Curvelet-based amplitude recovery & coherent-noise removal

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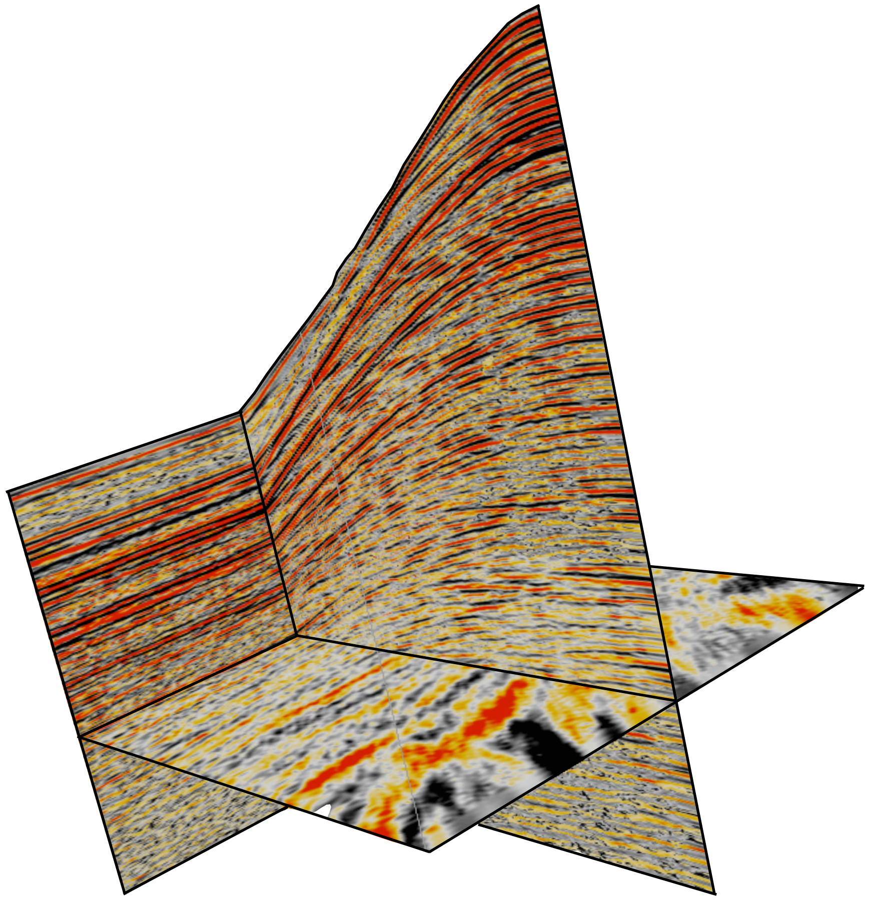

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Outline

II. Curvelet-based amplitude recovery & coherent-noise removal

- curvelet-domain matched filtering
- **applications:**
  - amplitude-recovery by scaled curvelet-domain sparsity promotion
  - conditioning with curvelet-domain scaling
  - primary-multiple matching
- Bayesian coherent signal separation by sparsity promotion
- **application:** primary-multiple separation

Goals & challenges

Goals:

Exploit ***multiscale*** & ***directional*** *transforms* to conduct seismic data processing---i.e, use "***microlocal"*** properties of curvelets to

- correct ***migration*** amplitudes & precondition ***migration*** operators
- smoothly-varying corrections to ***coherent*** noise predictions (e.g., surface-related multiples)

