A compressive sensing perspective on simultaneous Marine acquisition

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Collaboration

Joint work with:

- Haneet Wason
- Tim Y. Lin
- Felix J. Herrmann

Part 1: Simultaneous Marine acquisition

Part 2: Compressed sensing (CS) overview

Part 3: Simultaneous acquisition operators as CS matrices

Part 4: Experimental results

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Data acquisition is the bottleneck

- The size of acquired data in seismic imaging poses a fundamental shortcoming for imaging and inversion.
- Conventional Marine acquisition is carried out as a sequence of single-source experiments of the subsurface response.
- Suppose we deploy N_s sources and N_r receivers, and each source experiment decays after N_t time samples,
- Then the size of the conventional Marine survey is $N = N_s N_r N_t$.

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Toy example:

For a survey made up of 128 sources, 128 receivers with 512 time samples per source experiment, we have to collect $N = 8,388,608$ samples.

Simultaneous source acquisition

- Simultaneous source acquisition has emerged as a technique to reduce the amount of data acquired (Beasley; Berkhout; Hampson et al.; de Kok and Gillespie).
- During simultaneous acquisition, sources are fired before the response of the previous source fully decays while the receivers record continuously.
- The result is a single long "super-shot" in which responses of shots are mixed together.

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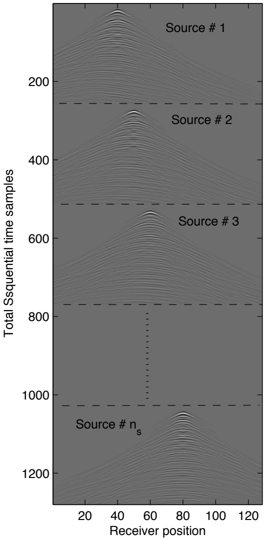

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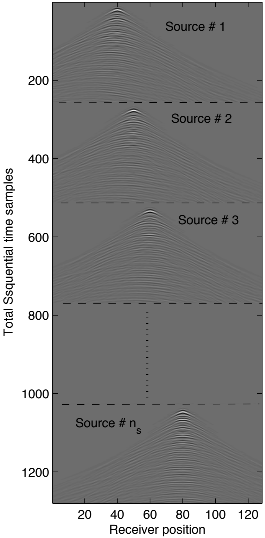
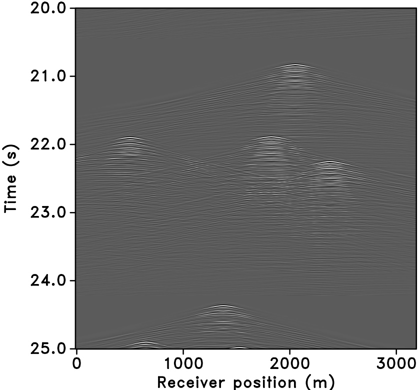




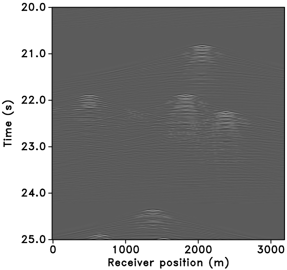
Simultaneous source acquisition

- In a Marine setting, source signatures can only consist of impulsive air-gun bursts at fixed energy.
- This results in cross-talk between the responses of the different sources which makes it difficult to estimate interference-free shot gathers.

Acquisition design challenges

- We want to be able to remove source interference.
- We want the scheme to be physically realizable.
- Choose the recovery setup that maximizes the recovered quality from the same number of samples.

