Compressive wavefield simulations

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Joint work with Yogi Erlangga, and Tim Lin

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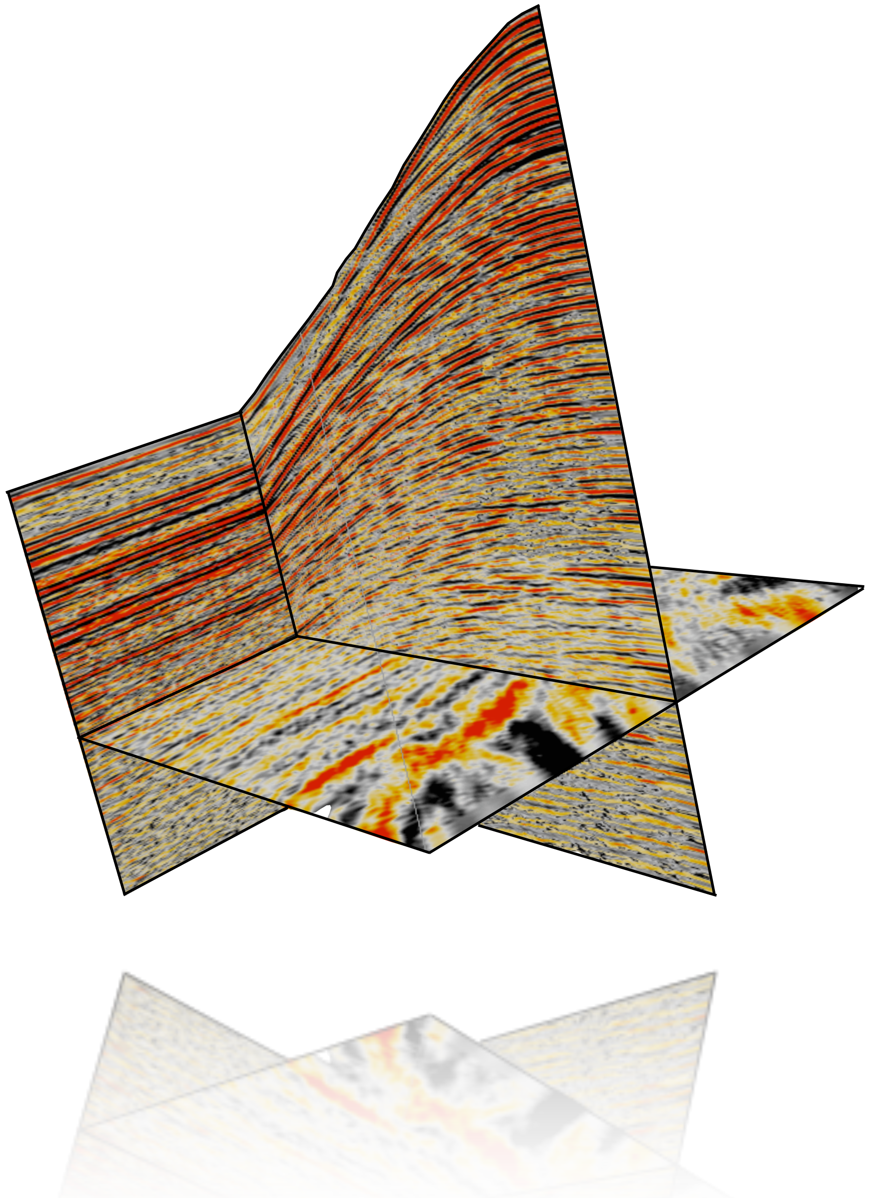
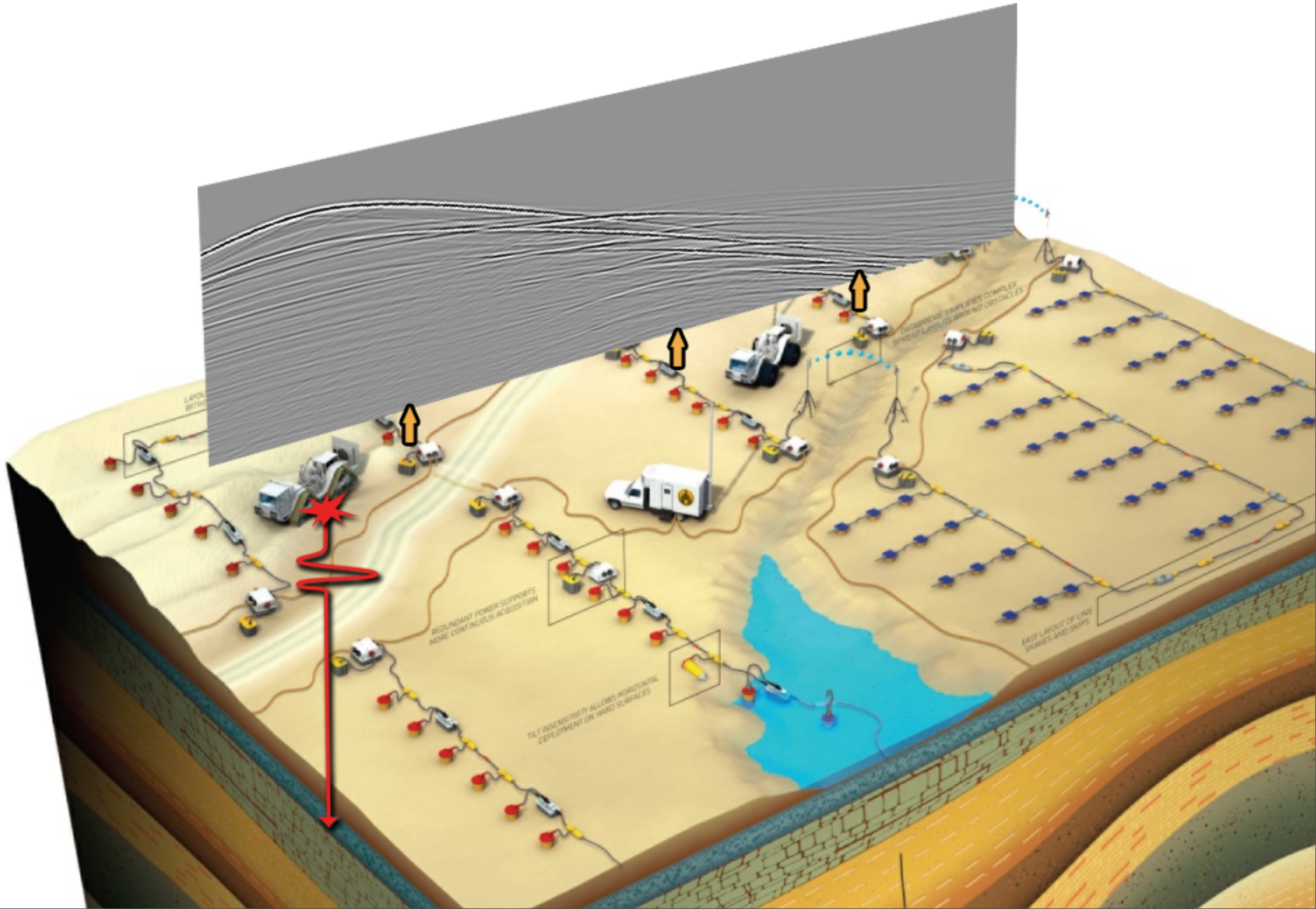