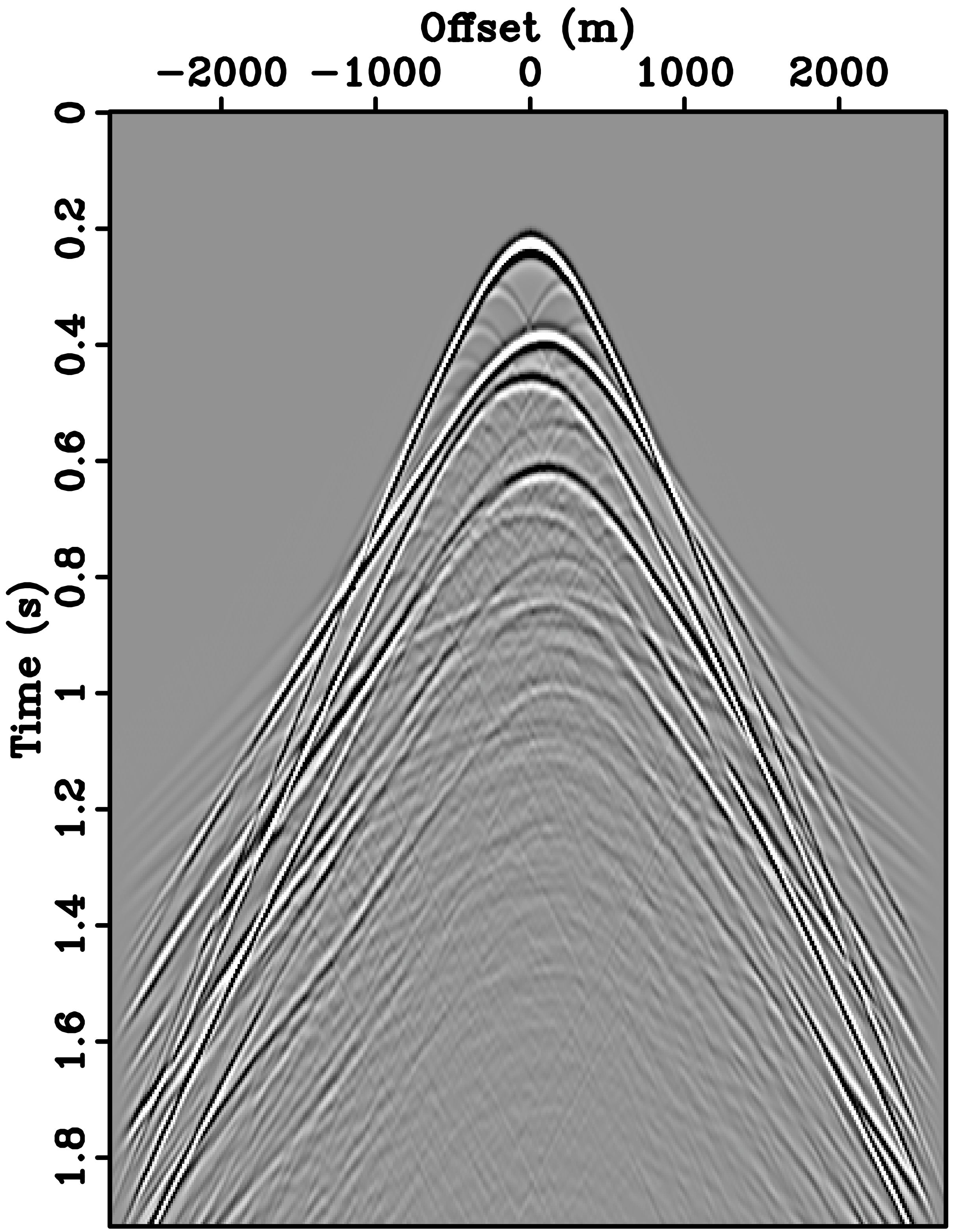
Leverage *curvelet-domain* ***adaptivity & sparsity*** to

- estimate diagonal curvelet-domain corrections by matching
  - migrated and remigrated images => approximation of the normal operator
  - "true" and predicted multiples => matching of predicted multiples to "true multiples"
- ***stably*** correct ***migration*** amplitude errors
- separate ***matched*** *coherent wavefield* constituents by *sparsity promotion*

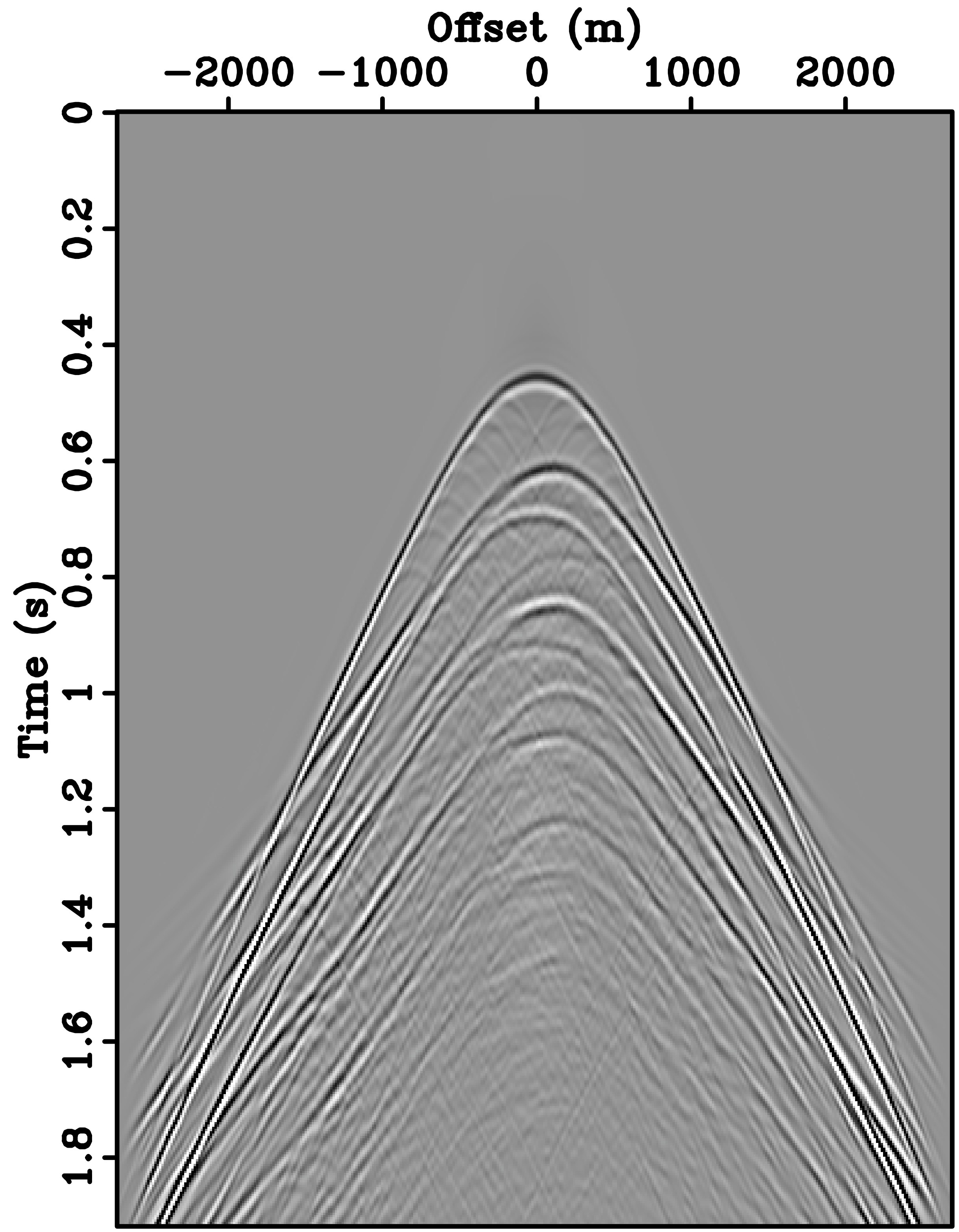
Challenges:

- *multidimensionality* of *wavefronts* set & existence of *conflicting* dips
- problem size & integration into existing workflows
  - black-box imaging & multiple prediction code

Example

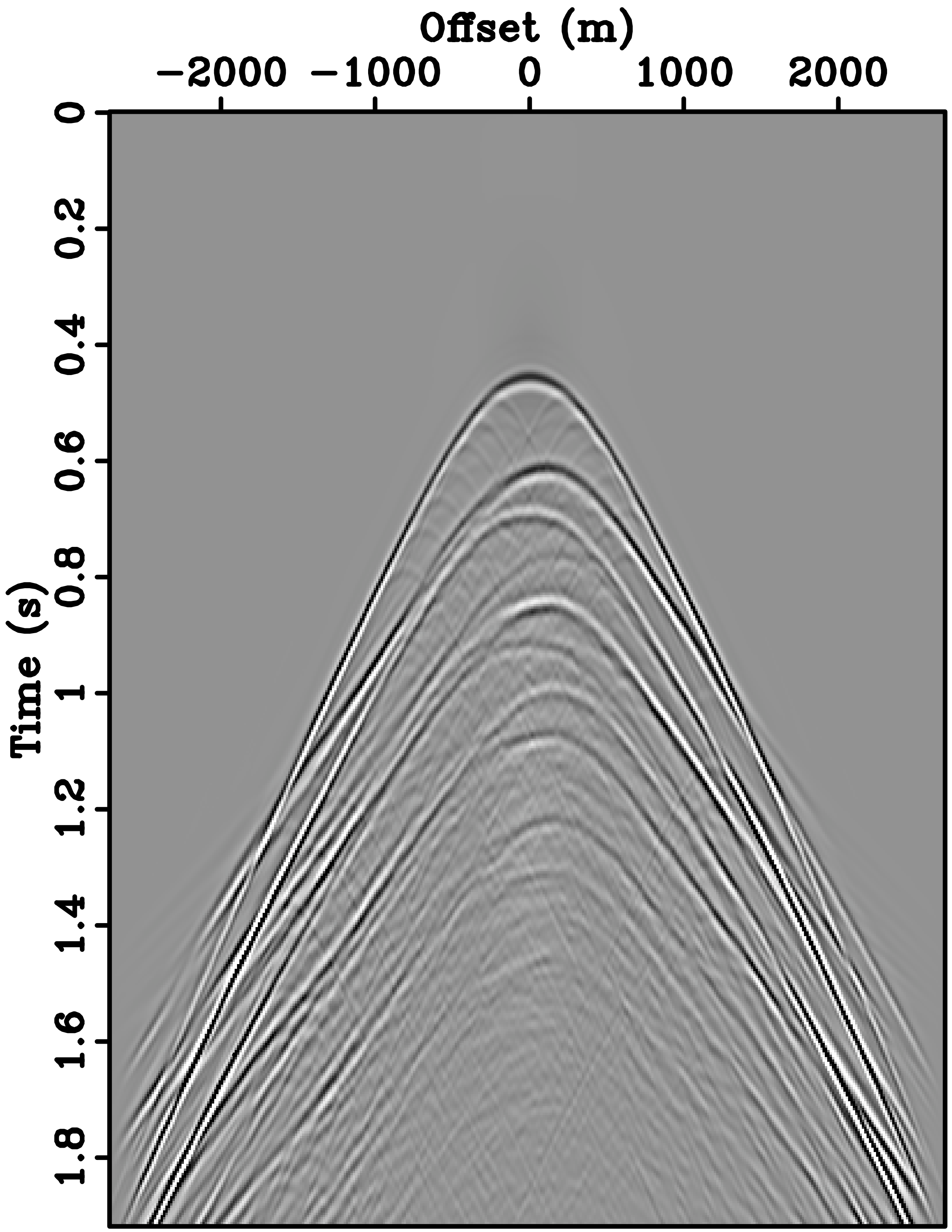


Total data

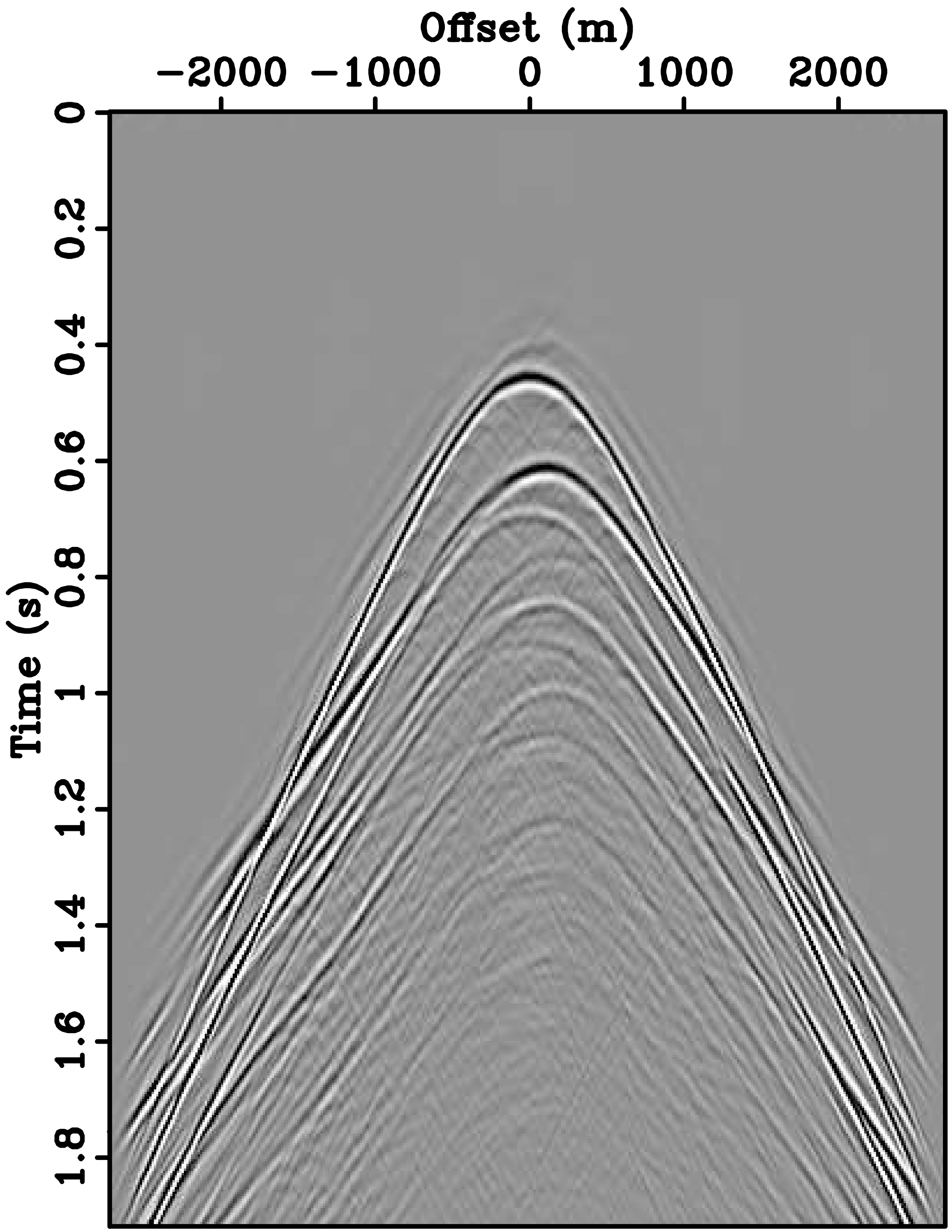


Single-term SRME multiples

Example



Single-term SRME multiples



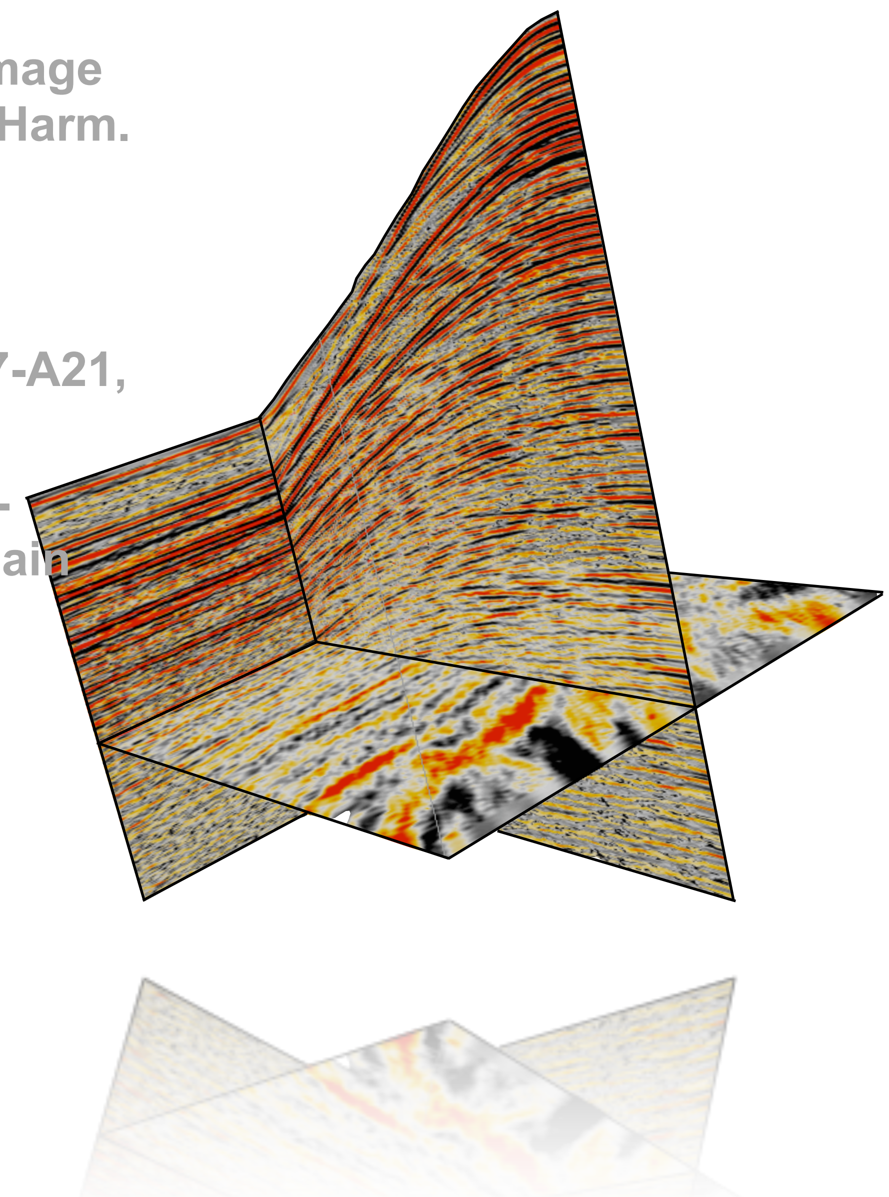
matched multiples

Curvelet-domain matched filter

Herrmann, F. J., Moghaddam, P. and Stolk, C. Sparsity- and continuity-promoting seismic image recovery with curvelet frames. App. & Comp. Harm. Analys., Vol. 24, No. 2, pp. 150-173, 2008.

Herrmann, F. J., Wang, D and Verschuur, D. J. Adaptive curvelet-domain primary-multiple separation. Geophysics, Vol 73, No. 3, pp. A17-A21, 2008.

