Three-term Amplitude-Versus-Offset (AVO) inversion revisited by Curvelet and Wavelet transforms

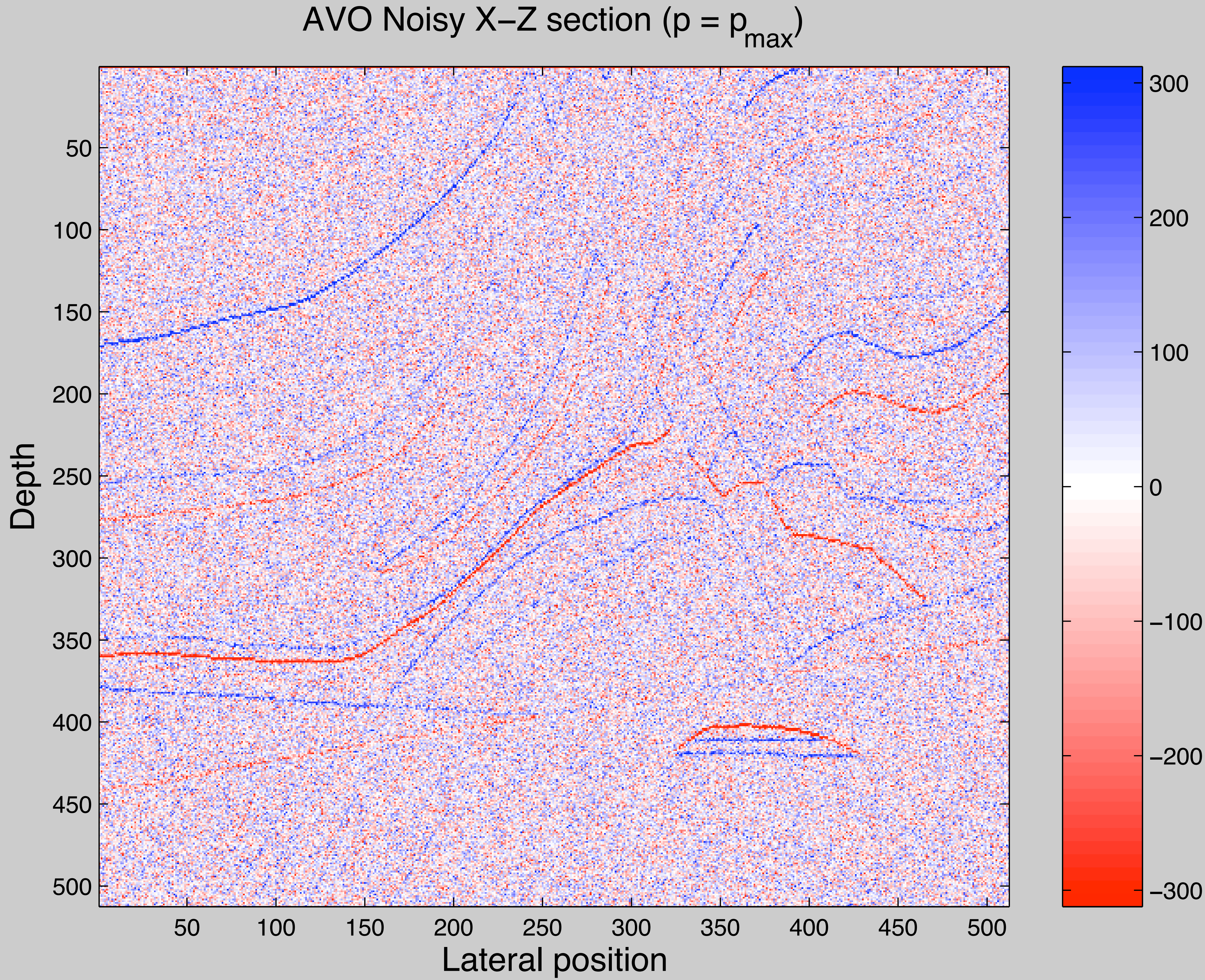
Gilles Hennenfent* & Felix Herrmann

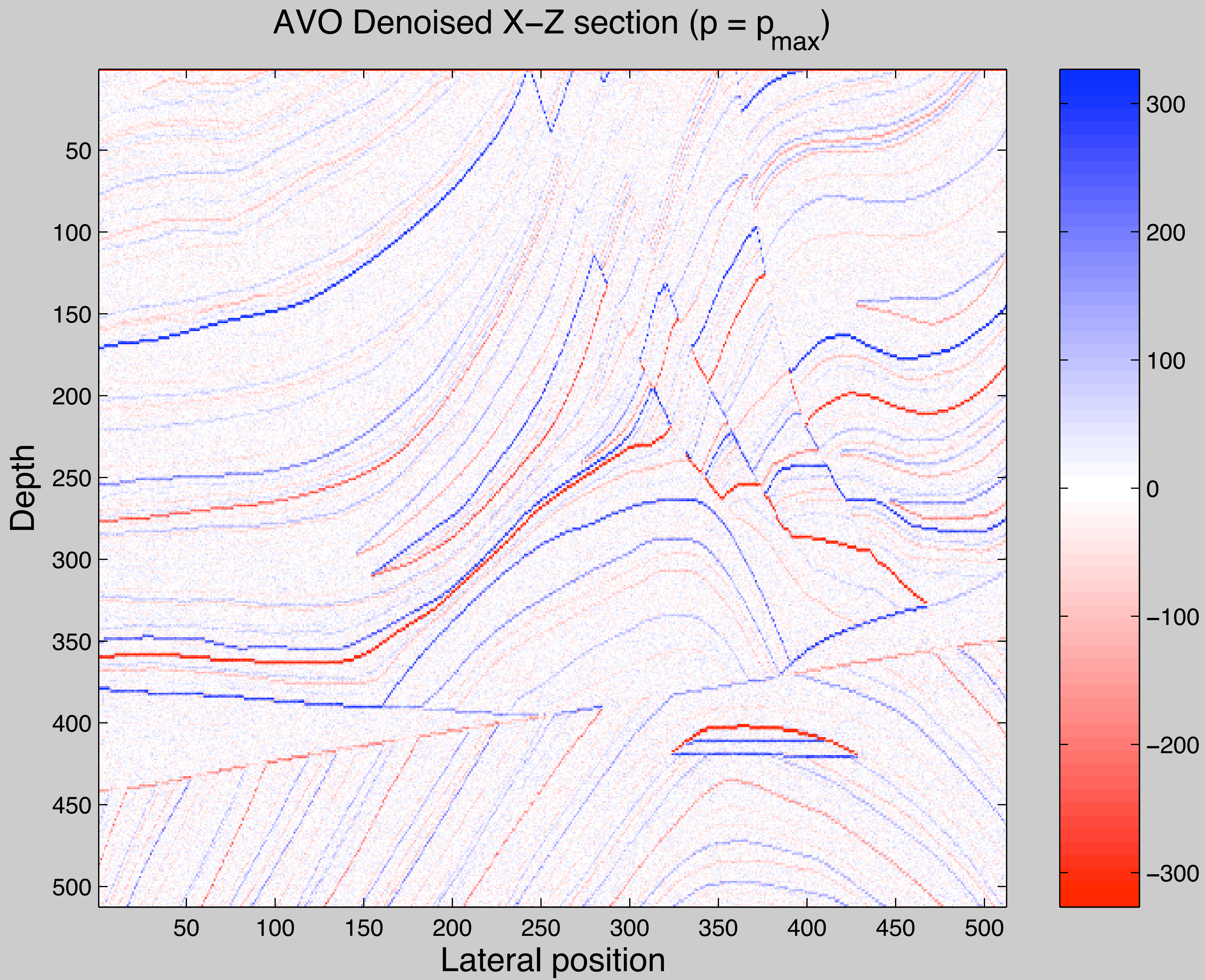
Earth & Ocean Sciences Department

University of British Columbia - Canada

Outline

- AVO forward & inverse problems
- How to improve using wavelet and curvelet frames?
    - In the data space
    - In the model space
- Angle Corrected Reflection Operator
- Discussion & Conclusions


AVO Noisy X−Z section (p = p_max)
Depth
Lateral position
50
100
150
200
250
300
350
400
450
500
300
200
100
0
−100
−200
−300



AVO Denoised X−Z section ($p = p_{max}$)
Depth
Lateral position
50
100
150
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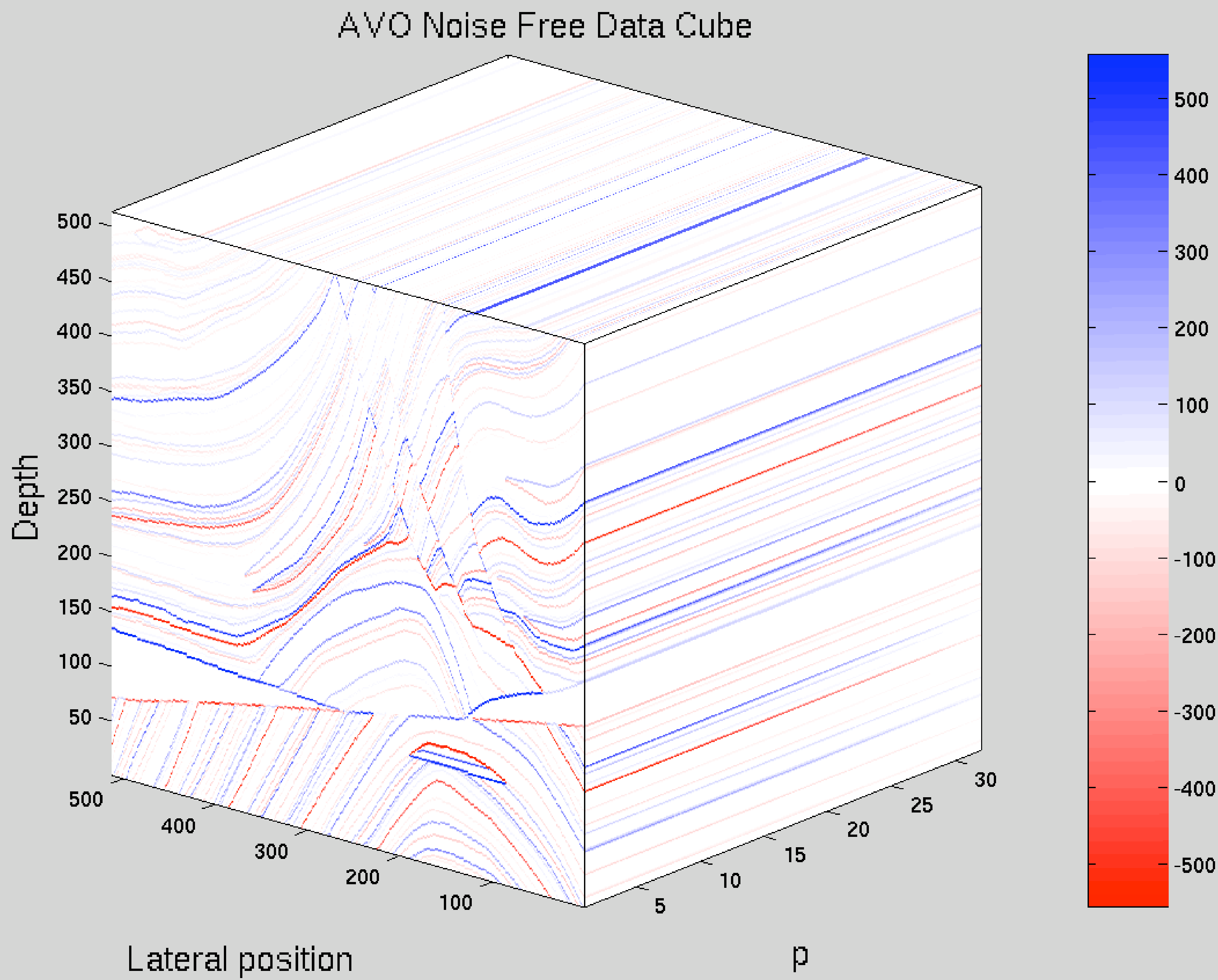
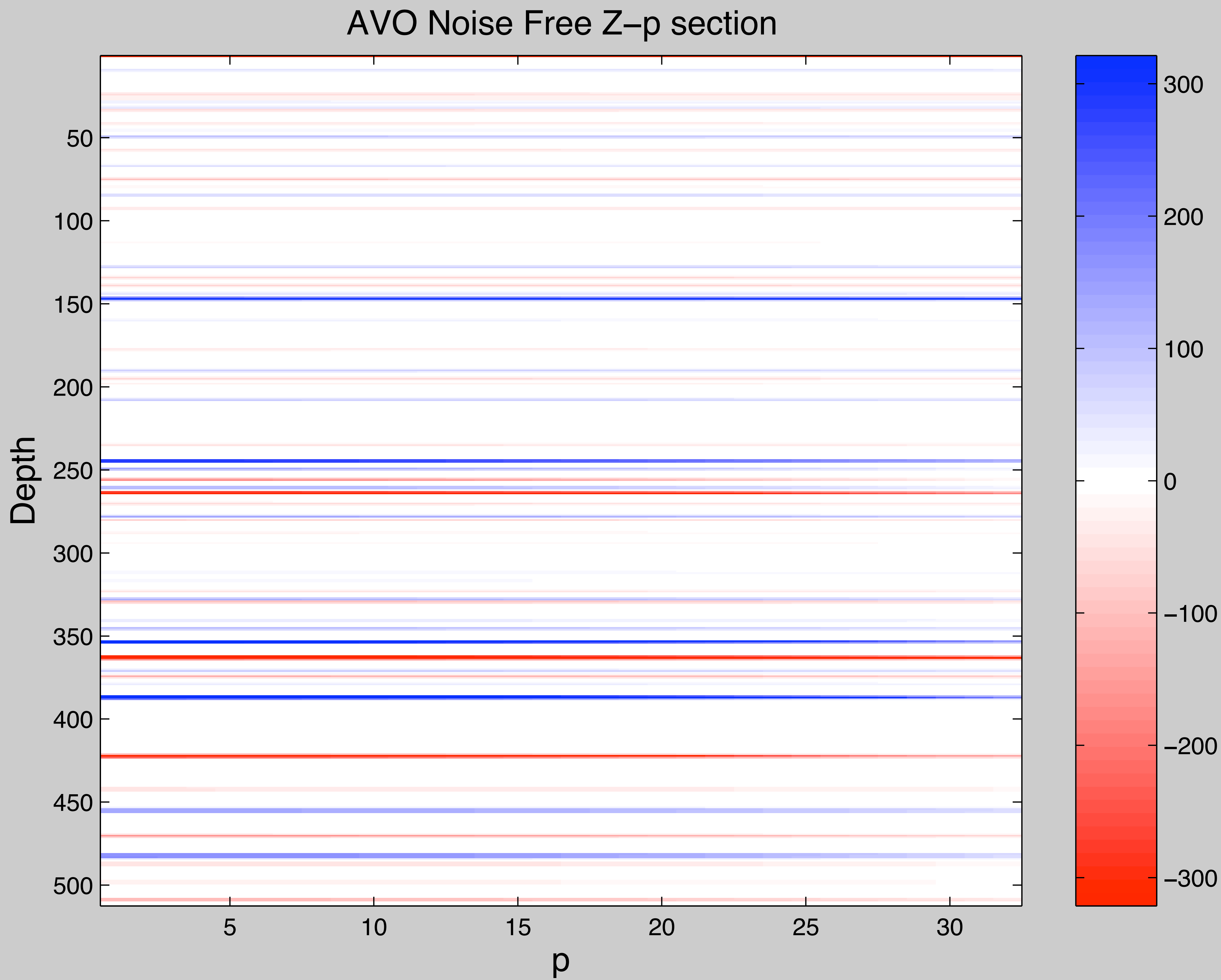
−300


