Curvelet-domain multiple elimination with sparseness constraints

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Context

SRME (Verschuur, Guitton, Berkhout, Weglein, Innanen)

Sparse Radon (Ulrych, Sacchi, Trad, etc.)

Pattern recognition (Spitz)

Redundant dictonaries/Morphological component separation/Pursuits (Mallat, Chen, Donoho, Starck, Elad)

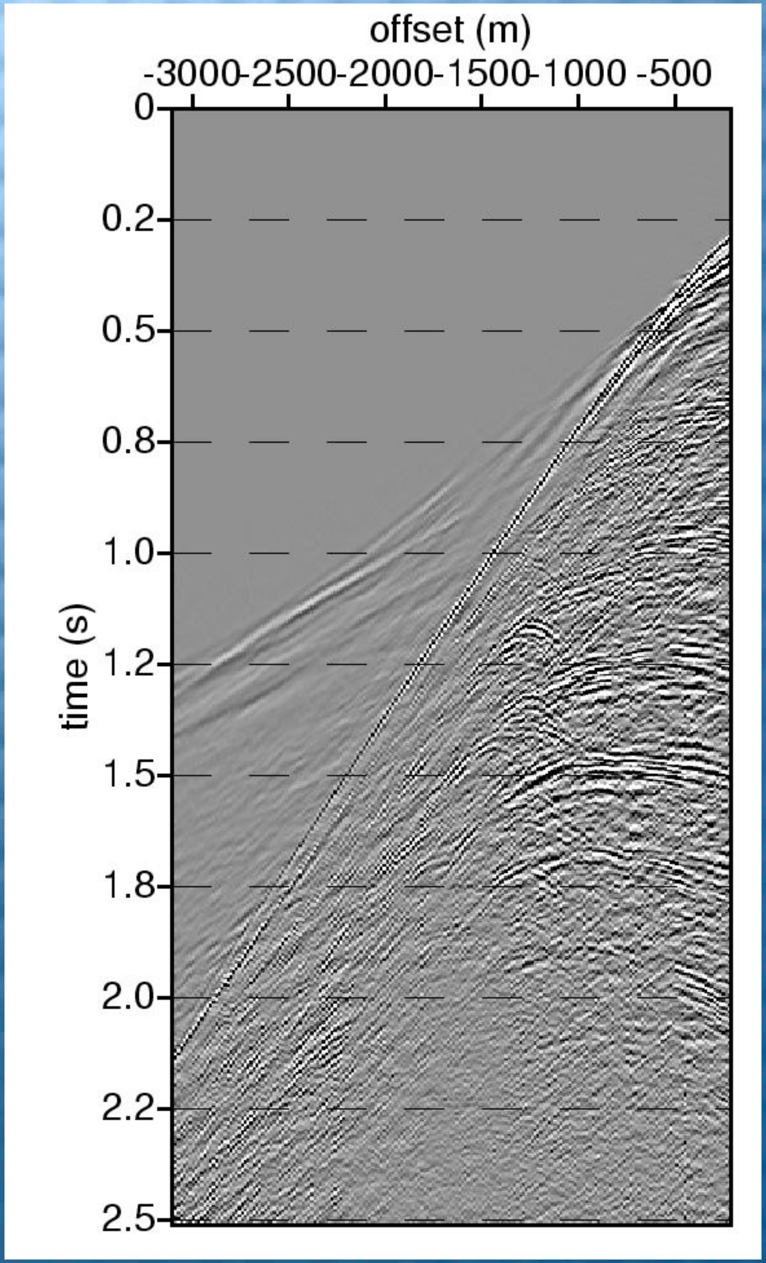
2-D/3-D Curvelets (Starck, Donoho, Candes, Demanet, Ying)

Motivation

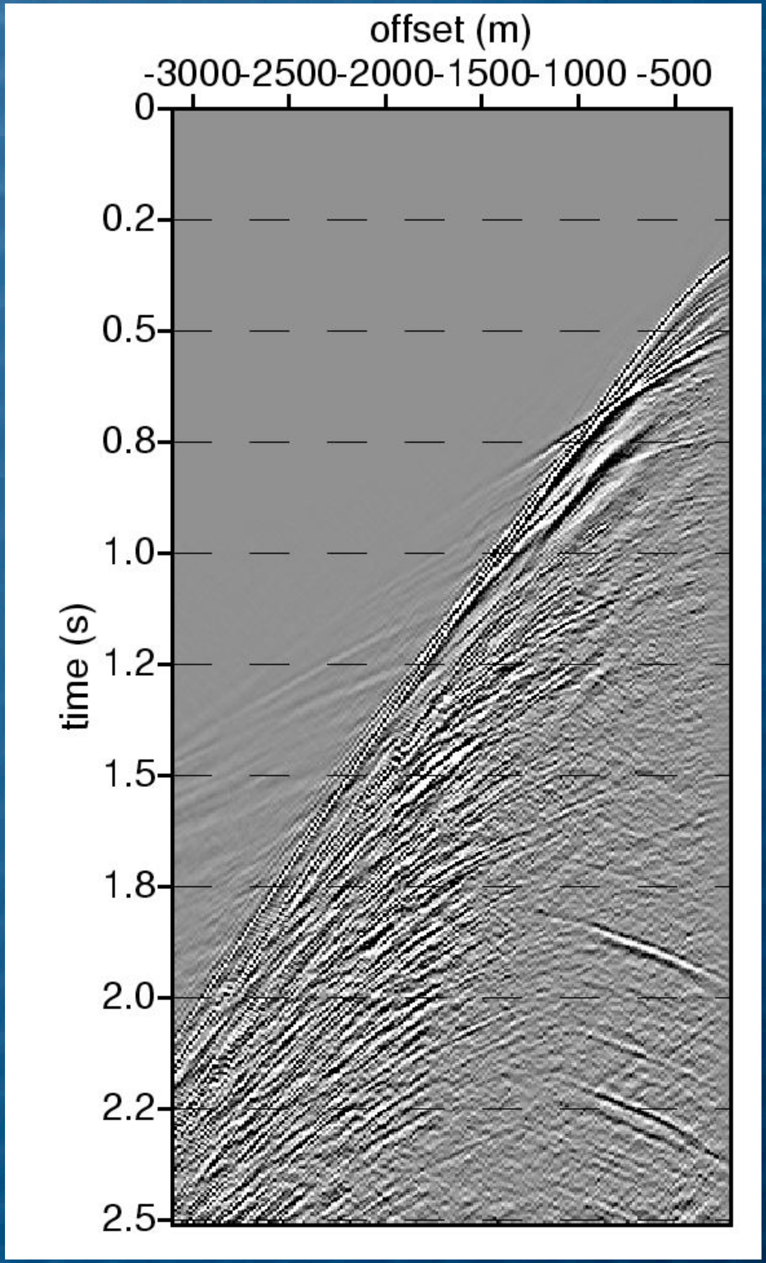
- **Geometry effects in 3D surface multiple prediction are a *bottle neck!***
- **New adaptive subtraction robust under**
  - **phase rotations**
  - **misalignments**
  - **incoherent noise**
- ***Use latest ‘optimal’ results from theoretical image processing***

- **Multiple elimination:**
  - **“denoising”/signal separation problem**
- **Solve with Curvelets (2-D/3-D)**
  - ***whiten* “Curvelet spectrum”**
  - **remove multiples with *thresholding***
  - ***separate*, *interpolate* & *denoise* with *iterative thresholding***
- **Effective when multiple prediction not too far from actual**

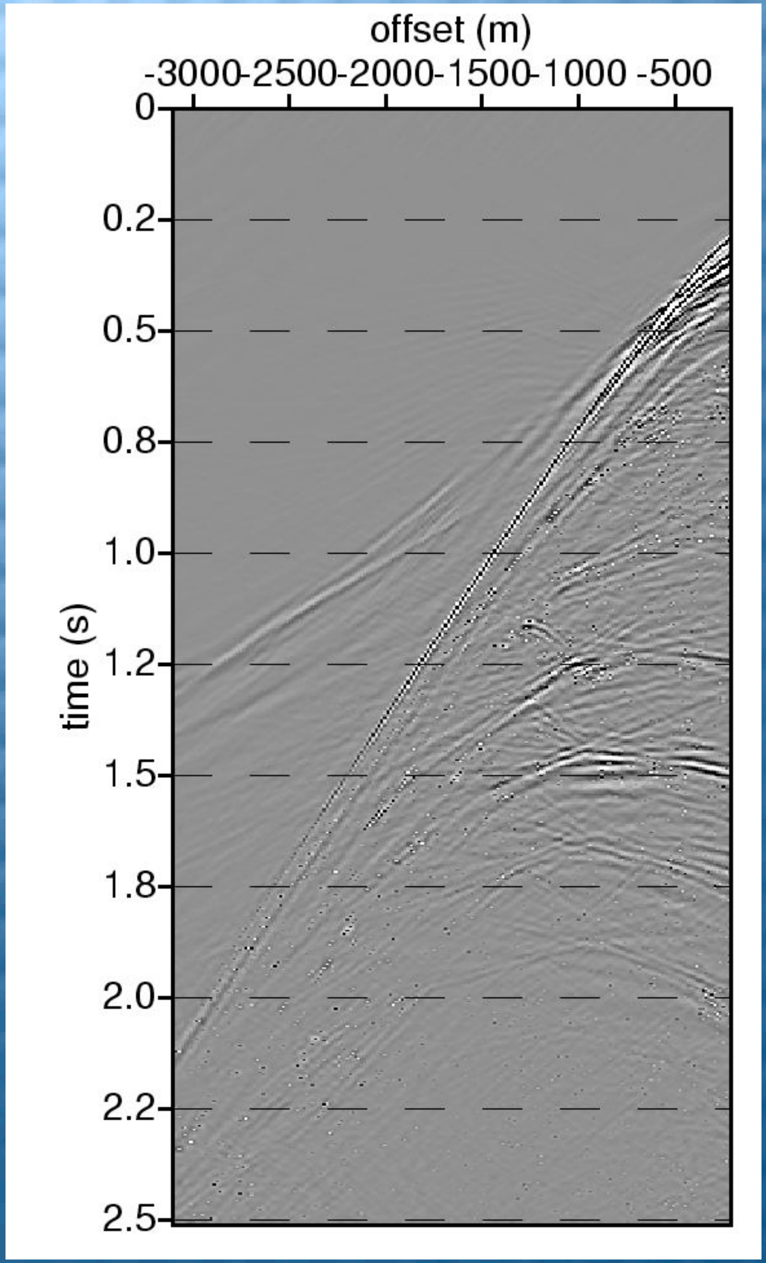
Subtraction with L2 norm



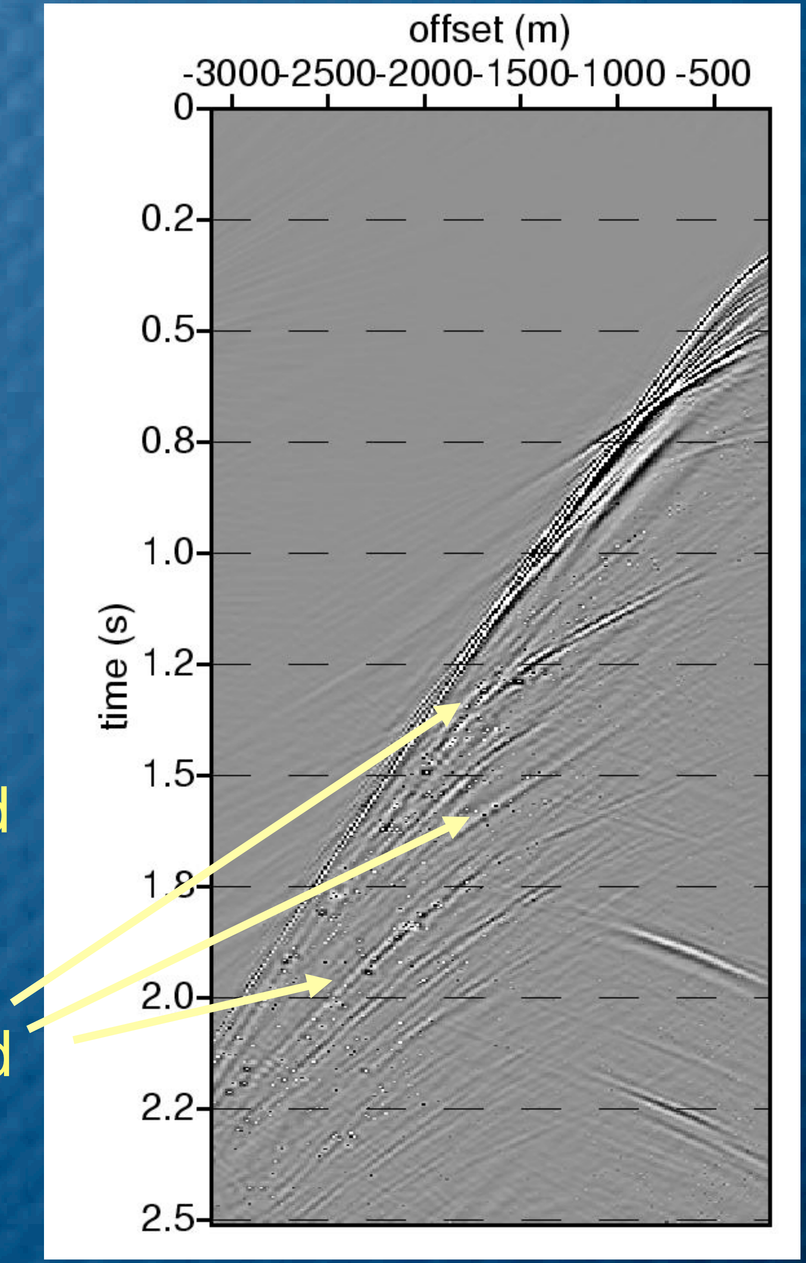
Output SRME multi-L2 subtraction



Multiple suppression with curvelets

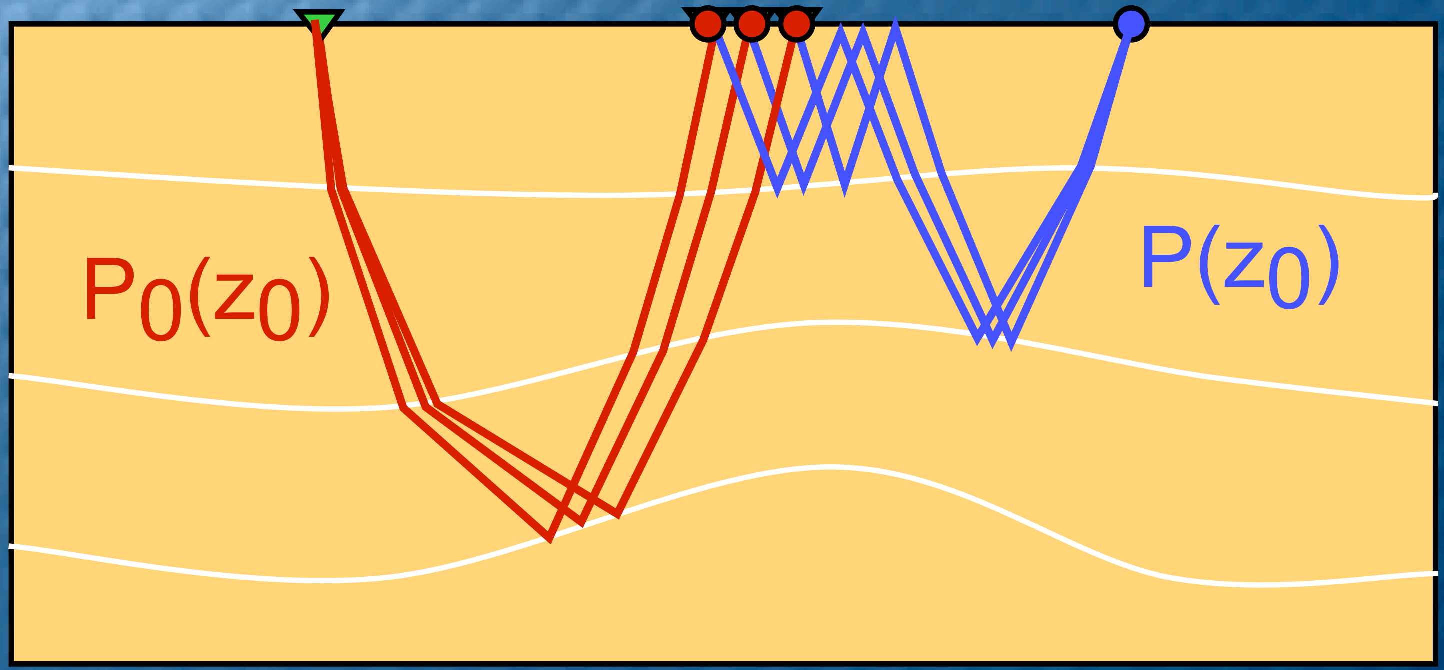


Output curvelet filtering with stronger threshold

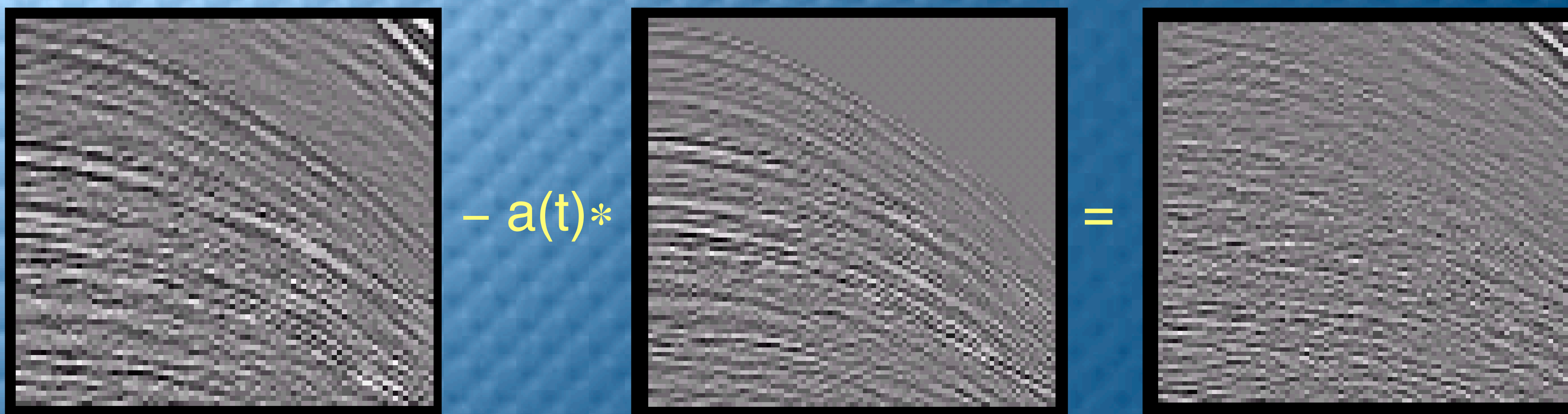


Surface multiple elimination

Multiple prediction:
data convolution along surface



L2 adaptive subtraction



Input data with multiples **Predicted multiples** **output data**

Adaptive subtraction based on minimum energy in the output

L2/L1 matched filter

Matched filter:

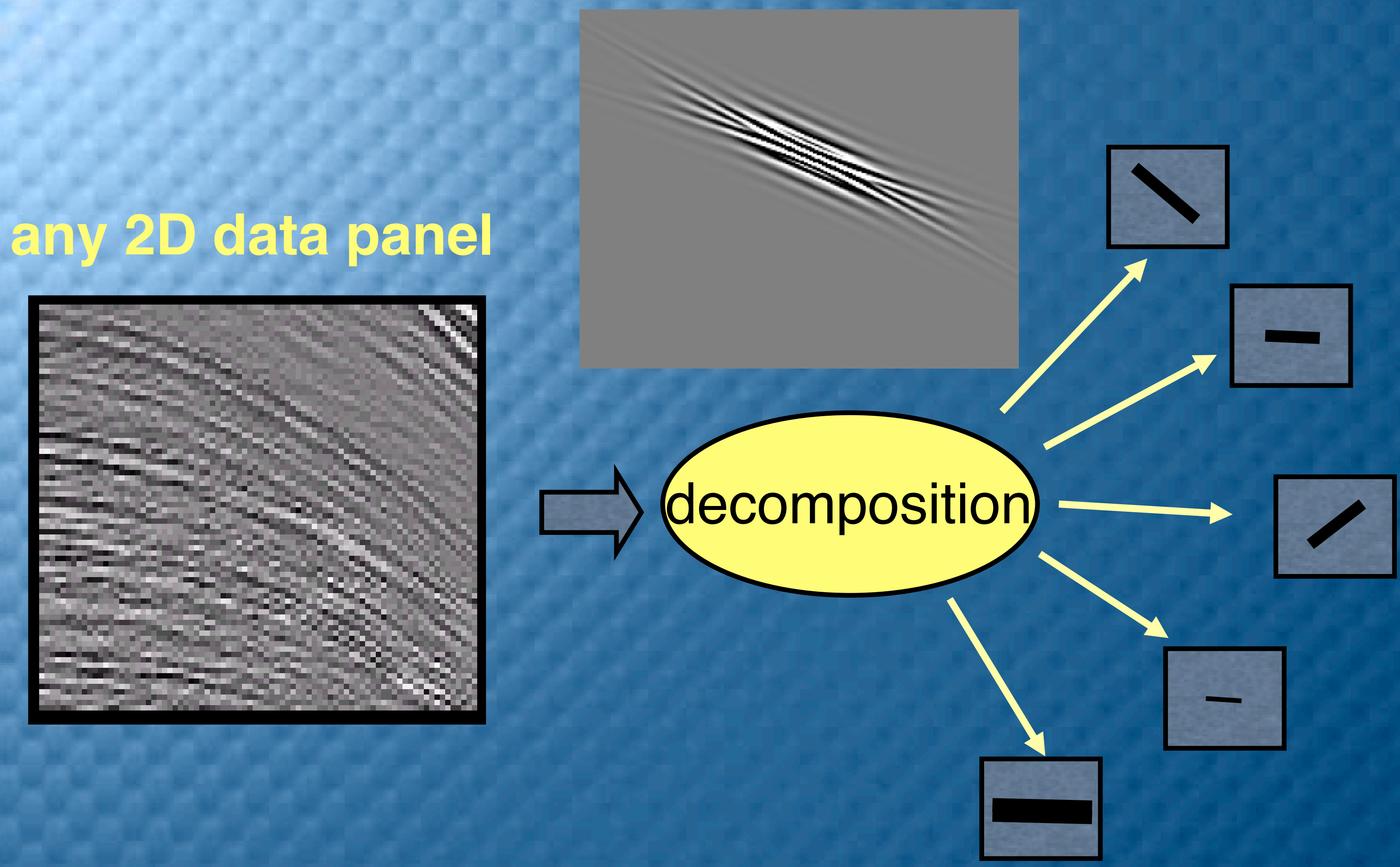
$$\underbrace{\hat{\mathbf{n}}}^{\text{denoised}} : \min_{\mathbf{\Phi}} = \| \underbrace{\mathbf{d}}_{\text{noisy data}} - \overbrace{\mathbf{\Phi}}^{\text{matched filter}} \overset{t}{*} \underbrace{\mathbf{m}}_{\text{pred. noise}} \|_p$$

- p=1 enhances sparseness
- residue is the denoised data
- risk of over fitting

May loose primary reflection events ...