Reza Shahidi and Felix J. Herrmann, Curvelet-domain matched filtering with frequency-domain regularization. SEG, 2009, Houston.



The forward model

Our curvelet-domain matched filtering is build on the following model:

$$g = \Psi(x, D) f$$

with Ψ a zero-order pseudodifferential operator (ΨDO) given by

$$(\Psi f)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} b(x,\xi) \hat{f}(\xi) \mathrm{d}\xi,$$

i.e., $|\partial_\xi^\alpha \partial_x^\beta b(x,\xi)| \leq C_{\alpha,\beta} \big(1 + |\xi|\big)^{m-|\alpha|}$ with $m = 0$.

We assume

- corrections by the symbol *b* vary *smoothly* as a function of *space* and *angle*

Approximate the action of Ψ by curvelet-domain scaling

- fast evaluation
- possibility to estimate diagonal approximation though *matching* during which the diagonal is computed by solving a nonlinear least-squares estimation problem

Diagonal approximation [F.J.H et. al ‘08]

Lemma 1. Suppose a is in the symbol class $S^0_{1,0}$, then, with C' some constant, the following holds

$$\|(\Psi(x,D) - a(x_\nu,\xi_\nu))\varphi_\nu\|_{L^2(\mathbb{R}^n)} \le C' 2^{-|\nu|/2}.$$

To approximate Ψ, we define the sequence $\mathbf{u} := (u_\mu)_{\mu\in\mathcal{M}} = a(x_\mu,\xi_\mu)$. Let $\mathbf{D}_\Psi$ be the diagonal matrix with entries given by $\mathbf{u}$. Next we state our result on the approximation of Ψ by $C^T\mathbf{D}_\Psi C$.