AVO Noise Free Data Cube
Depth
500
450
400
350
300
250
200
150
100
50
Lateral position
500
400
300
200
100
p
5
10
15
20
25
30
500
400
300
200
100
0
-100
-200
-300
-400
-500


AVO Noise Free Z−p section
Depth
p
50
100
150
200
250
300
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400
450
500
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300
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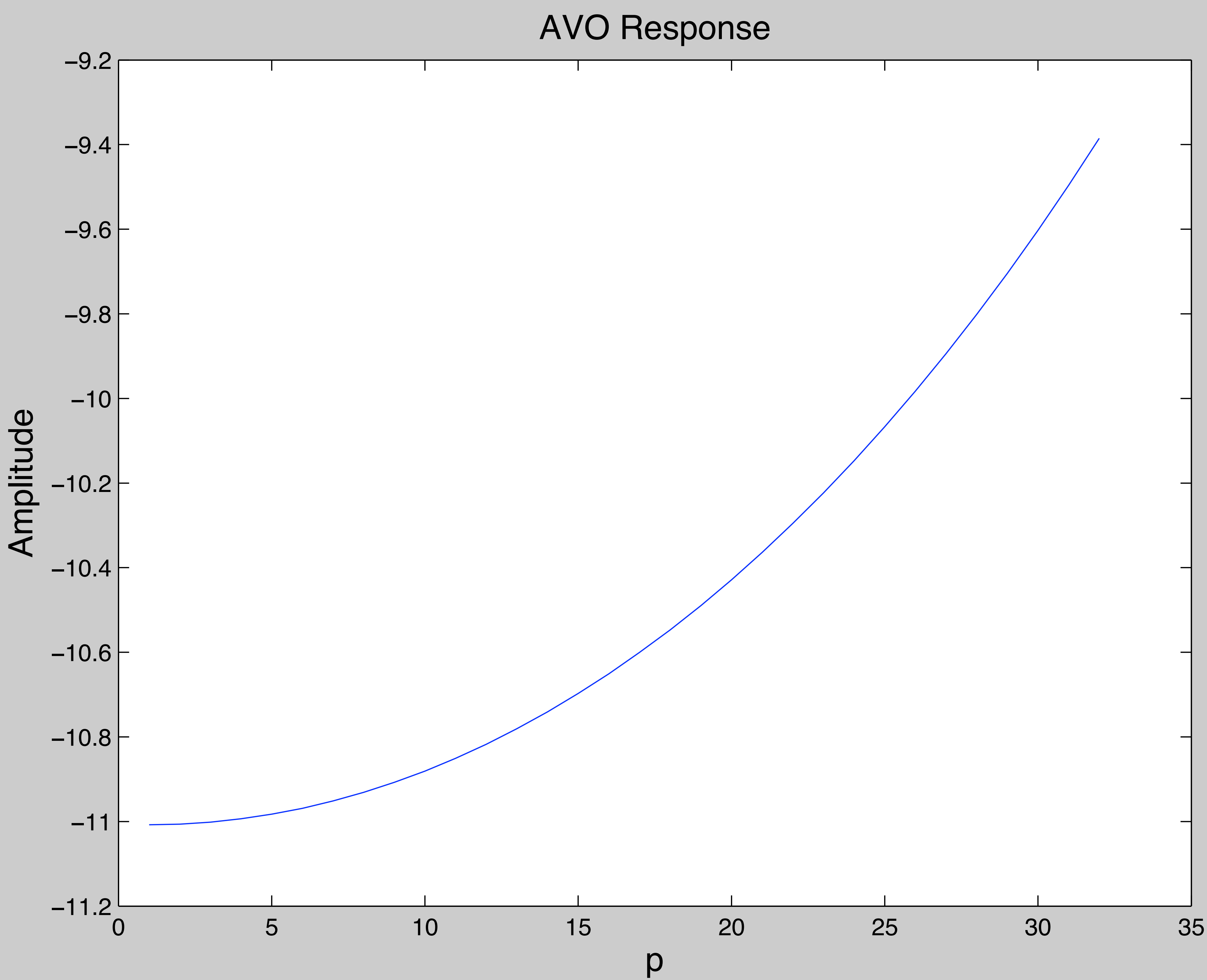
100
0
−100
−200
−300


AVO Response
Amplitude
p
−9.2
−9.4
−9.6
−9.8
−10
−10.2
−10.4
−10.6
−10.8
−11
−11.2
0
5
10
15
20
25
30
35

Linearized Zoeppritz Equation for Rpp

$$R_{pp}(p) = \frac{1}{2}(1 - 4\bar{c}_s^2 p^2)\frac{\Delta\rho}{\bar{\rho}} + \frac{1}{2\cos^2\theta}\frac{\Delta c_p}{\bar{c}_p} - 4\bar{c}_s^2 p^2 \frac{\Delta c_s}{\bar{c}_s}$$ *

$$\begin{bmatrix} R_{pp}(p_1) \\ \vdots \\ R_{pp}(p_N) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2}\frac{\bar{C}_p^2 p_1^2}{1-\bar{C}_p^2 p_1^2} & -2\frac{\bar{C}_p^2 p_1^2 \bar{C}_s^2}{\bar{C}_p^2} \\ \vdots & \vdots & \vdots \\ \frac{1}{2} & \frac{1}{2}\frac{\bar{C}_p^2 p_N^2}{1-\bar{C}_p^2 p_N^2} & -2\frac{\bar{C}_p^2 p_N^2 \bar{C}_s^2}{\bar{C}_p^2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\Delta \bar{Z}}{\bar{Z}} \\ \frac{\Delta \bar{C}_p}{\bar{C}_p} \\ \frac{\Delta \bar{\mu}}{\bar{\mu}} \end{bmatrix}$$

* Aki and Richards (1980)

AVO Forward Problem

$$d = Km$$

- Assume point scatterer:

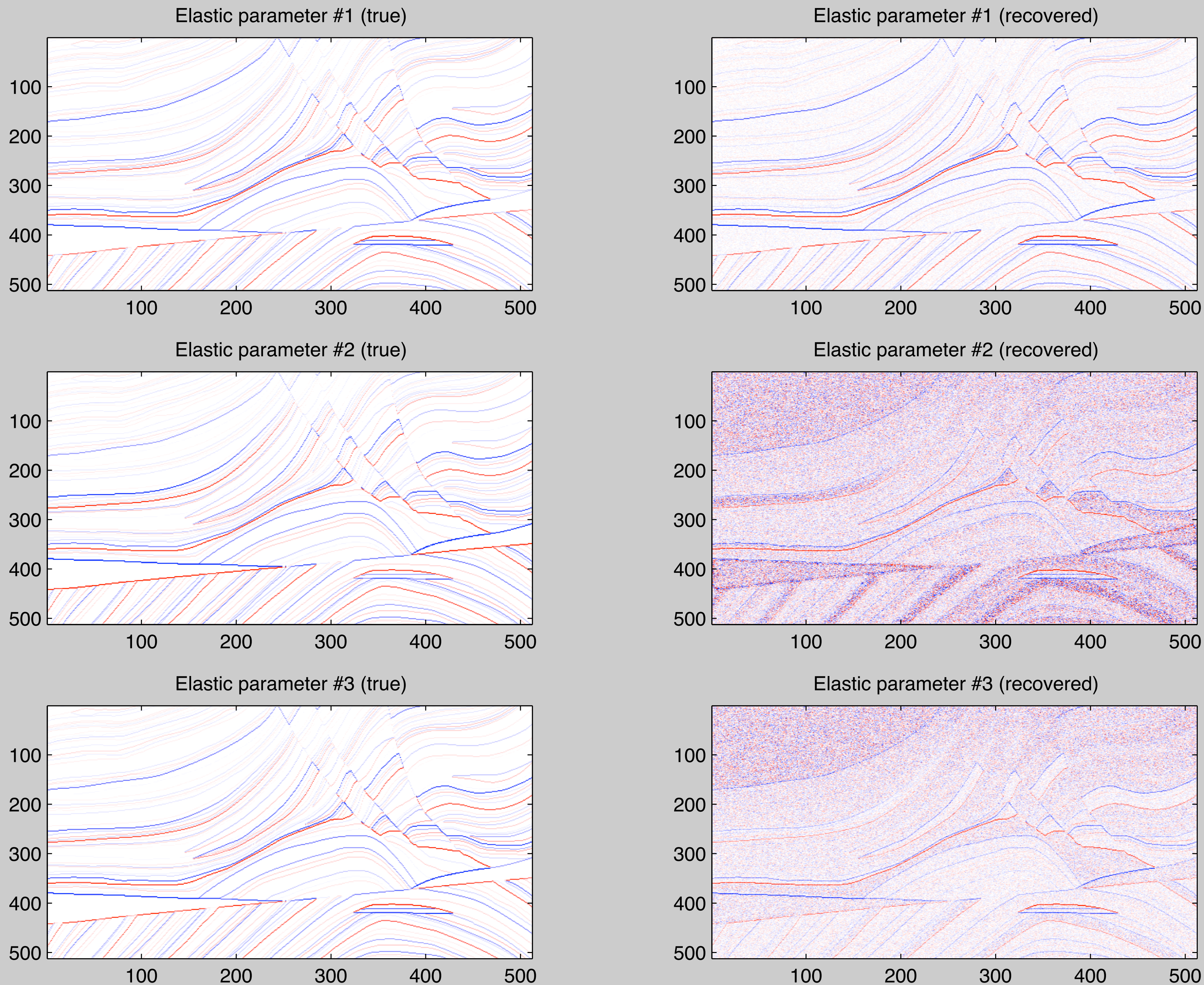
NO INFORMATION FROM LATERAL CONTINUITY ALONG REFLECTORS

- Real life:

$$d = Km + n$$

AVO Inverse Problem

- K ill-conditioned => inversion difficult
- Attempts to solve the problem:
  - $\min_m \frac{1}{2}\|d - Km\|_2^2$ (stopping criteria?)
  - $\min_m \frac{1}{2}\|d - Km\|_2^2 + \lambda\|m\|_2^2$ (smooth model)
  - $\min_m \frac{1}{2}\|d - Km\|_2^2 + \lambda\|\nabla m\|_2^2$ (smooth model)



Elastic parameter #1 (true)
Elastic parameter #1 (recovered)
Elastic parameter #2 (true)
Elastic parameter #2 (recovered)
Elastic parameter #3 (true)
Elastic parameter #3 (recovered)
100
200
300
400
500

- Data preprocessing (data side) by exploring:
  - CONTINUITY along reflectors (Curvelets)
  - SMOOTHNESS along p-axis* (Wavelets)
- Singularity-preserving inversion (model side):
  - Iterative thresholding algorithm with a sparsity constraint**
  - Basis function

* Sacchi (2003)

** Daubechies (2004)

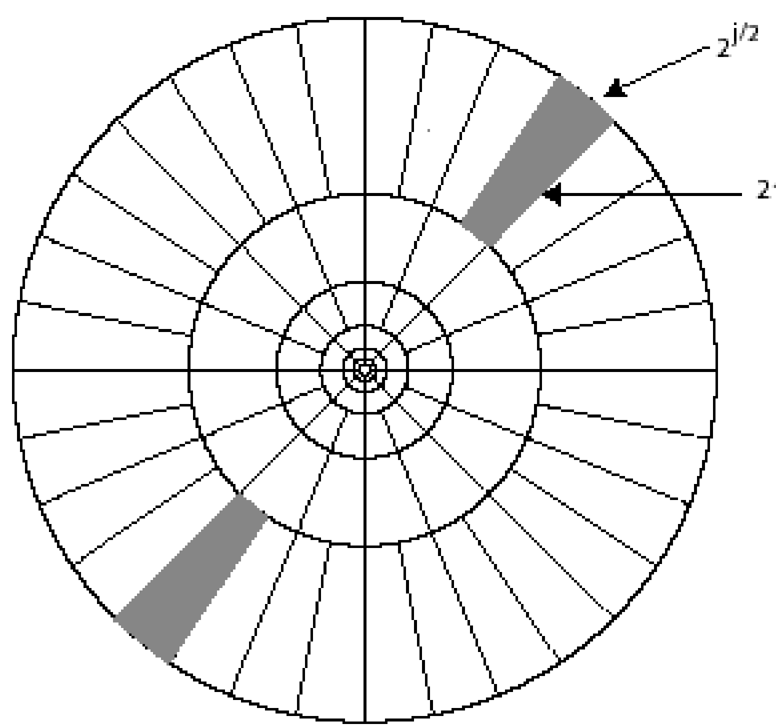
Curvelets

- Parabolic scaling principle:
  - $\text{length}^2 \propto \text{width}$
- Directional selective:
  - # orientations doubles every other scale
- Close to optimal for functions with singularities on C^2-curves: $$\|m - \hat{m}^C\| \propto \lambda \frac{(\log N)^3}{N^2}, \; N \to \infty$$

