Sparse least-squares seismic imaging with source **estimation** utilizing **multiples**

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Assumption: a *reasonably accurate* background velocity model is given

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Main messages

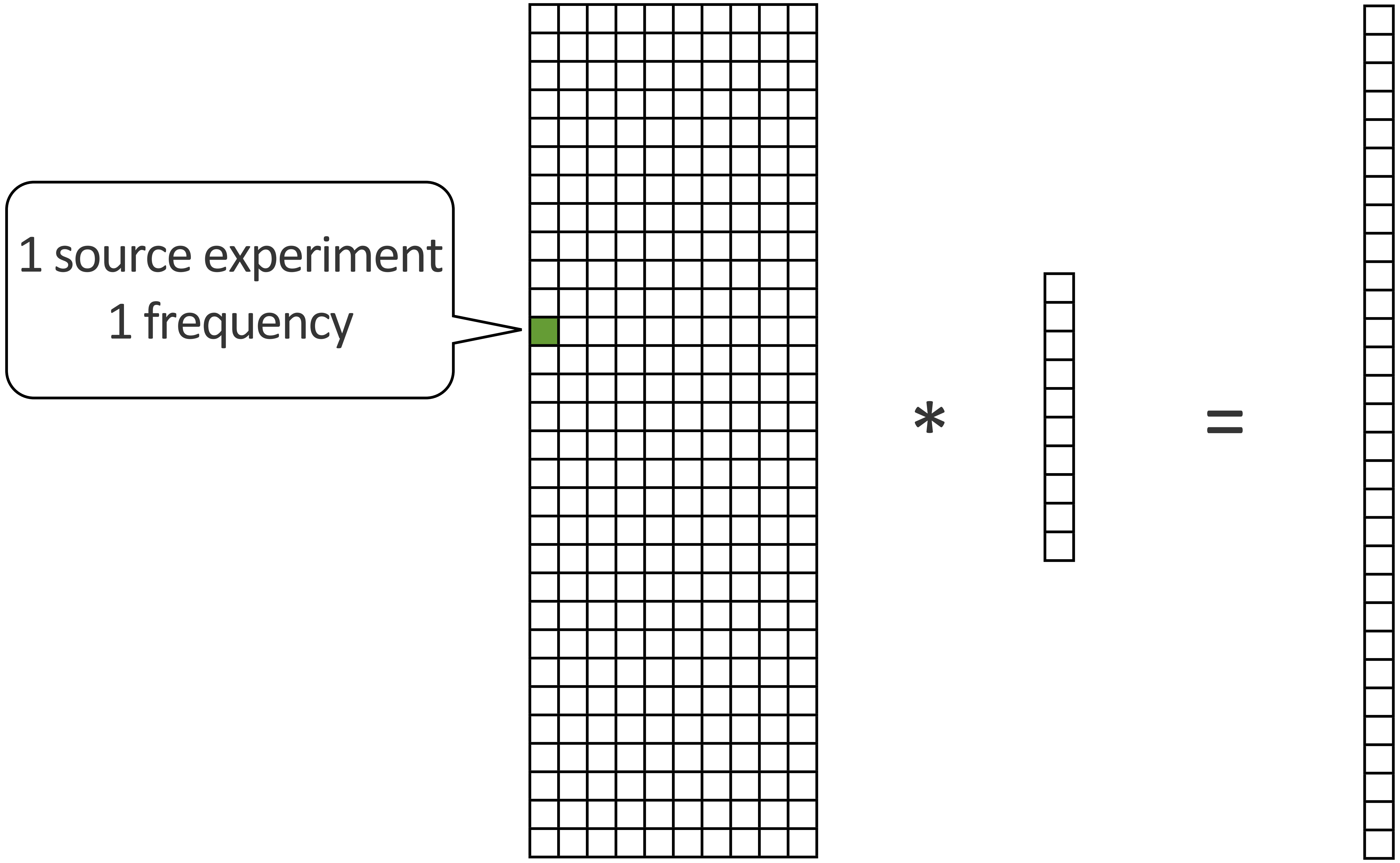
Demonstrate how least-squares migration

- can be carried out **efficiently**
- can be carried out **without** the knowledge of the **source wavelet**
- can make active use of **surface-related multiples** in the data

by sparsity-promotion accelerated by rerandomization

Efficient least-squares imaging by subsampling

LS imaging abstracted: solving a linear system of equations



$$\nabla \mathbf{F}[\mathbf{m}_0, \mathbf{S}] \quad \delta\mathbf{m} = \mathbf{d}$$

Reducing number of wave-equation solves

$$\mathbf{W} \quad \nabla\mathbf{F}[\mathbf{m}_0, \mathbf{S}] \quad \delta\mathbf{m} = \mathbf{W} \quad \mathbf{d}$$

Reducing number of wave-equation solves

- Linearity with respect to the sources:

$$\mathbf{W}\nabla\mathbf{F}[\mathbf{m}_0, \mathbf{S}] = \nabla\mathbf{F}[\mathbf{m}_0, \mathbf{W}\mathbf{S}] \doteq \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{S}}] = \mathbf{W}\mathbf{d} \doteq \underline{\mathbf{d}}$$

- $\mathbf{W}$ can have Gaussian-distributed entries
  - randomized **simultaneous** sources
- $\mathbf{W}$ can be a subset of the identity matrix
  - randomized **subset** of all sources

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Control source cross-talks by sparsity-promotion

$$\mathrm{BP}_\sigma : \quad \begin{aligned} &\operatorname*{argmin}_{\mathbf{x}} \|\mathbf{x}\|_1 \\ &\text{subject to} \quad \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2 \leq \sigma^2. \end{aligned}$$

$\mathbf{C}^*$: curvelet synthesis operator

w_i : spectra of source wavelet

Ω : frequency subset

Σ : (simultaneous) source subset

van den Berg and Friedlander, 2008; Herrmann and Li, 2012

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From l1 minimization to l1 constraint

$$\mathrm{LS}_\tau : \quad \begin{aligned} &\min_{\mathbf{x}} f(\mathbf{x}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2 \\ &\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau. \end{aligned}$$

compute τ by solving $\inf f(\mathbf{x})|_{\|\mathbf{x}\|_1\leq\tau} = \sigma^2$

Herrmann and Li, 2012; Tu and Herrmann, 2012; Tu, Li and Herrmann 2013; Tu and Herrmann 2015a

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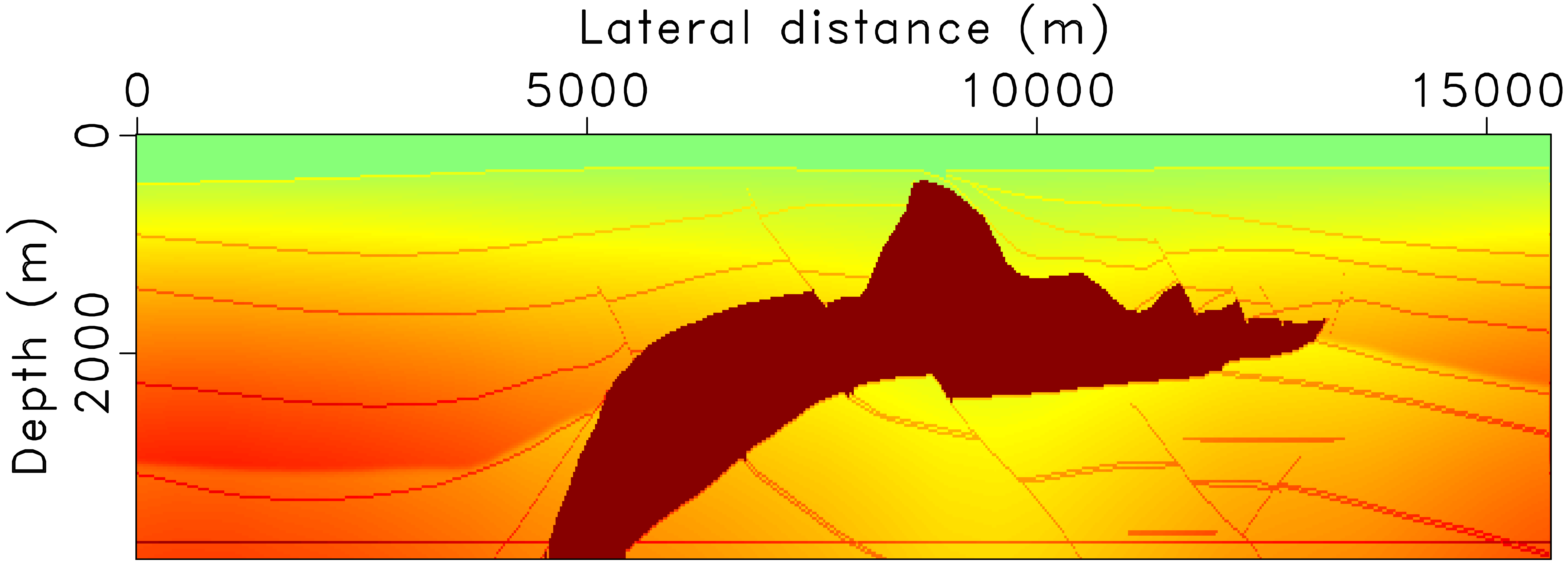
Going through *all your data* by drawing new samples

- drawing new $\mathbf{W}$ after each LS_τ subproblem is solved.
- statistically you make use of all your data
- benefits:
  - improving convergence in terms of model error decrease
  - improving robustness to modelling errors

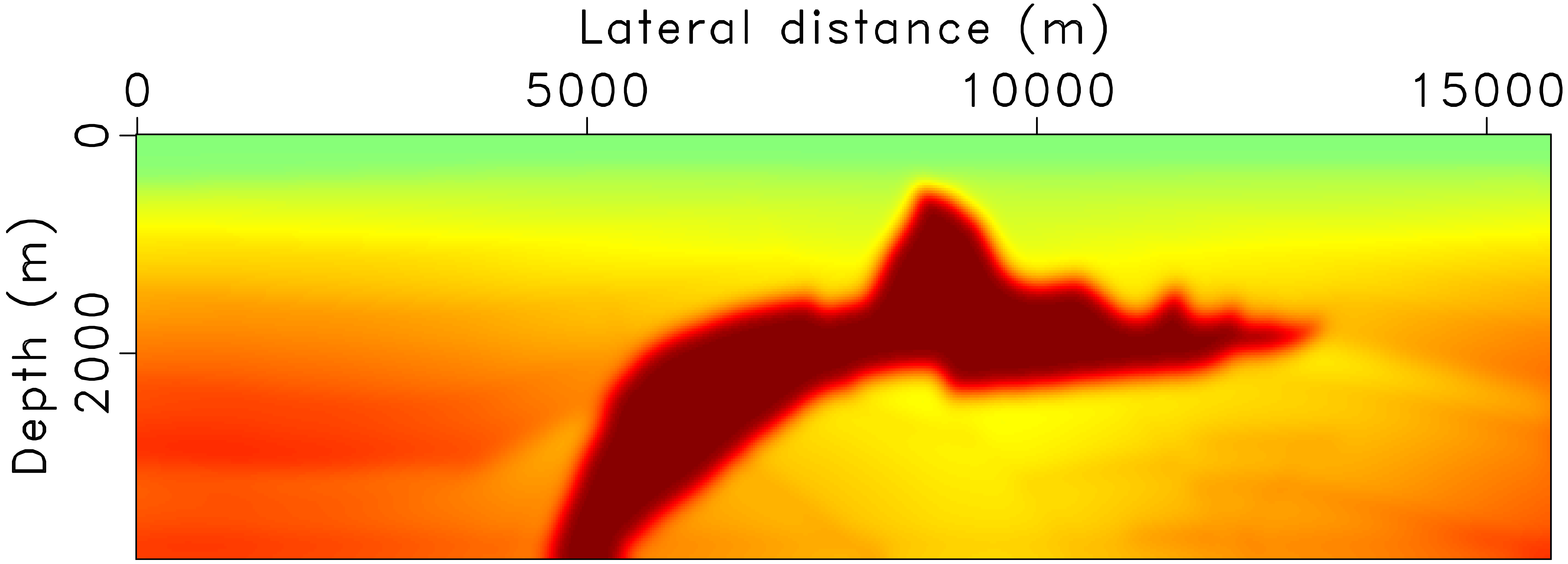
Experiment setup

- a 2D slice of the SEG/EAGE salt model, 3.9 km deep, 15.7 km wide, 24.38 m grid spacing
- *smooth* background model, including *smooth* salt boundaries
- 5 Hz Ricker wavelet, 8 s recording, 96 freq. samples
- 323 sources with 48.77 m spacing at 24.38 m depth
- forward modelling using iWave, inversion using in-house modelling
- using 15 frequencies, 15 simultaneous sources for fast inversion
- running for 60 iterations, simulation cost ~ 1 RTM with all sources and frequencies
- using the ***true*** wavelet for inversion

