Migration preconditioning with Curvelets

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Overview

1) **Objectives**
2) **Related Works**
3) **Problem Formulations**
4) **Results and comparisons**
5) **Conclusion**

Related Works

- Candes (2002), Edge preserving image reconstruction using Curvelet transform.
- Candes (2003), Curvelet and Fourier Integral Operators.
- Rickett (2003), Illumination-based migration
- Trad (2003), Sparse Radon Transform
- Claerbout (1994), Spectral Preconditioning

Problem Formulation

- The linear system of equation needs to be solved is:

$$d = Km + n$$

- Conventionally there are two approaches for solving above equation:

1) Iterative solver (GMRES, PCG,…)
2) Approximating the normal operator:

$$\hat{m} = (approx.(K^{*}K))^{-1}K^{*}d$$

Inversion

- A conventional form of inversion is:

$$\hat{m}: \min_{m} \frac{1}{2}\|d - Km\|_2^2 + \lambda J(m)$$

- This can be solved using following iterative solver:

$$\delta m = (K^* K + A)^{-1}(K^* d + B)$$

- Where:

$A = \nabla^2 \lambda J(m)$ and $B = \nabla \lambda J(m)$

Basic Questions

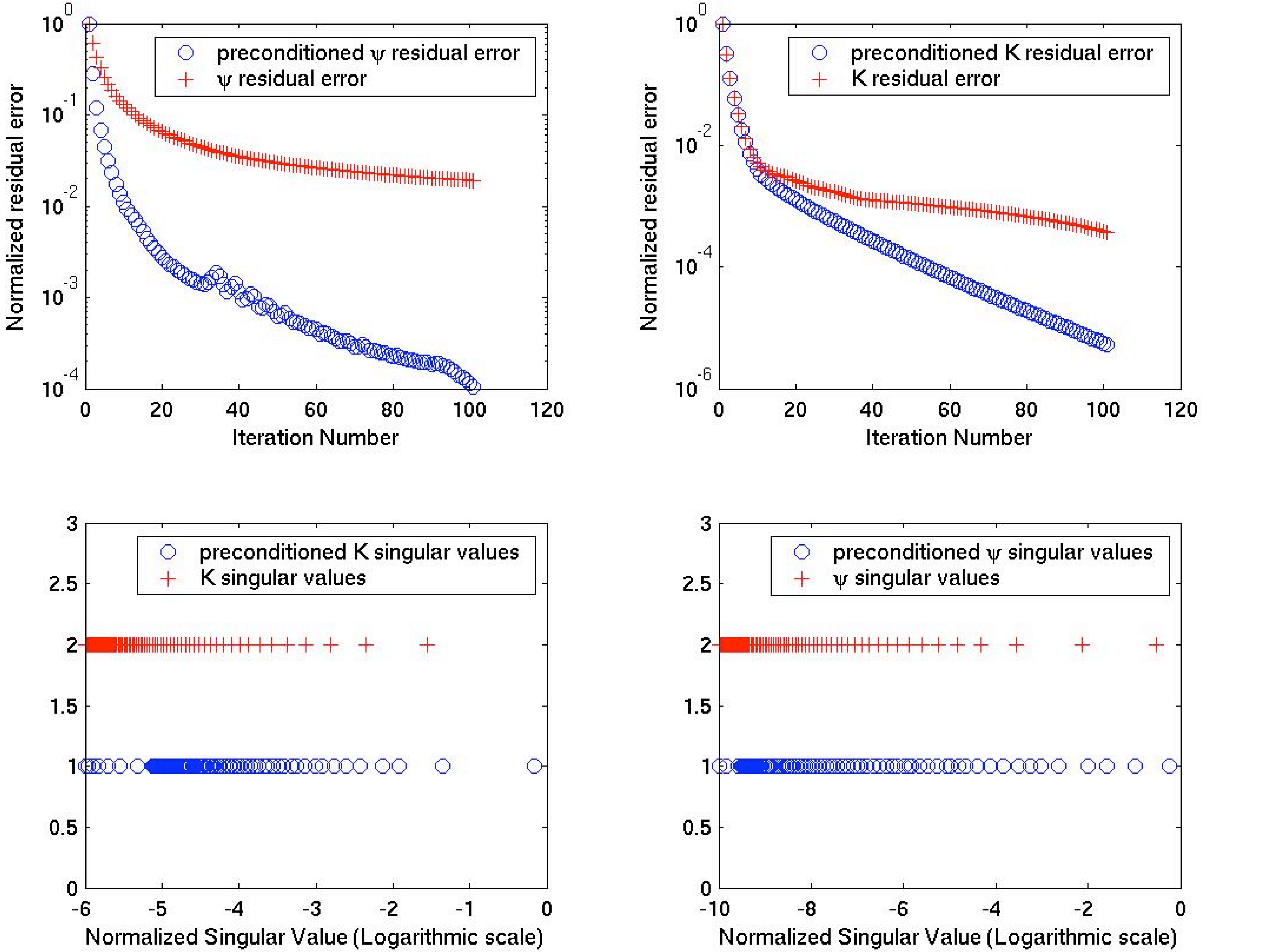
- How can we improve the structure of K?
- Which type of norm is better to use?
- How can we incorporate the noise information in solution as a typical inverse problem?
- Which solver is better to be used and how can we guarantee its convergence?

Preconditioning using Curvelet Transform

- We precondition migration and normal operator as:

$$K^{*} \Rightarrow \tilde{K}^{*} = CK^{*}C^{*} \qquad \psi = K^{*}K \Rightarrow \tilde{\psi} = CK^{*}KC^{*}$$

- The sparsity of both migration and normal operator increase after preconditioning.
- For preconditioned operator the singular values shifted away from zero and have tendency to concentrate in a point in spite of operator itself.
- The convergence rate for preconditioned normal operator faster than normal operator itself.



preconditioned ψ residual error
ψ residual error
Normalized residual error
Iteration Number
preconditioned K residual error
K residual error
Normalized residual error
Iteration Number
preconditioned K singular values
K singular values
Normalized Singular Value (Logarithmic scale)
preconditioned ψ singular values
ψ singular values
Normalized Singular Value (Logarithmic scale)

Preconditioned System

- We map the original system of equations to preconditioned as:

$$CK^*d = CK^*KC^*Cm + CK^*n$$

- Or:

$$\tilde{u} = \tilde{\psi}\tilde{m} + \tilde{n}$$

- Maximum Likelihood Solution:

$$\tilde{\psi}\tilde{m}_{ML} = \tilde{u}$$

- In ML solution priori knowledge about the model is ignored
- We are looking for a solution which contains priori information about model

First Guess!

