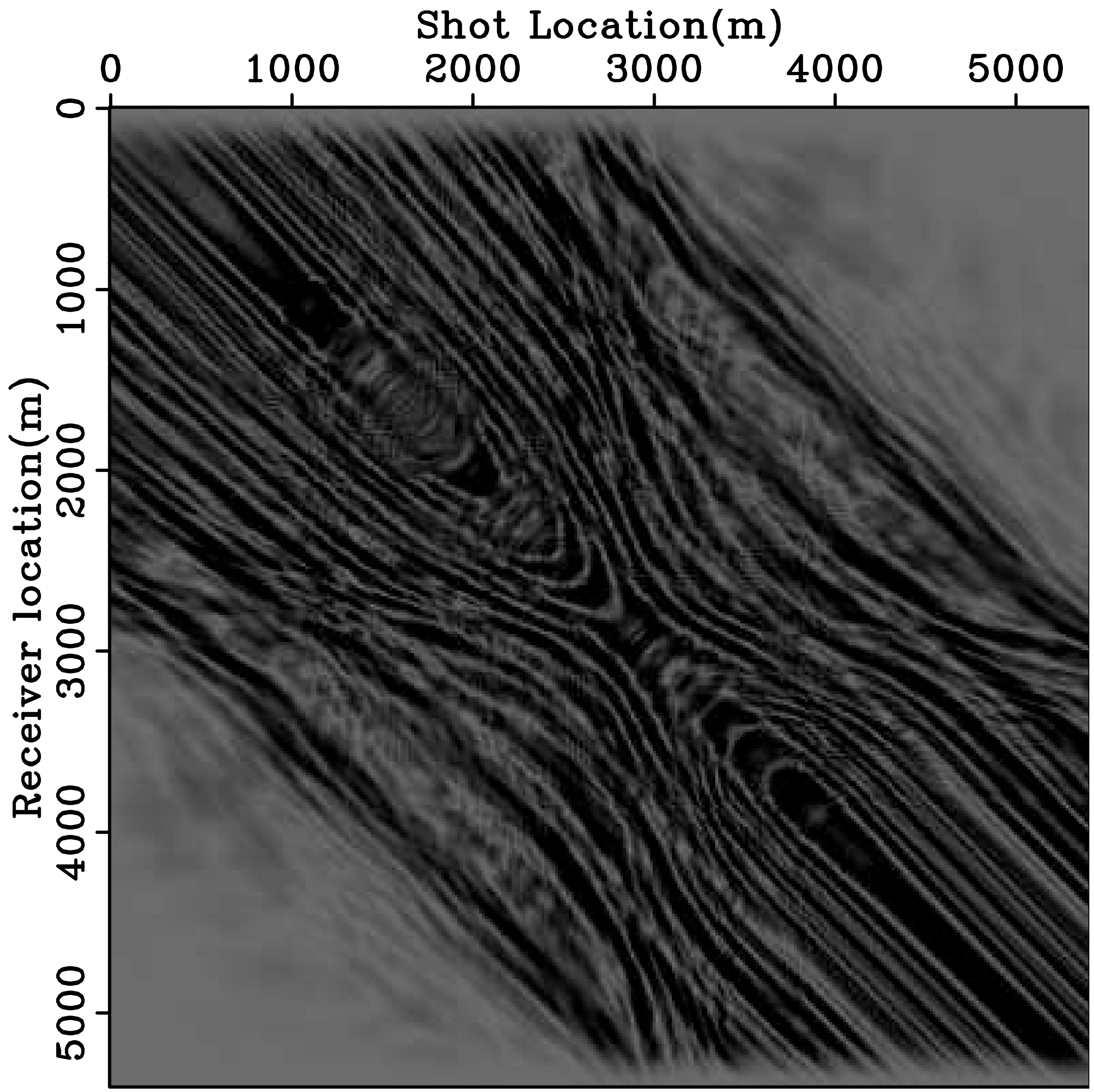
Primary operator



Frequency slice from data matrix with *dominant* primaries.

Primary operator



Frequency Slice (30Hz)

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Primary operator

Primaries to first-order multiples:

$$\boldsymbol{\Delta}\mathbf{p} \mapsto \mathbf{m}^1 = (\boldsymbol{\Delta}\mathbf{P}\boldsymbol{\mathcal{A}} *_{t,x} \boldsymbol{\Delta}\mathbf{p})$$

First-order multiples into primaries:

$$\mathbf{m}^1 \mapsto \boldsymbol{\Delta}\mathbf{p} \approx (\boldsymbol{\Delta}\mathbf{P}\boldsymbol{\mathcal{A}} \otimes_{t,x} \boldsymbol{\Delta}\mathbf{p})$$

with the acquisition matrix

$$\boldsymbol{\mathcal{A}} = \left(\boldsymbol{\mathcal{S}}^{\dagger}\mathbf{R}\boldsymbol{\mathcal{D}}^{\dagger}\right)$$

"inverting" for source and receiver wavelet wavelets geometry and surface reflectivity.

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Curvelet-based Focal transform

Solve with 3-D curvelet transform

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \text{and } \boldsymbol{\Delta}\mathbf{P} := \mathbf{F}^H \text{block diag}\{\boldsymbol{\Delta}\mathbf{p}\}\mathbf{F} \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{p} \\ \mathbf{p} &= \text{total data.} \end{aligned}$$

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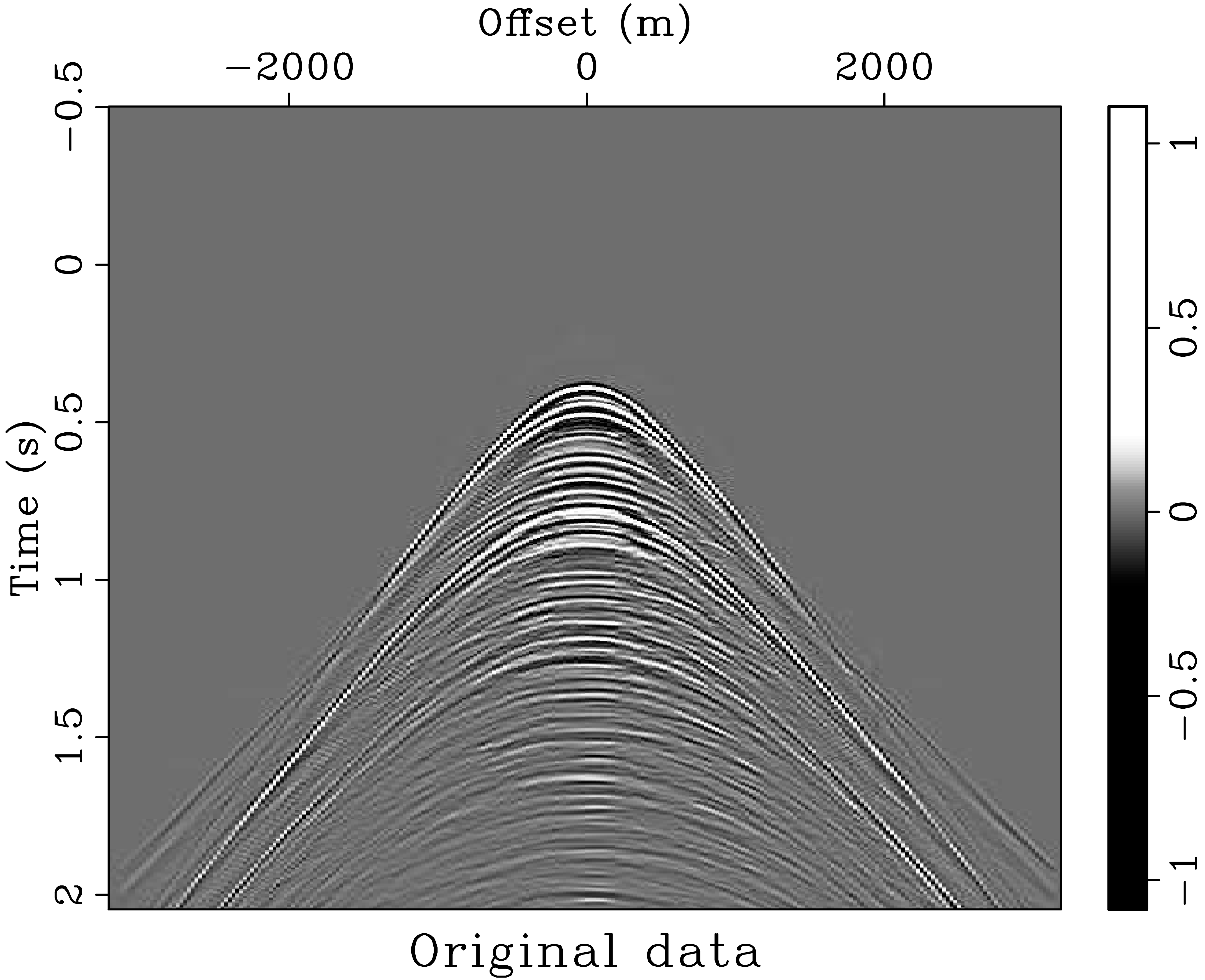
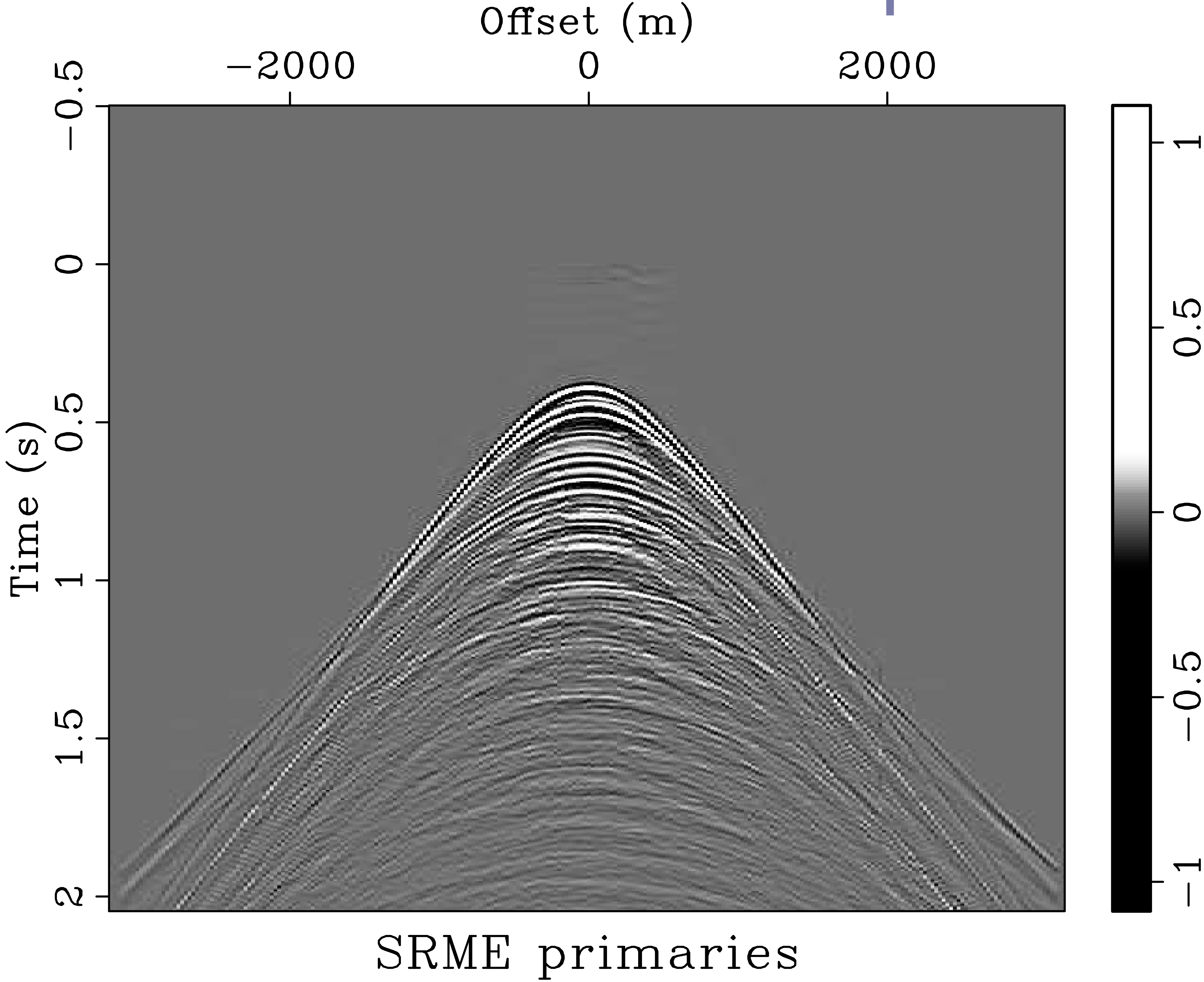
Total data



