SLIM

Seismic Laboratory for Imaging and Modeling

Sparseness-Constrained Data Continuation With Frames: Applications To Missing Traces & Aliased 2D/3D Signals

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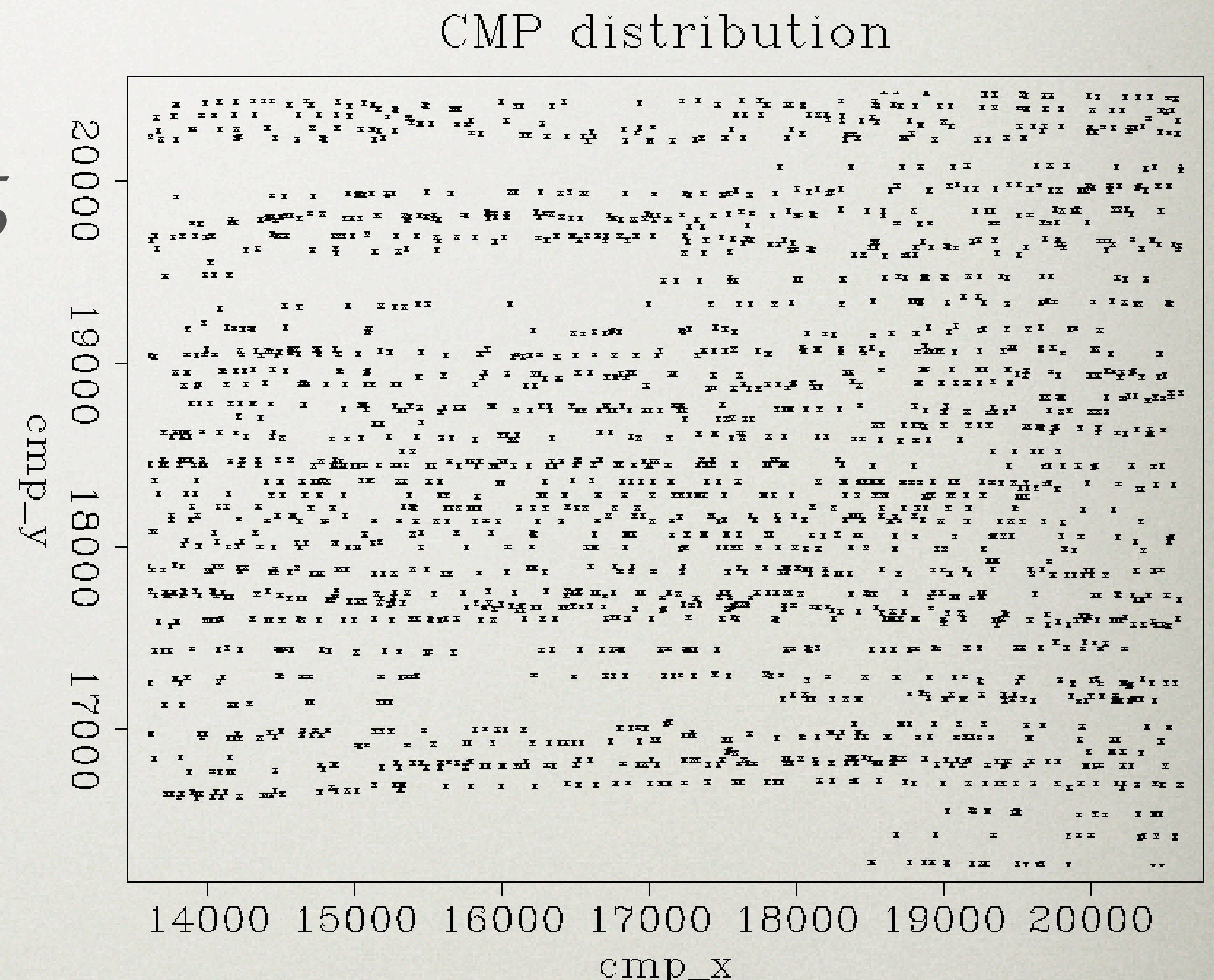
Introduction

- Seismic data typically irregularly (and sparsely) sampled along spatial coordinates

- Most multi-trace processing algorithms can **NOT** handle irregular sampled (and aliased) data



Courtesy S. Fomel

- Simple approaches

  linear interpolation

  binning

- Data mapping methods

  offset continuation [Stovas & Fomel, 1993]

  shot continuation [Schwab, 1993]

- Transform-based methods

  (nonuniform) discrete Fourier transform [Duijndam & Schonewille, 1999; Sacchi, 1996]

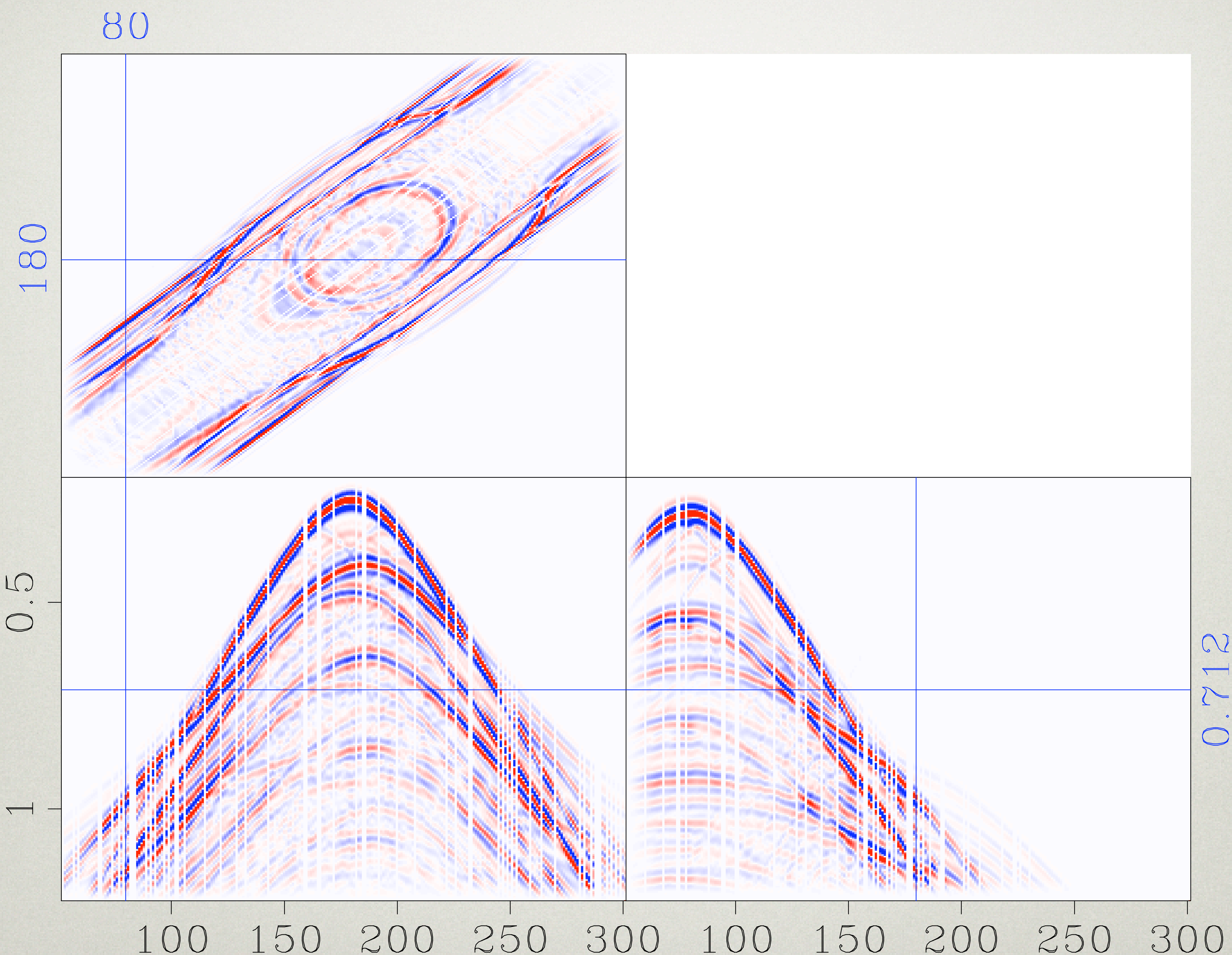
  Radon [Thorson & Claerbout, 1985; Hampson, 1986]

Objectives

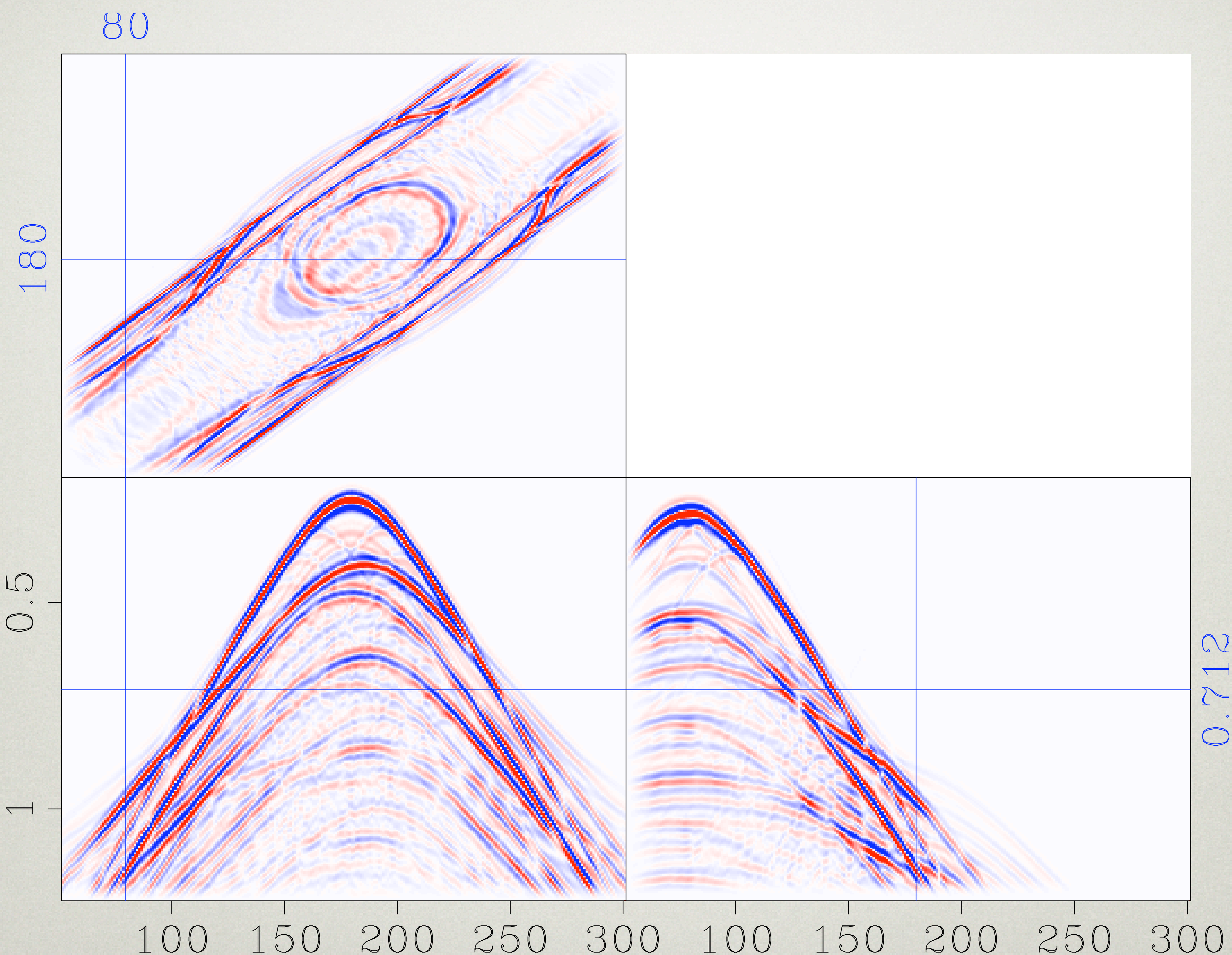
Focus: missing traces in otherwise regularly sampled data

Desired approach:

- does **NOT** require information on the velocity model or local dips,
- explores continuity along 2D/3D wavefronts.



data with missing traces



original data

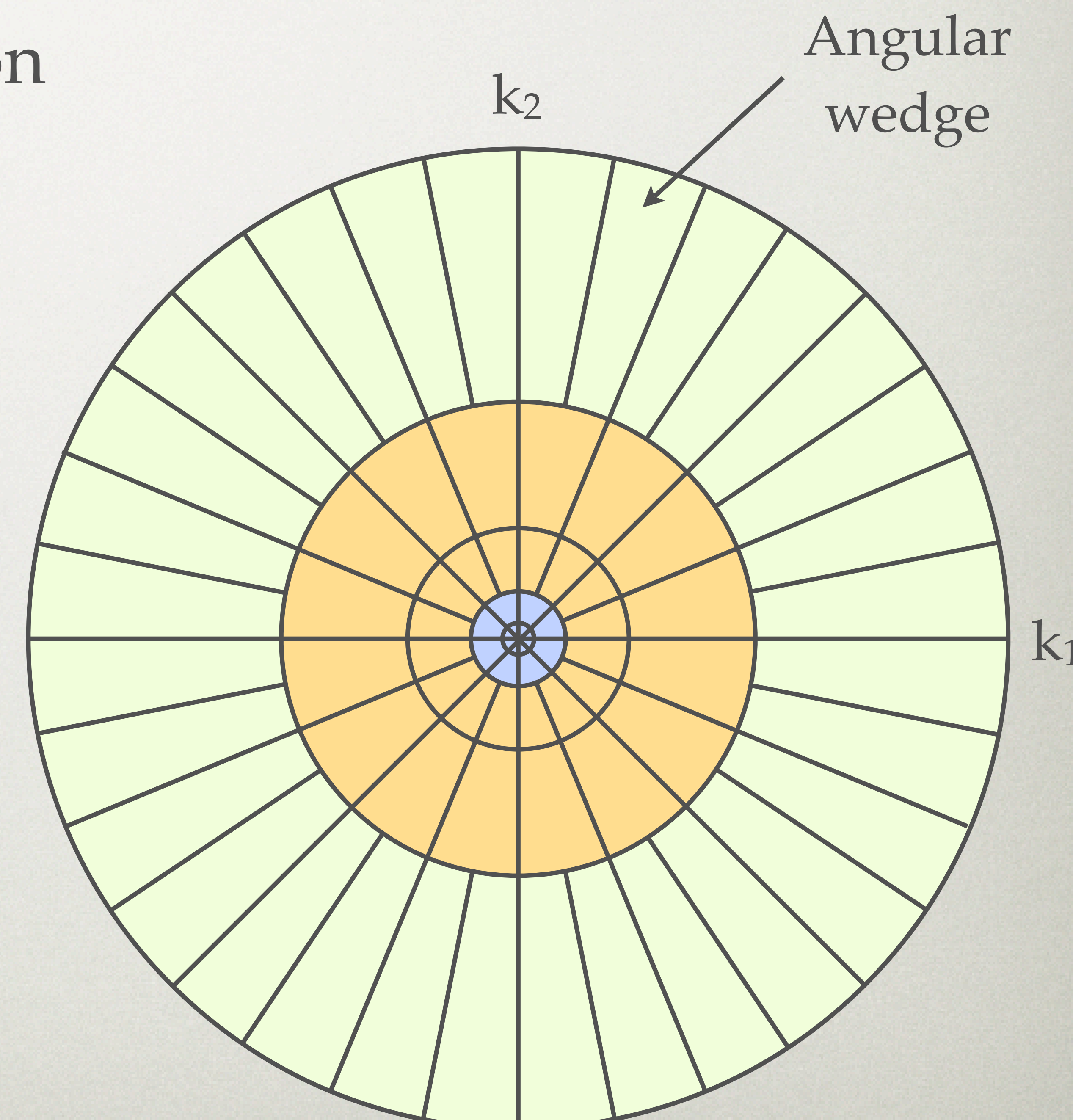
Organization

- Overview on the curvelet transform
- Seismic data regularization
- Examples
- Conclusions