Guitton&
Verschuur'04

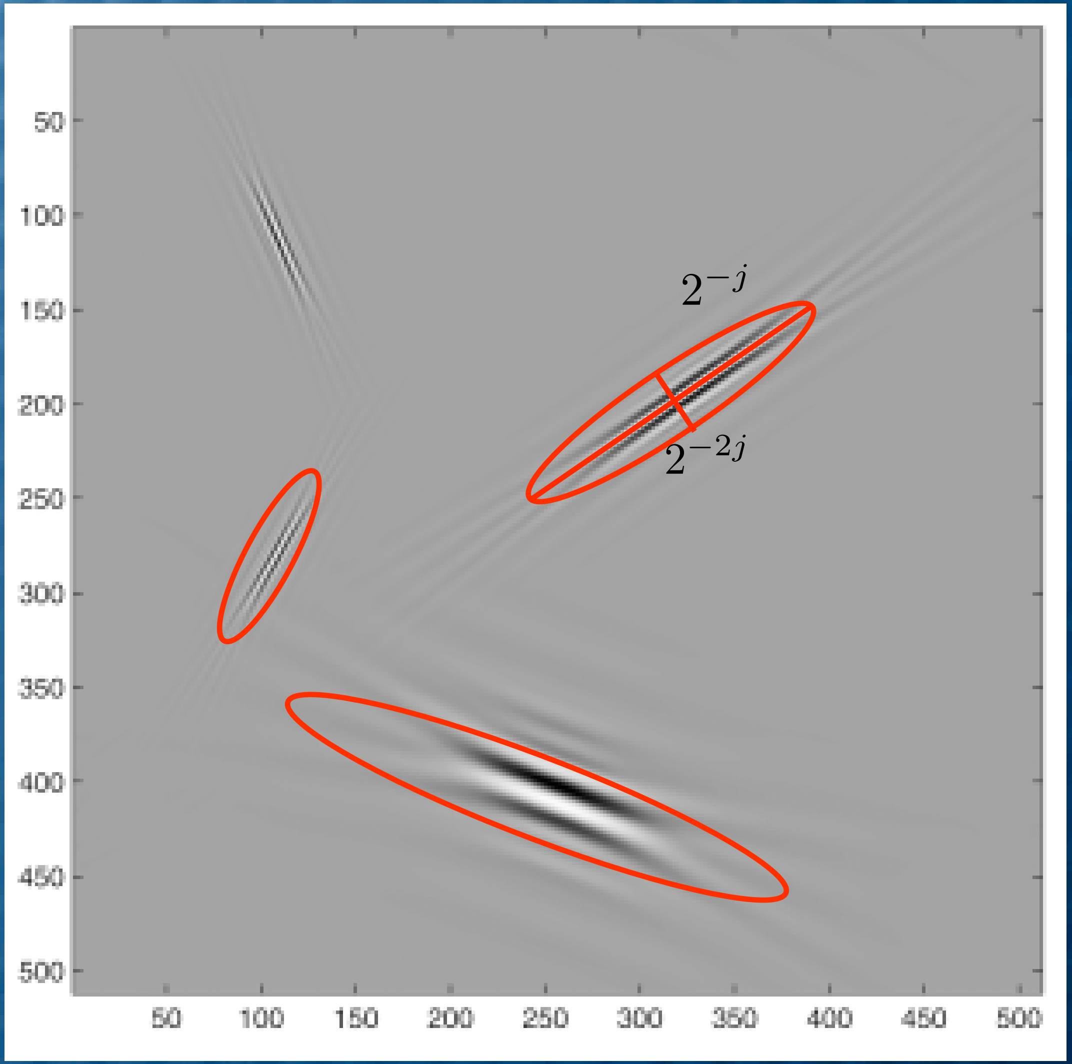
Curvelet domain



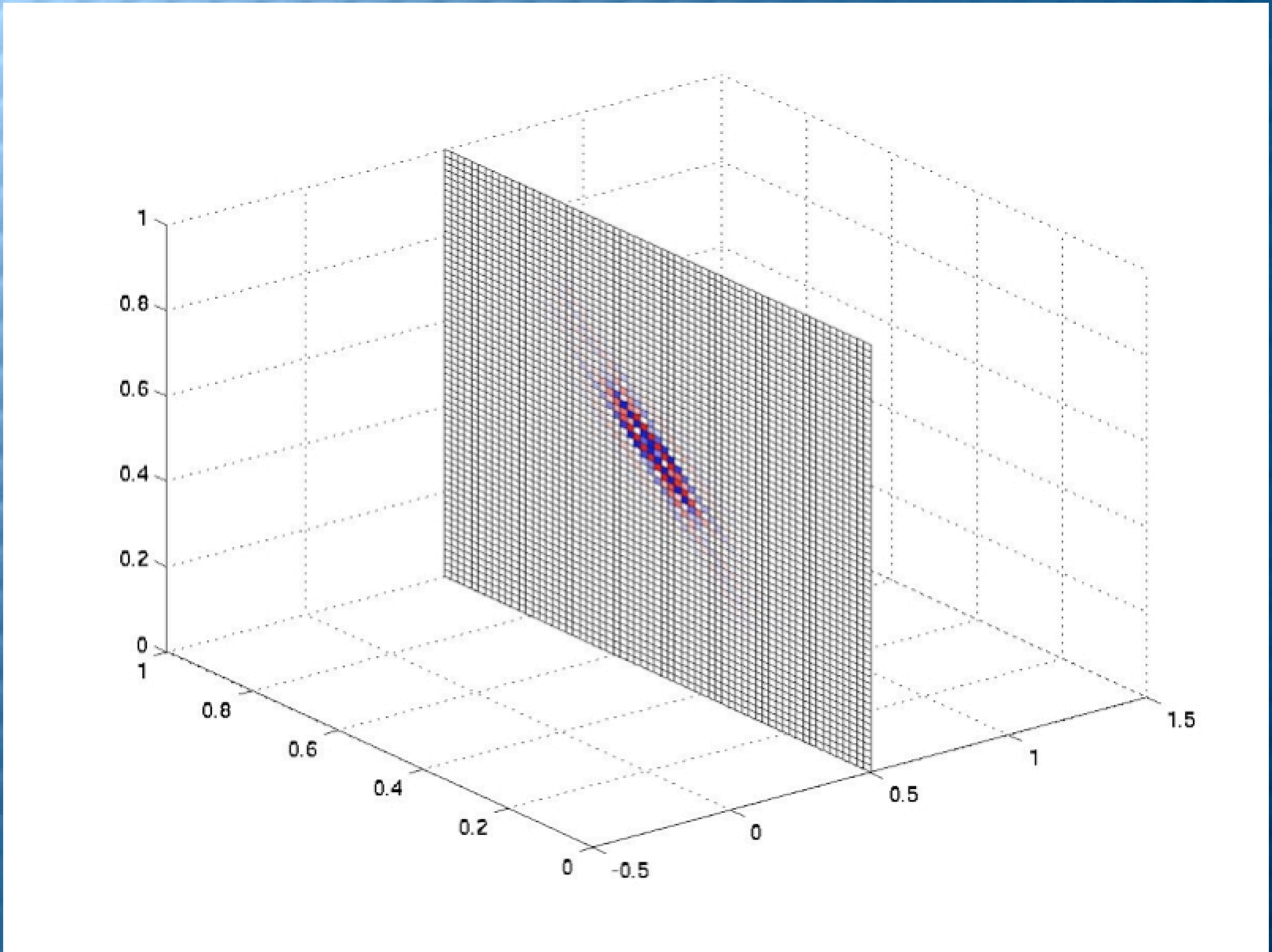
- **Almost orthogonal decomposition into multiscale basis functions with local frequency and local dip properties**
- **Natural basis for wave equations**
- **Consist of plane wavelets invariant under convolution**

Why curvelets

- Nonseparable
- Local in 2-D space
- Local in 2-D Fourier
- Anisotropic
- Multiscale
- Almost orthogonal
- Tight frame $\mathrm{B}^T\mathrm{B} = \mathrm{I}$
- Optimal

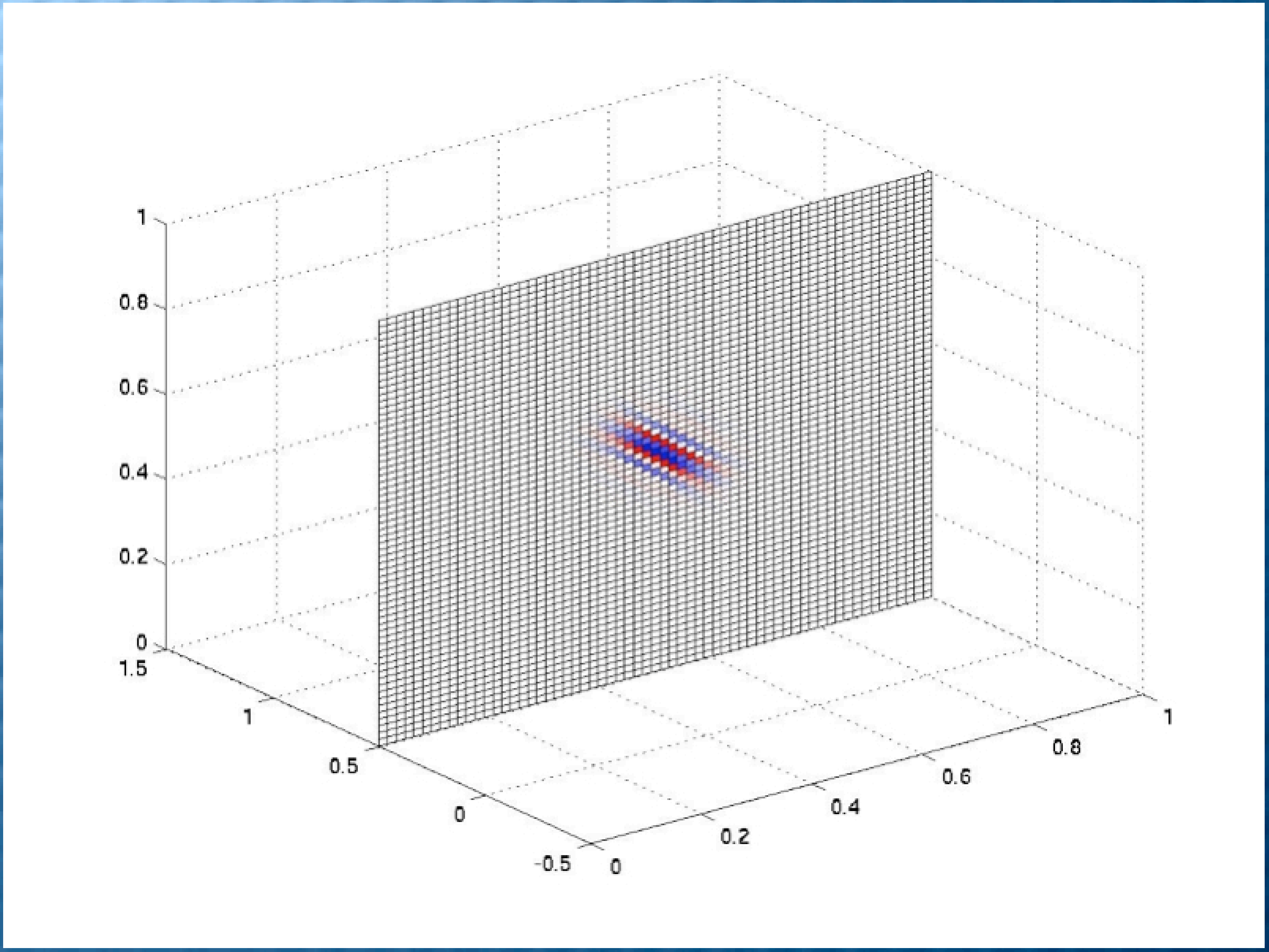


3-D Curvelets



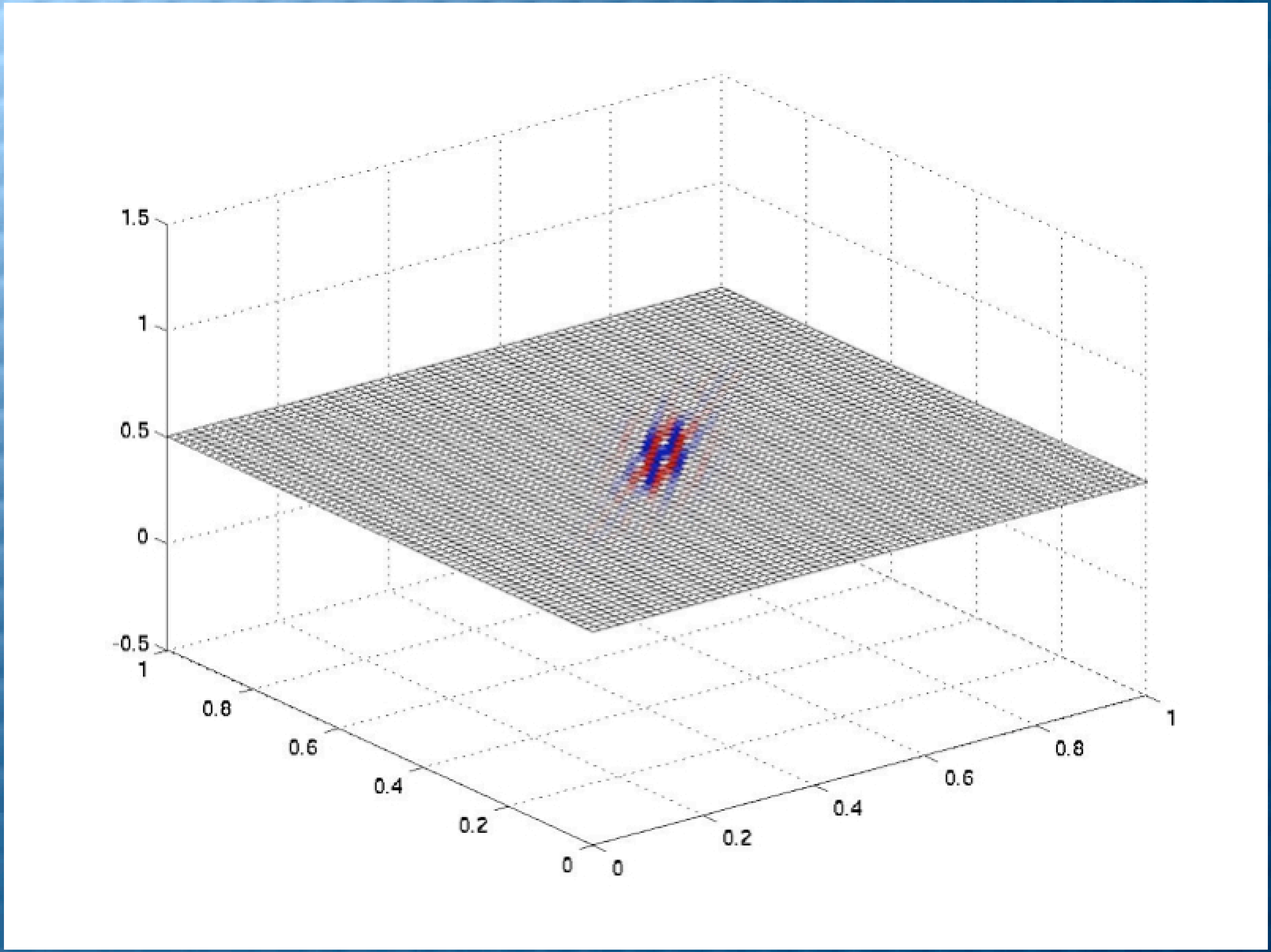
Thanks to Demanet & Ying

3-D Curvelets



Thanks to Demanet & Ying

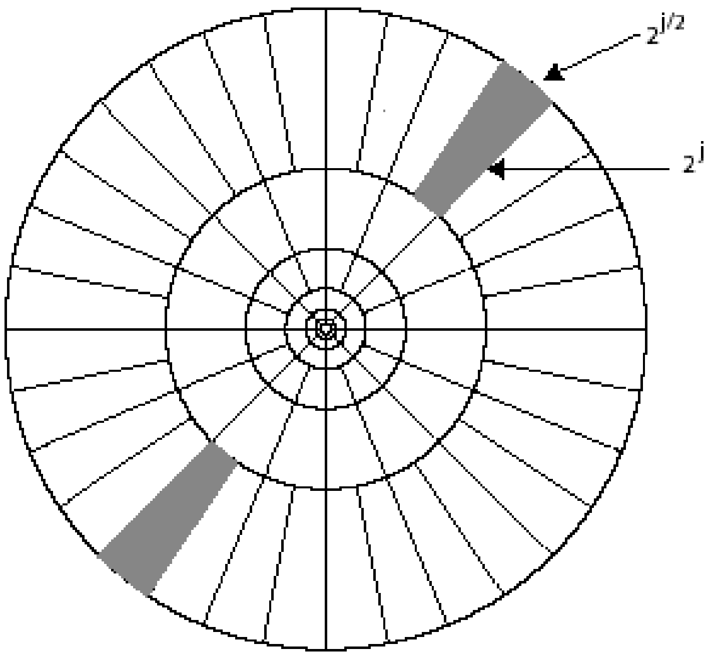
3-D Curvelets



Thanks to Demanet & Ying

Why curvelets

$$W_j = \{\zeta, \quad 2^j \le |\zeta| \le 2^{j+1}, |\theta - \theta_J| \le \pi \cdot 2^{\lfloor j/2 \rfloor}\}$$



second dyadic partitioning

source: Candes'01, Stein '90

Fourier/SVD/KL

$$||f - \tilde{f}_m^F|| \propto m^{-1/2}, \; m \to \infty$$

Wavelet

$$||f - \tilde{f}_m^W|| \propto m^{-1}, \; m \to \infty$$

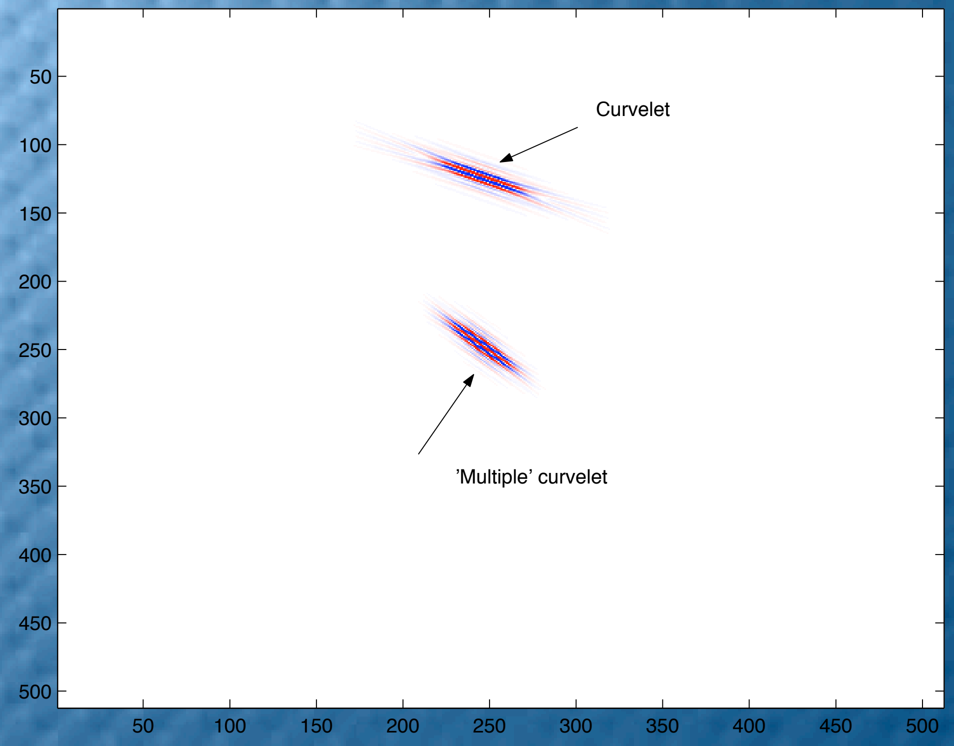
Optimal data adaptive

$$||f - \tilde{f}_m^A|| \propto m^{-2}, \; m \to \infty$$

Close to optimal Curvelet

$$||f - \tilde{f}_m^C|| \le C \cdot m^{-2} (\log m)^3, \; m \to \infty$$

Curvelet 'Multiples'



- **Almost diagonalize Green's functions (Candes & Demanet '04)**
- **Natural basis for wave equations**
- **Invariant under convolution, i.e. 'multiple multiple' = curvelet-like**
- **Curvelets sense *local* dip and *local* frequency content and can *discriminate* on these properties**