SRME estimate for the primaries



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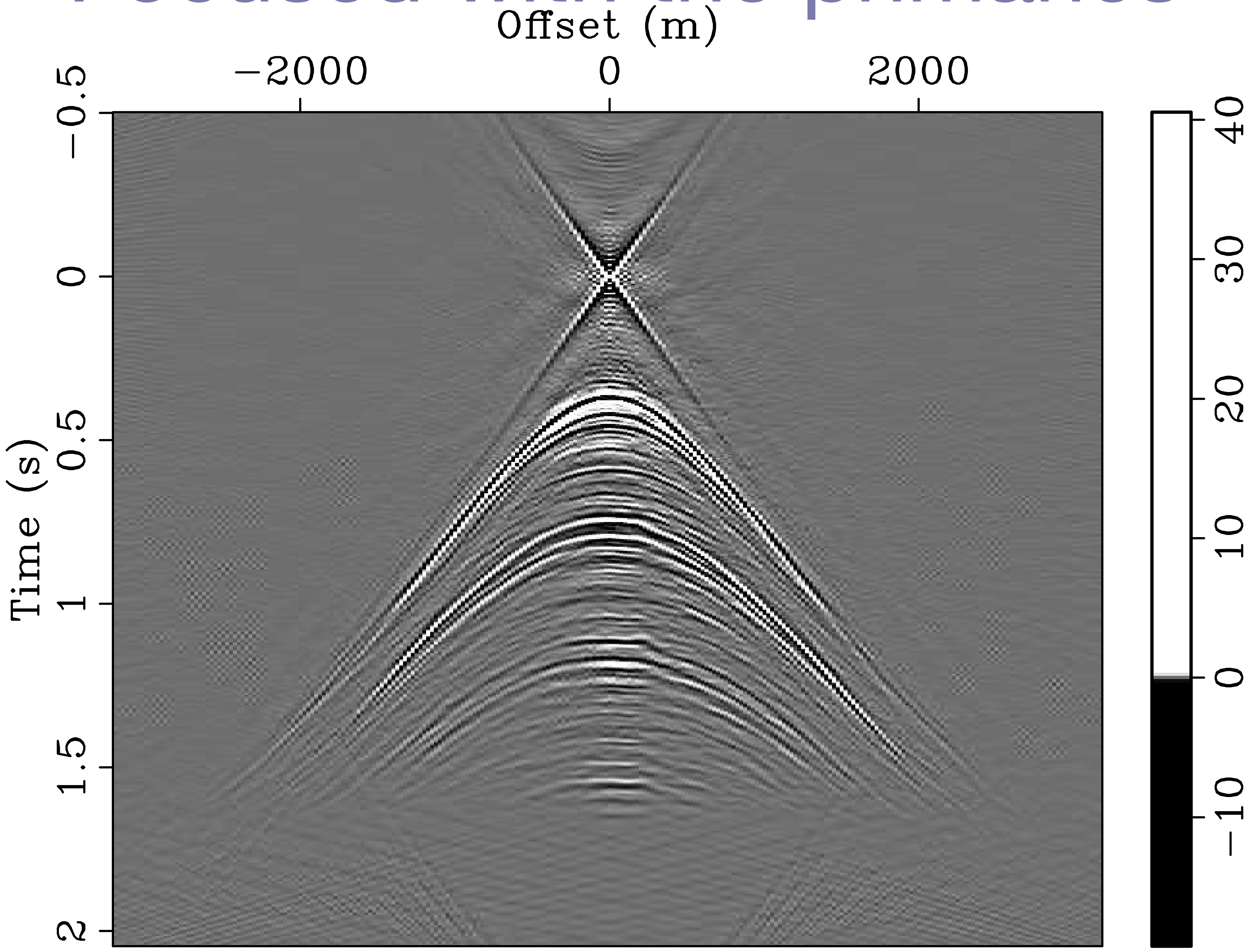
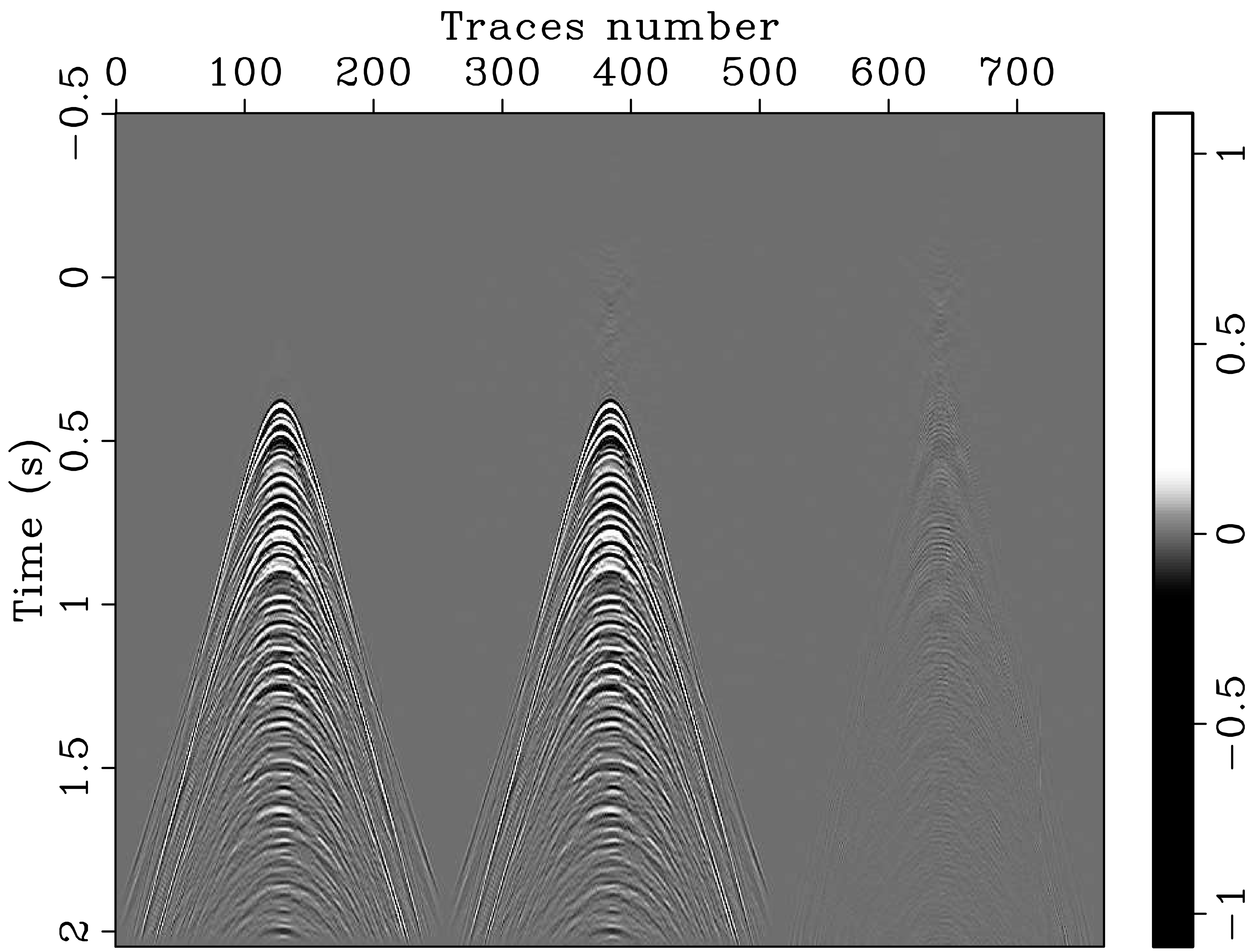
Focused with the primaries



Difference



Traces number
0
100
200
300
400
500
600
700
Time (s)
−0.5
0
0.5
1
1.5
2
1
0.5
0
−0.5
−1

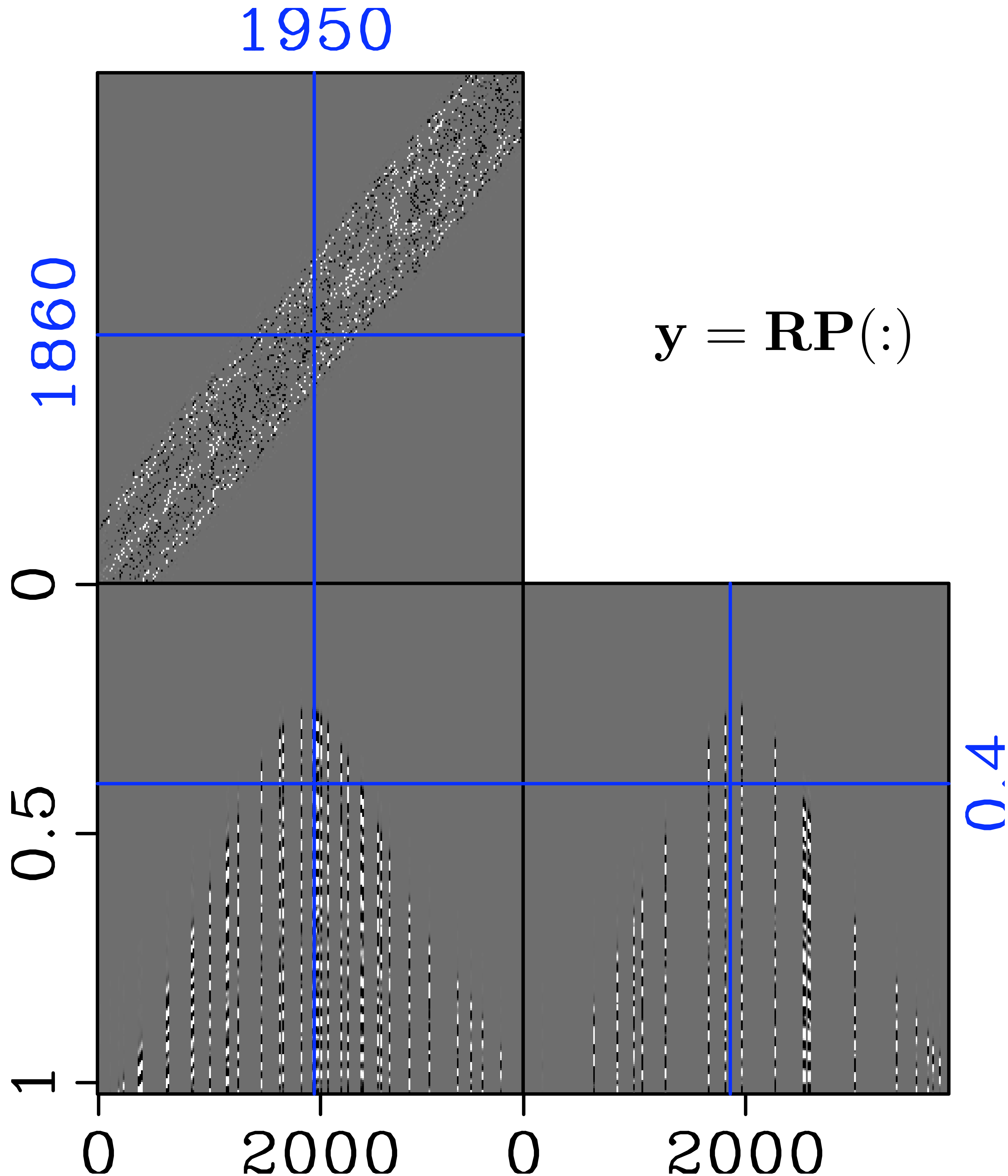
Recovery with focussing

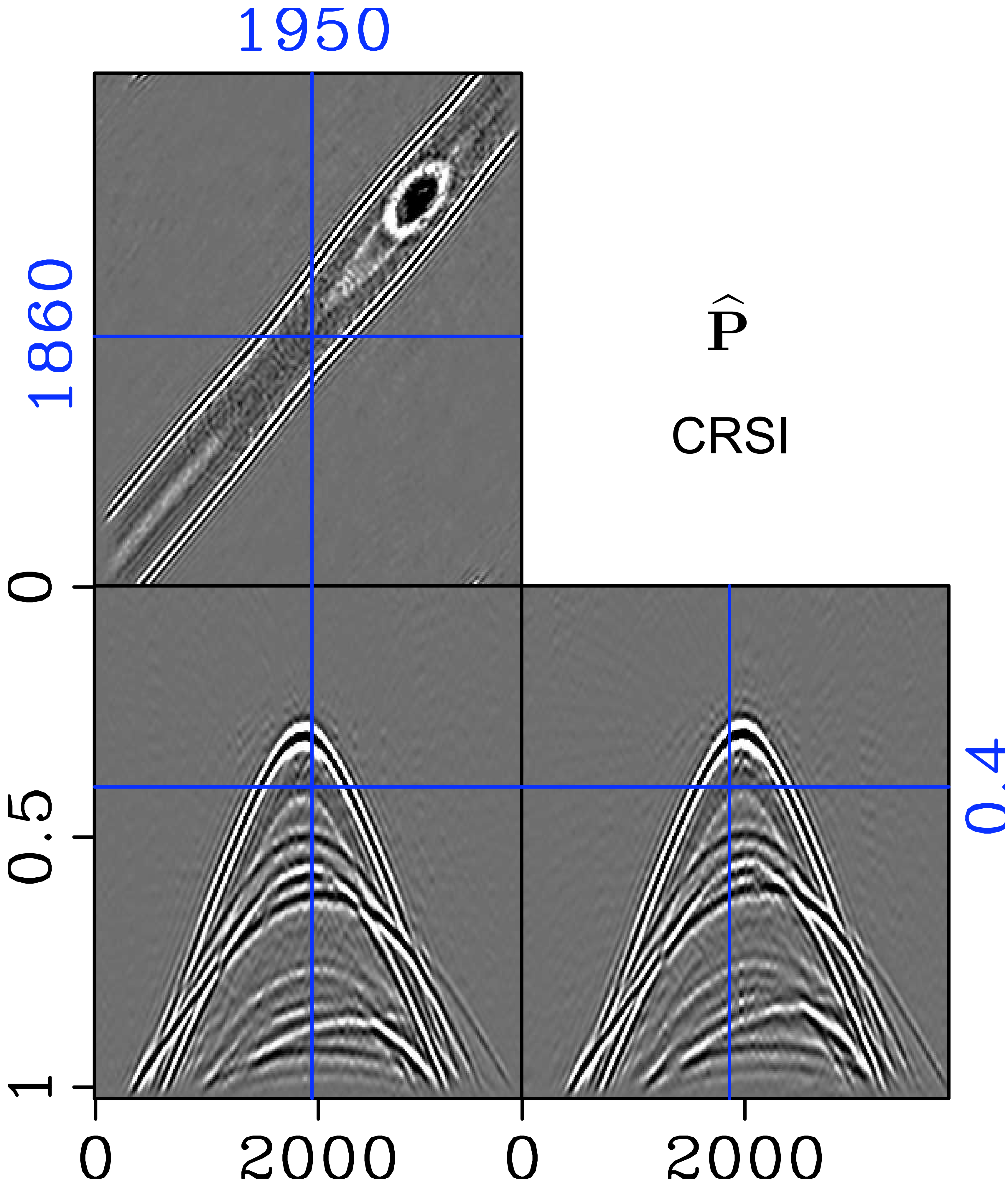
Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{S}^T &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{y} &= \mathbf{R}\mathbf{p} \\ \mathbf{R} &= \text{picking operator.} \end{aligned}$$