Curvelet Transform

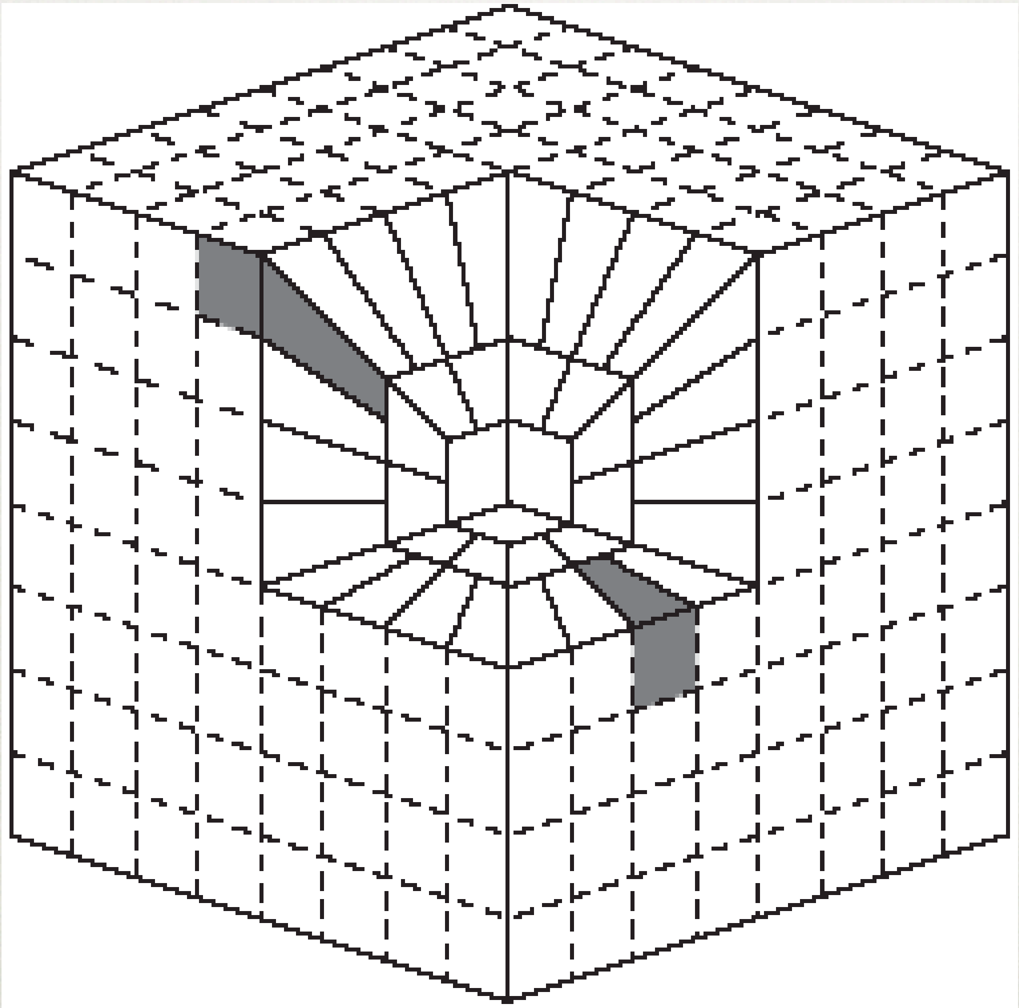
Tight frame decomposition

- ***multi-scale***
  tilling of the Fourier plane into second dyadic coronae
- ***multi-directional***
  coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- ***anisotropic***
  parabolic scaling principle
- ***local***



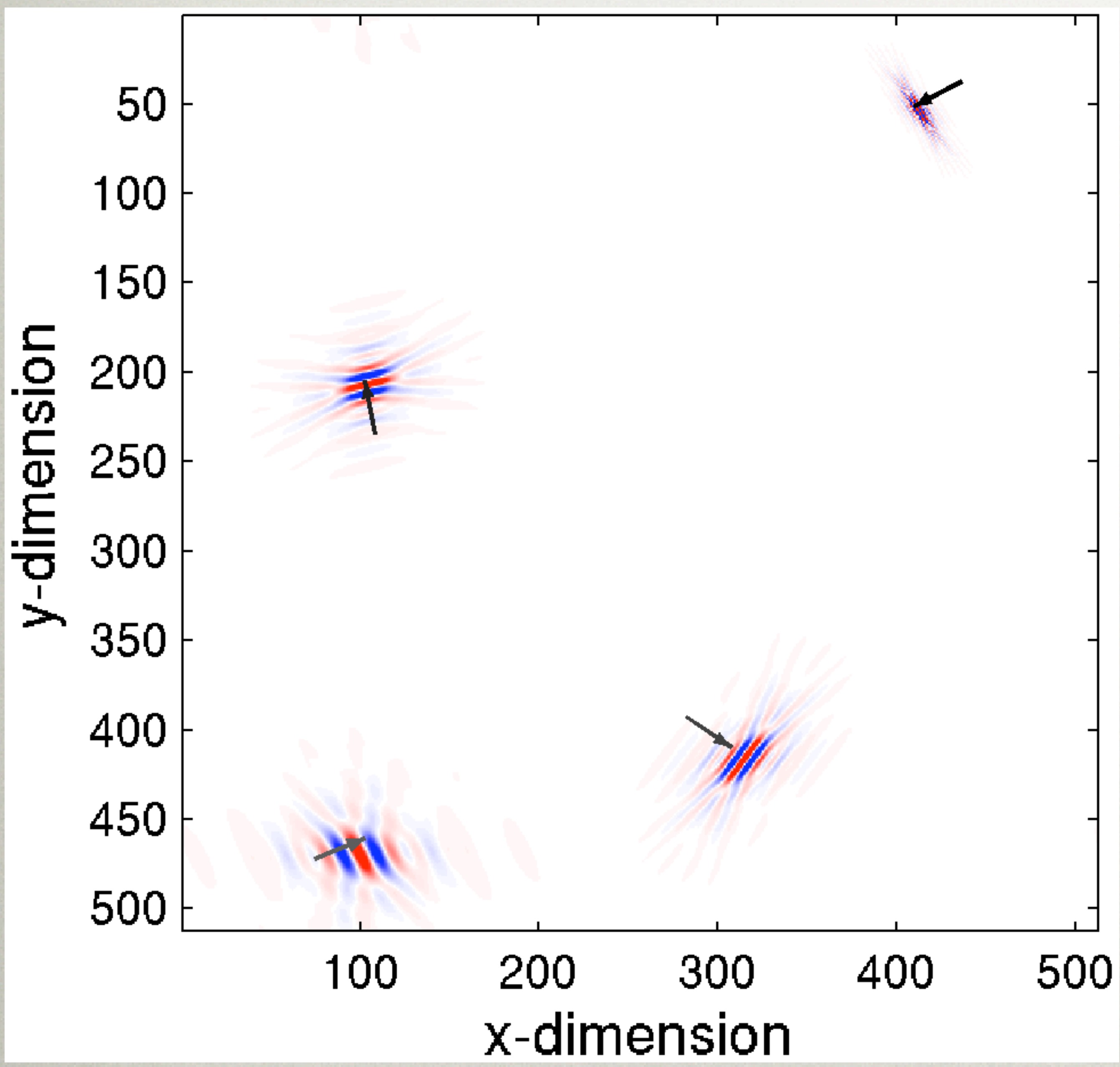
[Candès & Donoho, 2002]

3D Curvelets

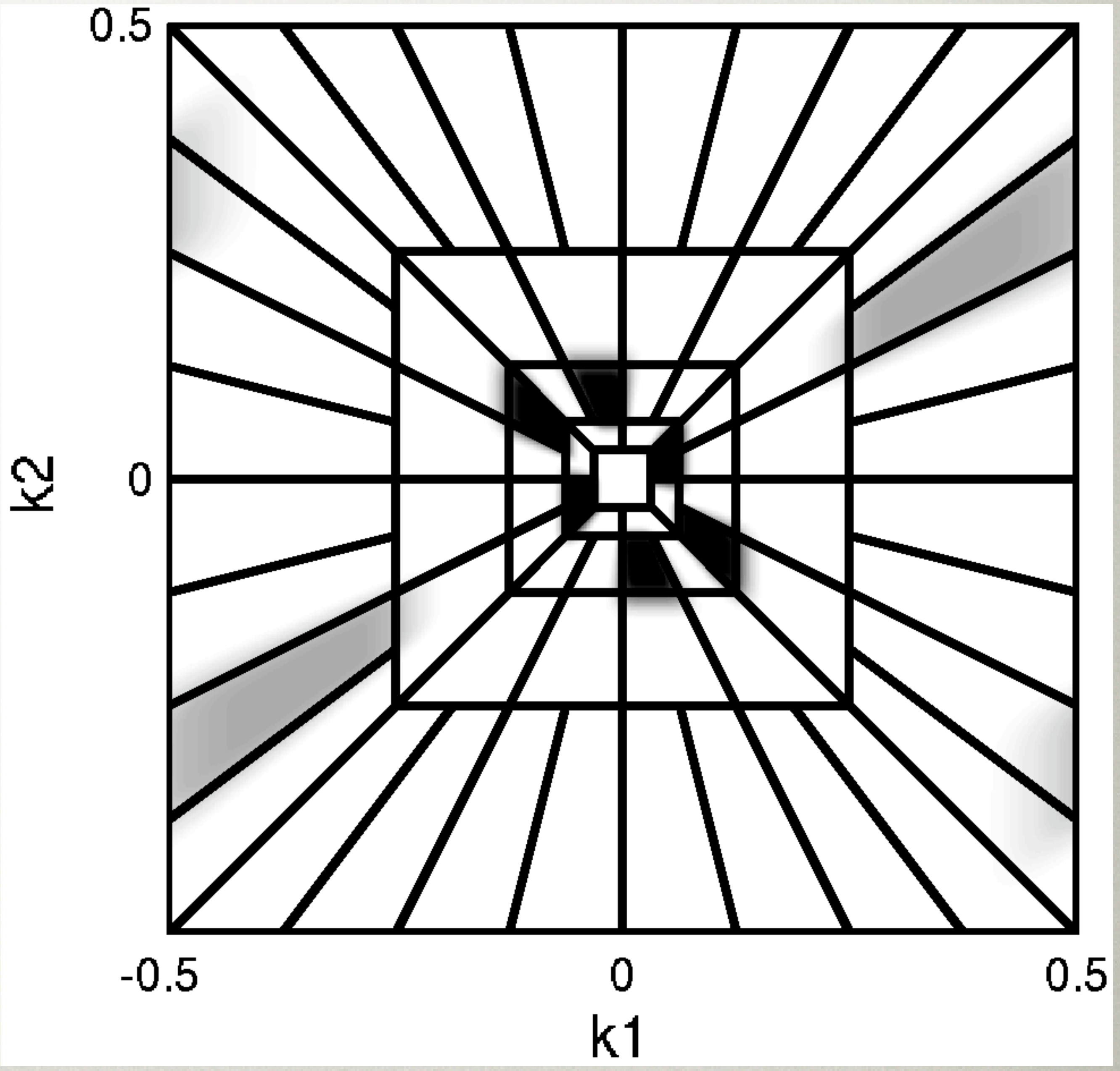


Courtesy [Candès et al., 2005; Ying et al., 2005]

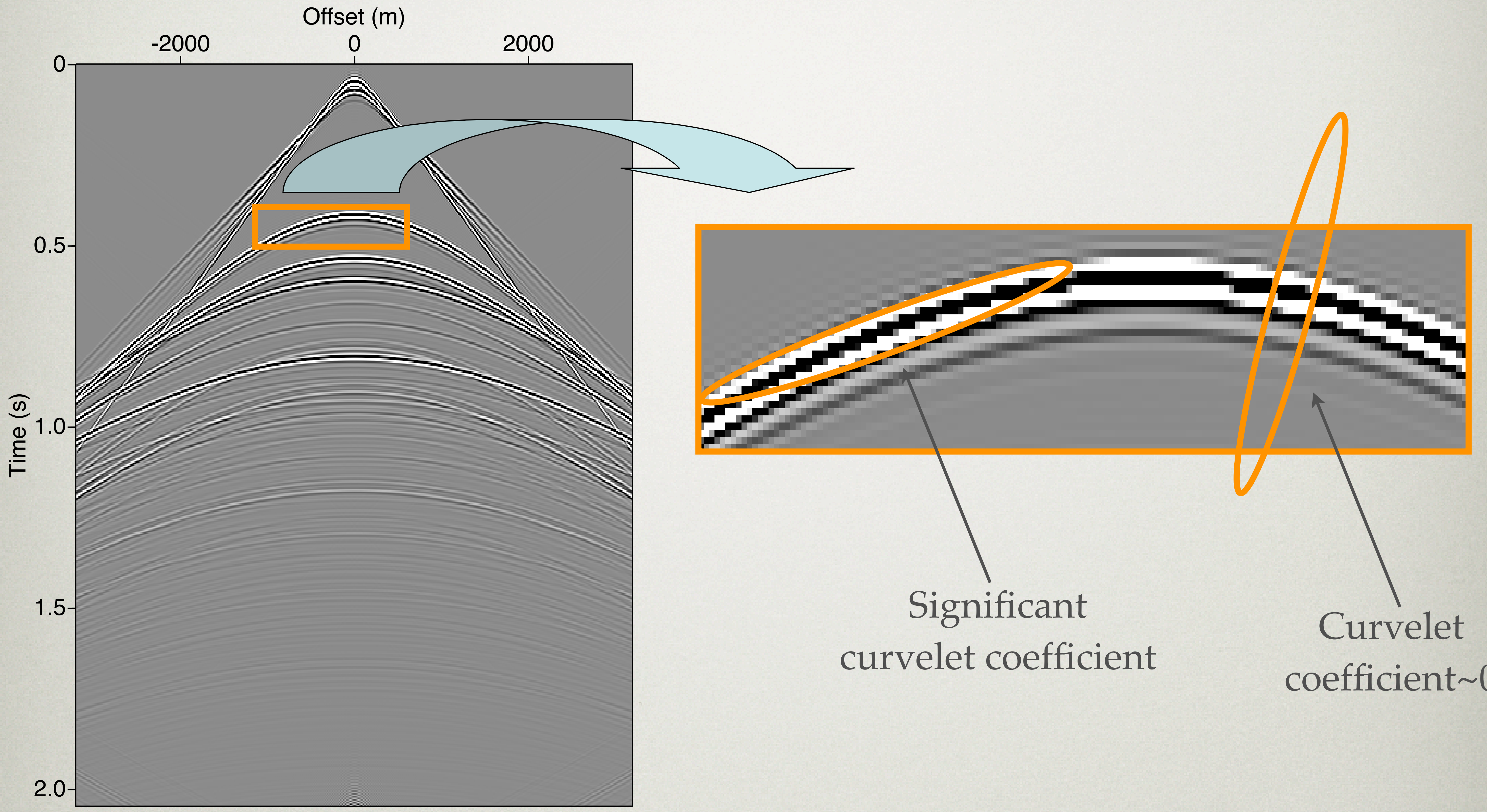
Curvelets



50
100
150
200
250
300
350
400
450
500
y-dimension
100
200
300
400
500
x-dimension