Candes (2002)

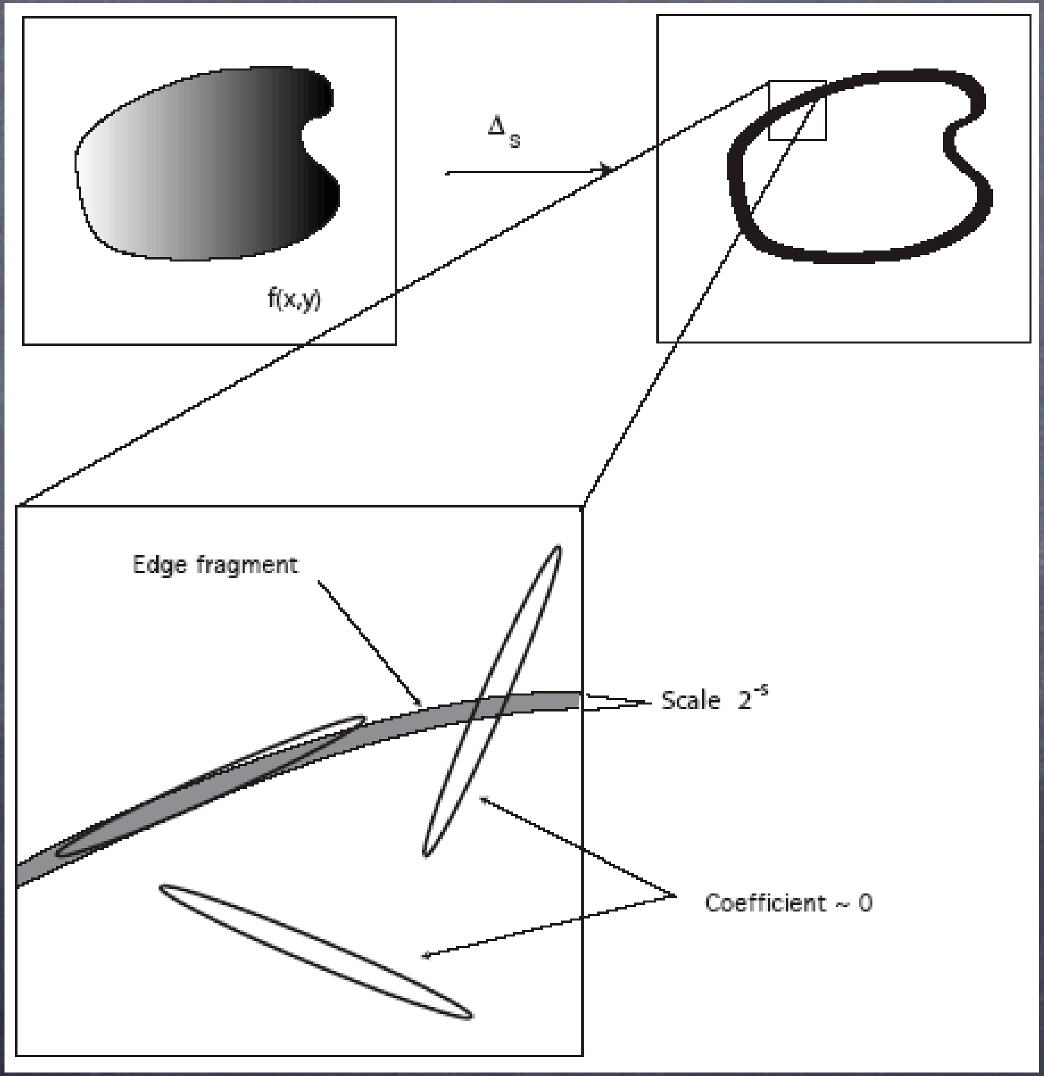
2D Frequency Domain Tilling

$$W_j = \{\zeta, \quad 2^j \le |\zeta| \le 2^{j+1}, |\theta - \theta_J| \le \pi \cdot 2^{\lfloor j/2 \rfloor}\}$$



Candes (2002)

Curvelets & Curves



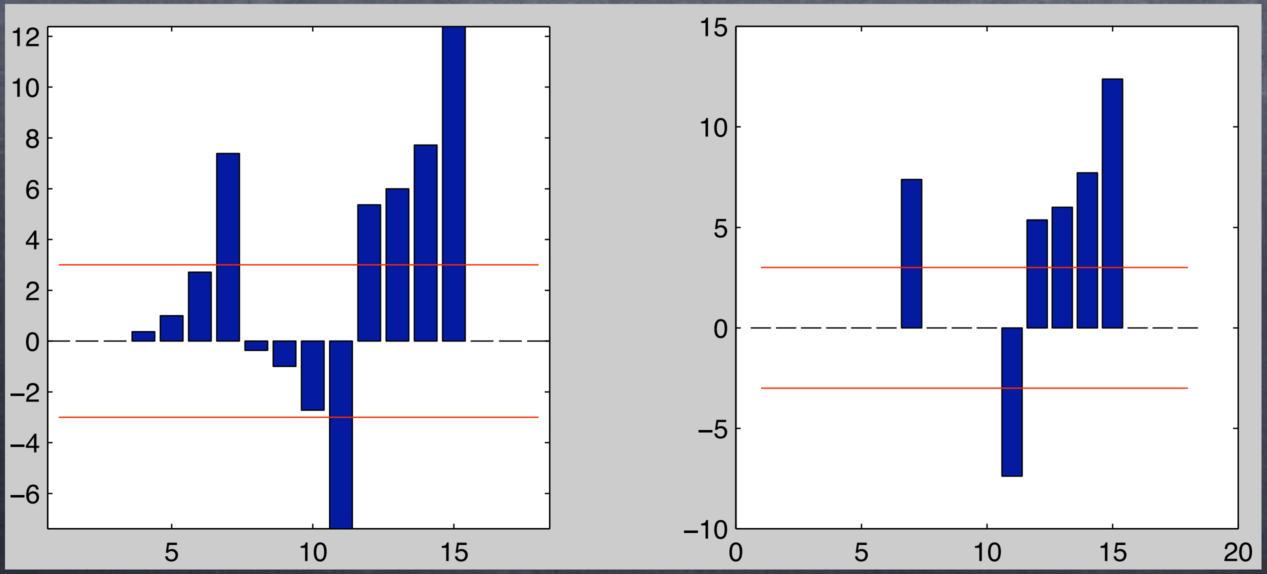
Candes (2002)

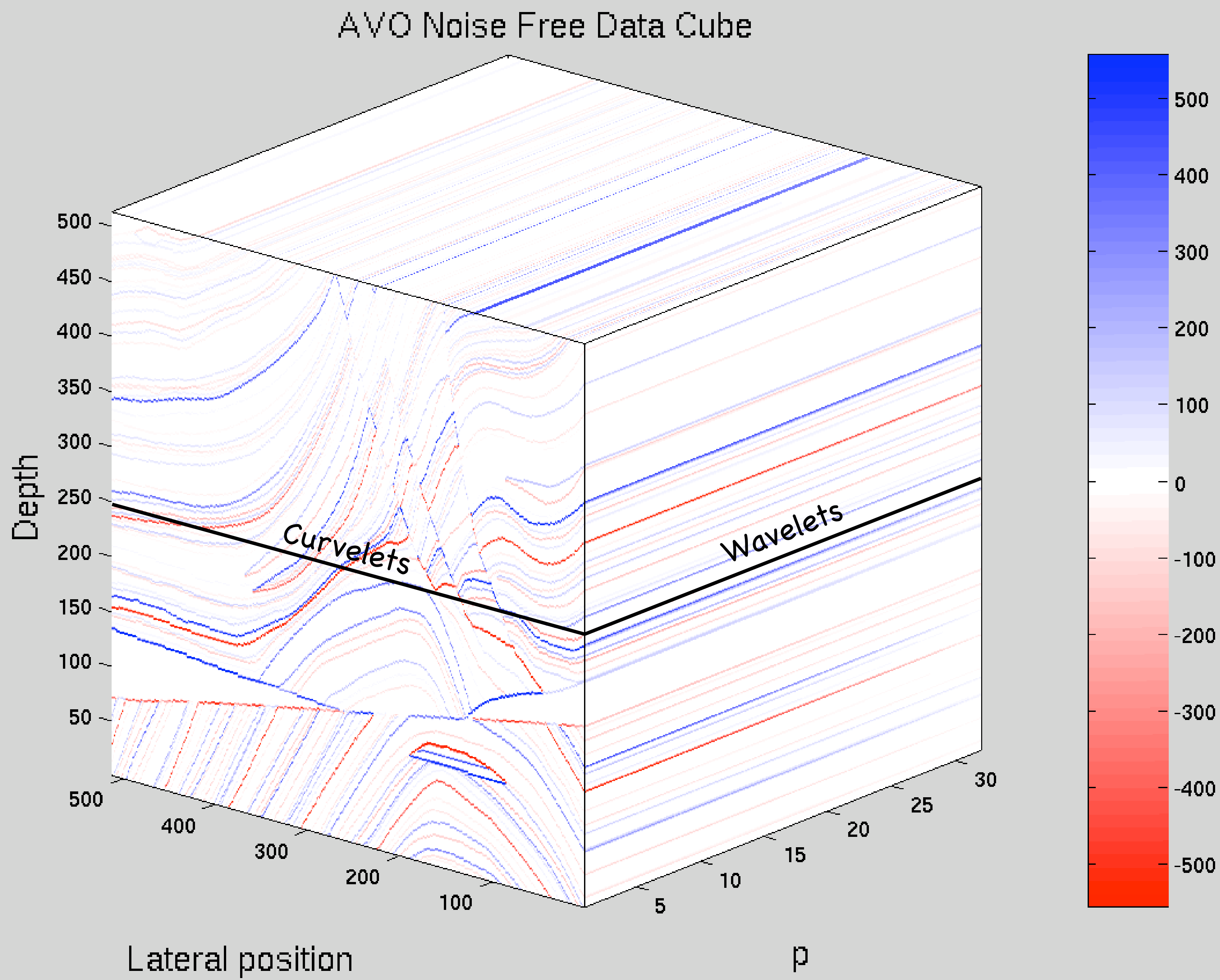
Data Preprocessing

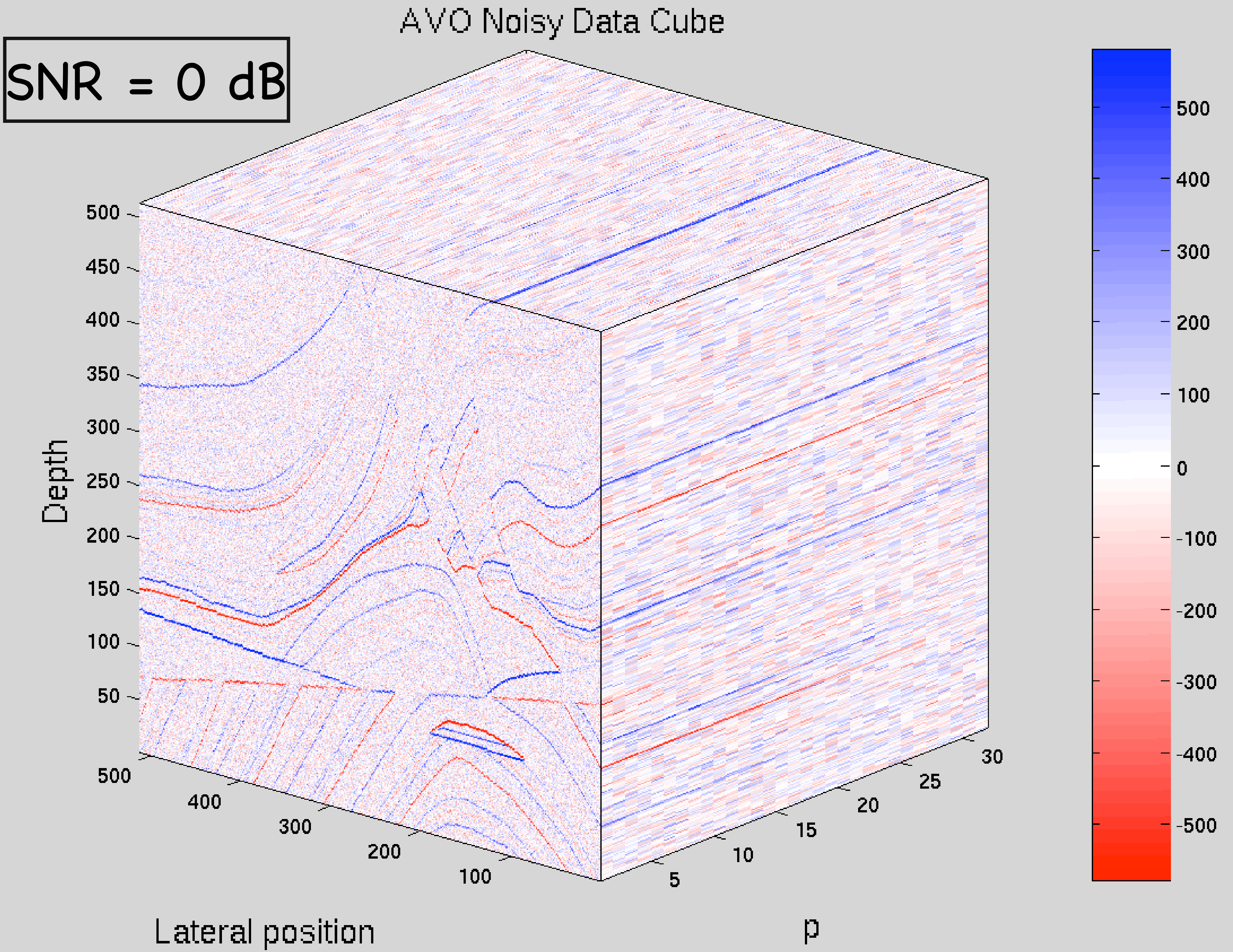
$$W_p C_{xz} d = W_p C_{xz} K m + W_p C_{xz} n$$