1950
1860
0
0.5
1
0
2000
0
2000
0.4
$\mathbf{y} = \mathbf{RP}(:)$



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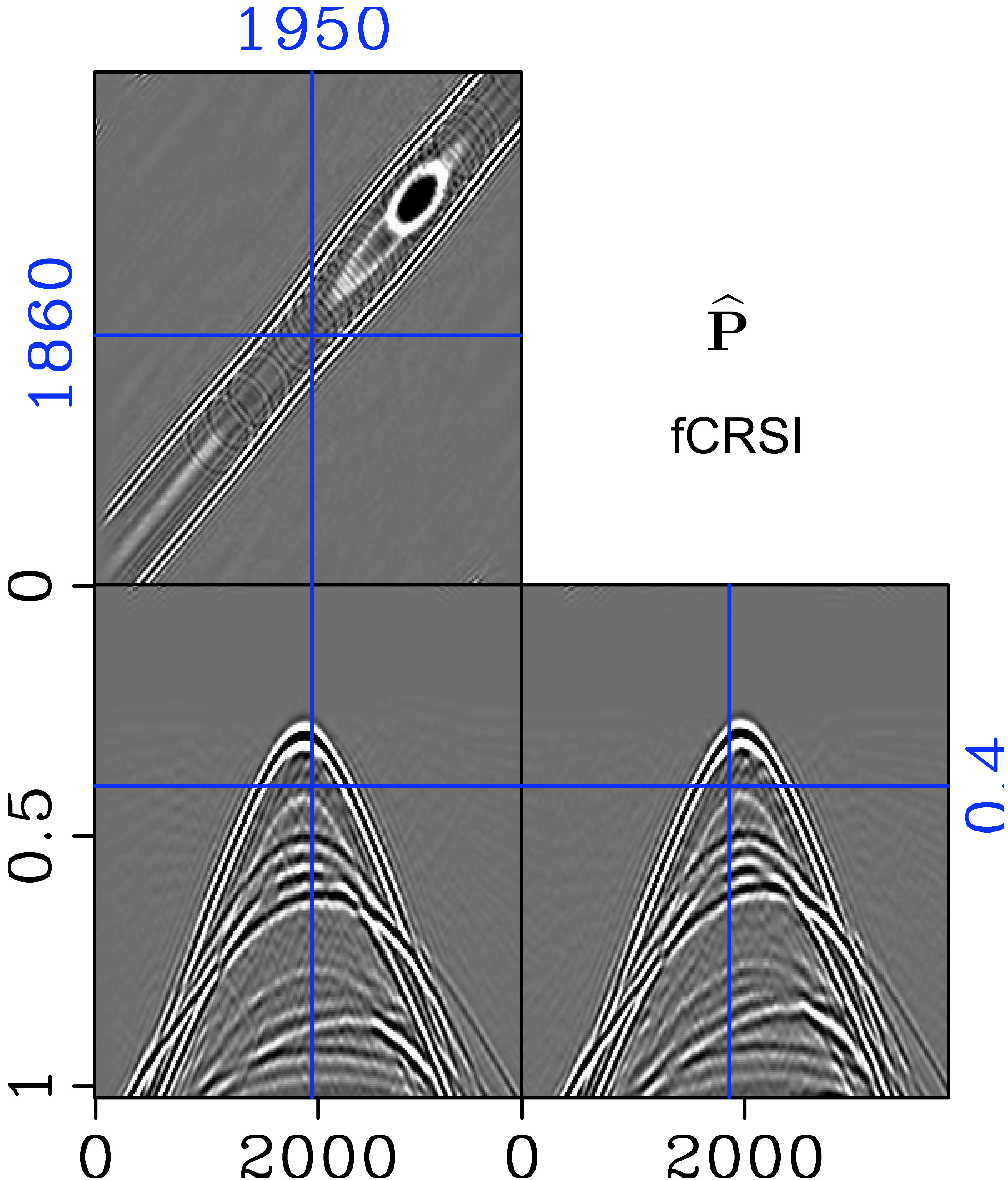
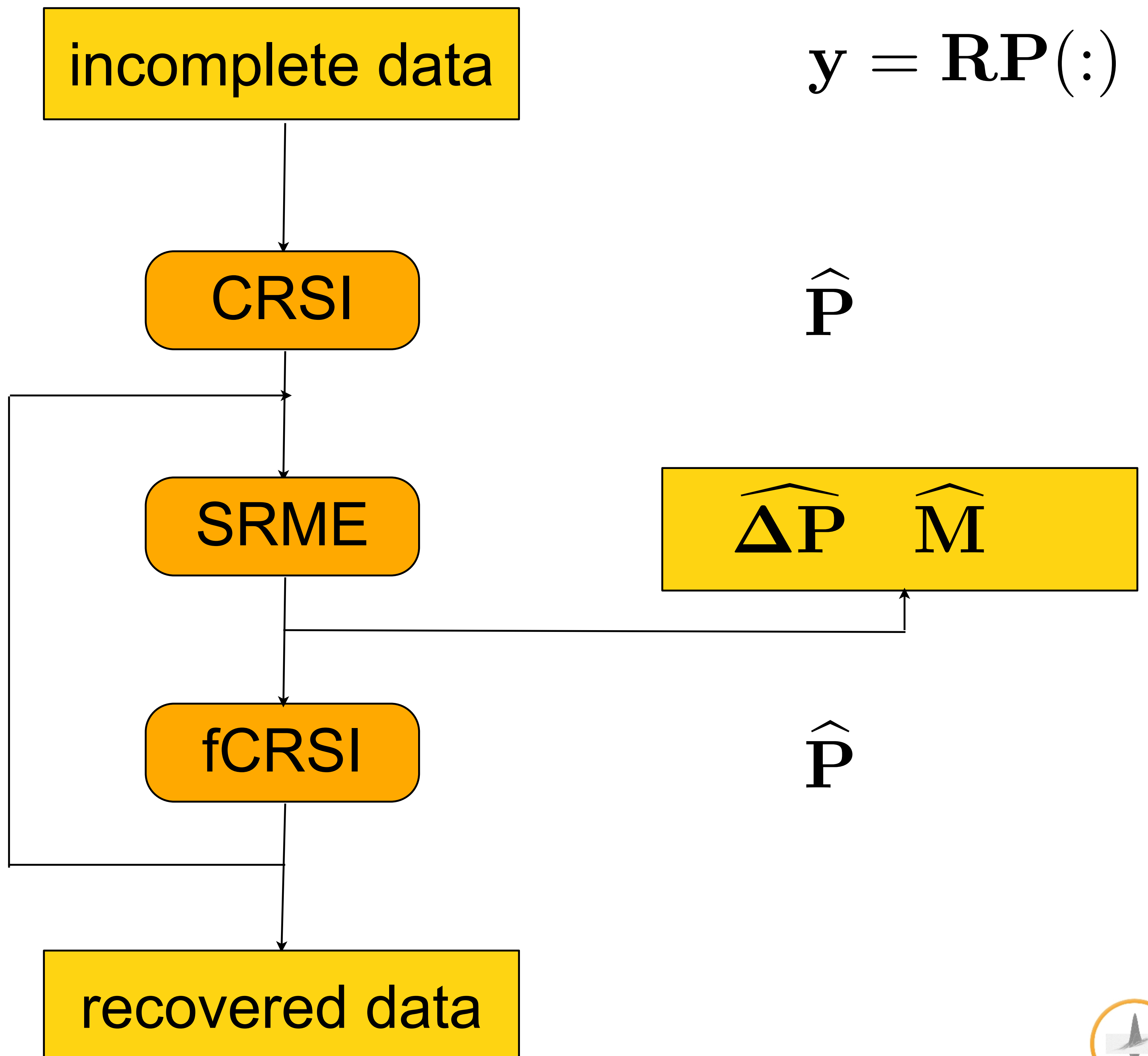


1950
1860
0
0.5
1
0.4
0
2000
0
2000
$\widehat{\mathbf{P}}$
fCRSI

Multiple prediction with fCRSI



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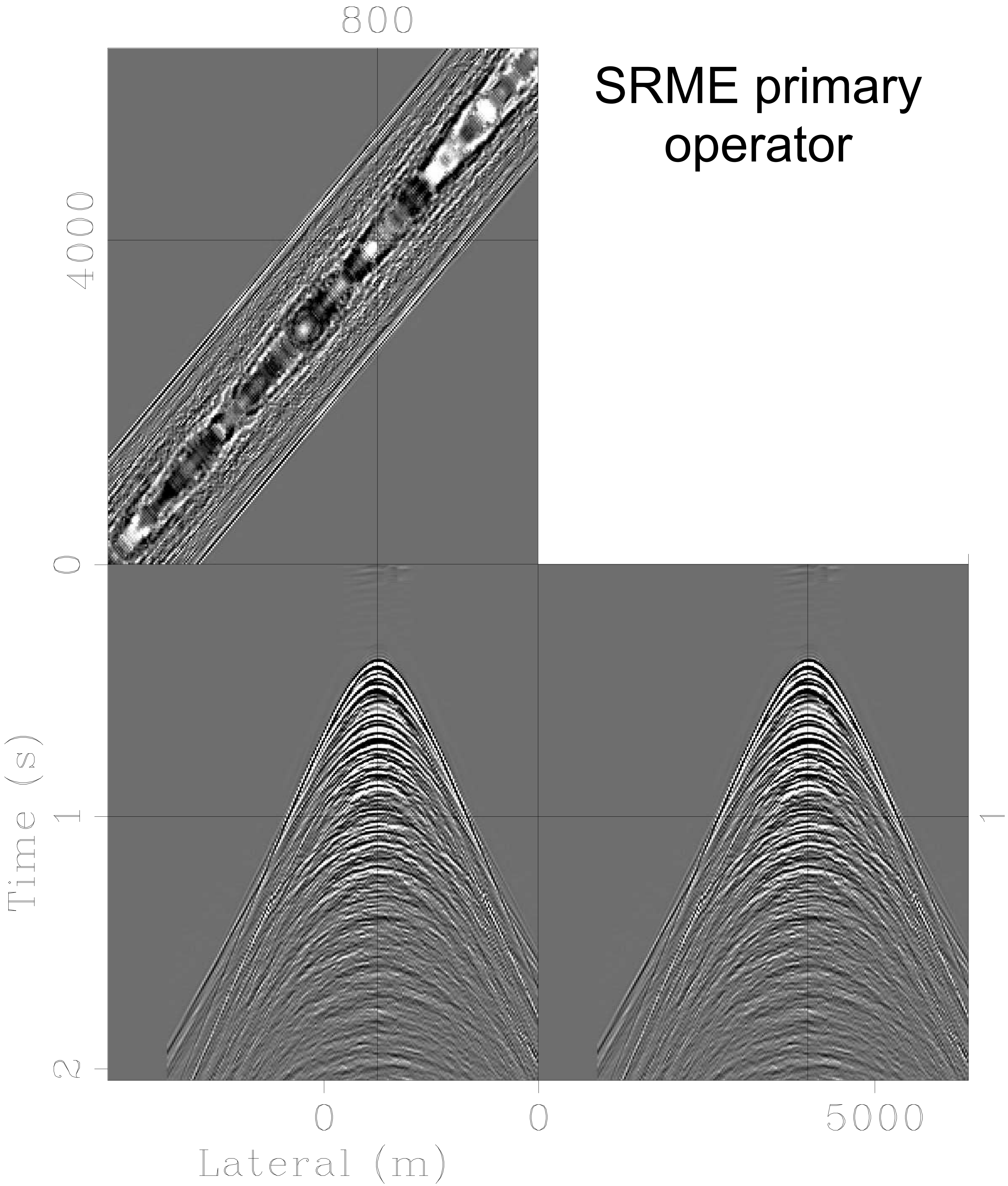
SRME primary operator



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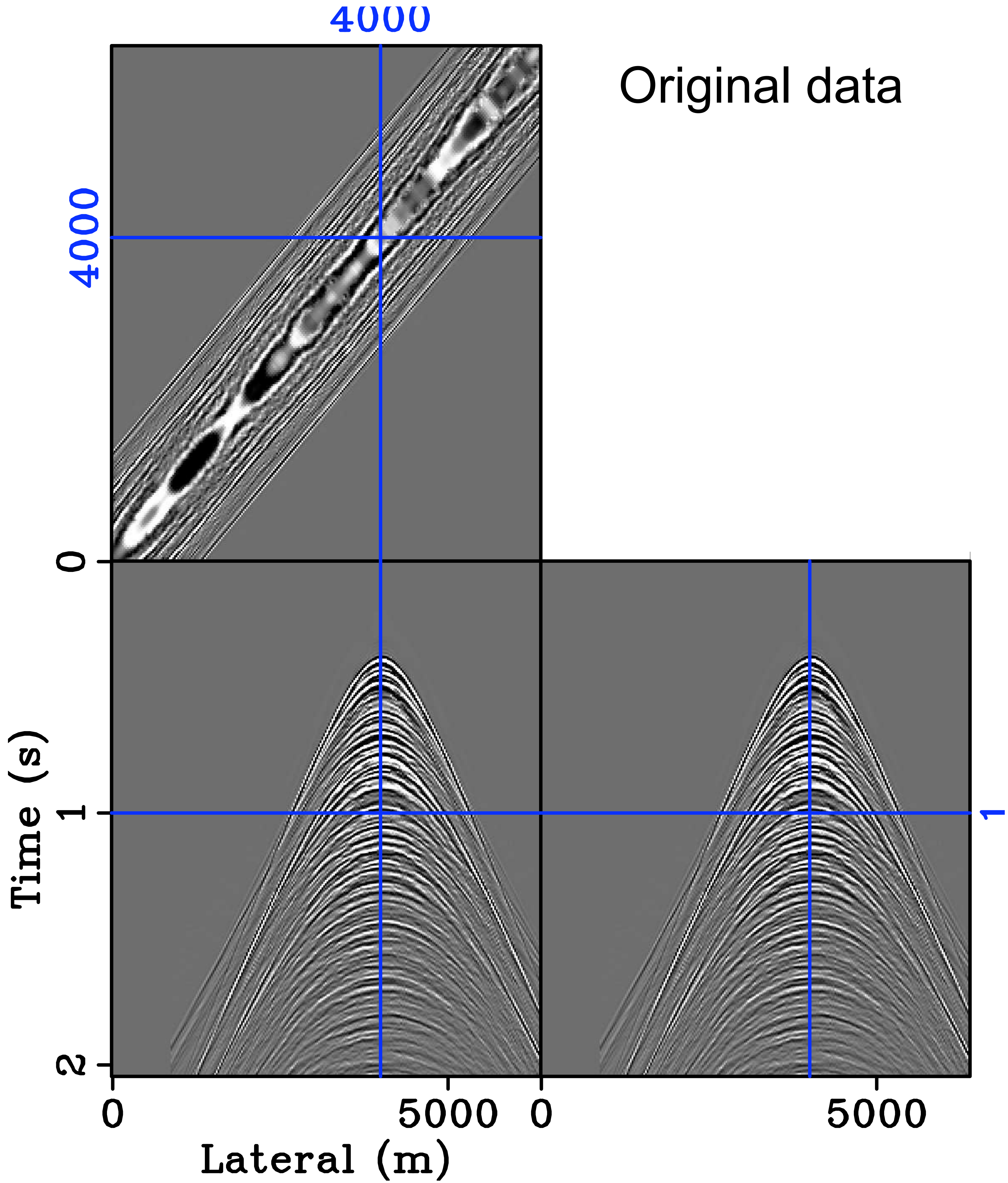
Wavefield reconstruction with fCRSI