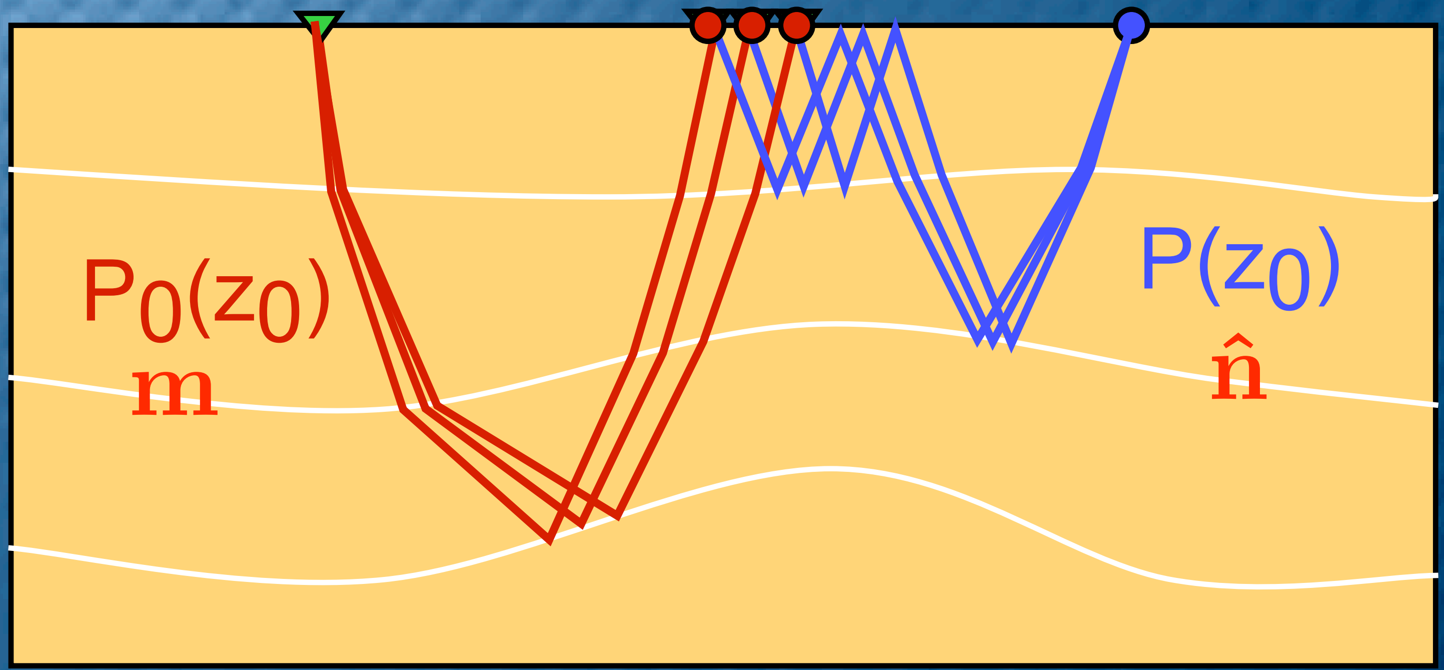
Curvelet adaptive subtraction

Remainder no longer primaries

Primaries are model m

Multiples are noise n

Predicted multiples are $\hat{\mathbf{n}}$



Colored denoising

$$\overbrace{\mathbf{d}}^{\text{noisy data}} = \underbrace{\mathbf{m}}_{\text{noise-free}} + \overbrace{\mathbf{n}}^{\text{col. noise}}$$

Denoising:

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{C}_n^{-1/2}(\mathbf{d}-\mathbf{m})\|_2^2 + J(\mathbf{m})$$

with covariance

$$\mathbf{C}_n \equiv \mathbf{E}\{\mathbf{n}\mathbf{n}^T\}$$

and both m, n **related to PDE**

Weighted thresholding

Covariance model & noise near diagonal:

$$\mathbf{BC}_{n\,\mathrm{or}\,m}\mathbf{B}^T \approx \mathbf{\Gamma}^2 \quad \text{near diagonal}$$

For ortho basis and app. noise prediction:

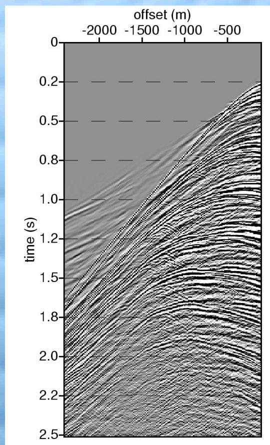
$$\boxed{\hat{m} = \mathbf{B}^T S_{\lambda\Gamma}\left(\mathbf{Bd}\right)}$$

equivalent to

$$\boxed{\hat{\mathbf{m}} = \mathbf{B}^T \arg\min_{\tilde{m}} \frac{1}{2}\|\mathbf{\Gamma}^{-1}\left(\tilde{\mathbf{d}} - \tilde{\mathbf{m}}\right)\|_2^2 + \|\tilde{\mathbf{m}}\|_{1,\lambda}}$$

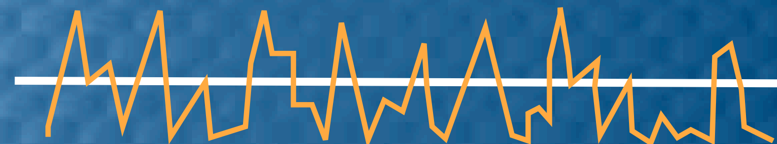
$$\tilde{\mathbf{d}} = \mathbf{Bd}, \quad \tilde{\mathbf{m}} = \mathbf{Bm} \quad \text{and} \quad \mathbf{\Gamma} = [\mathrm{diag}\{\mathrm{diag}\{\mathbf{B}\hat{\mathbf{n}}\}\}]^{1/2}$$