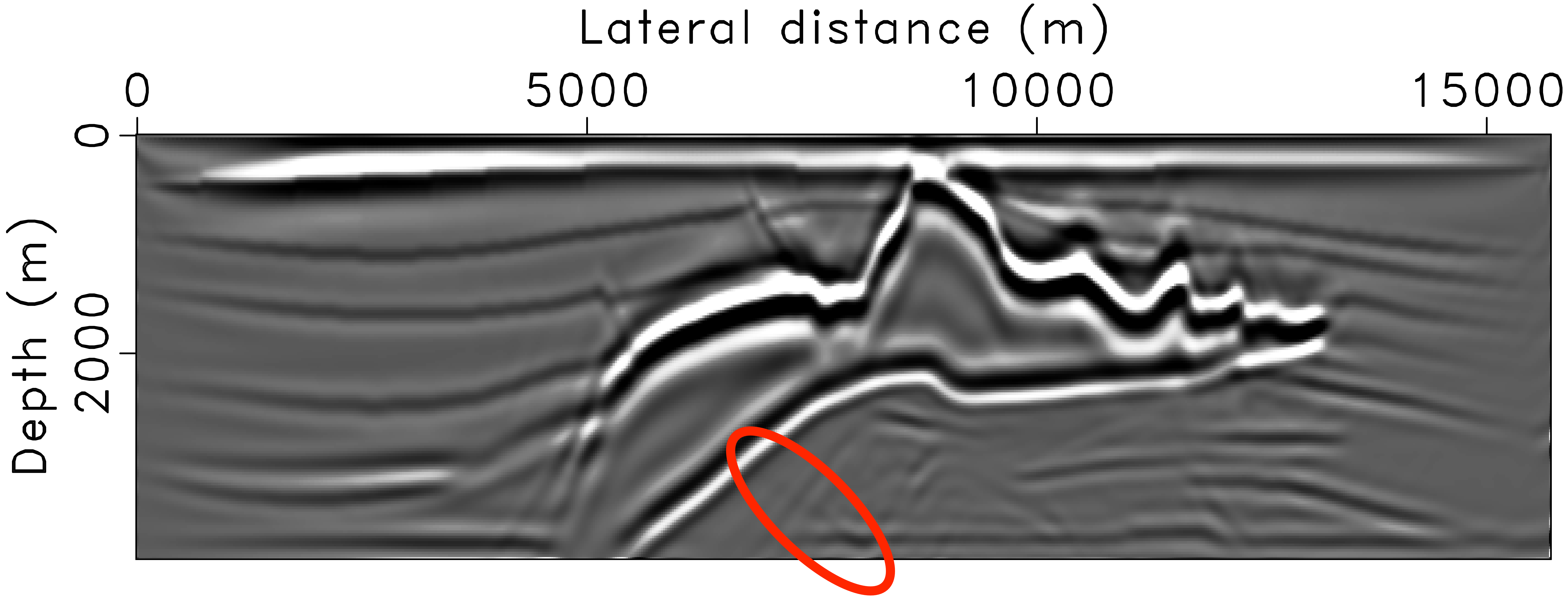
True model



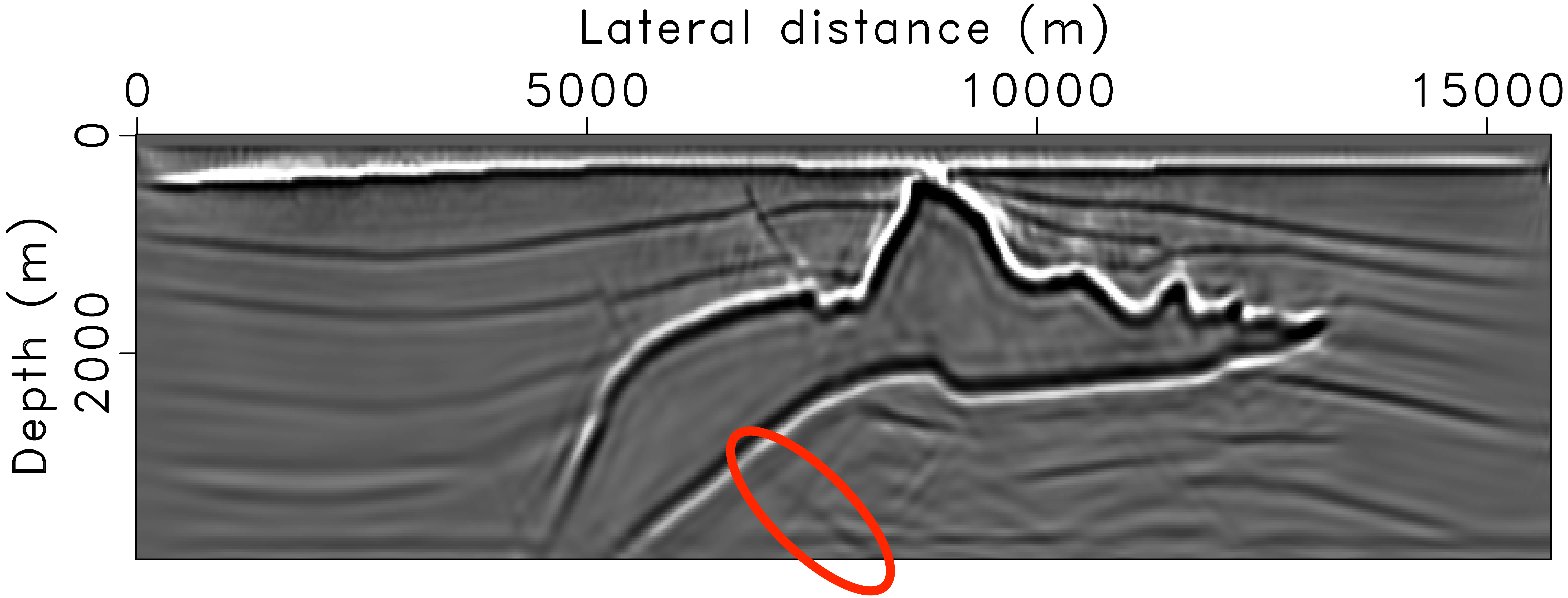
Background model



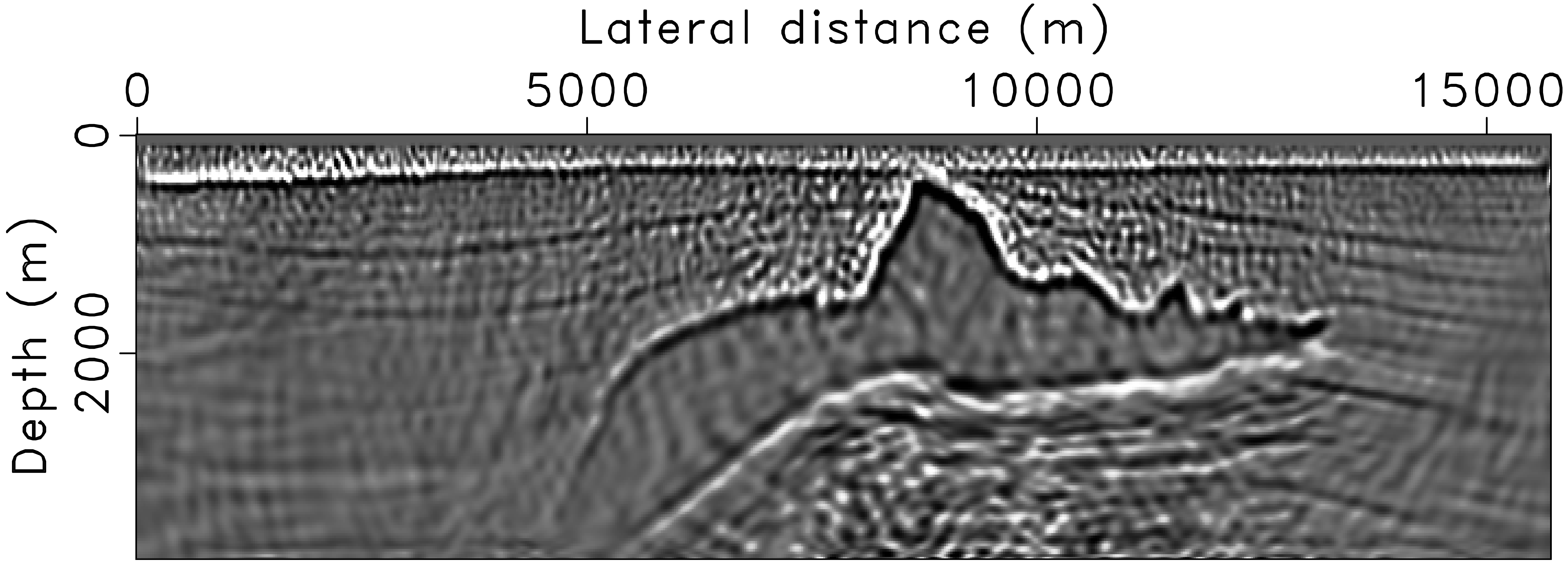
RTM image



Fast LS image w/ drawing new samples



Fast LS image w/o drawing new samples



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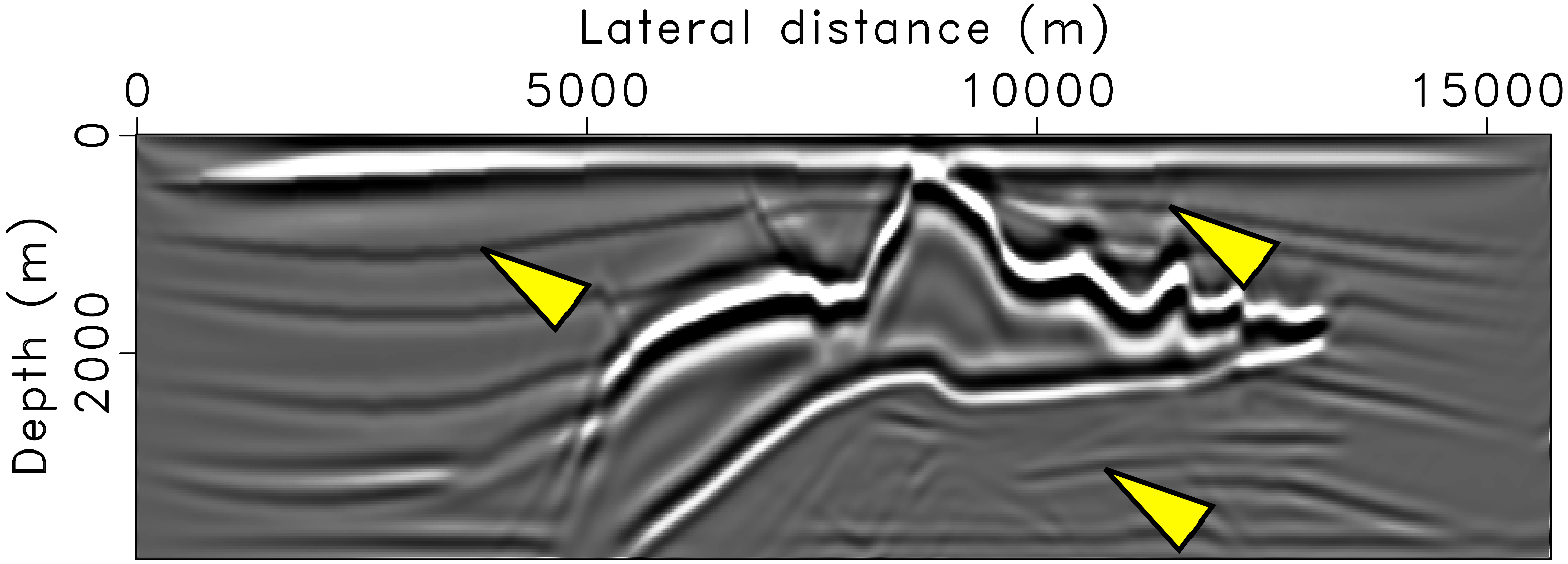
Challenge