$$\hat{\tilde{d}} = \Theta_\Gamma(W_p C_{xz} d) \approx W_p C_{xz} K m$$

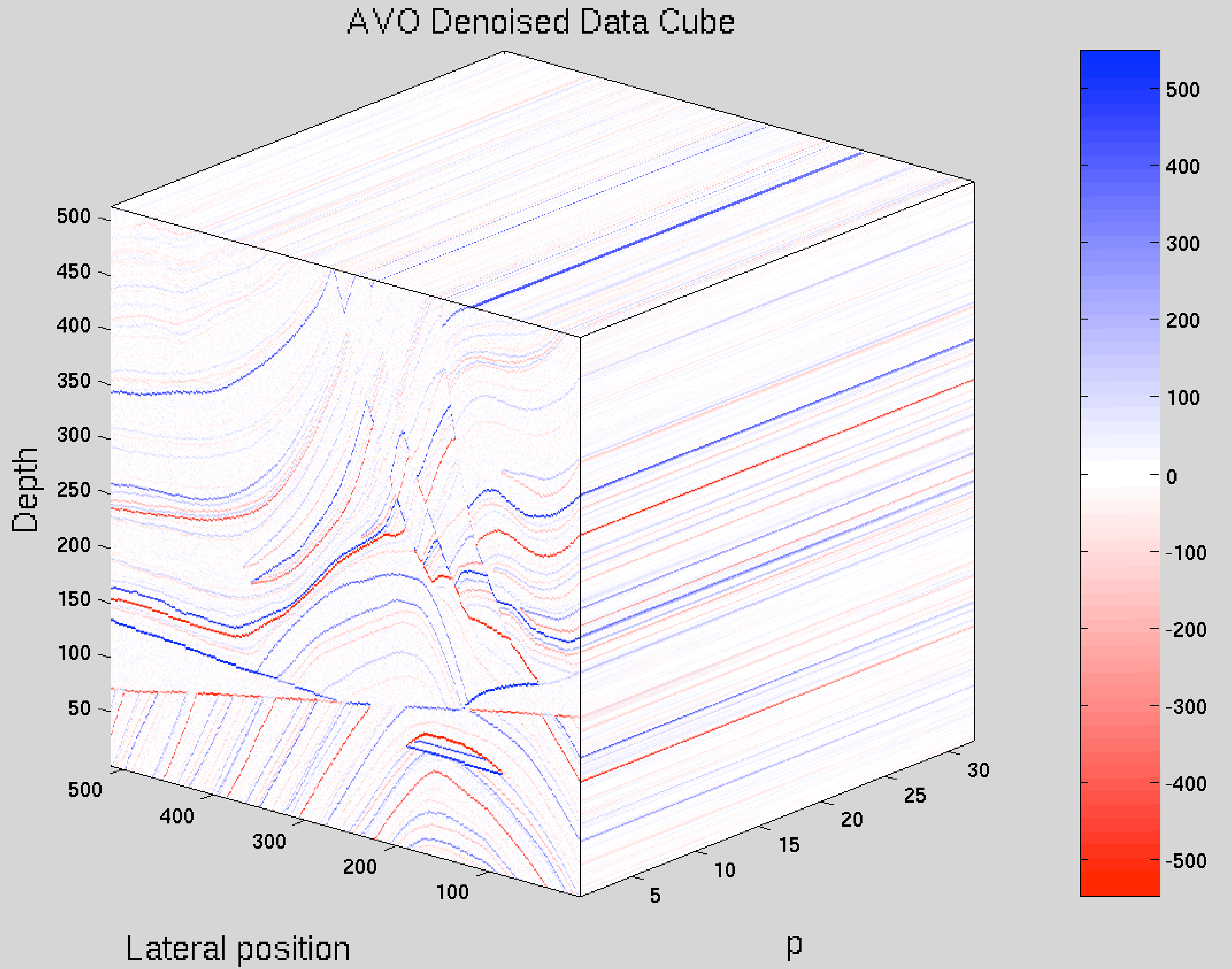
with $\Theta_\Gamma(u) = u I\{|u| > \Gamma\}$ (Hard threshold)



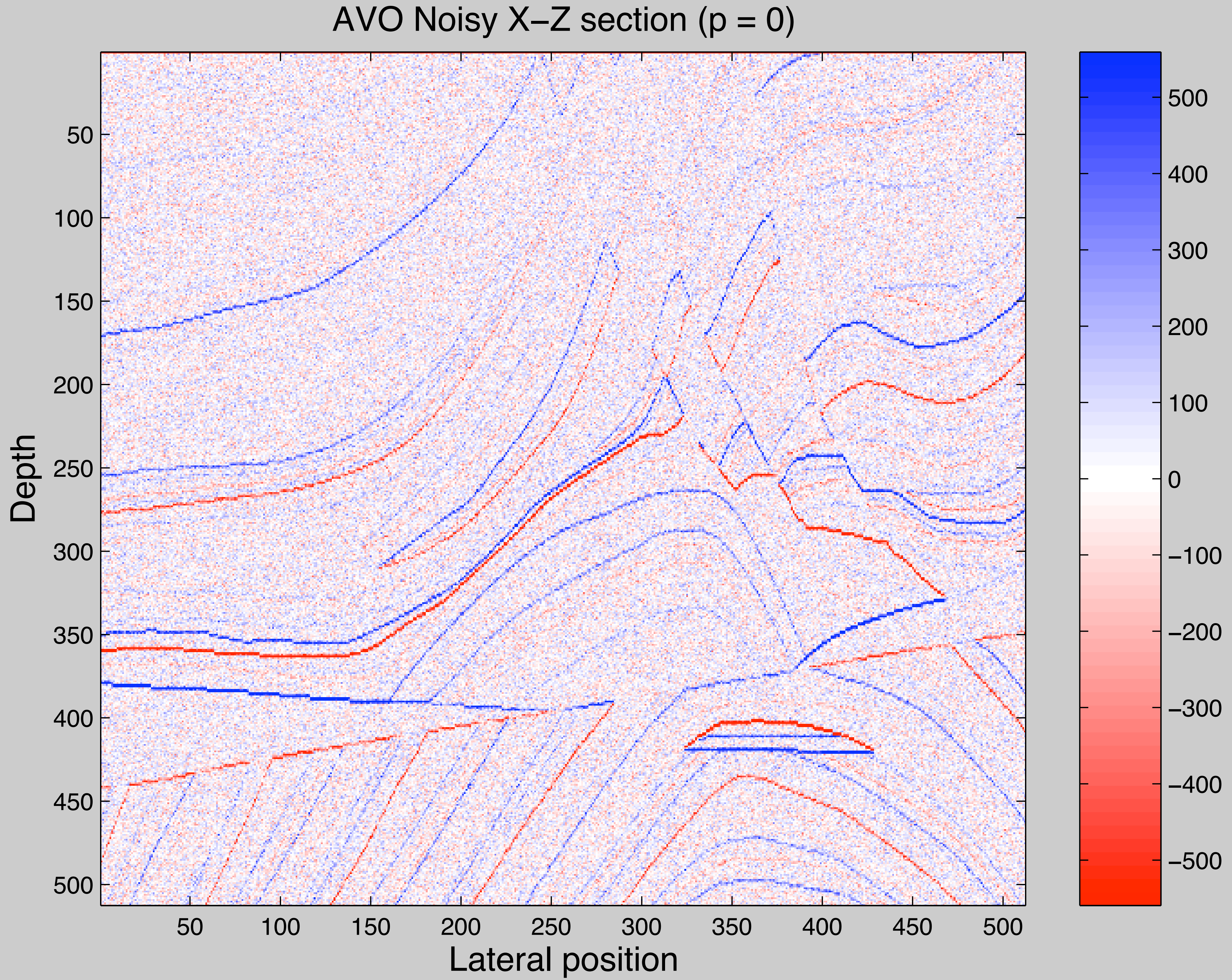

AVO Noise Free Data Cube
Depth
500
450
400
350
300
250
200
150
100
50
Lateral position
500
400
300
200
100
p
5
10
15
20
25
30
Curvelets
Wavelets
500
400
300
200
100
0
-100
-200
-300
-400
-500

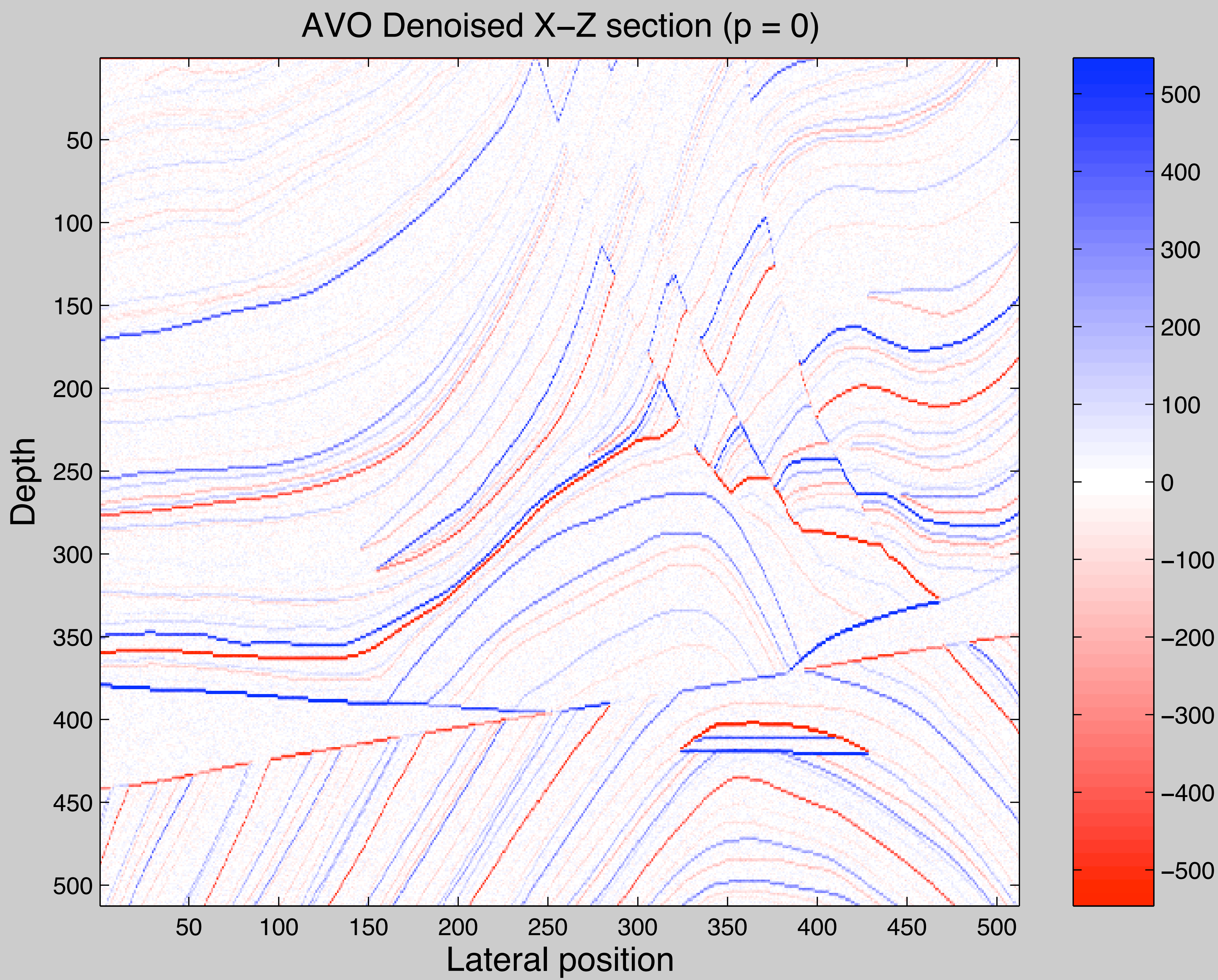


AVO Noisy Data Cube
SNR = 0 dB
Depth
500
450
400
350
300
250
200
150
100
50
Lateral position
500
400
300
200
100
p
5
10
15
20
25
30
500
400
300
200
100
0
-100
-200
-300
-400
-500



AVO Denoised Data Cube
Depth
500
450
400
350
300
250
200
150
100
50
Lateral position
500
400
300
200
100
p
5
10
15
20
25
30
500
400
300
200
100
0
-100
-200
-300
-400
-500


AVO Noisy X–Z section (p = 0)
Depth
Lateral position
50
100
150
200
250
300
350
400
450
500
500
400
300
200
100
0
−100
−200
−300
−400
−500



