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Random sampling: new insights into the reconstruction of coarsely-sampled wavefields

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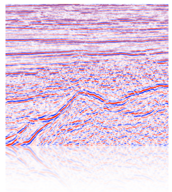

SEG International Exposition and 77th Annual Meeting
Regularization of Seismic Acquisition
Monday, September 24th, 2007

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Unleash the power of random sampling…

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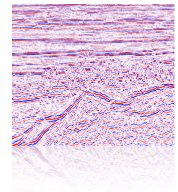

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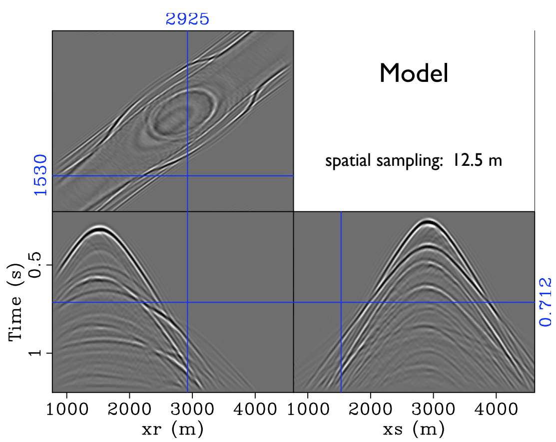
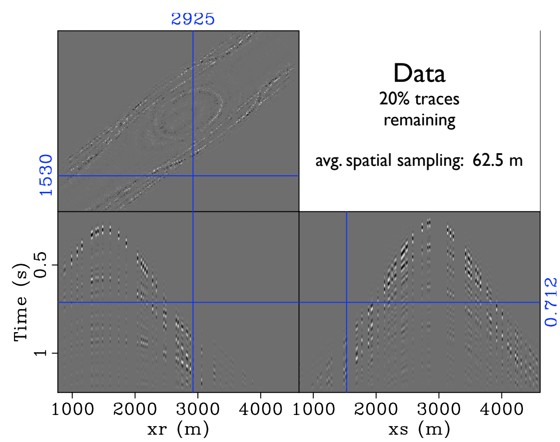


Model

spatial sampling: 12.5 m

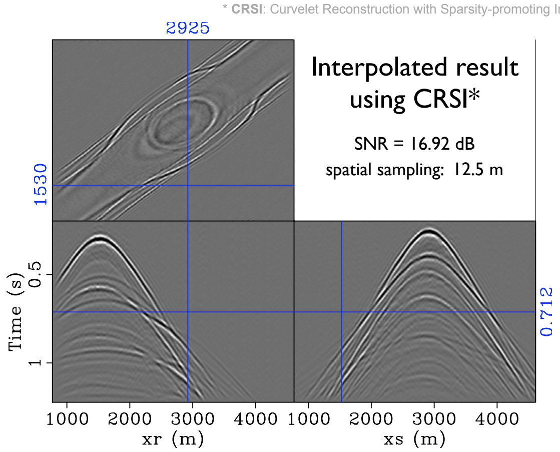


Data

20% traces remaining

avg. spatial sampling: 62.5 m

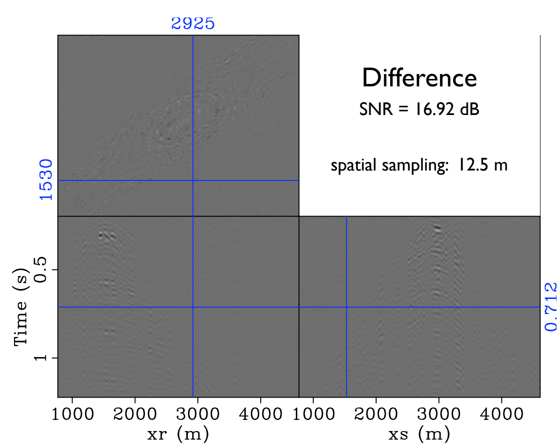
* **CRSI**: Curvelet Reconstruction with Sparsity-promoting Inversion



