Application of stable signal recovery to seismic data interpolation

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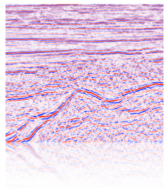

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SPNA 2: topics in seismic processing I
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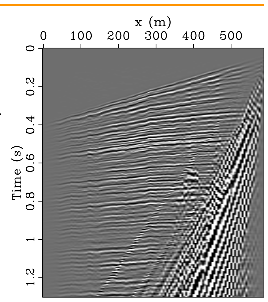
Motivation

- improve
  - multiple prediction & removal
  - aliased ground roll removal
  - imaging
- reduce acquisition cost & time
  - acquire less data

Approach

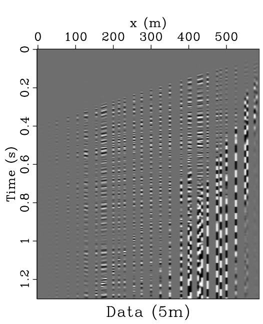
- exploit geometry of seismic data
  - **high dimensional**
    - typically 5D - i.e. time × source location × receiver location
  - very **strong geometrical structure** (i.e. wavefronts)
- provide **sampling criteria** for seismic data
  - how well can one expect to recover seismic data given an acquisition geometry? (interpolation of vintage survey)
  - what is the 'optimal' acquisition geometry in order to recover seismic data within a given accuracy? (sparse sampling scheme)

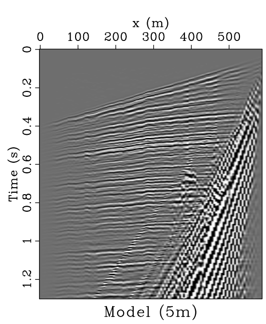


Agenda

- seismic data interpolation problem
  - forward & "classical" inverse problem
- Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)
  - compressibility as a prior
  - curvelets
- synthetic and real data examples

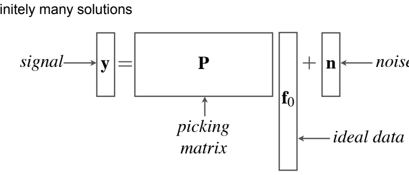
Seismic data interpolation problem





Forward & "classical" inverse problem

- (severely) underdetermined system of linear equations
  - infinitely many solutions



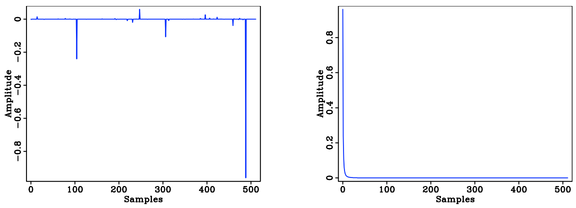
- among "classical" approaches
  - minimize energy (i.e. quadratic constraint)

$$\tilde{\mathbf{f}} = \arg\min_{\mathbf{f}} \frac{1}{2} \underbrace{\|\mathbf{y} - \mathbf{P}\mathbf{f}\|_2^2}_{\text{data misfit}} + \lambda \underbrace{\|\mathbf{L}\mathbf{f}\|_2^2}_{\text{energy constraint}}$$

Compressibility as a prior

- what is compressibility?
  - generalization of **sparsity**
  - **x** is compressible if its sorted entries decay sufficiently fast
  - compressible signals have small l_1 norm

$$\|\mathbf{x}\|_1 := \sum_i |\mathbf{x}|_{(i)}$$



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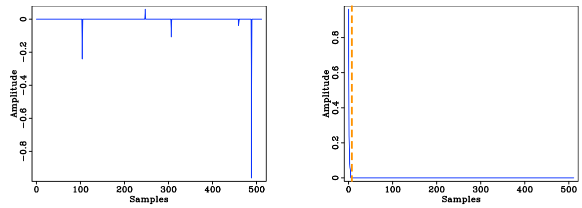


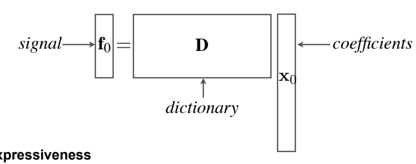
Compressibility as a prior

- why use sparsity/compressibility?
  - powerful property (i.e. extra piece of information about the signal)

Idea of promoting sparsity for geophysical problems is commonly attributed to Claerbout and Muir in 1973 and was further developed e.g. by Oldenburg who proposed to deconvolve seismic traces for reflectivity as sparse spike trains.