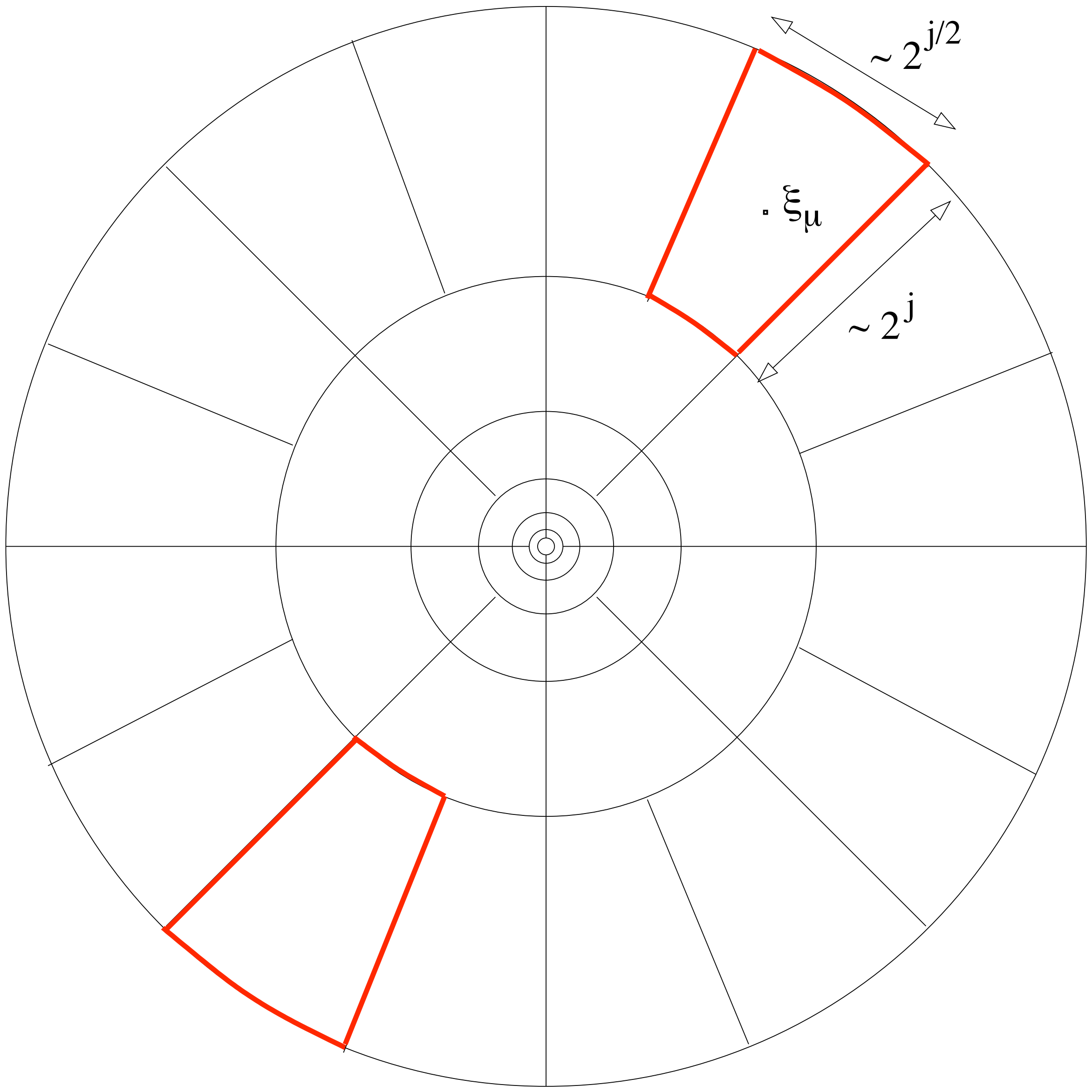
Theorem 1. The following estimate for the error holds

$$\|(\Psi(x,D) - C^T\mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \le C'' 2^{-|\mu|/2},$$

where C'' is a constant depending on Ψ.