Interpolated result using CRSI*

SNR = 16.92 dB

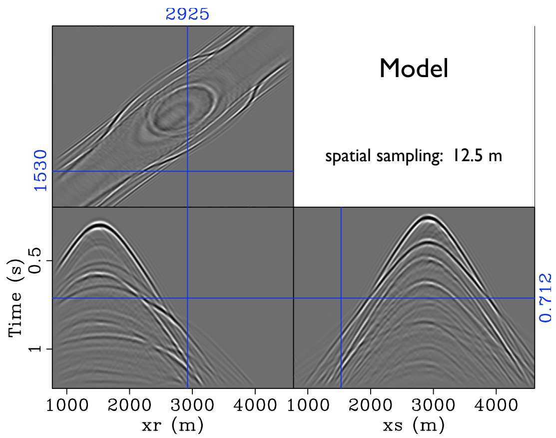
spatial sampling: 12.5 m

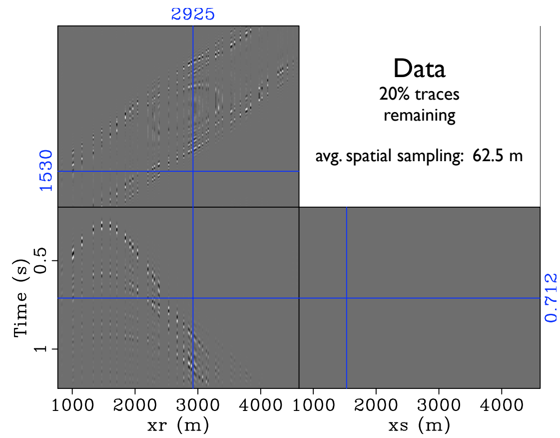


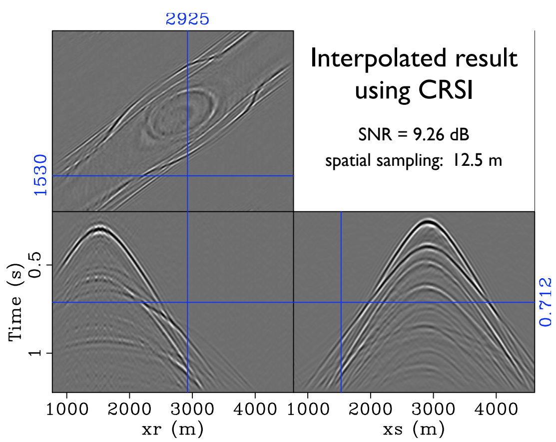
Difference

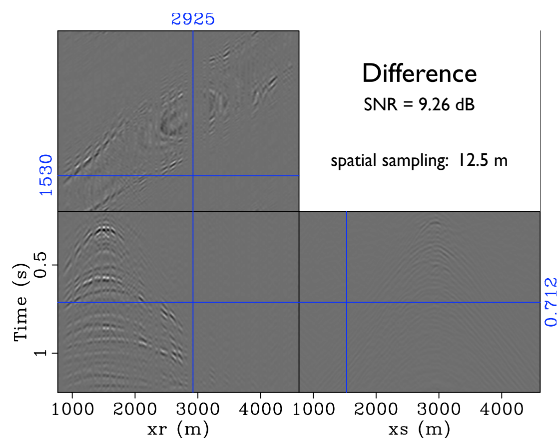
SNR = 16.92 dB

spatial sampling: 12.5 m









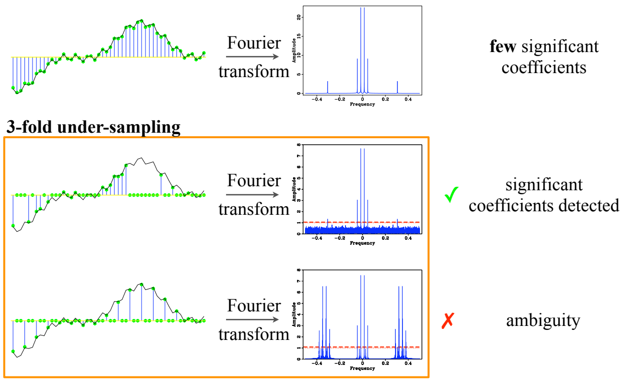
Motivation

- preliminary observations
  - 5-fold undersampled data in the time-source-receiver volume
    - missing traces at *irregular source-receiver locations*: **good reconstruction!**
    - missing traces at *irregular receiver locations*: **(much) less accurate reconstruction…**
- questions
  - what makes one case better than the other?
  - are acquisition irregularities really harmful to processing and imaging?
  - is there something to learn about favorable coarse acquisition geometries?
  - can the success of an interpolation method be (accurately) predicted based on the acquisition geometry?