0.5
0
k2
-0.5
0
0.5
k1



Offset (m)
-2000
0
2000
Time (s)
0
0.5
1.0
1.5
2.0
Significant
curvelet coefficient
Curvelet
coefficient~0

Nonlinear Approximation Rates

Optimal

$$\|\mathbf{f} - \mathbf{f}_p^O\|_2^2 \propto p^{-2}, \quad p \to \infty$$

Fourier

$$\|\mathbf{f} - \mathbf{f}_p^F\|_2^2 \propto p^{-1/2}, \quad p \to \infty$$

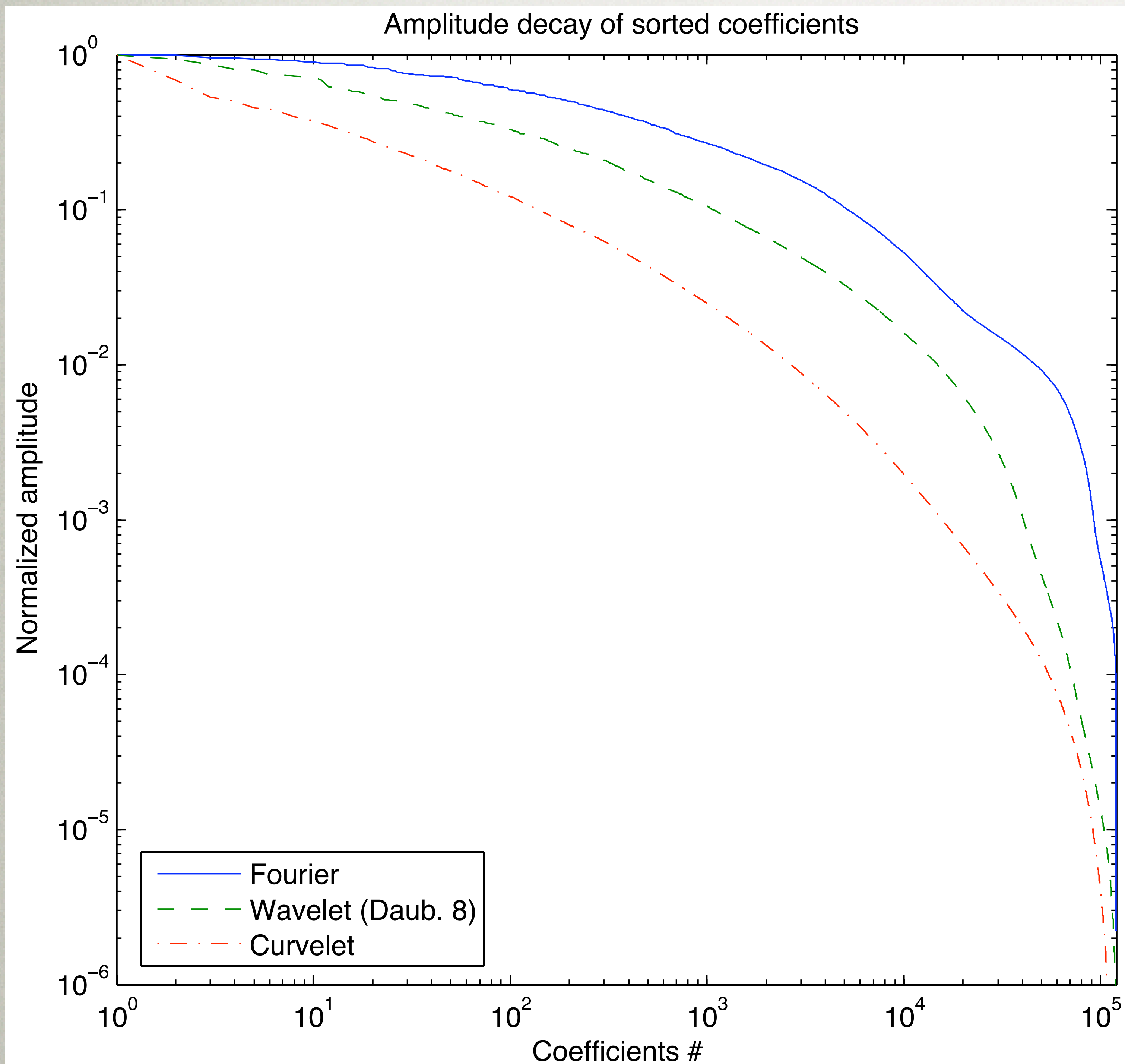
Wavelets

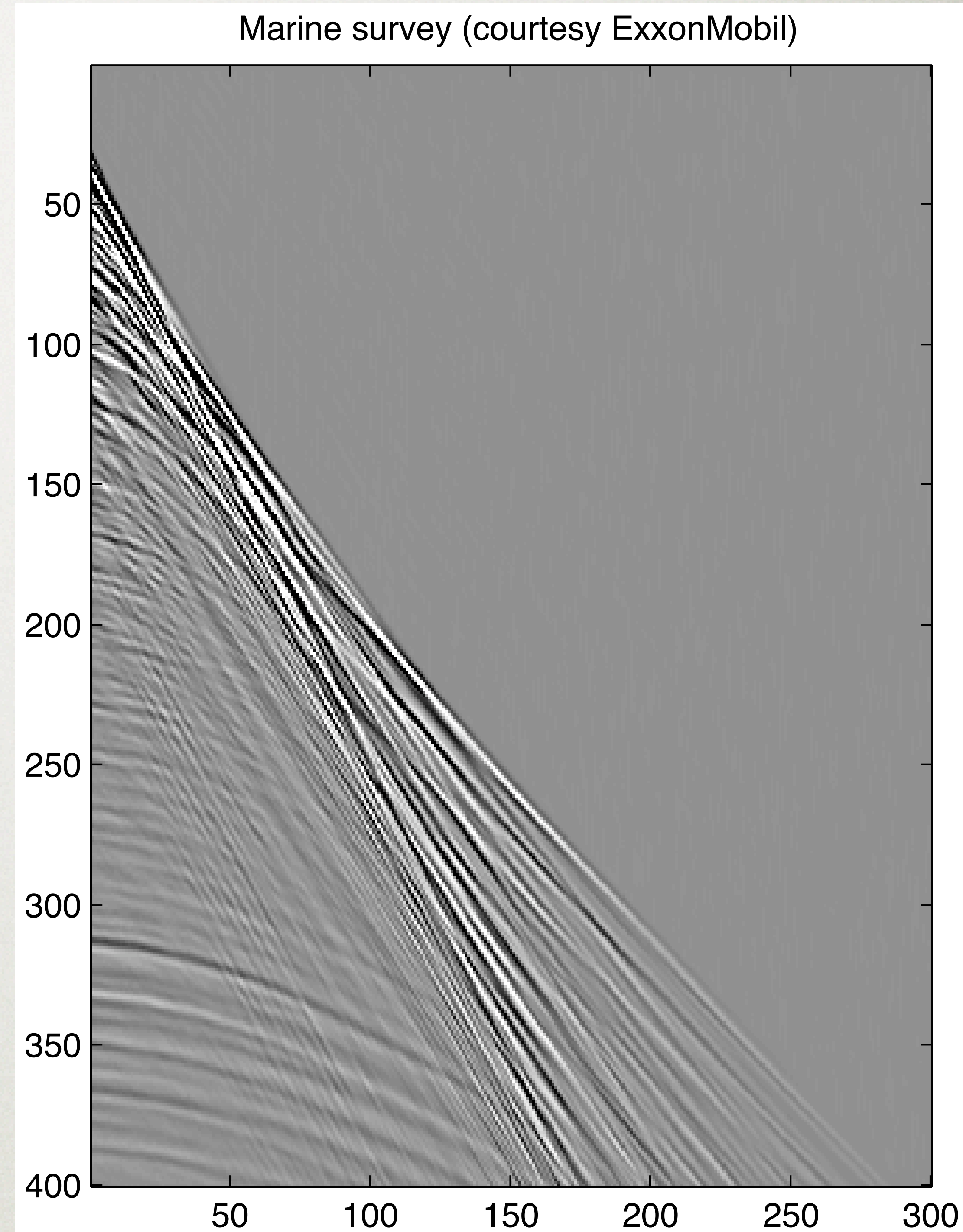
$$\|\mathbf{f} - \mathbf{f}_p^W\|_2^2 \propto p^{-1}, \quad p \to \infty$$

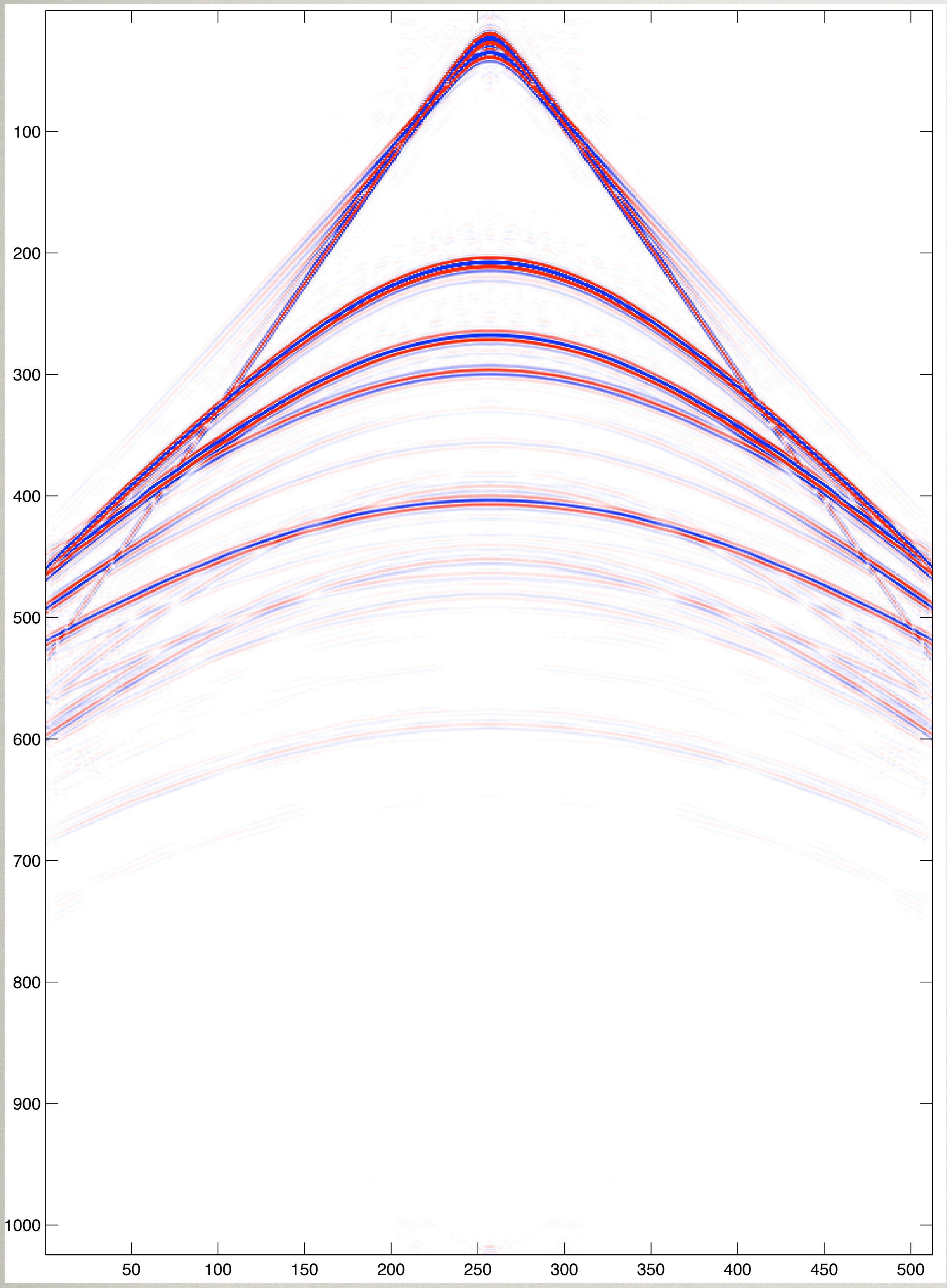
Curvelets

$$\|\mathbf{f} - \mathbf{f}_p^C\|_2^2 \leq C\ p^{-2}\ (\log p)^3, \quad p \to \infty$$

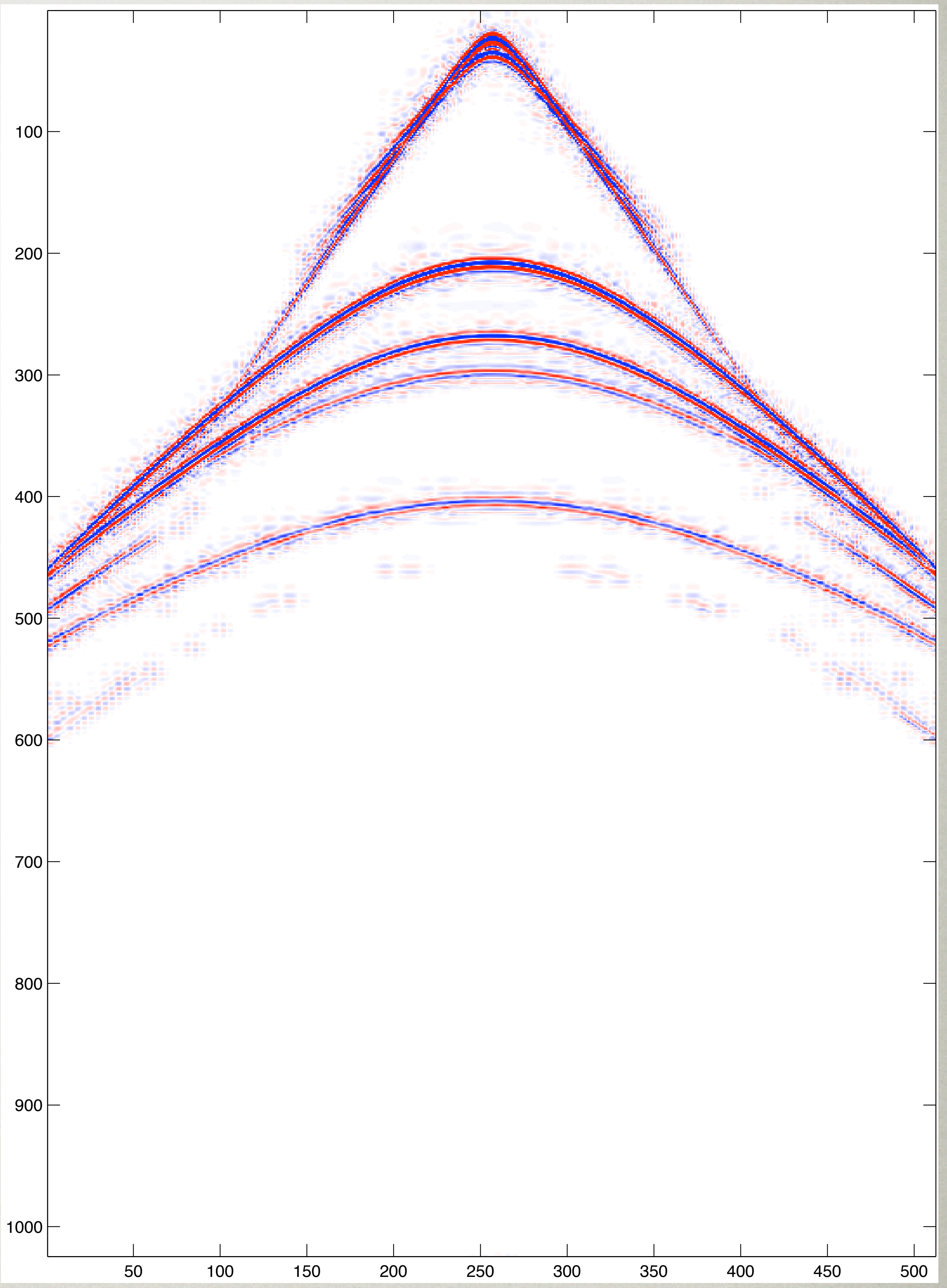
[Candès & Donoho, 2002]