Multiple suppression with curvelets



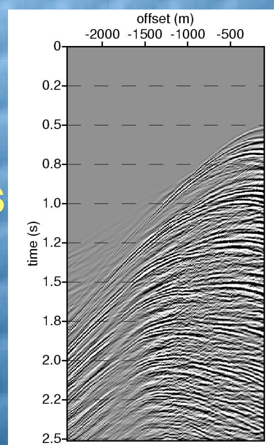
Input data with multiples

Curvelet transform



$\mathbf{Bd}$

Curvelet coeff. data



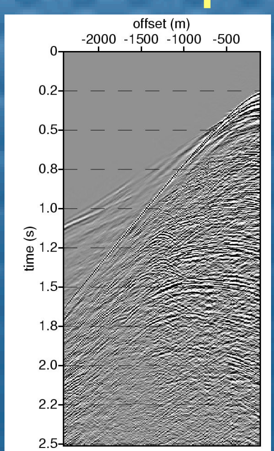
predicted multiples

Threshold

Γ

$|\mathbf{B}\hat{\mathbf{n}}|$

Curvelet coeff. pred. multiples



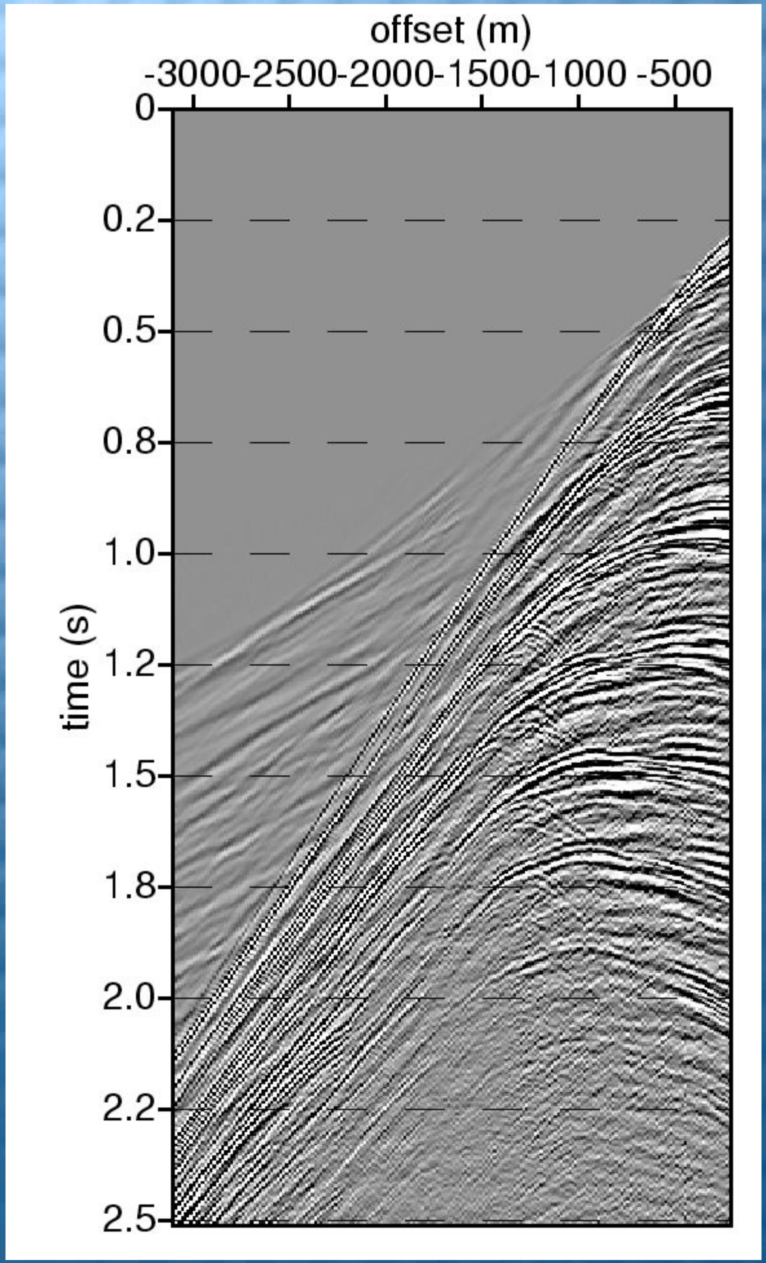
Filtered input data

Inv. curvelet transform

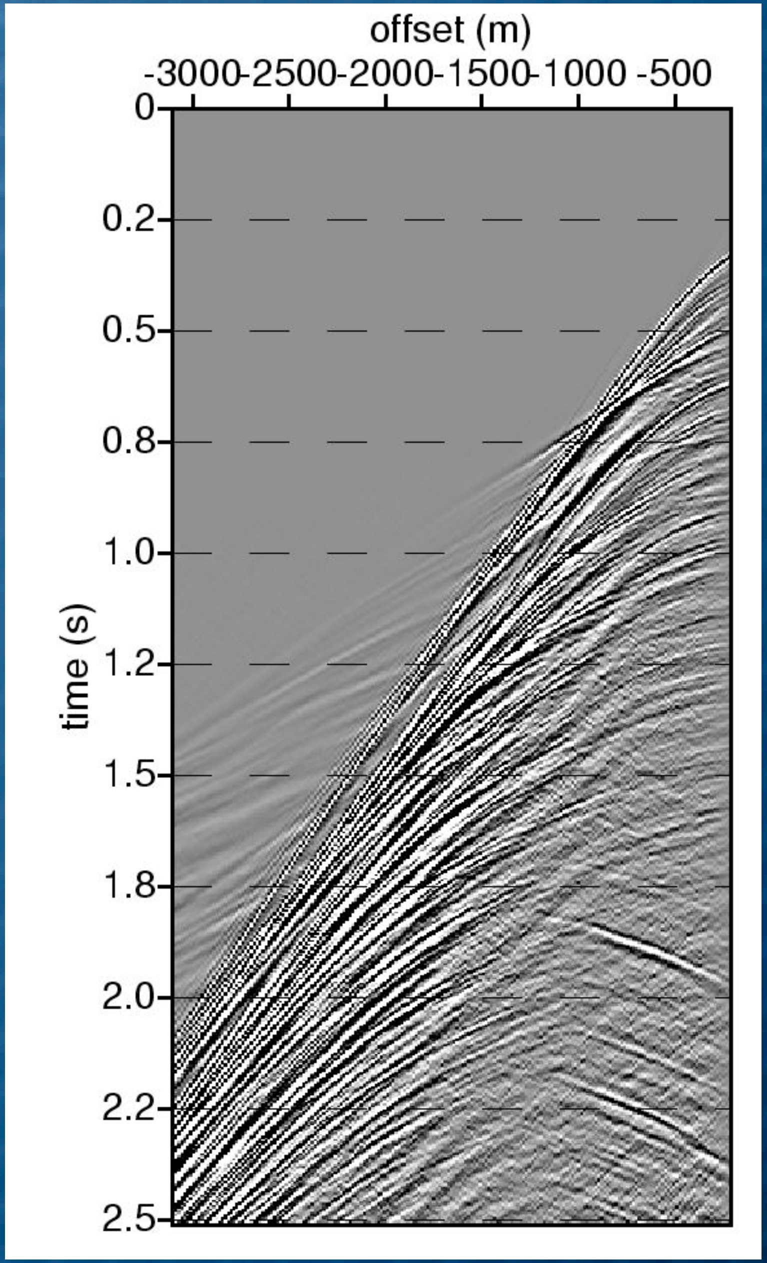
$S_{\lambda\Gamma}(\mathbf{Bd})$

Curvelet coeff. primaries

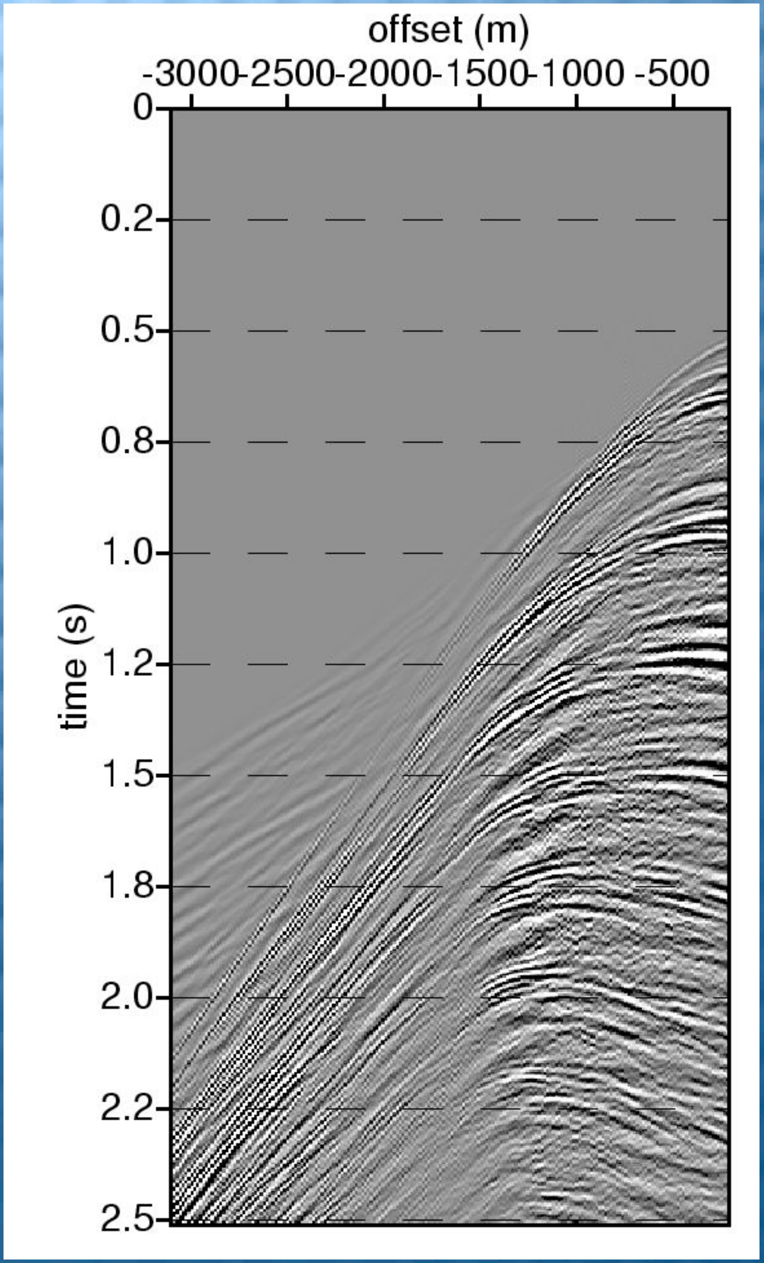
Subtraction with L2 norm



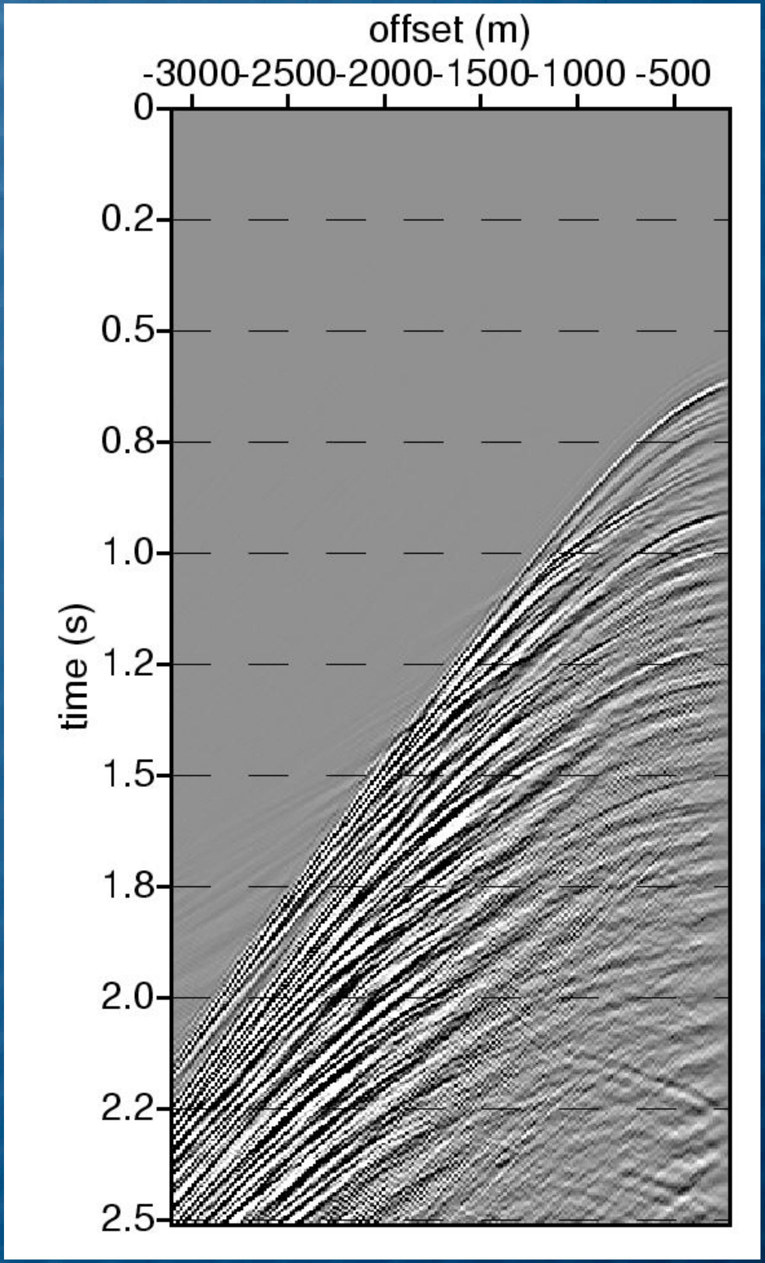
Input with multiples



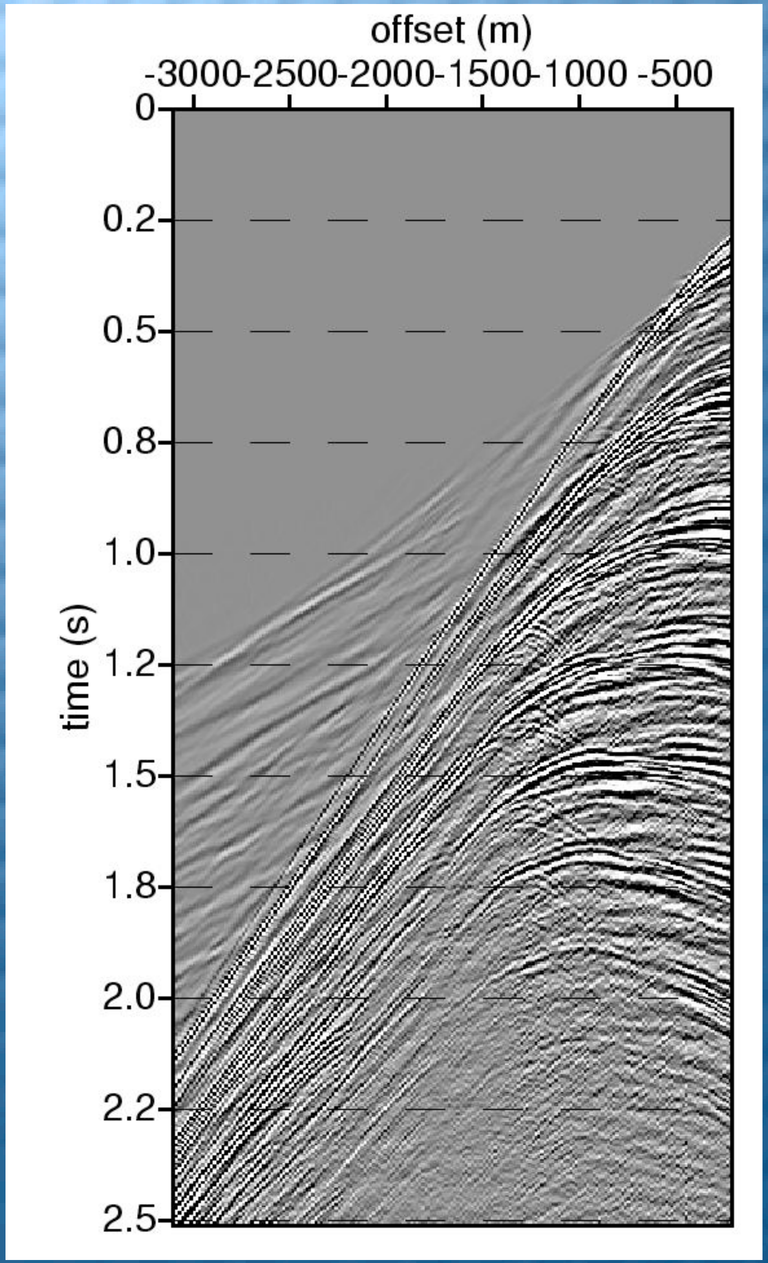
Subtraction with L2 norm



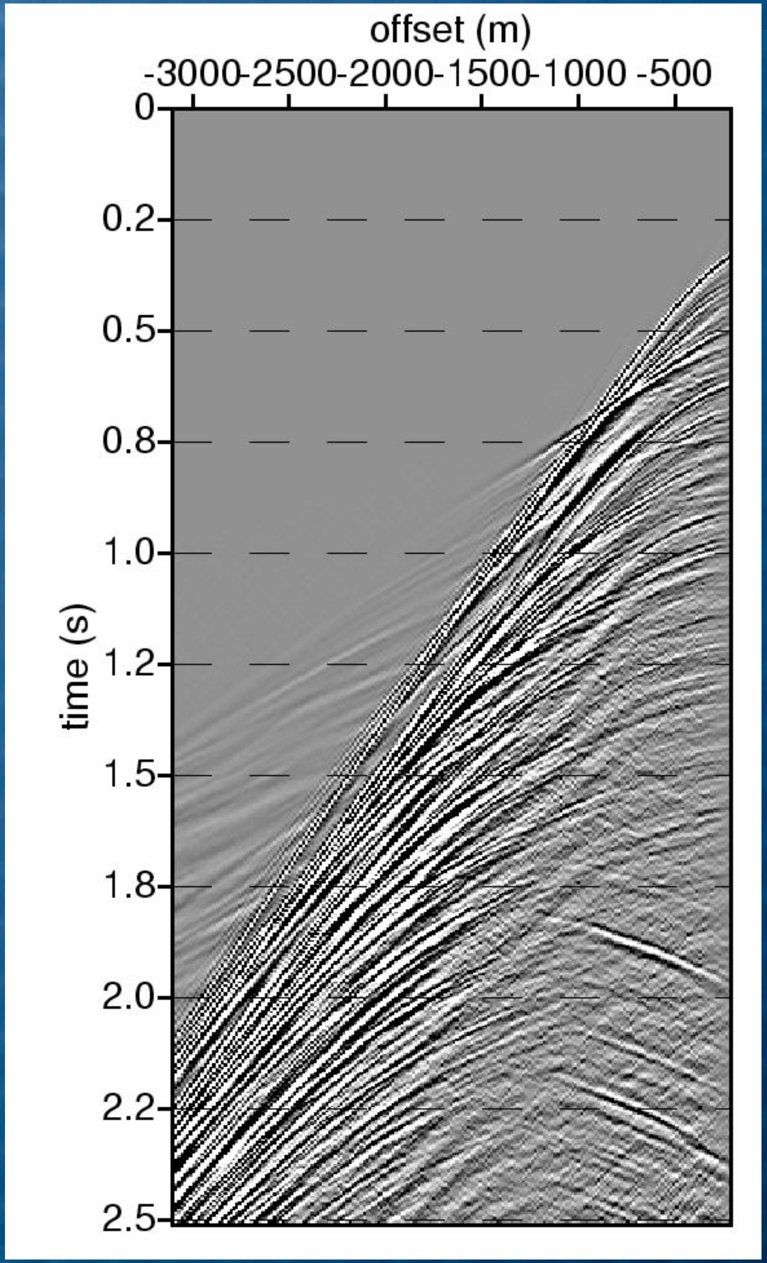
predicted multiples



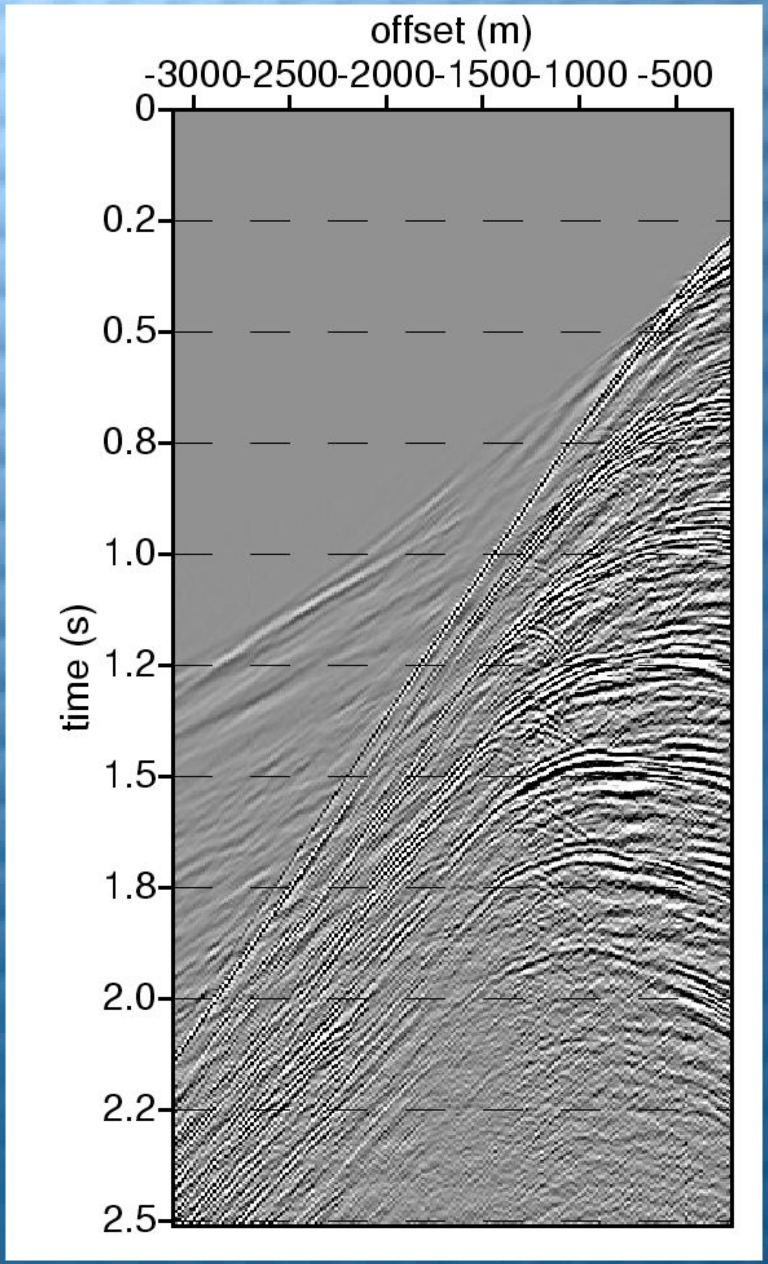
Subtraction with L2 norm



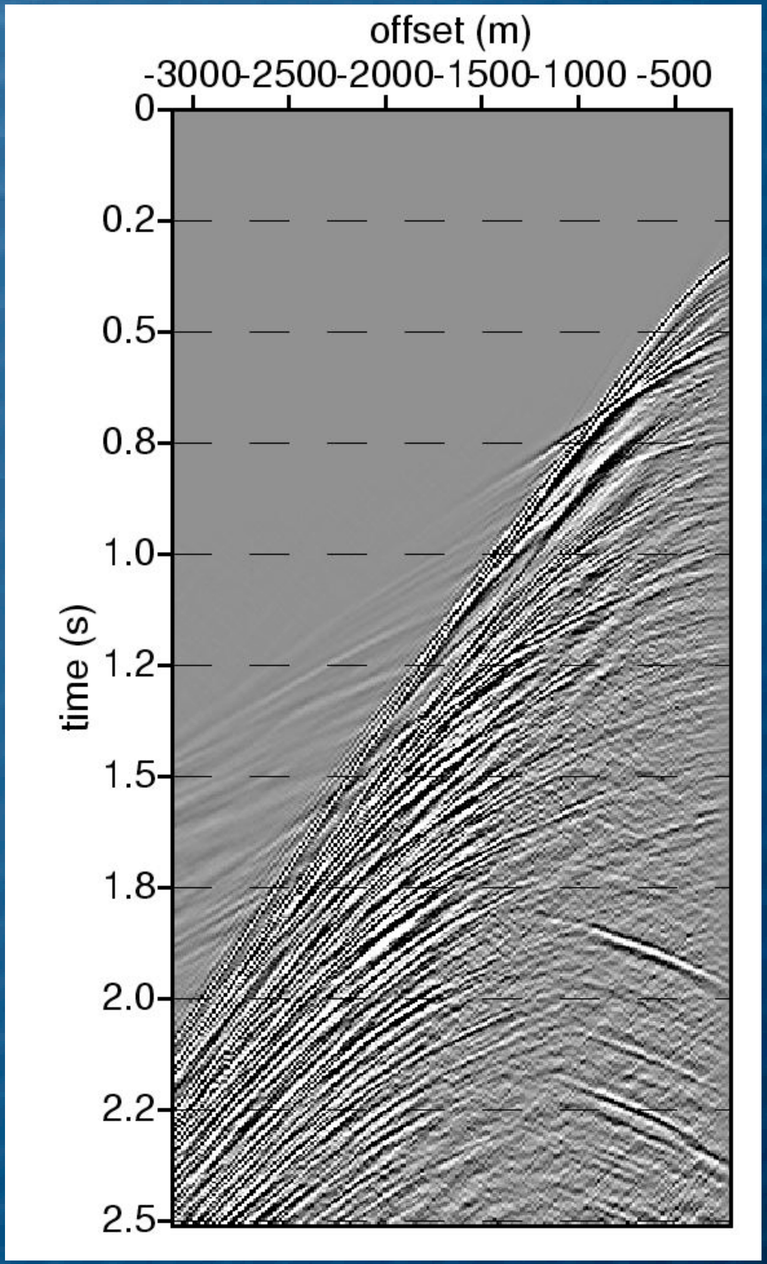
Input
with
multiples