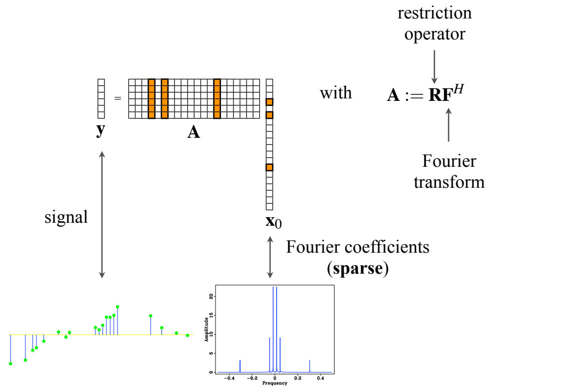
Previous art

- geophysical literature
  - Bednar, J. B., 1996, Coarse is coarse of course unless… The Leading Edge, **15**, 763 - 764.
  - Sun, Y., G. T. Schuster, and K. Sikorski, 1997, A quasi-Monte Carlo approach to 3D migration: Theory: Geophysics, **62**, 918 - 928.
  - Trad, D. O., and T. J. Ulrych, 1999, Radon transform: beyond aliasing with irregular sampling, 6th International Congress of the Brazilian Geophysical Society.
  - Abma, R. and N. Kabir, 2006, 3D interpolation of irregular data with a POCS algorithm: Geophysics, **71**, E91 – E97.
  - Zwartjes, P. M. and M. D. Sacchi, 2007, Fourier reconstruction of nonuniformly sampled, aliased data: Geophysics, **72**, V21–V32.
- other fields
  - Donoho, D. L., Y. Tsaig, I. Drori, and J.-L. Starck, 2006, Sparse solution of underdetermined linear equations by stagewise orthogonal matching pursuit: Technical report, Stanford Statistics Department. TR-2006-2.
  - Dippe, M. and E. Wold, 1992, Stochastic sampling: theory and application: Progress in computer graphics, **1**, 1 – 54.

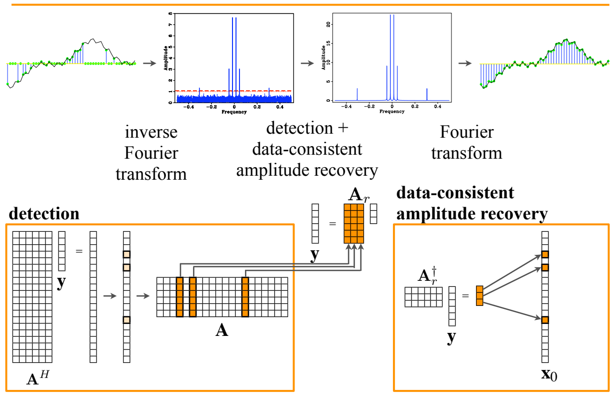
Simple example



Forward problem



Naive sparsity-promoting recovery



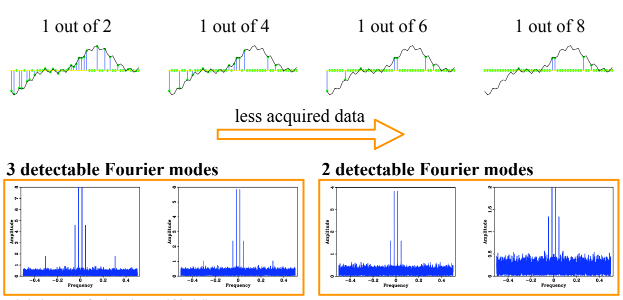
Observations

- 3-fold undersampling
  - random: significant coefficients **detected!**
  - regular: **ambiguity** between significant coefficients and aliases
- random undersampling
  - recovery ≈ denoising + amplitudes correction
  - (accurate) recovery of the coefficients above the “noise” level

[Donoho et al ‘06]

Undersampling “noise”

- “noise”
  - due to $\mathbf{A}^H\mathbf{A} \neq \mathbf{I}$
  - defined by $\mathbf{A}^H\mathbf{A}\mathbf{x}_0 - \alpha\mathbf{x}_0 = \mathbf{A}^H\mathbf{y} - \alpha\mathbf{x}_0$