- Hard-Thresholding of migrated noisy data in Curvelet domain:

$$\hat{\tilde{u}} = \theta_T(\tilde{u}) = \theta_T(CK^* d)$$

Where: $T = \lambda\Gamma$ and $\Gamma = \sqrt{diag(\tilde{\psi})}$

- Curvelet domain Hard-Thresholding is a minimax denoising approach
- This process keeps only the events which are lying on the curves as a priori information

Constrained Optimization

- Optimization Problem:

$$\hat{m}: \min_{m} J(m) \;\; s.t. \;\; \left|\tilde{\psi}\tilde{m} - \hat{\tilde{u}}\right|_{\mu} \le e_{\mu}$$

- Or approximately:

$$\hat{m}: \min_{m} J(m) \;\; s.t. \;\; \left|diag(\tilde{\psi})\tilde{m} - \hat{\tilde{u}}\right|_{\mu} \le e_{\mu}$$

- and tolerance defined by:

$$e_{\mu} = \begin{cases} \Gamma_{\mu} & \tilde{u}_{\mu} \ge \lambda\Gamma_{\mu} \\ \lambda\Gamma_{\mu} & \tilde{u}_{\mu} \le \lambda\Gamma_{\mu} \end{cases}$$

- with e_{μ} threshold and noise-dependent *tolerance* on Curvelet coefficients
- with λ define the control parameter

Solution of Constrained Optimization

- An augmented Lagrangian method is implemented to solve optimization problem
- L1 norm is employed as J(m)
- For each subproblem a Steepest Decent method is employed
- Initial value for model is the solution of all-constraints-zero, which is the minimax denoising solution

Algorithm

$k = 0$

$m_0, \Lambda_0^{\pm}, \mu_0$ % Initial Values

% Sub Optimum Loop

$$g_k = \nabla_m \ell_A(m_k, \Lambda_k^{\pm}; \mu_k) \quad \text{\% Gradient}$$

$$\tau_k = \arg\left[\min_{\tau>0} \ell_A(m_k - \tau g_k, \Lambda_k^{\pm}; \mu_k)\right] \quad \text{\% Line Search}$$

$$m_{k+1} = m_k - \tau_k g_k \quad \text{\% Update m}$$

% *Update* Λ *and* μ

Setting the Initial Values

- Since the number of iteration is limited, setting the initial values which are near to final solution is very important
- The initial value for model is all-constraints-zero solution in our optimization problem:

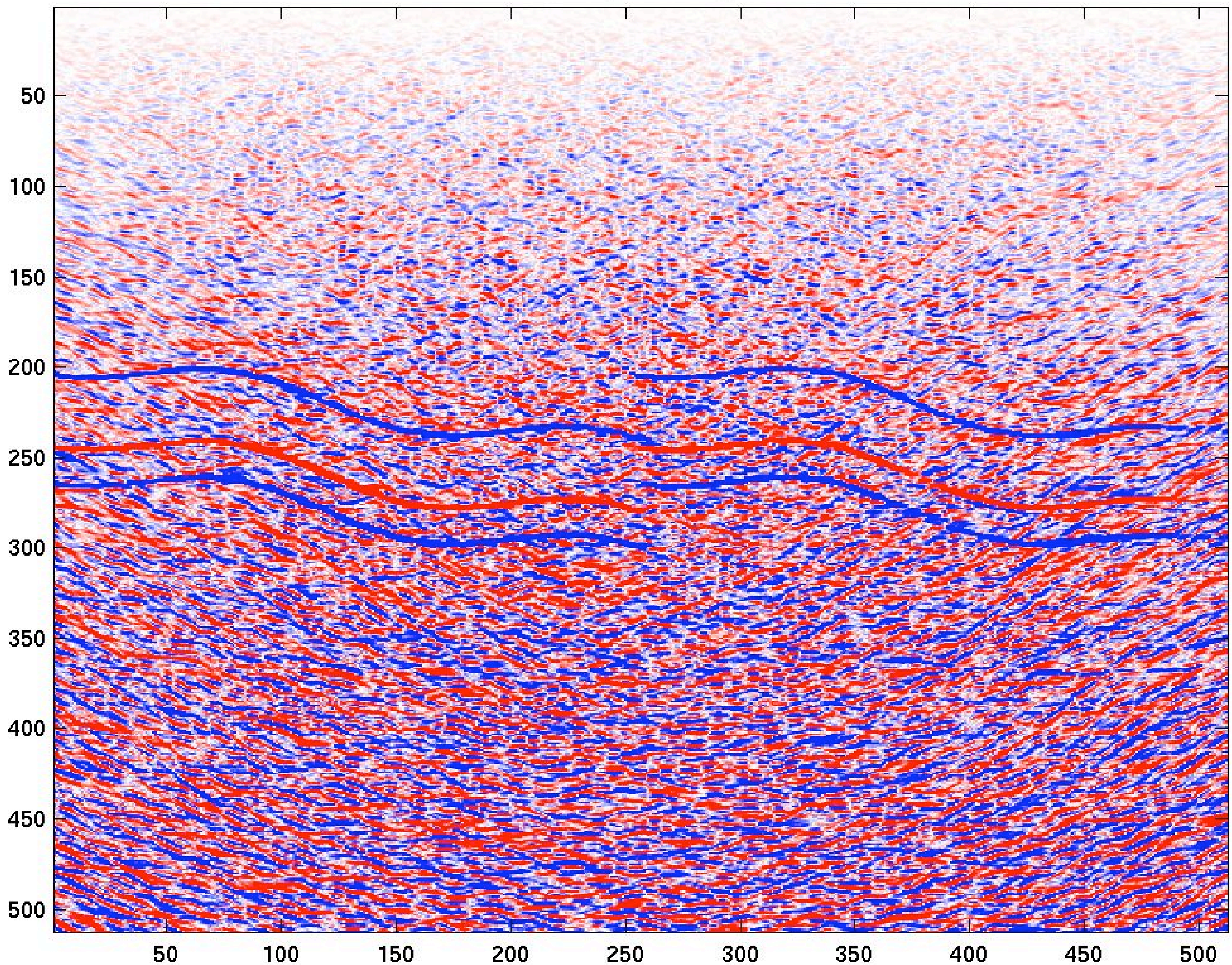
$$\tilde{\psi}\tilde{m}_0 = \hat{\tilde{u}}$$

- The initial value for Lagrange-multipliers can be in this form:

$$\Lambda_0^{\pm} = \pm\tilde{\psi}C\nabla_m J(m_0)$$

- In approximate form these value can be set as: $diag(\tilde{\psi})\tilde{m}_0 = \hat{\tilde{u}}$ $\quad \Lambda_0^{\pm} = \pm diag(\tilde{\psi})C\nabla_m J(m_0)$