Original data



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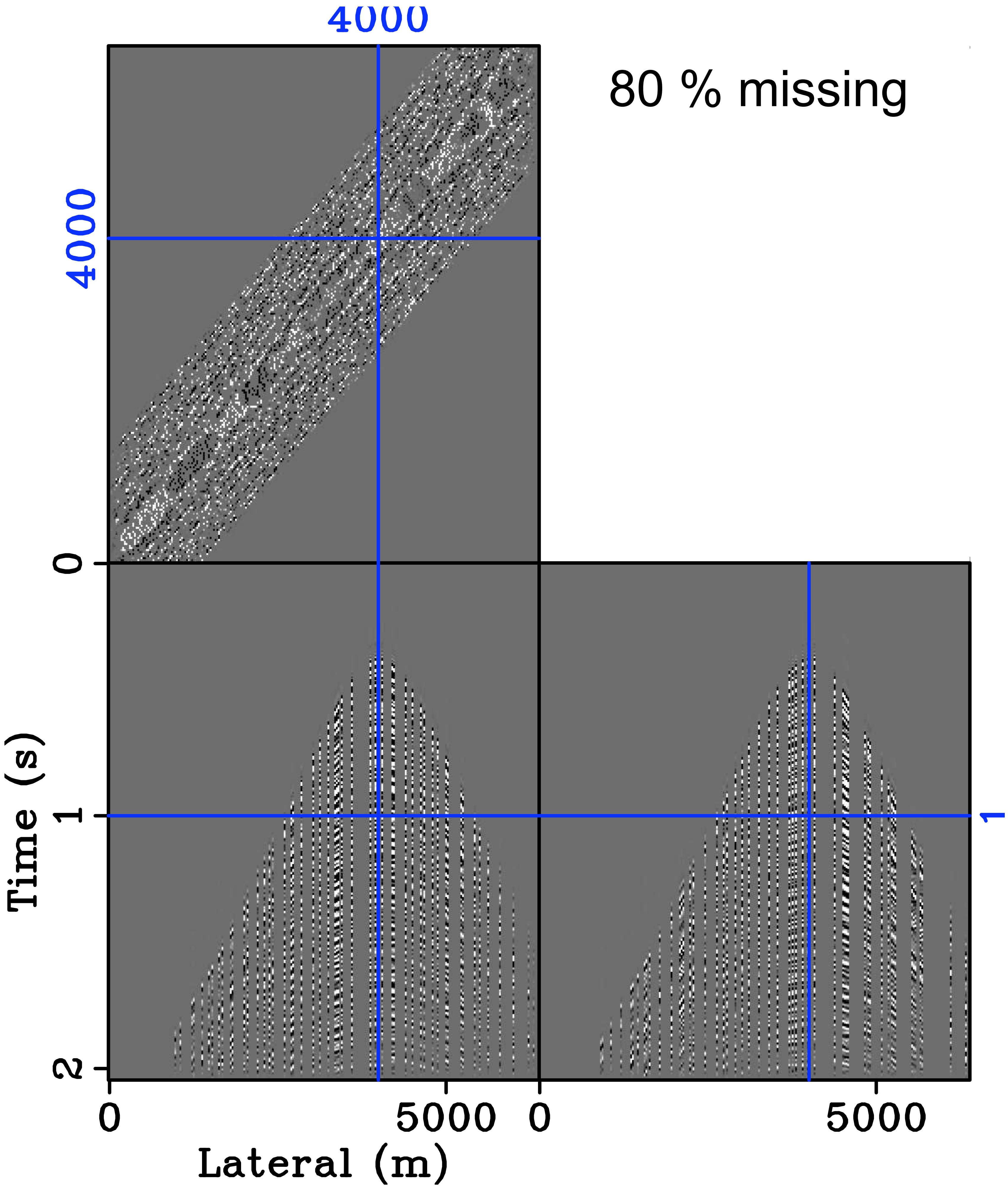
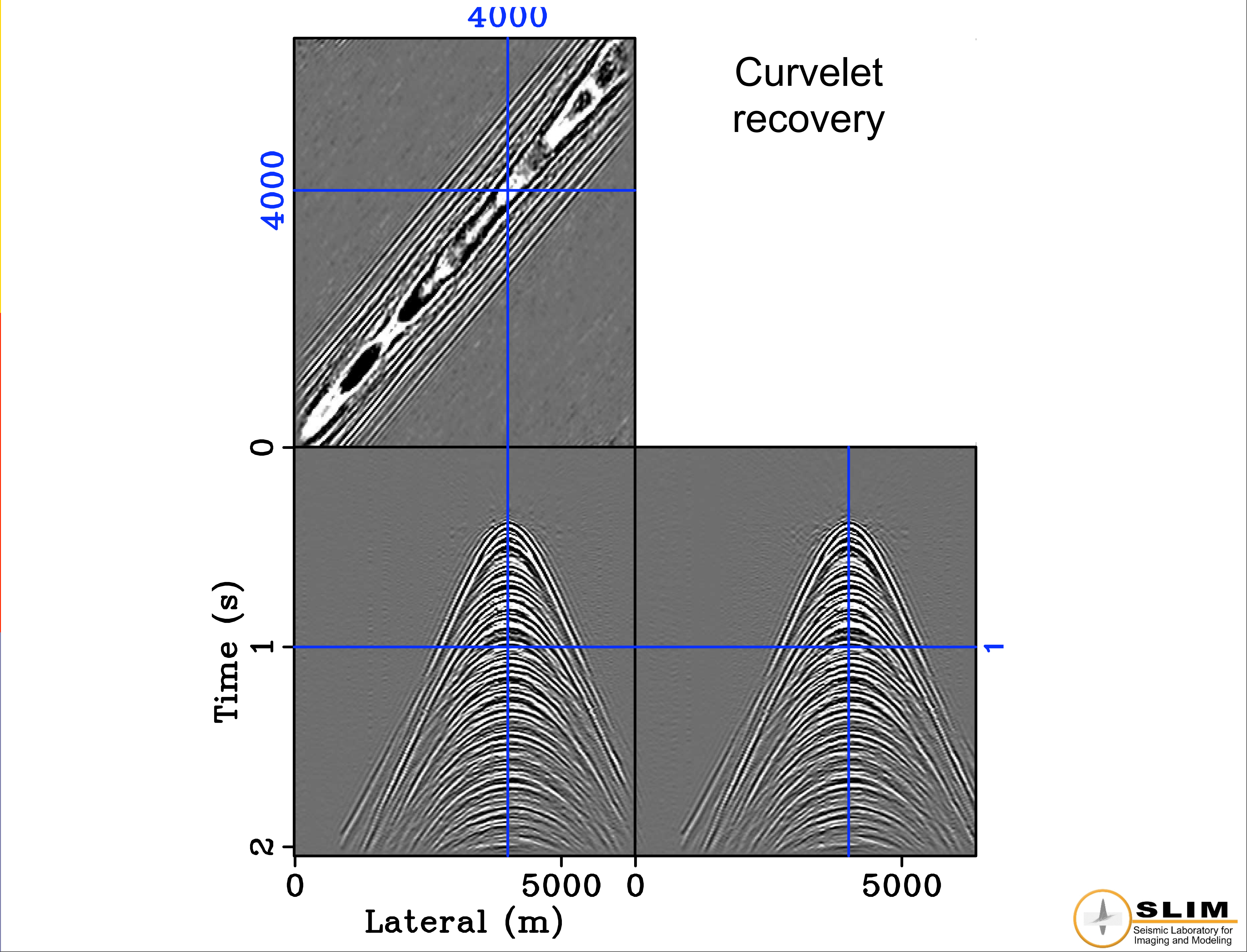


4000
80 % missing
4000
0
1
2
Time (s)
1
0
5000
0
5000
Lateral (m)


Curvelet
recovery
4000
4000
0
1
2
1
Time (s)
0
5000
0
5000
Lateral (m)
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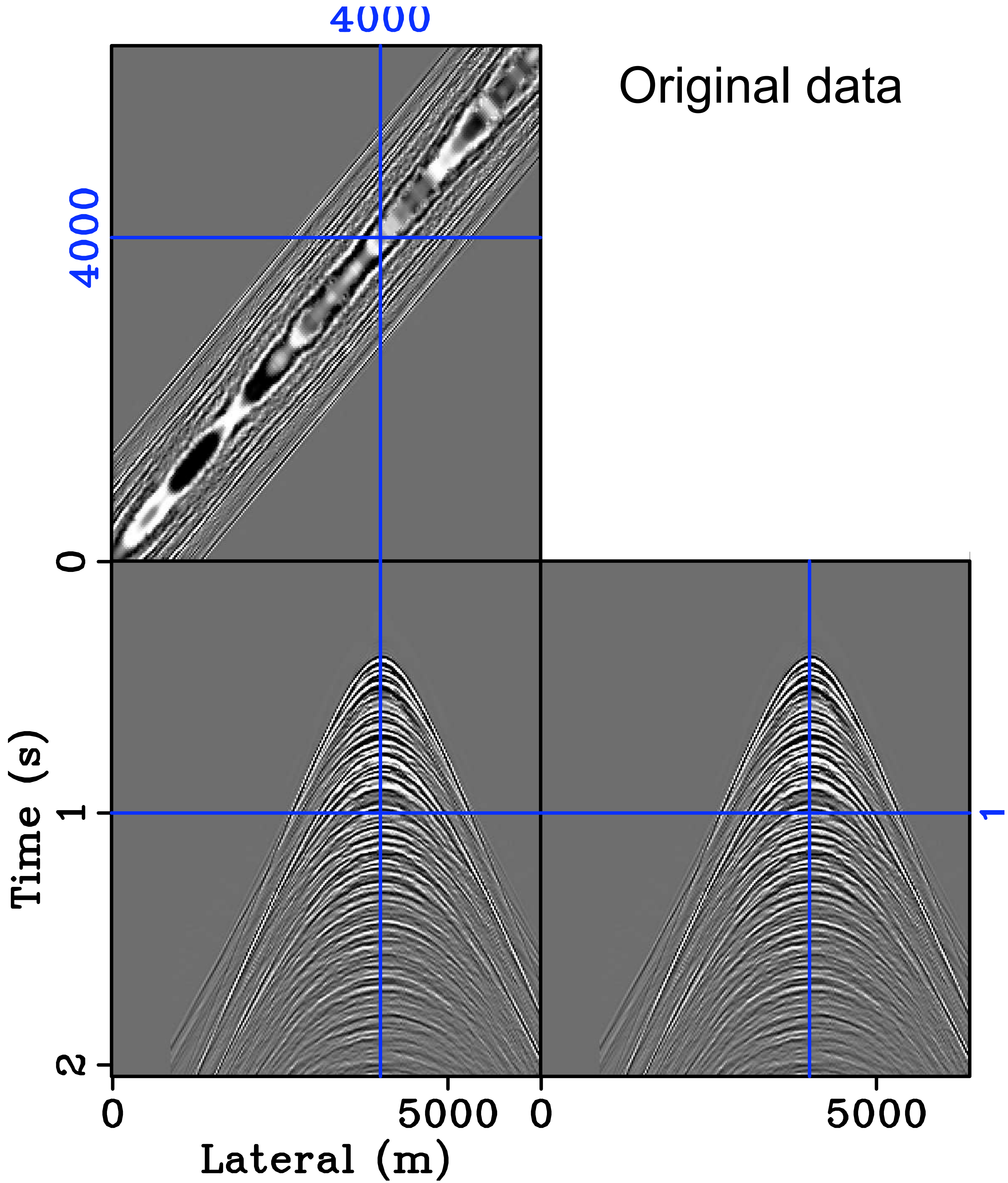
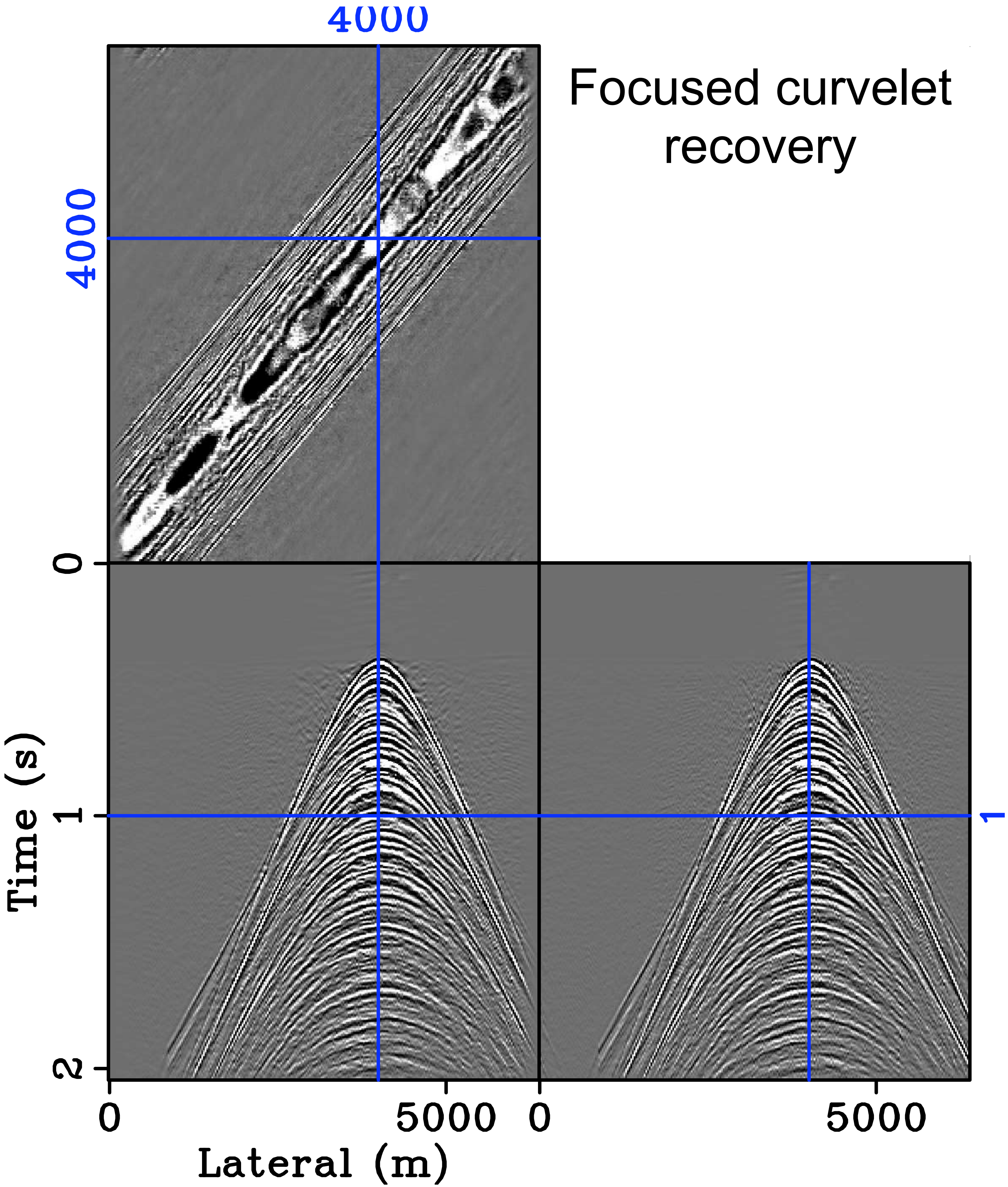
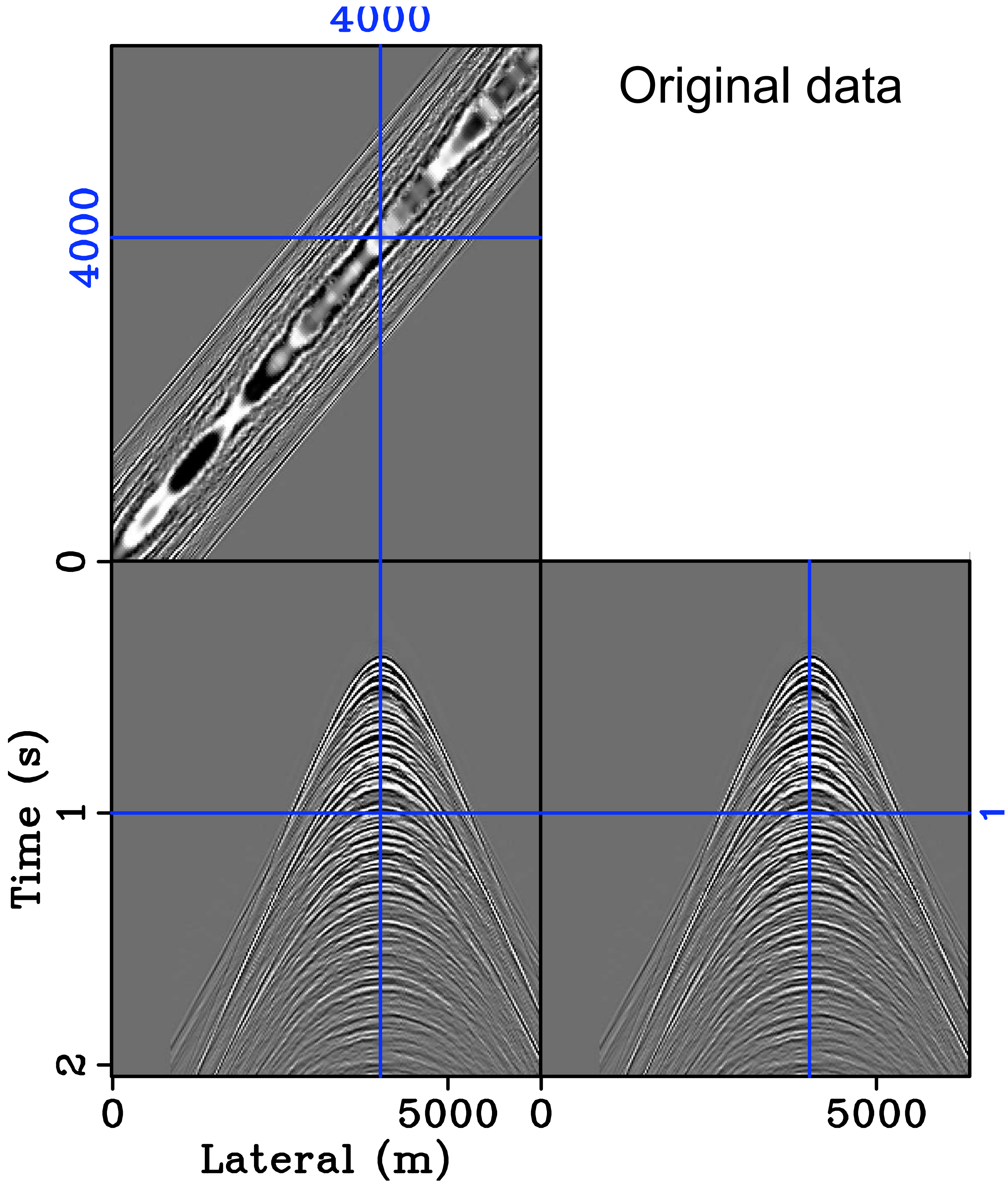
Imaging and Modeling