Marseille, May 19, 2009

Seismic acquisition

Individual shots



Individual shots

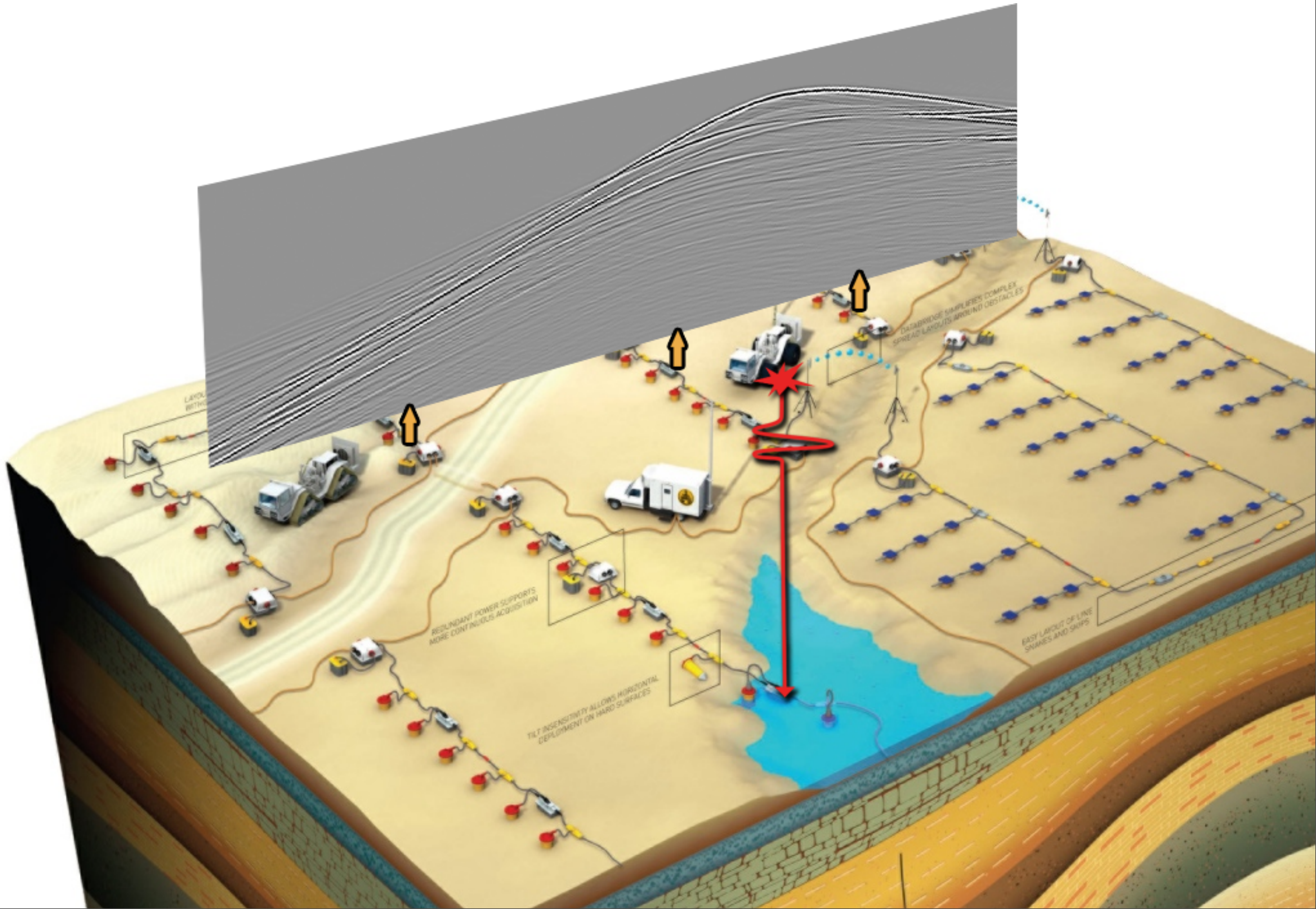
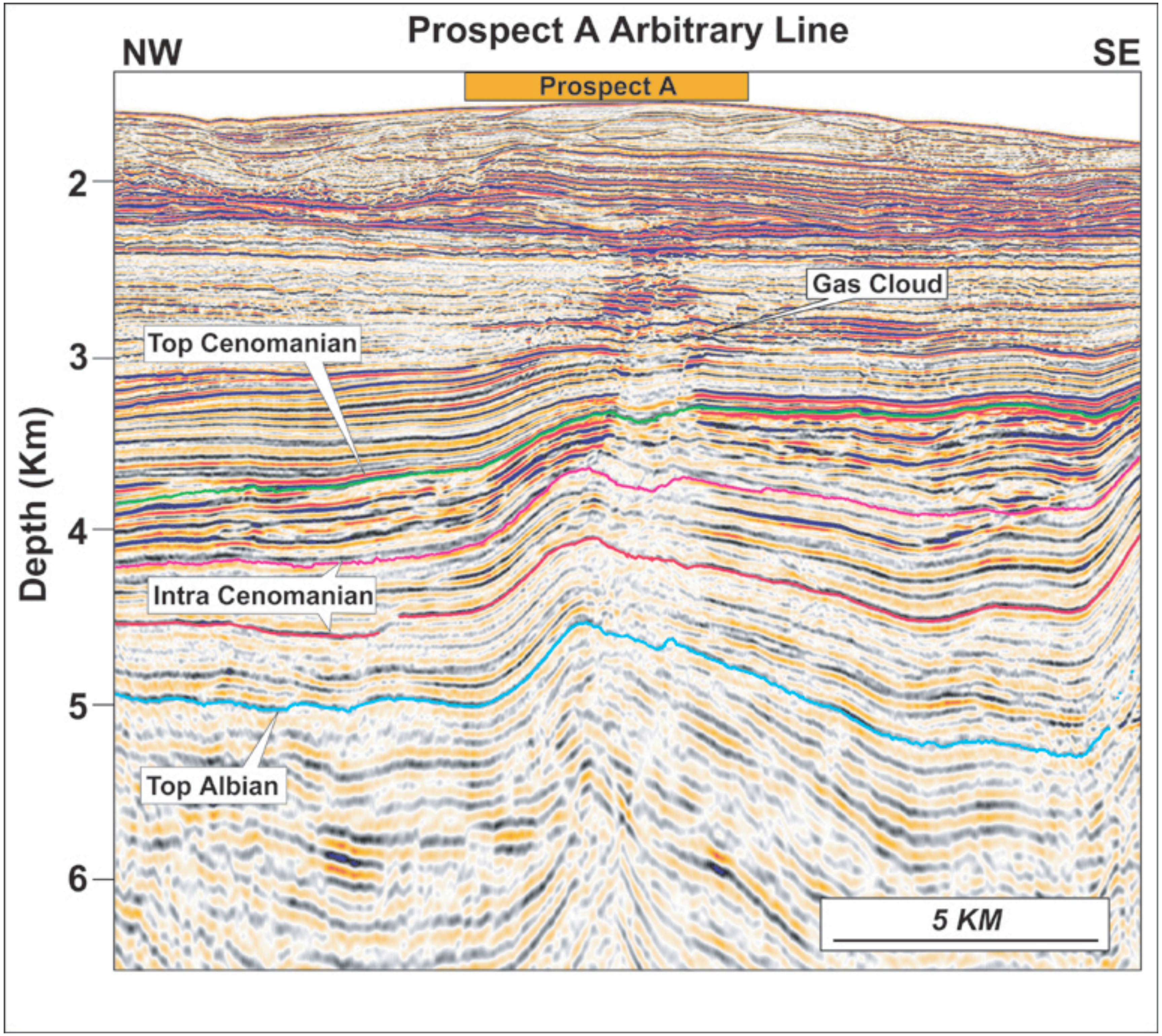


image courtesy vancoenergy (www.vancoenergy.com)

After imaging



Essentials of seismic inversion

Simulation:

sources

residue (data)

$$\mathbf{H}[\mathbf{m}]\mathbf{U} = \mathbf{Q} \quad \text{and} \quad \mathbf{H}^*[\mathbf{m}]\mathbf{V} = \mathbf{R}$$

discretized PDE (Helmholtz)

variables (Earth)

solution (seismic wavefield)

adjoint solution (seismic wavefield)

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- Oscillatory high-dimensional solutions that are ***extremely*** expensive to compute
- Inversion (e.g. via Gauss-Newton) requires multiple solves
- Number of blocks in **H** and number of **rhs** determine simulation & acquisition costs

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discretized PDE (Helmholtz) → $\mathbf{H}$; **variables (Earth)** → $\mathbf{m}$; **solution (seismic wavefield)** → $\mathbf{U}$; **adjoint solution (seismic wavefield)** → $\mathbf{V}$

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Imaging:

image volume ↓ · **multi-D 'cross-correlation'** ↓

$$\boldsymbol{\delta}\mathbf{I}(x_s, x_r, t) = (\mathbf{U} \circ \mathbf{V})$$

$$\delta\mathbf{m}(x_s = x_r, t = 0) = \mathrm{diag}\{\boldsymbol{\delta}\mathbf{I}\}$$

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- Explicit matrix evaluations part of the KKT system
- Only interested in diagonal (focused energy)
- Penalize off diagonals (impose focusing) as part of an extended PDE constrained optimization problem