Compressible representations

- seek
  - **simplicity**
    - signal $\mathbf{f}_0$ is built as a linear combination of **few atoms** from dictionary **D**

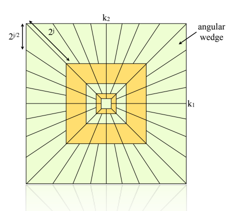


  - **expressiveness**
    - each selected atom significantly contributes to the construction of $\mathbf{f}_0$ (i.e. energy of the signal $\mathbf{f}_0$ is concentrated in **few significant coefficients**)

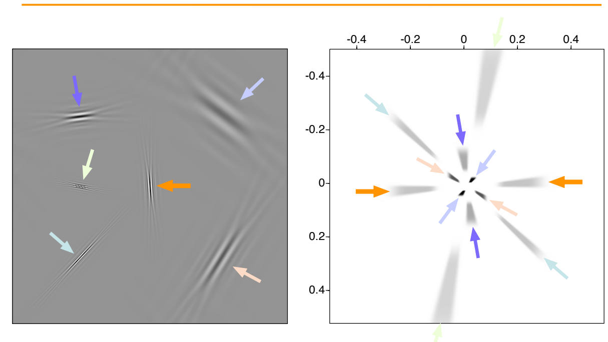
Representations for seismic data

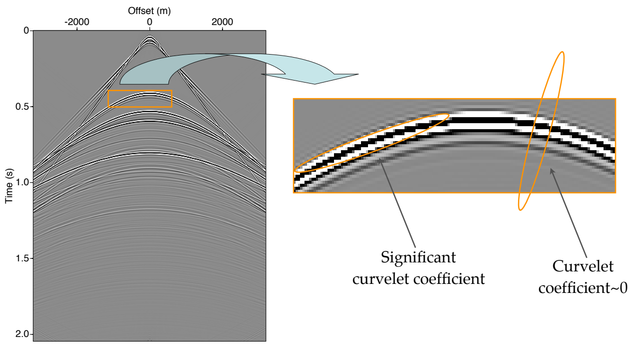
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

- curvelet transform
  - **multi-scale:** tiling of the FK domain into dyadic coronae
  - **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
  - **anisotropic:** parabolic scaling principle
  - **local**



2D curvelets





Sparsity-promoting inversion*

- reformulation of the problem

$$\text{signal} \longrightarrow \mathbf{y} = \mathbf{PC}^H \, \mathbf{x}_0 + \mathbf{n} \longleftarrow \text{noise}$$

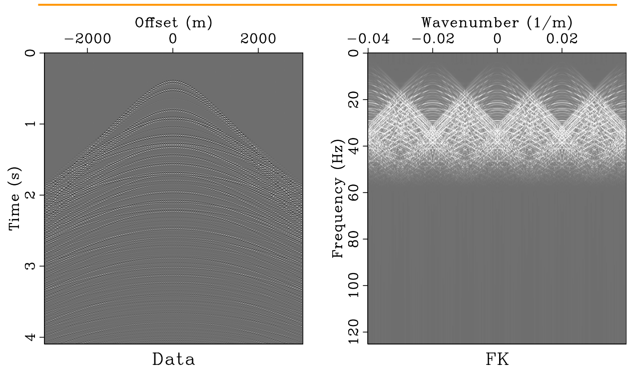
curvelet representation of ideal data ($\mathbf{x}_0$)

- Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)
  - look for the **sparsest/most compressible, physical** solution ← KEY POINT OF THE RECOVERY

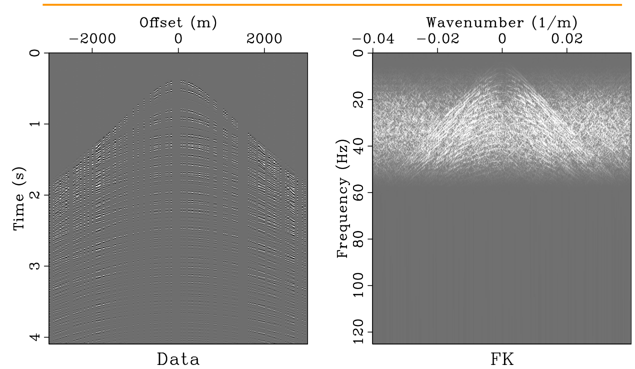
$$(P_1)\begin{cases} \tilde{\mathbf{x}} = \arg\overbrace{\min_{\mathbf{x}} \|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{y} - \mathbf{PC}^H\mathbf{x}\|_2}^{\text{data misfit}} \leq \varepsilon \\ \tilde{\mathbf{f}} = \mathbf{C}^H\tilde{\mathbf{x}} \end{cases}$$