Original data



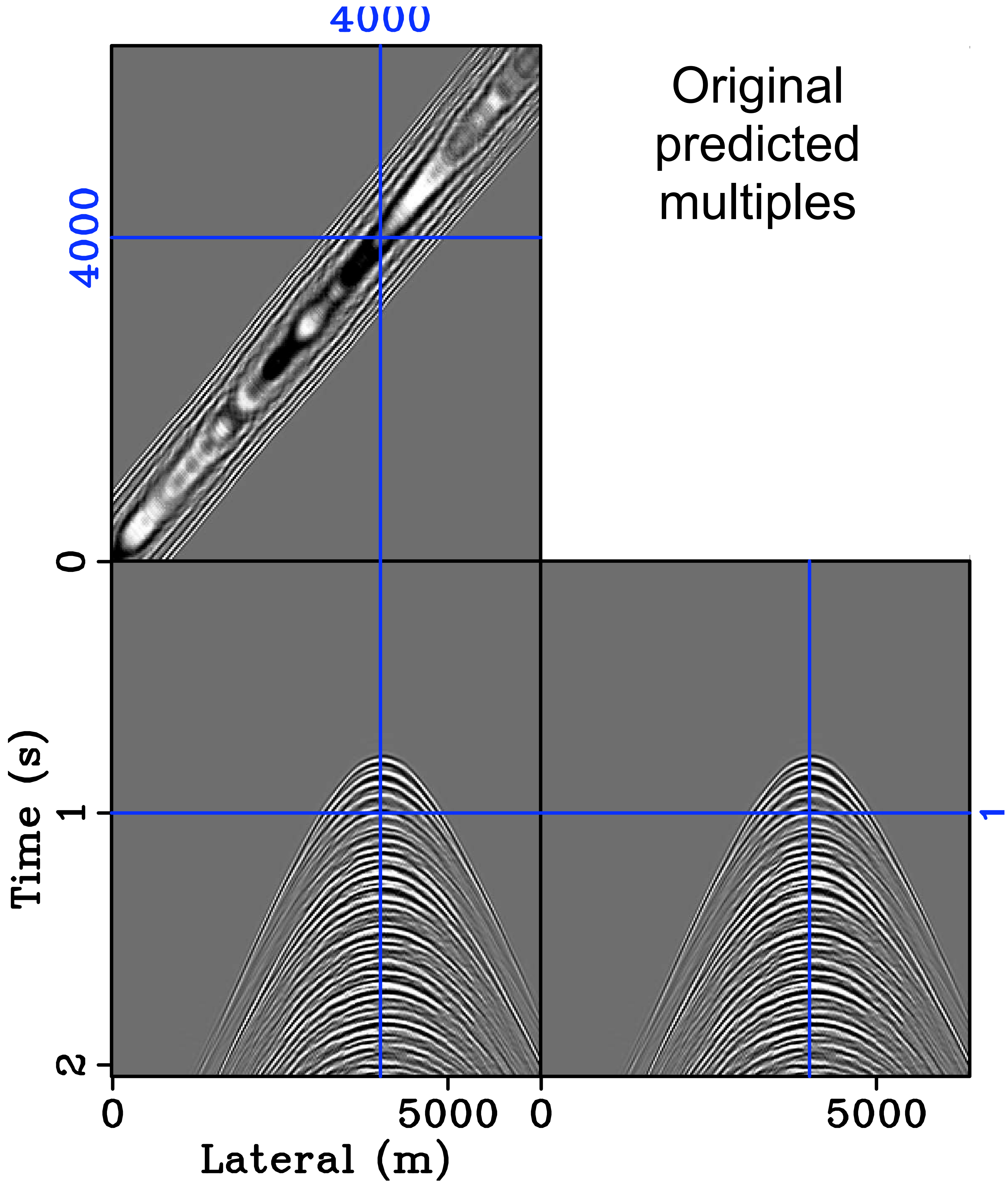
Focused curvelet recovery





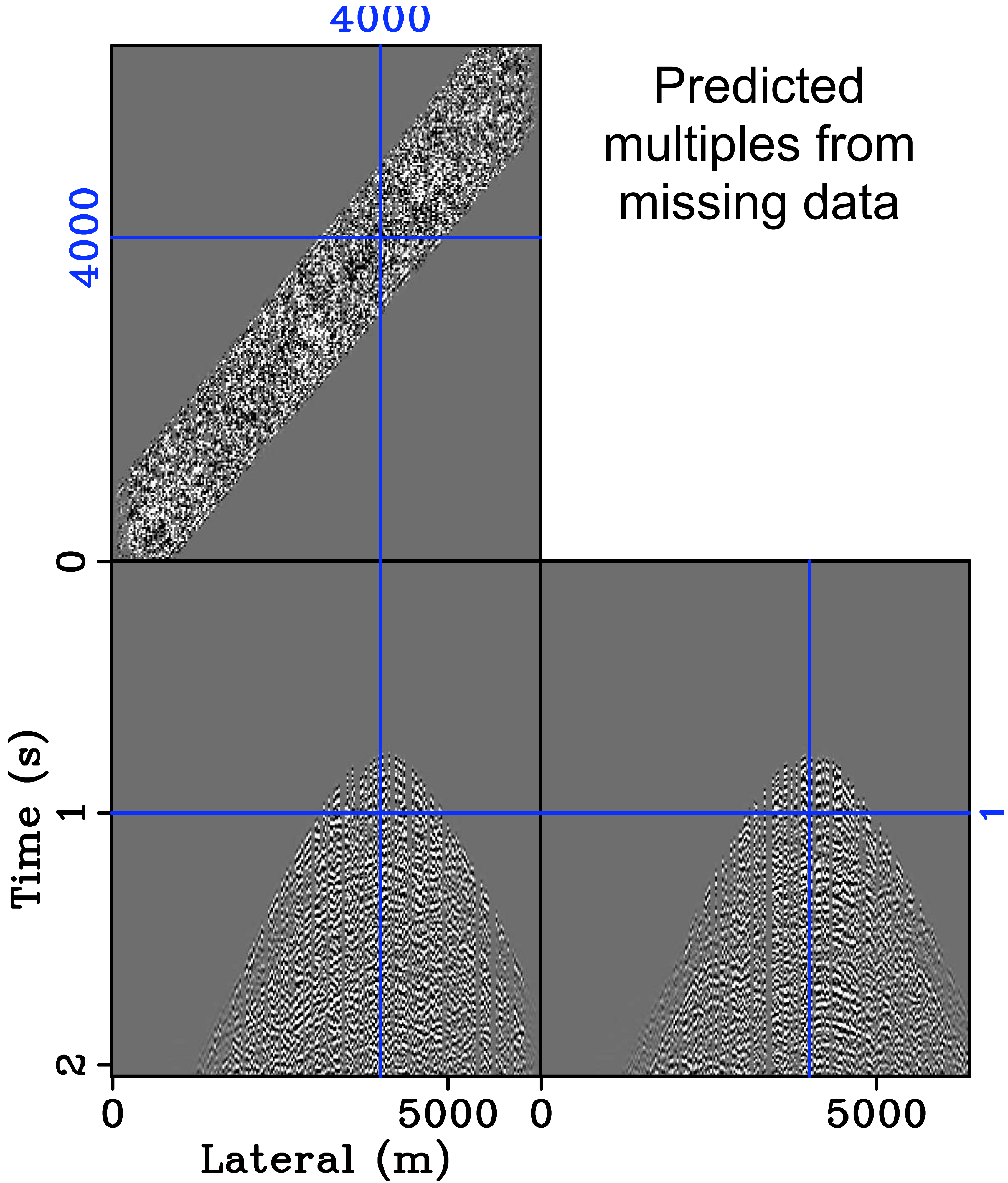
Original data

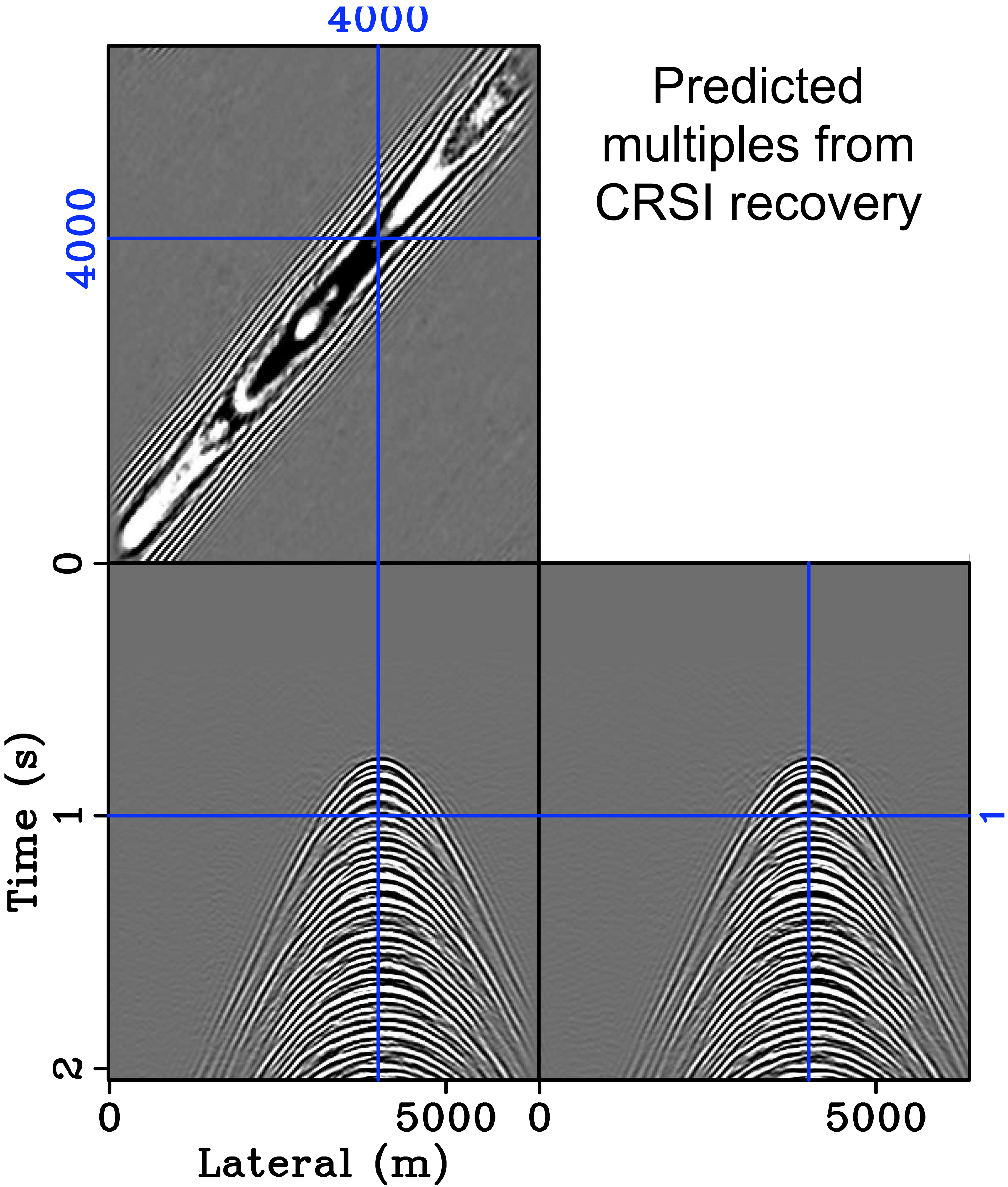
Multiple prediction



Original predicted multiples

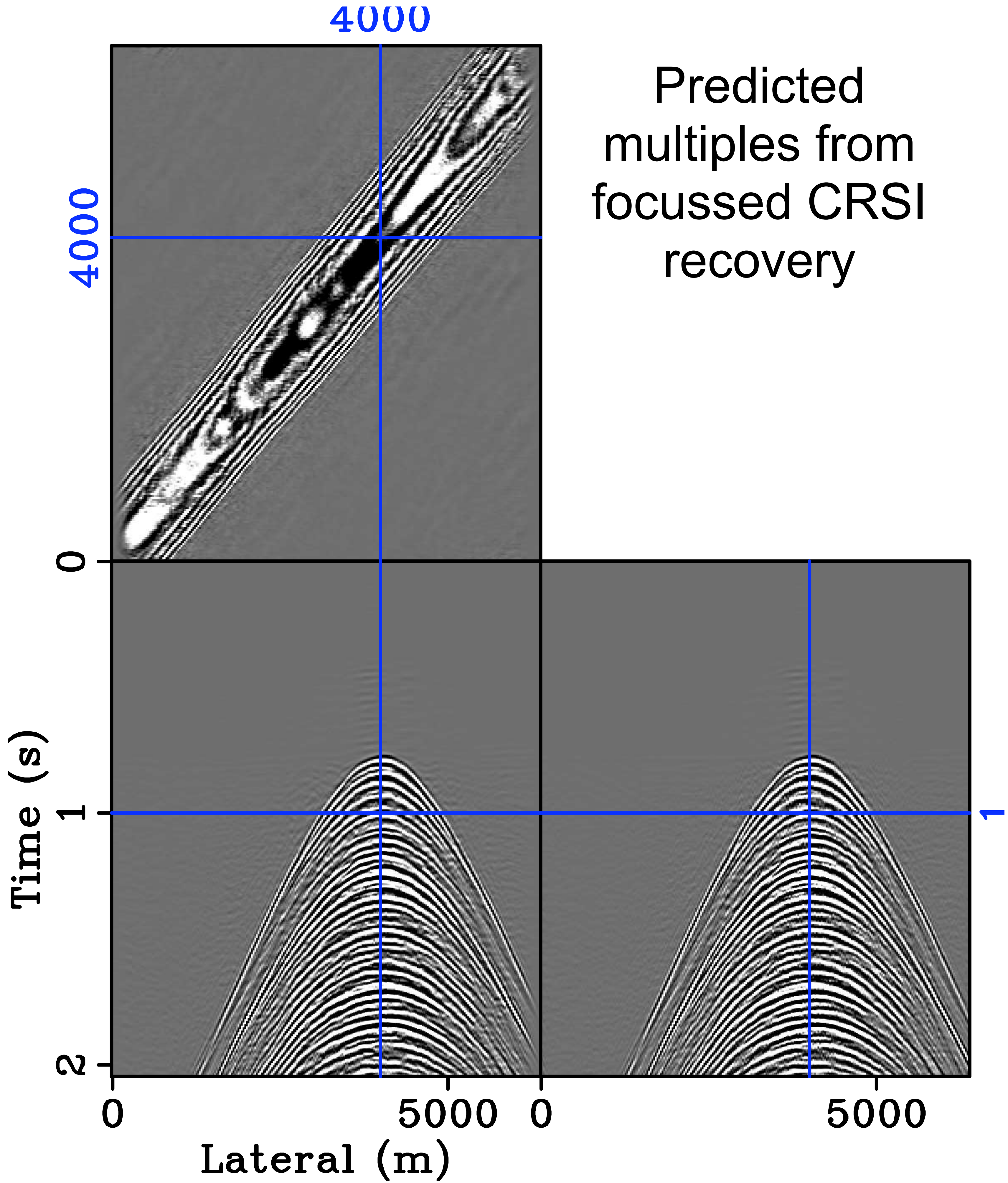
Predicted multiples from missing data



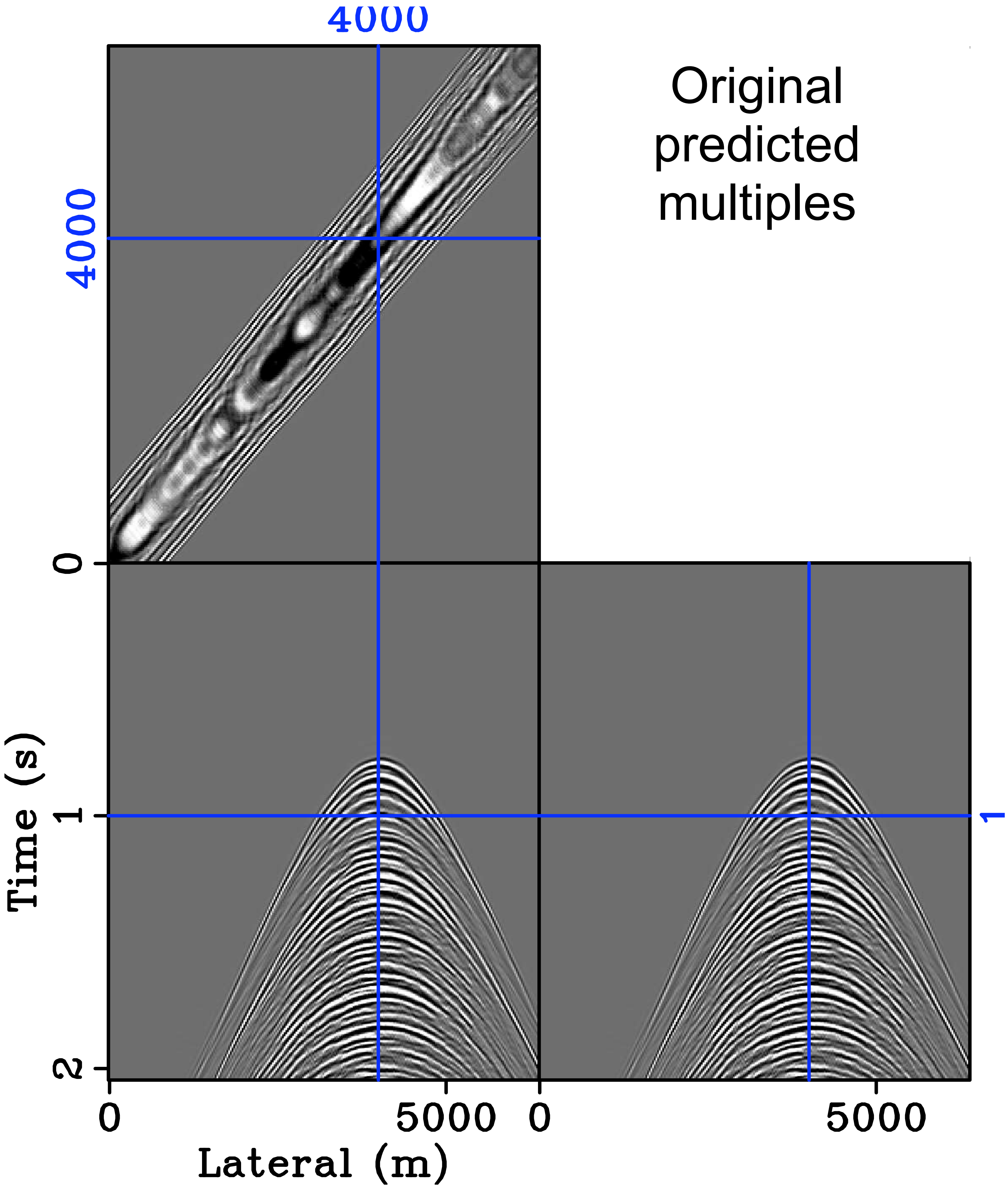