Sneak Peek



SNR = 11.1dB

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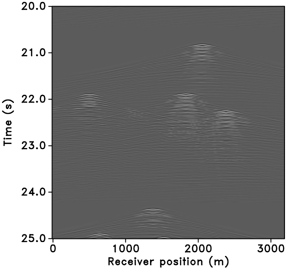

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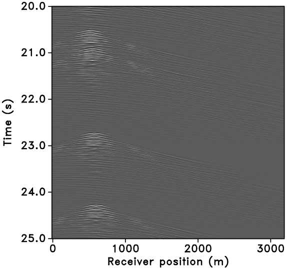

SNR = 10.5dB

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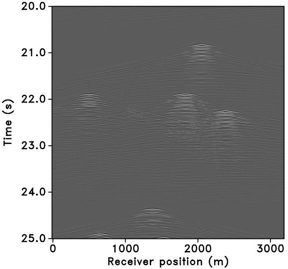

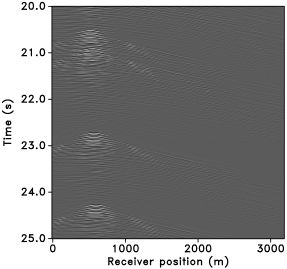

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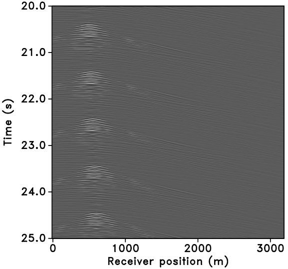

SNR = 4.39dB

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Part 2: Compressed sensing (CS) overview

Part 3: Simultaneous acquisition operators as CS matrices

Part 4: Experimental results

Compressed Sensing

- Compressed Sensing is an acquisition paradigm for signals that admit *sparse* or nearly sparse representations in some transform domain.
- Consider a signal $f \in \mathbb{R}^N$, $f = S^H x$, where S is a transform matrix and x is the coefficient vector.
- We can approximate f by the signal $\bar{f}$ using the k-largest coefficients of x.
- Given $n \ll N$ linear and noisy measurements $b = RMf + e$.
- Let $A = RMS^H$, it is possible to approximate x from the measurements b if
  - A obeys certain conditions: RIP, mutual coherence.
  - x is sufficiently sparse: depends on the recovery algorithm.

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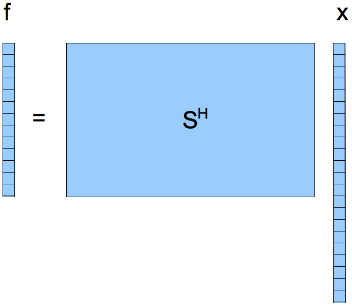

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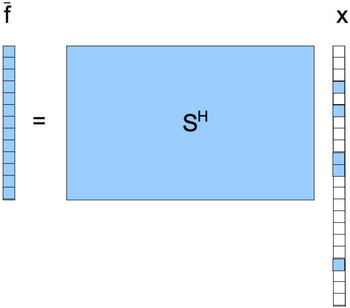

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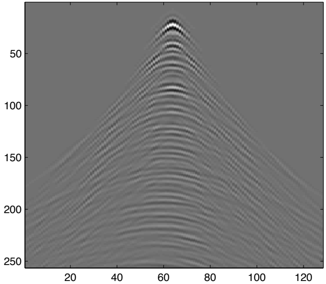

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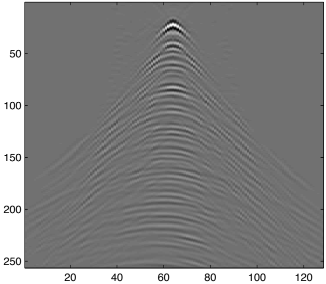

2% curvelet coefficients, SNR = 15.17dB

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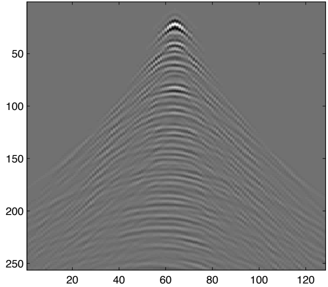

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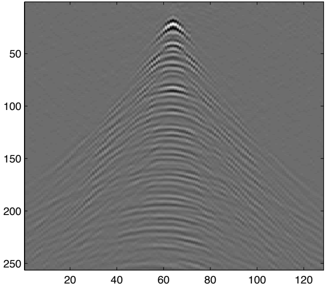