AVO Denoised X−Z section (p = 0)
Depth
50
100
150
200
250
300
350
400
450
500
Lateral position
50
100
150
200
250
300
350
400
450
500
500
400
300
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100
0
−100
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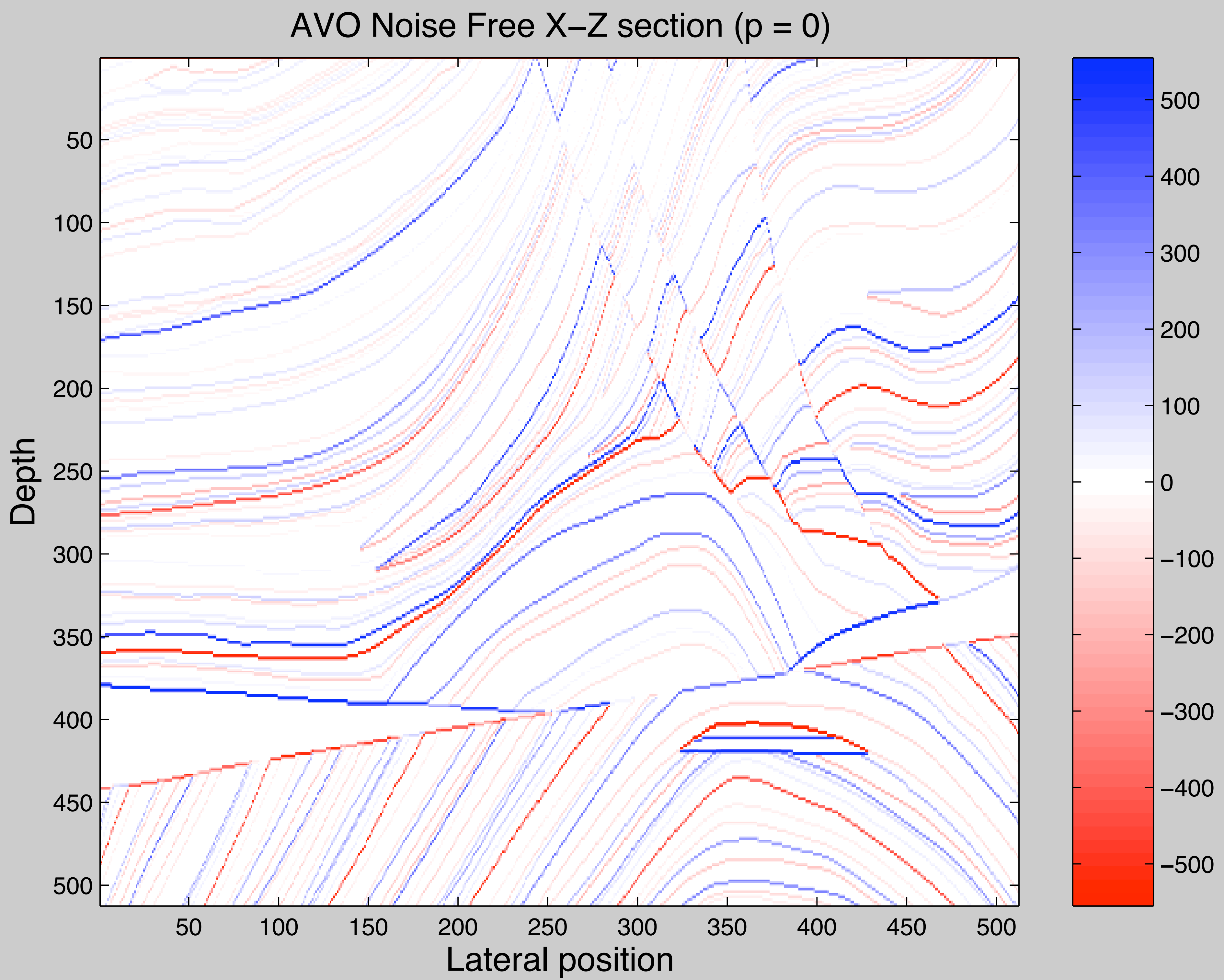
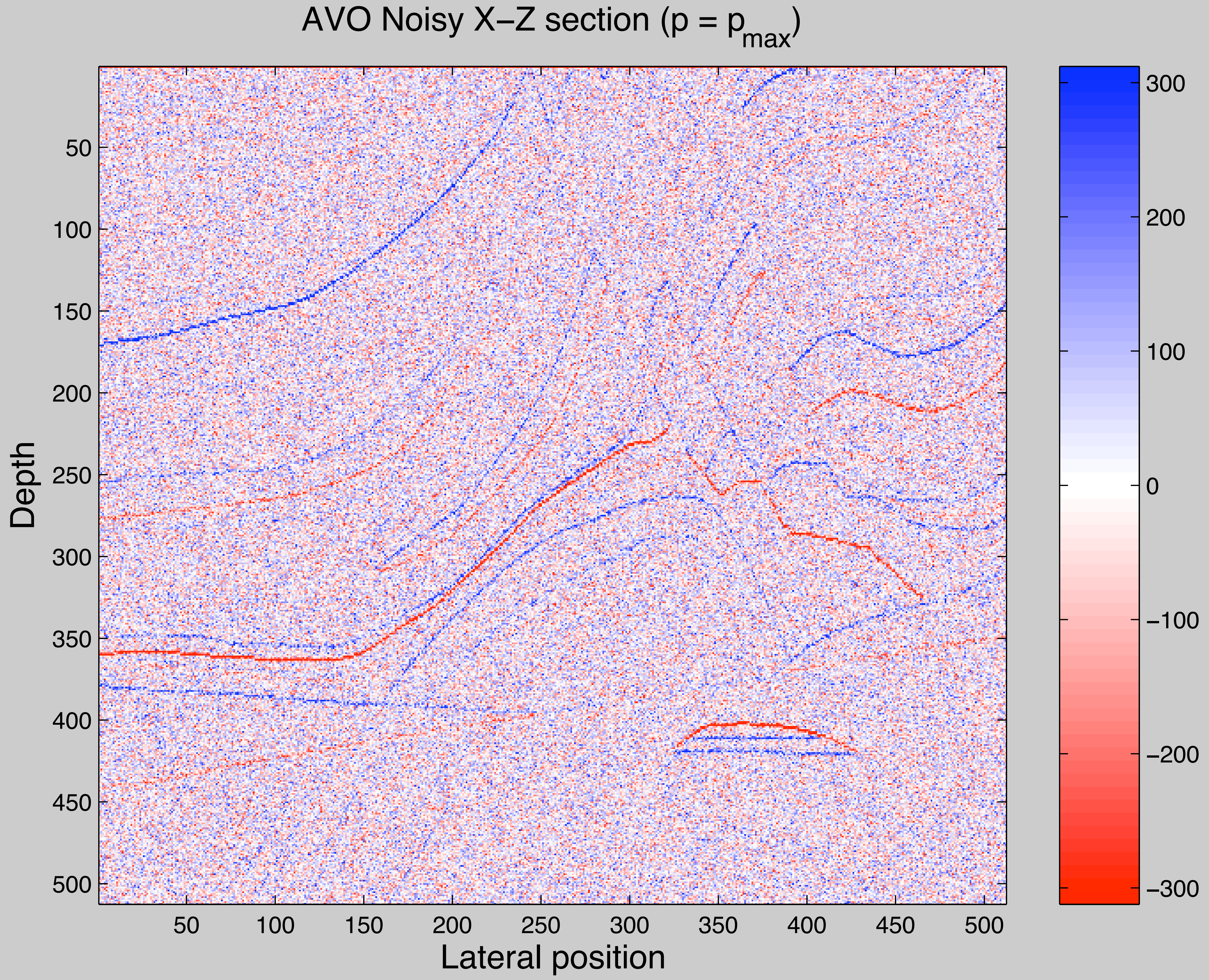
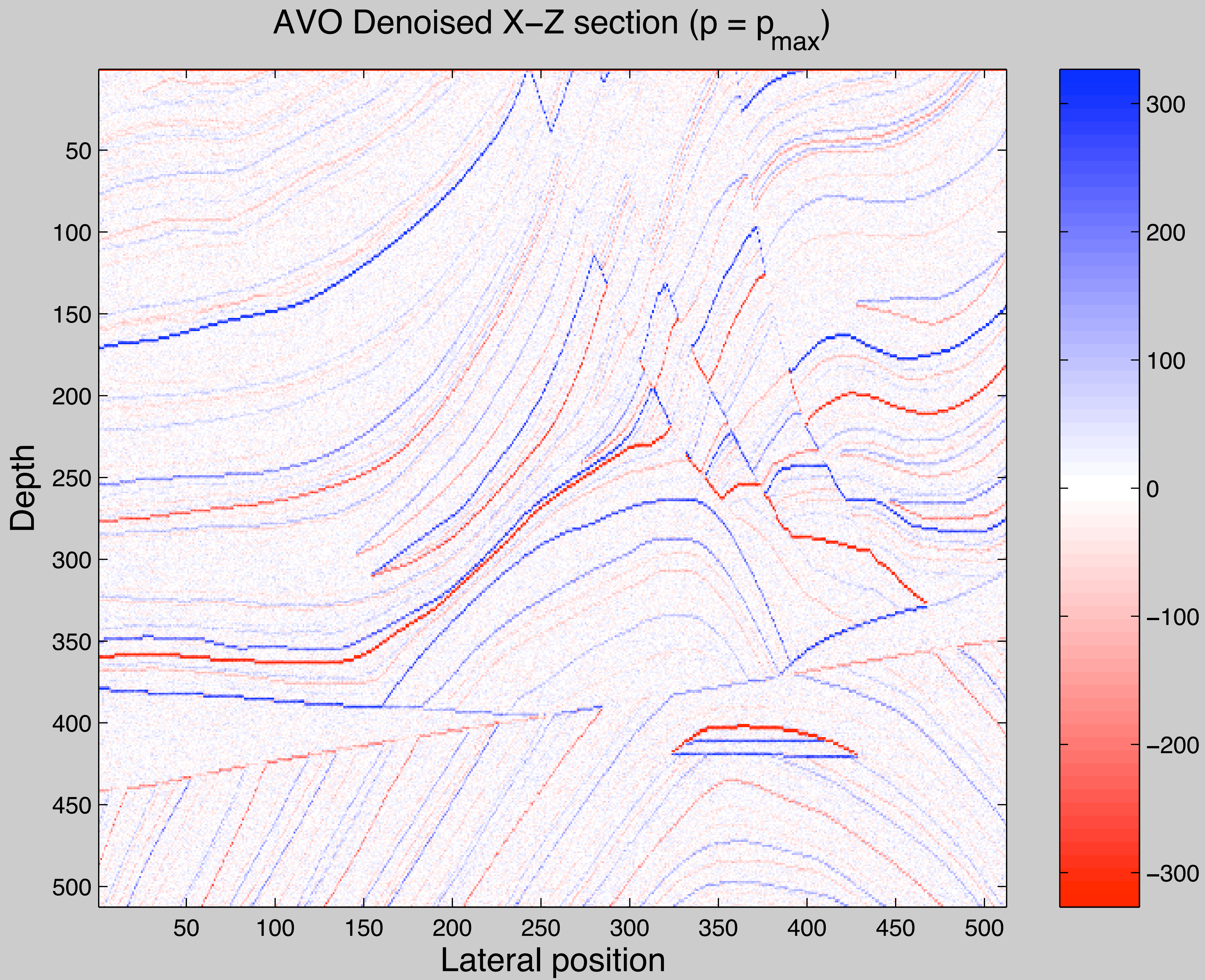

AVO Noise Free X−Z section (p = 0)
Depth
Lateral position
50
100
150
200
250
300
350
400
450
500
500
400
300
200
100
0
−100
−200
−300
−400
−500


AVO Noisy X−Z section (p = p_max)
Depth
Lateral position
50
100
150
200
250
300
350
400
450
500
300
200
100
0
−100
−200
−300