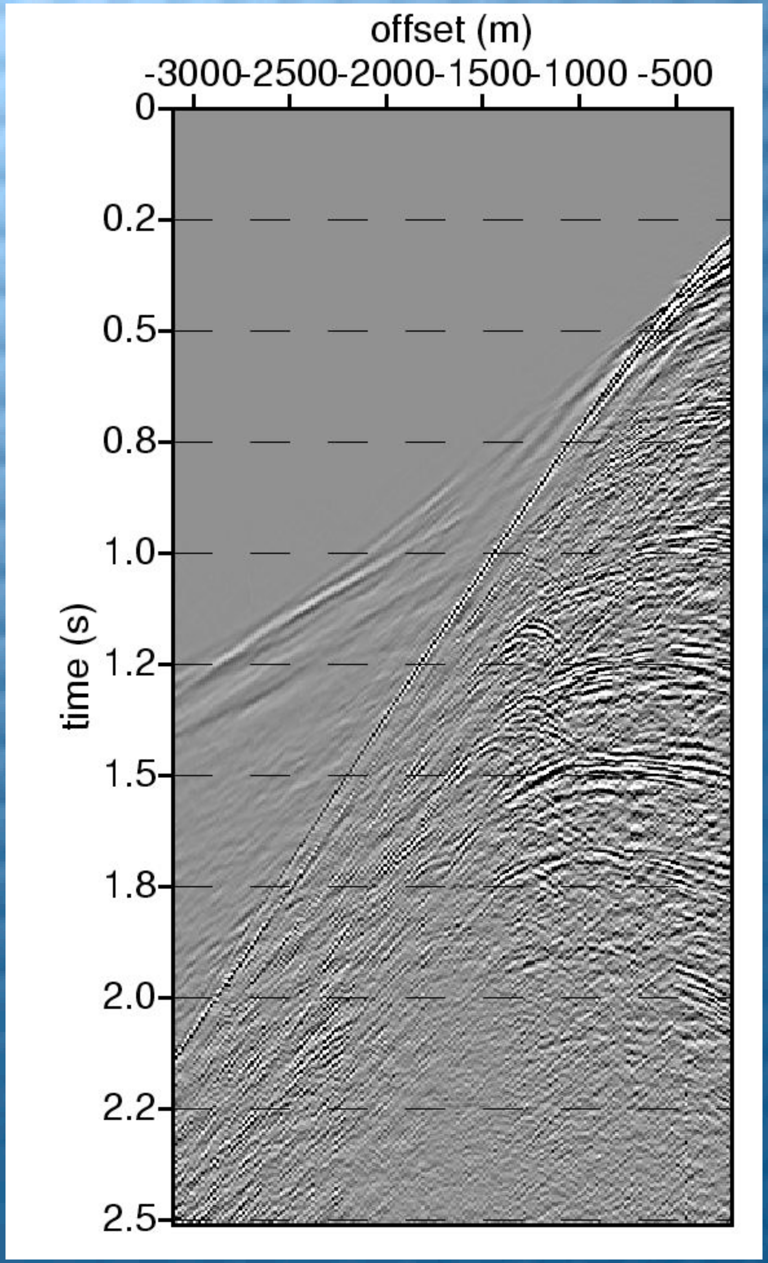
Subtraction with L2 norm



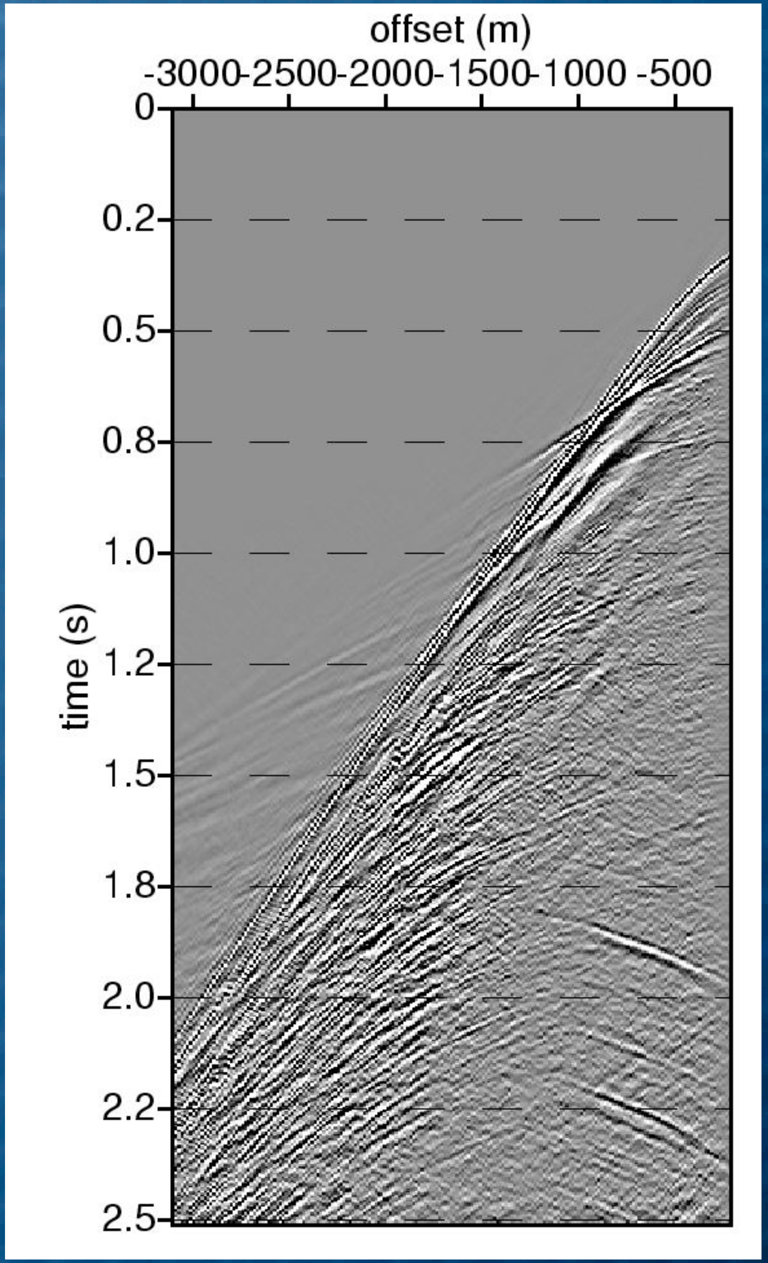
Output
SRME
L2
subtraction



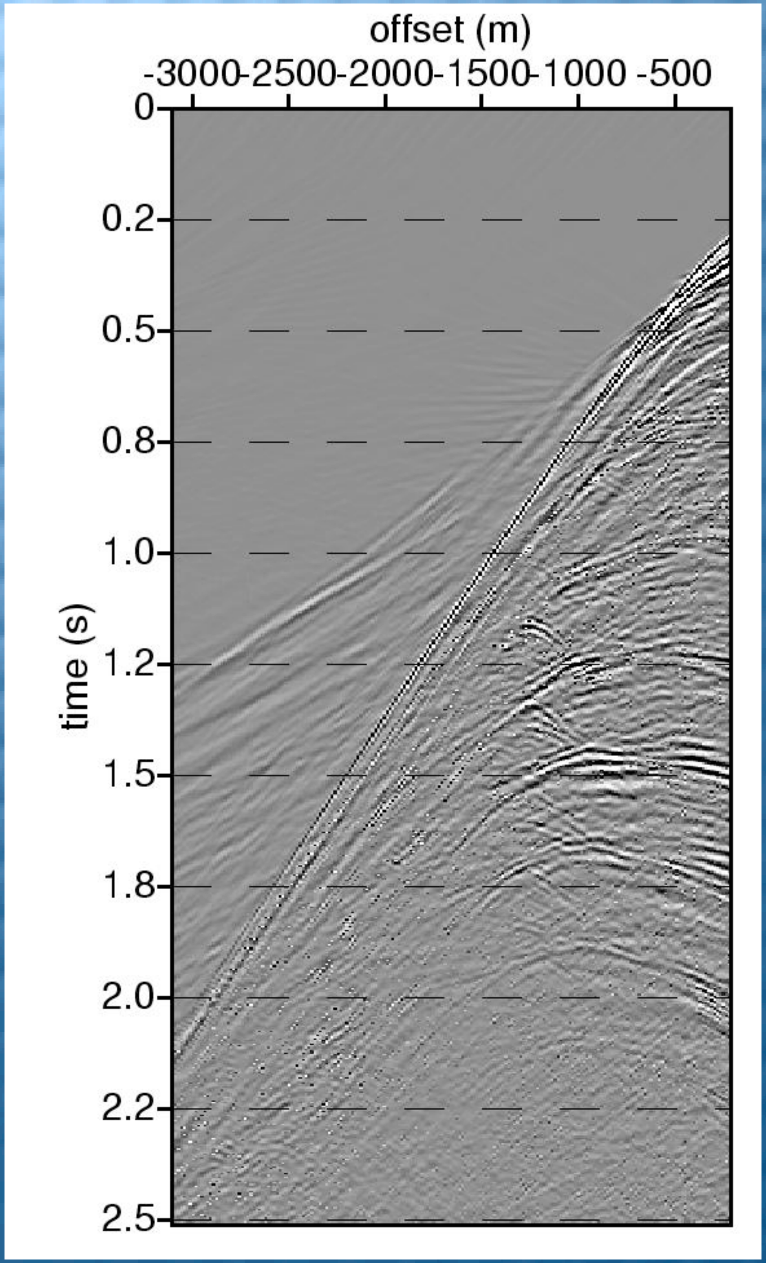
Subtraction with L2 norm



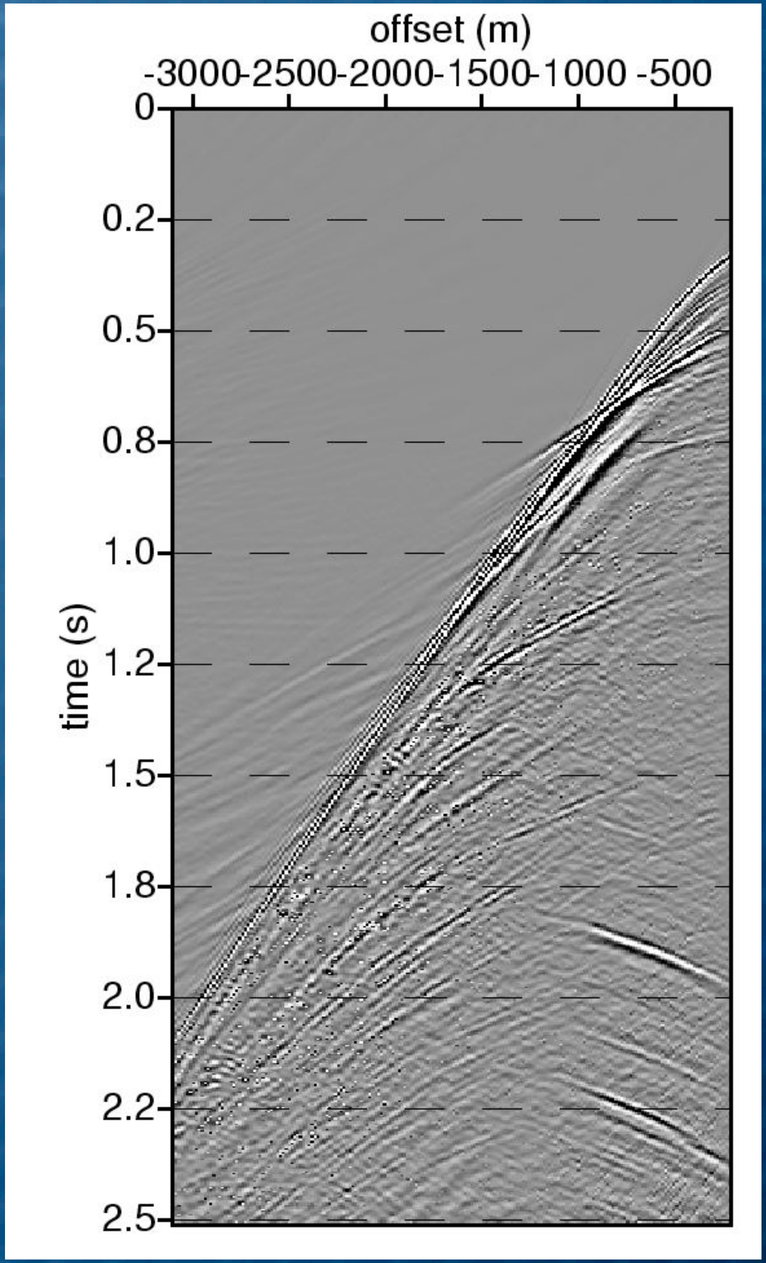
Output
SRME
multi-L2
subtraction



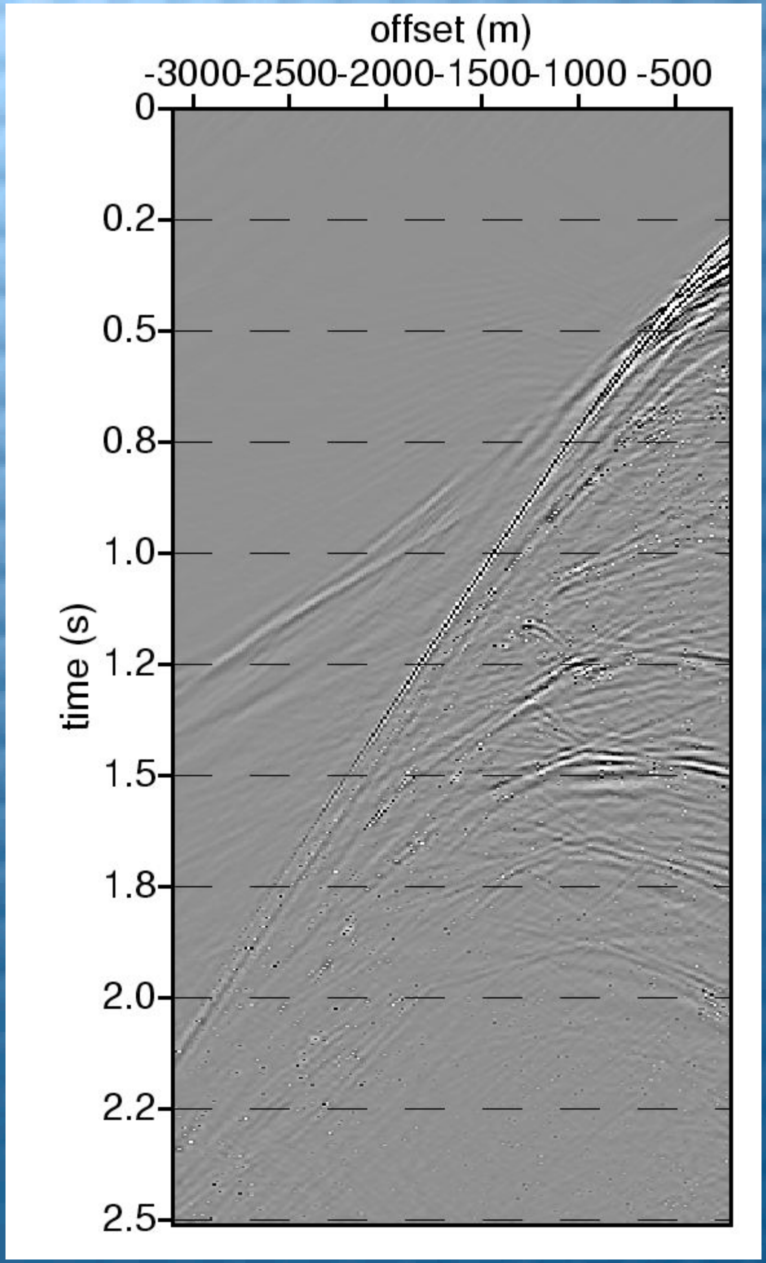
Multiple suppression with curvelets



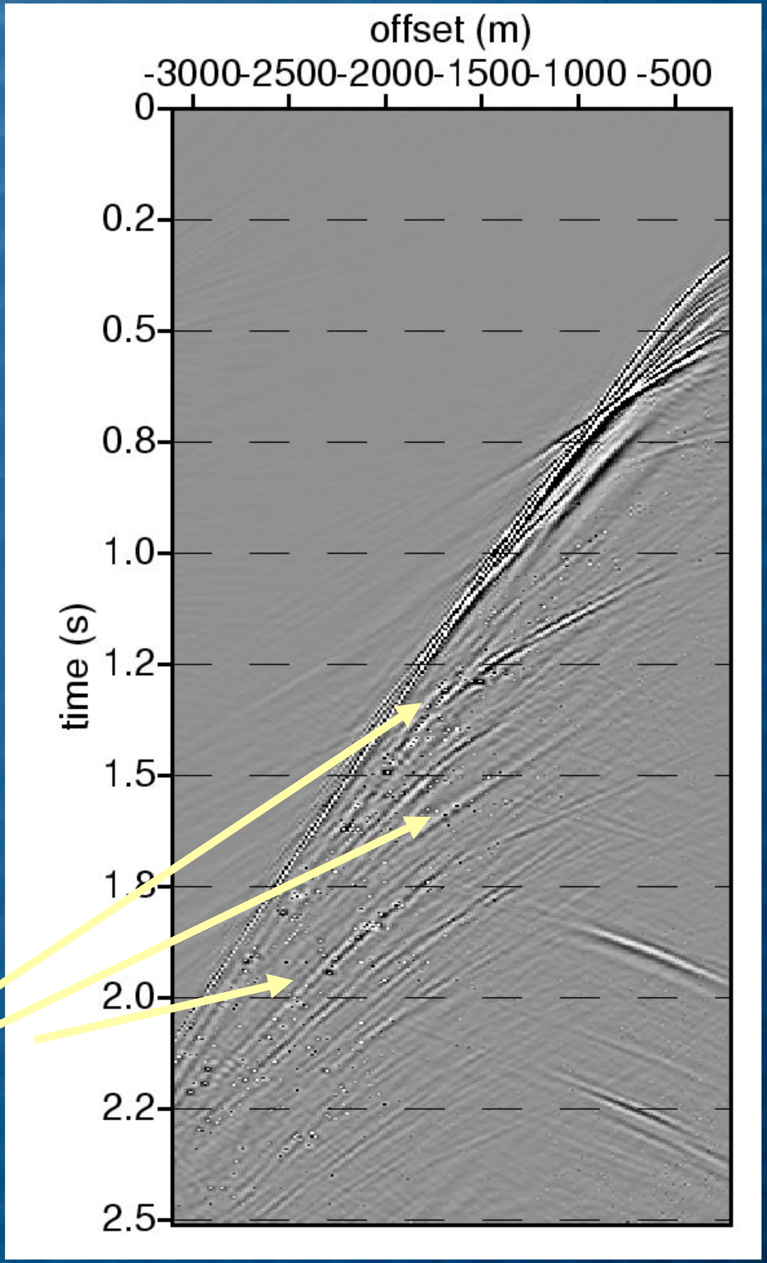
Output curvelet filtering



Multiple suppression with curvelets

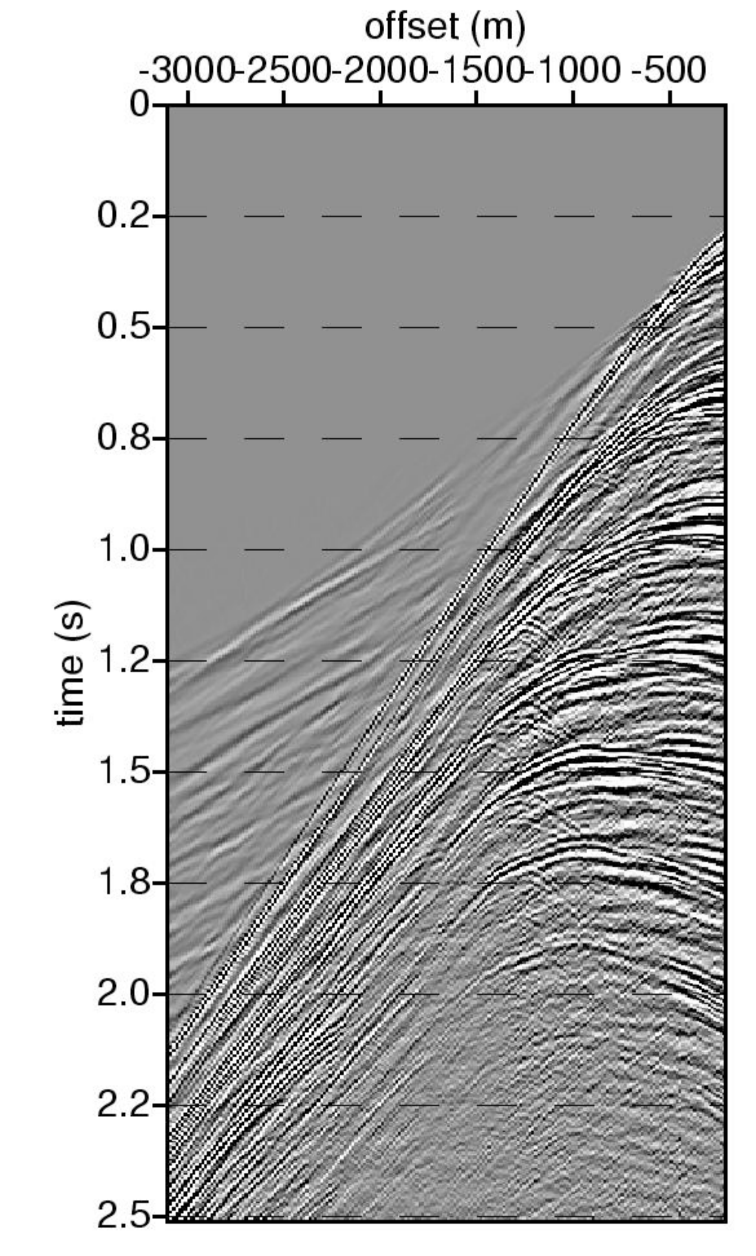


Output curvelet filtering with stronger threshold

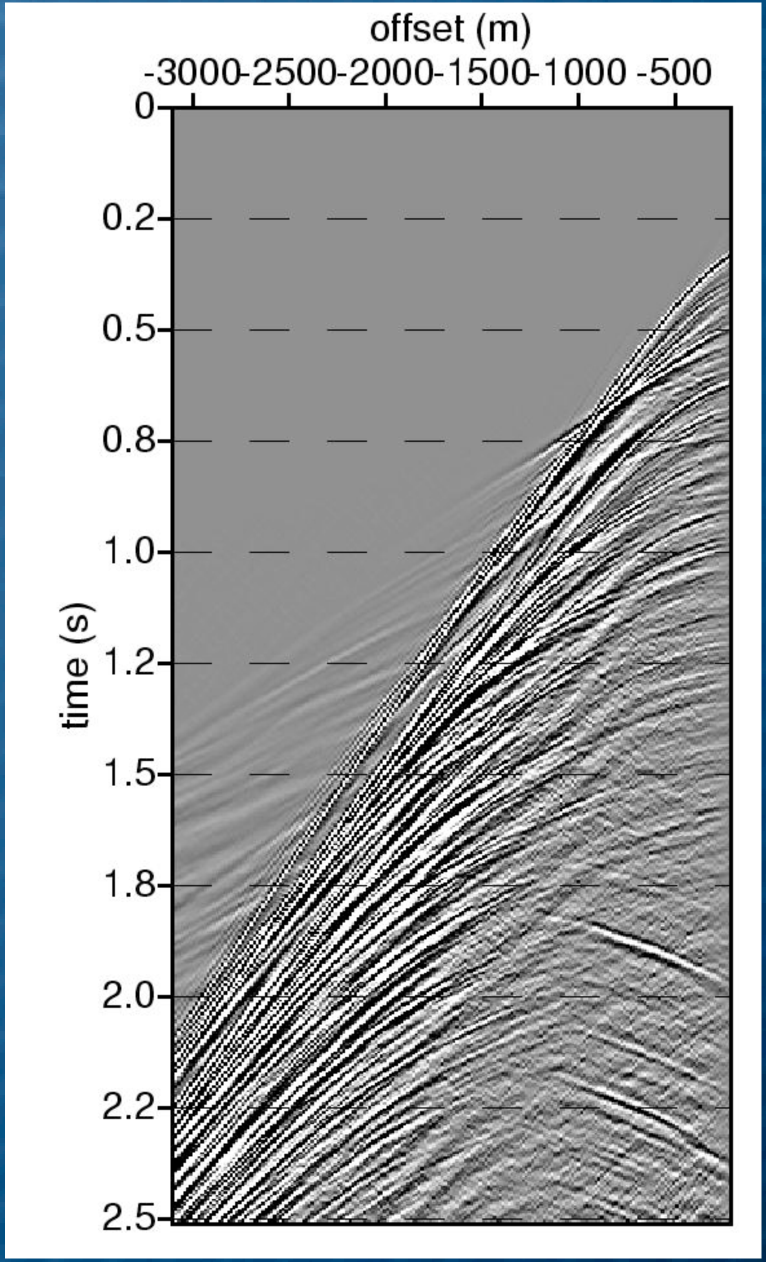


Preserved primaries

Subtraction with L2 norm

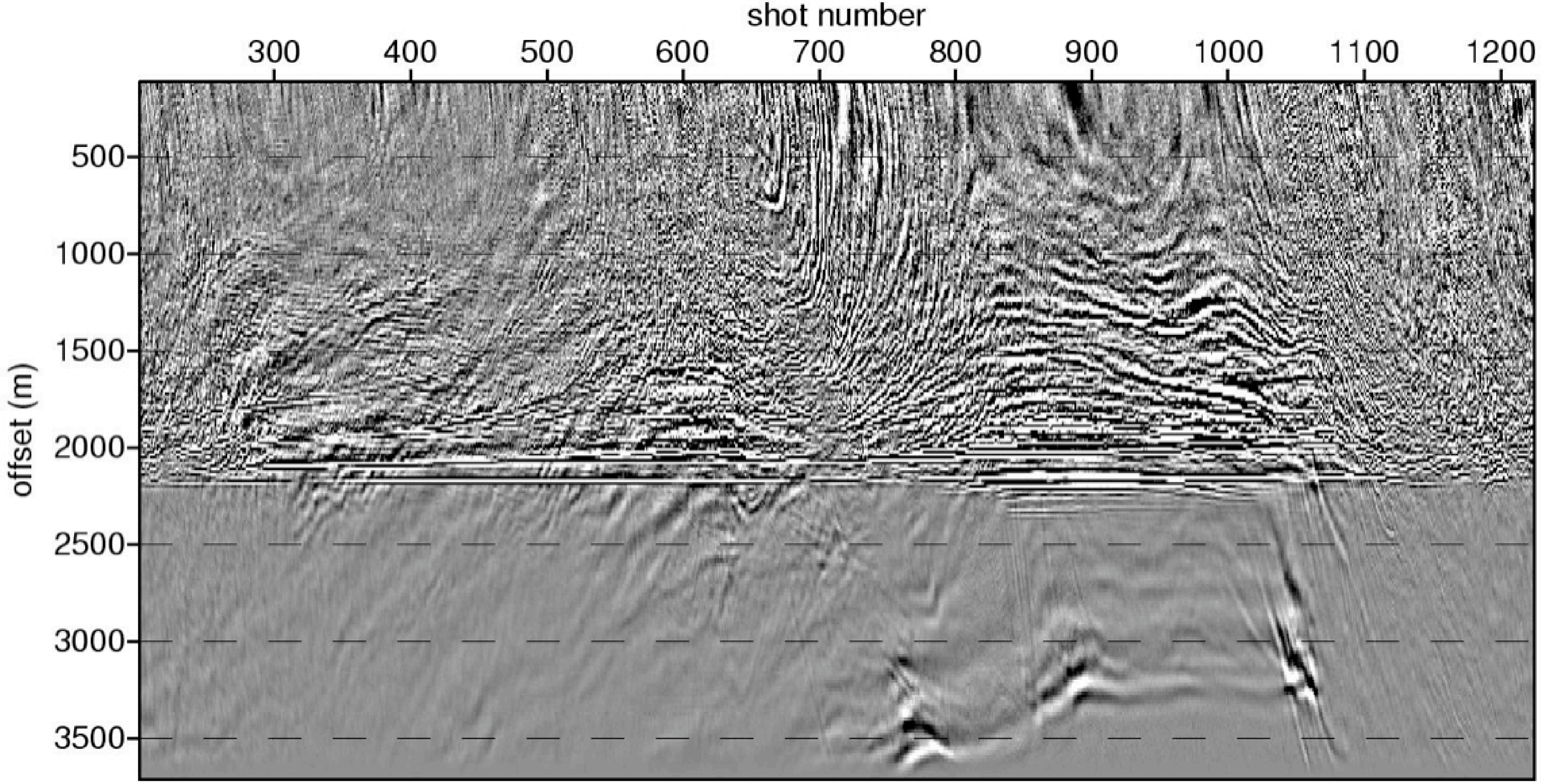


Input
with
multiples



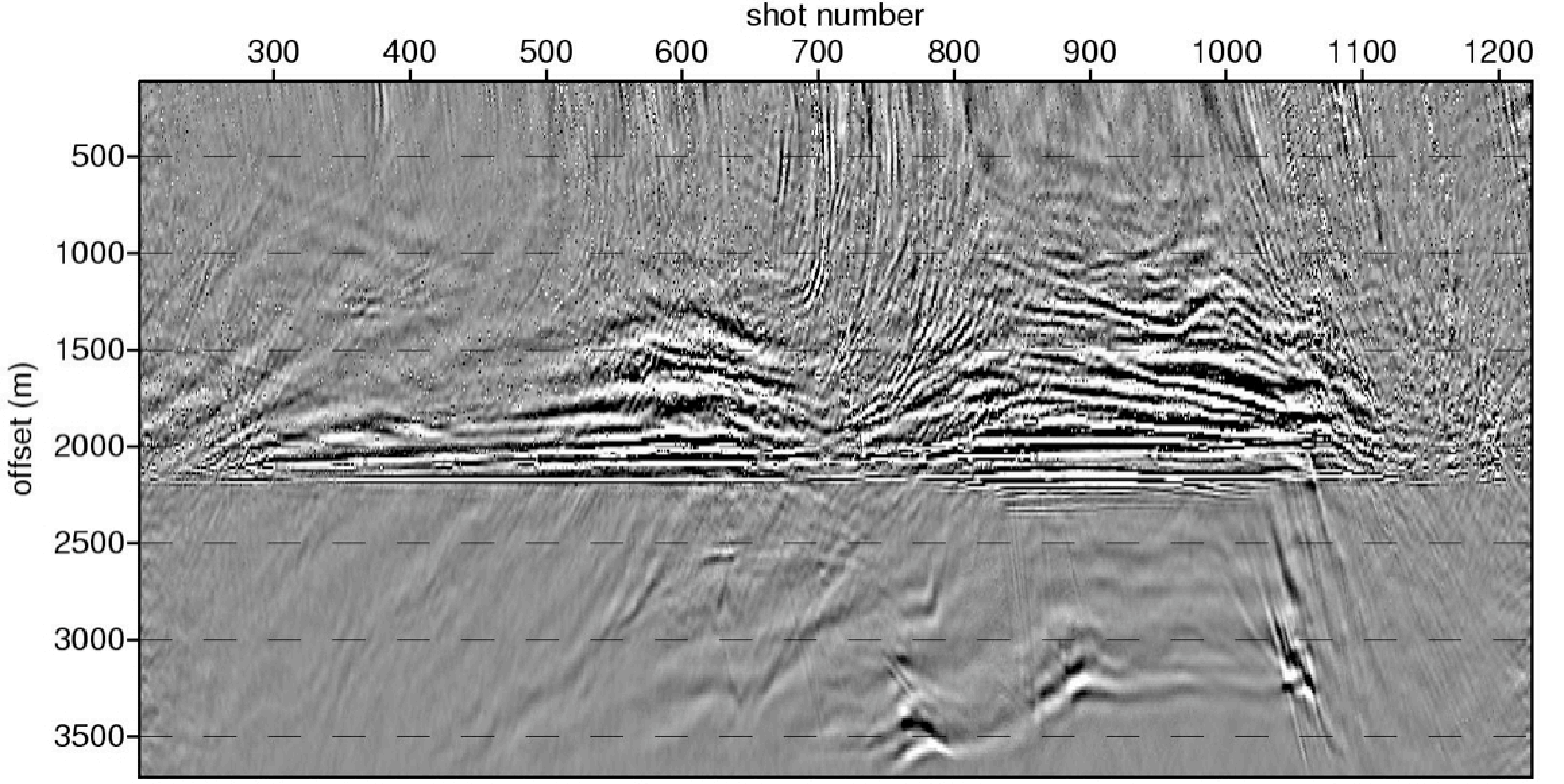
Time slices

L2



Time slices

curvelet



Observations

Used 3D SRME to predict noise

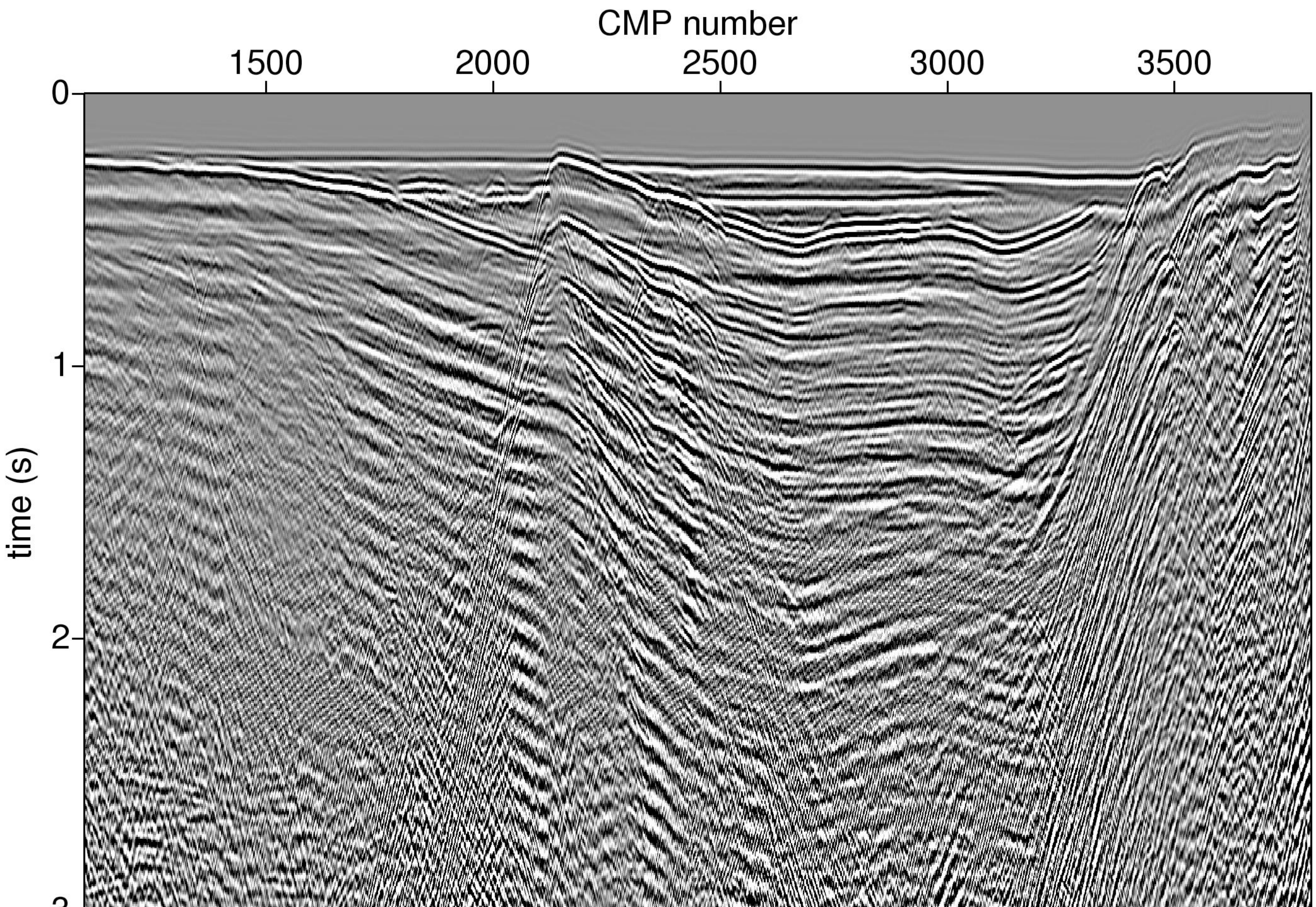
Can use other noise predictions (e.g. Radon)