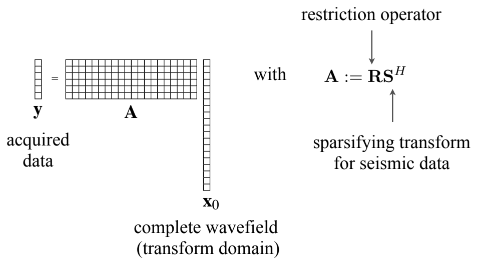


Further observations & comments

- random undersampling
  - size of the undersampling “noise” is a function of the undersampling factor
    - the less data acquired, the higher the “noise” level
  - for increasing undersampling
    - the largest coefficients remain detectable for the longest
  - for given undersampling
    - fixed number of recoverable coefficients
    - the more energy these significant coefficients carry compared to the total energy, the better the recovery ⇒ **need of an efficient representation for seismic data**

[Candès et al ‘05]
[Donoho ‘06]

Sparsity-promoting wavefield reconstruction



Interpolated data given by $\tilde{\mathbf{f}} = \mathbf{S}^H \tilde{\mathbf{x}}$ with

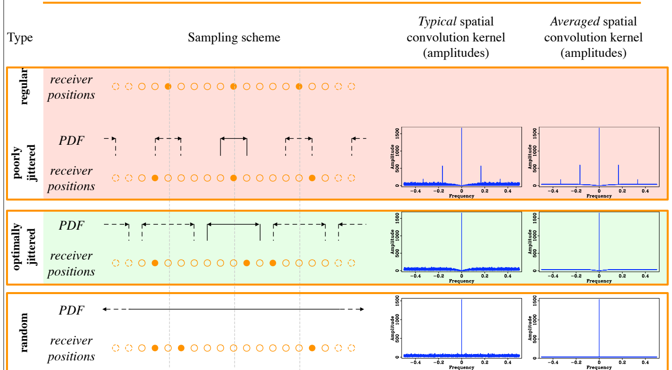
$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$

[Sacchi et al '98]
[Xu et al '05]
[Zwartjes and Sacchi '07]
[Herrmann and Hennenfent '07]

Nonlinear wavefield sampling

- *sparsifying transform*
  - typically **localized** in the time-space domain to handle the complexity of seismic data
    - curvelet transform (dyadic-parabolic partition of the f-k domain)
    - [windowed Fourier transform]
- *sampling scheme*
  - generates incoherent random undersampling "noise" in the sparsifying domain
  - do not create large gaps
    - because of the limited spatiotemporal extend of transform elements used for the reconstruction
- *sparsity-promoting solver*
  - requires few matrix-vector multiplications

Discrete random jittered undersampling

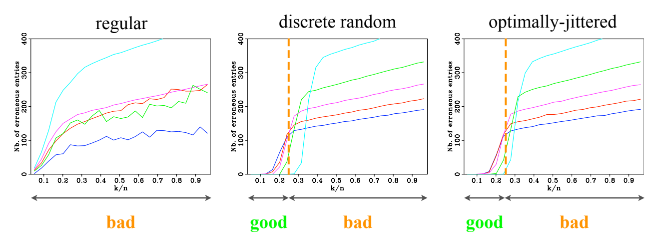


[Hennenfent and Herrmann '07]

Recovery from random vs. opt.-jittered data

- *k*-sparse signal of length *N* in the Fourier domain
- *n* observations in the time domain
  - $n = N/\gamma$ with the undersampling factor γ ranging from 2 to 6

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$



[Hennenfent and Herrmann '07]

Curvelet Recovery w Sparsity-promoting Inversion

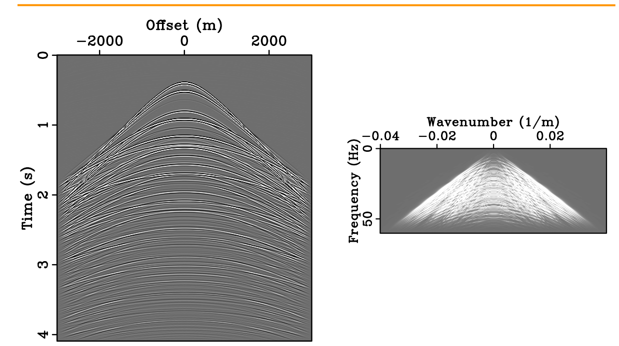
- Interpolated data given by $\tilde{\mathbf{f}} = \mathbf{C}^H \tilde{\mathbf{x}}$ with