AVO Denoised X−Z section (p = p_max)
Depth
Lateral position
50
100
150
200
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300
350
400
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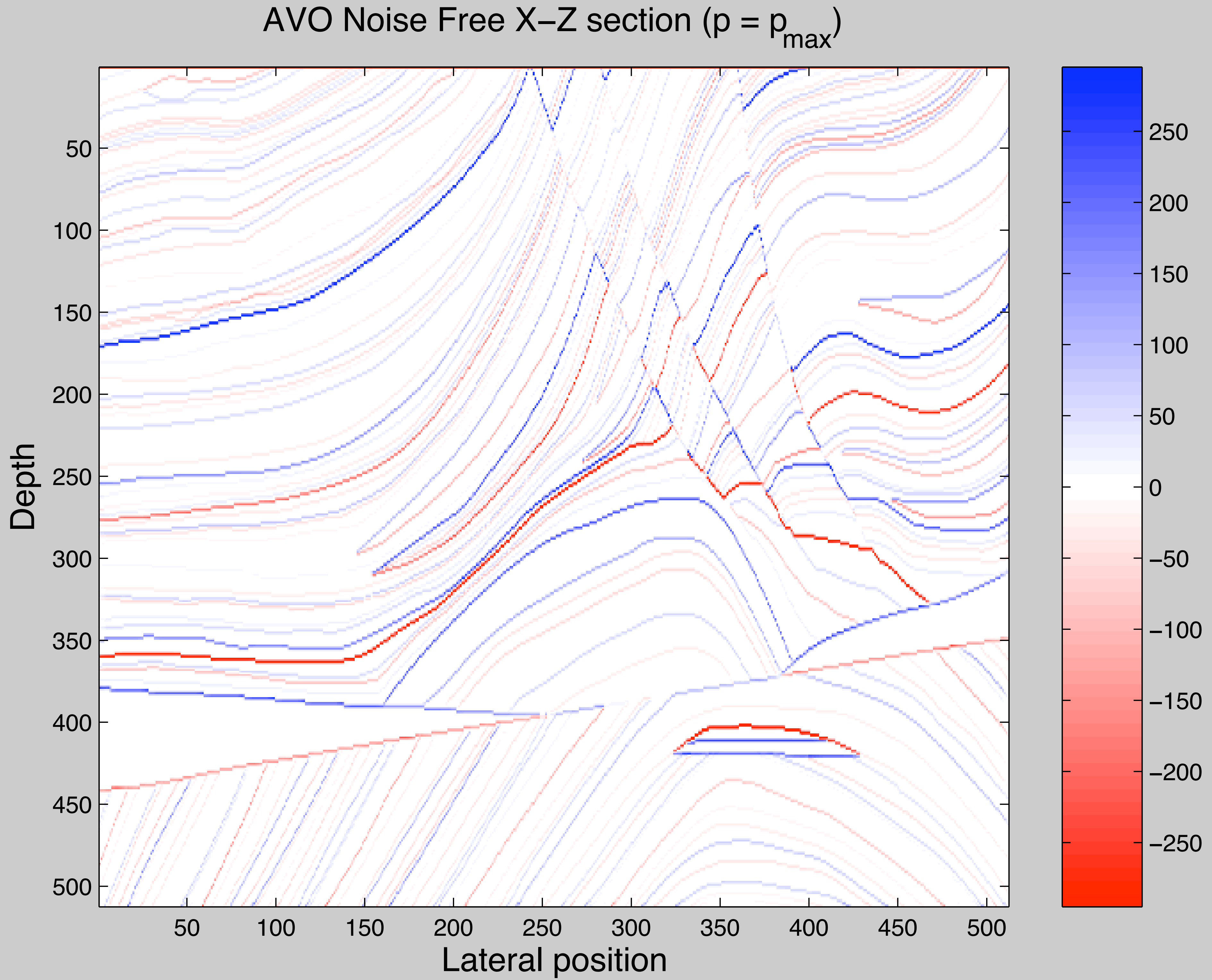
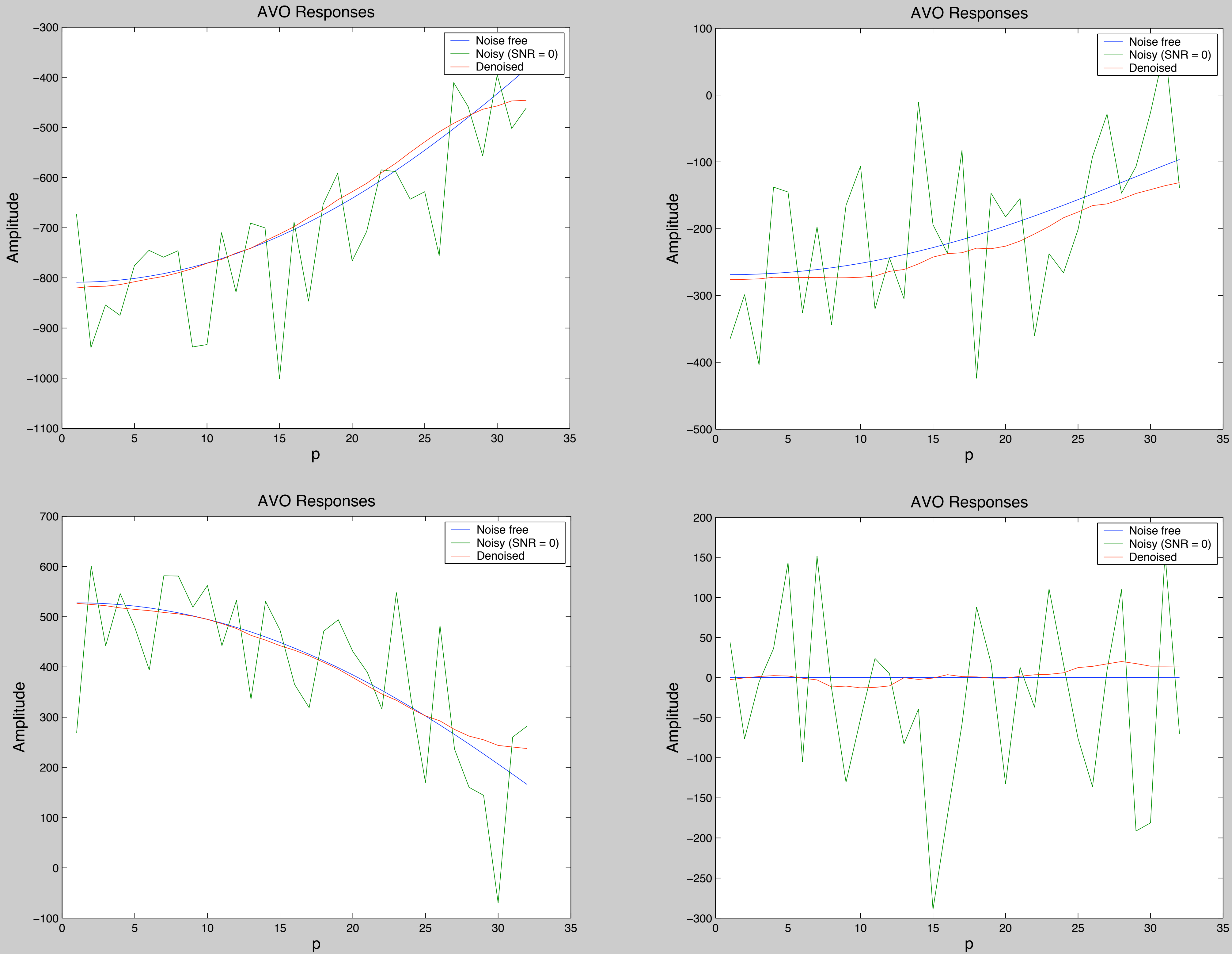
350
400
450
500
300
200
100
0
−100
−200
−300


AVO Noise Free X–Z section (p = p_max)
Depth
Lateral position
50
100
150
200
250
300
350
400
450
500
250
200
150
100
50
0
−50
−100
−150
−200
−250

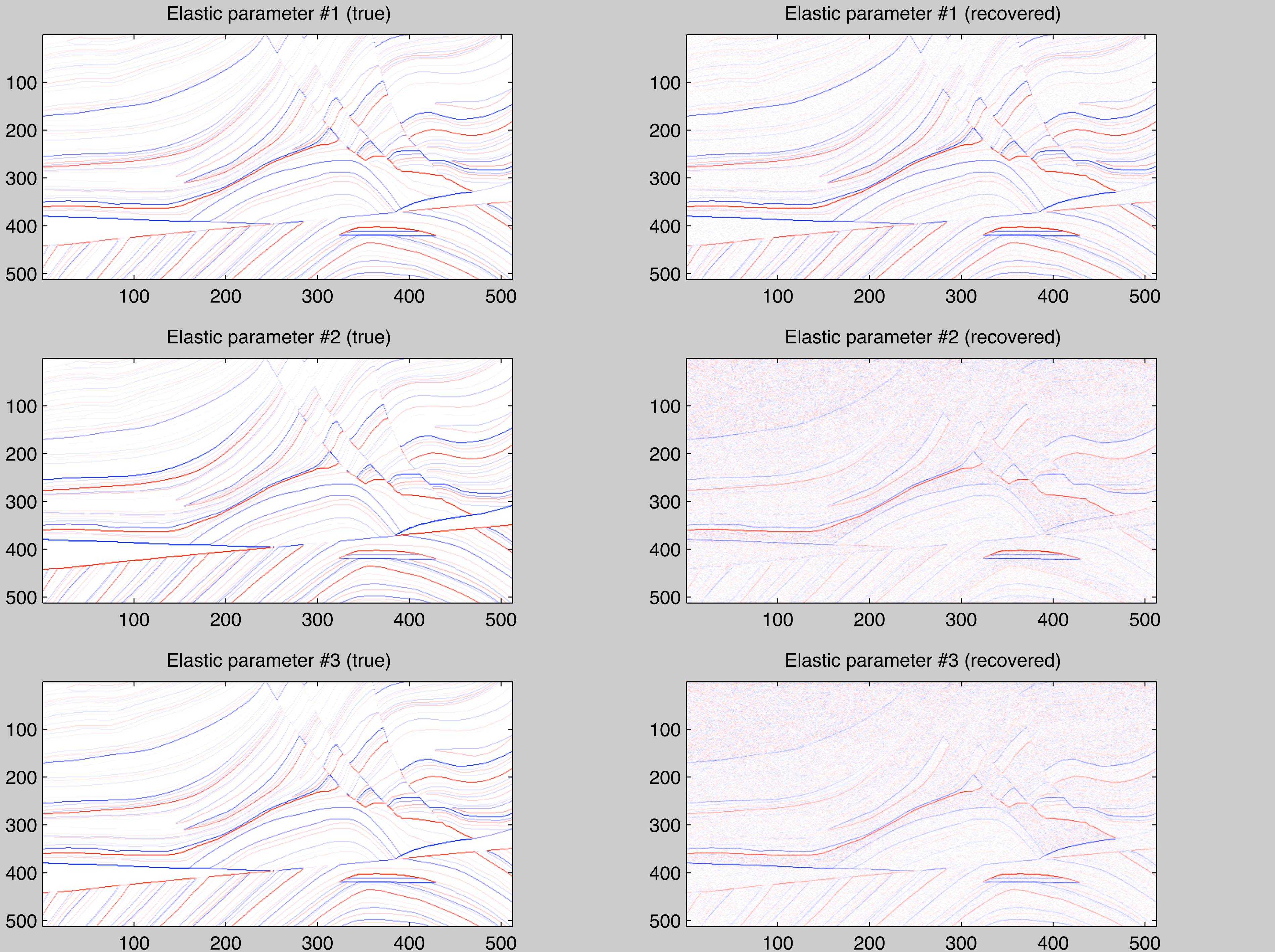


AVO Responses
Noise free
Noisy (SNR = 0)
Denoised
Amplitude
p
−300
−400
−500
−600
−700
−800
−900
−1000
−1100
0
5
10
15
20
25
30
35
AVO Responses
Noise free
Noisy (SNR = 0)
Denoised
Amplitude
p
100
0
−100
−200
−300
−400
−500
0
5
10
15
20
25
30
35
AVO Responses
Noise free
Noisy (SNR = 0)
Denoised
Amplitude
p
700
600
500
400
300
200
100
0
−100
0
5
10
15
20
25
30
35
AVO Responses
Noise free
Noisy (SNR = 0)
Denoised
Amplitude
p
200
150
100
50
0
−50
−100
−150
−200
−250
−300
0
5
10
15
20
25
30
35

SNR Improvement

$$\boxed{SNR = -10\log_{10}(\frac{\sigma^2_{signal}}{\sigma^2_{noise}})}$$

- p = 0:
  - SNR(noisy data) = 1.60 dB
  - SNR(denoised data) = 14.23 dB
- p = pmax:
  - SNR(noisy data) = -4.51 dB
  - SNR(denoised data) = 6.96 dB



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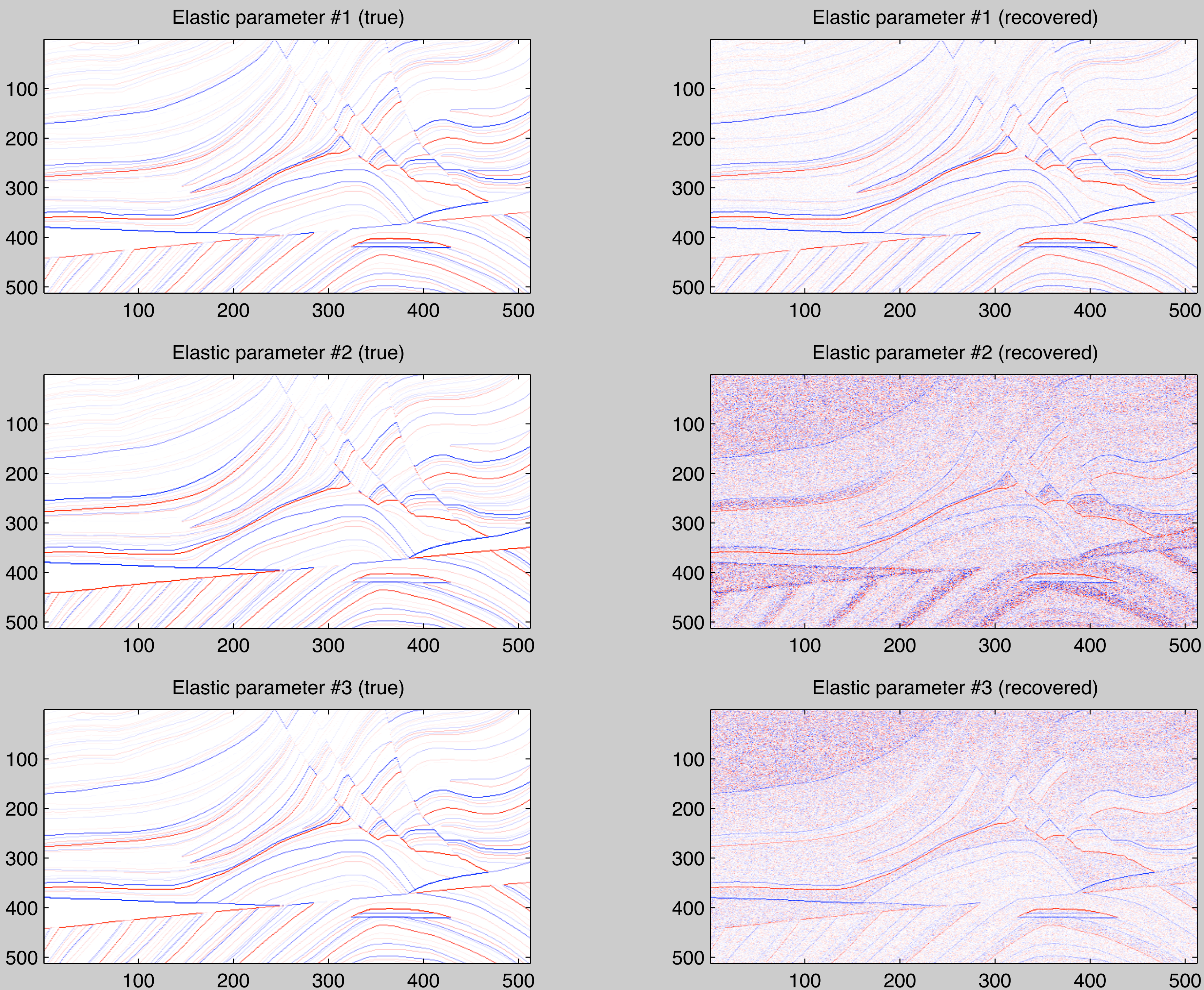
Elastic parameter #3 (recovered)
100
200
300
400
500



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Elastic parameter #2 (recovered)
Elastic parameter #3 (true)
Elastic parameter #3 (recovered)
100
200
300
400
500