We do **NOT *subtract*** rather ***mute***

- put to zero or preserve coherent features
- less sensitive to errors

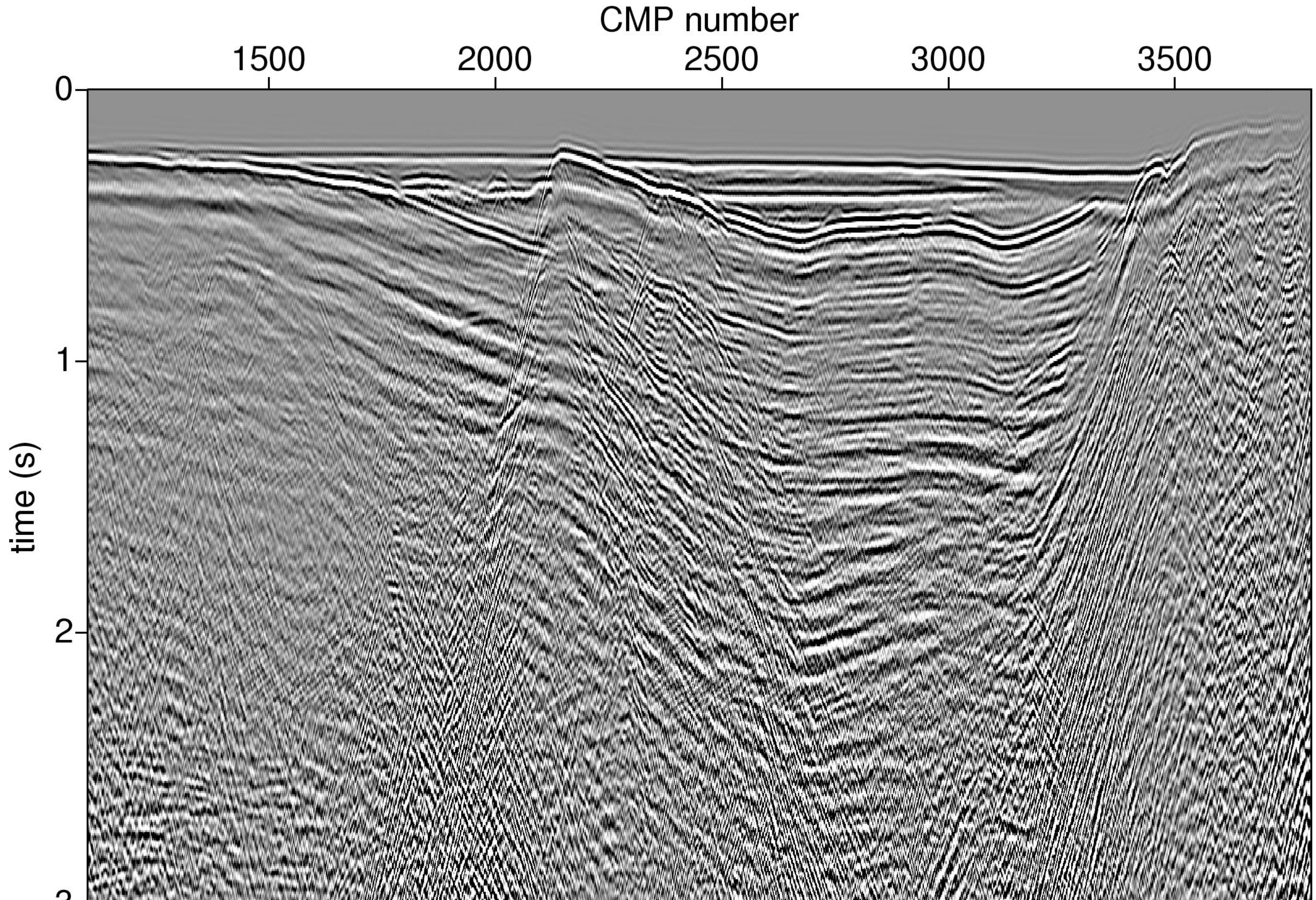
Preserve the edges & primaries!

Stack section with multiples



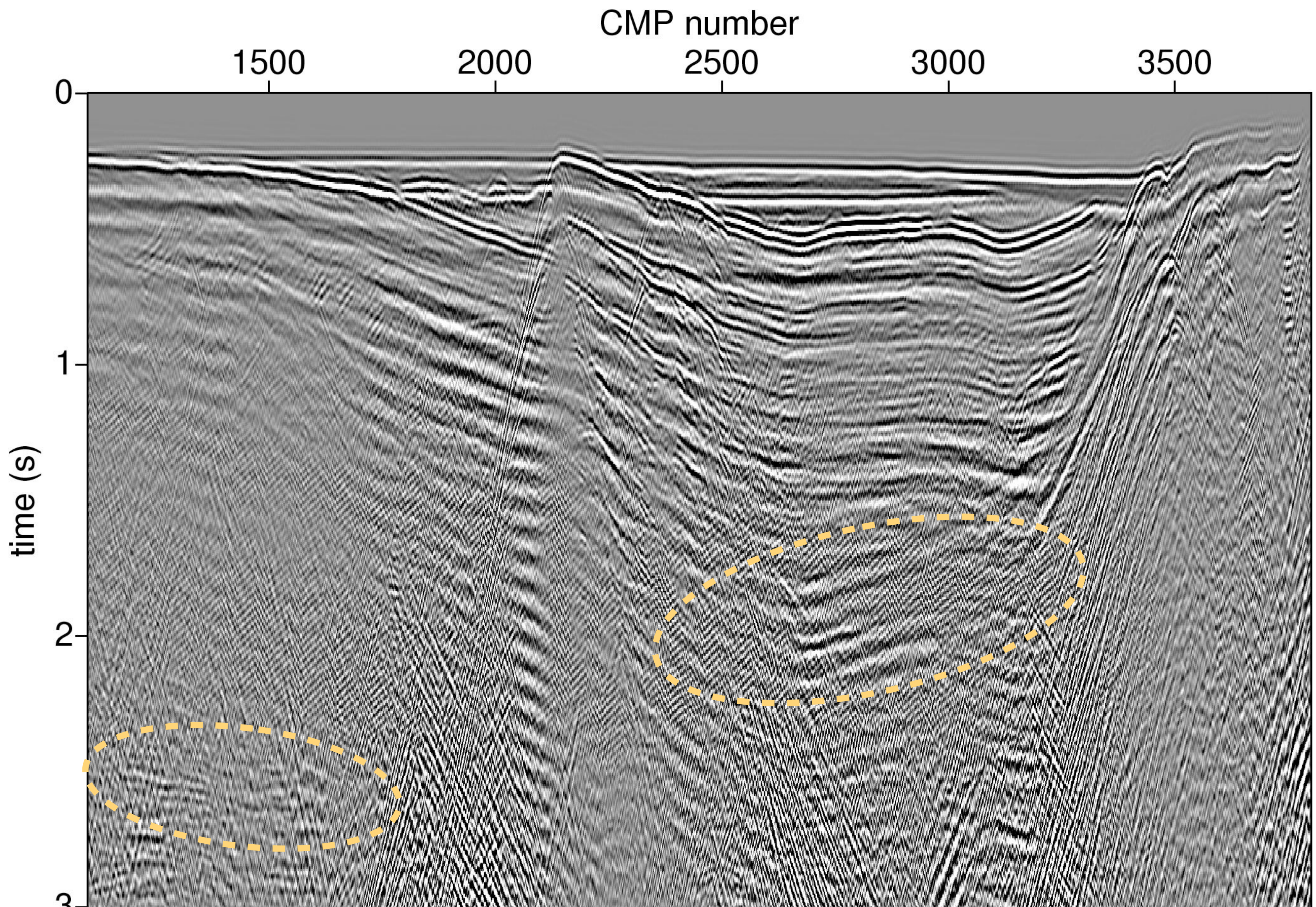
Data from offshore Scotland courtesy Total

Stack section after L.S. subtraction

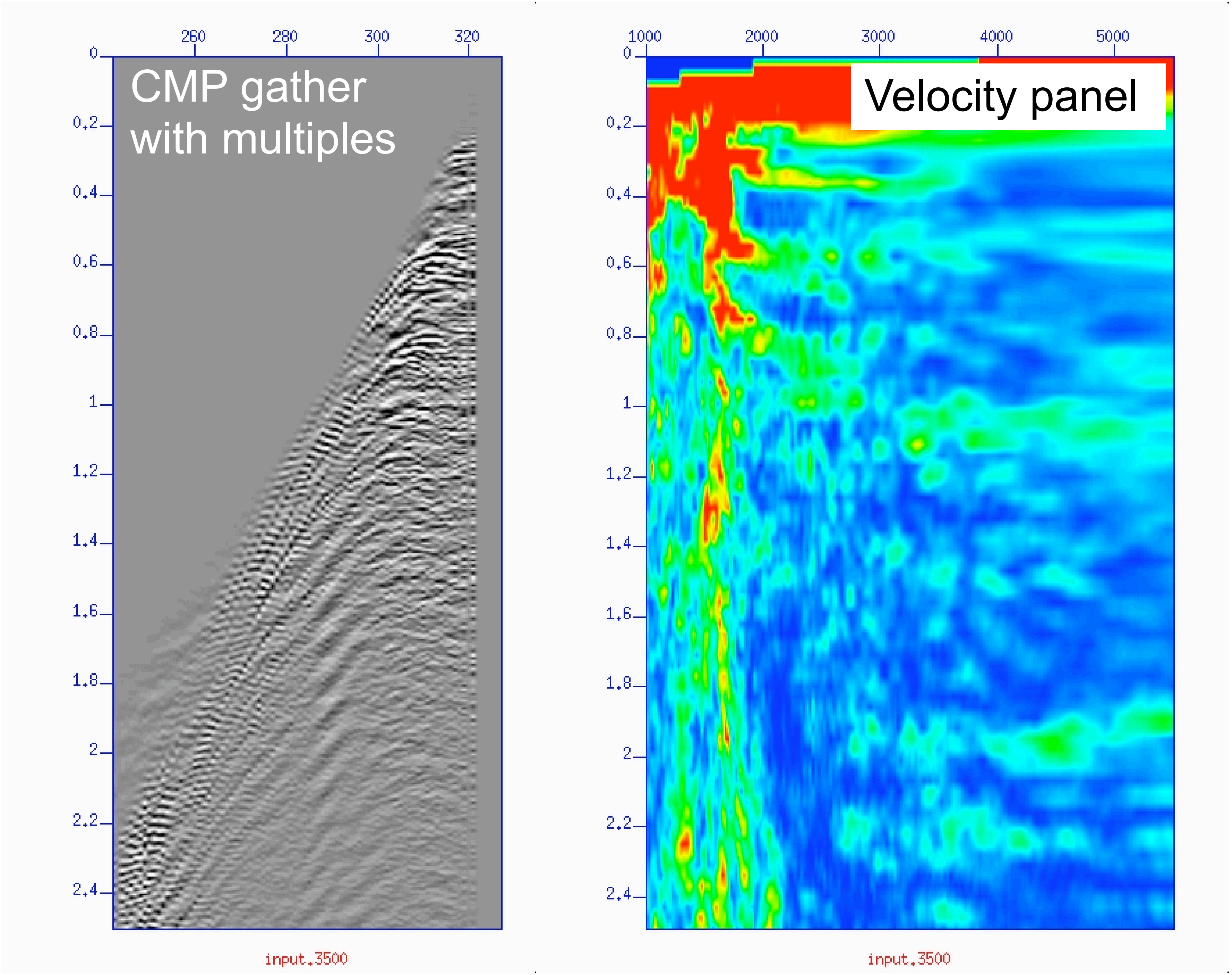


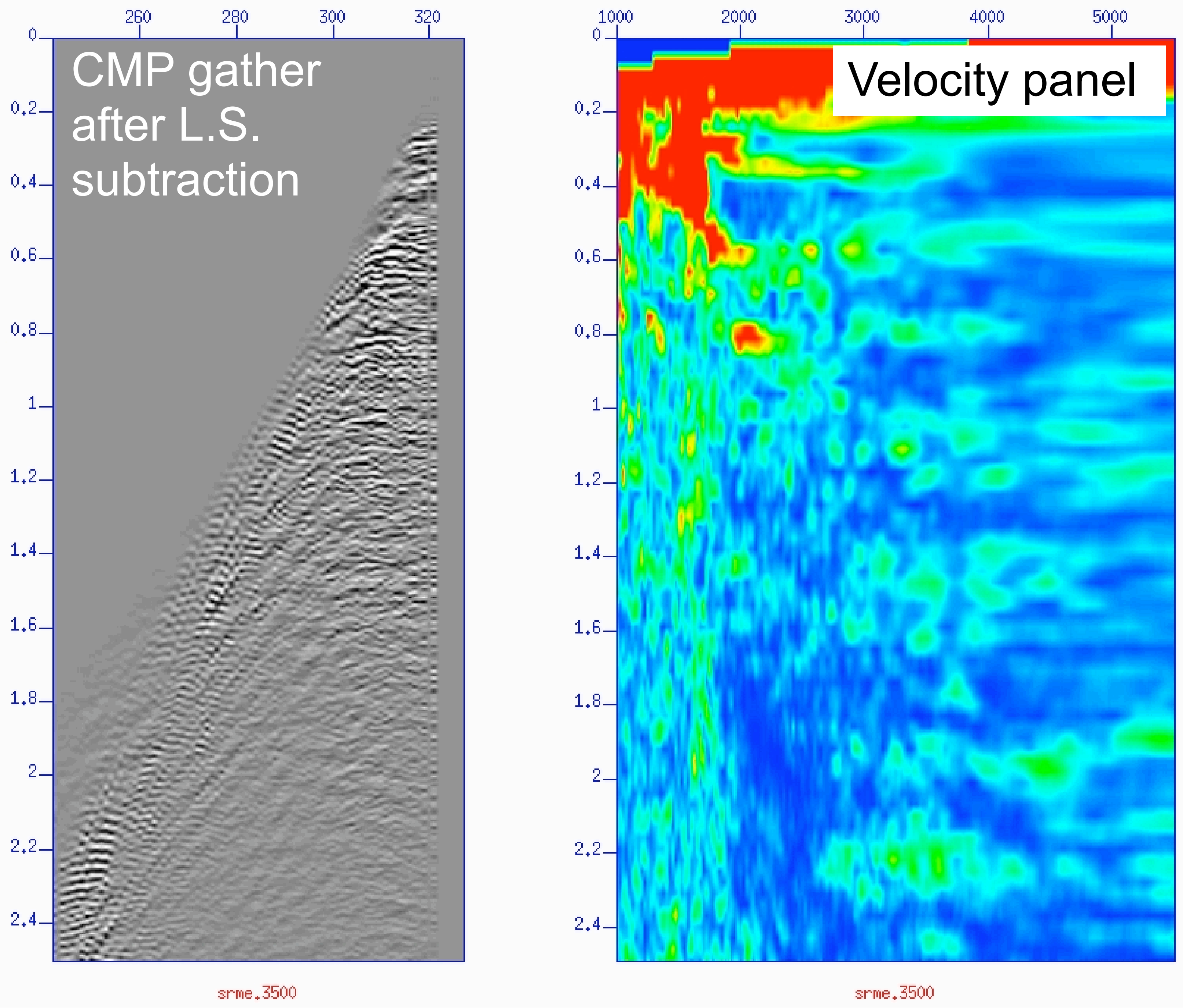
Data from offshore Scotland courtesy Total

Stack section after curvelet-based subtraction



Data from offshore Scotland courtesy Total


CMP gather with multiples
Velocity panel
260
280
300
320
1000
2000
3000
4000
5000
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
input.3500
input.3500


CMP gather after L.S. subtraction
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1.2
1.4
1.6
1.8
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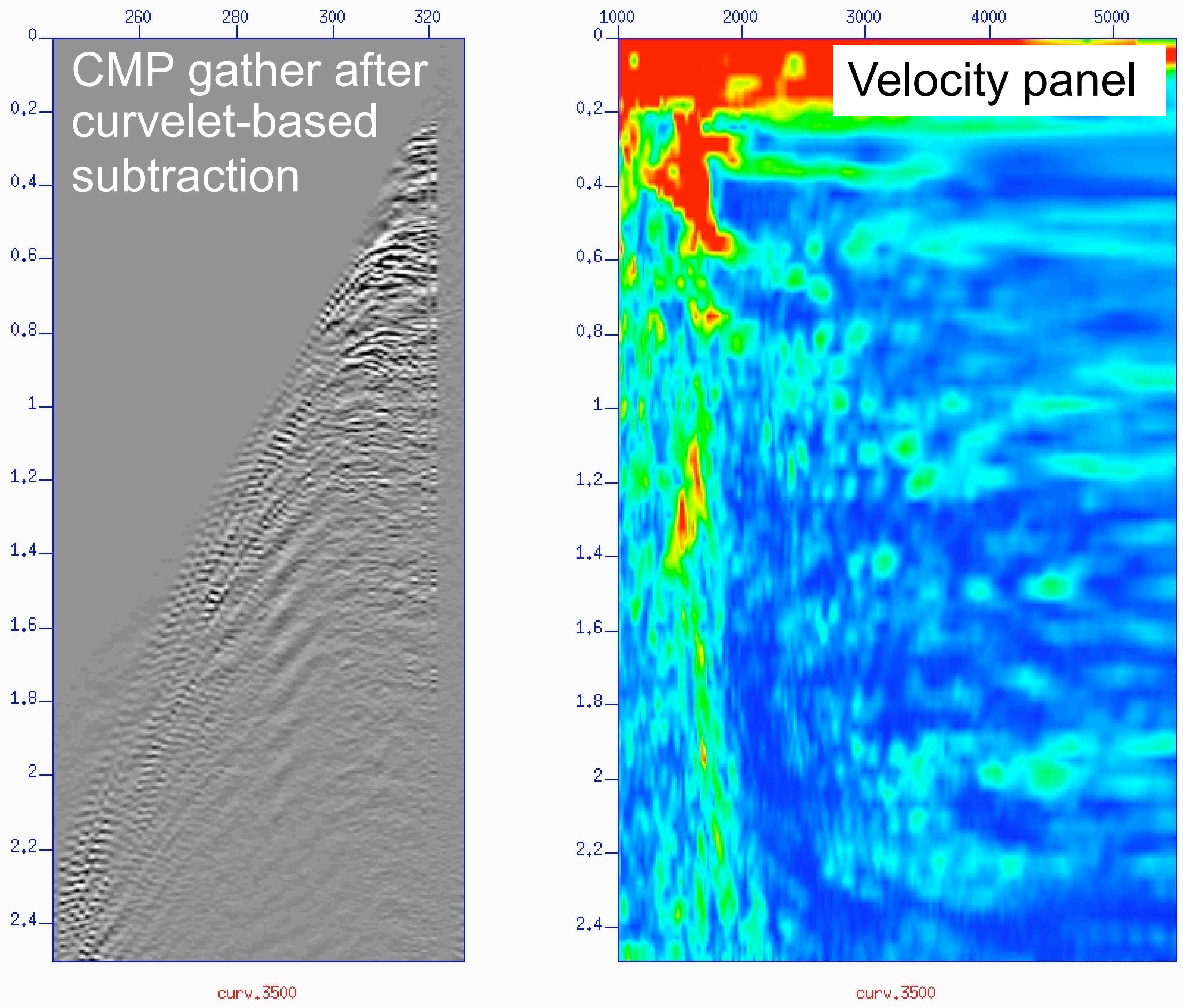
2.2
2.4
srme.3500
srme.3500