* inspired by *Stable Signal Recovery (SSR) theory* by E. Candès, J. Romberg, T. Tao & *Fourier Reconstruction with Sparse Inversion (FRSI)* by P. Zwartjes

Sampling & aliasing

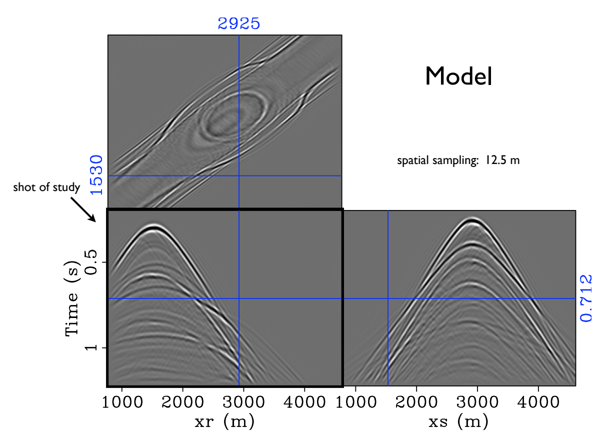


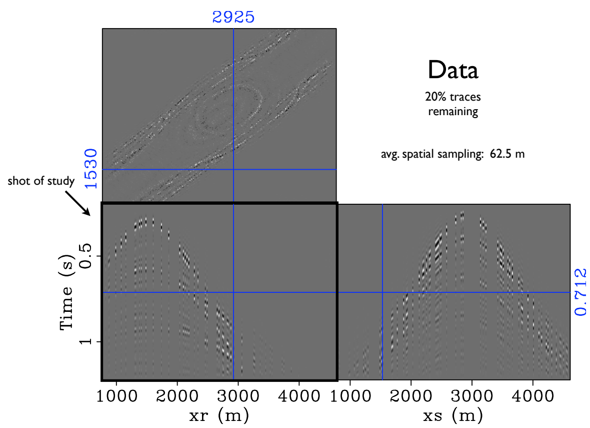
From aliasing to noise



Examples

- synthetic: Delphi dataset
  - uplift from image to volume (time × source location × receiver location) interpolation
  - influence of missing data structure
- real: ExxonMobil test dataset
  - challenging land data
    - ground roll
      - slow (i.e. weak minimum velocity constraint)
      - strong (~ 30 dB stronger than signal)
    - underlying de-aliasing problem