Amplitude decay of sorted coefficients
Normalized amplitude
Coefficients #
Fourier
Wavelet (Daub. 8)
Curvelet


Marine survey (courtesy ExxonMobil)



1% reconstruction from
curvelet coefficients



1% reconstruction from
wavelet coefficients

Seismic Data Regularization

Forward problem

$$\mathbf{Pd} = \mathbf{PFx} + \mathbf{Pn}$$

Inverse problem

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{P}(\mathbf{d} - \mathbf{Fx})\|_2^2 + \lambda J(\mathbf{x})$$

Use sparsity of seismic data
in the curvelet domain

Our approach

Curvelet synthesis operator

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{P}(\mathbf{d} - \boxed{\mathbf{C}^H\mathbf{x}})\|_2^2 + \lambda\|\mathbf{x}\|_1$$

Interpolated data

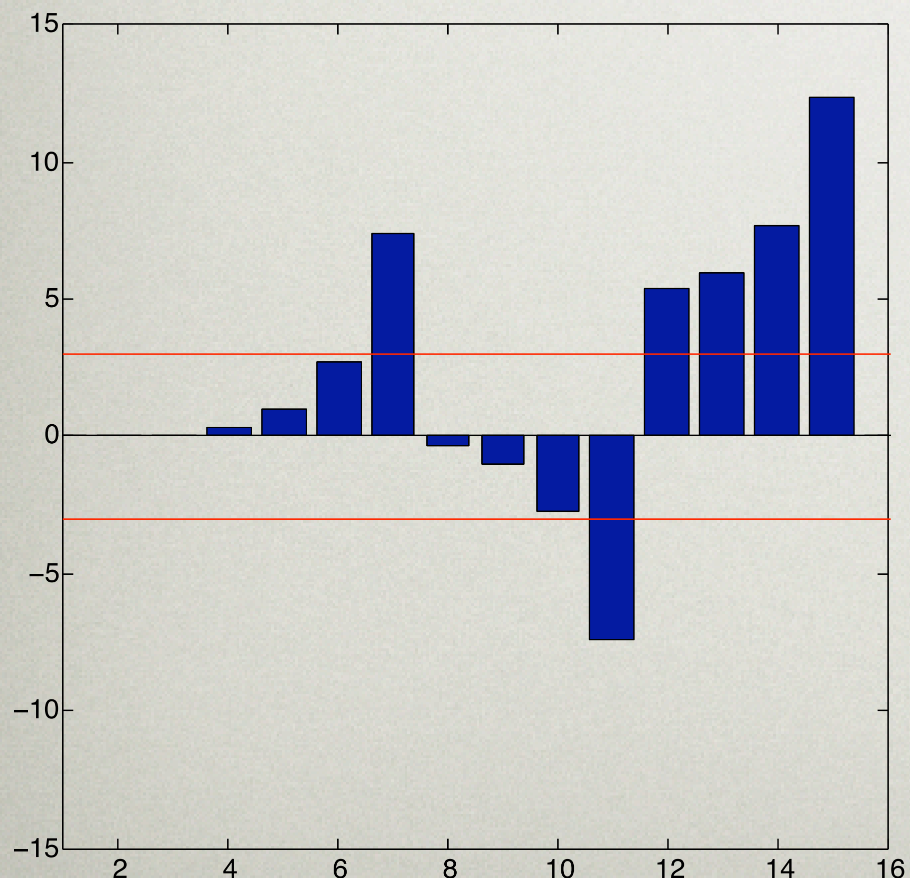
Sparsity in the curvelet domain

Soft-thresholded Landweber iteration [Daubechies, 2005]

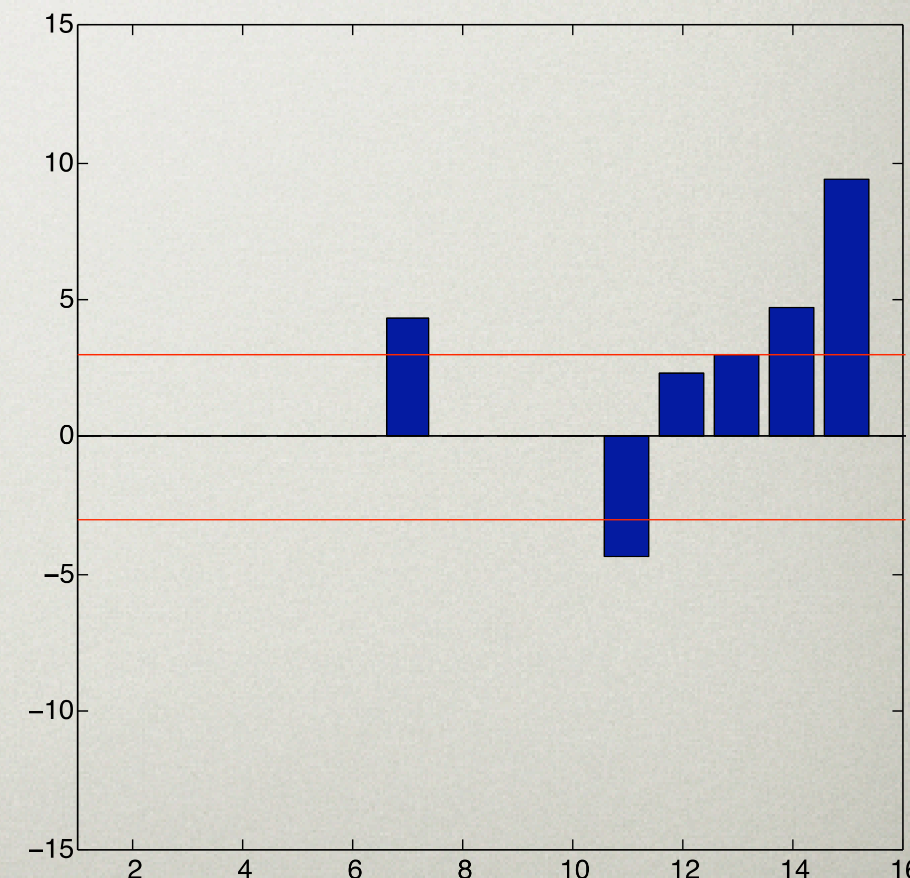
$$\mathbf{x}^{m+1} = \mathcal{S}^s_{\lambda_m}[\mathbf{x}^m + \mathbf{C}\mathbf{P}(\mathbf{d} - \mathbf{C}^H\mathbf{x}^m)]$$

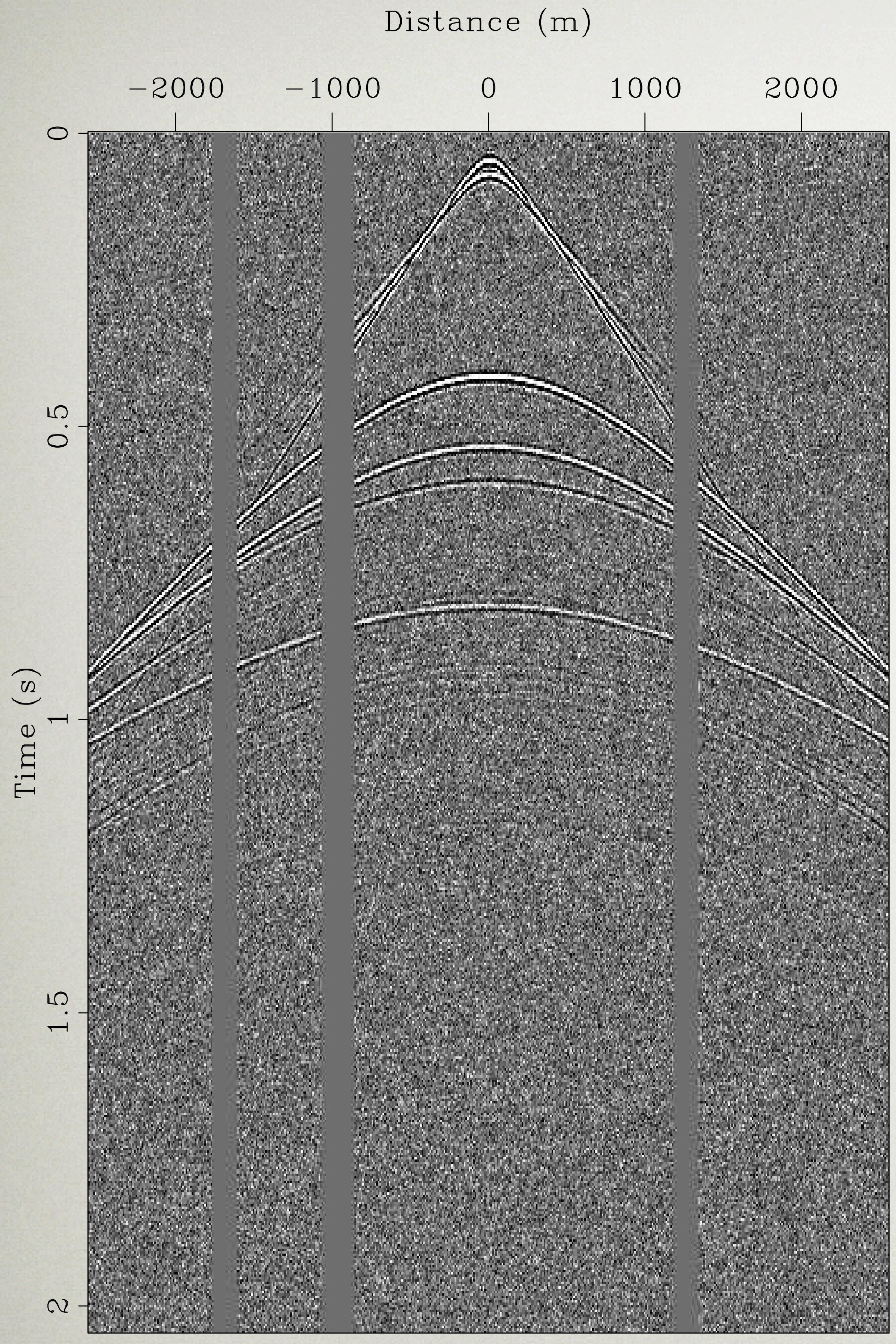
Soft Threshold

$$S^s_\lambda(x) = \begin{cases} x - \operatorname{sign}(x)\lambda & |x| \geq \lambda \\ 0 & |x| < \lambda \end{cases}$$