Curvelet's parametrization

Tiling the ξ space



Estimation matched filter

After discretization action of Ψ can be approximated by

$$\mathbf{f} \approx \mathbf{B}\mathbf{g} \text{ with } \mathbf{B} \approx \mathbf{C}^H \mathrm{diag}\left(\mathbf{b}\right)\mathbf{C}, \quad \{b\}_{\mu\in\mathcal{M}} \geq 0$$

Given **f** and **g**, the *diagonal* **b** can be estimated with a *global nonlinear* least-squares estimation procedure [Symes ’08, F.J.H et. al. ‘08]

$$\tilde{\mathbf{z}} = \arg\min_{\mathbf{z}} \frac{1}{2}\|\mathbf{f} - \mathbf{C}^T \mathrm{diag}\left(\mathbf{C}\mathbf{g}\right) e^{\mathbf{z}}\|_2^2 + \gamma \|\mathbf{L}_{\mathcal{C}} e^{\mathbf{z}}\|_2^2$$

- $\mathbf{L}_{\mathcal{C}}$ *curvelet-domain* sharpening operator that promotes ***phase-space*** smoothness
- guarantees the solution to be *positive*
- handles *conflicting* dips by using *non-separable* curvelets

$$\tilde{\mathbf{f}}_{\mathrm{matched}} = \mathbf{B}\mathbf{g} \text{ with } \mathbf{B} = \mathbf{C}^* \mathrm{diag}\left(e^{\tilde{\mathbf{z}}}\right)\mathbf{C}$$

Estimation matched filter

Solve the system

$$\begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \approx \begin{bmatrix} \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{g}\} \\ \gamma \mathbf{L}_{\mathcal{C}} \end{bmatrix} e^{\mathbf{z}}$$

or

$$\mathbf{y} \approx \boldsymbol{\mathcal{F}}_{\gamma}[\mathbf{z}]$$

Minimize with limited-memory BFGS [Nocedal '89]

$$J(\mathbf{z}) = \frac{1}{2} \|\mathbf{y} - \boldsymbol{\mathcal{F}}_{\gamma}[\mathbf{z}]\|_2^2$$

with the gradient [Vogel '02]

$$\mathrm{grad}\, J(\mathbf{z}) = \mathrm{diag}\{e^{\mathbf{z}}\} \left[\mathbf{F}_{\gamma}^T \left(\mathbf{F}_{\gamma} e^{\mathbf{z}} - \mathbf{y}\right)\right]$$