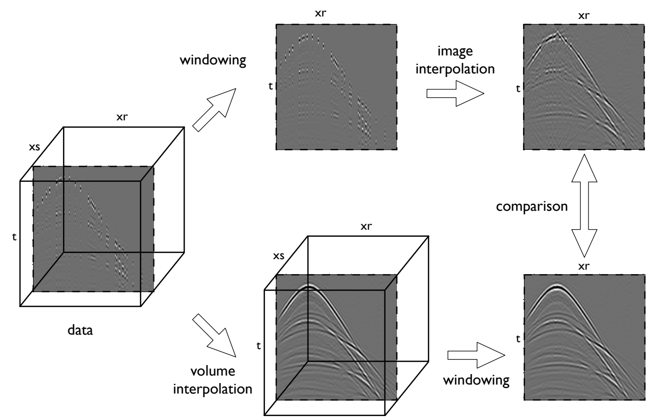


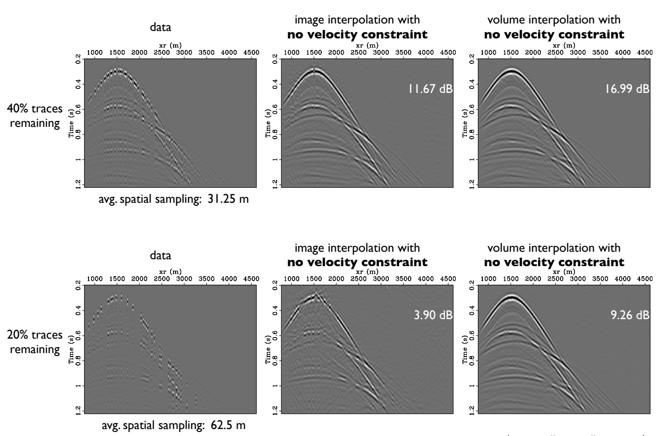


Data

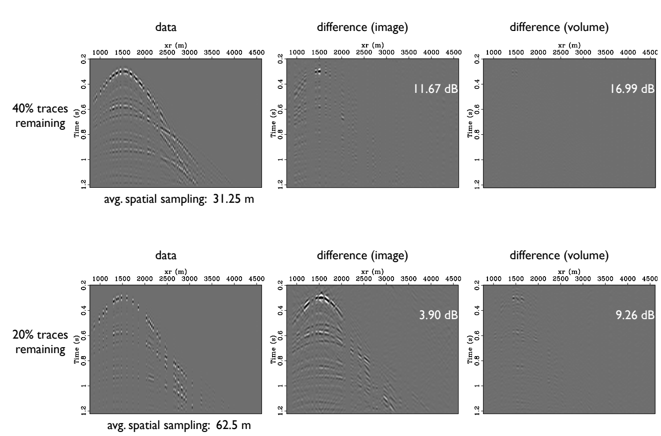
20% traces remaining

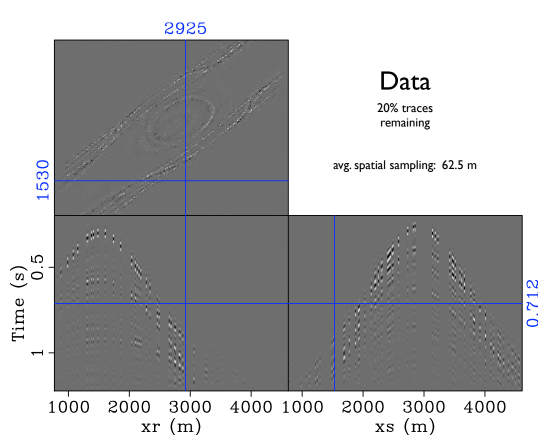
avg. spatial sampling: 62.5 m





$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\text{model}\|_2}{\|\text{reconstruction error}\|_2}\right)$$





Data

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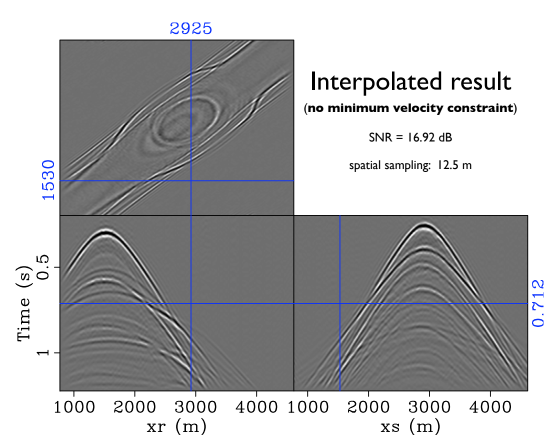


Interpolated result

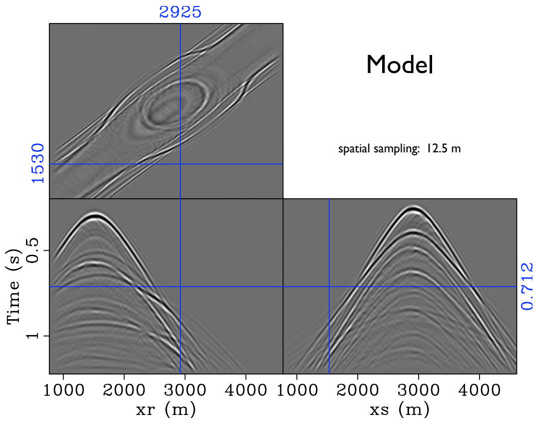
(no minimum velocity constraint)

SNR = 16.92 dB

spatial sampling: 12.5 m

Model

spatial sampling: 12.5 m

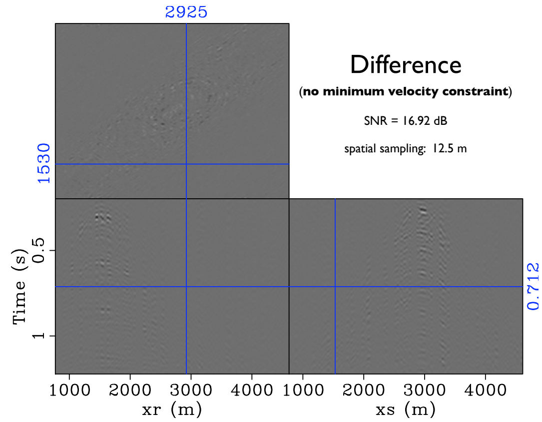


Difference

(no minimum velocity constraint)