14.8% Fourier coefficients, SNR = 13.91dB

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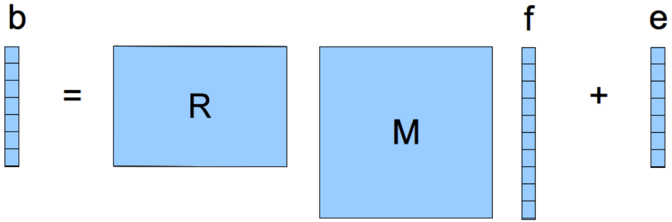

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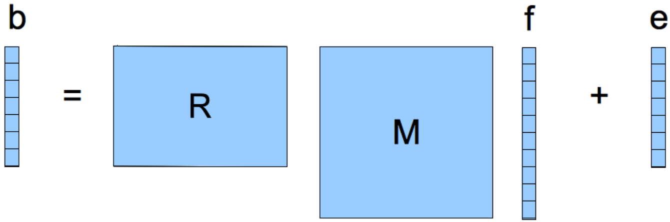

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Measurement matrix conditions

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- The mutual coherence measures the orthogonality of all columns of A.

Definition: Mutual Coherence (Donoho and Elad; Bruckstein et al.)

The mutual coherence is equal to the largest inner product between between the normalized columns of A

$$\mu(A) = \max_{1 \leq i \neq j \leq P} \frac{|a_i^H a_j|}{(\|a_i\|_2 \cdot \|a_j\|_2)}.$$

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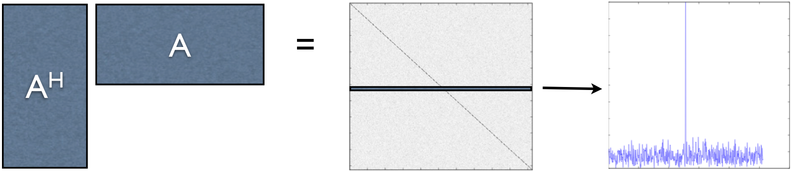

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- There are precise conditions on A that guarantee that the theorem holds.
- The restricted isometry property (RIP) of order k indicates whether every group of k columns of A are nearly orthogonal.

Definition: Restricted Isometry Property (RIP) (Candés and Tao)

The RIP constant $\delta_k \in (0,1)$ is defined as the smallest constant such that $\forall x \in \Sigma_k^N$

$$(1-\delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\delta_k)\|x\|_2^2,$$

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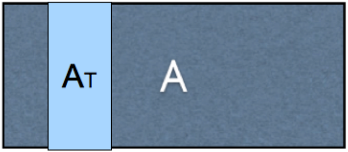

- RIP is equivalent to saying that for any set T of size k, the symmetric matrix $A_T^H A_T$ is positive definite with eigenvalues in $[1-\delta_k, 1+\delta_k]$.

Sparsity conditions

- The recoverable sparsity k of x depends on the recovery algorithm.

- Optimization Algorithms
  - $\min_{x \in \mathbb{R}^N} \|x\|_0 \quad \text{subject to } \|Ax - b\|_2 \leq \|e\|_2, \qquad k < n/2$
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Compressed sensing and simultaneous Marine acquisition

- Simultaneous acquisition is exactly a compressed sensing problem.
- The objective is to recover the high-dimensional sequential shot record f from the lower-dimensional "supershot" record $b = RMf$.
- Formulate the acquisition process in terms of the sampling operator RM.
- We want to construct the sampling operator RM such that:
  1. $A = RMS^H$ satisfies the CS recovery conditions.
  2. RM is physically realizable.
- Recover the sequential shot record by finding $\tilde{f} = S\tilde{x}$, where

$$\tilde{x} = \arg\min_x \|x\|_1 \quad \text{s.t.} \quad Ax = b.$$

- *Conventional recovery from simultaneous sources is performed by applying the adjoint A^H to b followed by median filtering in the midpoint-offset domain.