Impediments & solution strategy

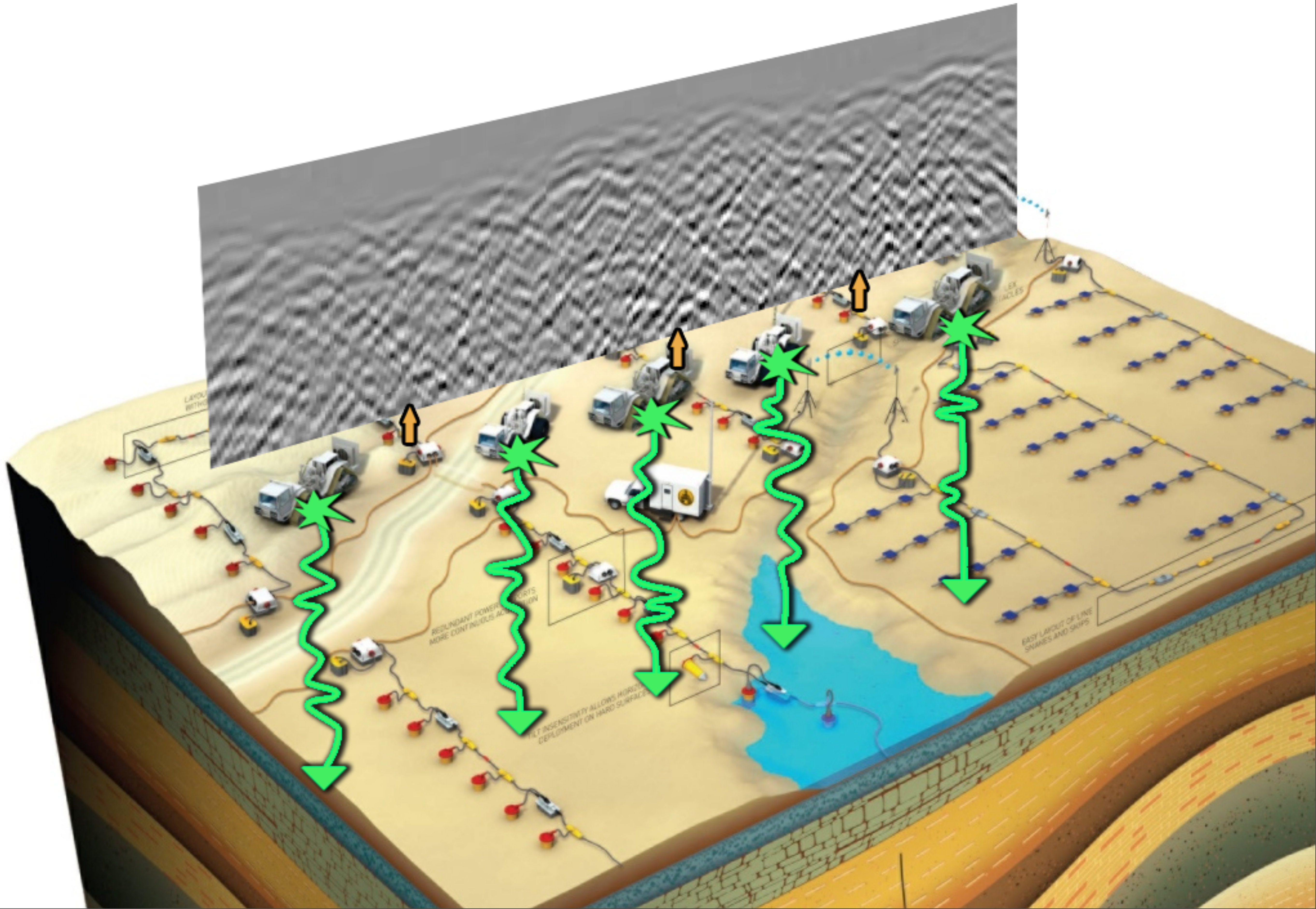
- Acquisition, modeling & inversion **costs** are proportional to the **size** of *data* and *model*
  - curse of dimensionality (d=5 for data)
  - redundancy in data
  - high geometrical structure (wavefronts)
- Computation of image volumes through focusing requires explicit storage of wavefields
  - redundancy (d=7 for image volume, d=3 for model)
  - object of interest is low dimensional and sparse (compressible) in ***phase*** space
  - high geometrical structure (sheet-like singularities)
- **Bottom line: We are drowning in data that may exceed Petabytes**
- **Solution:** use *embedding/compressive* sampling techniques
  - to reduce the size for the Helmholtz system part of forward modeling
  - to reduce the size of the explicit matrix evaluations part of imaging
  - exploit geometrical structure & focusing = joint sparsity promotion with ***curvelets***

Wavefield computations

$$\overbrace{\begin{bmatrix} \mathcal{H}_{\omega_1} & 0 & & \\ 0 & \mathcal{H}_{\omega_2} & \ddots & \\ & \ddots & \ddots & 0 \\ & & 0 & \mathcal{H}_{\omega_{n_f}} \end{bmatrix}}^{\mathbf{H}} \overbrace{\begin{bmatrix} \mathbf{U}_{\omega_1} \\ \mathbf{U}_{\omega_1} \\ \vdots \\ \mathbf{U}_{n_f} \end{bmatrix}}^{\mathbf{U}} = \overbrace{\begin{bmatrix} \mathbf{B}_{\omega_1} \\ \mathbf{B}_{\omega_2} \\ \vdots \\ \mathbf{B}_{n_f} \end{bmatrix}}^{\mathbf{B}} \quad \Rightarrow \quad \mathbf{HU} = \mathbf{B}$$

- Matrix-free preconditioned indirect solver based on multilevel Krylov with deflation [Erlanga, Nabben, '08, Erlanga and F.J.H, '08]
- Solution gives multidimensional wavefield $\mathbf{u}(x_s, x_r, t)$
- Block-diagonal structure **H** and multiple rhs are amenable to CS as long as CS sampling matrix ***commutes*** with **H**
- Corresponds to simultaneous acquisition
  - replaces *impulsive* individual sources by *simultaneous* randomized sources
  - reduces number *simultaneous* sources (rhs) & *angular* frequencies (blocks)

Simultaneous & continuous sources



Sparse recovery

$$\mathbf{P_1}: \begin{cases} \mathbf{y} & = \mathbf{RMd} \\ \tilde{\mathbf{x}} & = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{Ax} = \mathbf{y} \\ \mathbf{A} & = \mathbf{RMS}^* \\ \tilde{\mathbf{d}} & = \mathbf{S}^*\tilde{\mathbf{x}} \end{cases}$$

Challenges:

- large to extreme large system size (number of unknowns is 2^{25} for a really small problem)
- find proper subsampling matrix that is physically realizable and numerically fast
- find proper sparsifying transforms that balances ***sparsity*** with **mutual coherence**

Solver:

- bring in as many entries per iteration as possible
- projected gradient with root finding method ($\mathrm{SPG}\ell_1$, Friedlander & van den Berg, ‘07-’08)
- few matrix-vector multiplies
- use matrix-free implementations where possible

CS sampling matrix

Subsample along source and frequency coordinates

Use ***fast*** transform-based sampling algorithms such as ***scrambled Fourier*** [Romberg, '08] or ***Hadamard*** ensembles [Gan et. al., '08]

$$\mathbf{RM} = \overbrace{\begin{bmatrix} \mathbf{R}_1^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_1^{\Omega} \\ \vdots \\ \mathbf{R}_{n_{s'}}^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_{n_{s'}}^{\Omega} \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_2^* \operatorname{diag}\left(e^{\hat{i}\boldsymbol{\theta}}\right) \otimes \mathbf{I}\right) \mathbf{F}_3}^{\text{random phase encoder}},$$

$$\theta_w = \text{Uniform}([0, 2\pi])$$

- Different random restriction for each $n_s' \ll n_s$ simultaneous experiments
- Restriction reduces system size
- Different from implementations of sampling matrices based on Kronecker-products
- Numerical complexity CS sampling