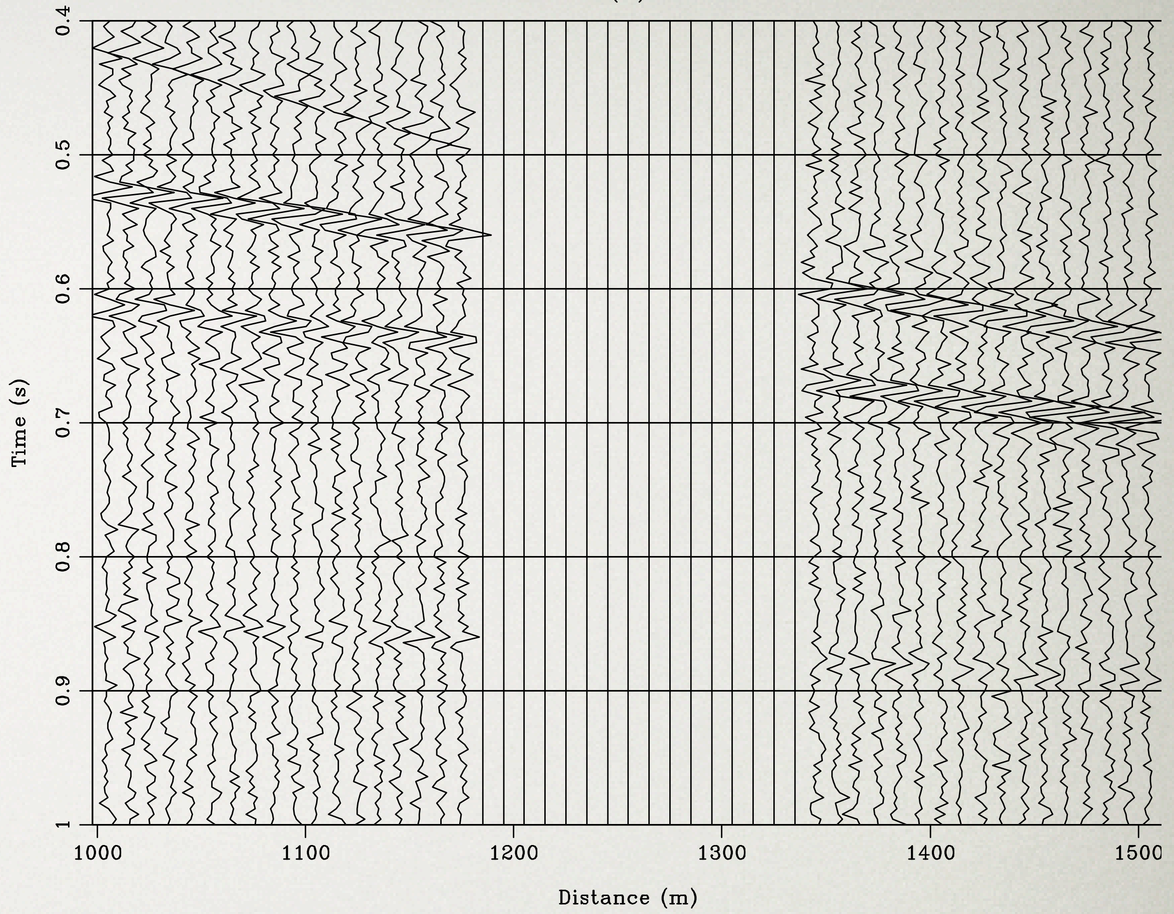
$$\xrightarrow{\mathcal{S}^s_\lambda}$$



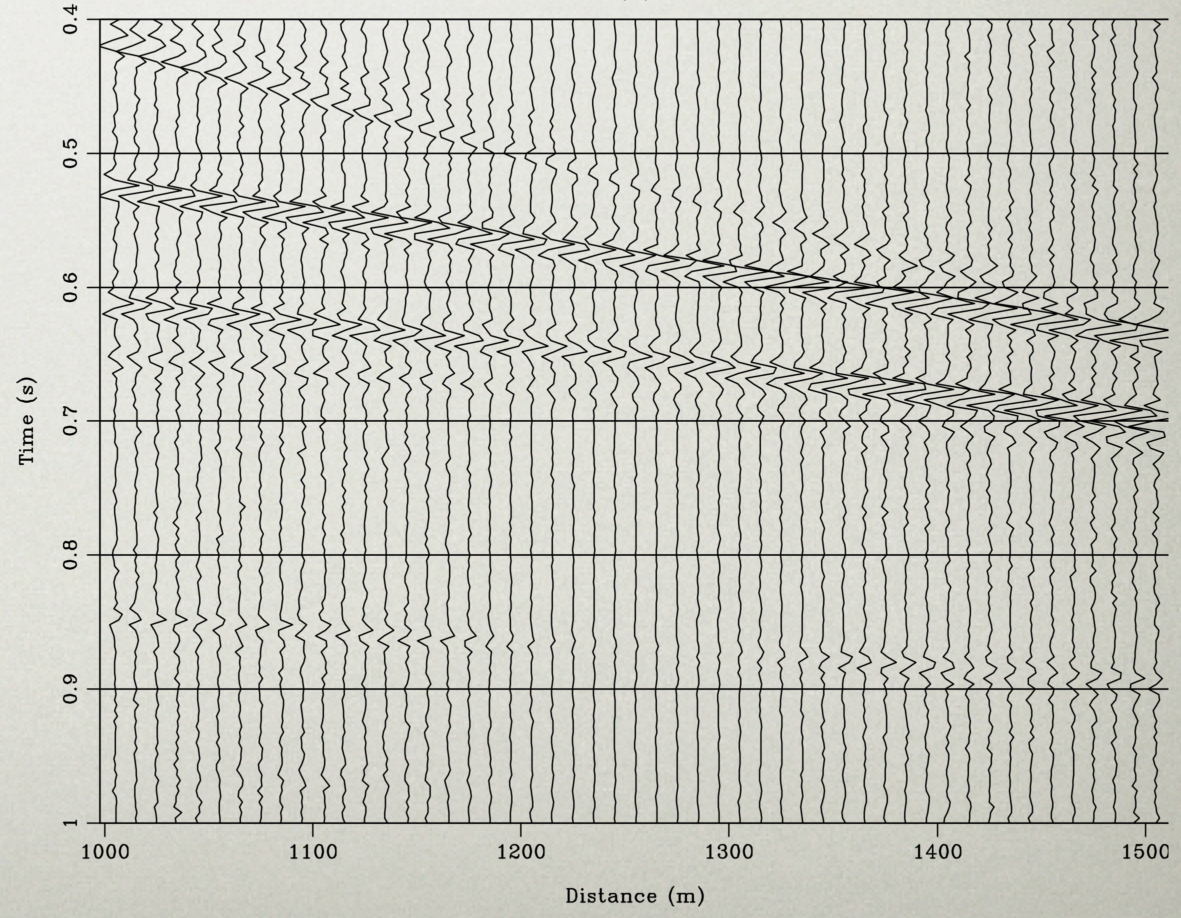

Distance (m)
−2000
−1000
0
1000
2000
Time (s)
0
0.5
1
1.5
2

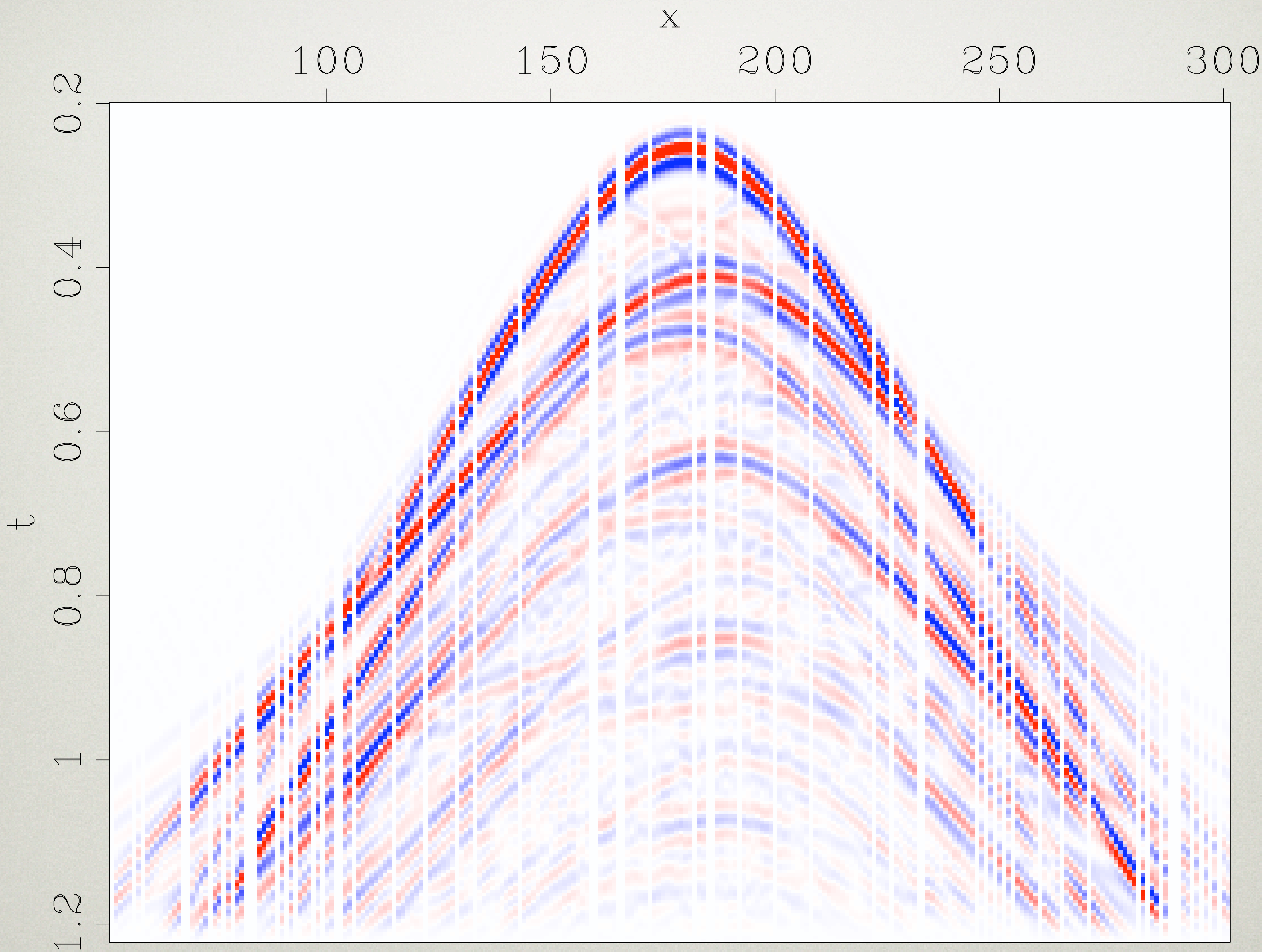
(a)

(b)

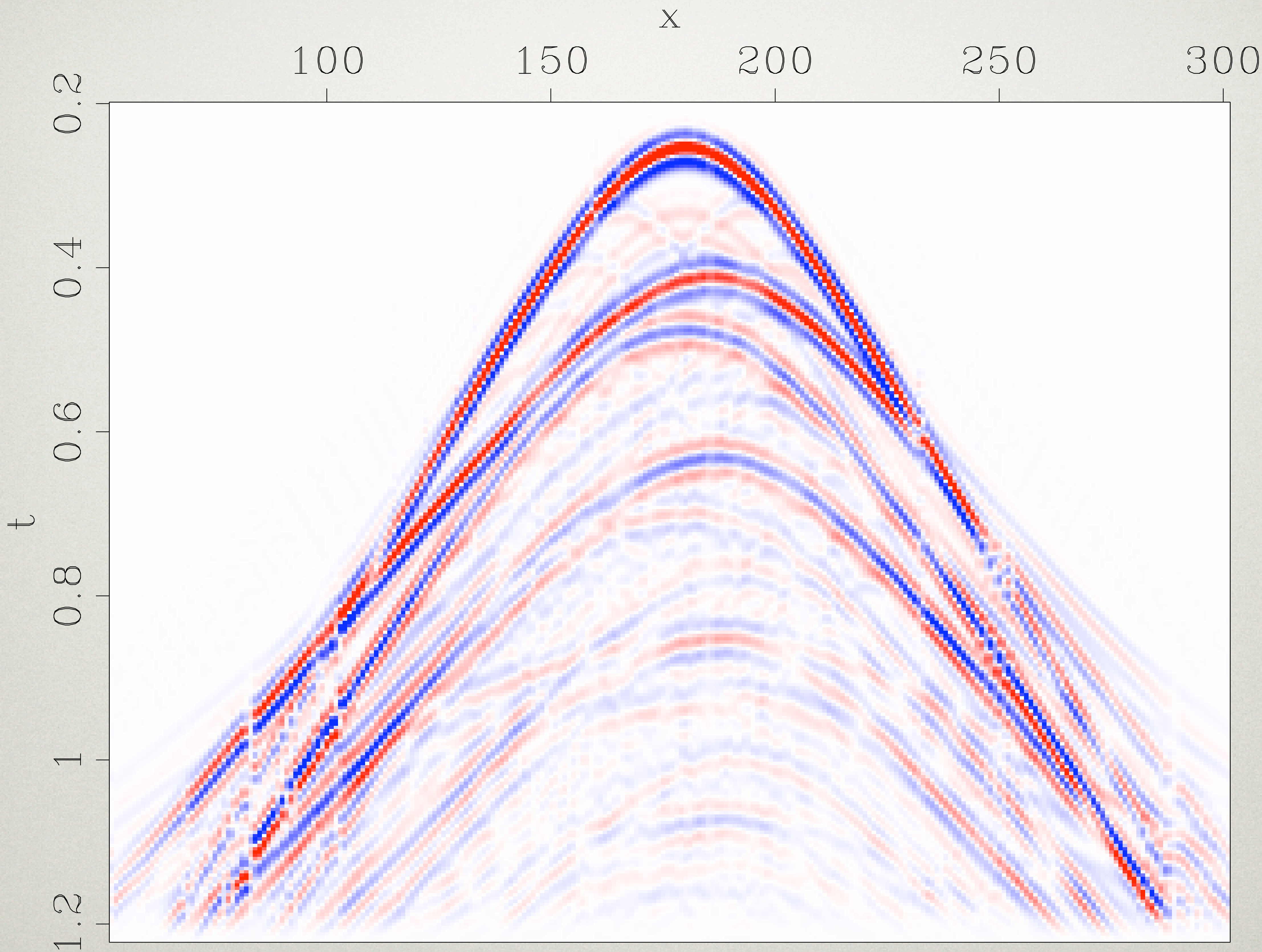

Time (s)
0.4
0.5
0.6
0.7
0.8
0.9
1
1000
1100
1200
1300
1400
1500
Distance (m)

(c)

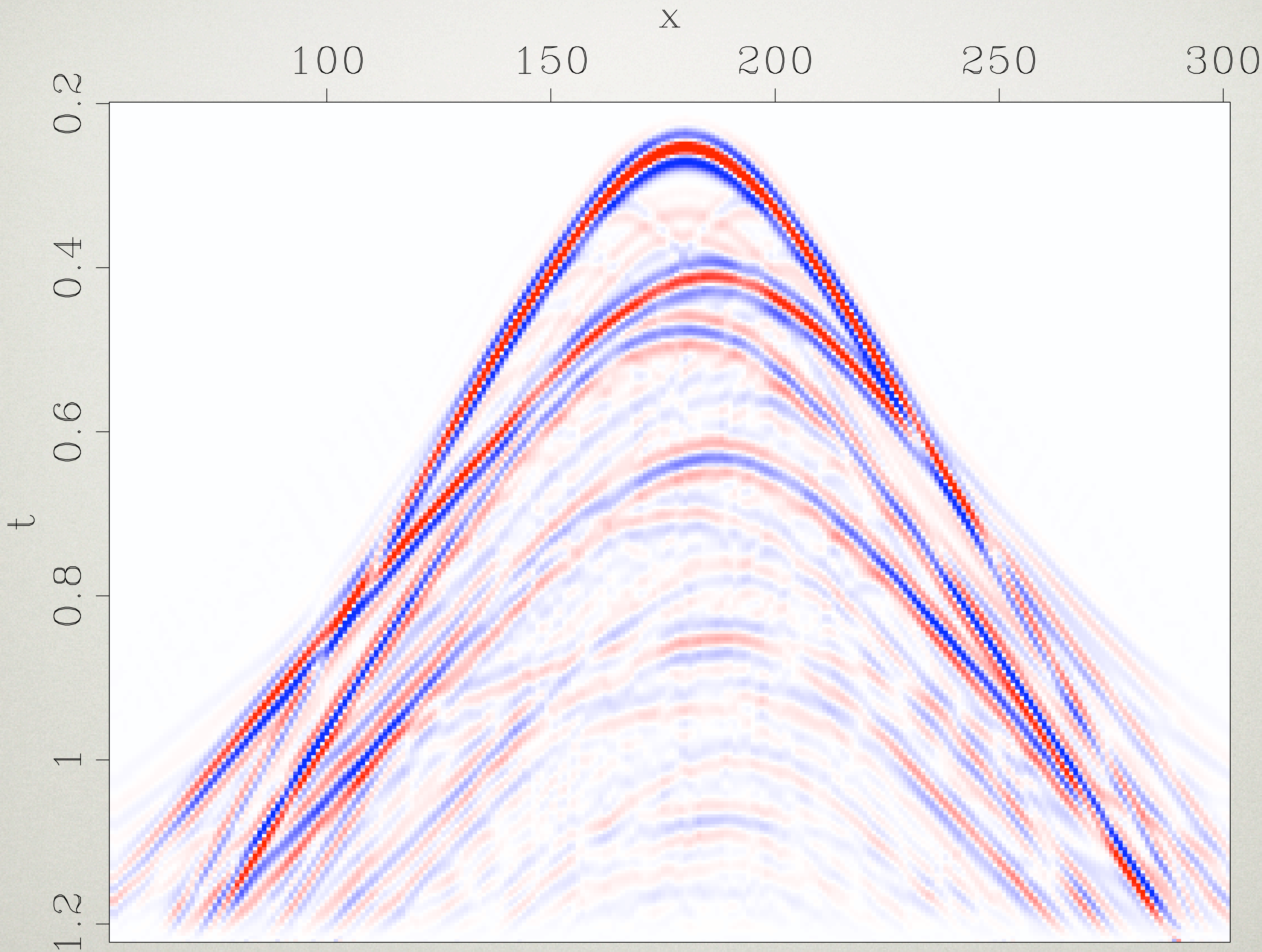

Time (s)
0.4
0.5
0.6
0.7
0.8
0.9
1
1000
1100
1200
1300
1400
1500
Distance (m)