Noisy Image



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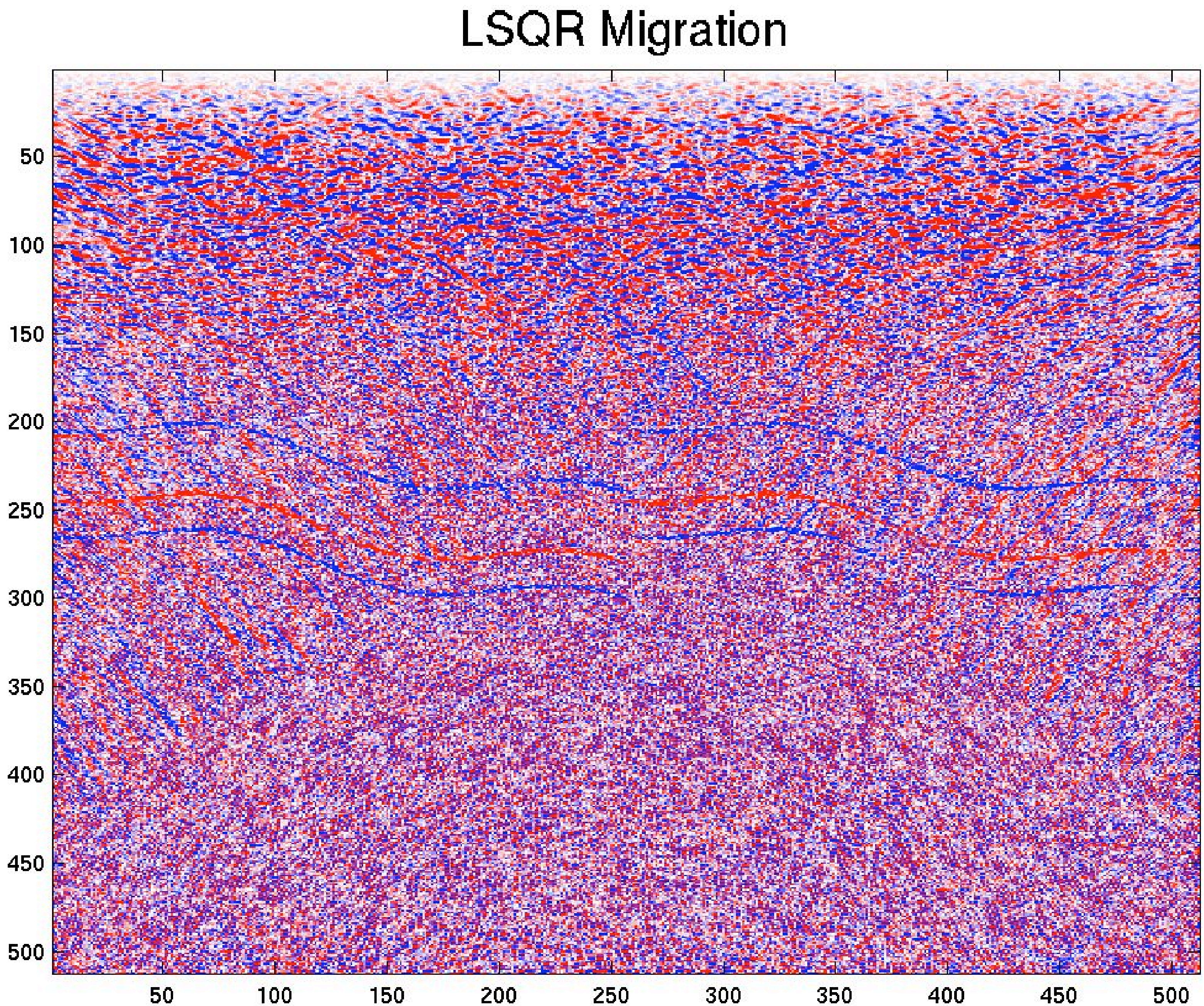
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