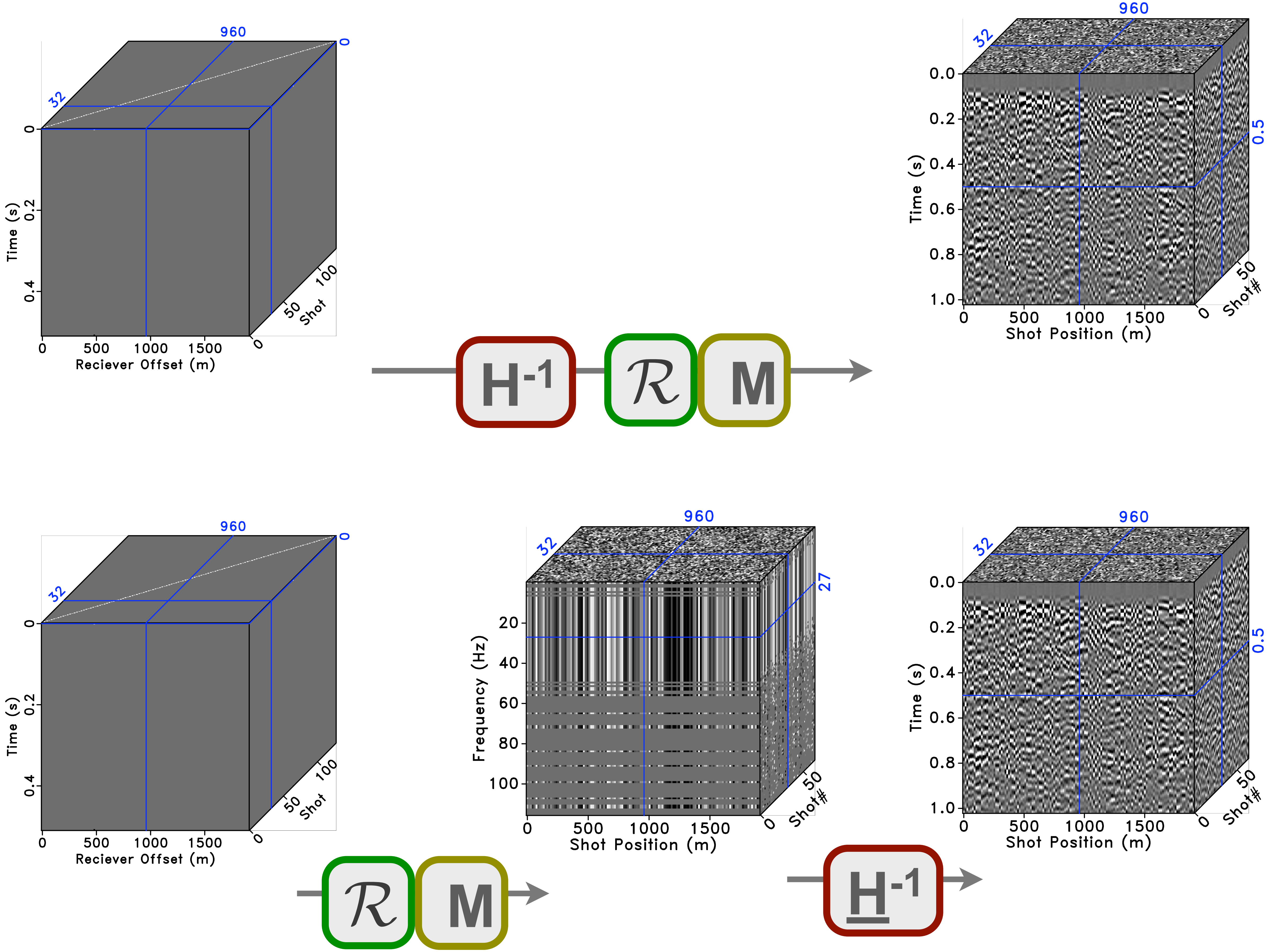
$$\boxed{\mathcal{O}(n^3 \log n)}$$

Source-solution sampling equivalence

$$\begin{cases} \mathbf{B} = \mathbf{D}^* \underbrace{\mathbf{s}}_{\text{single shots}} \\ \mathbf{H}\mathbf{U} = \mathbf{B} \\ \mathbf{y} = \mathbf{RMDU} \end{cases} \iff \begin{cases} \underline{\mathbf{B}} = \underline{\mathbf{D}}^* \underbrace{\mathbf{RMs}}_{\text{simul. shots}} \\ \underline{\mathbf{HU}} = \underline{\mathbf{B}} \\ \underline{\mathbf{y}} = \underline{\mathbf{DU}} \end{cases}$$

- Show equivalence between
  - CS sampling the ***full*** solution for separate single-source experiments
  - Solution of ***reduced*** system after CS sampling the collective single-shot source wavefield **s**
- Have to show that

$$\mathbf{y} = \underline{\mathbf{y}}$$



960
32
0
Time (s)
0.2
0.4
0
500
1000
1500
Reciever Offset (m)
50
100
Shot
H-1
R
M
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
Shot Position (m)
0.5
Shot#
Frequency (Hz)
20
40
60
80
100
27

Sparsifying transform

- Use fast discrete 2-D Curvelet transform based on wrapping [Demanet '06] along shot and receiver coordinates
  - compresses highly geometrical features of monochromatic wavefields
  - incoherent with compressive-sampling matrix that acts along the source coordinate

- Use fast discrete wavelet transform along the time coordinate
  - compresses front-like features arriving along the time direction
  - reasonable incoherent with sampling of angular frequencies

- Combine both transforms through a **Kronecker** product

$$\mathbf{S} = \mathbf{C}_{2d} \otimes \mathbf{W}$$

- Numerical complexity *sparsifying* transform

$$\boxed{\mathcal{O}(n^3 \log n)}$$

Complexity analysis

Assume discretization size in each dimension is *n*, and

$$n_s = n_t = n_f = \mathcal{O}(n)$$

Time-domain finite differences:

- $\mathcal{O}(n^4)$ in 2-D
- large constants

Multilevel-Krylov preconditioned [Erlangga, Nabben, FJH, '08]

- $\mathcal{O}(n^4) = n_f n_s n_{it} \mathcal{O}(n^2)$ with $n_{it} = \mathcal{O}(1)$
- small constants

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Complexity analysis cont'd

Cost sparsity promoting optimization dominated by matrix-vector products

- Sparsity transform is $\mathcal{O}(n^3 \log n)$
- Gaussian projection $\mathcal{O}(n^3)$ per frequency
- **Cost** $\mathcal{O}(n^4)$, which does not lead to asymptotic improvement

Use fast transforms (e.g. Random Convolutions by Romberg '08)

- fast projection in time & shot directions: $\mathcal{O}(n \log n)$
- **Cost** $\mathcal{O}(n^3 \log n)$ instead of $\mathcal{O}(n^4)$

Bottom line: Computational cost for the ℓ_1-solver is less $(\mathcal{O}(n^3 \log n)$ vs. $\mathcal{O}(n^4))$ than the cost of solving Helmholtz

- smaller memory imprint
- cost reduction dependent on complexity = transform-domain sparsity of the solution

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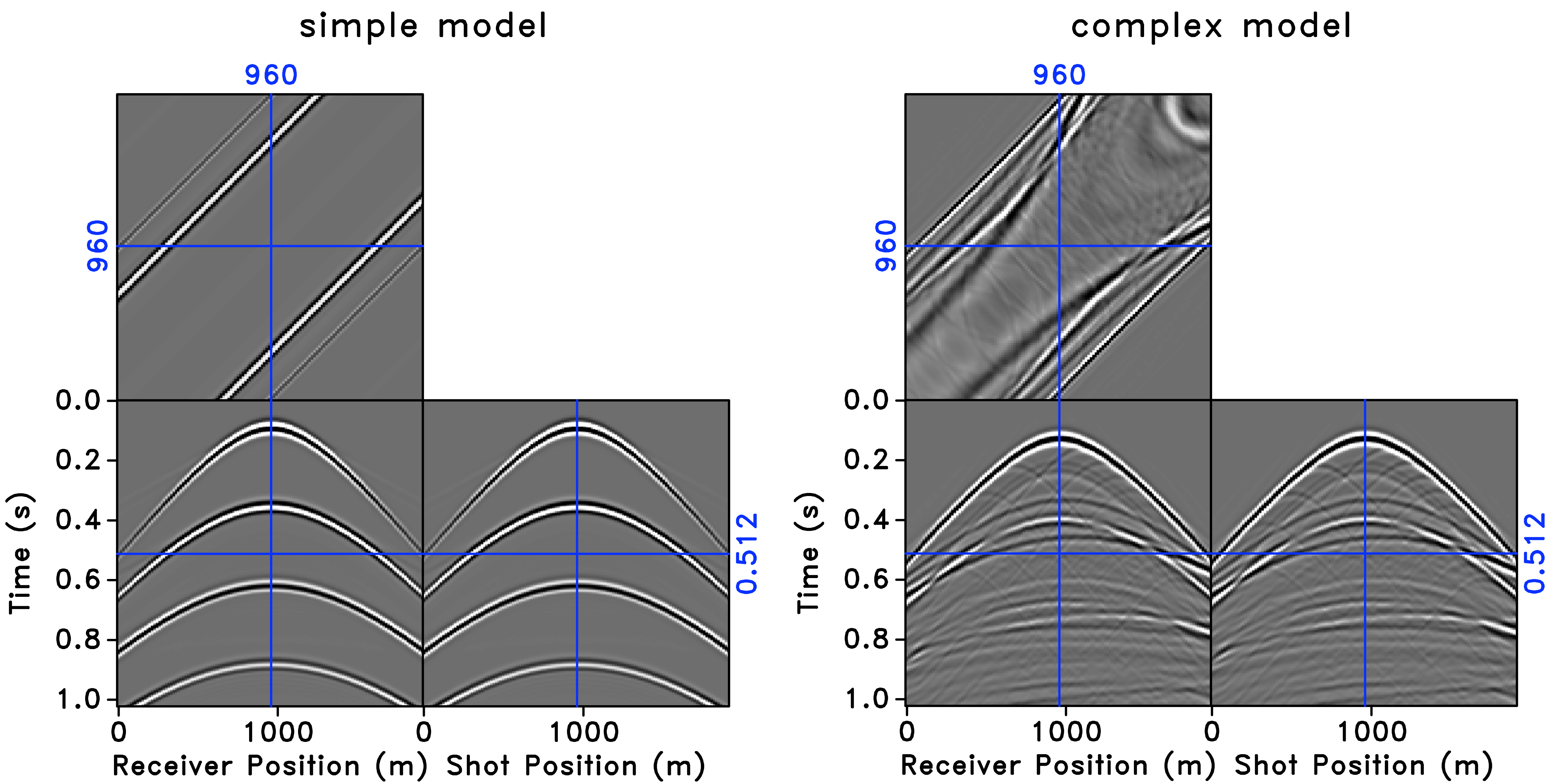
Green’s functions