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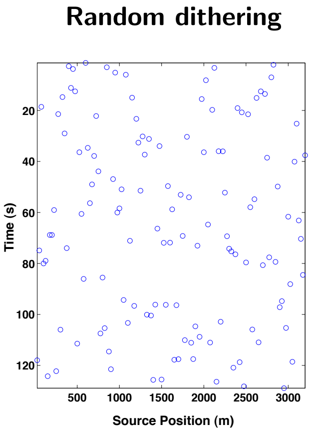
Random dithering

- Typically, matrices with i.i.d Gaussian random entries satisfy the CS recovery conditions.
- In the Marine setting, a binary $0 - 1$ matrix with i.i.d Bernoulli entries is the closest to Gaussian that we can physically realize.



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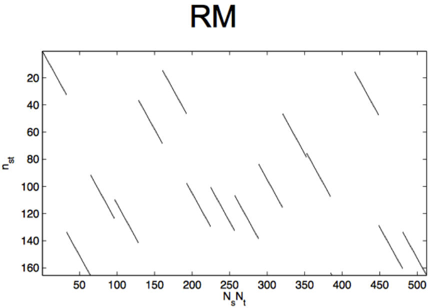

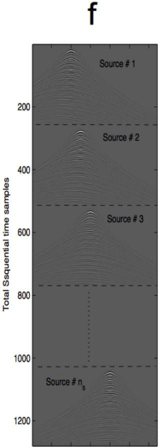

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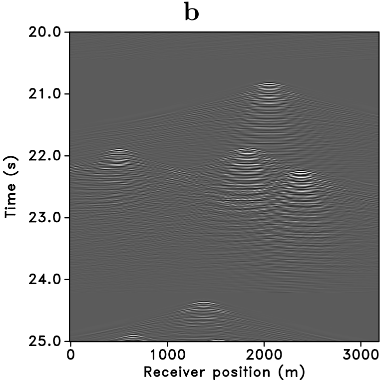

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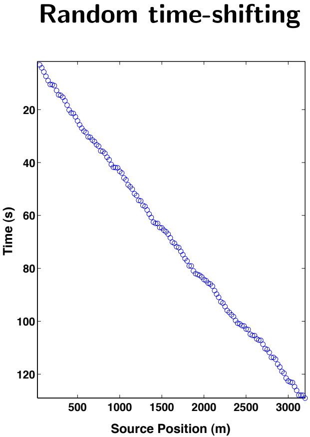
- Although random-dithering can be achieved physically, it requires an airgun located at each source location, which can be costly if not practically infeasible.

Random time-shifting

- We envisage an acquisition system involving several vessels with airguns swarming over the ocean-bottom array.
- To avoid collisions amongst these vessels, we sort the random source positions with respect to survey time.



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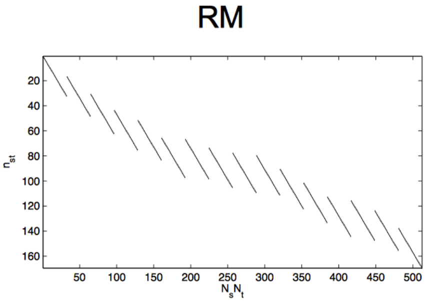

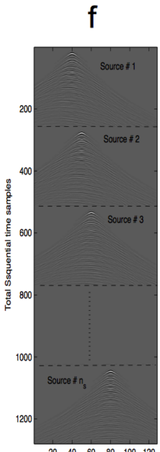

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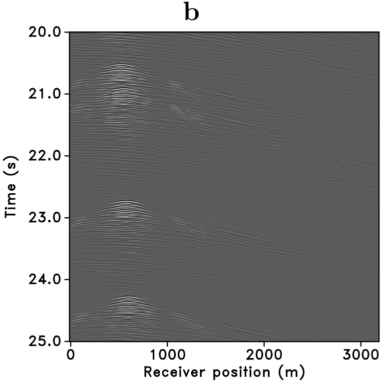

Constant time-shifting

- Random time-shifting retains the randomness necessary for CS recovery, albeit at a lower order than random dithering.
- To emphasize the importance of randomization, we include the case where we simply decrease the intershot time delays.