CMP gather after curvelet-based subtraction
Velocity panel
260
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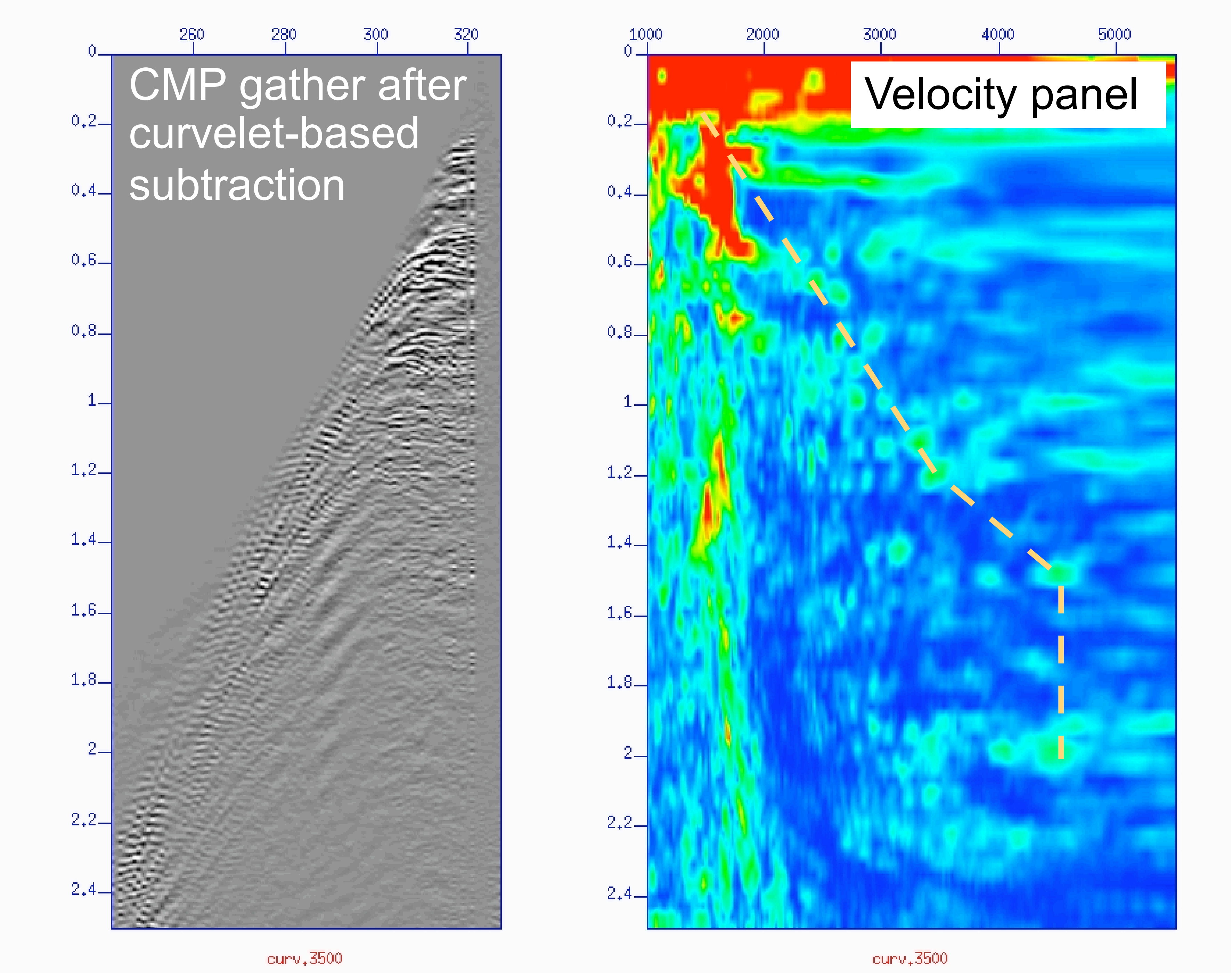
curv.3500
curv.3500



CMP gather after curvelet-based subtraction
Velocity panel
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0.6
0.8
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1.6
1.8
2
2.2
2.4
curv.3500
curv.3500

Iterative thresholding

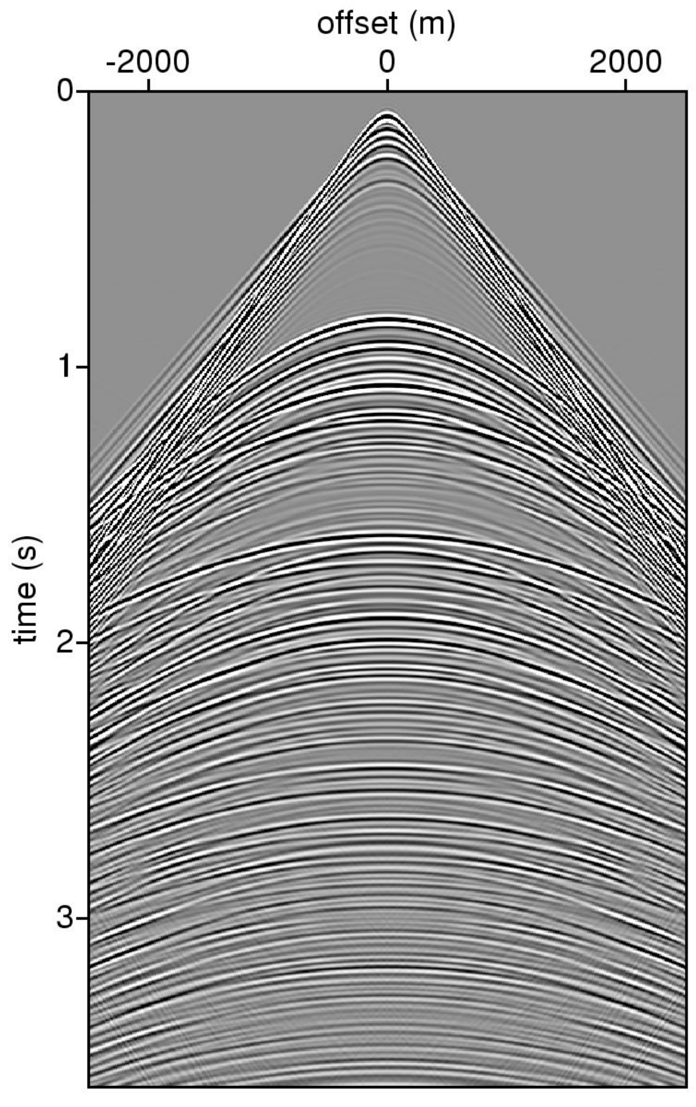
Curvelets are Frames:

- **redundant (factor 7.5-4)**
- **thresholding does *not* solve:**

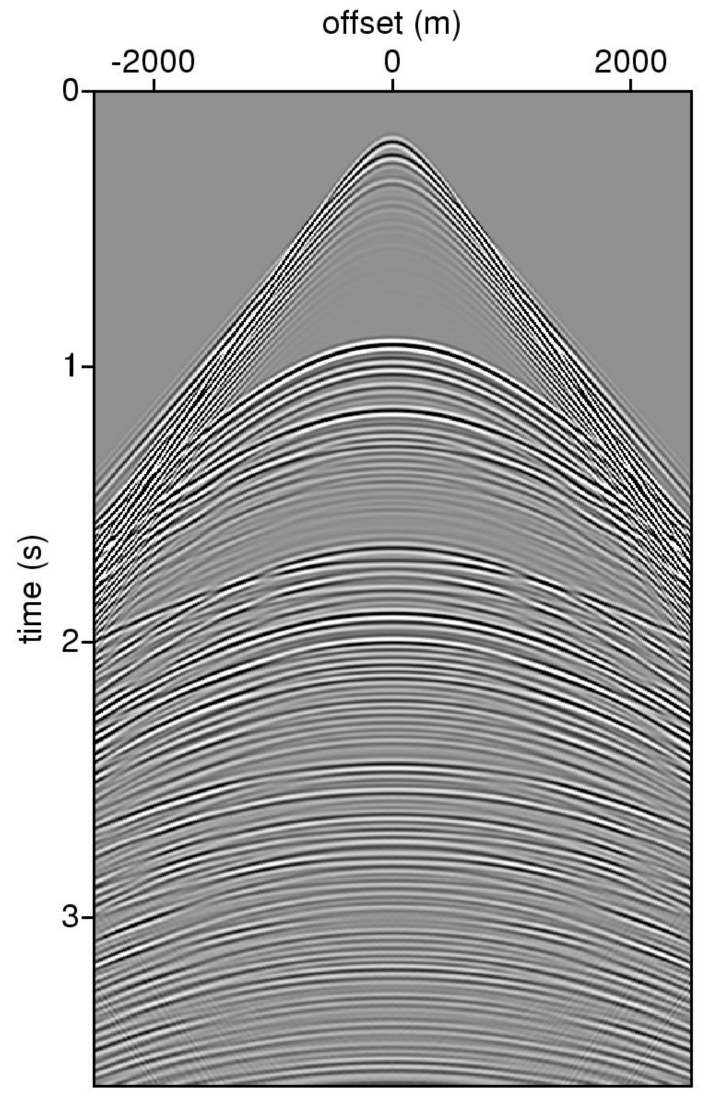
$$\hat{\mathbf{m}} = \mathbf{B}^T \arg\min_{\tilde{m}} \frac{1}{2} \|\mathbf{\Gamma}^{-1} \left(\tilde{\mathbf{d}} - \tilde{\mathbf{m}}\right) \|_2^2 + \|\tilde{\mathbf{m}}\|_{1,\lambda}$$

Alternative formulation by iterative thresholding!