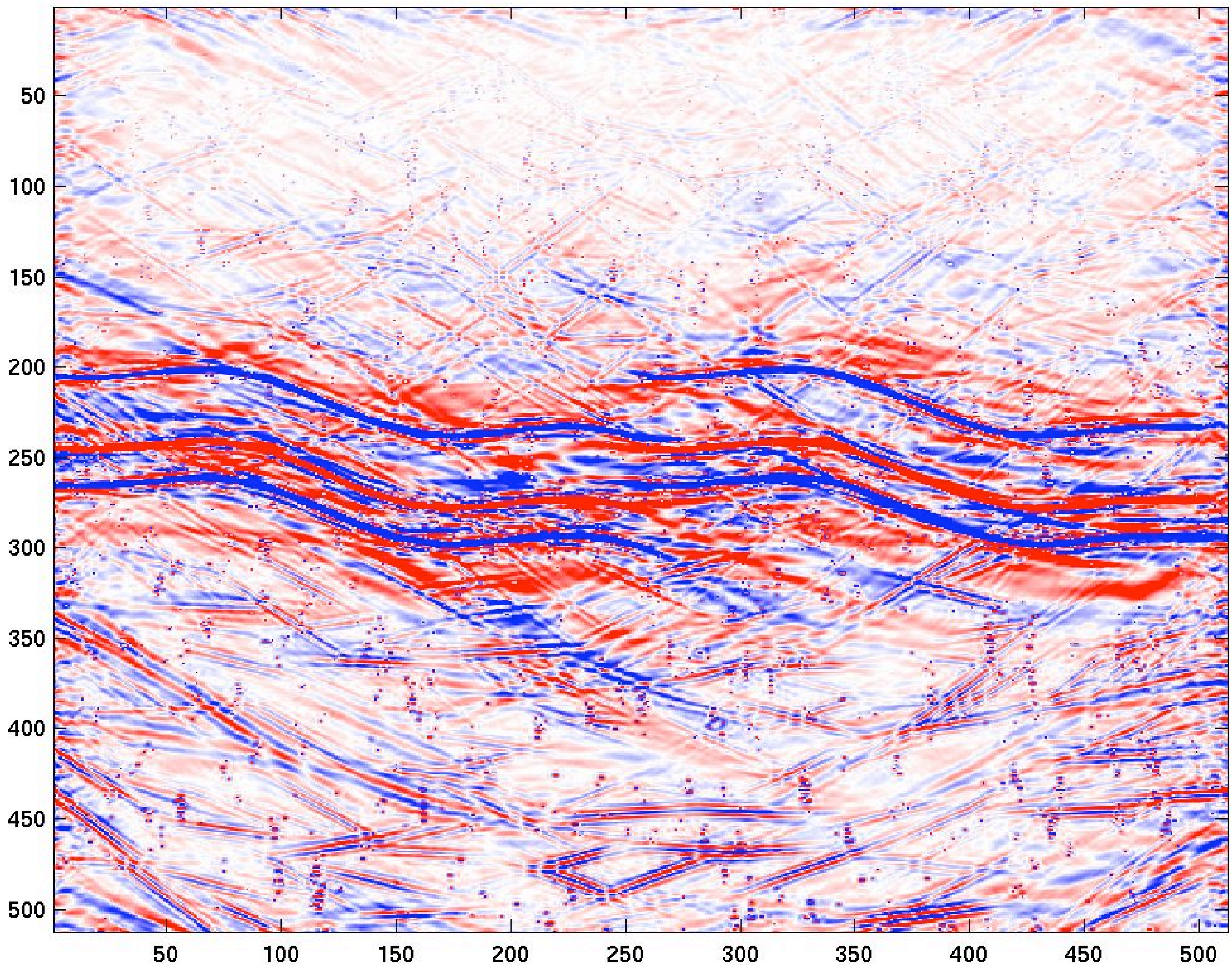
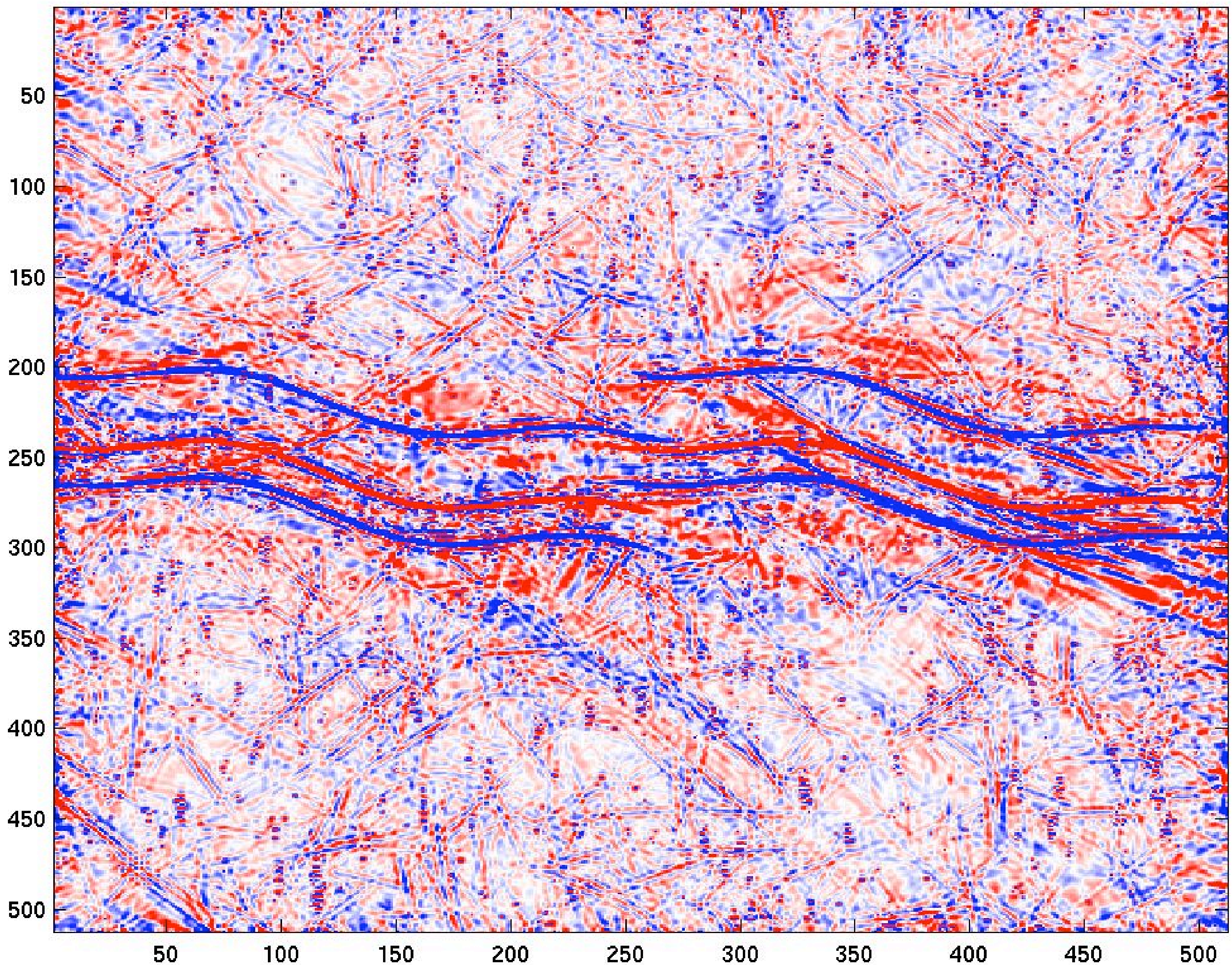
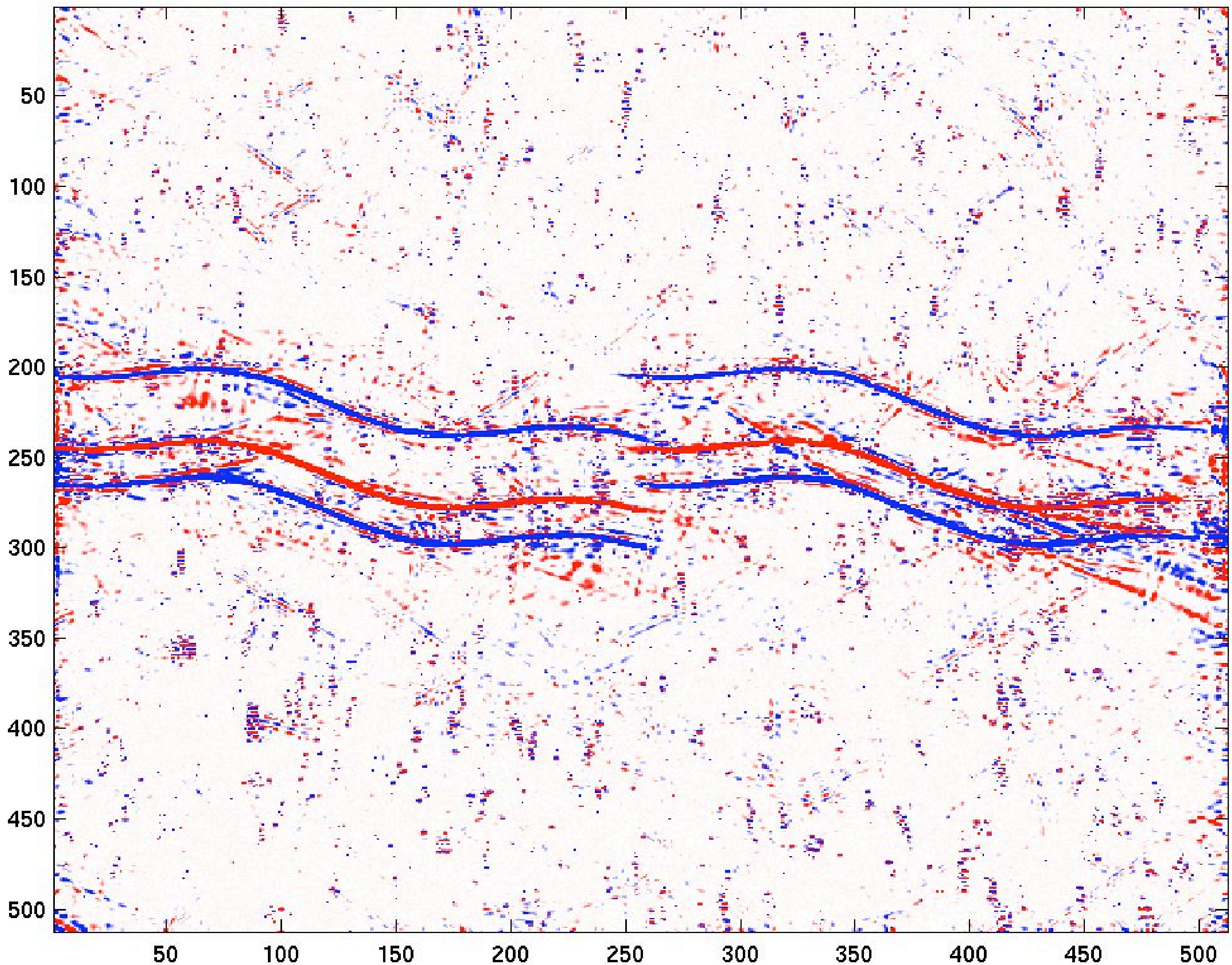
Threshold Denoised