SNR = 16.92 dB

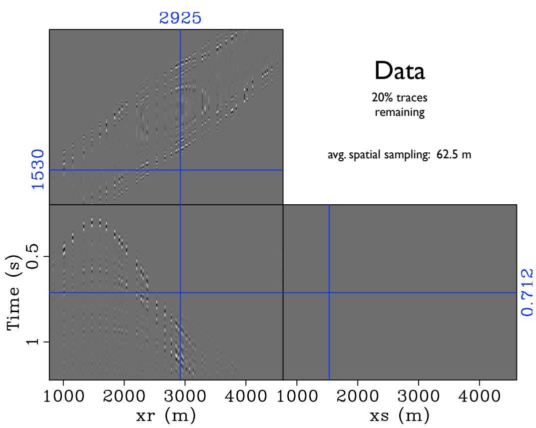
spatial sampling: 12.5 m



Data

20% traces remaining

avg. spatial sampling: 62.5 m

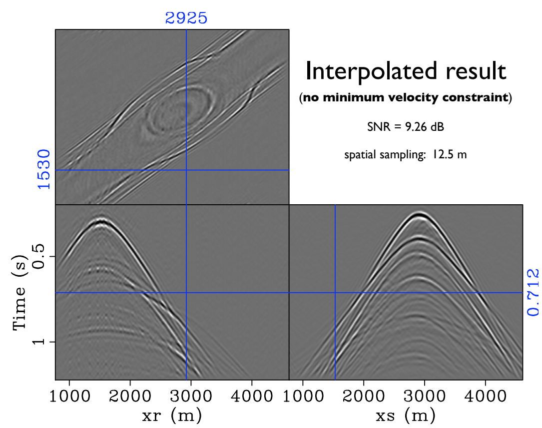


Interpolated result

(no minimum velocity constraint)

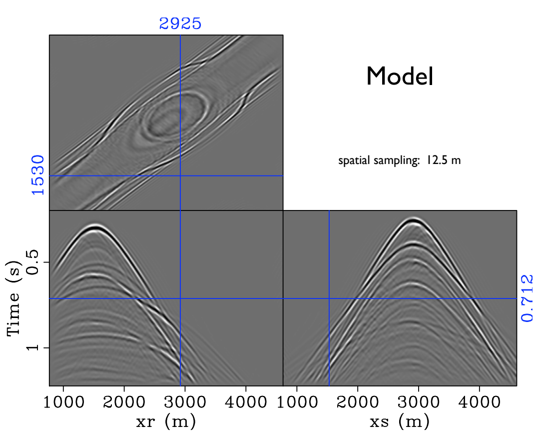
SNR = 9.26 dB

spatial sampling: 12.5 m



Model

spatial sampling: 12.5 m

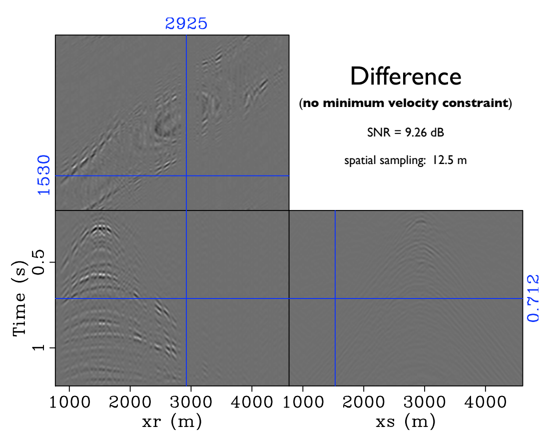


Difference

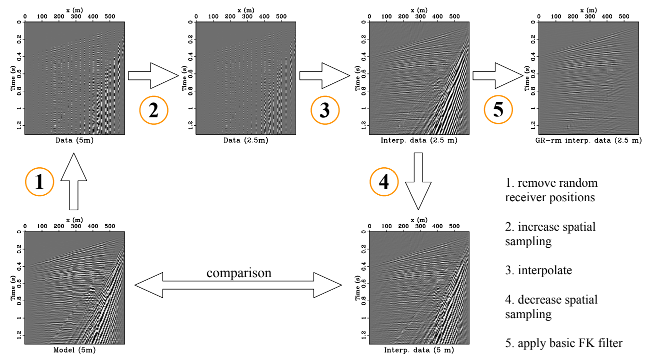
(no minimum velocity constraint)