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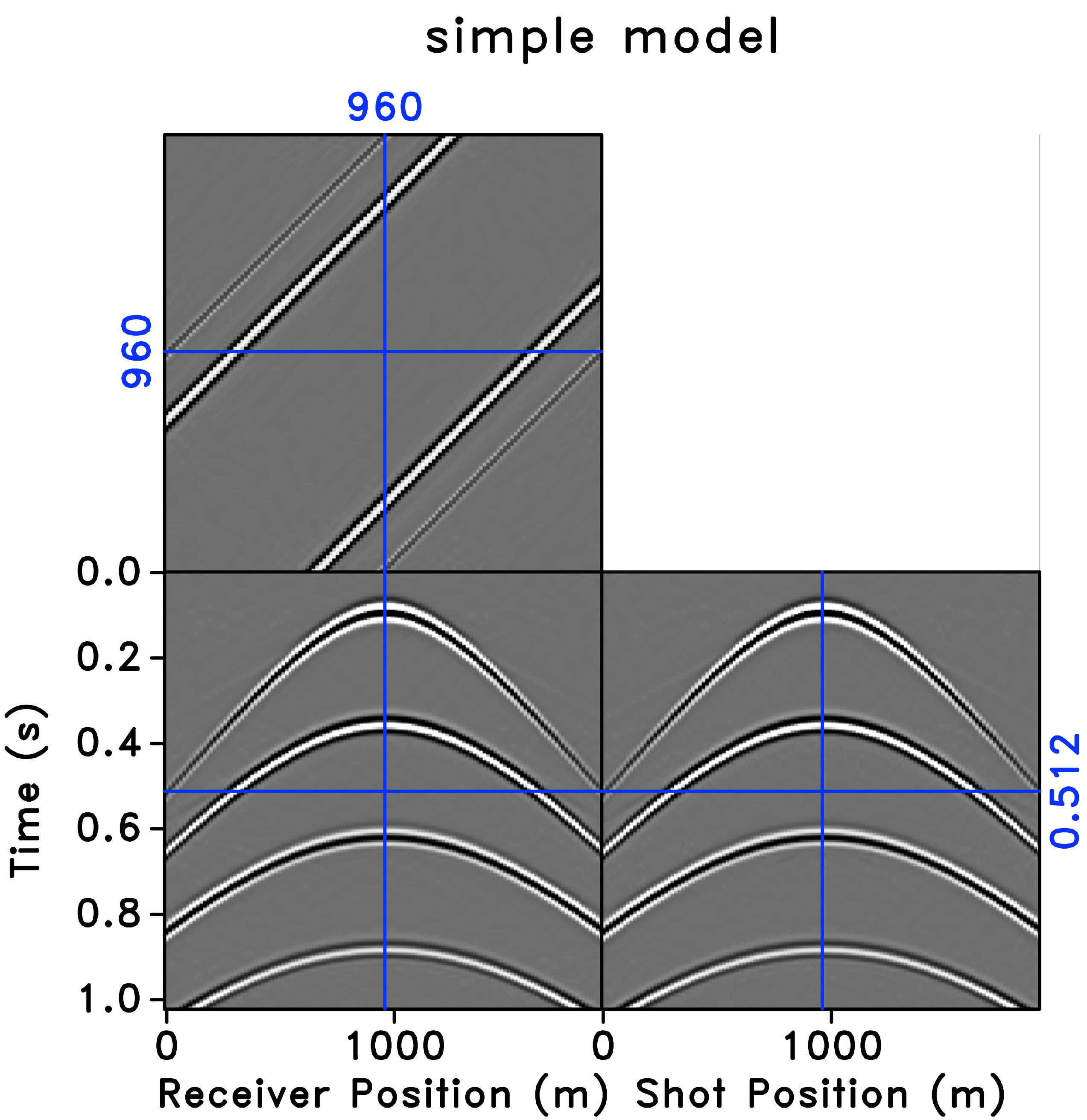
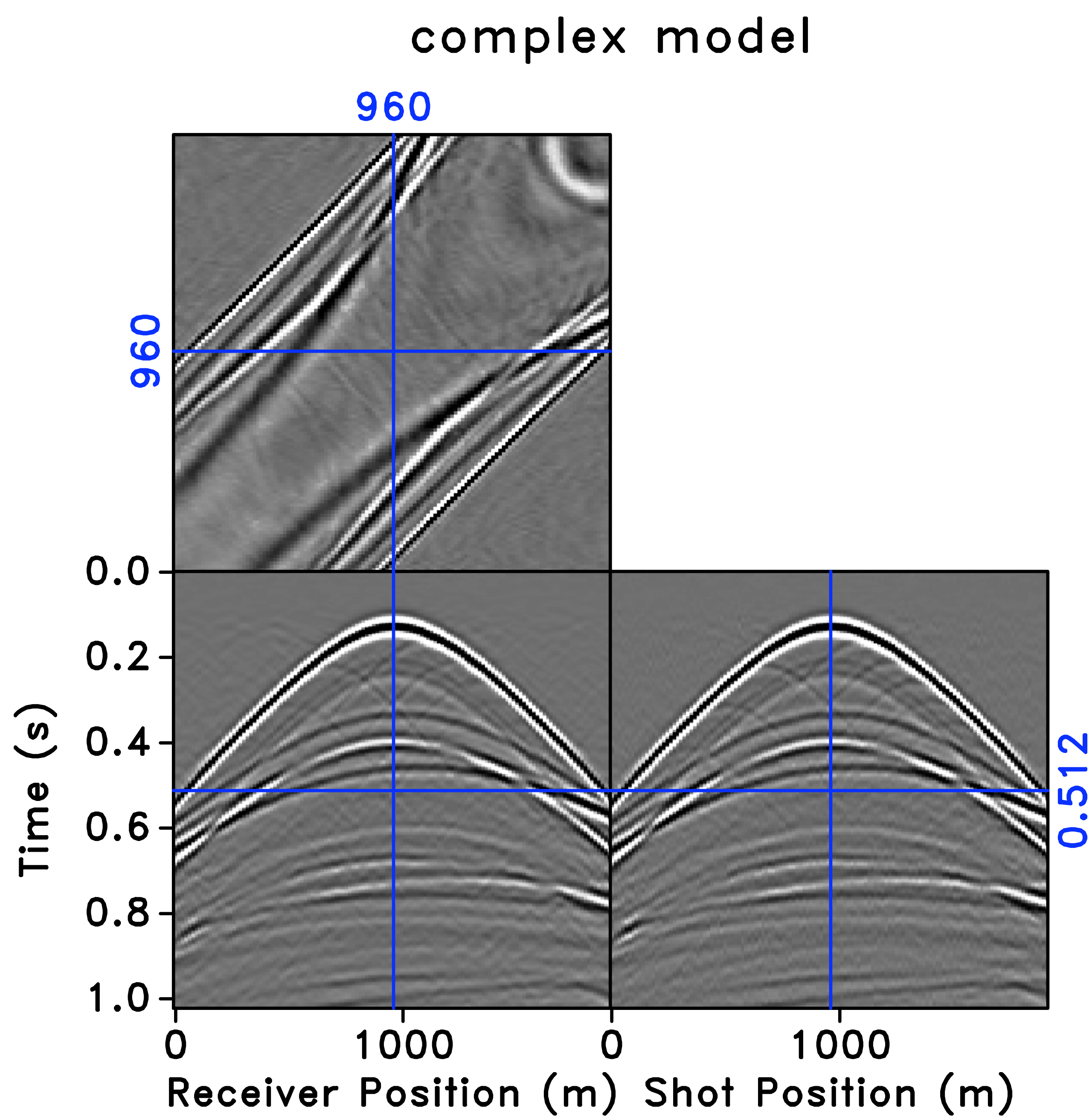
Recovered data



28.1dB



18.2dB

300 SPGL1 iteration

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The next step

- **Fast matrix computations**
  - Improved Approximation Algorithms for Large Matrices via Random Projections by Tamás Sarlós, ’08

$$\mathbf{AB} \approx \mathbf{A} \left(\mathbf{RM}\right)^{*} \left(\mathbf{RM}\right) \mathbf{B}$$

  - Faster Least Squares Approximation by Petros Drineas et. al., ’08

$$\arg\min_{\mathbf{X}} \|\mathbf{B} - \mathbf{AX}\|_2 \approx \arg\min_{\mathbf{X}} \|\mathbf{RM}\left(\mathbf{B} - \mathbf{AX}\right)\|_2$$

- **Joint sparsity-promotion with mixed (1,2) norm minimization**
  - Joint-sparse recovery from multiple measurements by E. van den Berg and M. Friedlander, ‘09

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{B}\|_{2,2} \leq \sigma,$$

- Based on Johnson-Lindenstrauss type embeddings.