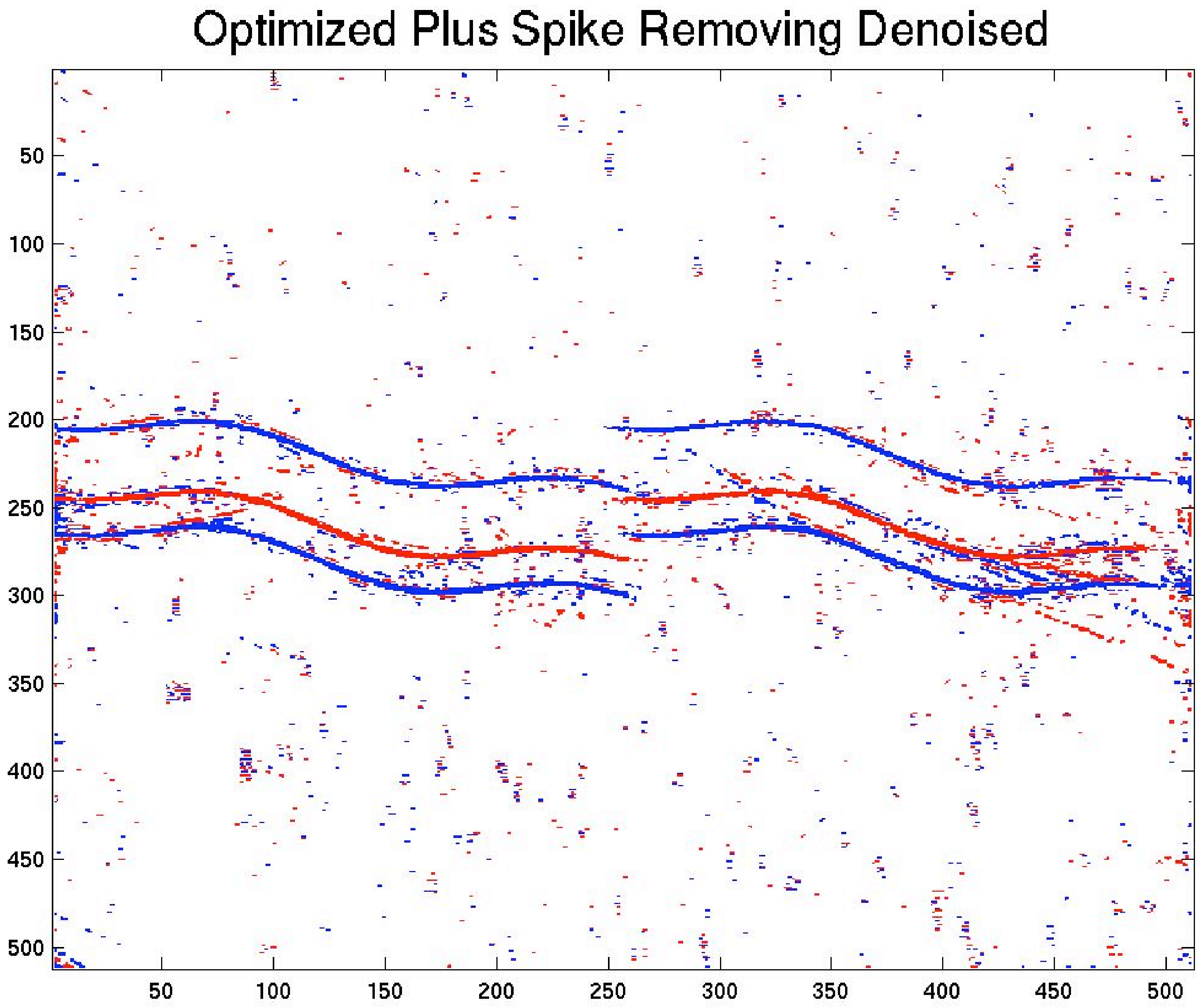


Thresholded and Corrected Denoised



Optimized Denoised




Optimized Plus Spike Removing Denoised
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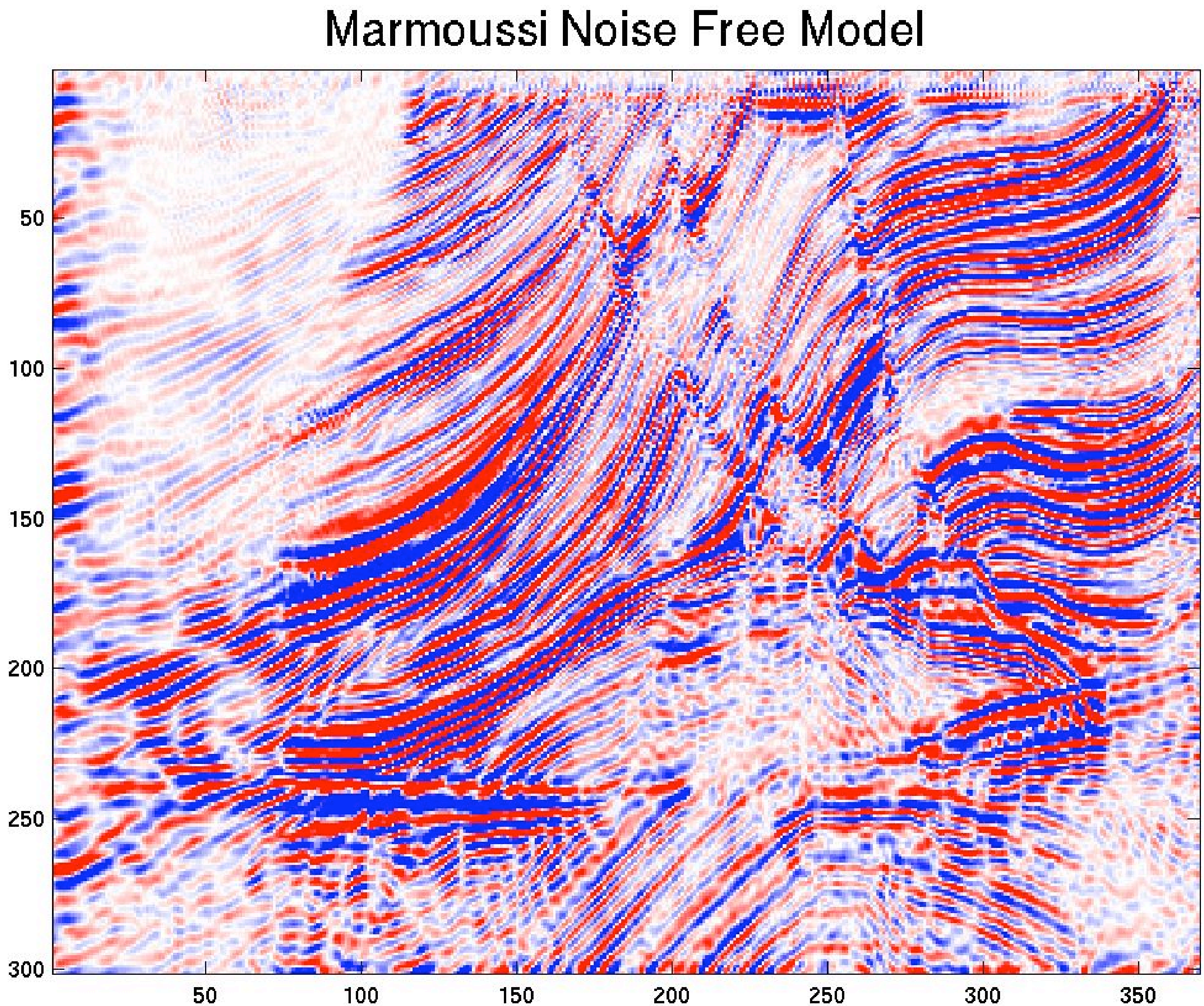
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