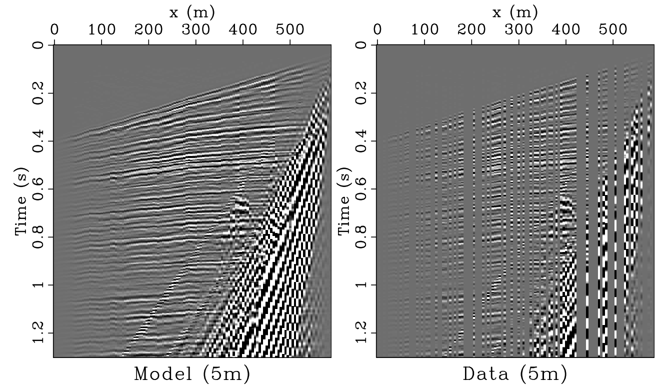
SNR = 9.26 dB

spatial sampling: 12.5 m

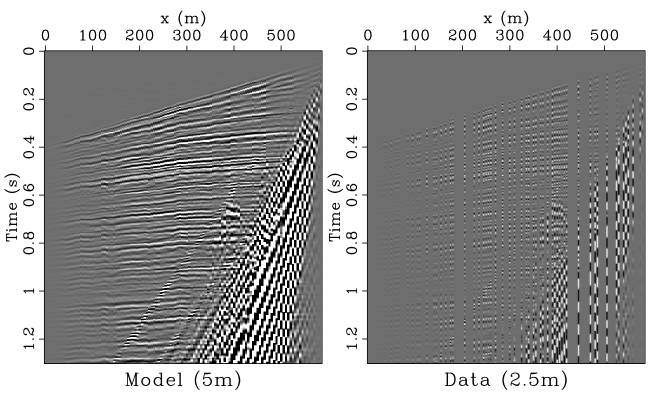


Experiment

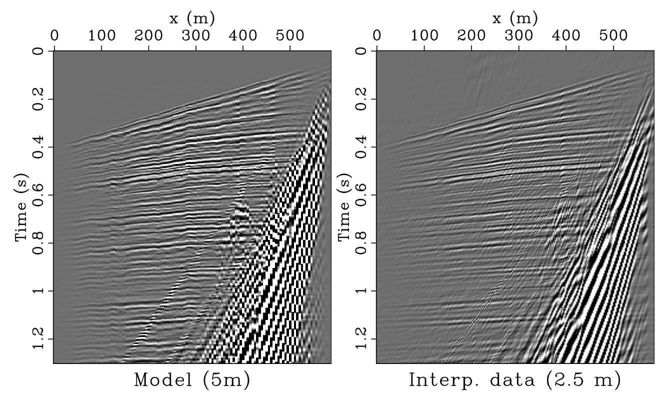


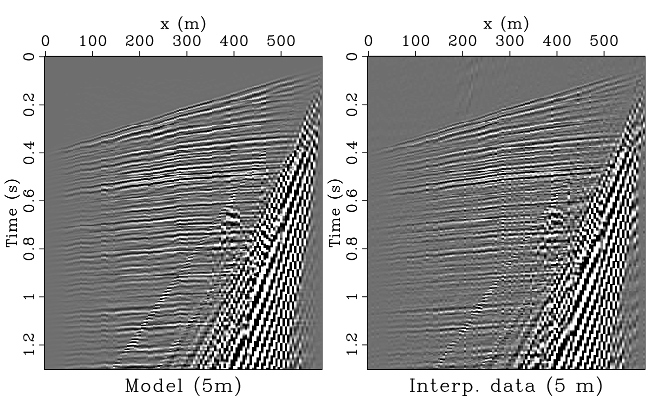


avg. spatial sampling: 10 m

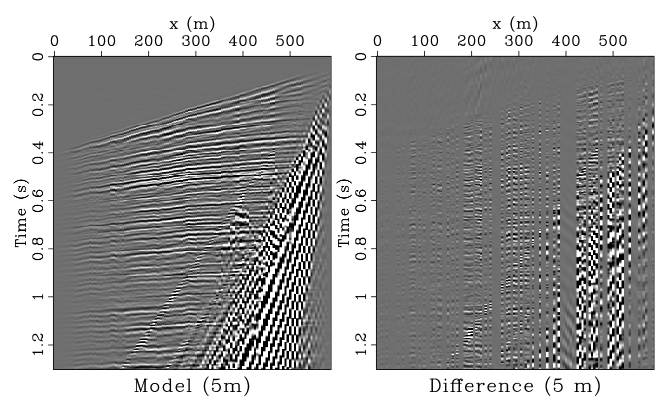


avg. spatial sampling: 10 m





SNR = 7.79 dB



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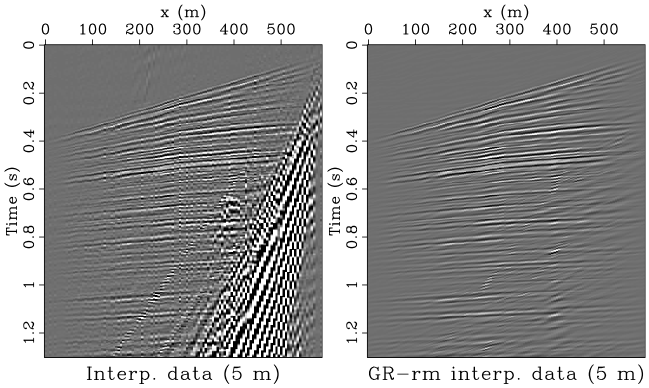
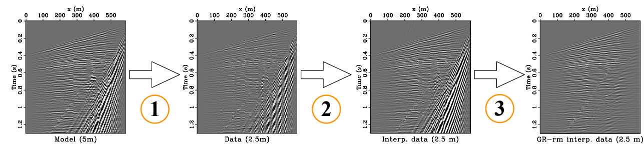
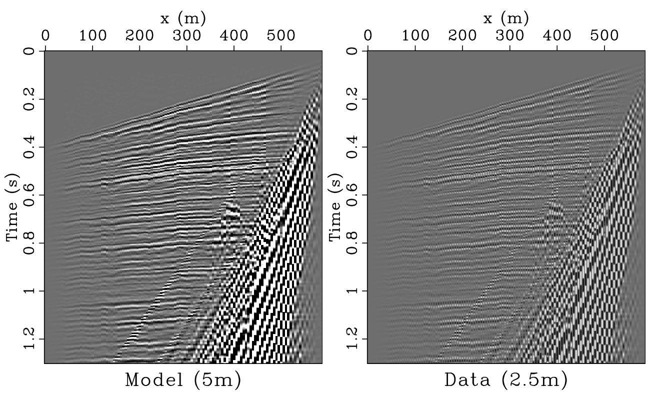


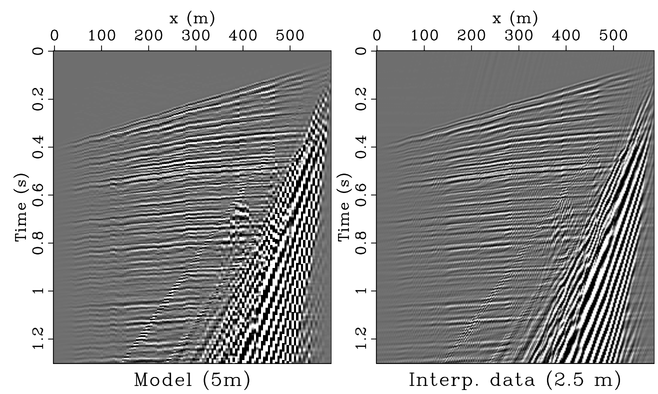
(basic FK filtering)

Experiment



1. increase spatial sampling
2. interpolate
3. apply basic FK filter





Conclusions

- **curvelets** exploit the very strong geometrical structure of seismic data
- **compressibility** is a powerful property (i.e. extra piece of information about the signal) that offers striking benefits
- **randomness** in the structure of missing data significantly helps recovery
- **Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)** performs well
  - synthetic: Delphi dataset
    - from 62.5 m to 12.5 m
    - significant uplift from image to volume interpolation
    - significant influence of the structure of missing data
  - real: ExxonMobil test dataset
    - from 10 m to 2.5 m
    - CRSI interpolates both signal & noise (i.e. ground roll)

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