Example shallow water environment

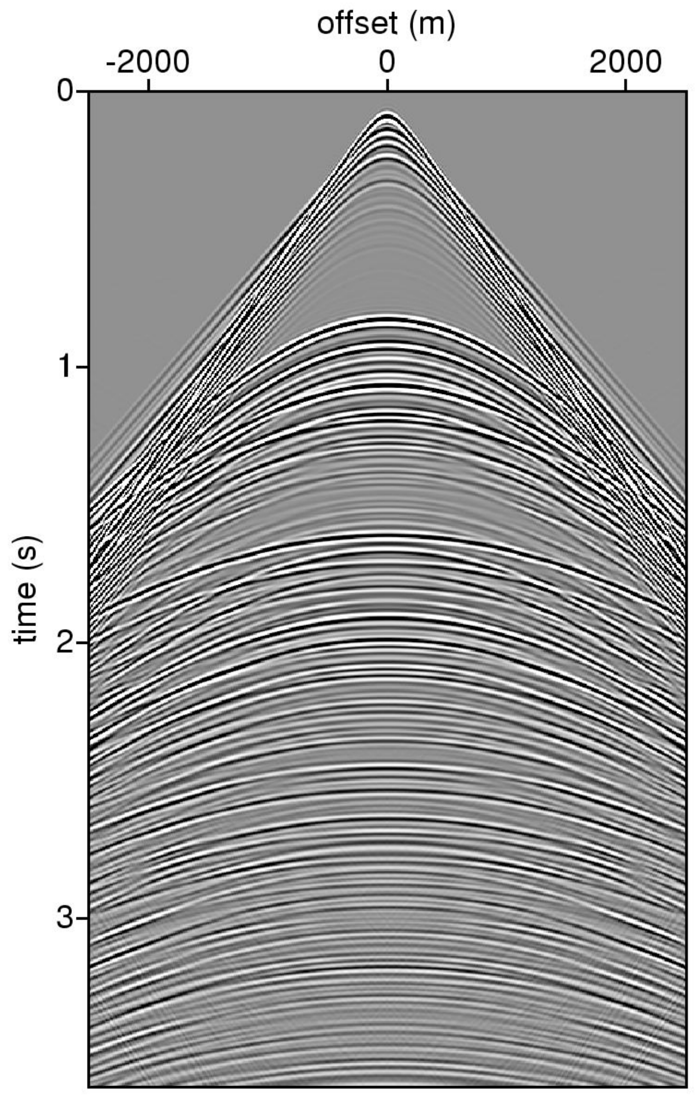


Input data with multiples

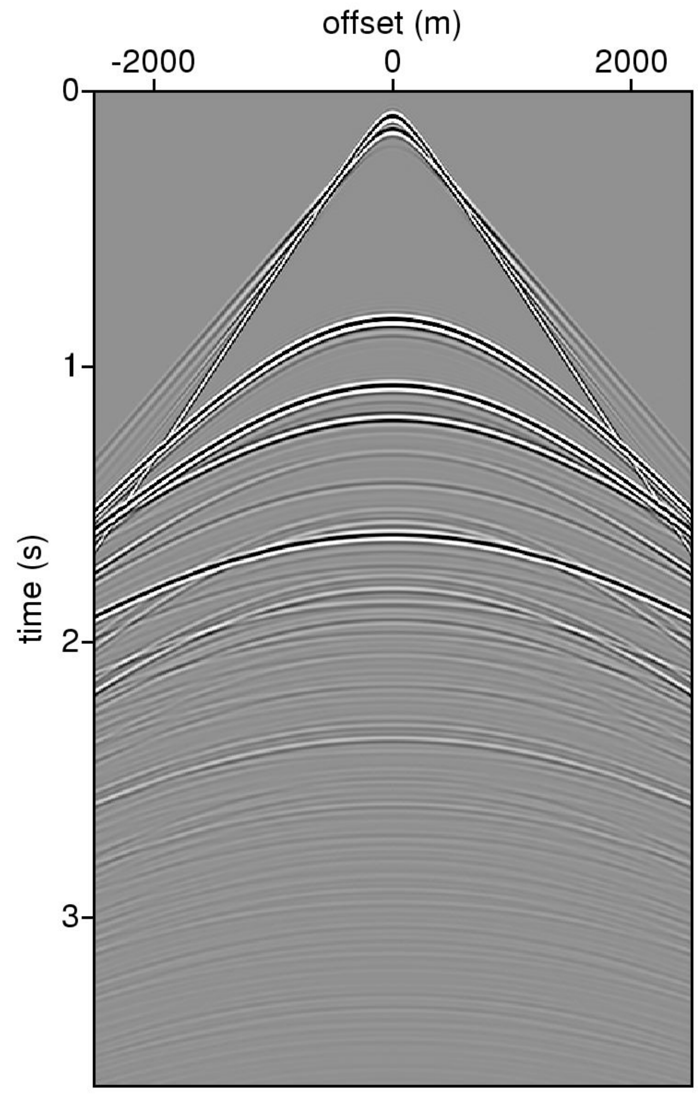


True surface multiples

Example shallow water environment

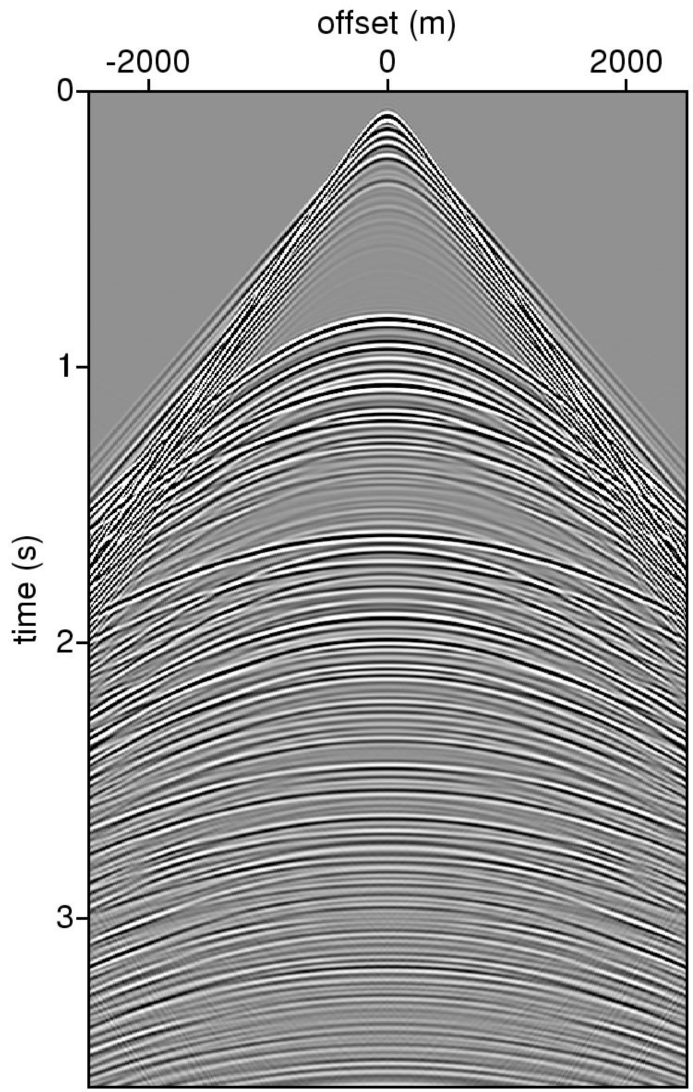


Input data with multiples

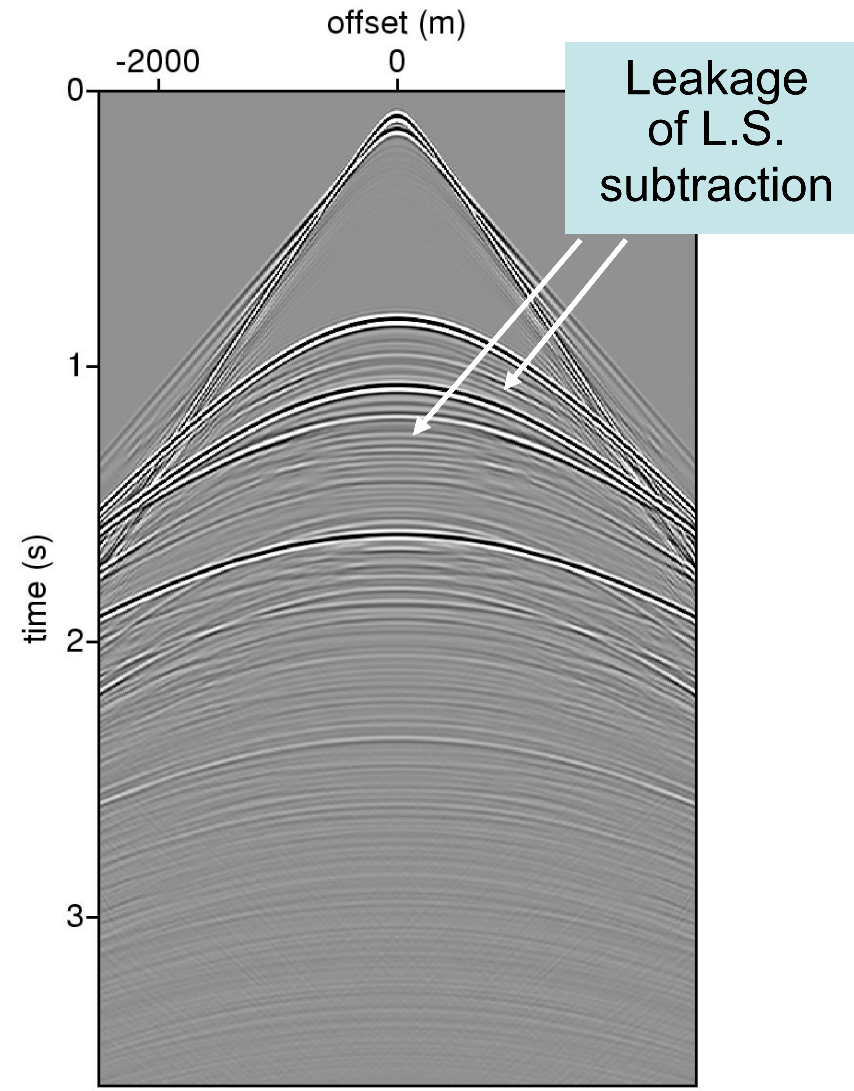


True primaries

Example shallow water environment

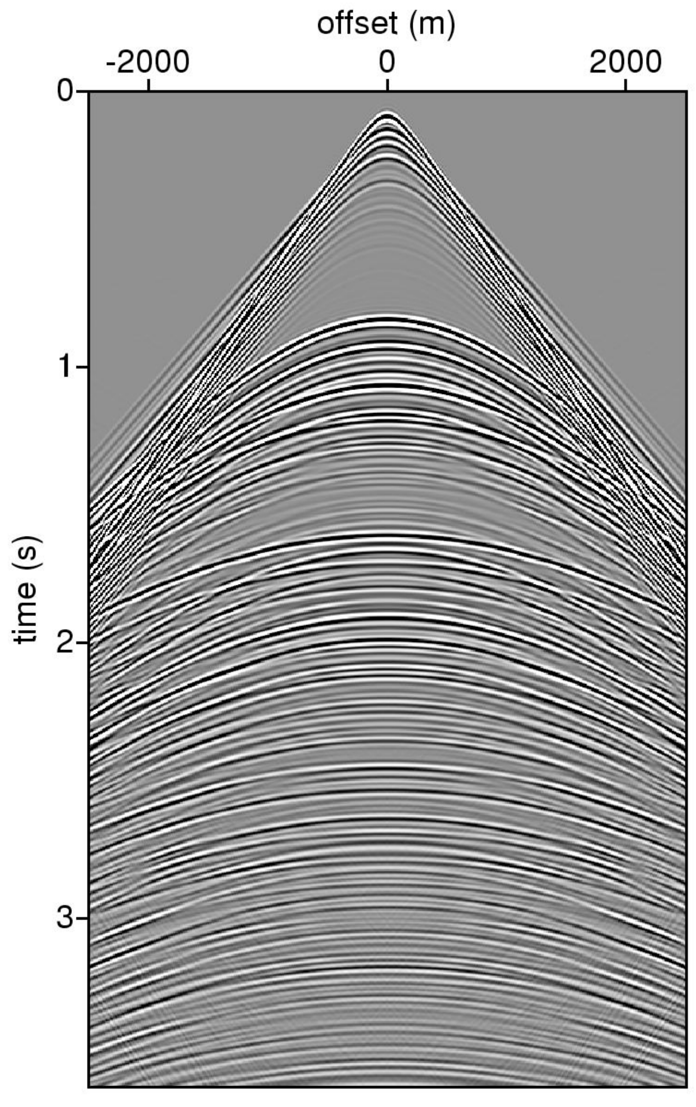


Input data with multiples

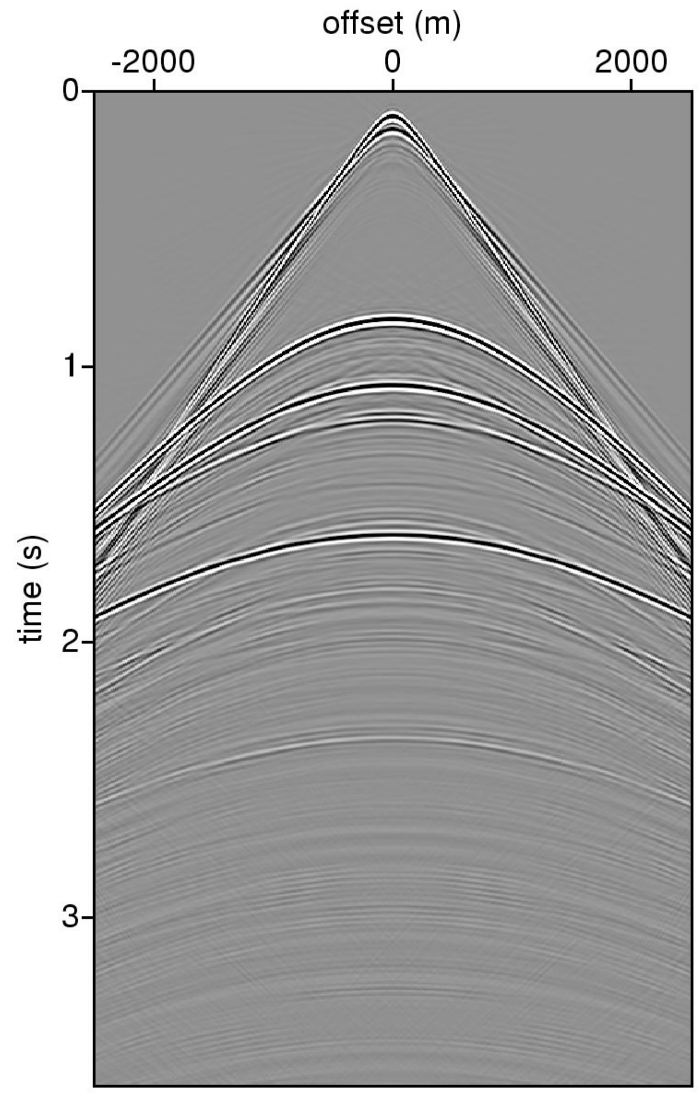


Result L.S. subtraction

Example shallow water environment

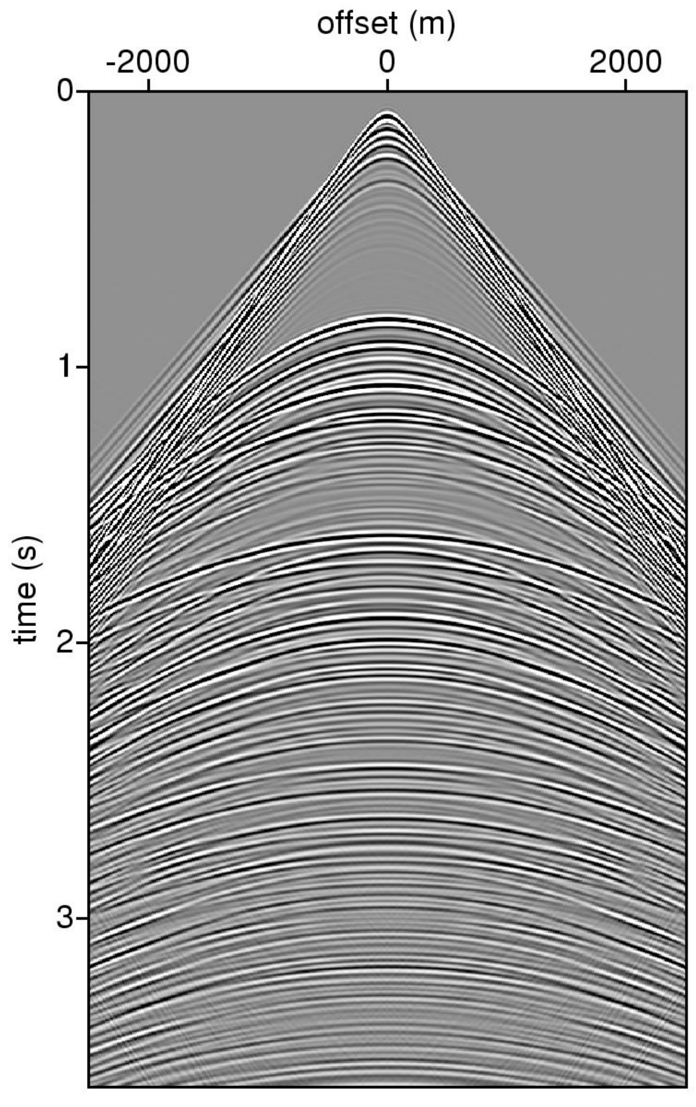


Input data with multiples

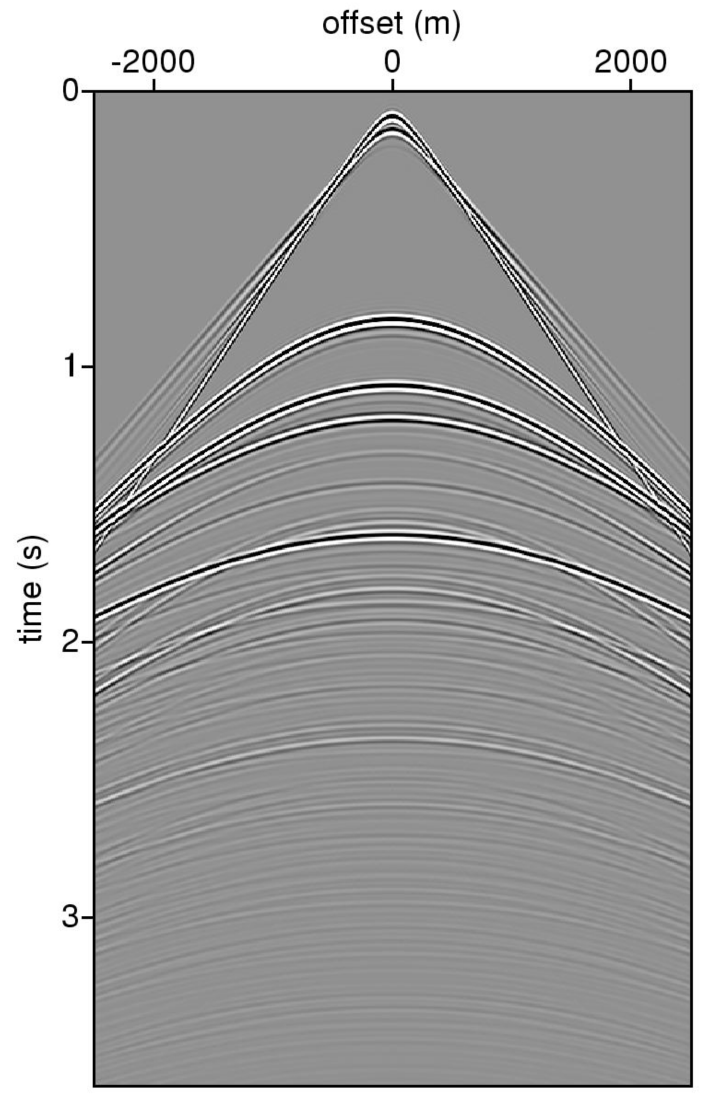


Result curvelet-based subtraction

Example shallow water environment

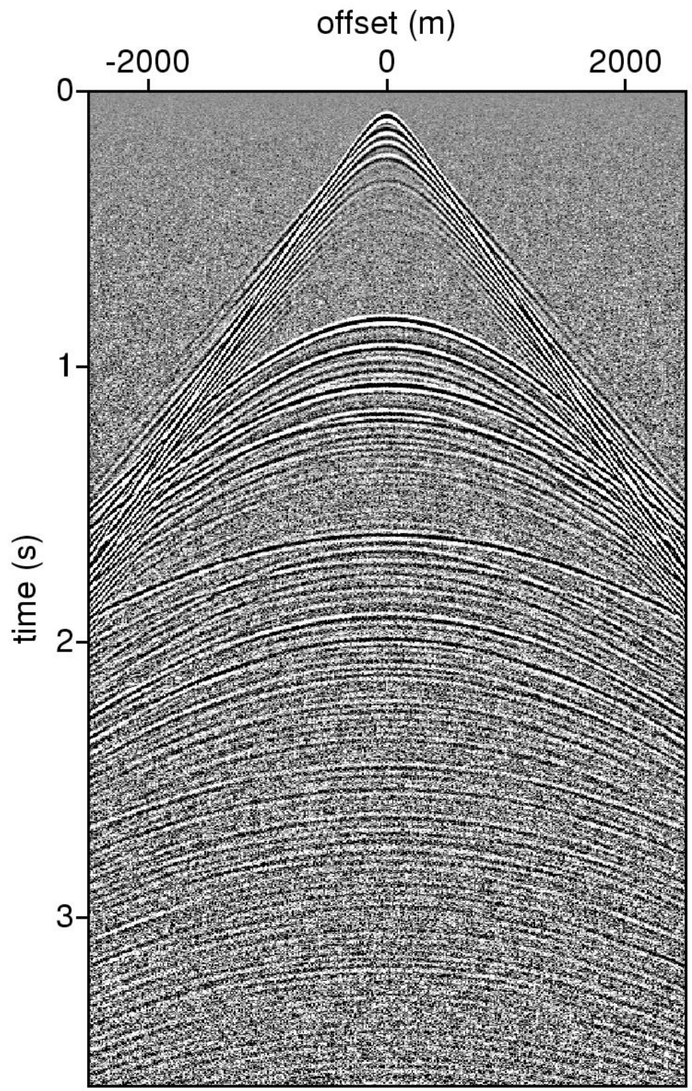


Input data with multiples

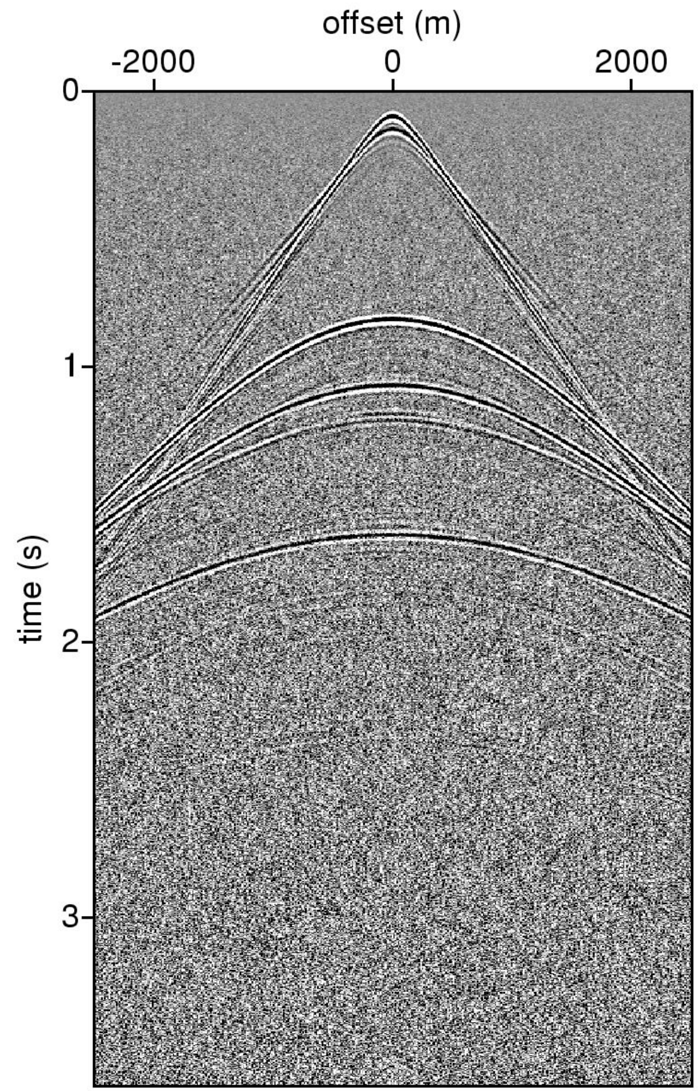


True primaries

Example shallow water environment

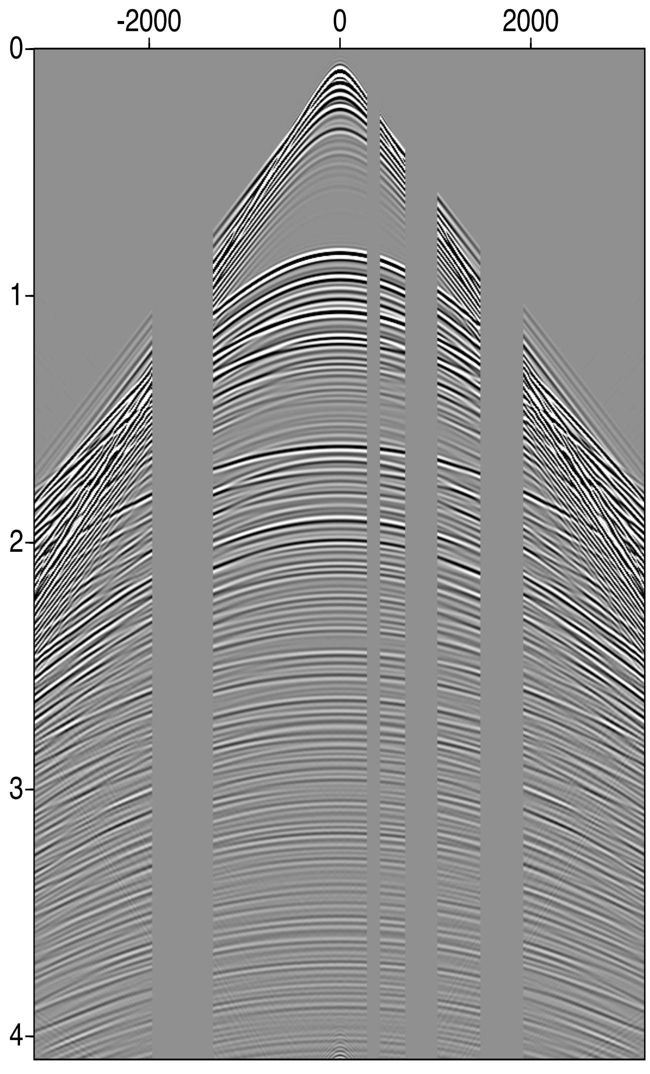


Input data with multiples and noise

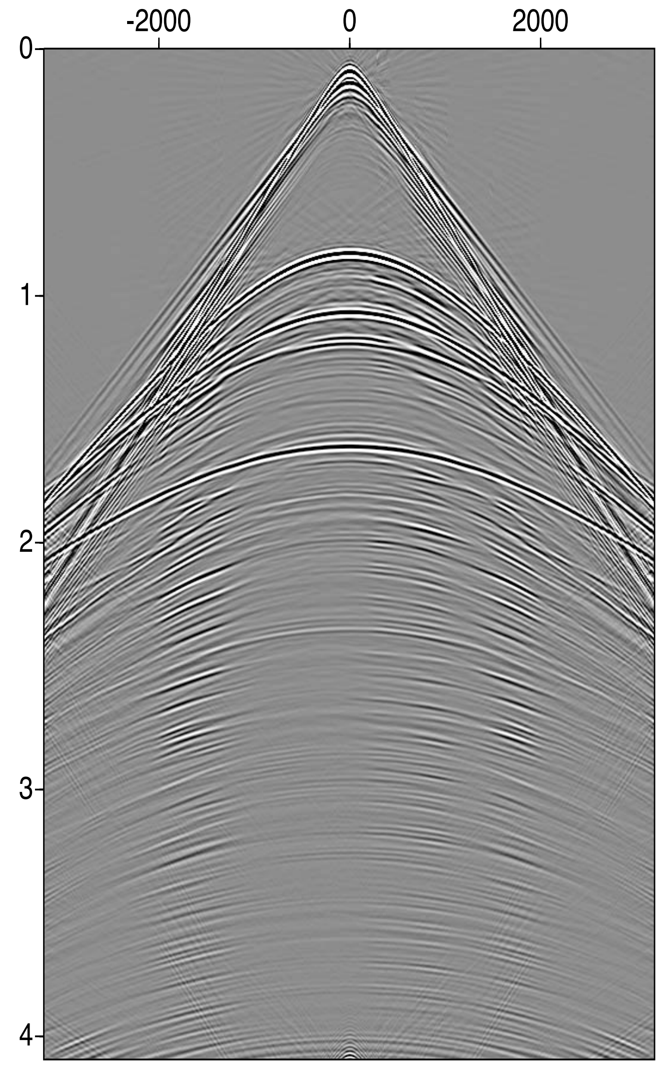


Result curvelet-based subtraction

Example shallow water environment

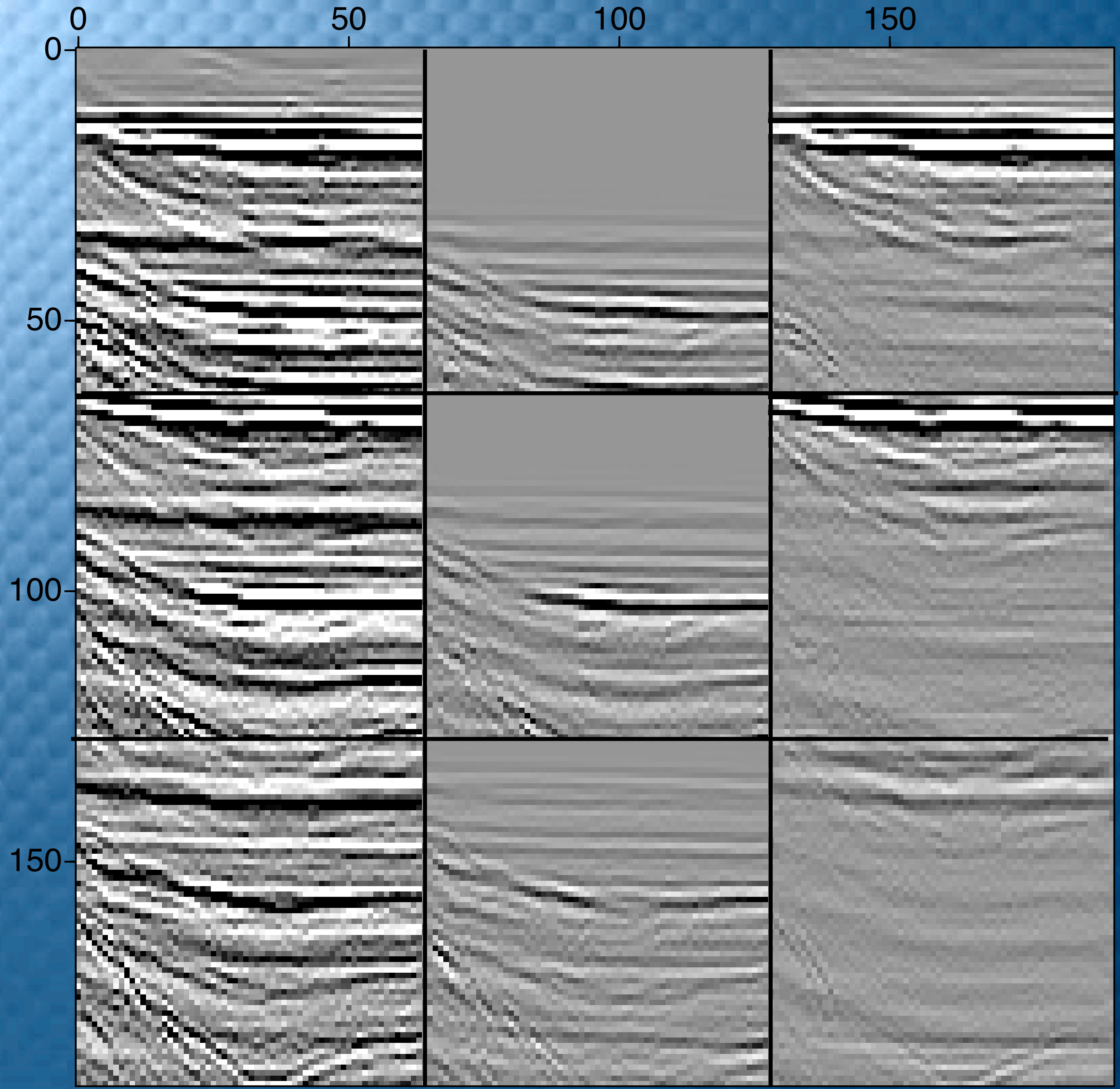


missing data with multiples



estimated primaries

3-D Curvelets



Conclusions

- For 3D SRME the acquisition geometry determines the prediction quality and possibilities
- New domains extend the adaptive subtraction toolbox and hence the quality of the end result
- Non-linear Curvelet thresholding adds robustness
    - under incoherent noise
    - under missing data
- Extended to 3-D

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Frank Kempe (Cray) for conducting the 3D SRME tests on the Cray-X1 and George Stephenson (Cray) for his support

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