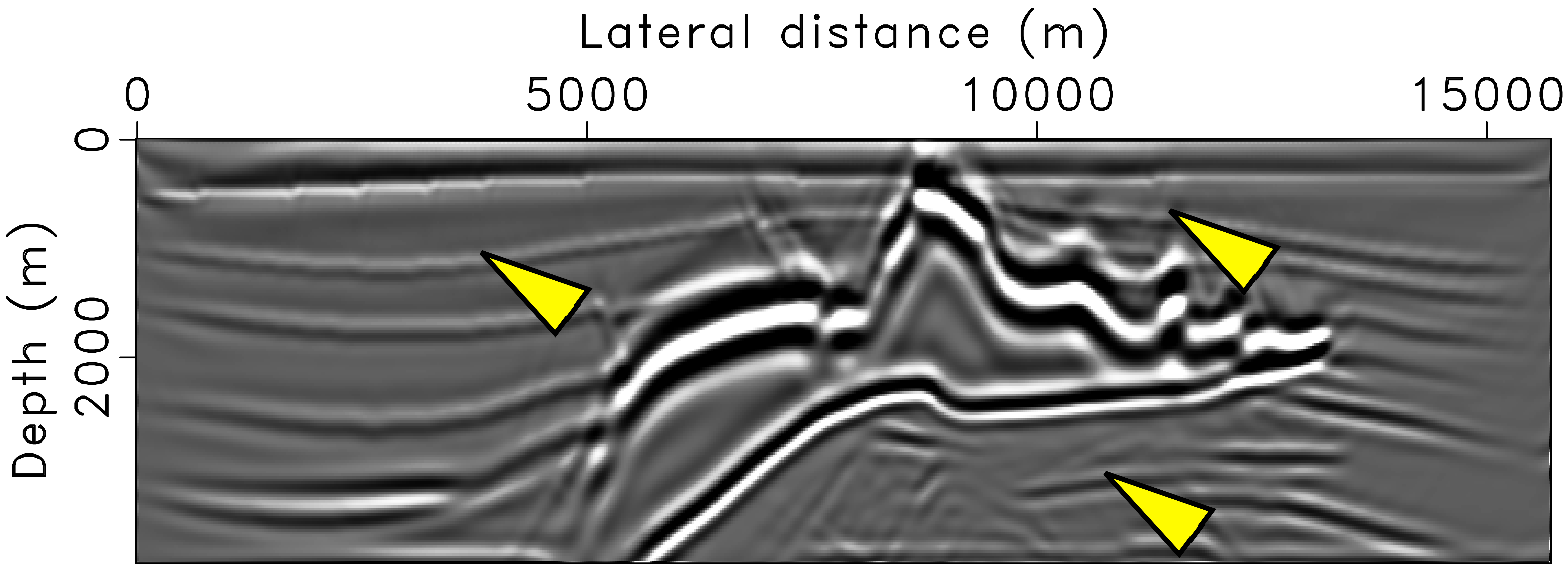
What if the source wavelet is not known, or an estimation of it contains errors such as a shift in time?

Imaging with source estimation

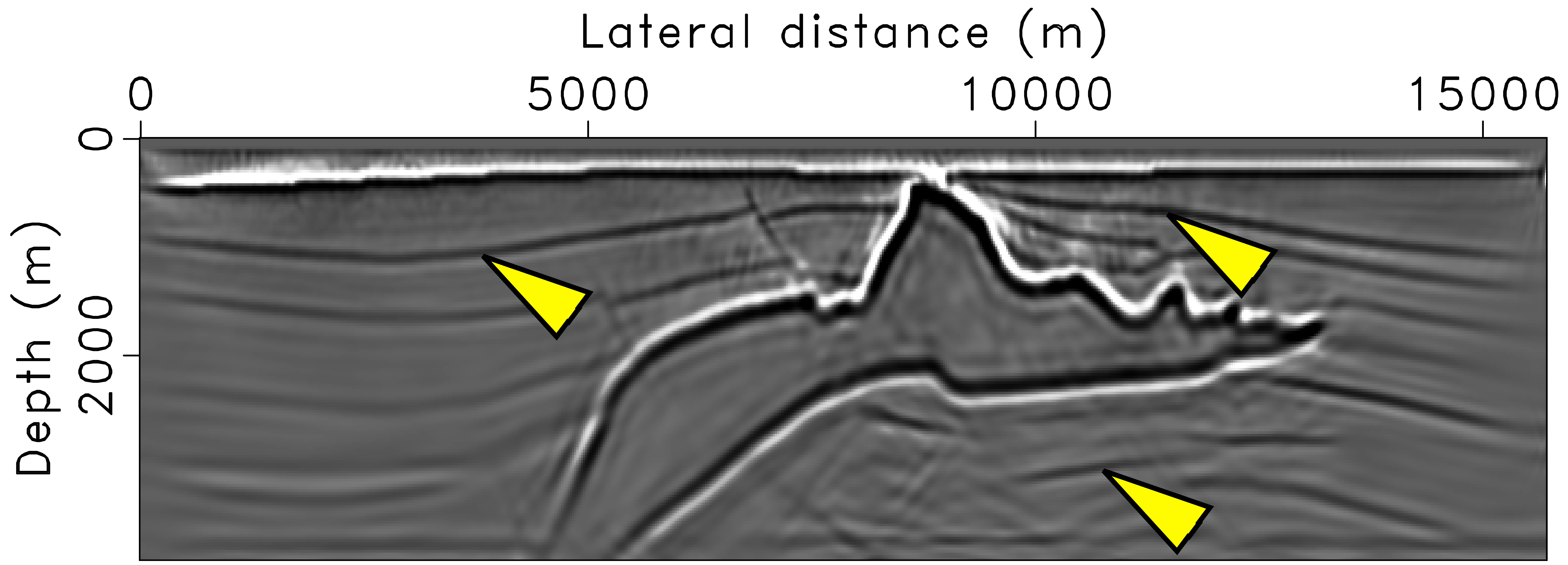
RTM image w/ true source wavelet



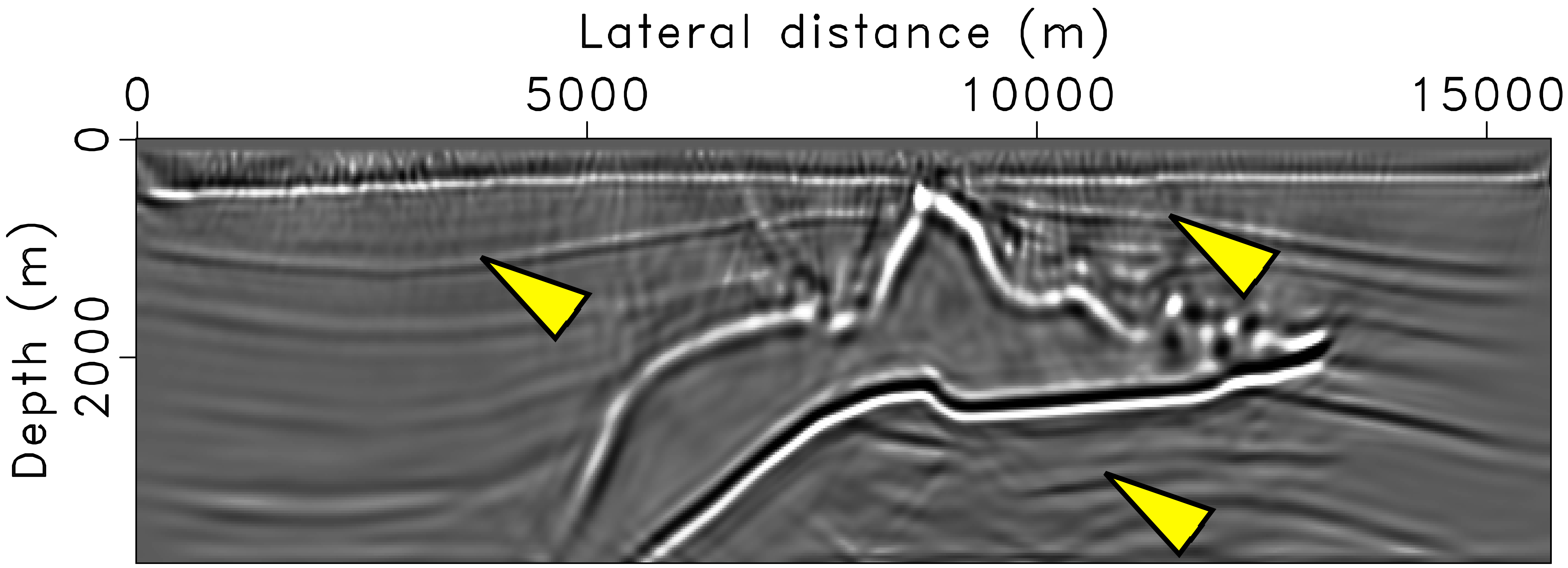
RTM image w/ wrong source wavelet (0.1 s shift)



Fast LS image **w/** true source wavelet



Fast LS image **w/** wrong source wavelet (0.1 s shift)



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Solution: “wavelet-free” fast LS migration

By borrowing ideas from source estimation using *variable projection*

- known as the separable *non-linear* least-squares problem

To tightly integrate variable projection into our fast LS migration formulation:

- simultaneously invert source wavelet and the image

Problem formulation with unknown source wavelet

$$\min_{\mathbf{x},\boldsymbol{w}} f(\mathbf{x},\boldsymbol{w}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2$$

$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau.$$

Challenges

The core gradient step becomes

$$\mathbf{x}^{k+1} = \mathcal{P}_{\mathcal{X}}[\mathbf{x}^k + \lambda \nabla_{\mathbf{x}} f(\mathbf{x}, \boldsymbol{w})|_{\mathbf{x}=\mathbf{x}^k, \boldsymbol{w}=\boldsymbol{w}^k}]$$

with

$$\mathcal{X} \doteq \{\mathbf{x} : \|\mathbf{x}\|_1 \leq \tau\}.$$

Challenges:

- evaluation of the gradient
- computing the sparsity level

Pratt R. G., 1999; Aravkin and van Leeuwen, 2012

Gradient descent using variable projection

With an estimate of the solution vector $\mathbf{x}$, the source estimates can be obtained by:

$$\widetilde{w}_i(\mathbf{x}) = \frac{\sum_{j\in\Sigma} < \underline{\mathbf{d}}_{i,j}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}{\sum_{j\in\Sigma} < \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}.$$

Then the optimization problem is reduced to:

$$\min_{\mathbf{x}} \overline{f}(\mathbf{x}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, \widetilde{w}_i(\mathbf{x})\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2$$

$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau,$$

with $\nabla_{\mathbf{x}}\overline{f}(\mathbf{x}) = \nabla_{\mathbf{x}} f(\mathbf{x}, \widetilde{\boldsymbol{w}}(\mathbf{x}))$.

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Computing the sparsity level

nonlinear LS_τ :

$$\min_{\mathbf{x},\boldsymbol{w}} f(\mathbf{x},\boldsymbol{w}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2$$

$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau.$$

↕

nonlinear BP_σ :

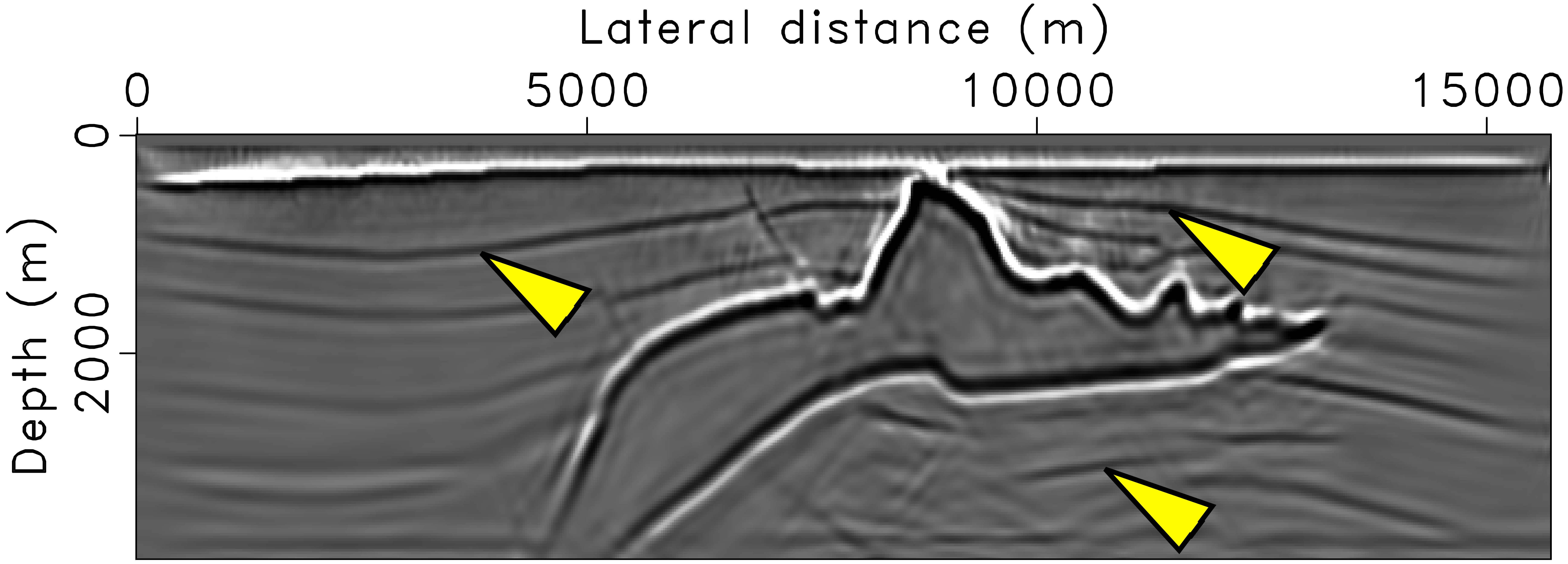
$$\operatorname*{argmin}_{\mathbf{x},\boldsymbol{w}} \|\mathbf{x}\|_1$$

$$\text{subject to} \quad \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2 \leq \sigma^2.$$