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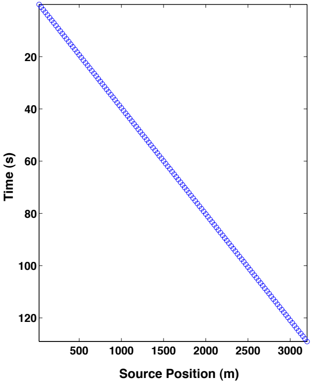

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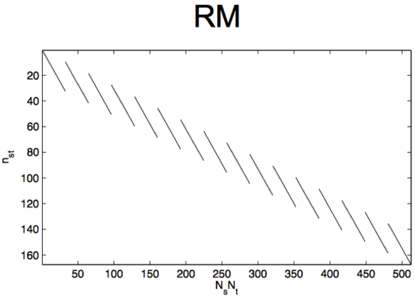

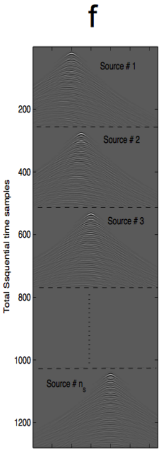

Constant time-shifting

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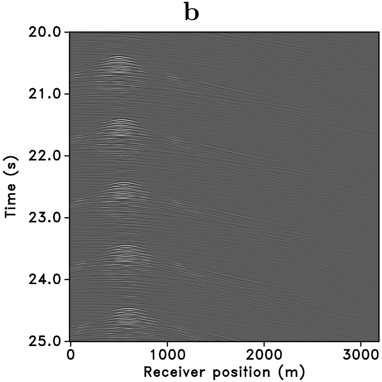
- To emphasize the importance of randomization, we include the case where we simply decrease the intershot time delays.



Part 1: Simultaneous Marine acquisition

Part 2: Compressed sensing (CS) overview

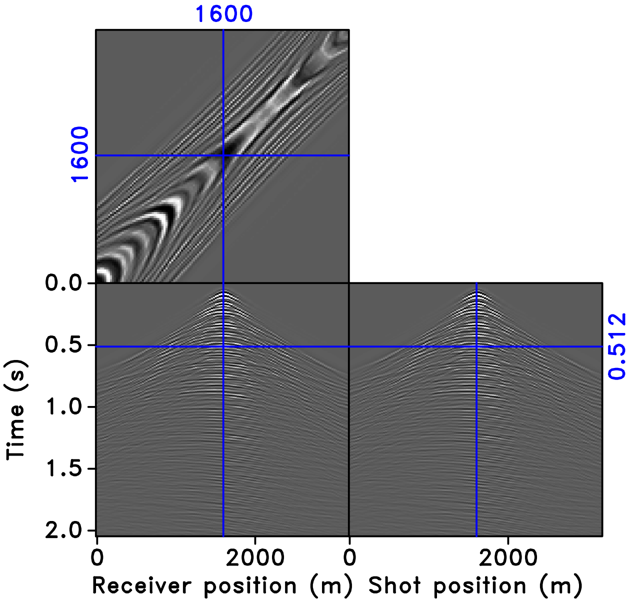
Part 3: Simultaneous acquisition operators as CS matrices