Predicted multiples from CRSI recovery

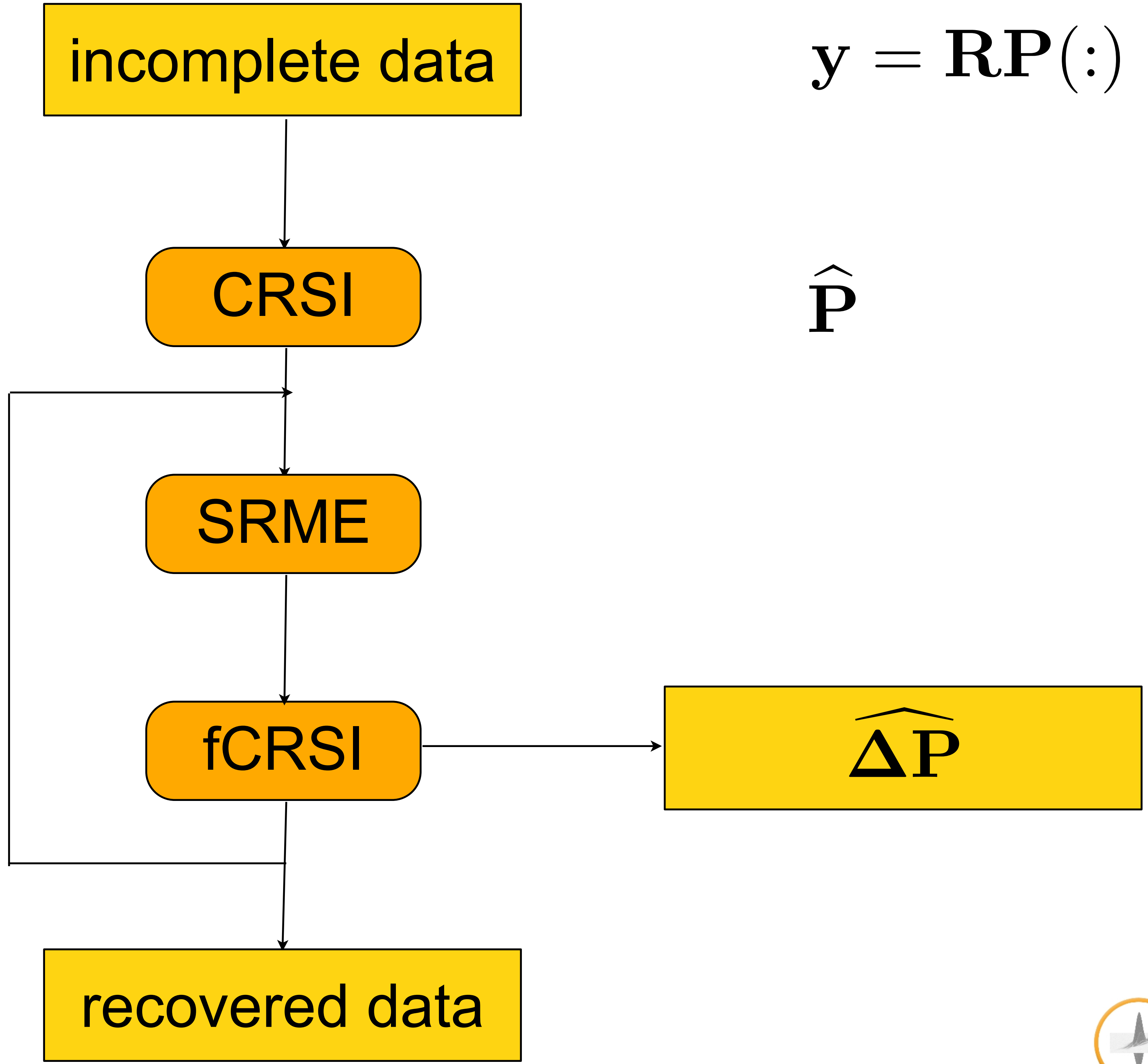
Predicted multiples from focussed CRSI recovery



Original predicted multiples



Primary prediction with fCRSI



$$\mathbf{y} = \mathbf{R}\mathbf{P}(:)$$

$$\widehat{\mathbf{P}}$$

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Curvelet-based Focal transform