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{R}\mathbf{C}^H\mathbf{x}$$

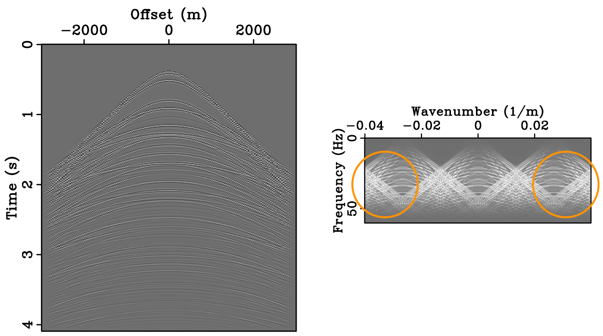
- sparsity-promoting solver
  - Iterative Soft Thresholding with cooling (ISTc)

[Herrmann and Hennenfent '07]

Model



Regular 3-fold undersampling



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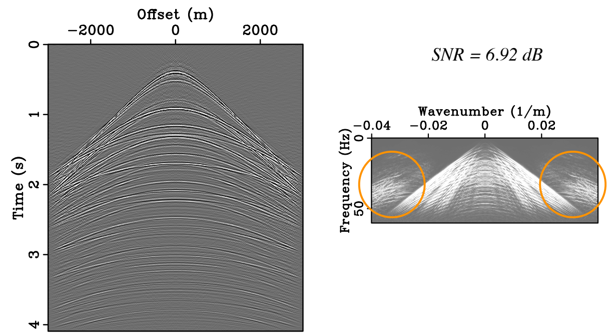
CRSI from regular 3-fold undersampling



SNR = 6.92 dB

$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

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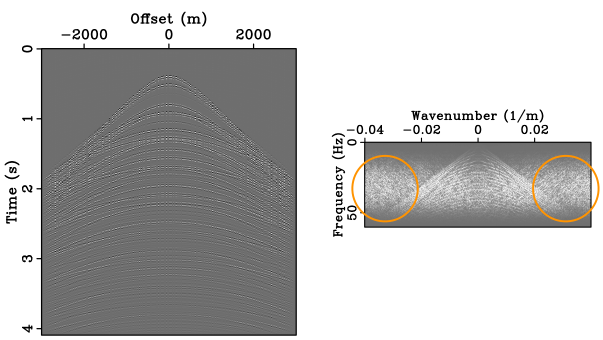
Optimally-jittered 3-fold undersampling



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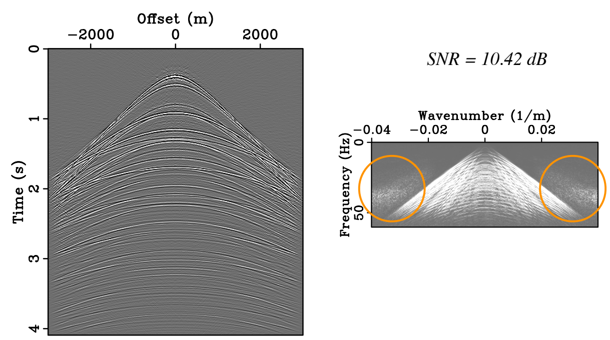
CRSI from opt.-jittered 3-fold undersampling



SNR = 10.42 dB

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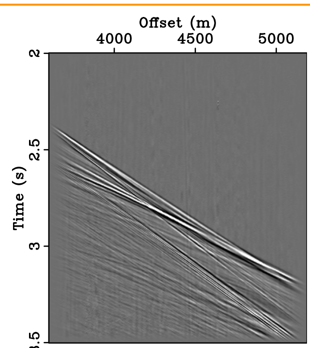
Model