Part 4: Experimental results

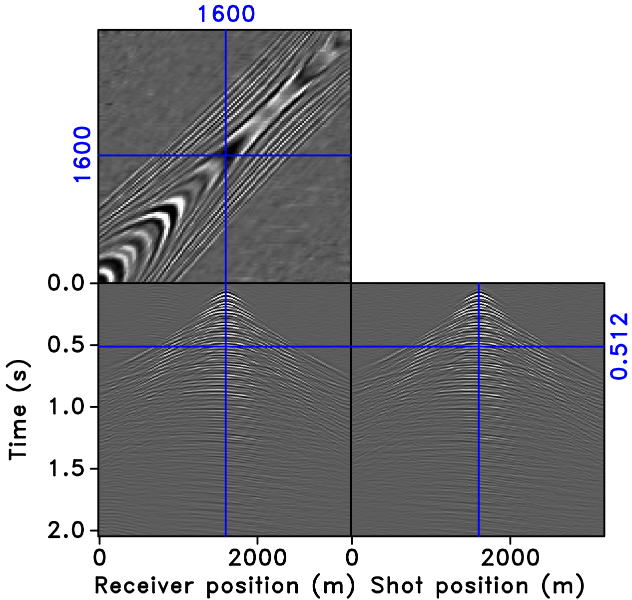
Experimental setup

- We apply the three sampling schemes; random dithering, random time-shifting, and constant time-shifting, on a seismic line from the Gulf of Suez.
- The fully sampled sequential data has $N_s = 128$ sources, $N_r = 128$ receivers, and $N_t = 512$ time samples.
- The subsampling ratio achieved through simultaneous acquisition is $\gamma = 0.5$.
- We recovery prestack data from sequential sources using ℓ_1 minimization with 3D curvelets and compare the recovery with 3D Fourier as the sparsifying transforms.