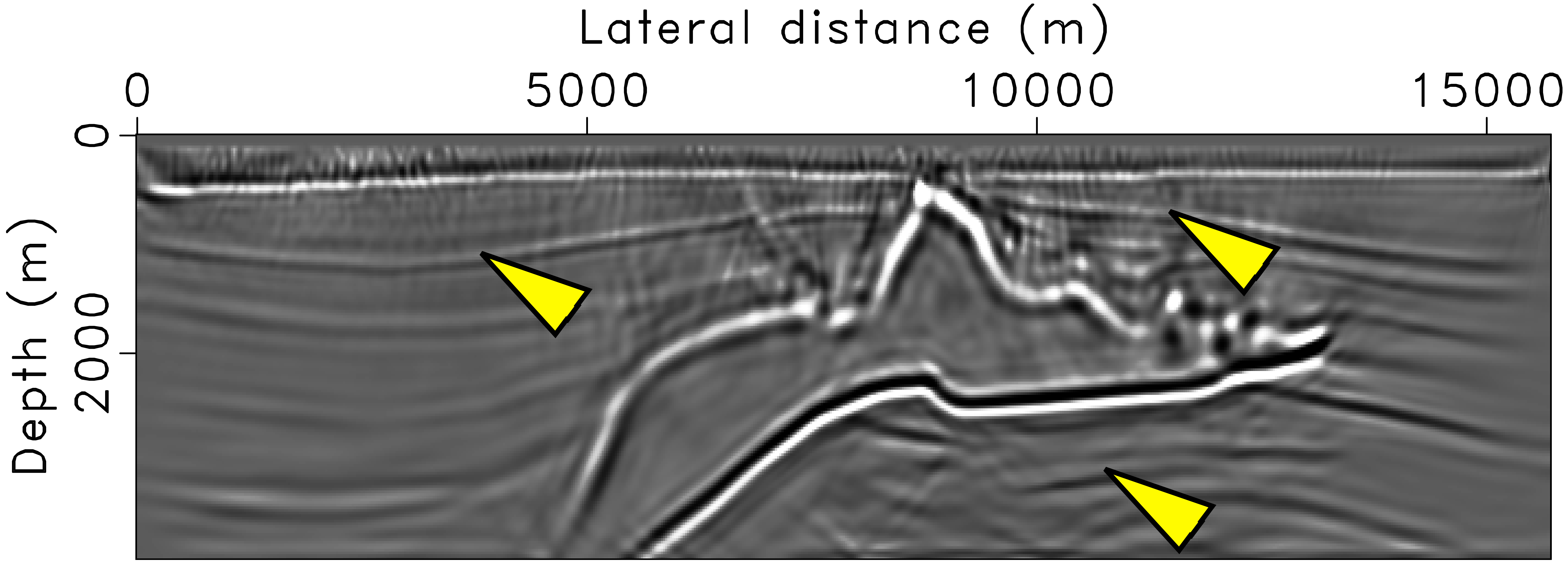
↓

compute τ by solving $\inf f(\mathbf{x},\mathbf{w})|_{\|\mathbf{x}\|_1\leq\tau} = \sigma^2$

Fast LS image w/ true source wavelet

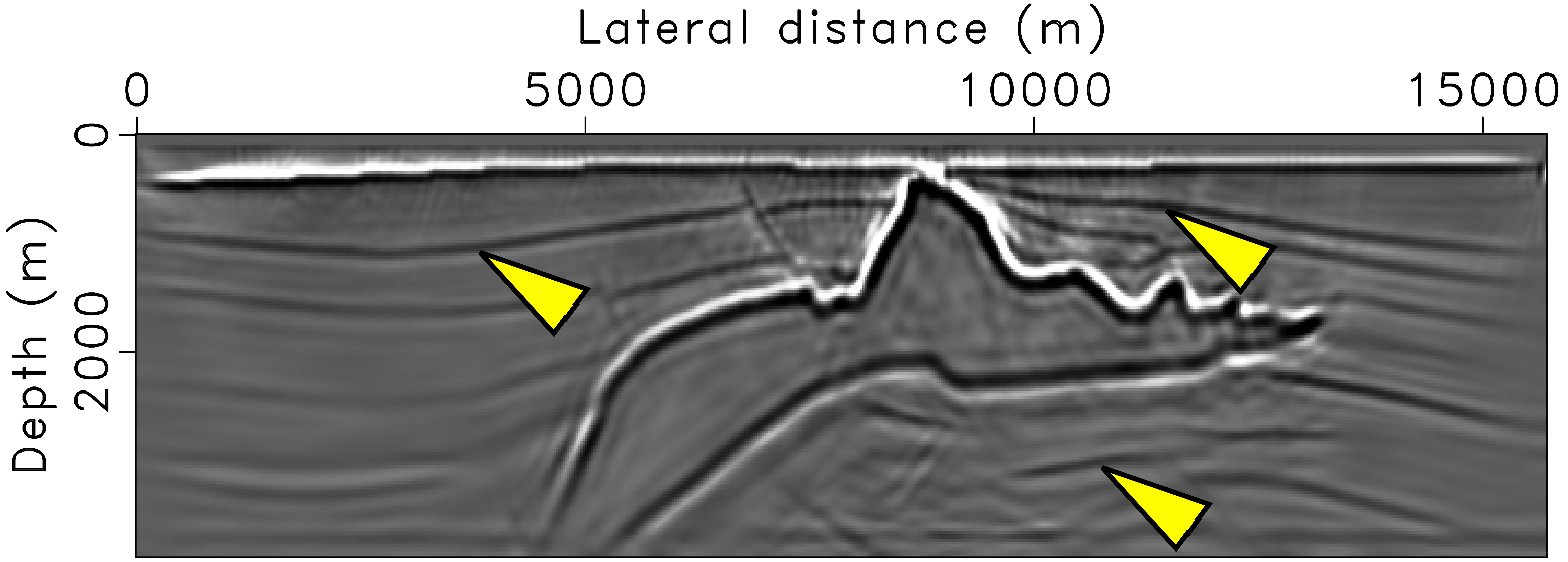


Fast LS image **w/** wrong source wavelet (0.1 s shift)

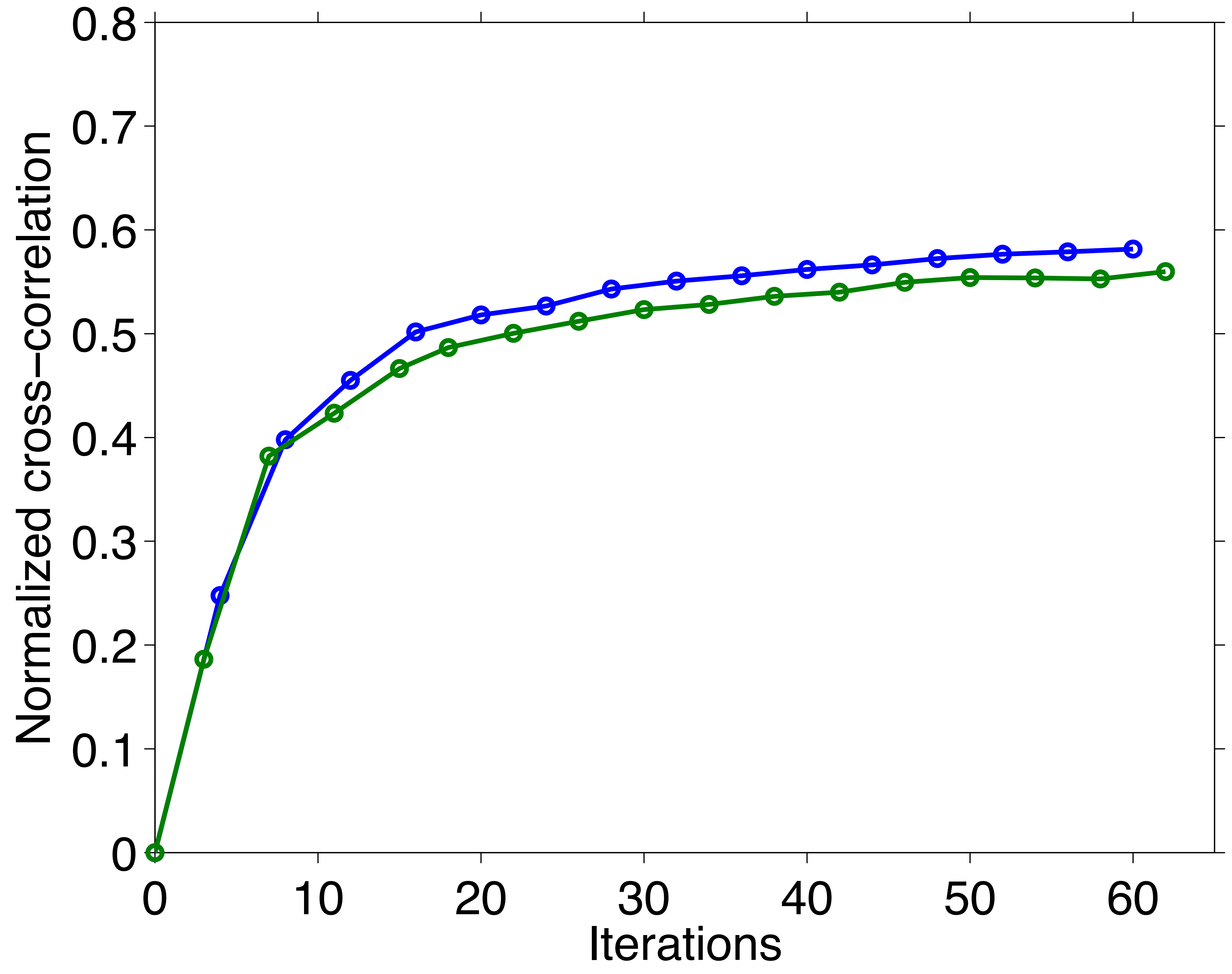


Fast LS image w/ source estimation

[initial guess has 0.25 s shift & flat spectra]



Convergence analysis



Normalized cross-correlation:

$$\mathrm{NCC}(\mathbf{v}_1, \mathbf{v}_2) = \frac{< \mathbf{v}_1, \mathbf{v}_2 >}{\|\mathbf{v}_1\|_2 \|\mathbf{v}_2\|_2}$$

blue: ***true*** source wavelet

green: source ***estimation***

Challenge

Non-deterministic scaling **ambiguity**:

$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i \in \Omega} \sum_{j \in \Sigma} \| \underline{\mathbf{d}}_{i,j} - \nabla \mathbf{F}[\mathbf{m}_0, w_i \underline{\mathbf{s}}_j] \mathbf{C}^* \mathbf{x} \|_2^2$$

$$= f(\alpha \mathbf{x}, \frac{1}{\alpha} \boldsymbol{w})$$

Utilizing surface-related multiples

Experiments setup

- model cropped from the sedimentary part of the Sigsbee 2B model, 3.8 km deep, 6 km wide, 7.62 m grid spacing
- 15 Hz Ricker wavelet, ~8 s recording time, 311 freq. samples
- 261 co-located sources/receivers, fixed spread, 22.86 m spacing, 7.62 m deep
- 10% frequencies & 10% sources for fast inversion
- run for ~50 iterations, simulation cost ~1 RTM with all sources and frequencies (or ~1.5 RTM with source estimation)