Wavefield focusing

Define linear mid-point/offset coordinate transformation

$$\boldsymbol{\delta}\mathbf{I}'(m,h,t) = \mathbf{T}^{\Delta h}_{(x_s,x_r)\mapsto(m,h)}\boldsymbol{\delta}\mathbf{I}(x_s,x_r,t),$$

with $m = \frac{1}{2}(x_s + x_r) \quad \text{and} \quad h = \frac{1}{2}(x_s - x_r)$

Penalize ***defocusing*** via minimizing [Symes, '09]

$$\|\mathsf{P}_h\mathbf{I}'(\cdot,\, h)\|_2 \text{ with } \mathsf{P}_h\cdot = \mathbf{h}\cdot$$

an *annihilator* that increasingly *penalizes* non-zero offsets.

Remark: conventional imaging principle

$$\delta\mathbf{m} = \boldsymbol{\delta}\mathbf{I}'(\cdot, h=0, t=0)$$

Compressive wavefield inversion with focussing

Compressively sample augmented system

$$\begin{aligned} \mathbf{RM}\left(\mathbf{U}^* \circ \mathbf{S}^*\mathbf{X}\right) &\approx \mathbf{RMV}^T \\ \mathsf{P}_h\mathbf{X} &\approx \mathbf{0} \end{aligned} \quad \text{or} \quad \mathbf{AX} \approx \mathbf{B}$$

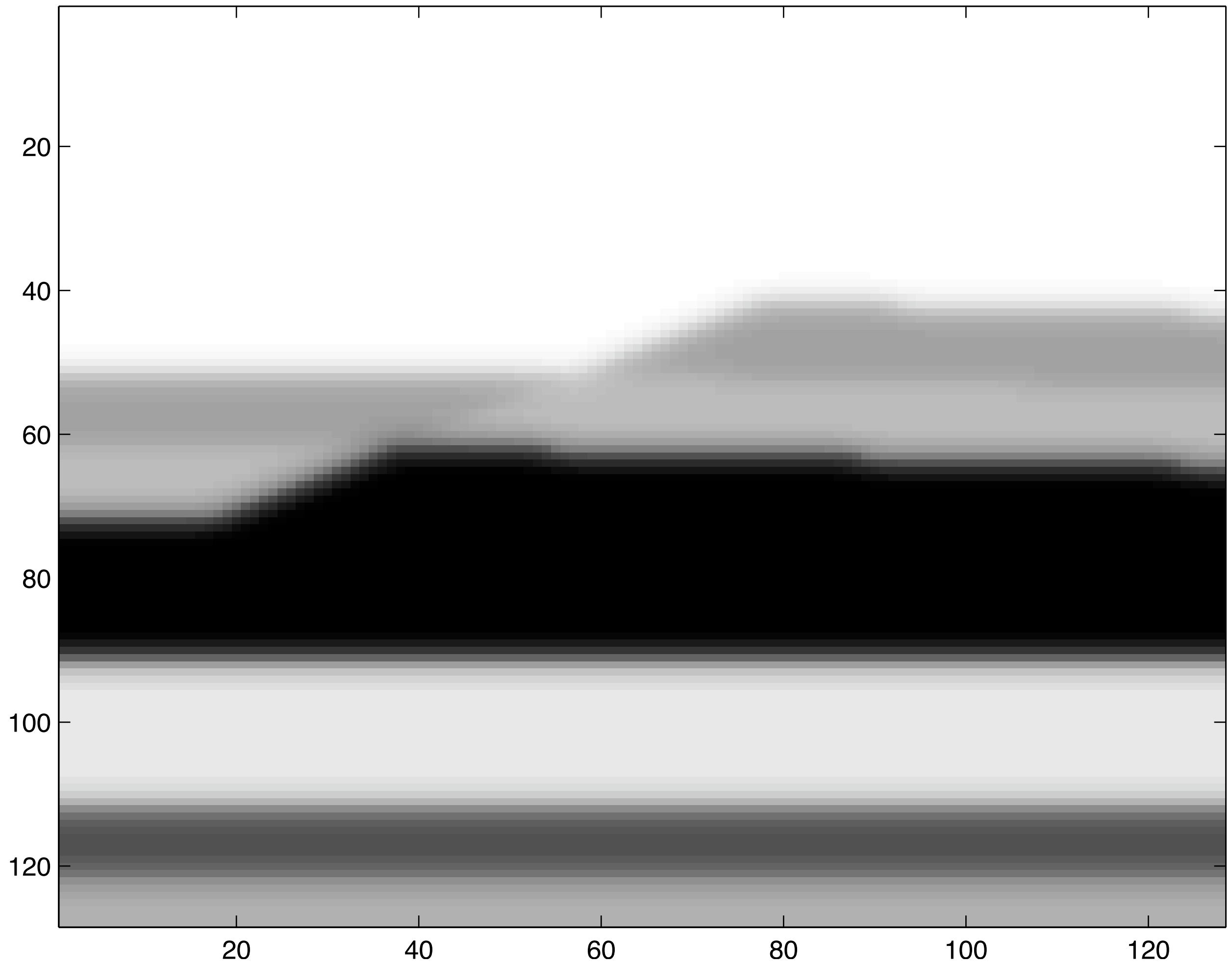
Recover focused solution by mixed (1,2)-norm minimization

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{B}\|_{2,2} \leq \sigma,$$

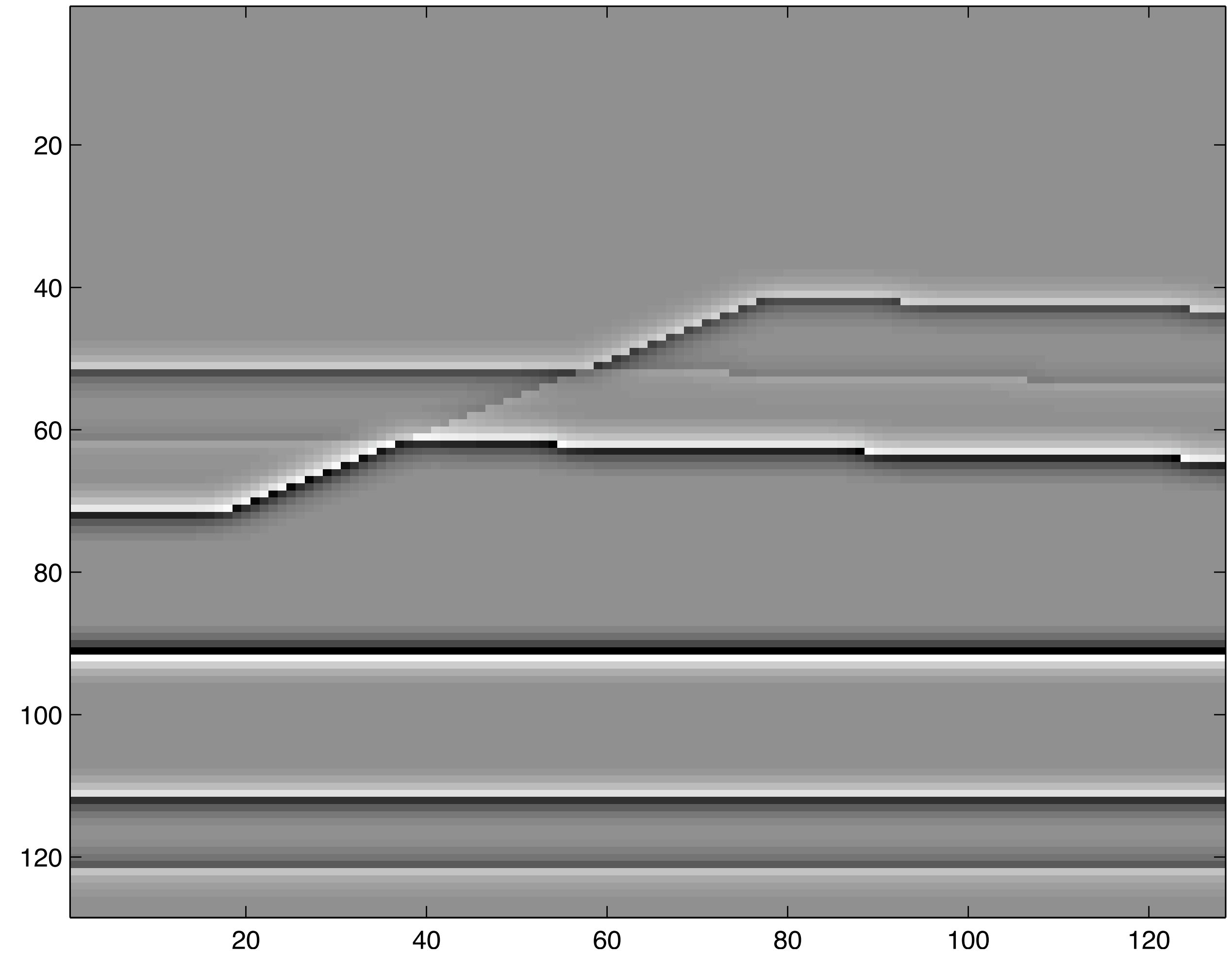
with $$\|\mathbf{X}\|_{1,2} := \sum_{i \in \text{rows}(\mathbf{X})} \|\text{row}_i(\mathbf{X})^*\|_2$$

and $$\|\mathbf{X}\|_{2,2} := \left(\sum_{i \in \text{rows}(\mathbf{X})} \|\text{row}_i(\mathbf{X})^*\|_2^2 \right)^{\frac{1}{2}}.$$