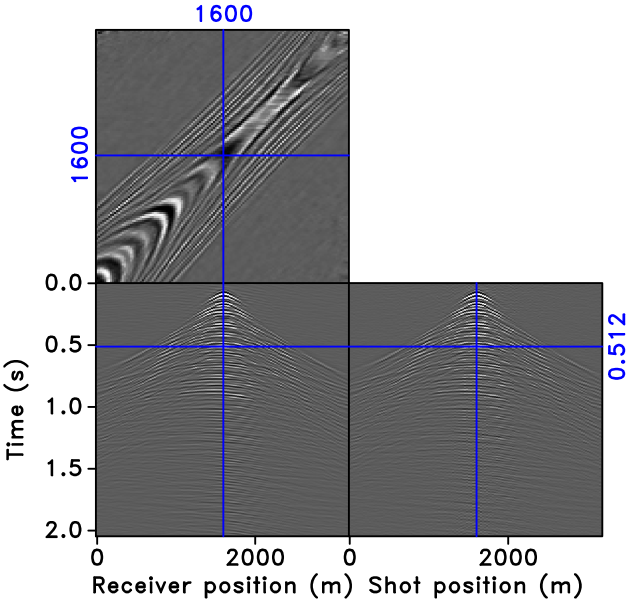
Original data



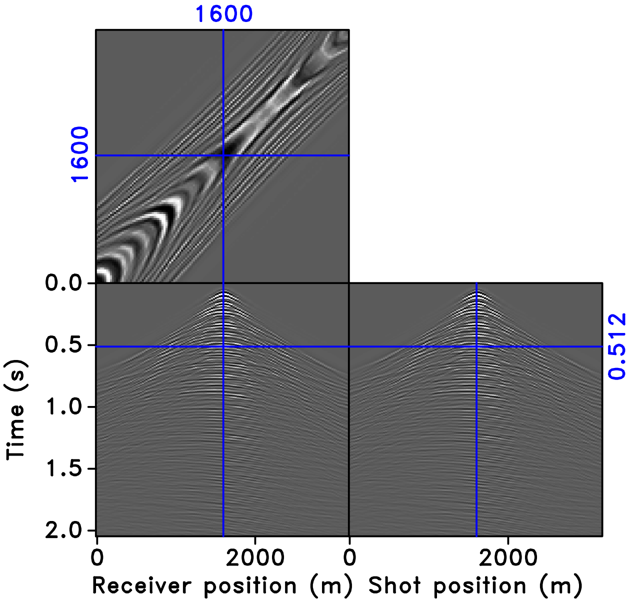
Random dithering - SNR = 11.1dB



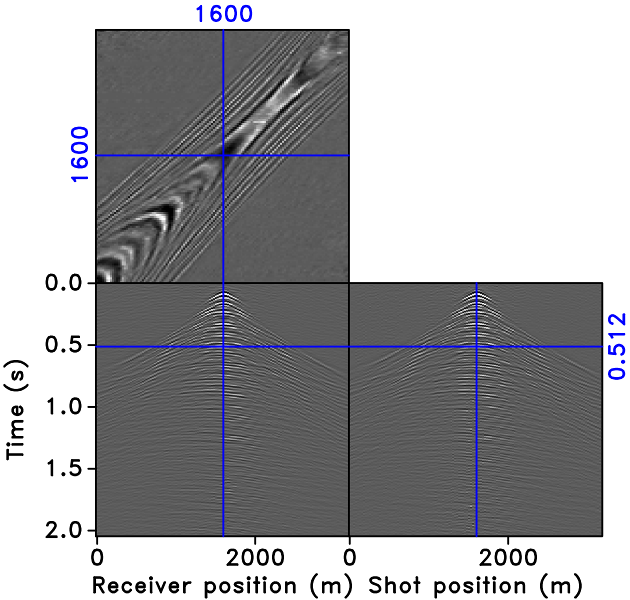
Random time-shifting (curvelet) - SNR = 10.5dB



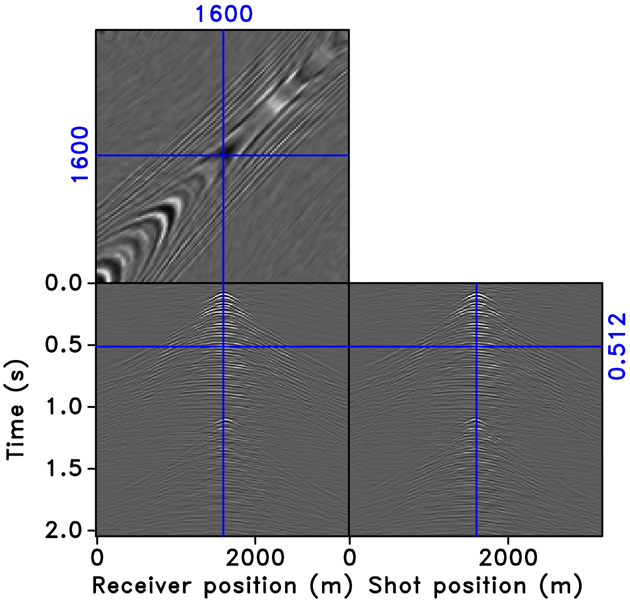
Original data



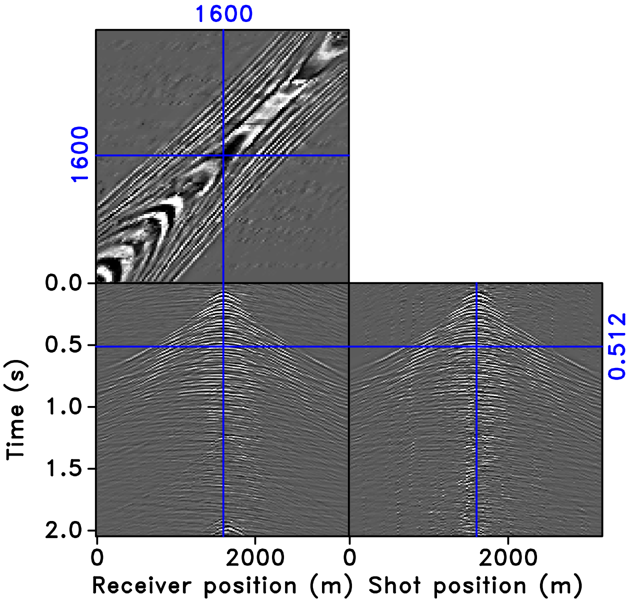
Random time-shifting (Fourier) - SNR $= 8.15$dB