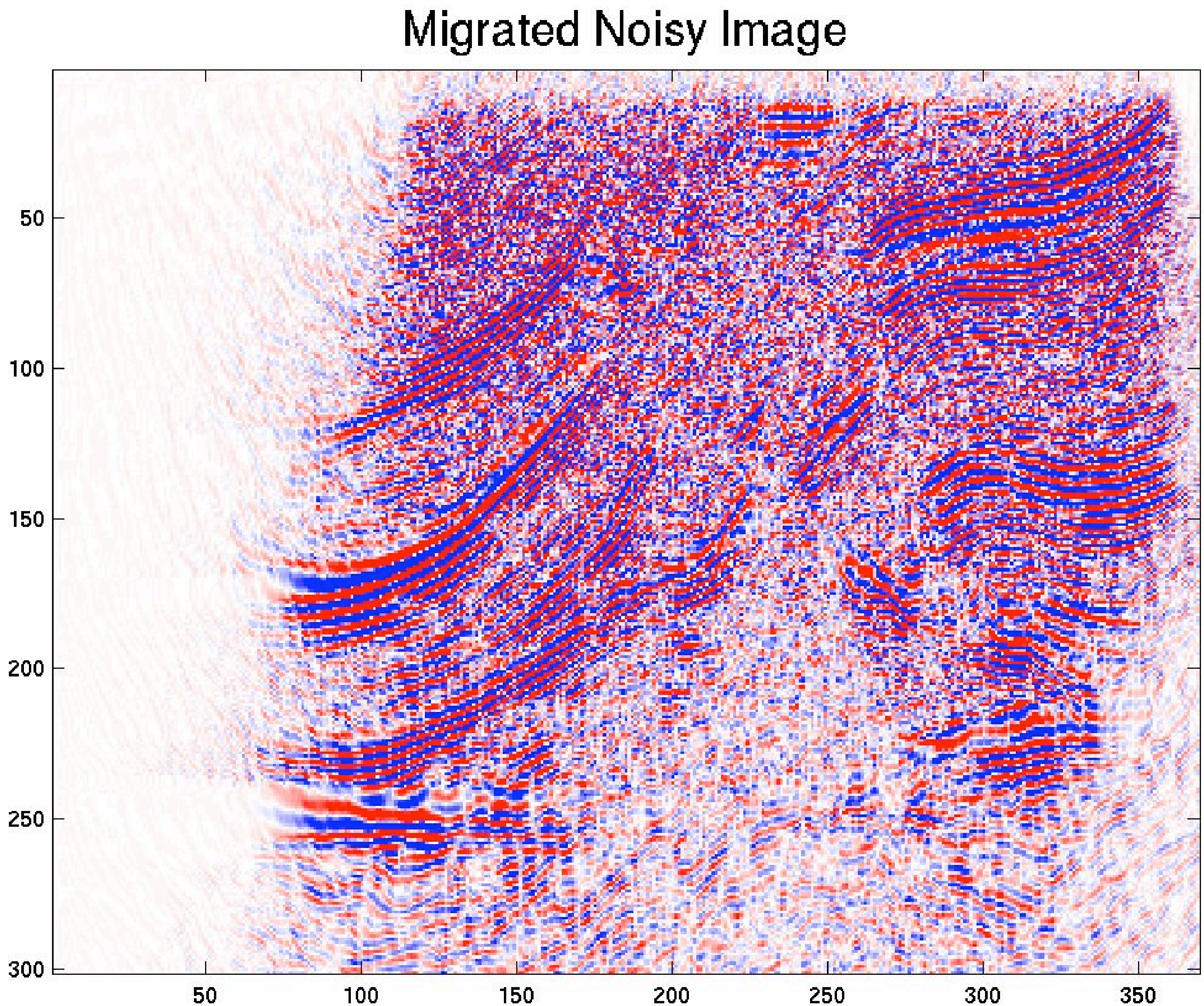
Migrated Noisy Image
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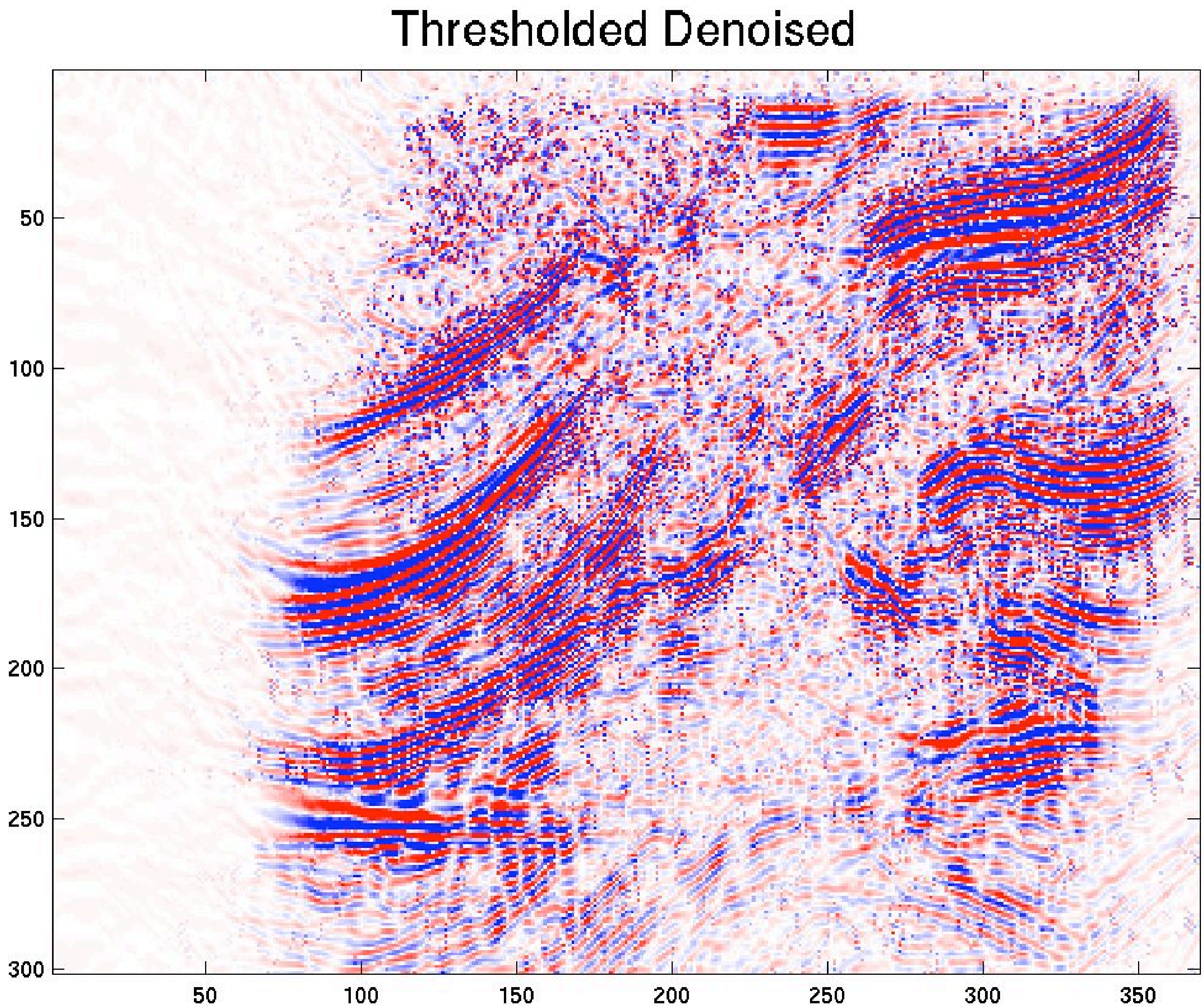
Thresholded Denoised
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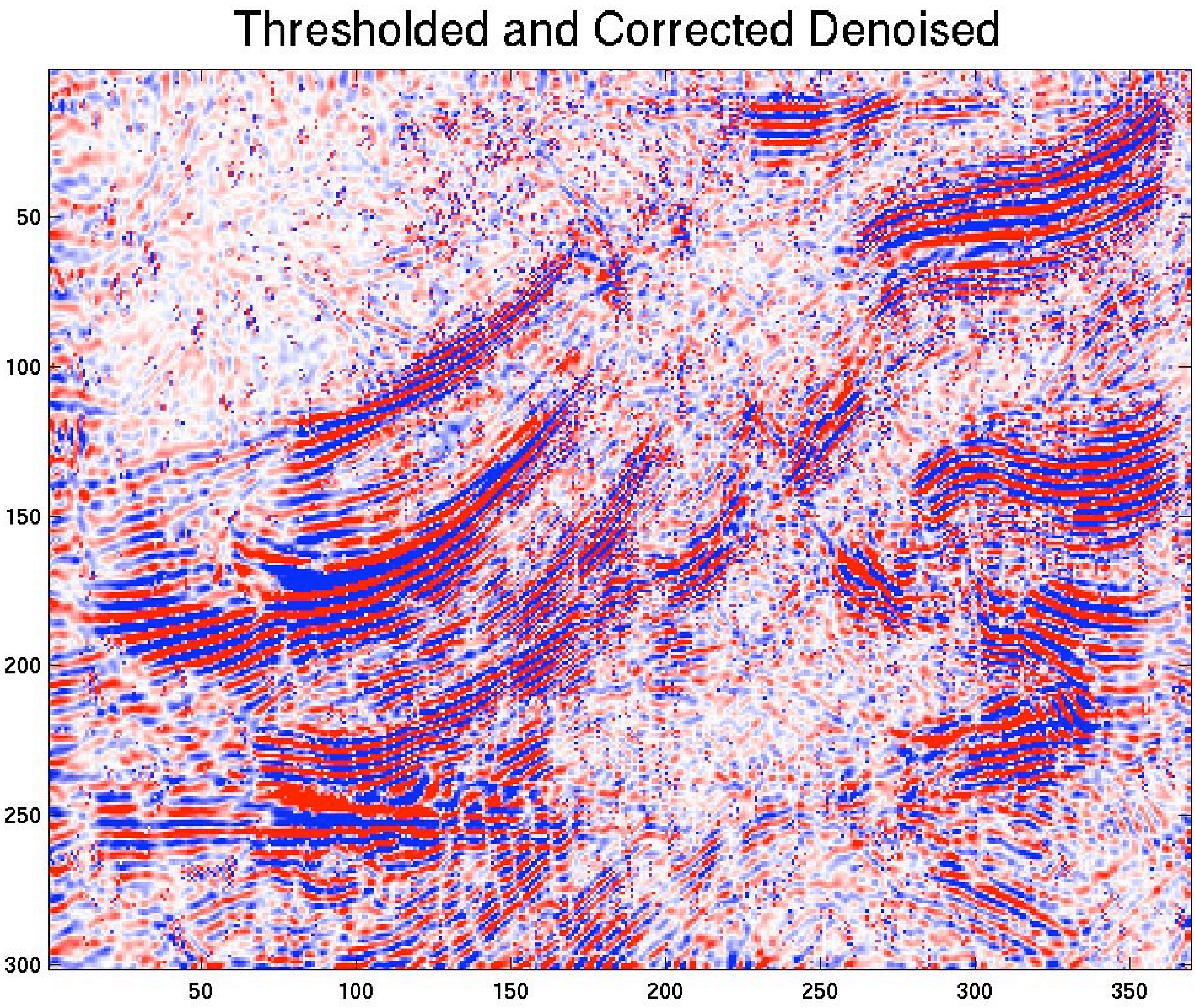