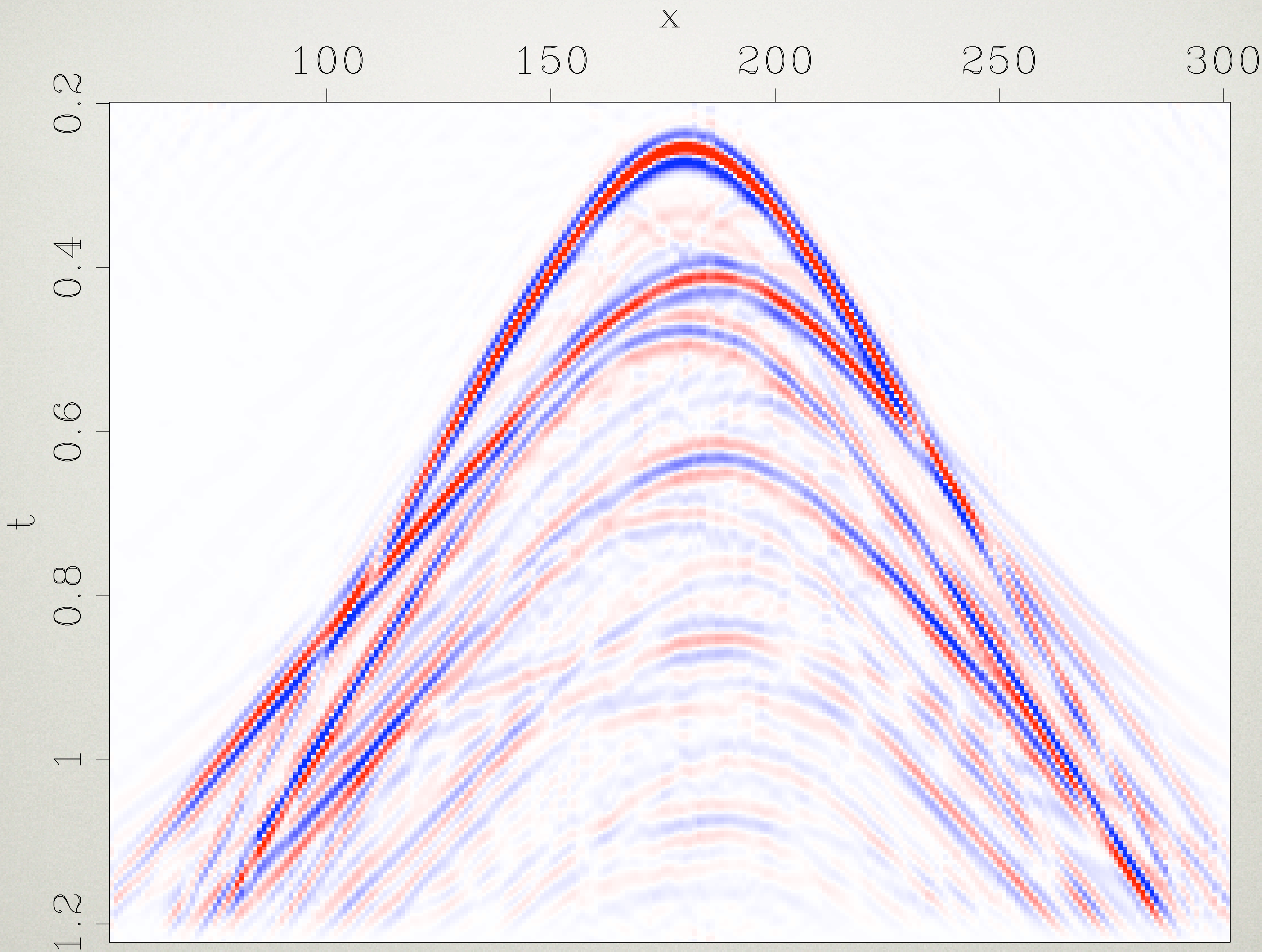
data with missing traces



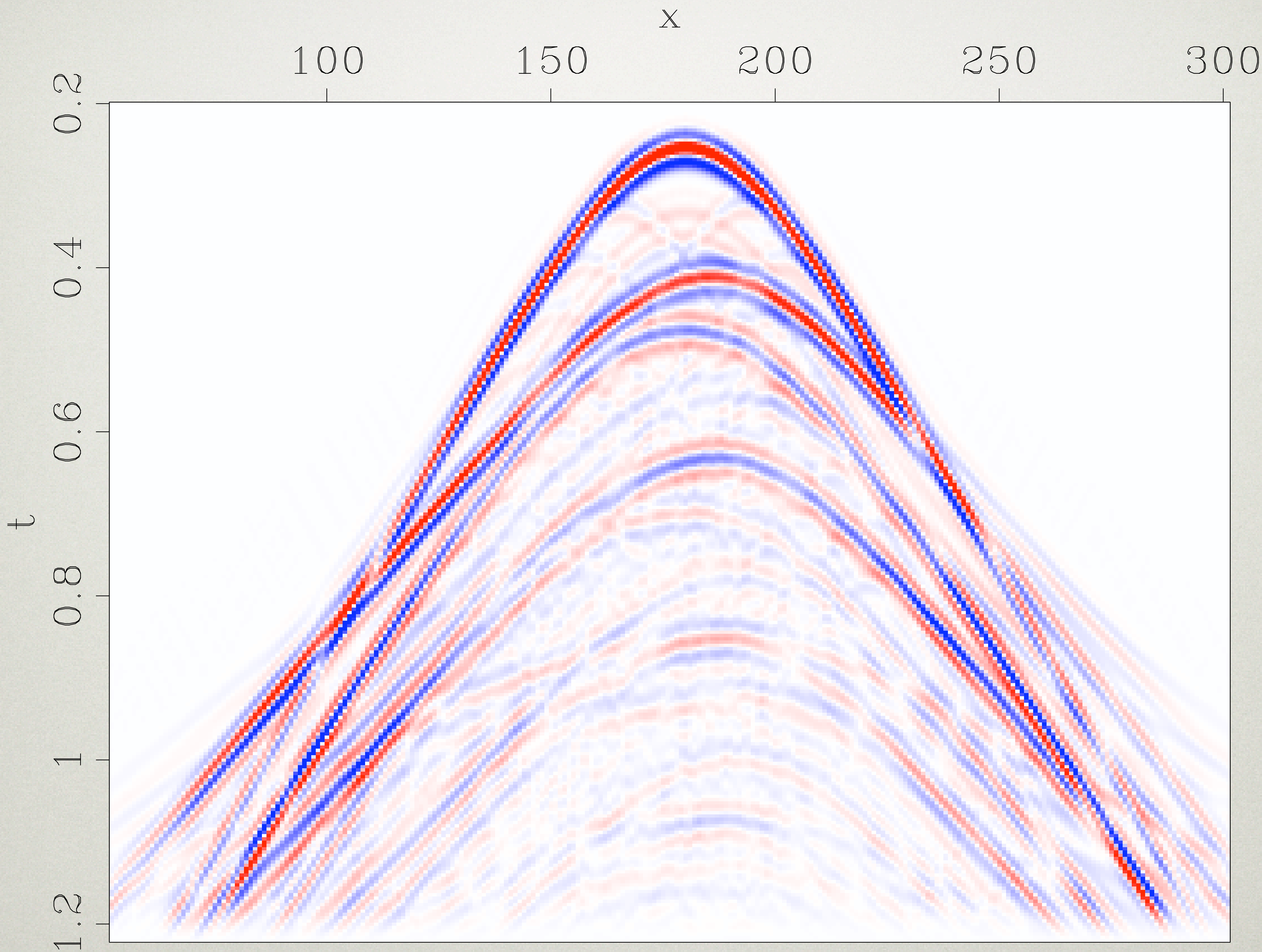
“standard” interpolated data



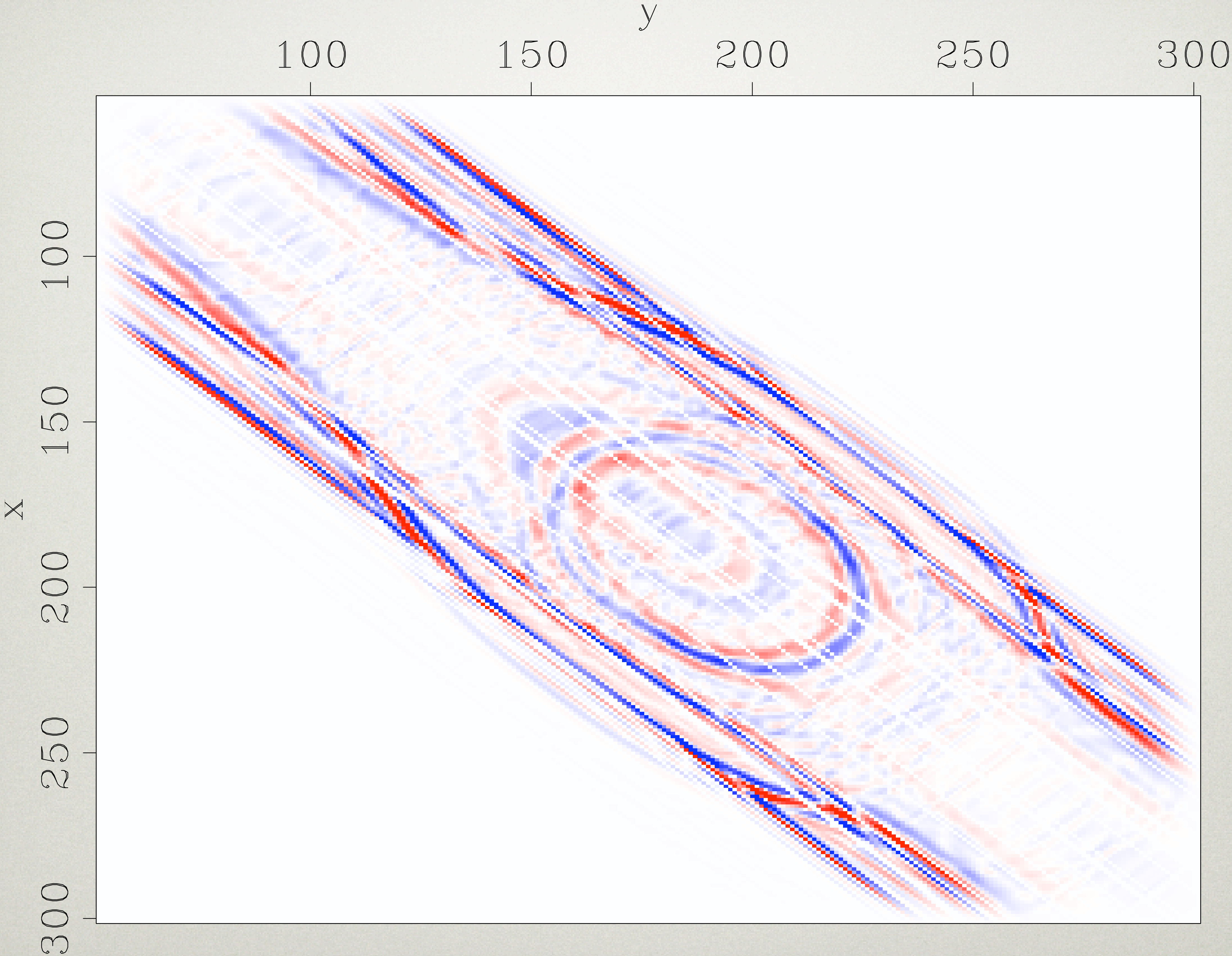
plane-wave destruction interpolated data