Constant time-shifting - SNR = 4.39dB

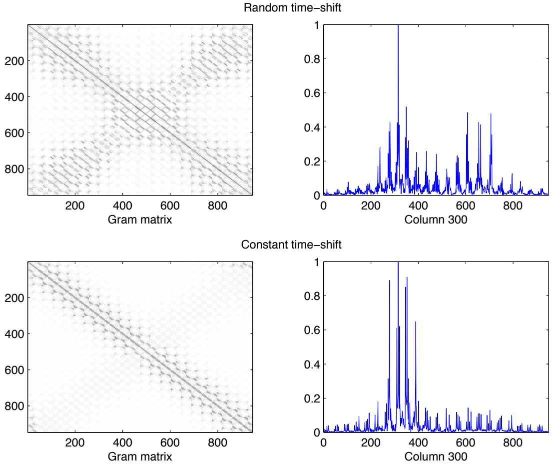


Adjoint operator with median filtering - SNR $= 5.04$dB



Why does the random time-shift operator work?

Mutual coherence of $A = RMS^H$ (curvelet), $k < \frac{1}{2}(1 + \frac{1}{\mu(A)})$



Why does the random time-shift operator work?

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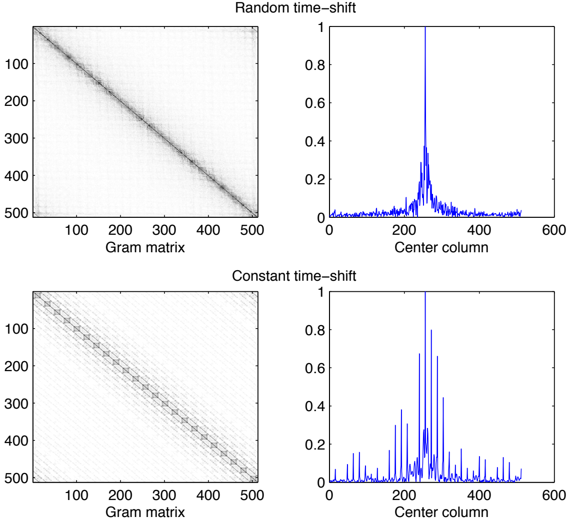
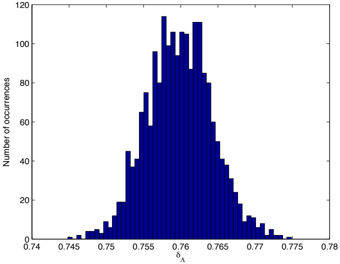


Why does the random time-shift operator work?

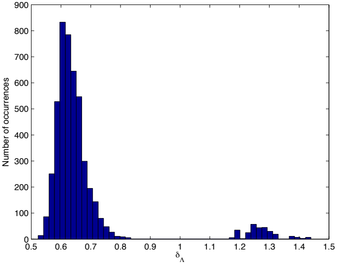
Monte Carlo estimation of the RIP constant.

$$(1-\delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\delta_k)\|x\|_2^2, \quad \delta_k \in (0,1)$$

Curvelet



Fourier



Conclusion

- We identified simultaneous marine acquisition as a linear subsampling system and analyze it using metrics from Compressed Sensing.
- We quantitatively verified that more randomness in acquisition and more compressible transforms improve the mutual coherence and restricted isometry constants, which predict a higher reconstruction quality.
- We demonstrate that with a 50% reduction in acquisition cost, we are able to recover the sequential shot records with high fidelity.
- This is a first step towards predicting the interaction between simultaneous source acquisition design and reconstruction fidelity.

Obrigado

Questions?

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