Example

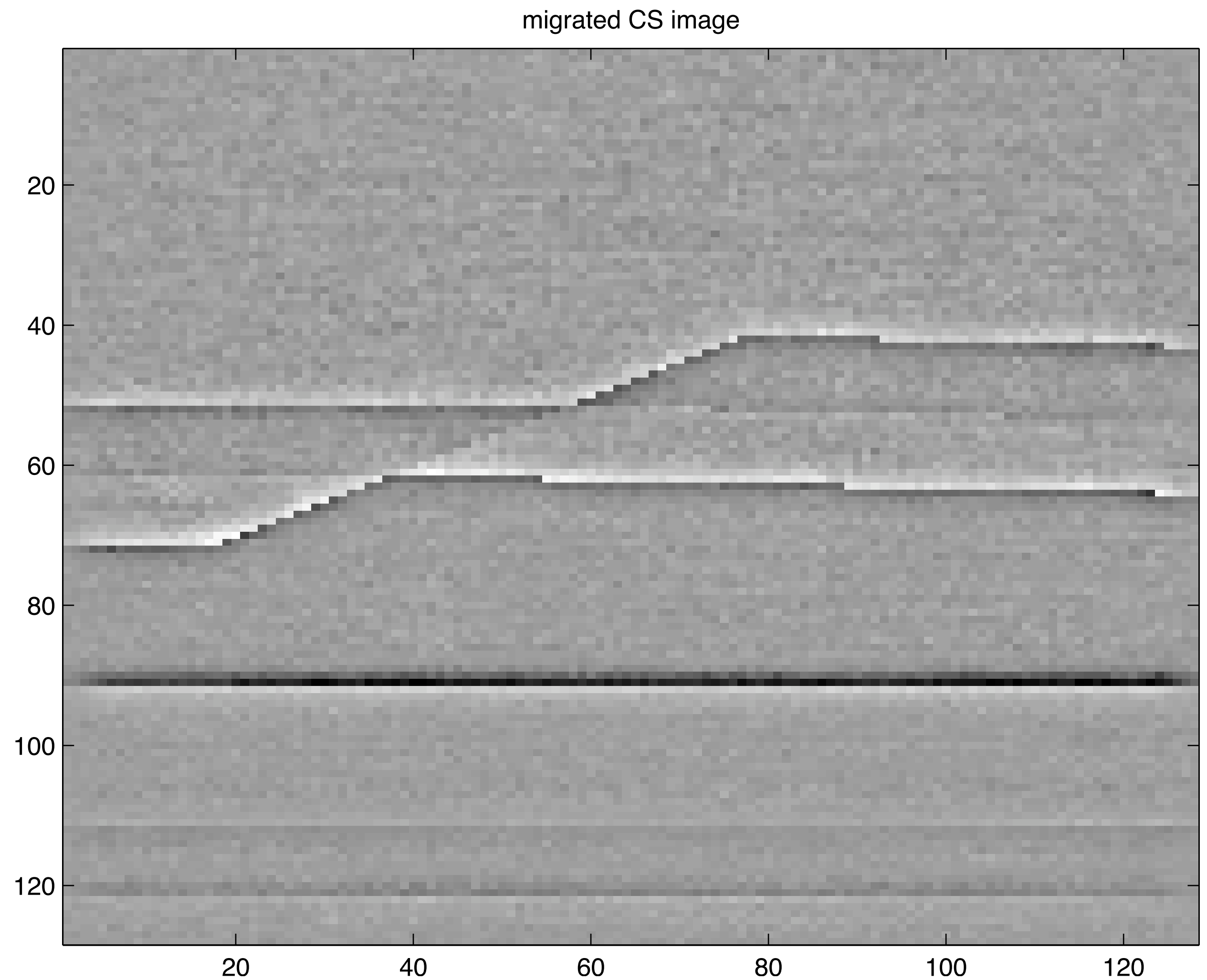


background velocity model

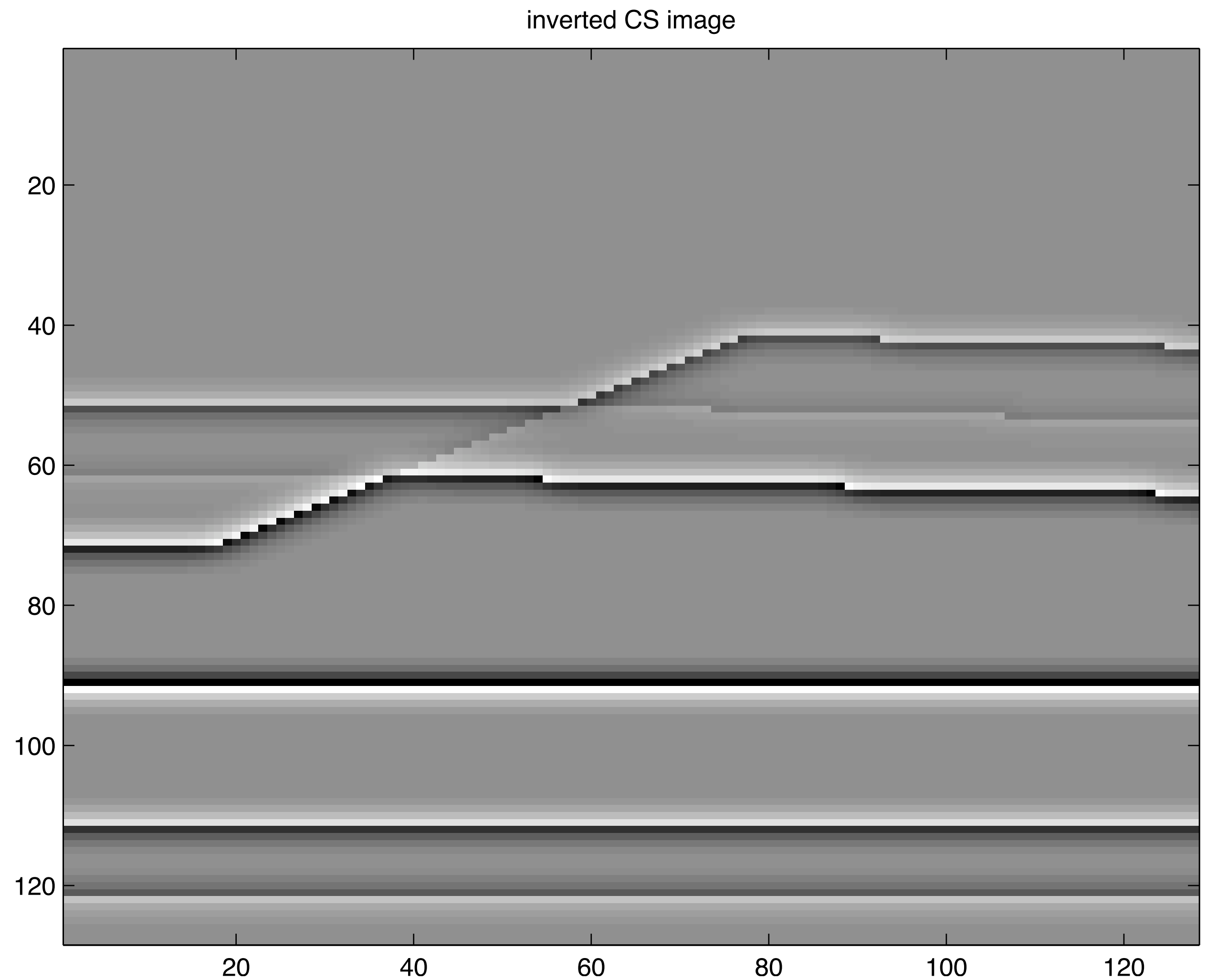


perturbation

Example



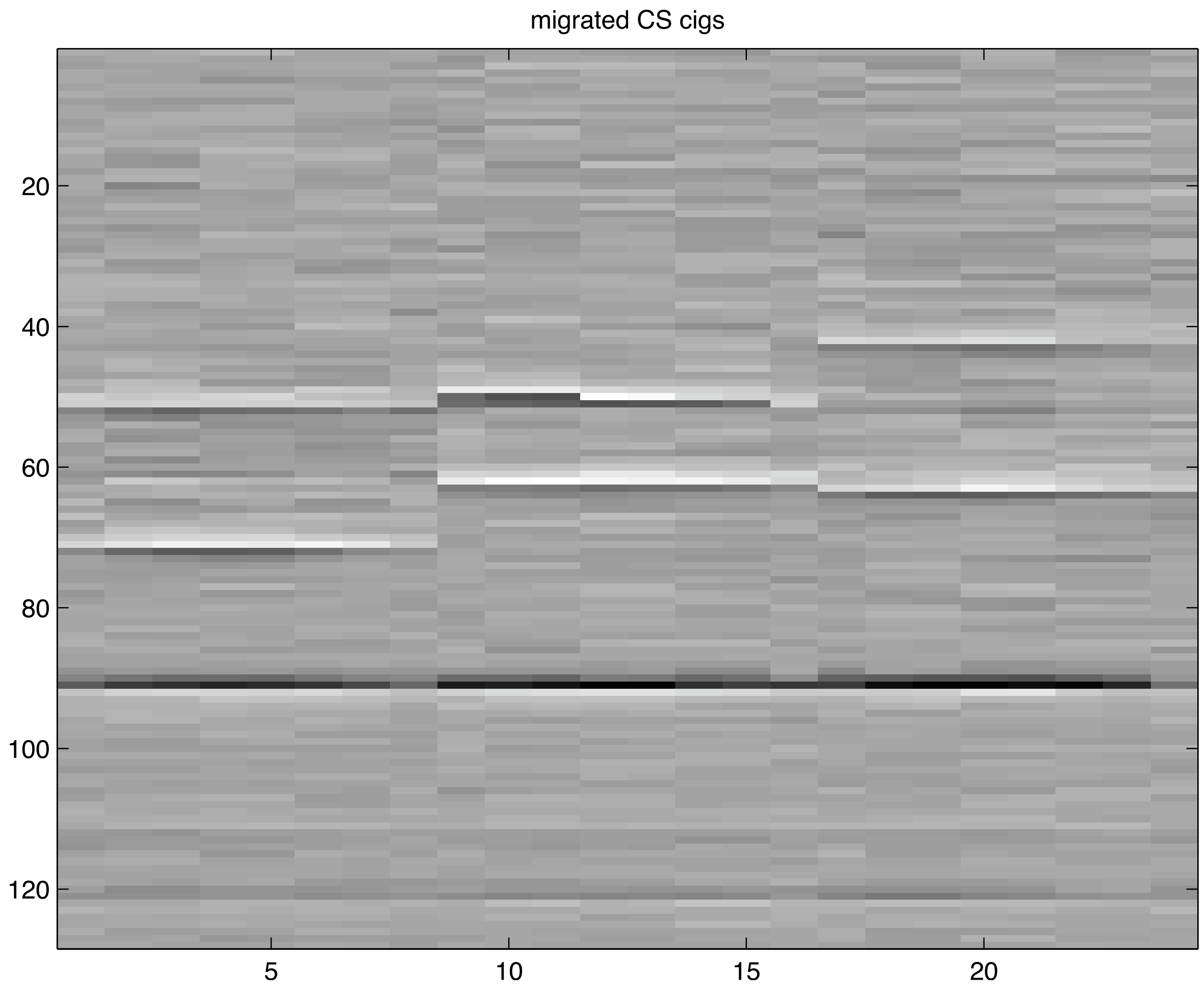
matched filter



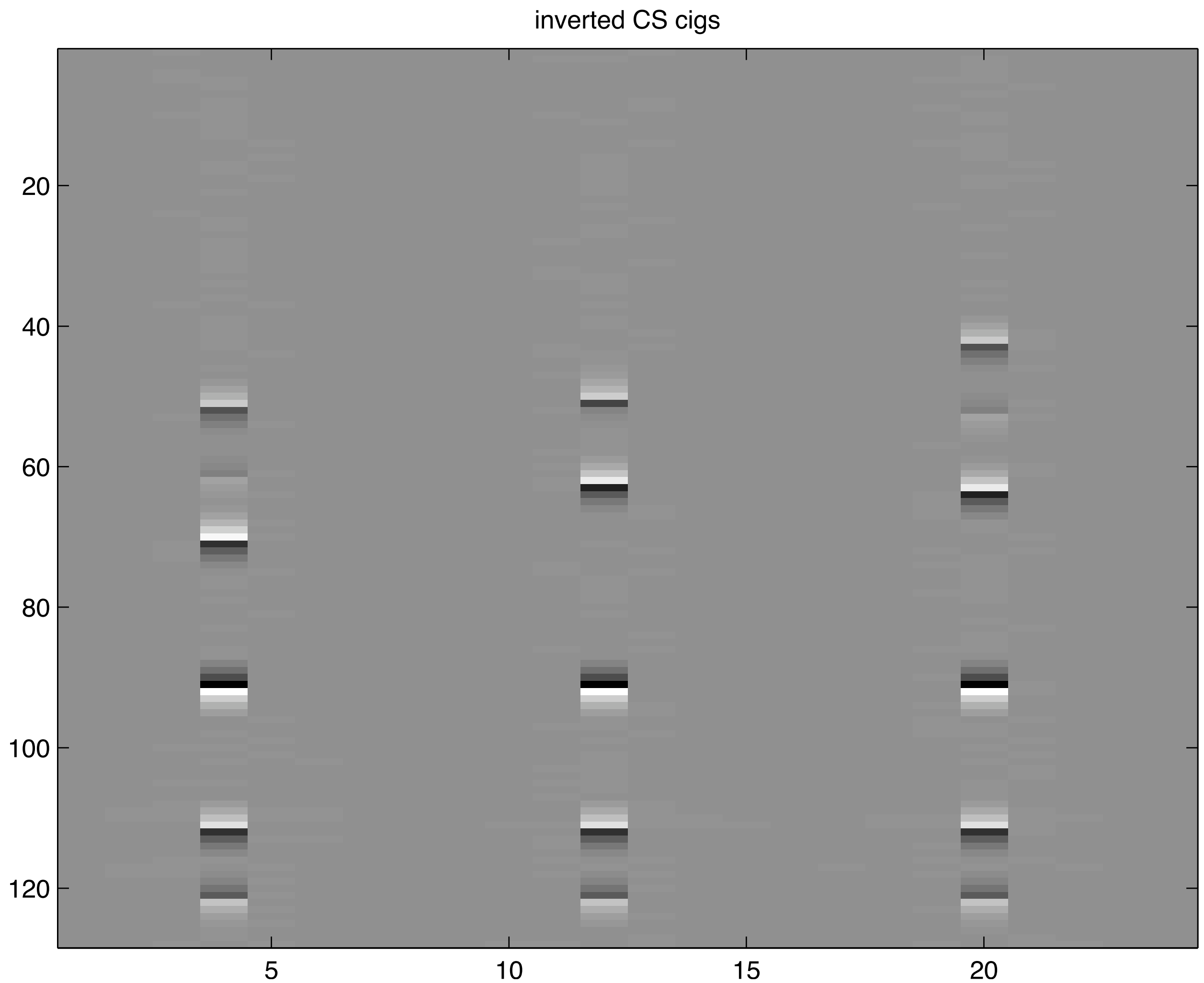
wavefield inversion

Recovery from 64-fold subsampling ...

Example



correlation based



wavefield inversion

Common-image gathers are focussed.

Conclusions & outlook

- **CS** provides a **new** *linear sampling paradigm*
  - ***degree*** of *subsampling commensurate* with transform-domain ***sparsity***
  - subsampling of seismic data volumes
    - missing source-receiver locations
    - simultaneous acquisition
  - *subsampling* of solutions to PDEs

- **CS** leads to
  - “acquisition” of *smaller* data volumes that carry the ***same*** *information* or
  - to ***improved*** *inferences* from data using the *same* resources
  - feasibility of nonlocal extensions of Helmholtz solvers that are otherwise impossible

- Bottom line: **acquisition & numerical modeling costs** are **no** longer determined by the ***size*** of the ***discretization*** but by the ***transform-domain compressibility*** of the **solution ...**

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- E. Candes and the Curvelab team

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and... Thank you!