Illustration: primary wave propagation

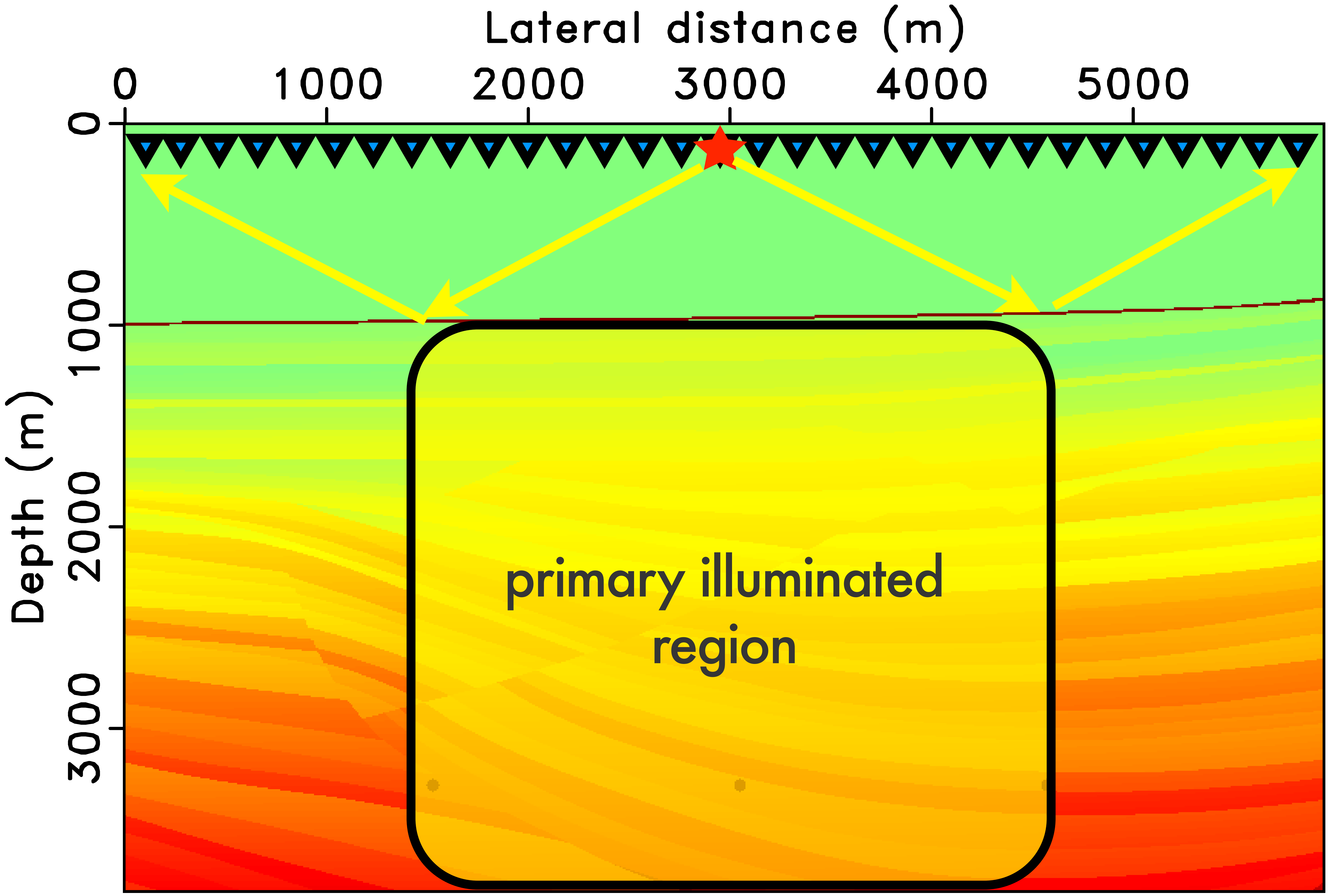
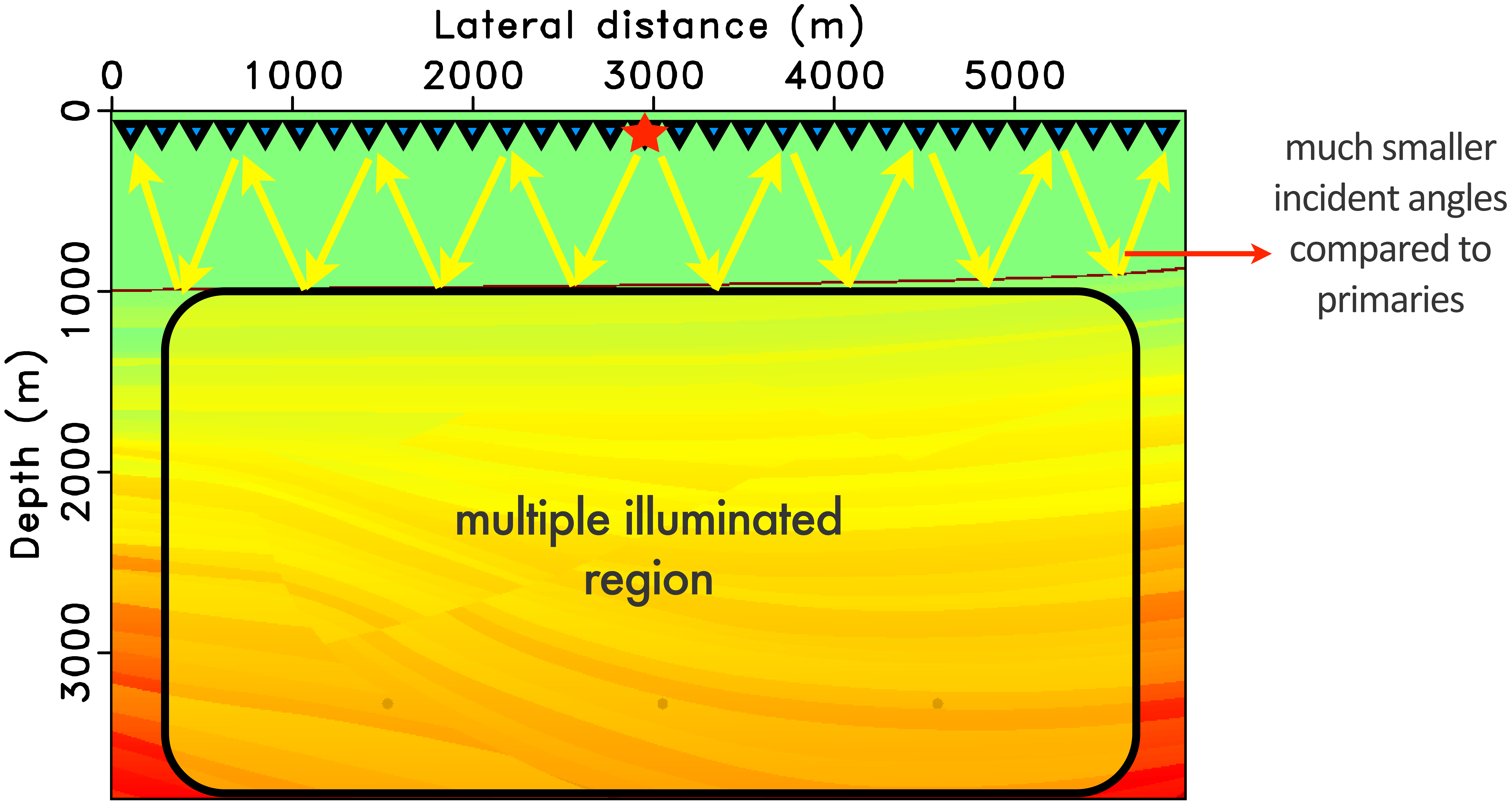
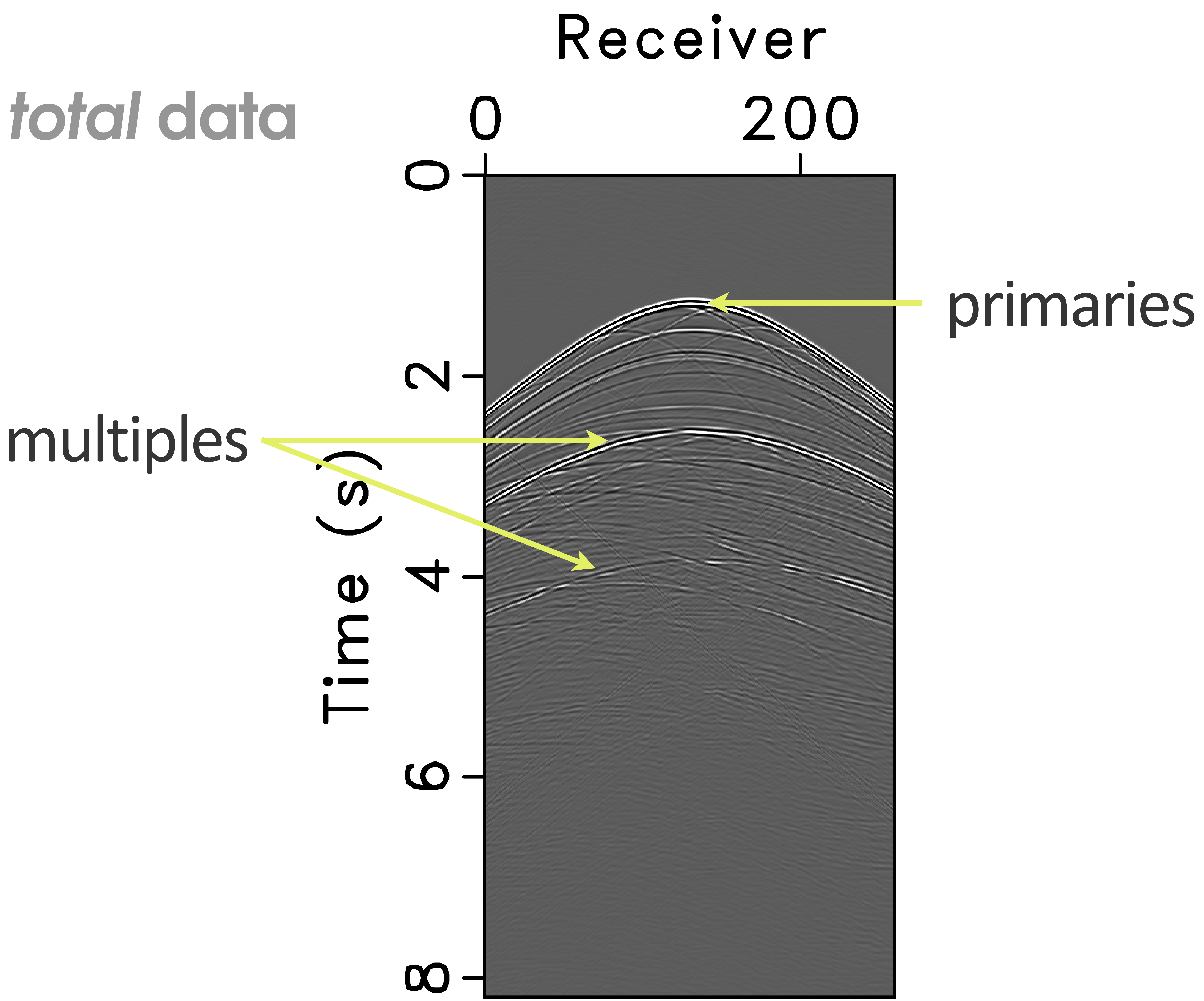