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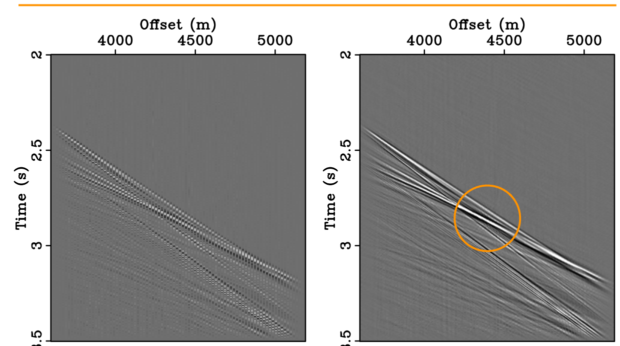
Regular 3-fold undersampling



SNR = 12.98 dB

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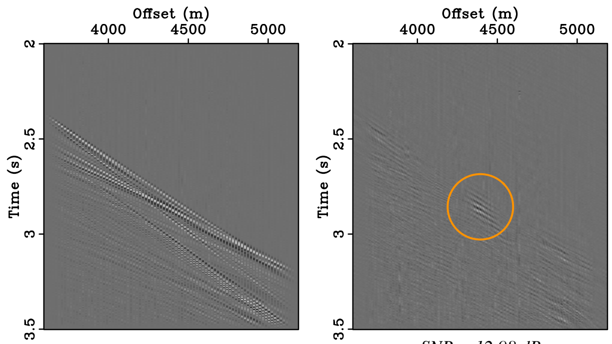
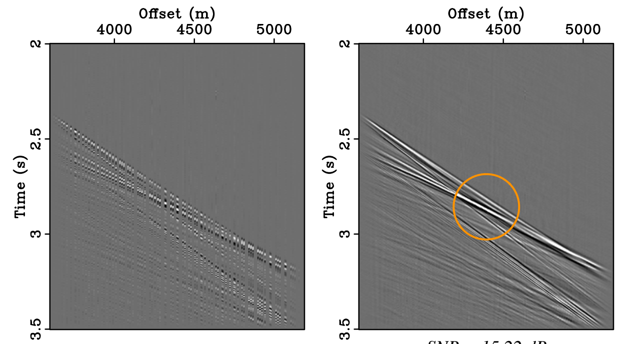
Regular 3-fold undersampling



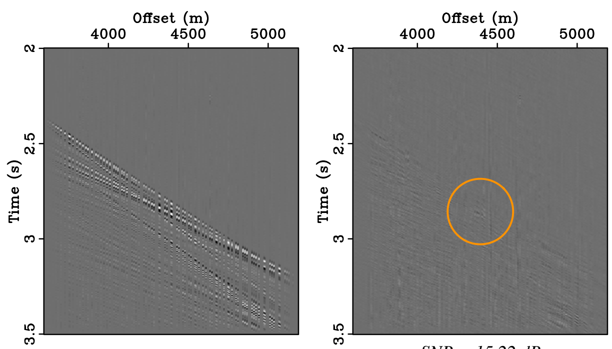
SNR = 12.98 dB

Optimally-jittered 3-fold undersampling



SNR = 15.22 dB

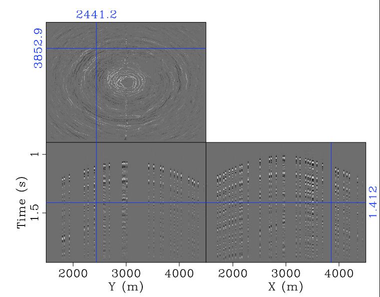
Optimally-jittered 3-fold undersampling



SNR = 15.22 dB

Is jittered undersampling practical?

- field data
  - typ. irregularly sampled
  - no large gaps when possible



Conclusions

- *sparsity* is a powerful property that offers striking benefits for signal reconstruction BUT it is not enough
- in the sparsifying domain, *interpolation is a denoising problem*
  - regular undersampling: **harmful coherent undersampling "noise"**, i.e., aliases
  - random & optimally-jittered undersamplings: **harmless incoherent random undersampling "noise"**
- nonlinear wavefield sampling
  - sparsifying transform: **curvelet transform**
  - coarse sampling scheme: **optimally-jittered undersampling**
  - sparsity-promoting solver: **iterative soft thresholding with cooling**

Acknowledgments

- SLIM team members
  - S. Ross Ross, H. Modzelewski, and C. Brown for SLIMpy (slim.eos.ubc.ca/SLIMpy)
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