Thresholded and Corrected Denoised
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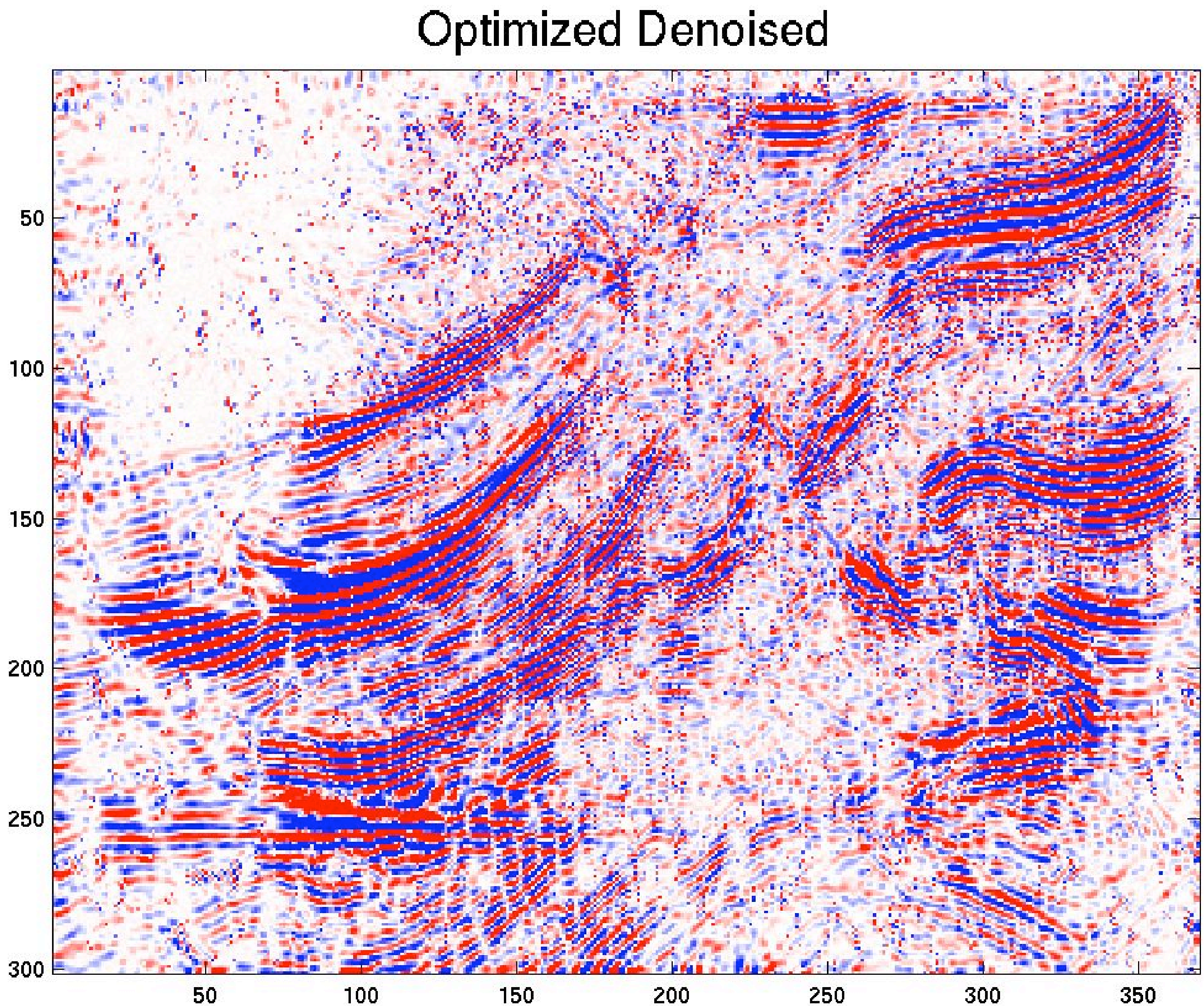