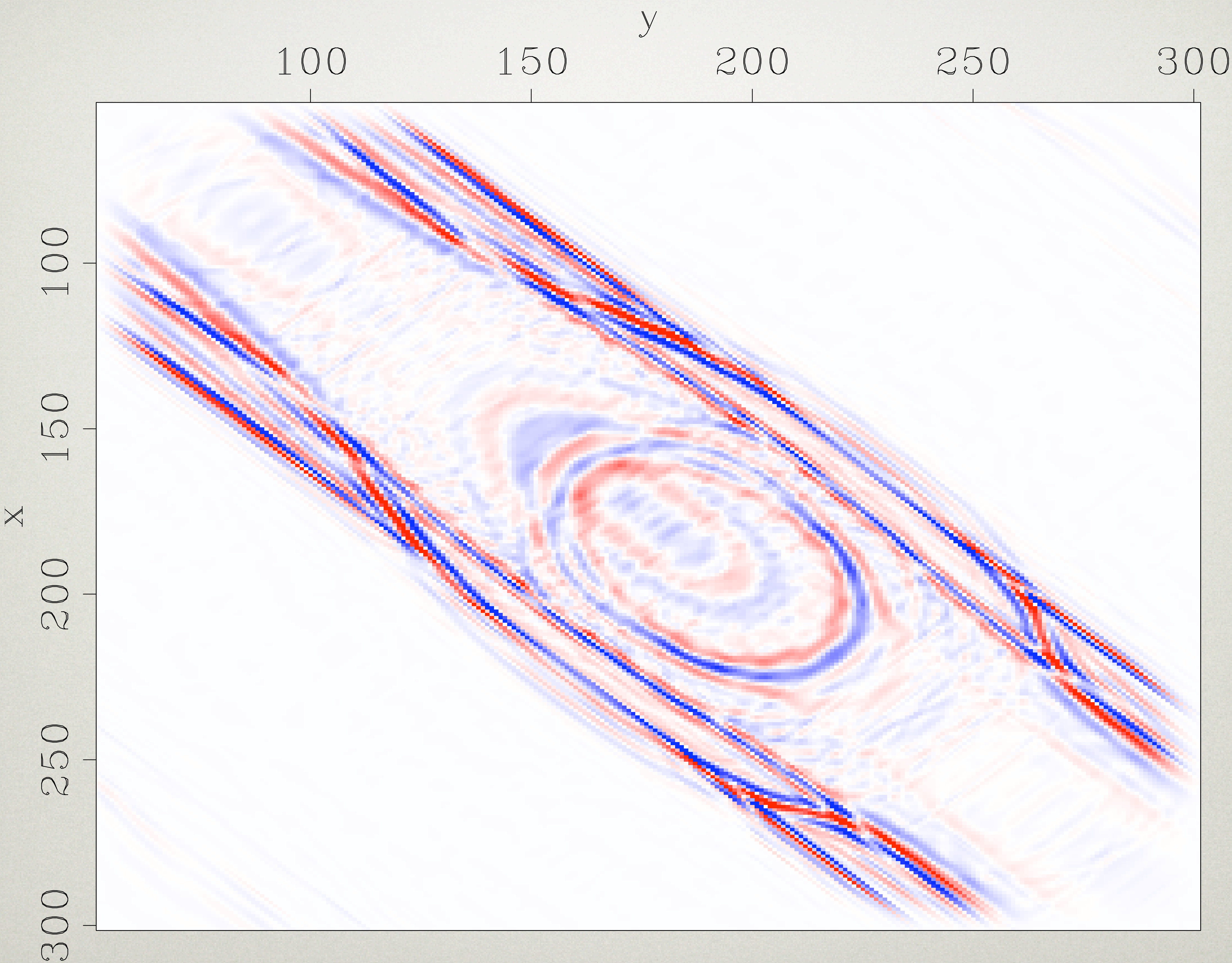
curvelet interpolated data



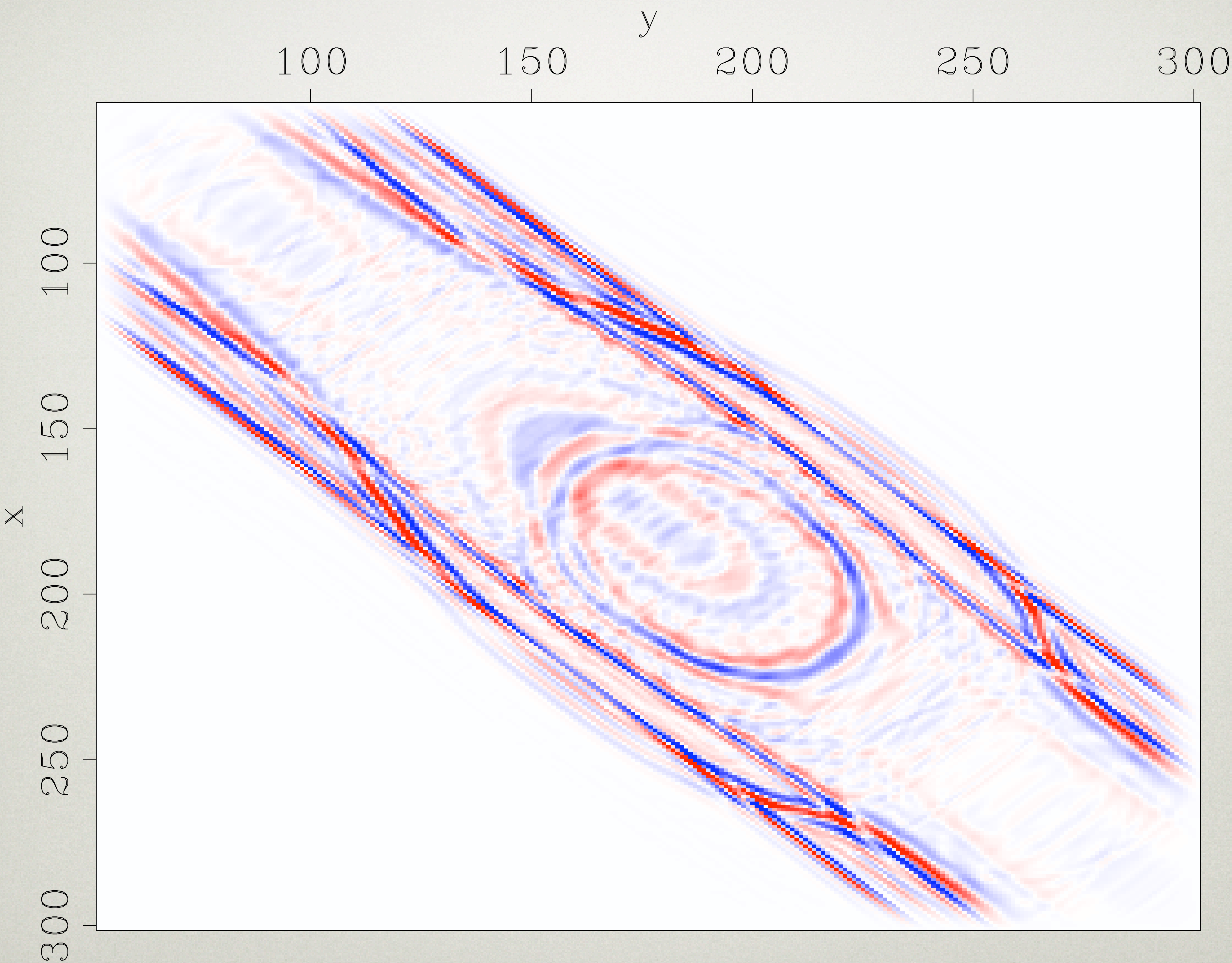
original data



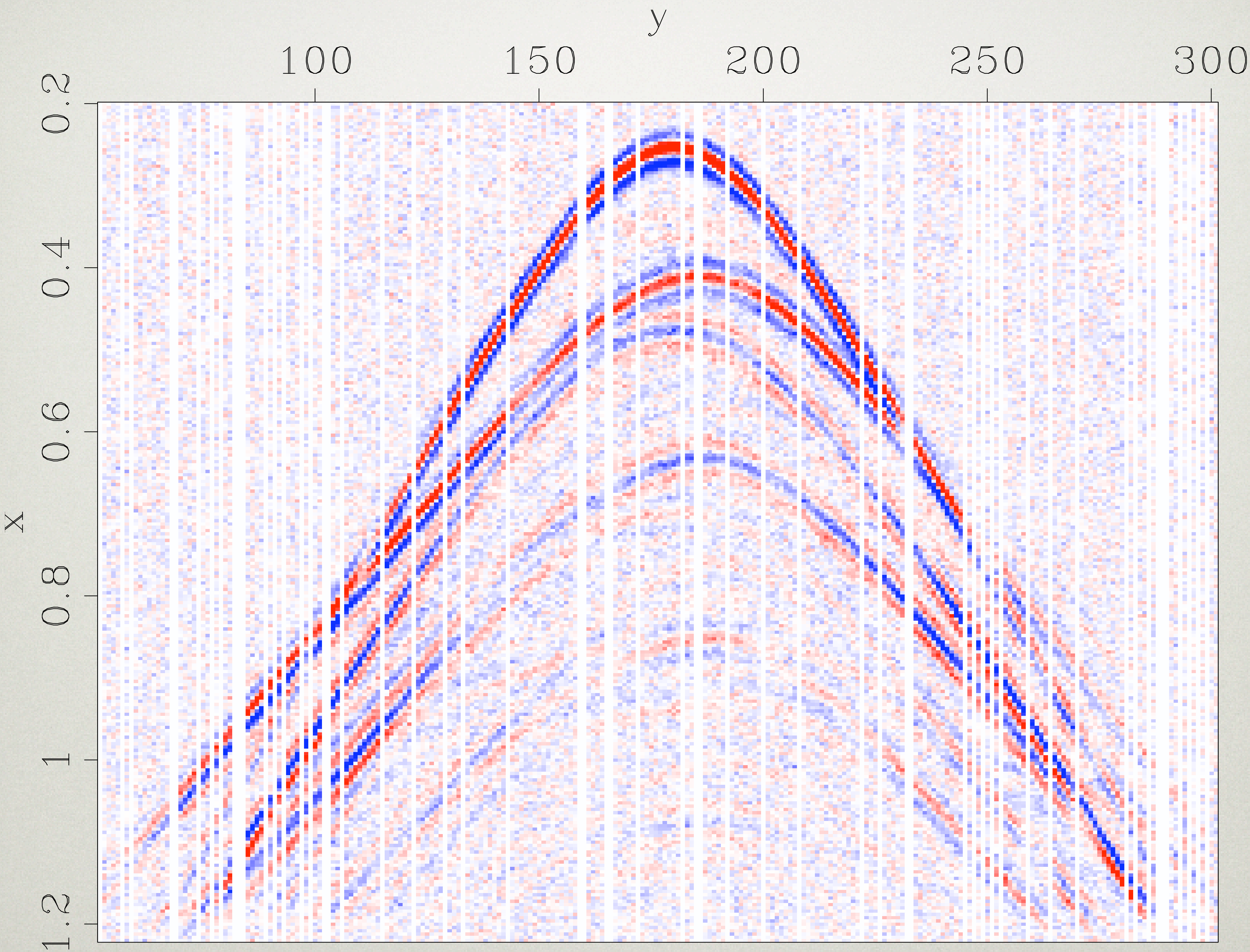
data with missing traces



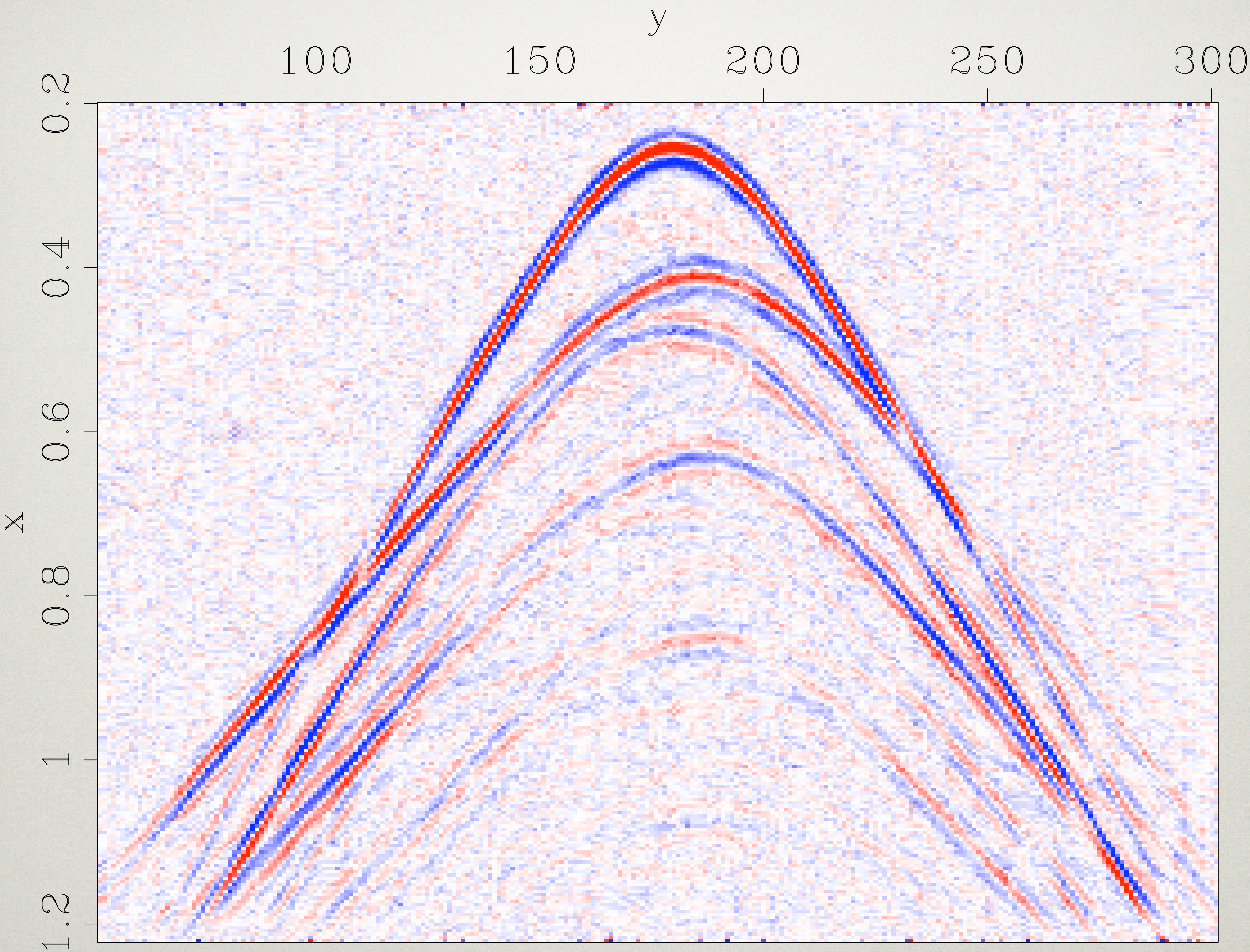
curvelet interpolated data



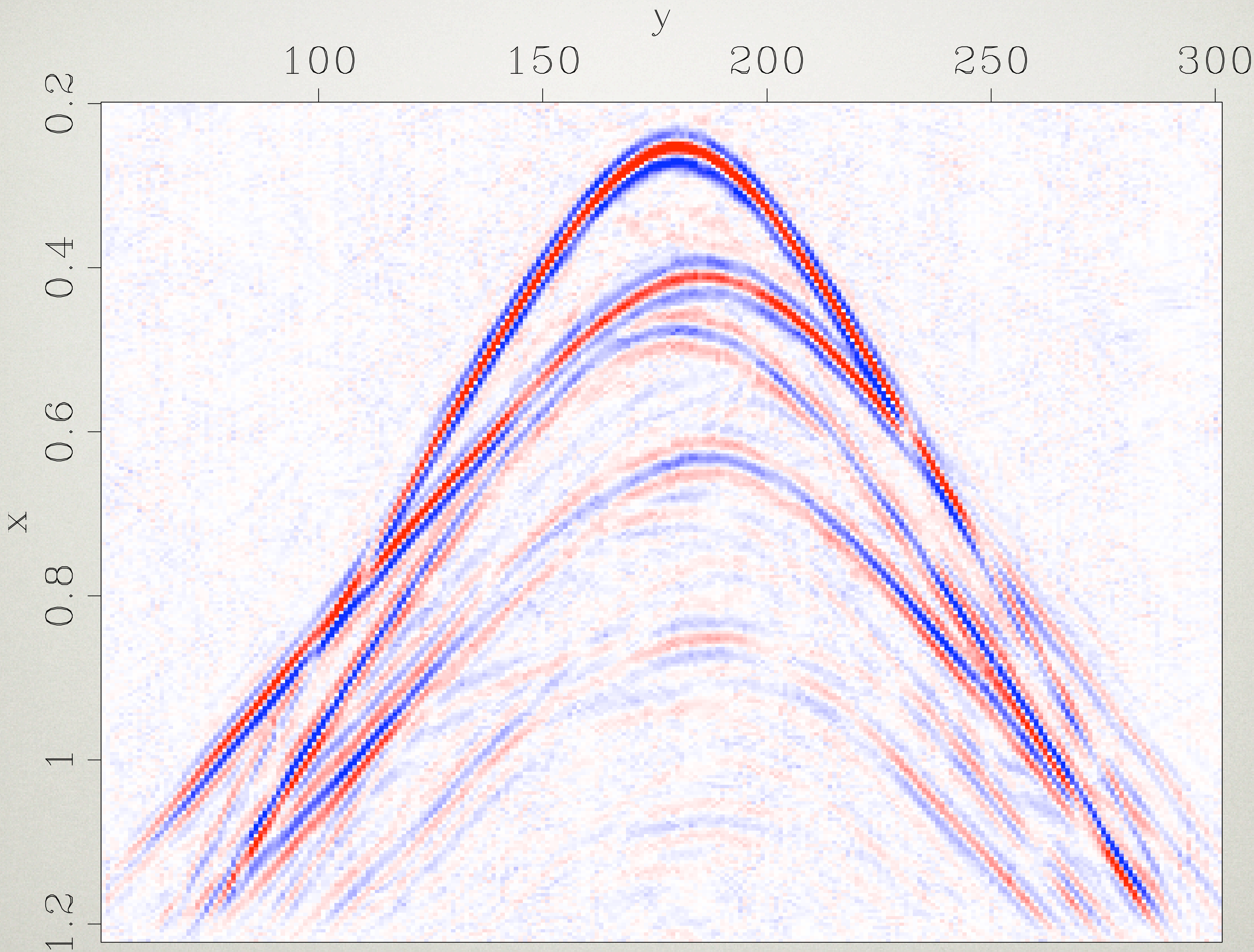
original data



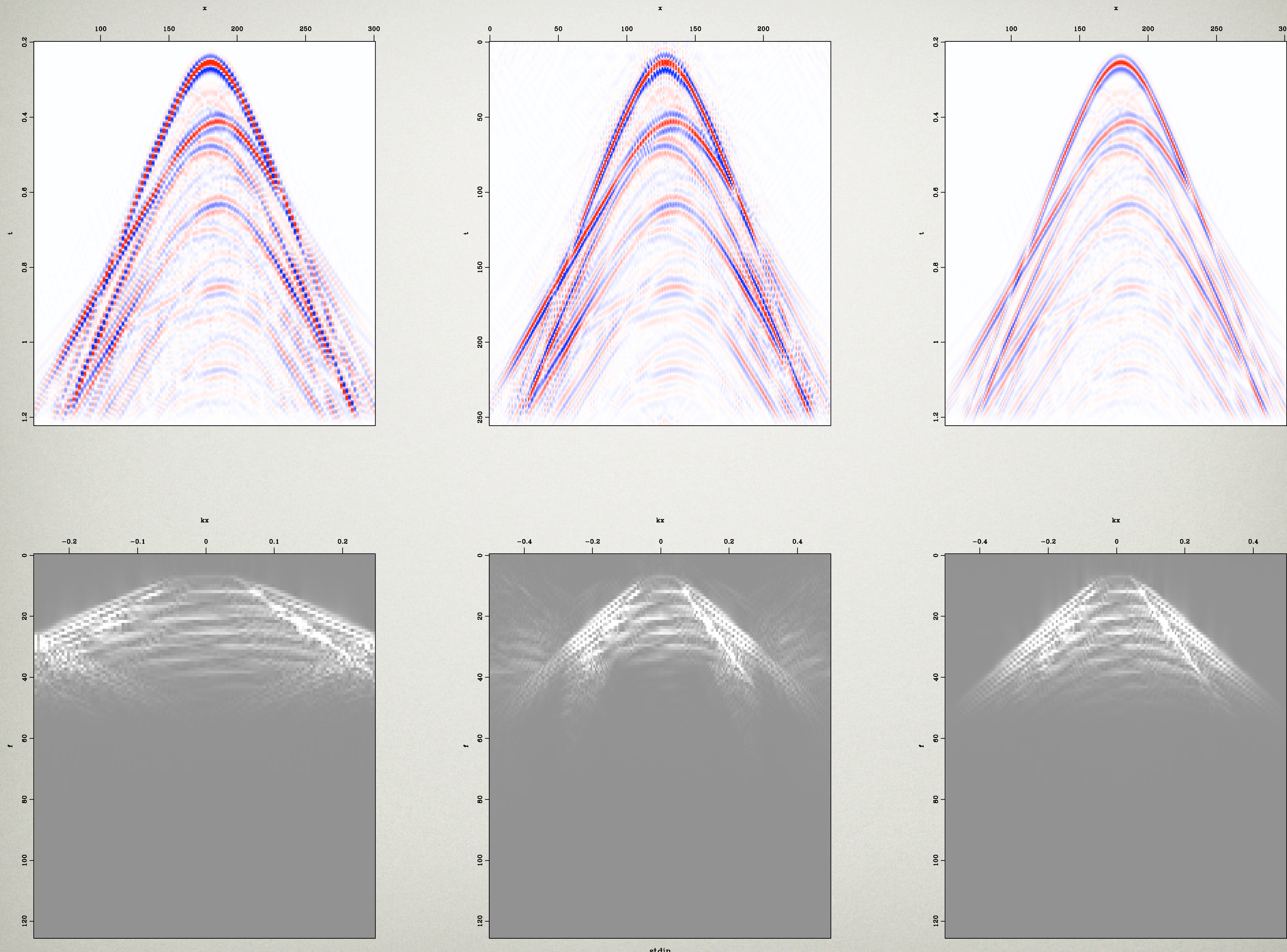
noisy data with missing traces

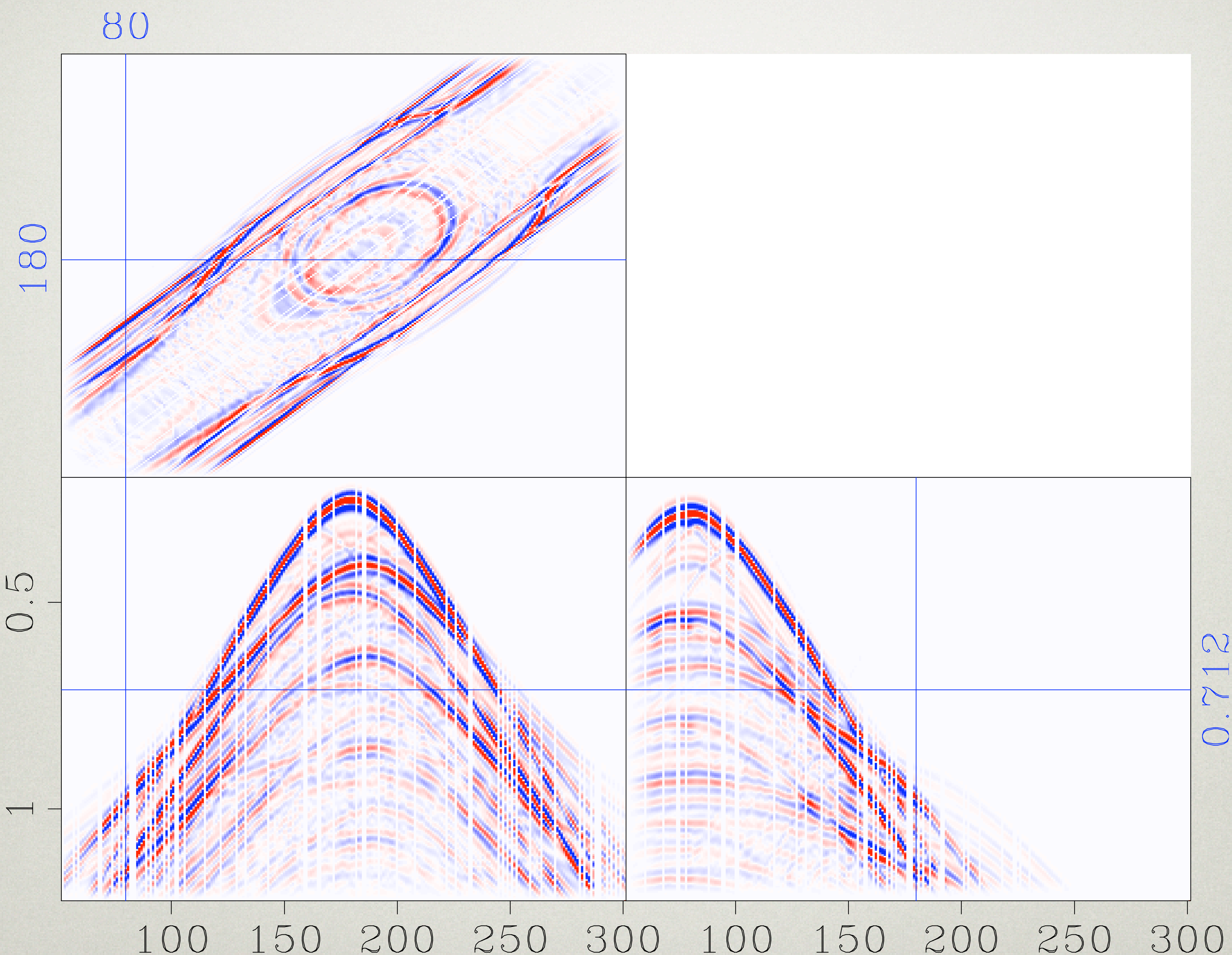


plane-wave destruction interpolated data

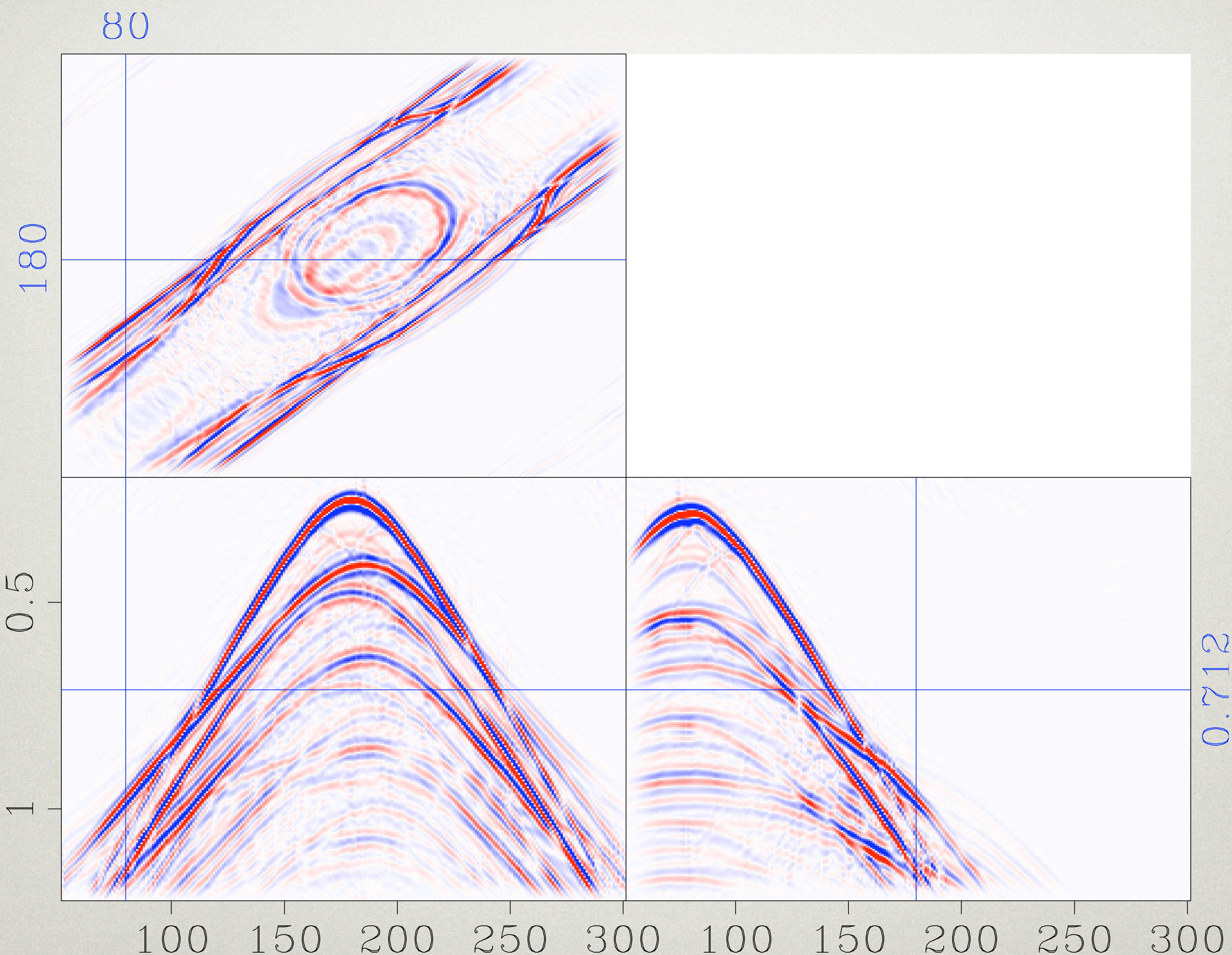


curvelet interpolated data