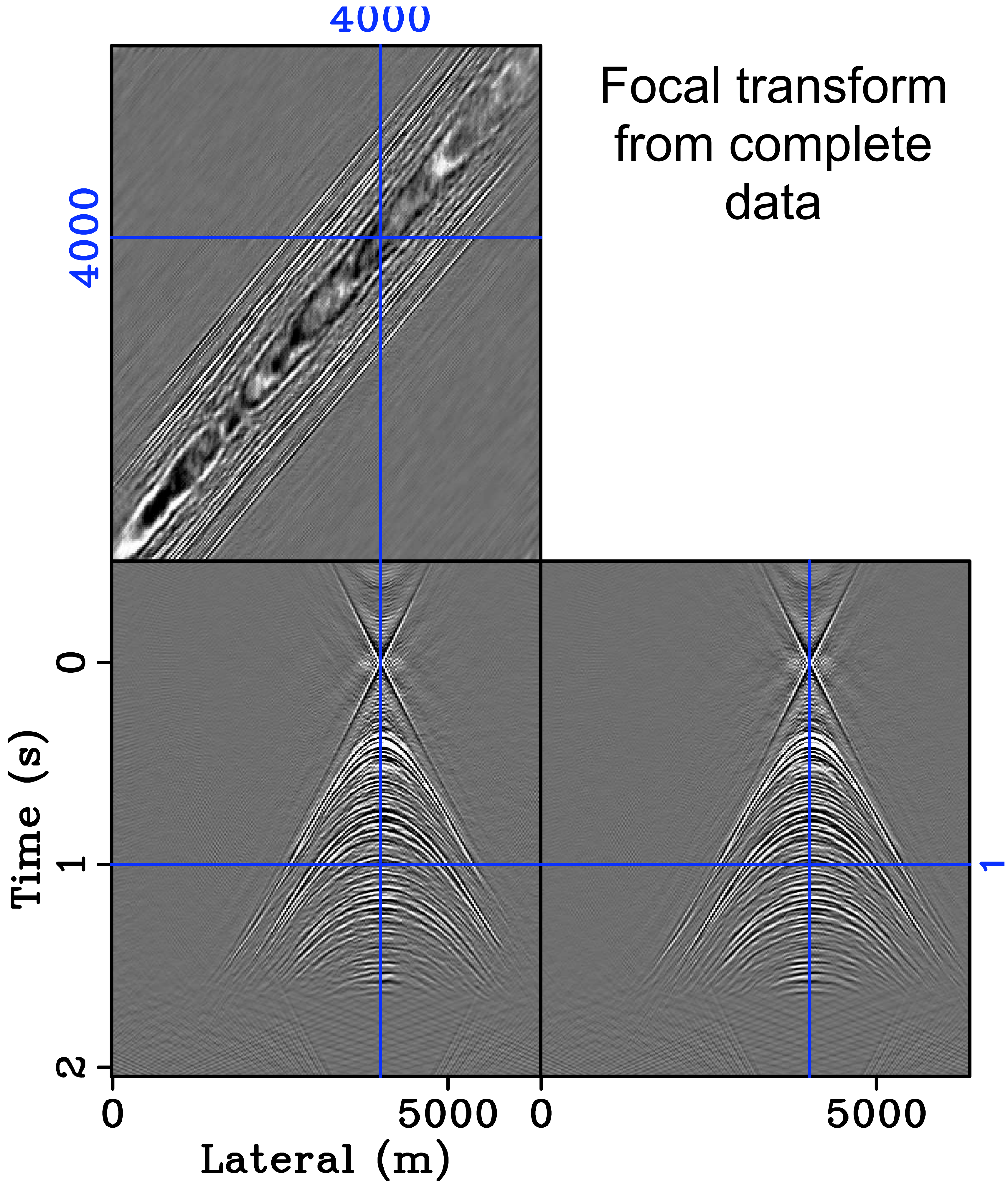
Solve

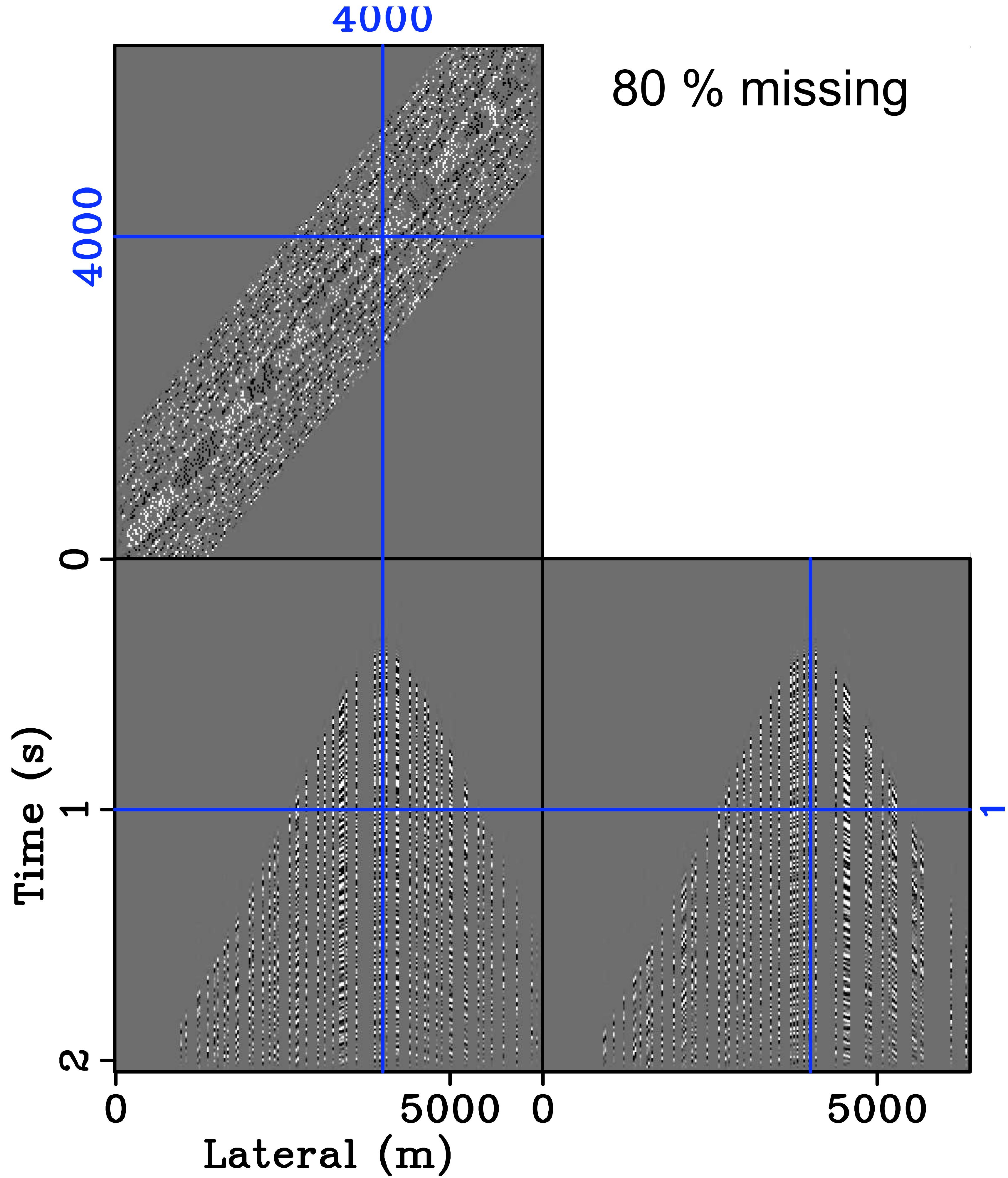
$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{\Delta P C}^T \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data} \\ \tilde{\mathbf{f}} &= \text{focused data.} \end{aligned}$$

Focal transform from complete data





80 % missing
4000
4000
0
1
2
1
Time (s)
0
5000
0
5000
Lateral (m)
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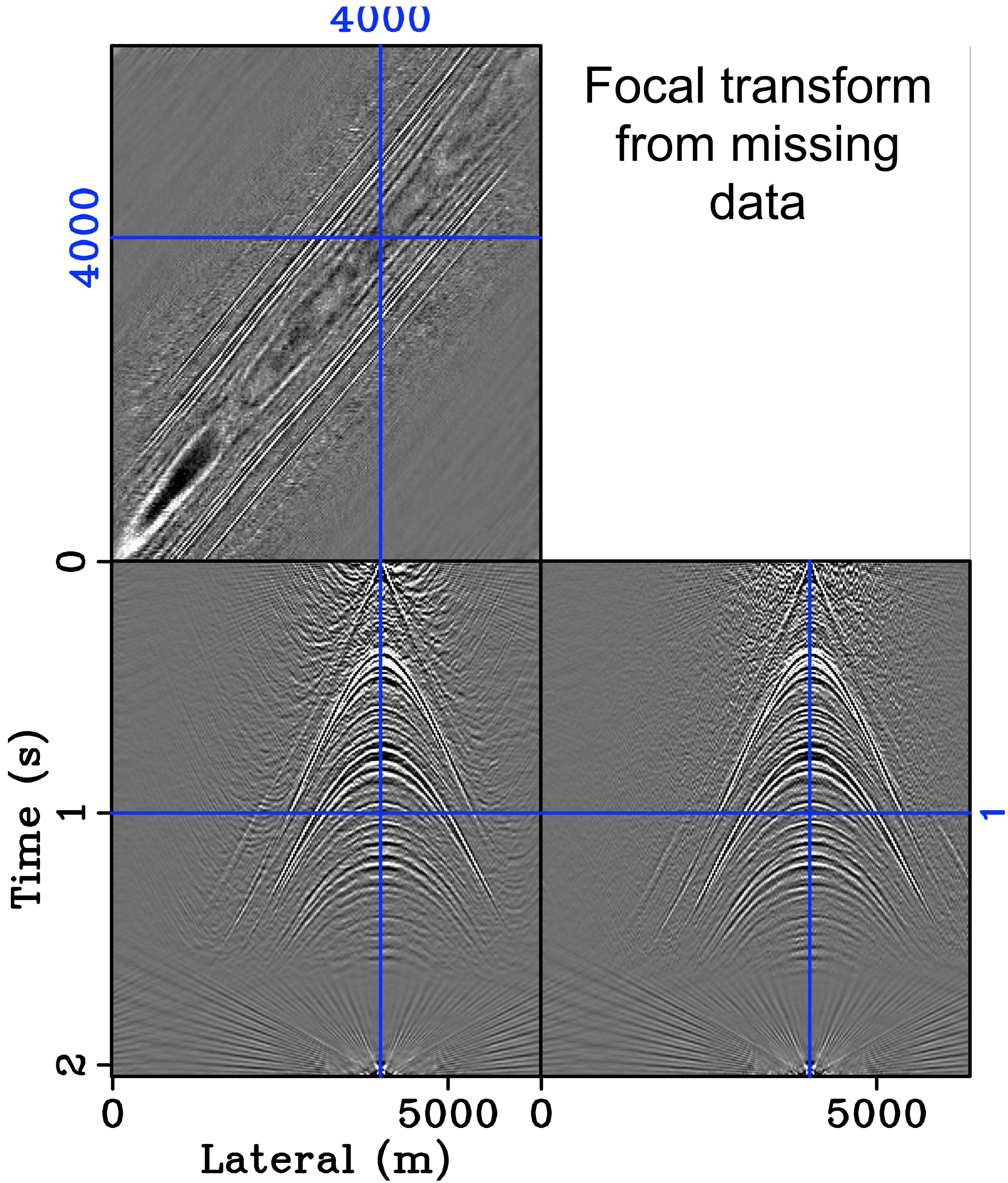
Focal transform from missing data



An encore ... preliminary results for the data inverse

Curvelet-based seismic data inverse

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{p}}^{-1} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with $\mathbf{A} := \mathbf{P}\mathbf{C}^T$

$$\mathbf{S}^T := \mathbf{C}^T$$

$$\mathbf{y} = \hat{\mathbf{I}}$$

$\mathbf{p}$ is the data to be inverted

Curvelet-sparsity regularized *data inverse* computed for the *whole* data volume ......

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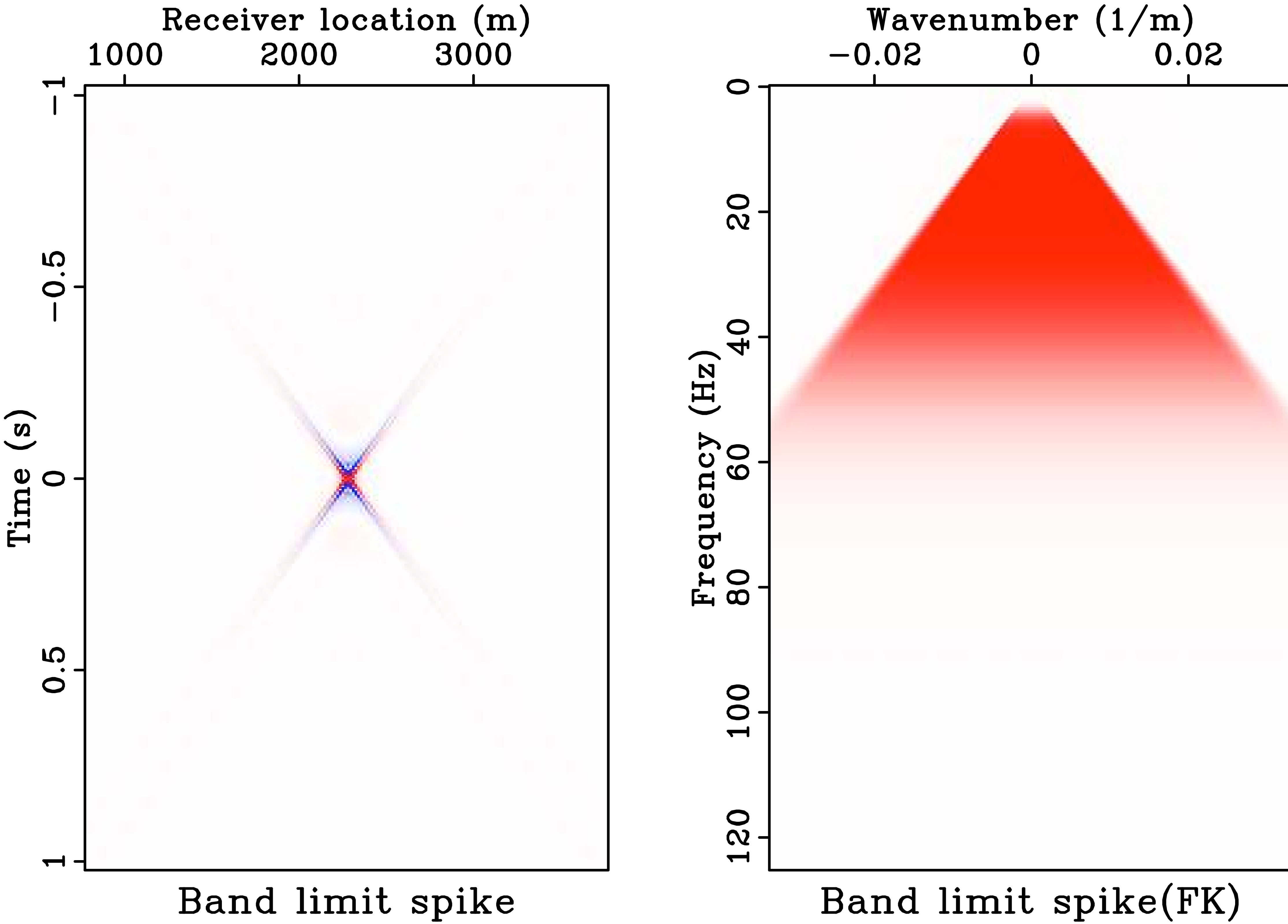
Curvelet-based seismic data inverse