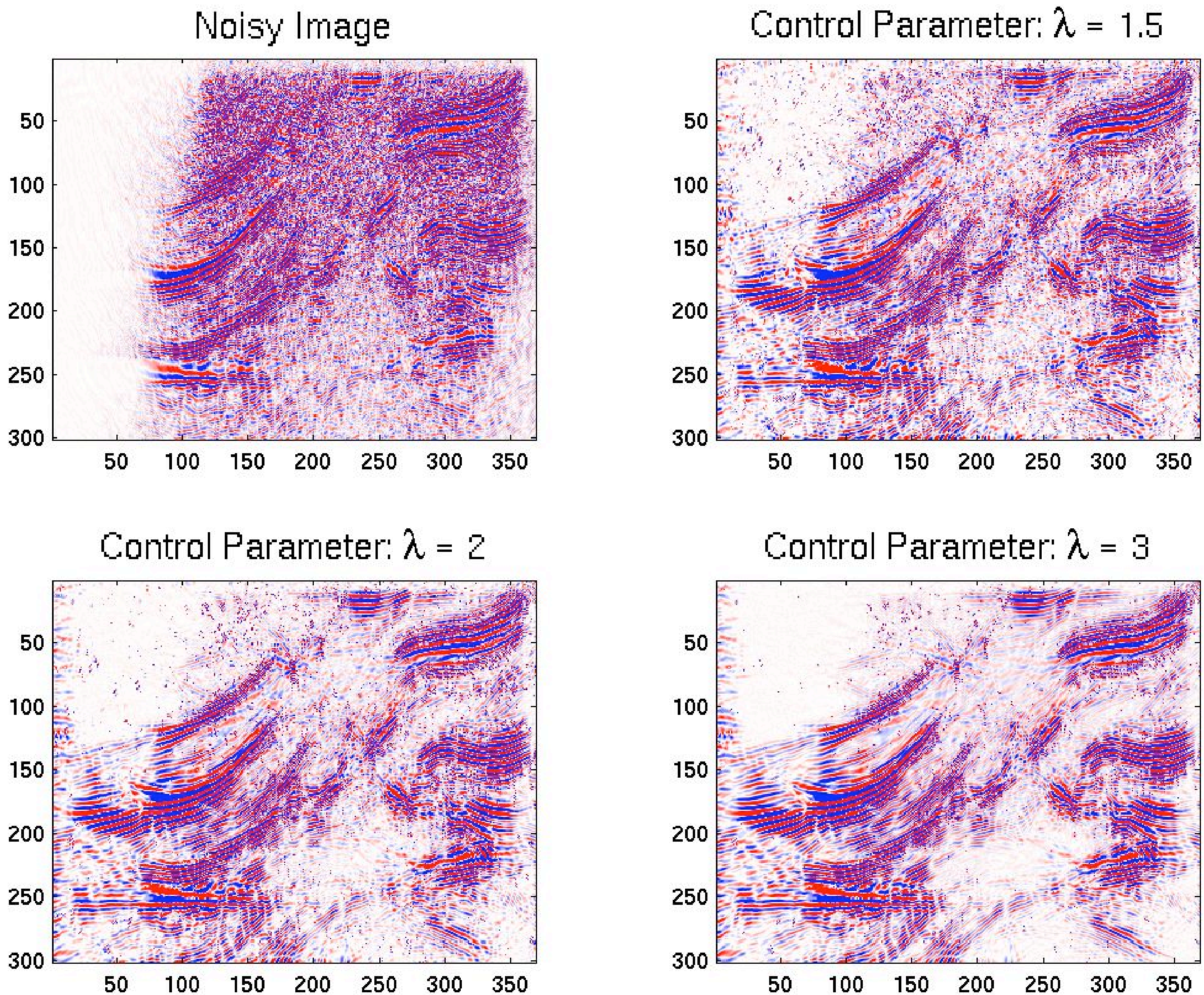
Noisy Image
Control Parameter: λ = 1.5
Control Parameter: λ = 2
Control Parameter: λ = 3
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Conclusion

- Preconditioning using Curvelet Transform introduced and it shows enhancement in the characteristics of migration and normal operators.
- A new constrained optimization is constructed by imposing sparsity on the model subject to bounded constraints defined by noise level
- An augmented-Lagrangian method to solve the optimization problem is implemented
- Results show Significant improvement rather than Least Square migration