Singularity-Preserving Inversion

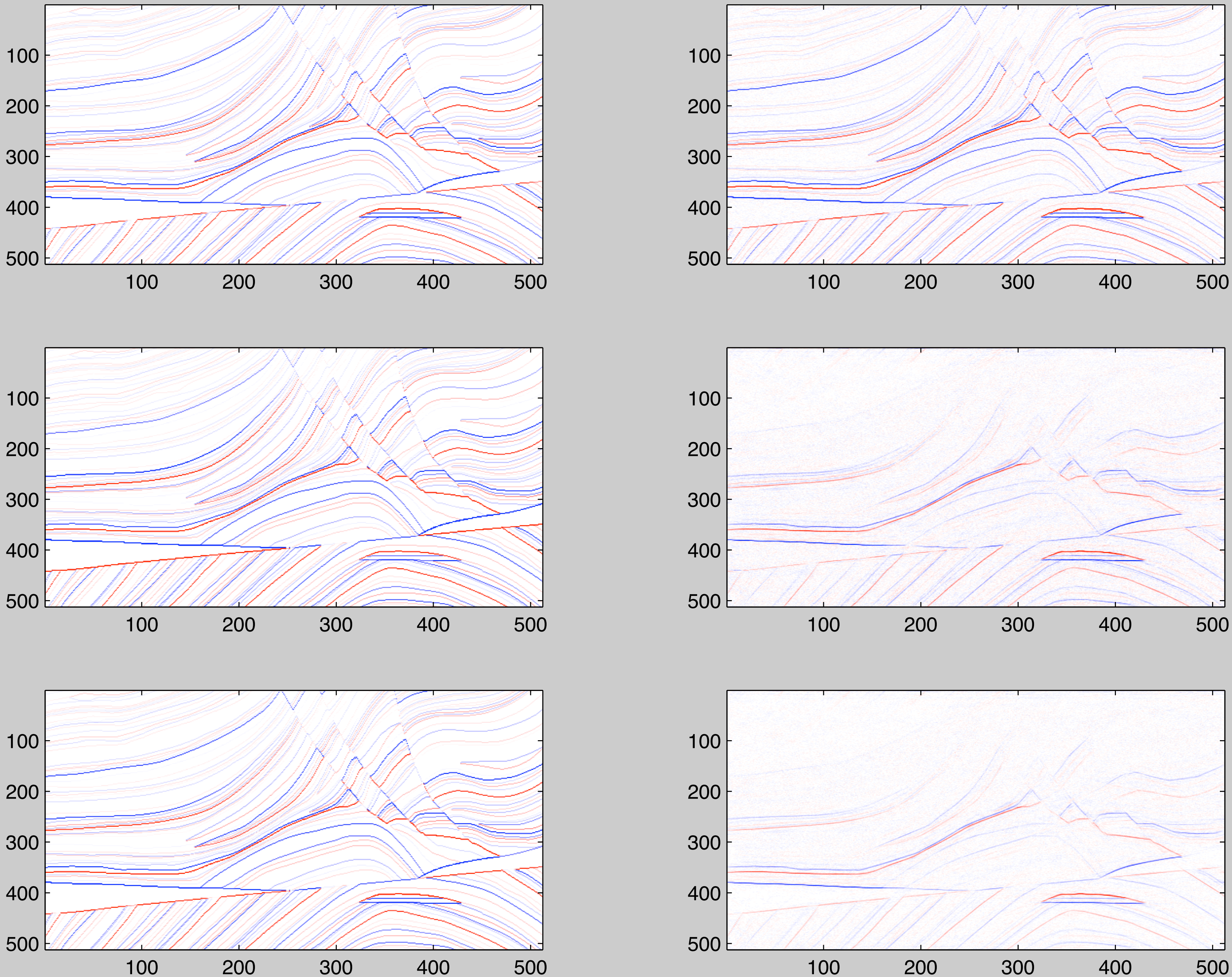
$$d = Fx + n$$

where $F = KC^*$ & $m = C^*x$

$$\boxed{\min_x \|d - Fx\|_2^2 + \|x\|_{1,w}}$$

$$x^n = S_w(x^{n-1} + F^*(d - Fx^{n-1}))$$ *

* Daubechies (2004) - Thresholded Landweber iteration


100
200
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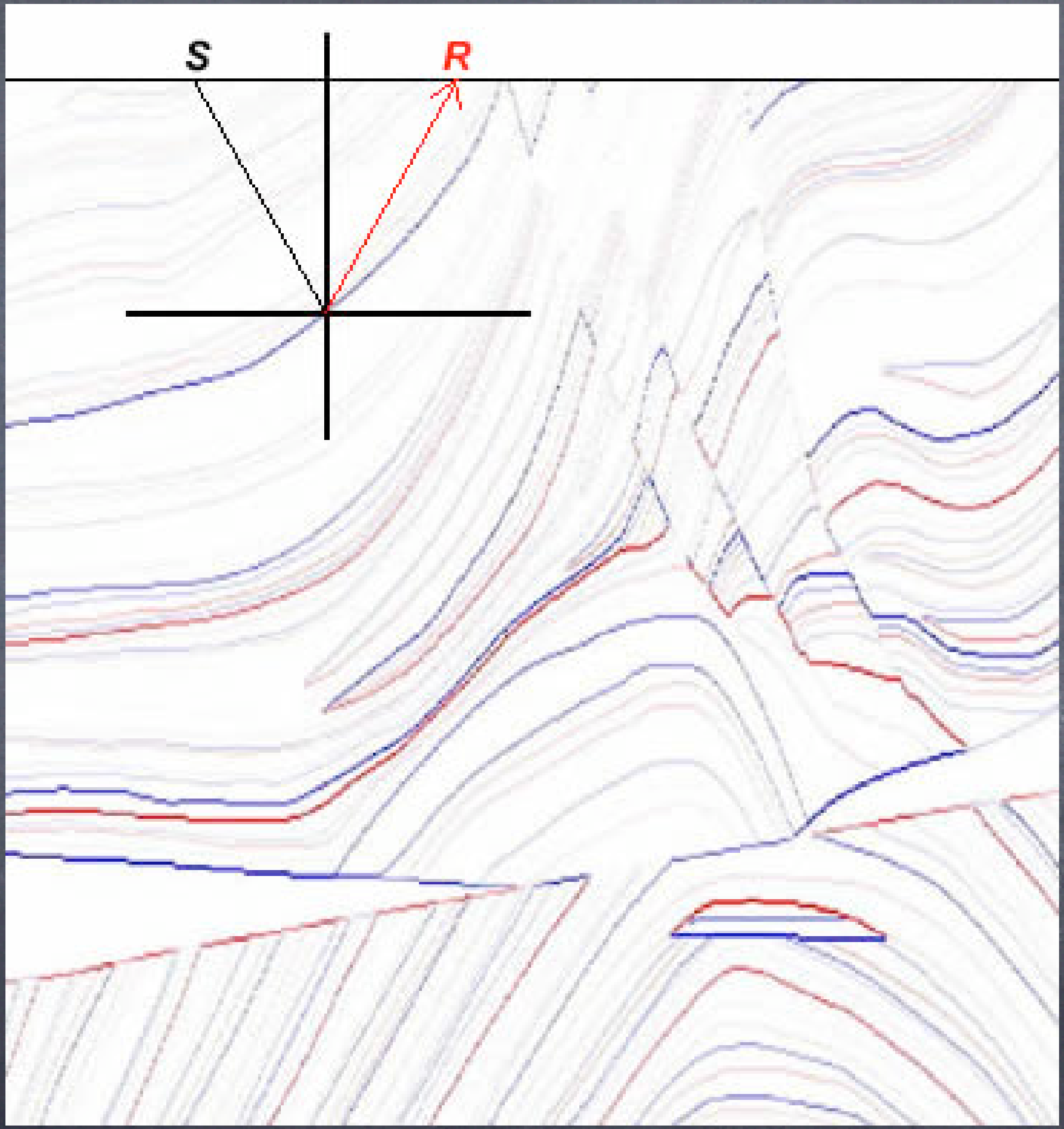
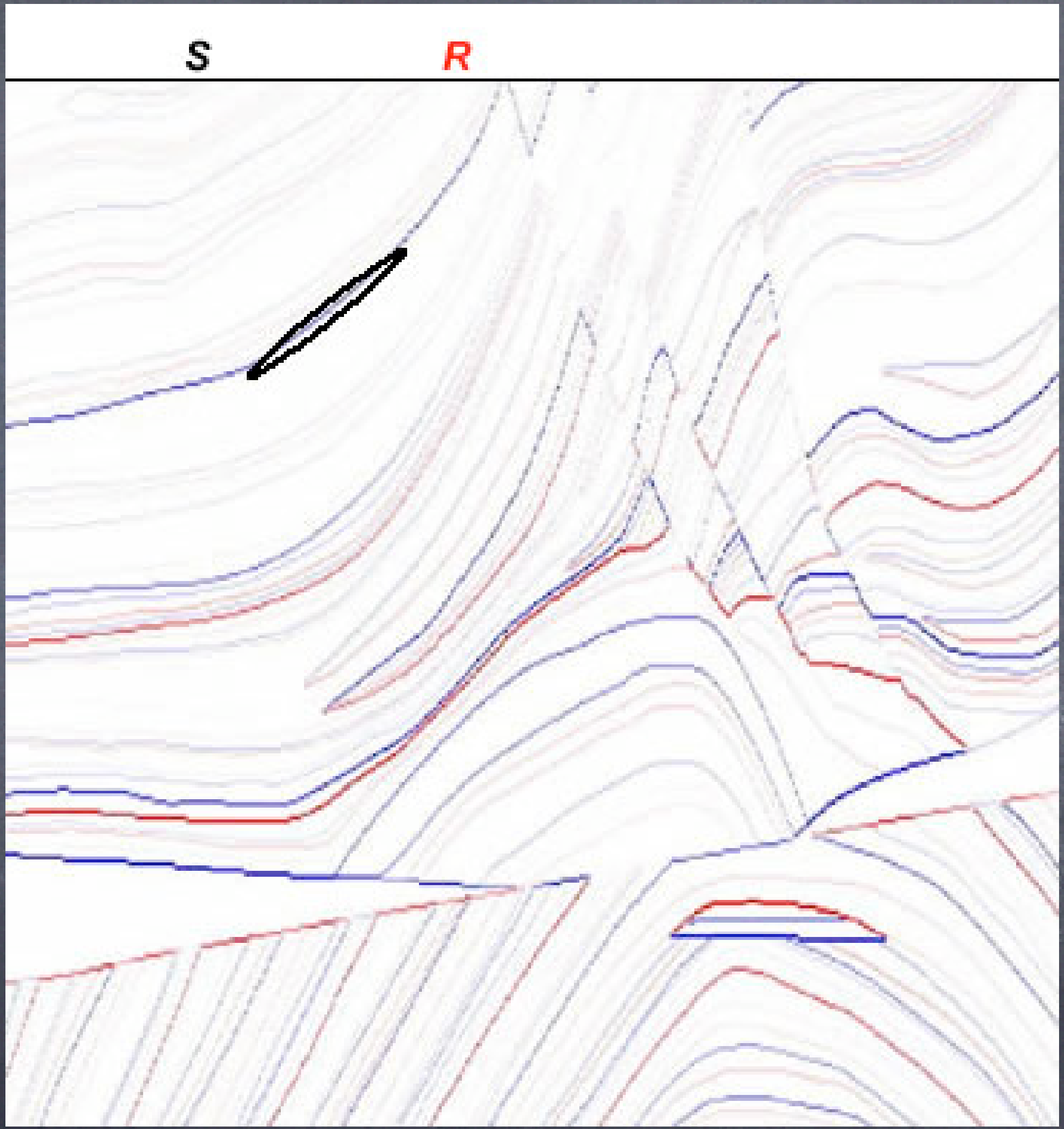
100 200 300 400 500