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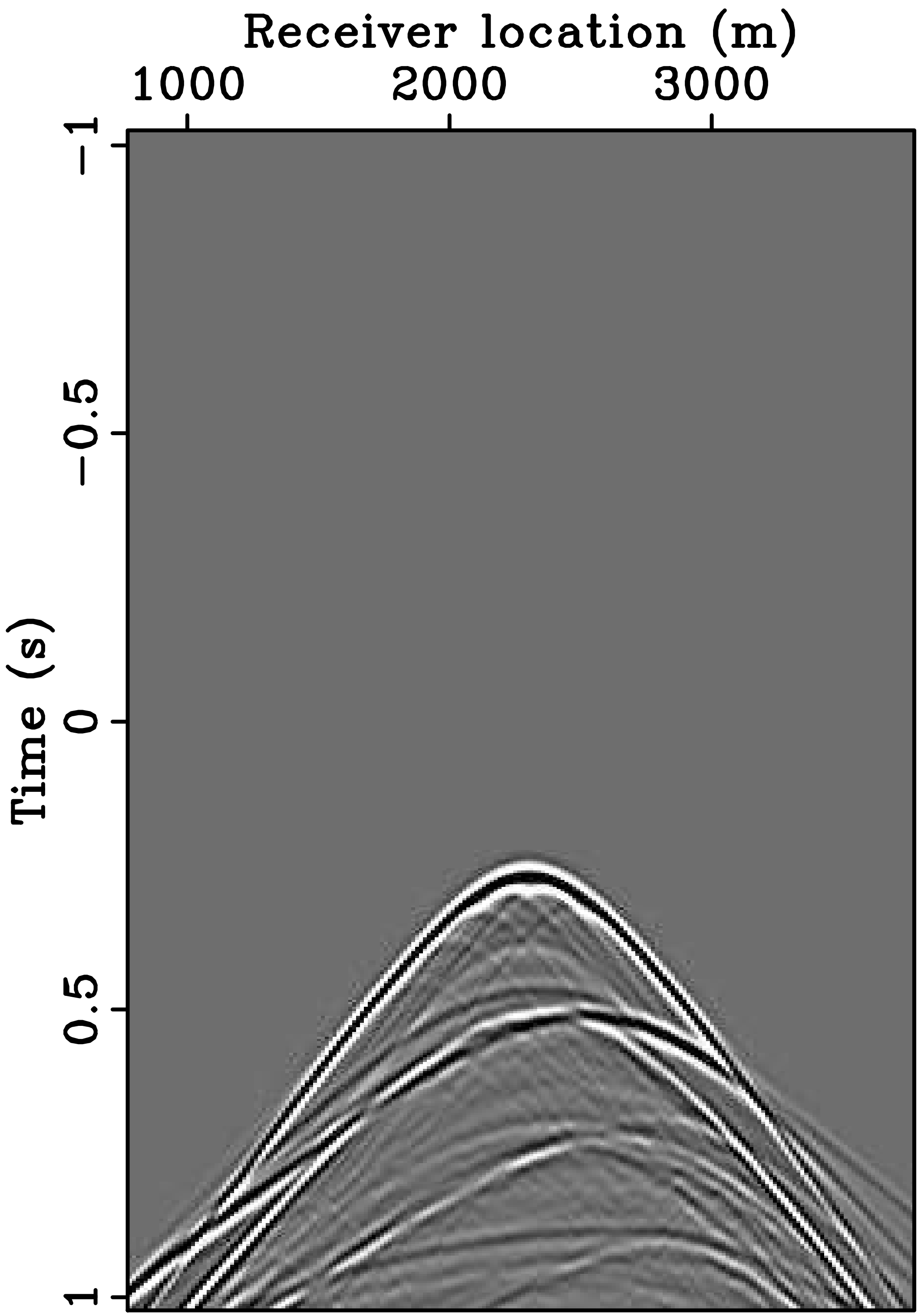
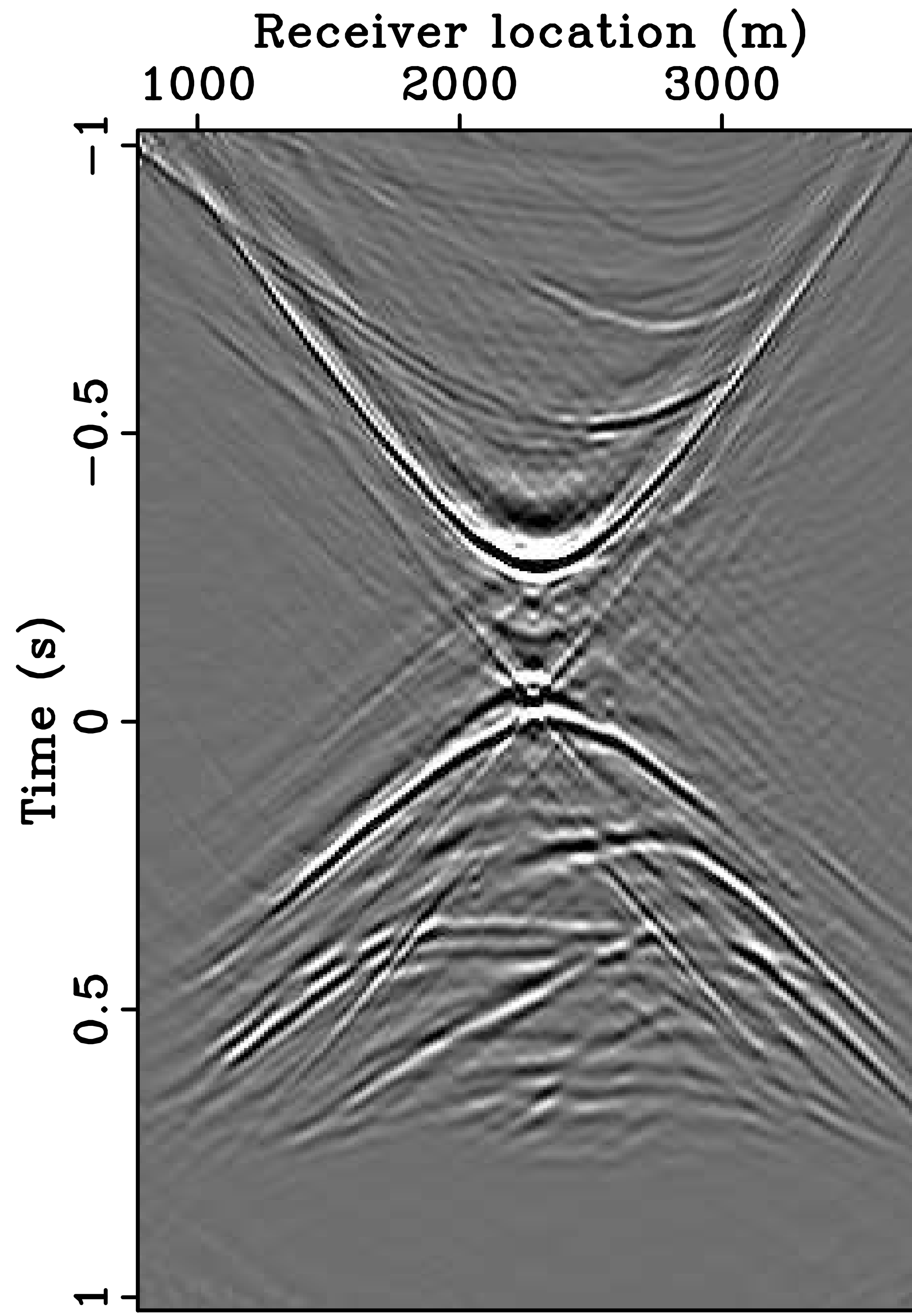
Curvelet-based seismic data inverse



Data



Data inverse

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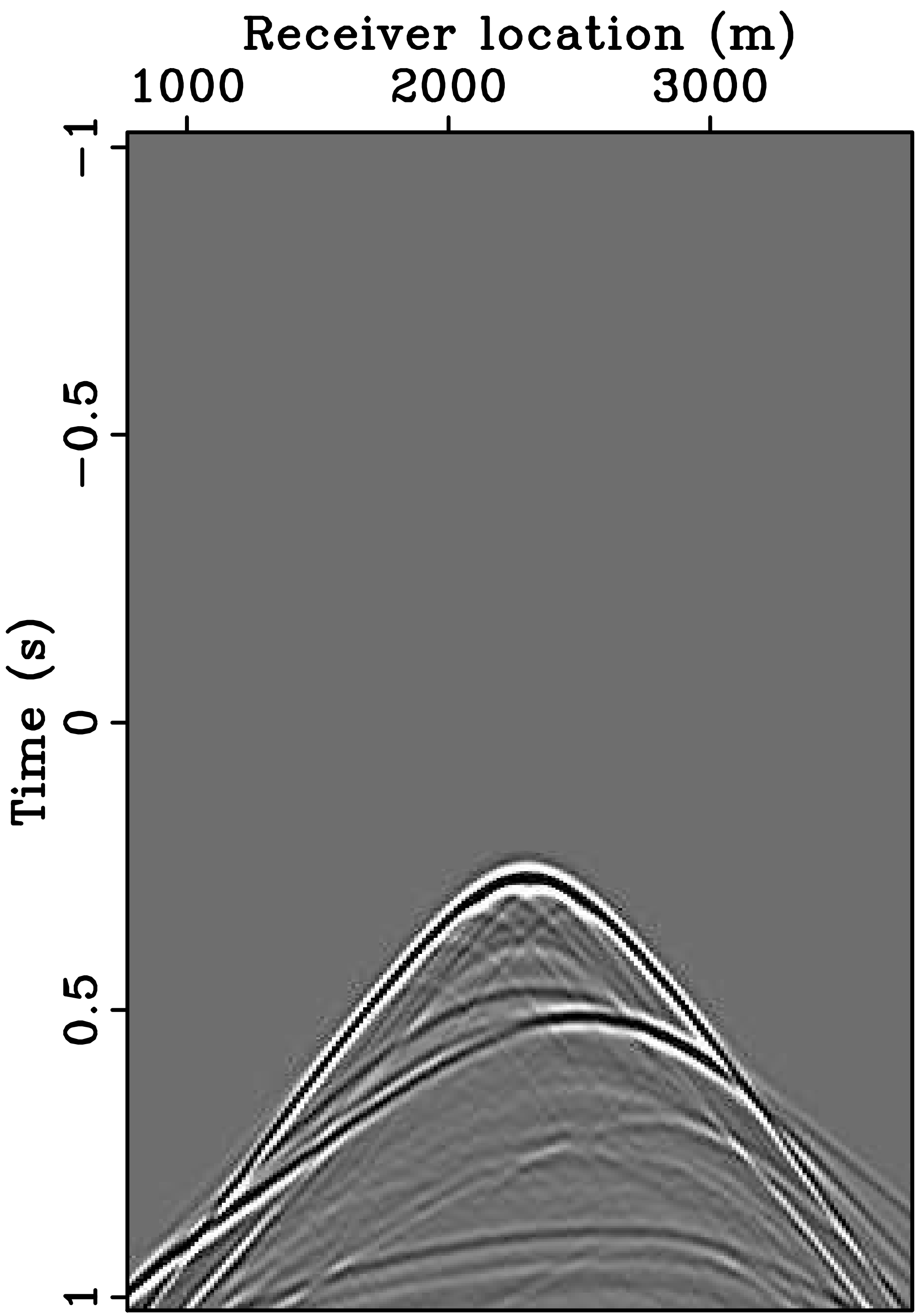
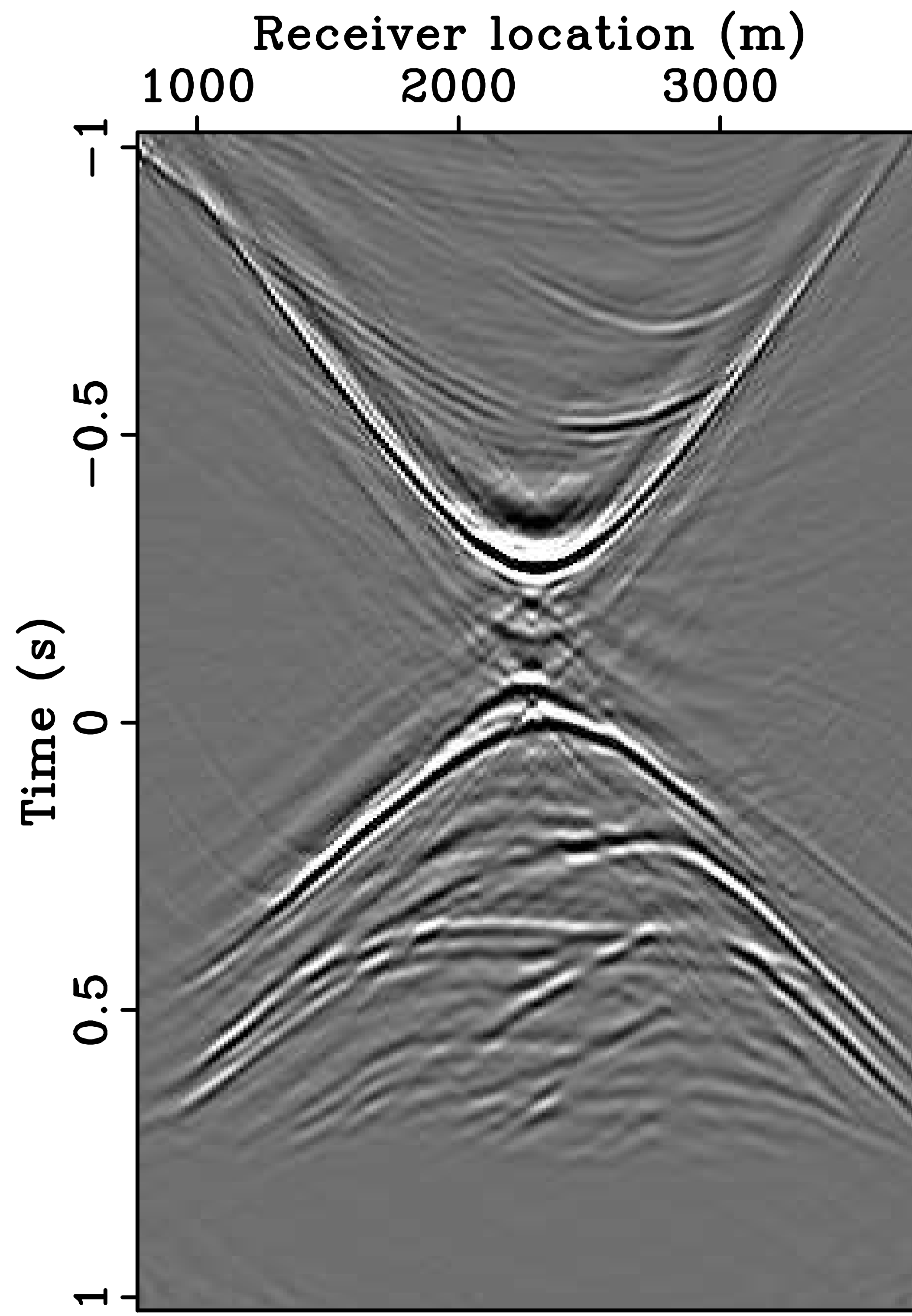
Curvelet-based seismic data inverse



Primaries



Primaries inverse

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Conclusions

CRSI

- recovers data by curvelet sparsity promotion
- uses *sparsity* as a *prior*

Focused CRSI

- incorporates additional *prior* information
- strips interaction with the surface <=> more *sparsity*
- improves the recovery and hence predicted multiples
- precursor of migration-based CRSI

Results of curvelet-based computation of the data inverse are encouraging.

Acknowledgments

The authors of CurveLab (Demanet, Ying, Candes, Donoho)

Dr. Verschuur for his synthetic data and the estimates for the primaries.

The SLIM team Sean Ross Ross, Cody Brown and Henryk Modzeleweski for developing SLIMPy: operator overloading in python

These results were created with Madagascar developed by Dr. Fomel.

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