Illustration: surface-multiples propagation

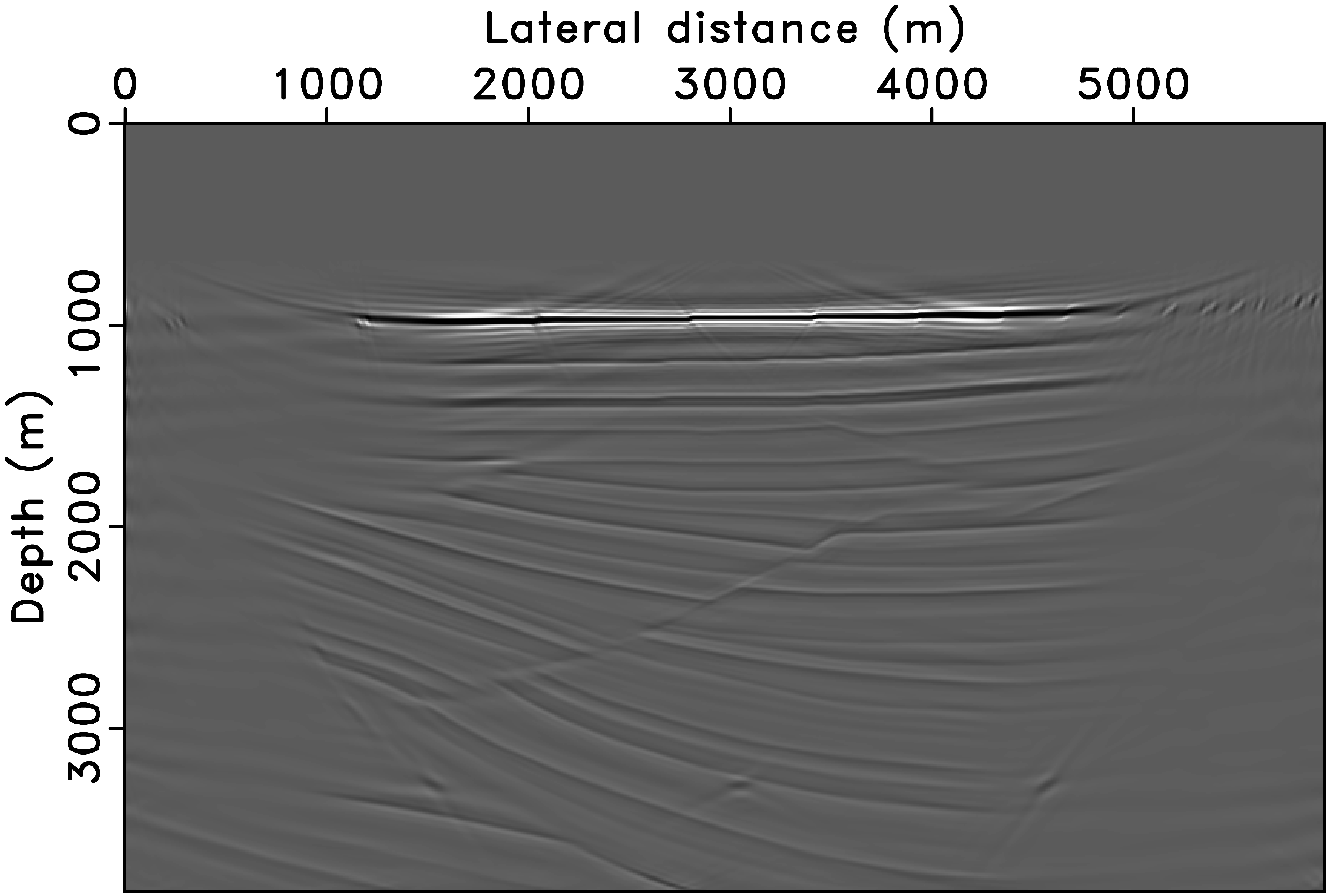
[Each receiver serves as a virtual secondary source!]



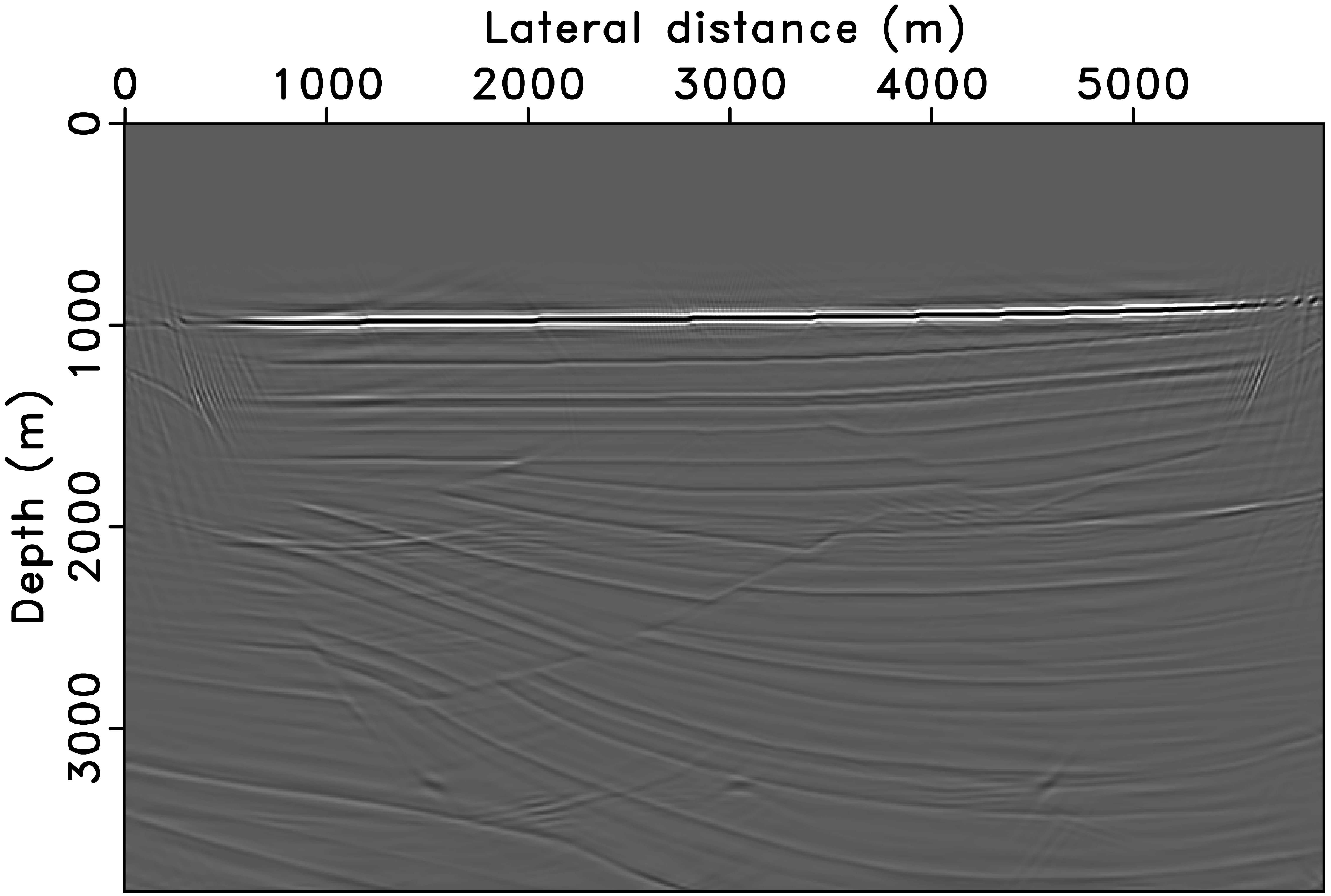
A shot-gather of *total* data



Inversion using primaries



Inversion using multiples



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Integrating SRME relation into linearized modelling

$$
\begin{aligned}
\mathbf{D}_i &\approx \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}, \mathbf{I}](\mathbf{S}_i - \mathbf{D}_i) \\
&= \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{P}_s^*\mathbf{I})(\mathbf{S}_i - \mathbf{D}_i) \\
&= \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{P}_s^*(\mathbf{S}_i - \mathbf{D}_i)) \\
&\doteq \nabla\mathcal{F}_i[\mathbf{m}_0, \mathbf{S}_i - \mathbf{D}_i].
\end{aligned}
$$

→ Dense matrix products

→ Wave-equation solves with total downgoing data

Tu et al., 2013; Tu and Herrmann, 2015b

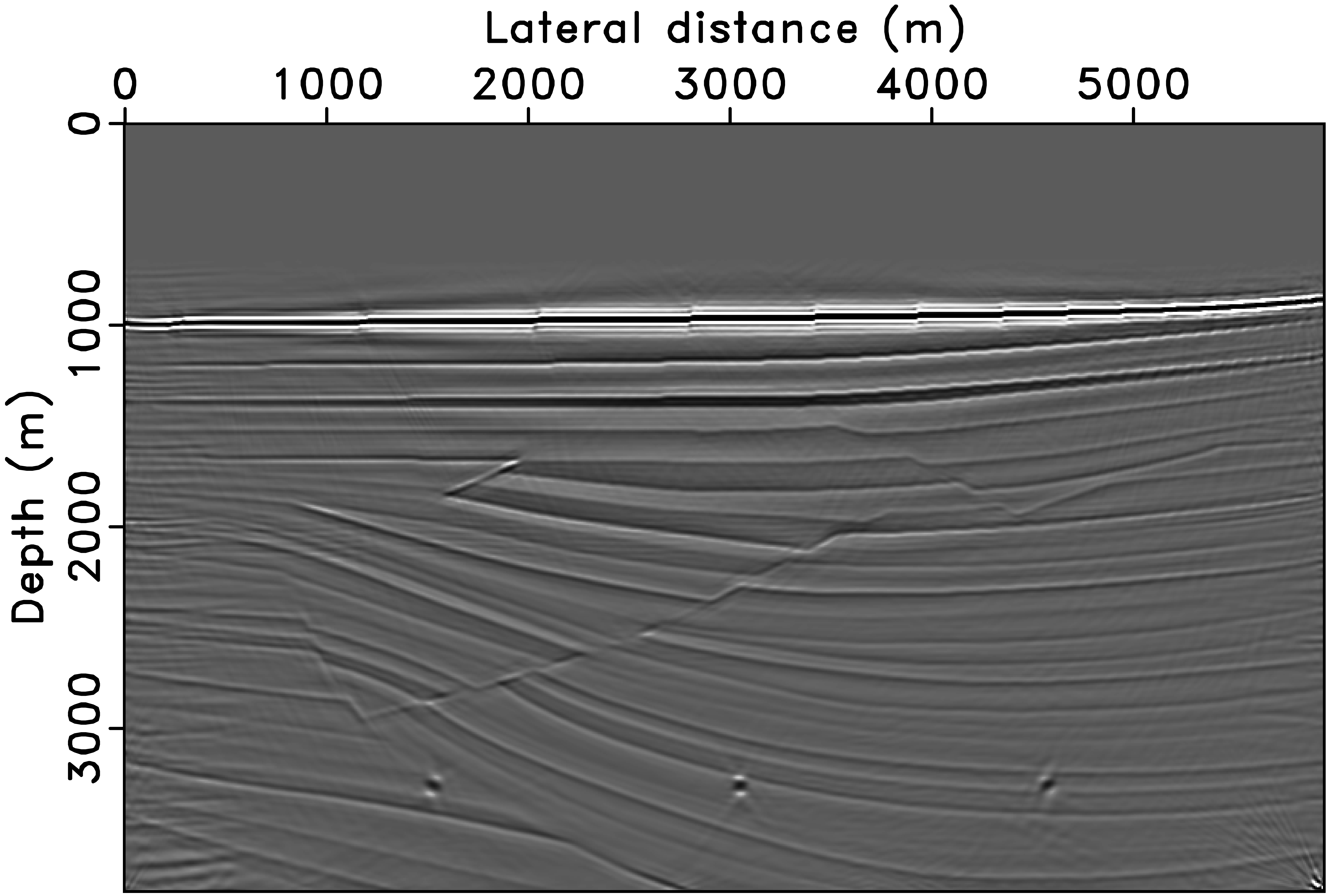
Incorporating surface-related multiples in the objective

$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j - \underline{\mathbf{d}}_{i,j}]\mathbf{C}^*\mathbf{x}\|_2^2$$

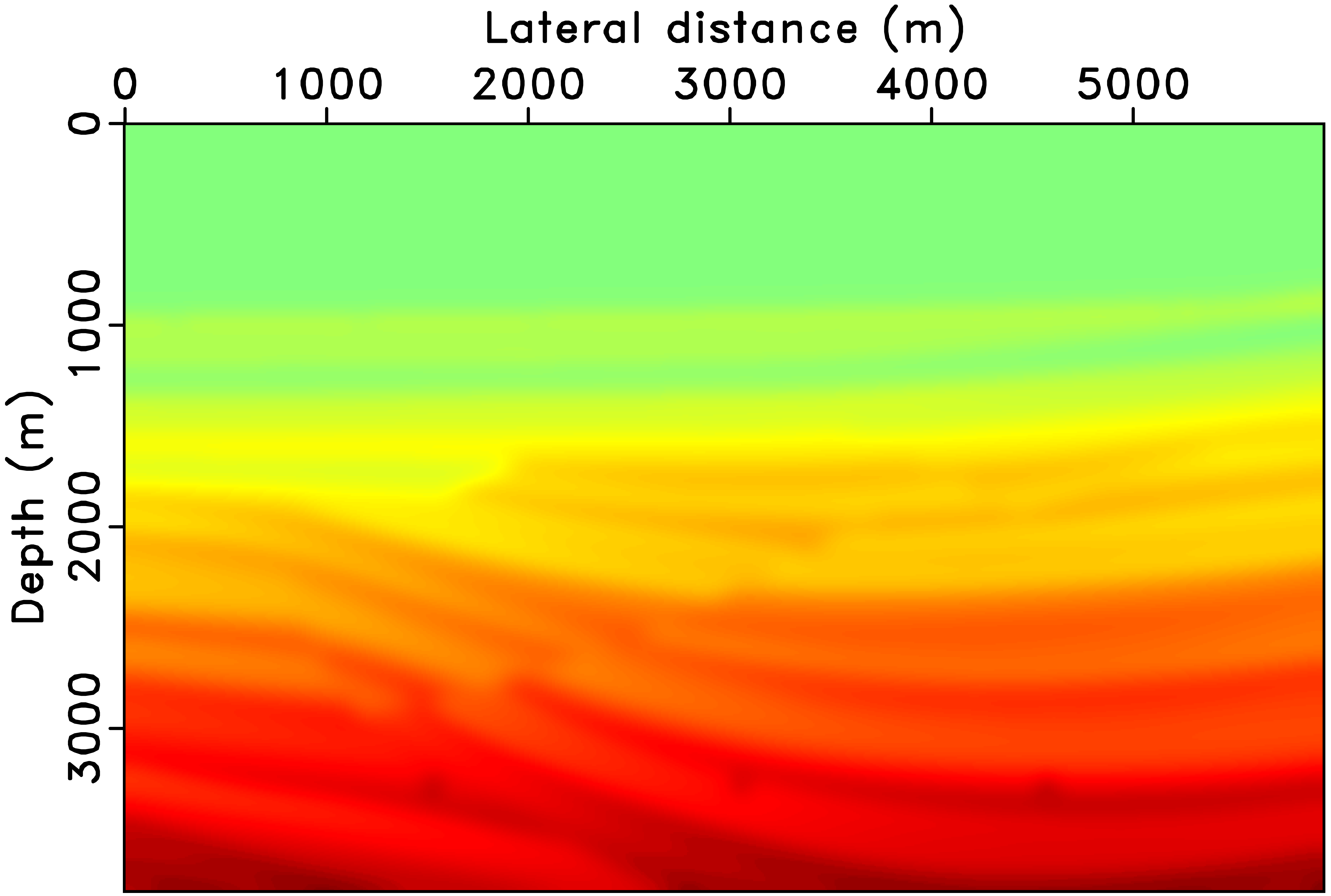
Definite source estimates:

$$\tilde{w}_i(\mathbf{x}) = \frac{\sum_{j\in\Sigma} < \underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, -\underline{\mathbf{d}}_{i,j}]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}{\sum_{j\in\Sigma} < \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}$$

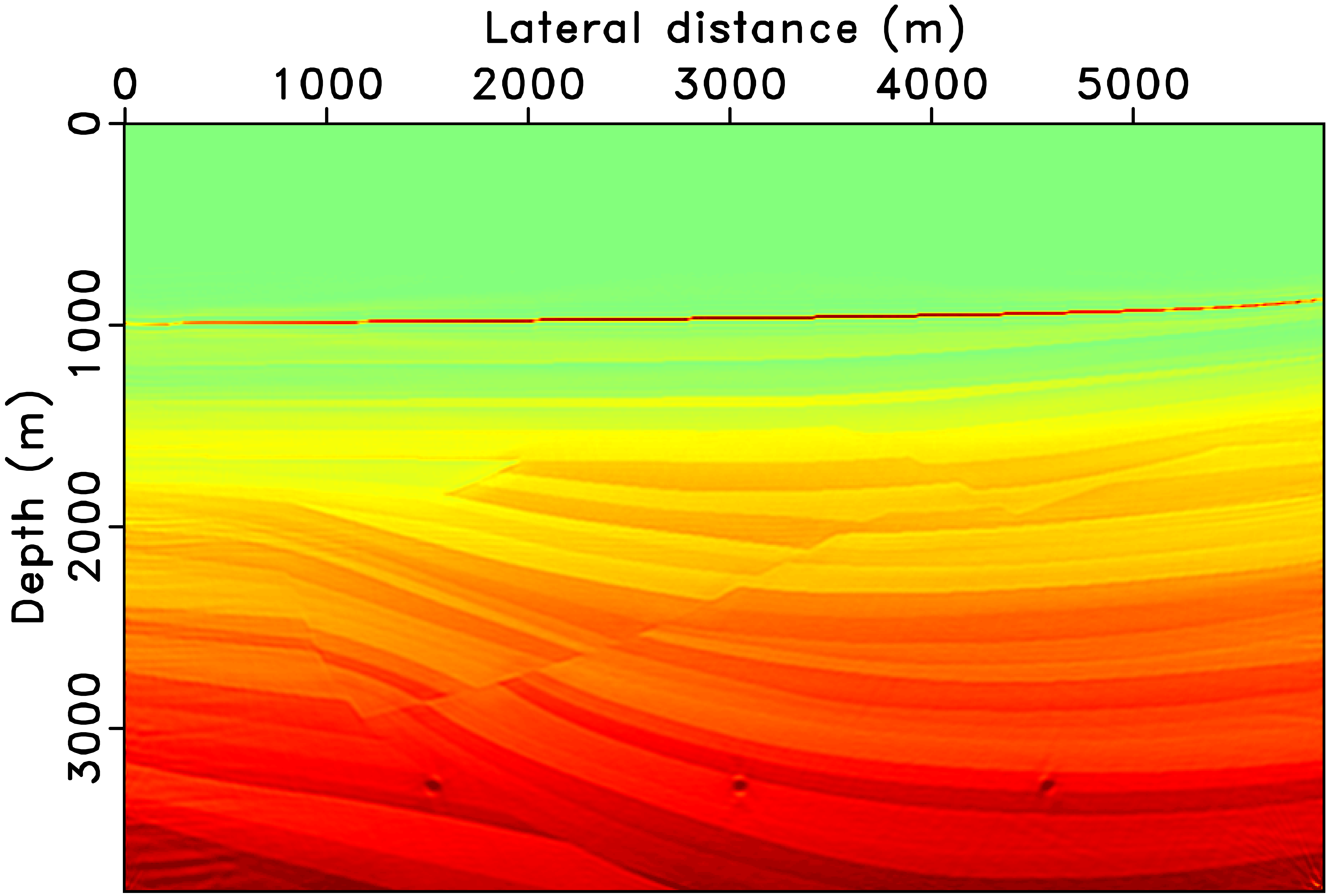
Inversion of ideal data with source estimation



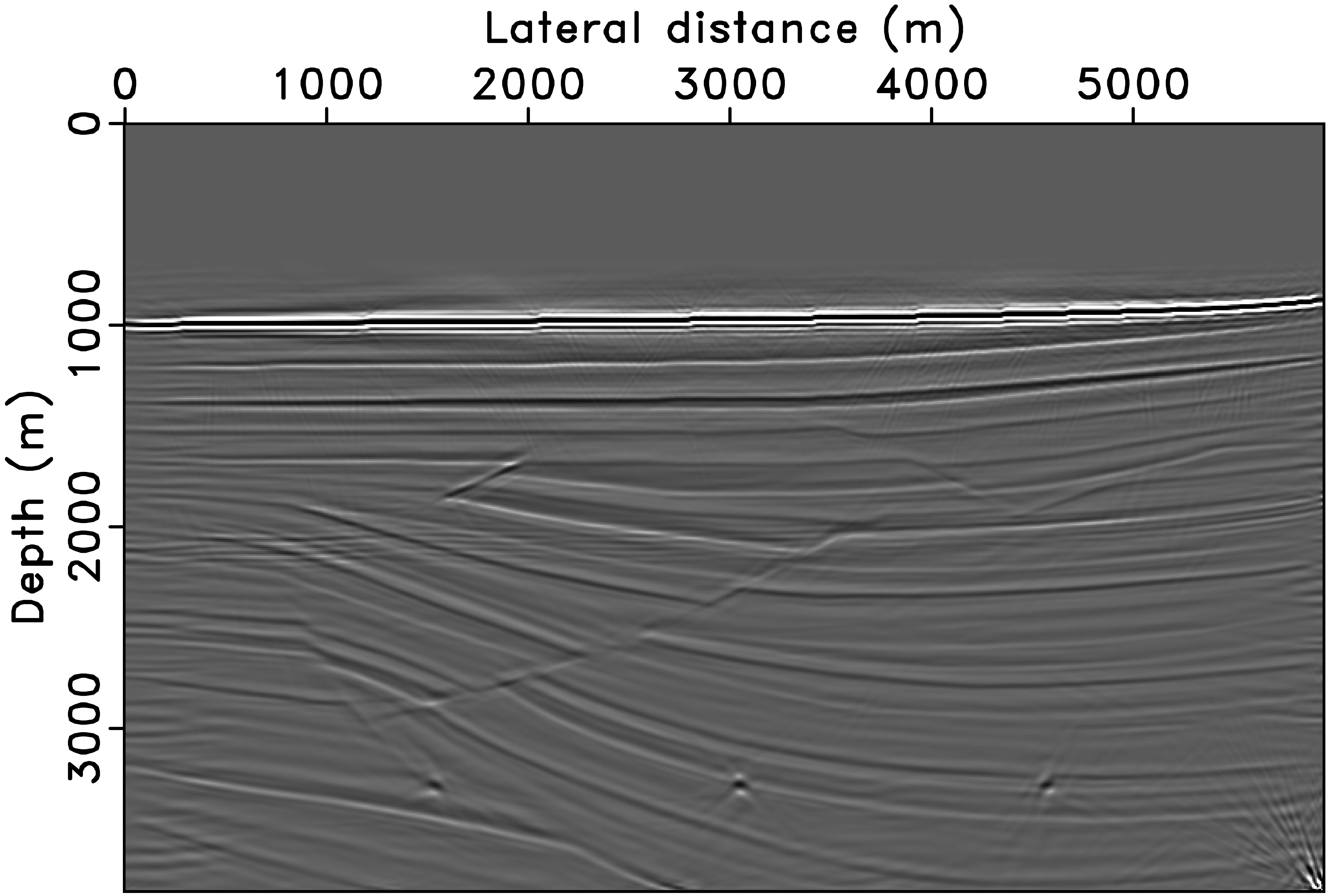
Background model



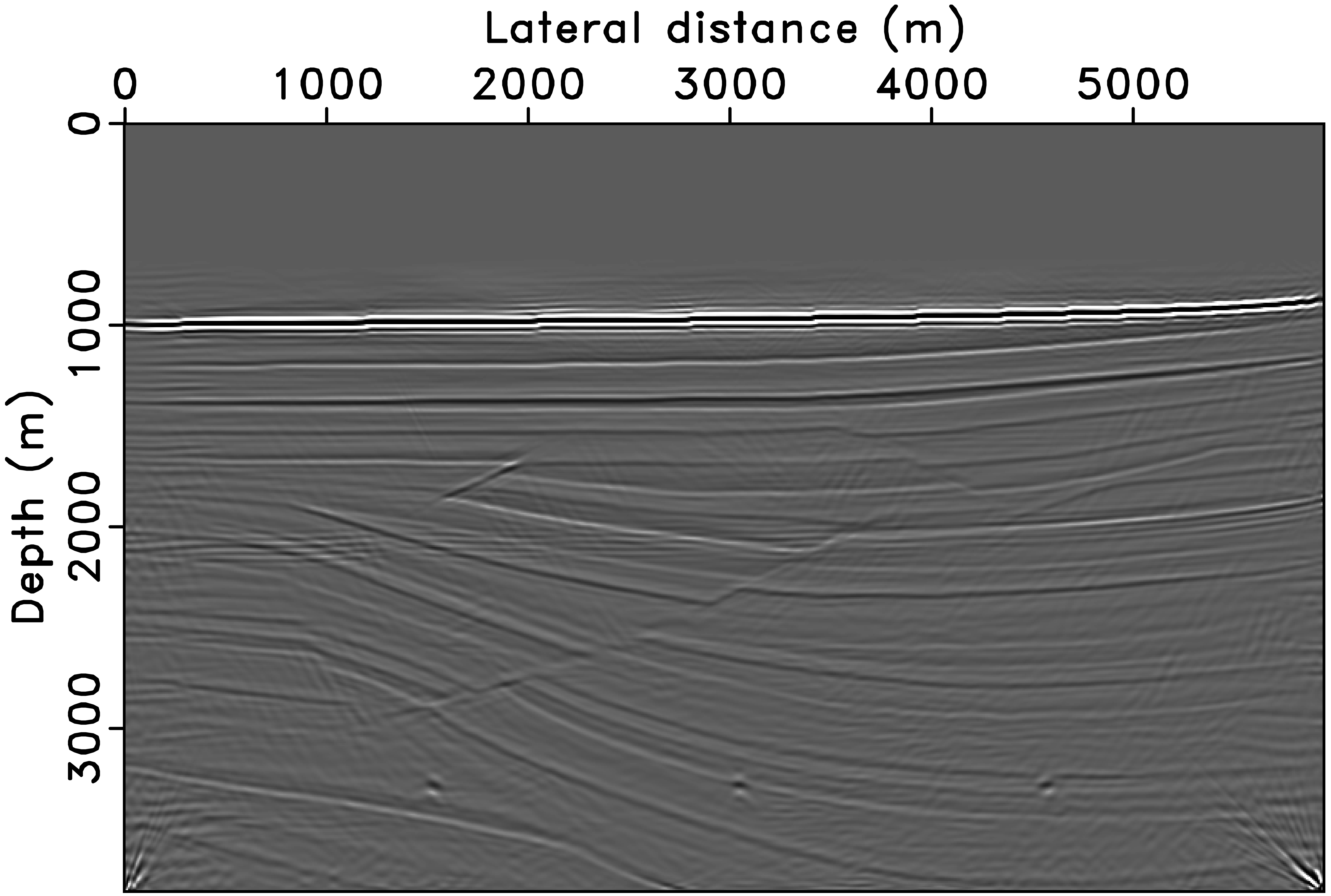
Adding image back to the background model