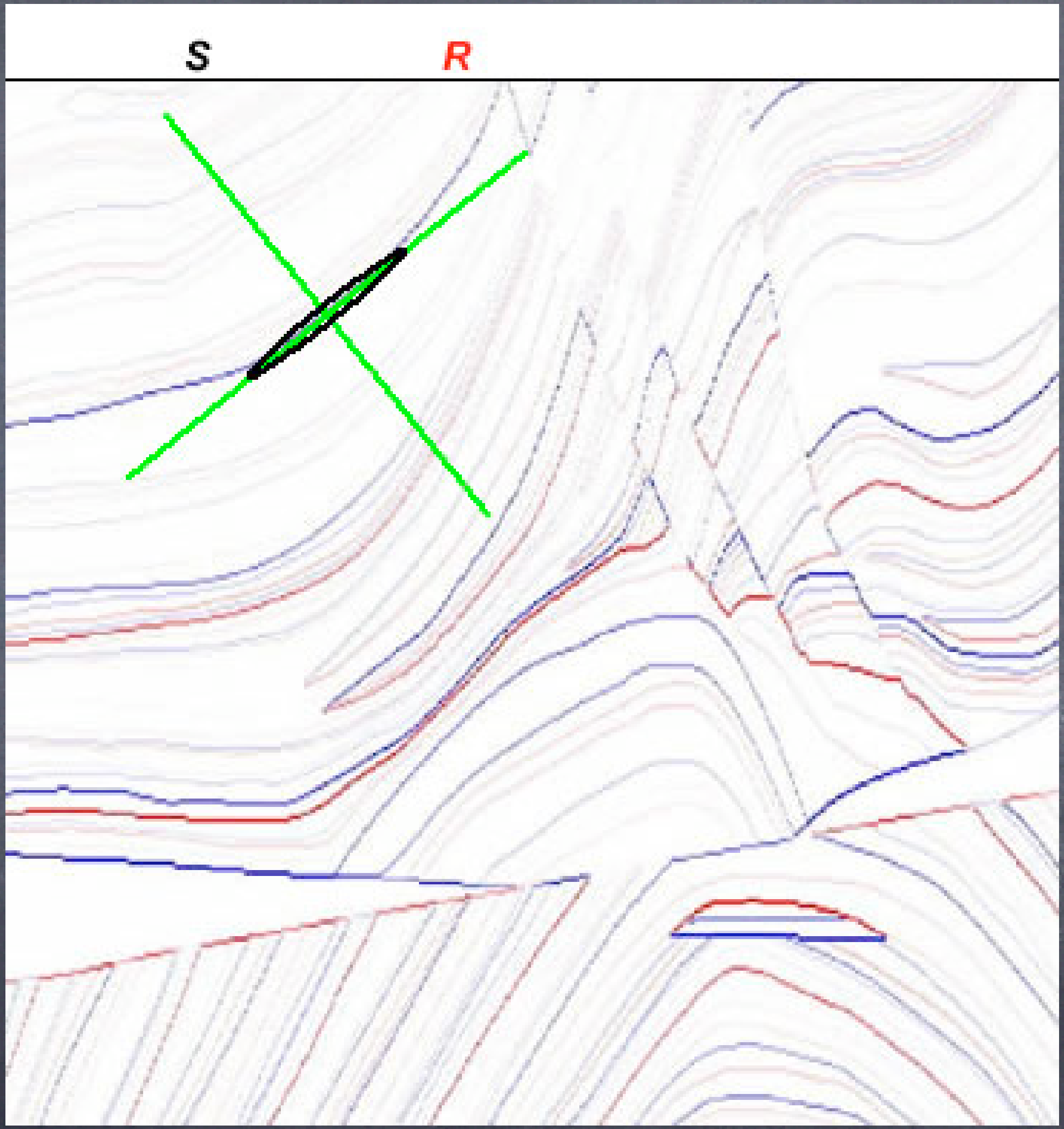
Angle Corrected Reflection Operator

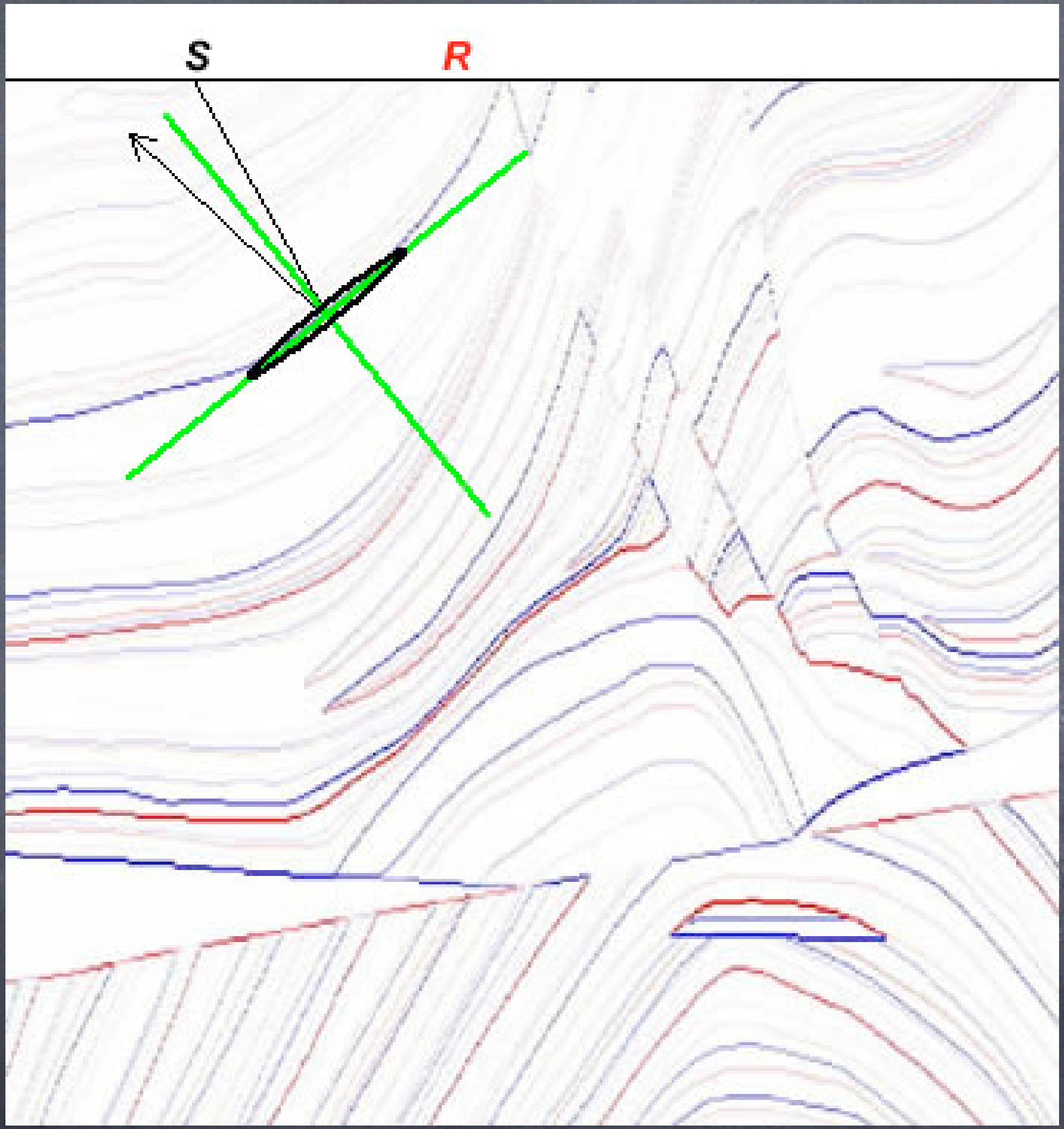
- Use curvelets to find LOCAL orientation
  - Alternative way to find geological dip
- Correct for the angle when computing the Rpp
  - LOCAL rotation of coordinate system

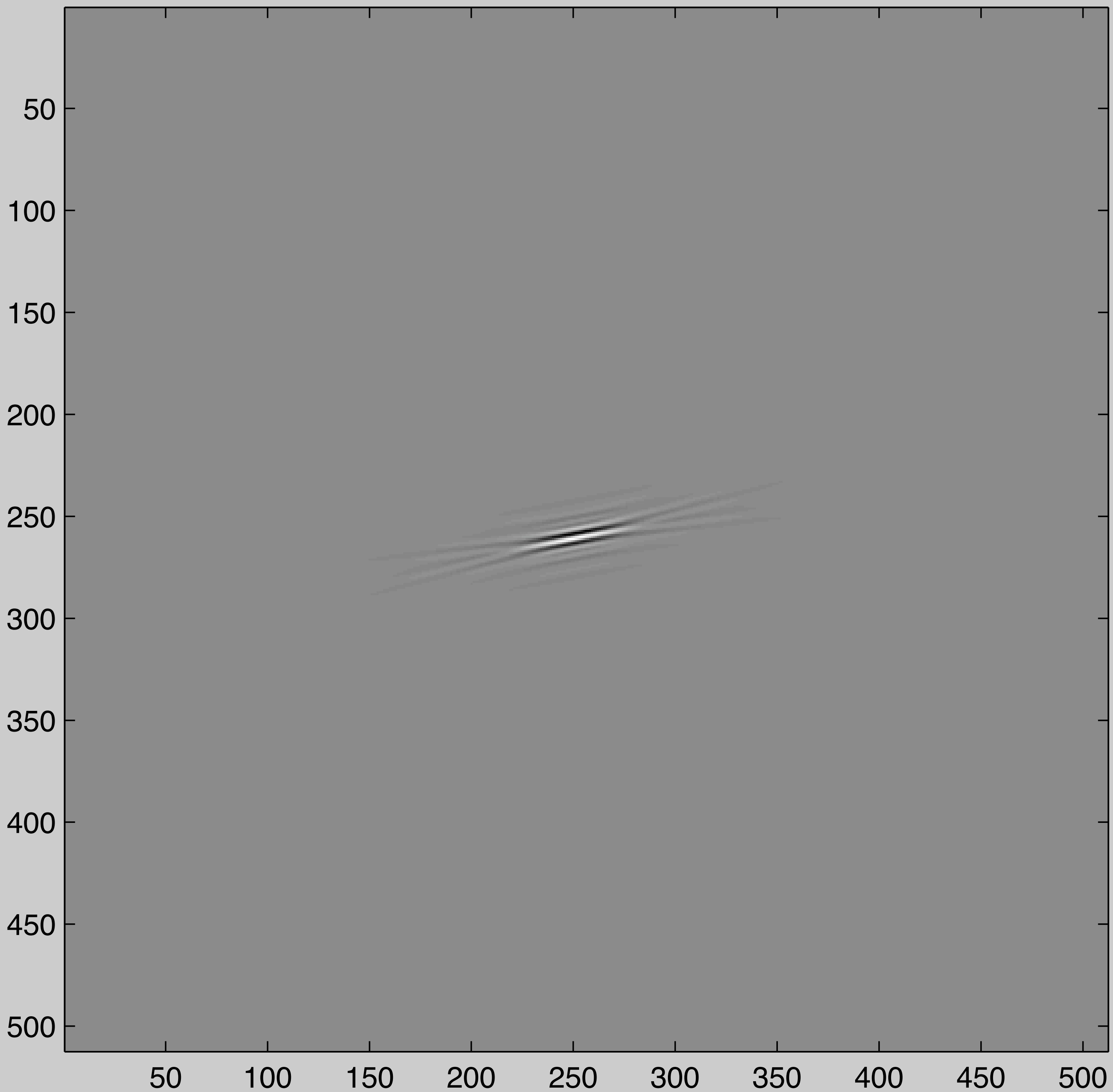

S
R



S
R


S
R


S
R


50
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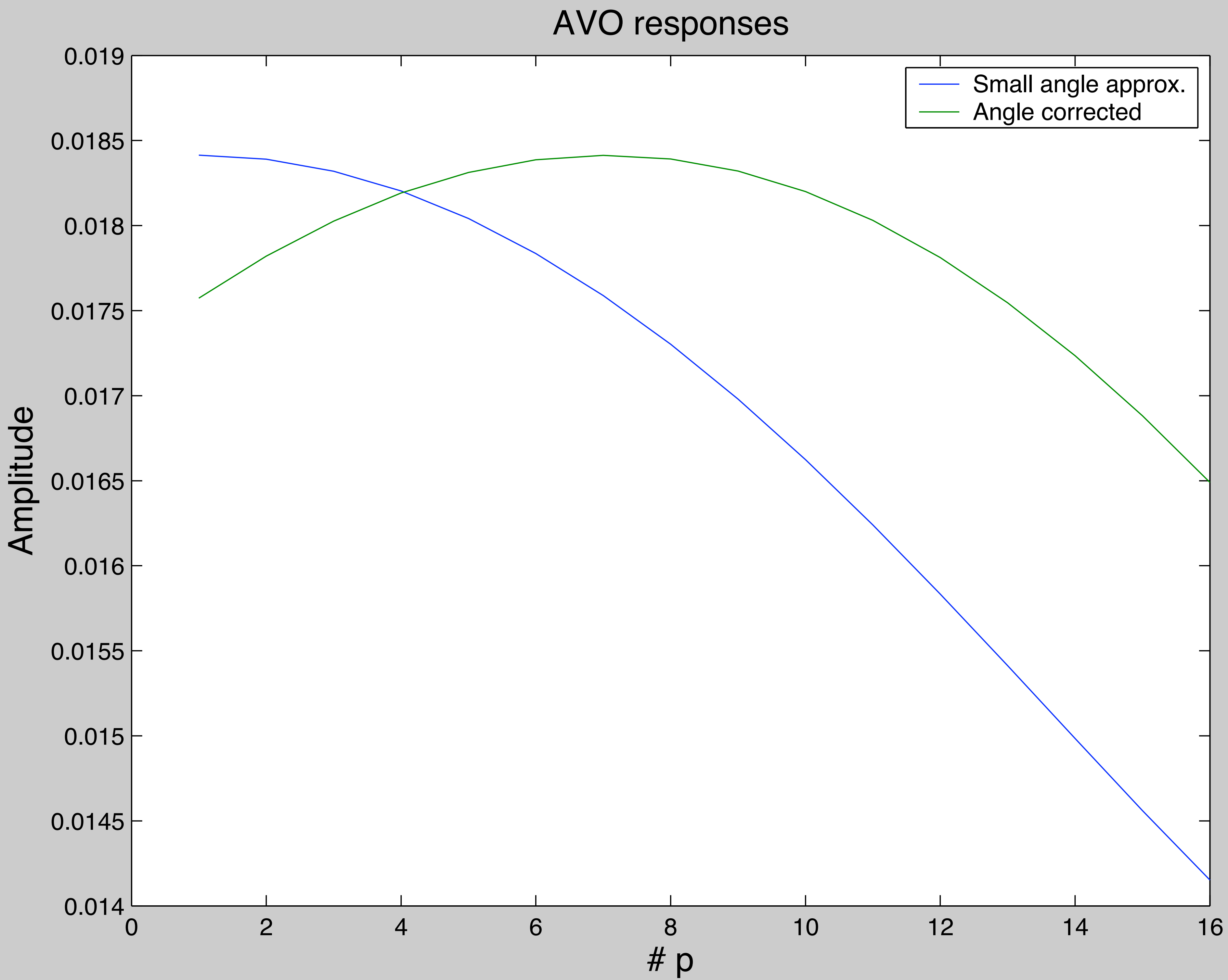
400
450
500


AVO responses
Small angle approx.
Angle corrected
Amplitude
p
0.019
0.0185
0.018
0.0175
0.017
0.0165
0.016
0.0155
0.015
0.0145
0.014
0
2
4
6
8
10
12
14
16

Acknowledgments

- NSERC
- ChevronTexaco (ChaRM sponsor)
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Earth and Ocean Sciences at UBC

Gilles Hennenfent*

ghennenfent@eos.ubc.ca

http://wigner.eos.ubc.ca/~hegilles/

Felix Herrmann

fherrmann@eos.ubc.ca

http://www.eos.ubc.ca/public/people/faculty/F.Herrmann.html