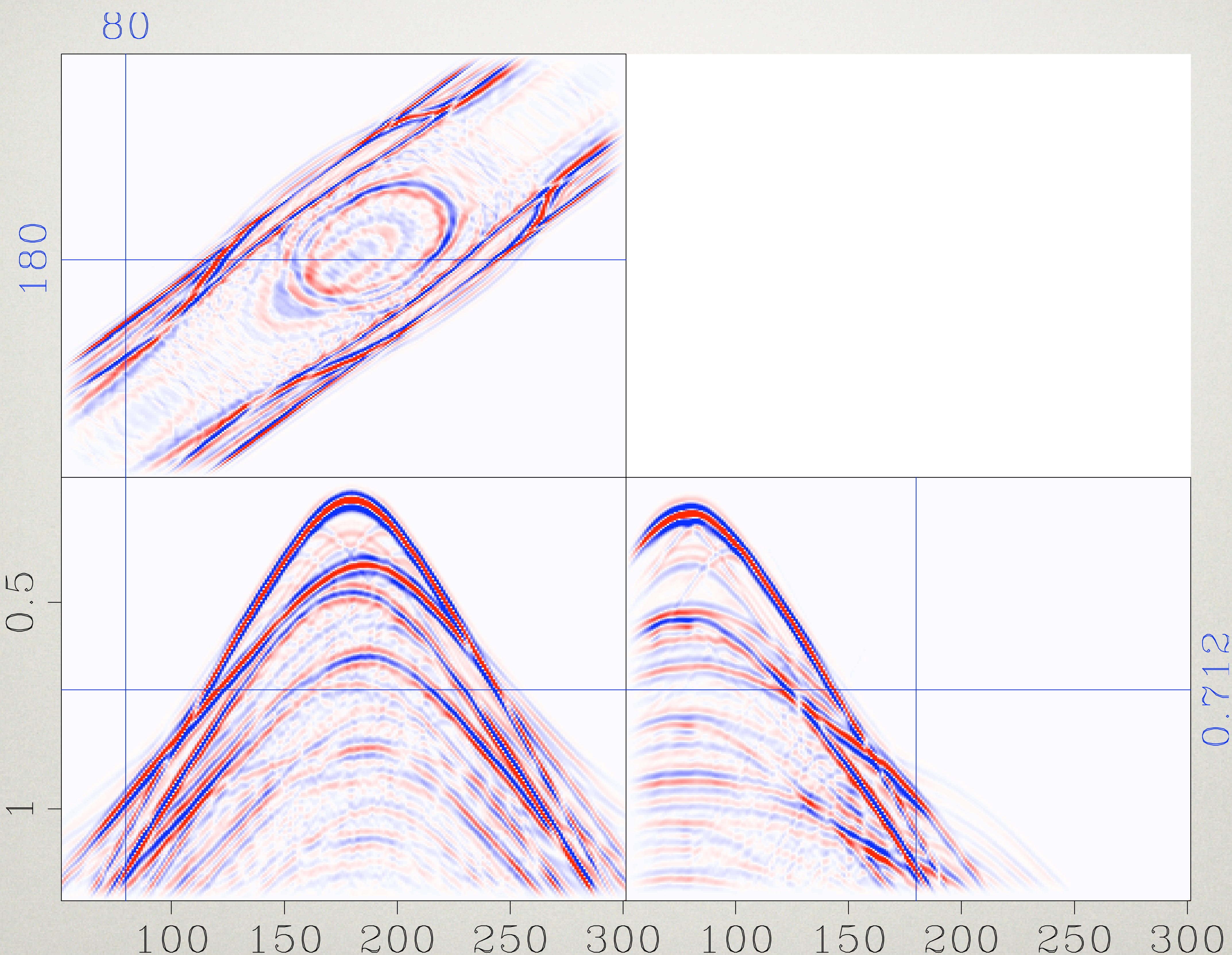

x
t
kx
f
stdin



data with missing traces



curvelet interpolated data



original data

Conclusions

Our approach:

- does **NOT** require information on the velocity model or local dips,
- explores continuity along 2D/3D wavefronts,
- robust under noise,
- based on fast transform FDCT (N log N).

Acknowledgments

- The authors of CurveLab (E. Candès, L. Demanet, L. Ying),
- S. Fomel for giving us the opportunity to test and use his new processing software RSF,
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