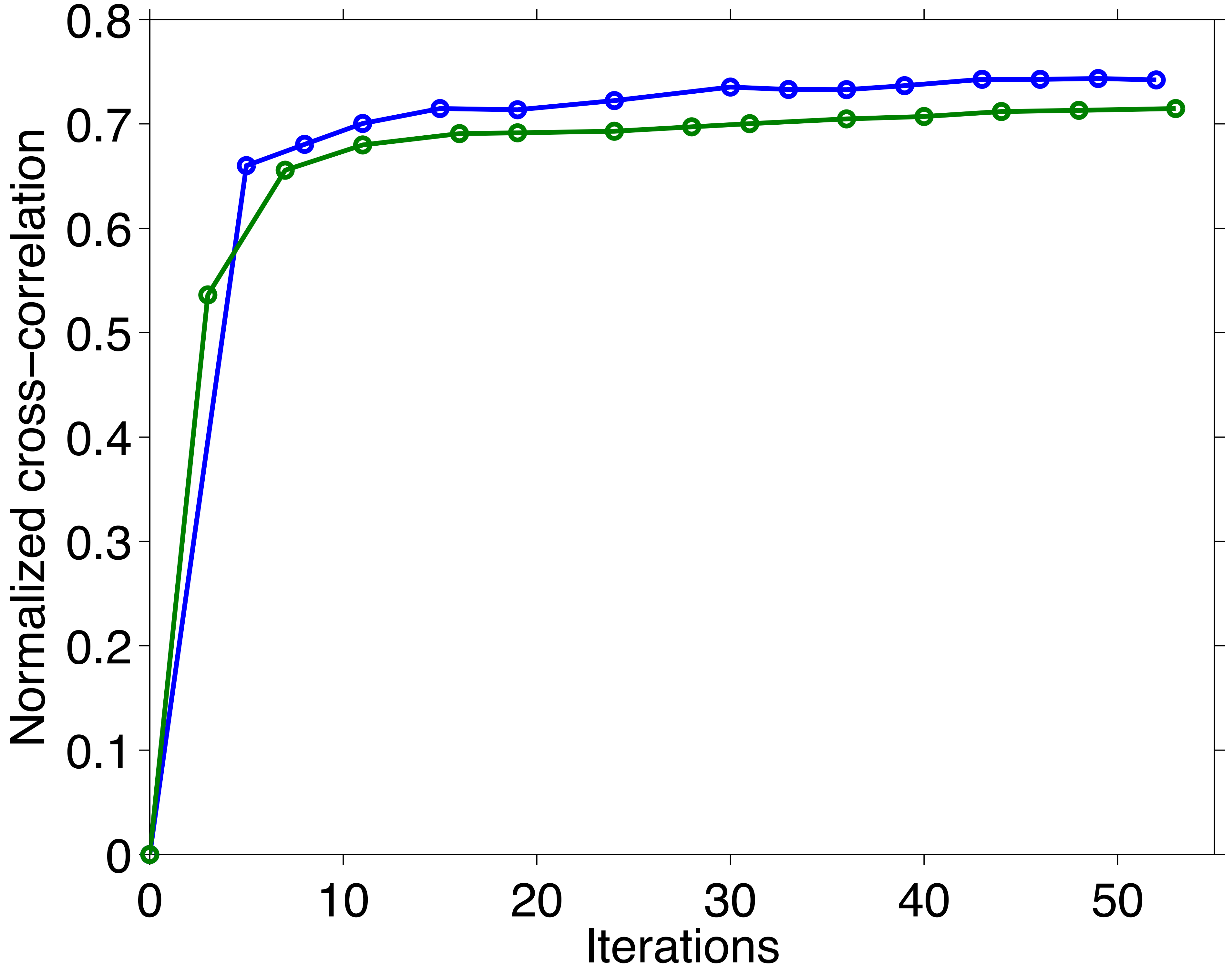
Inversion of iWave data with true source wavelet



Inversion of iWave data with source estimation



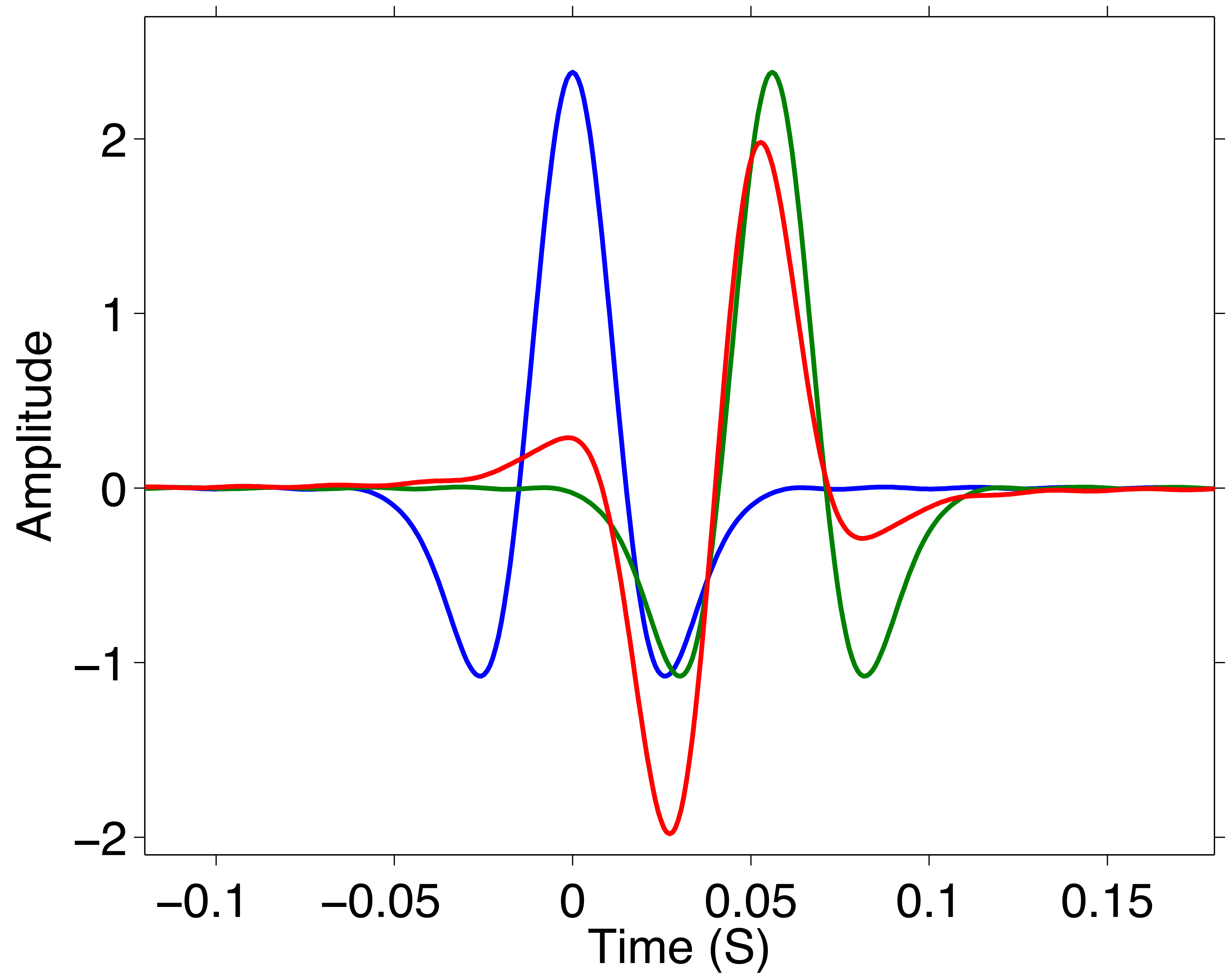
Convergence analysis

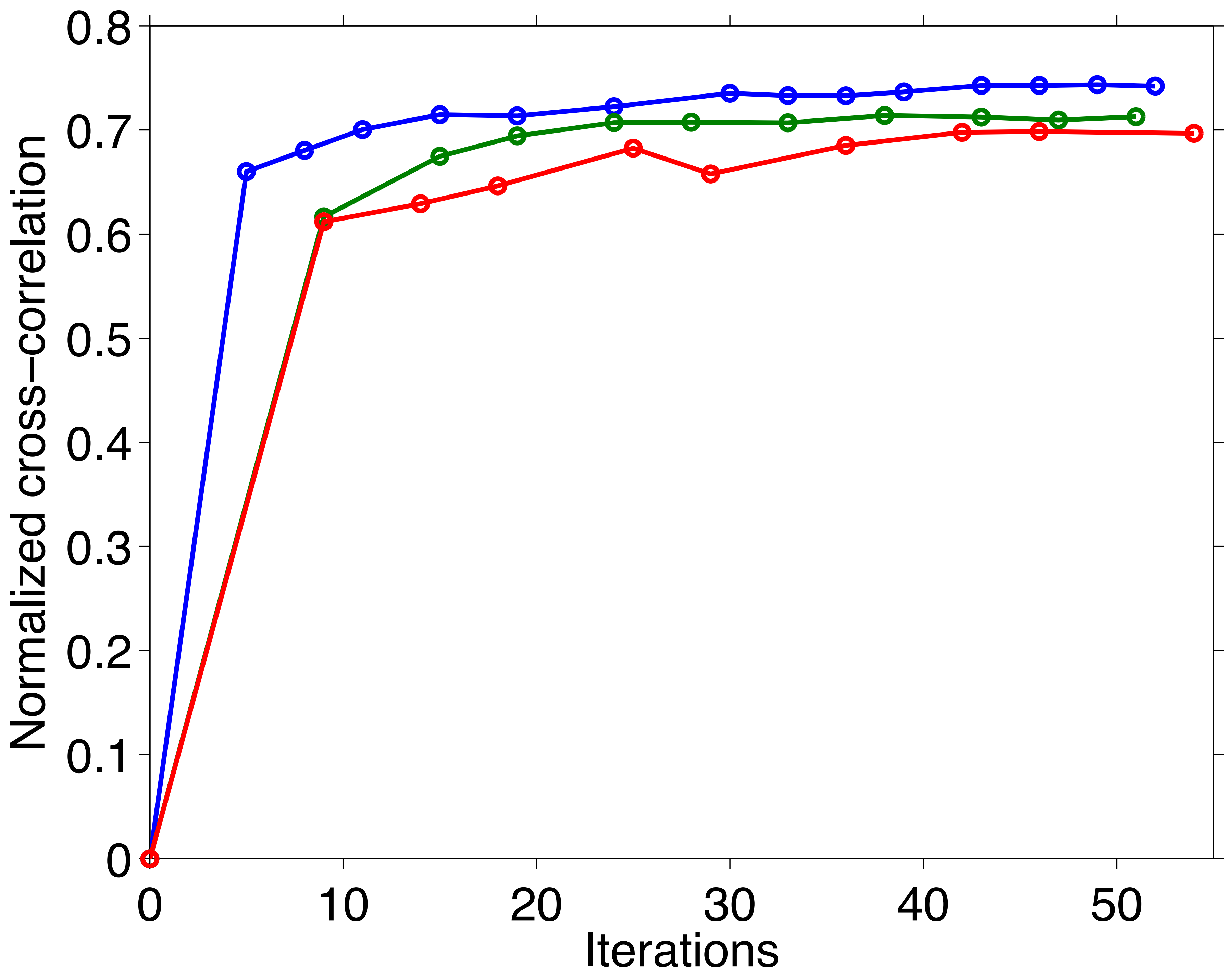


blue: ***true*** source wavelet

green: source ***estimation***

Robustness to initial wavelet guesses





Conclusions

Least-squares migration can be carried out

- **efficiently**
- without the knowledge of the source wavelet by **estimating** sources on-the-fly using variable projection.
- with contributions from surface-related multiples
  - increased illumination
  - resolved scaling ambiguity

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https://www.slim.eos.ubc.ca/

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