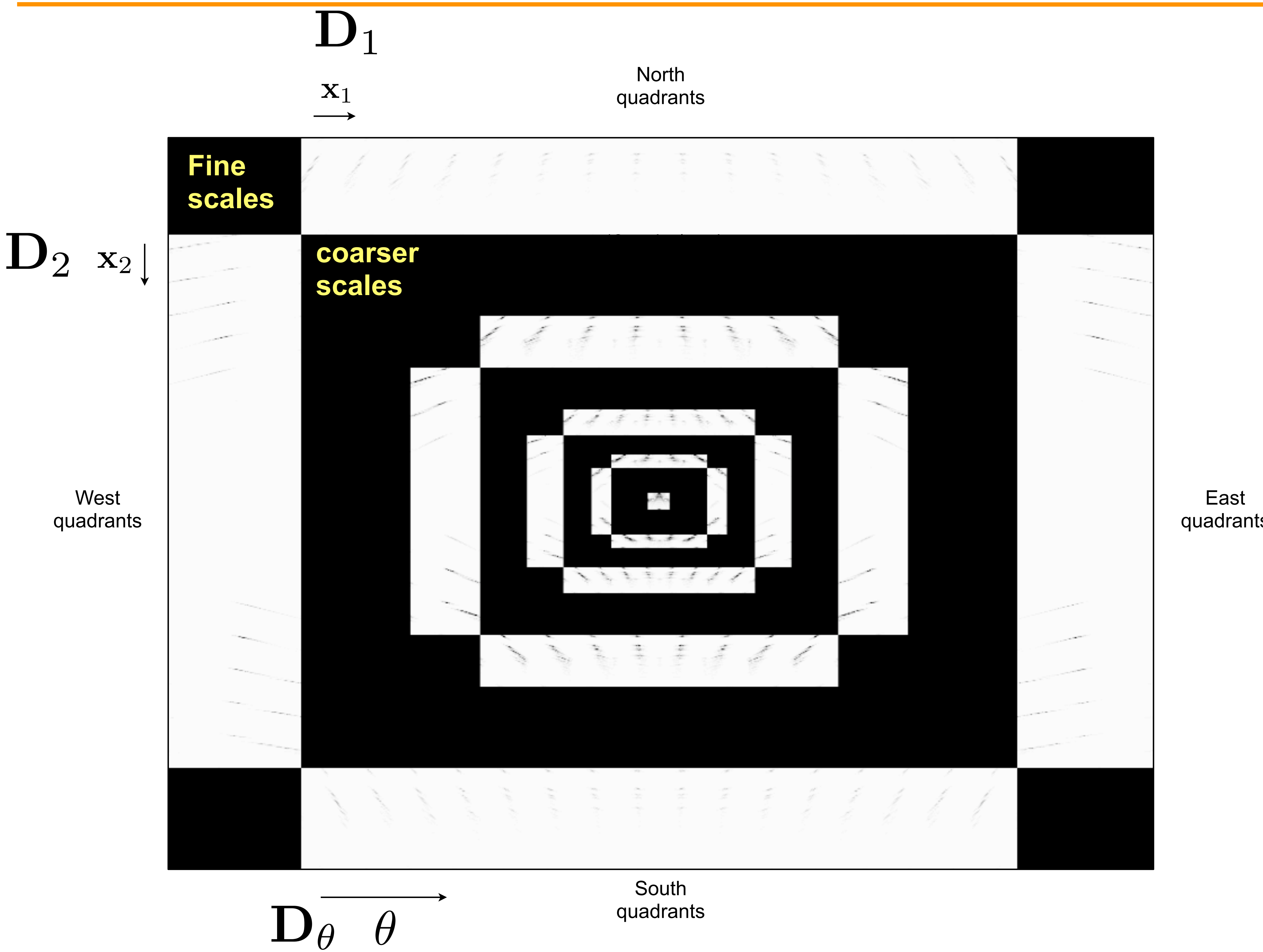
Phase-space regularization

Curvelet-domain *sharpening* operator

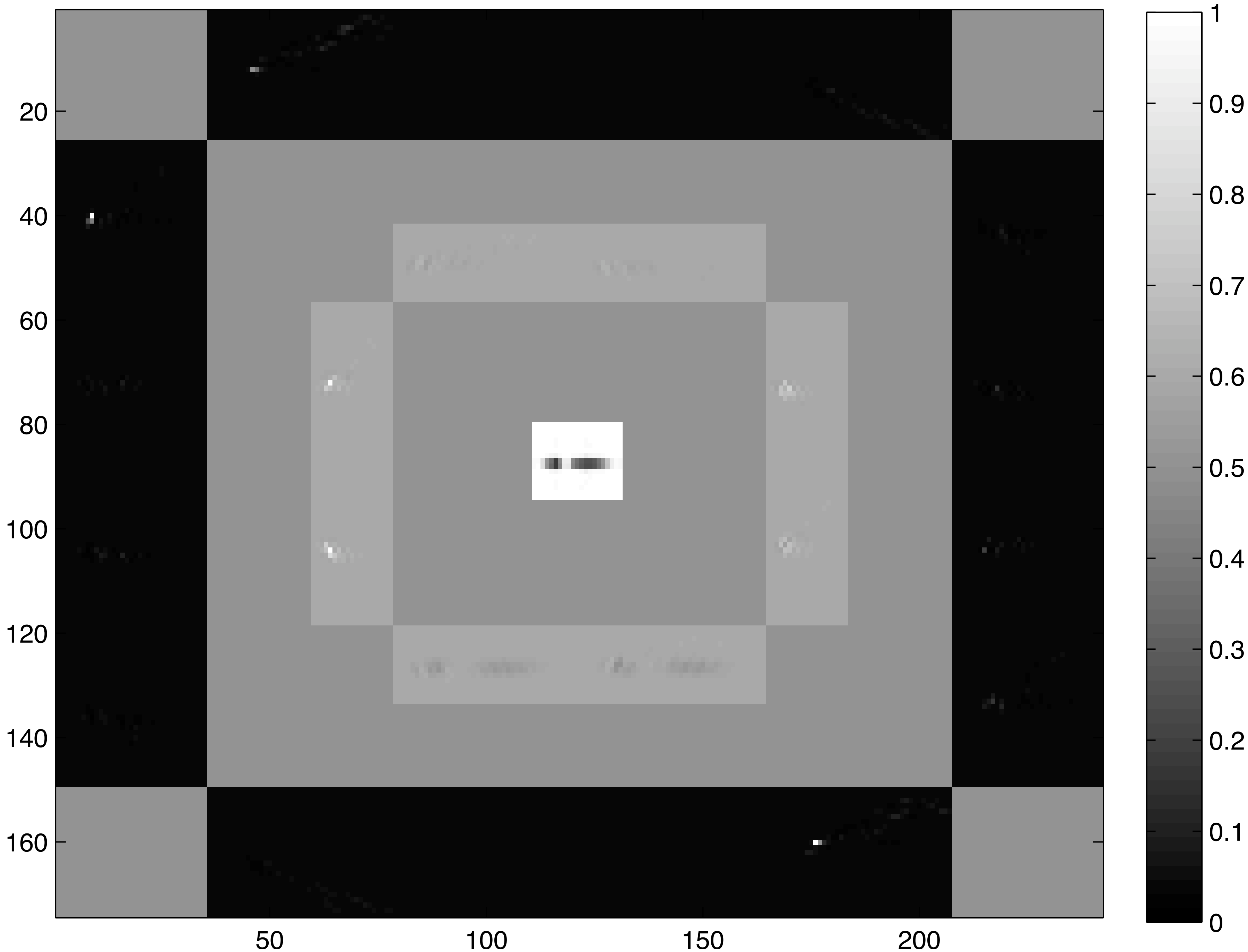
$$\mathbf{L}_{\mathcal{C}} = \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \mathbf{D}_\theta^T \end{bmatrix}^T$$

- *First-order differences* in *space* and *angle* directions for each scale
- *Regularization* parameter controls phase-space *smoothness*
- Limits overfitting
- Assures positivity with nonlinear least-squares ...

Phase-space regularization

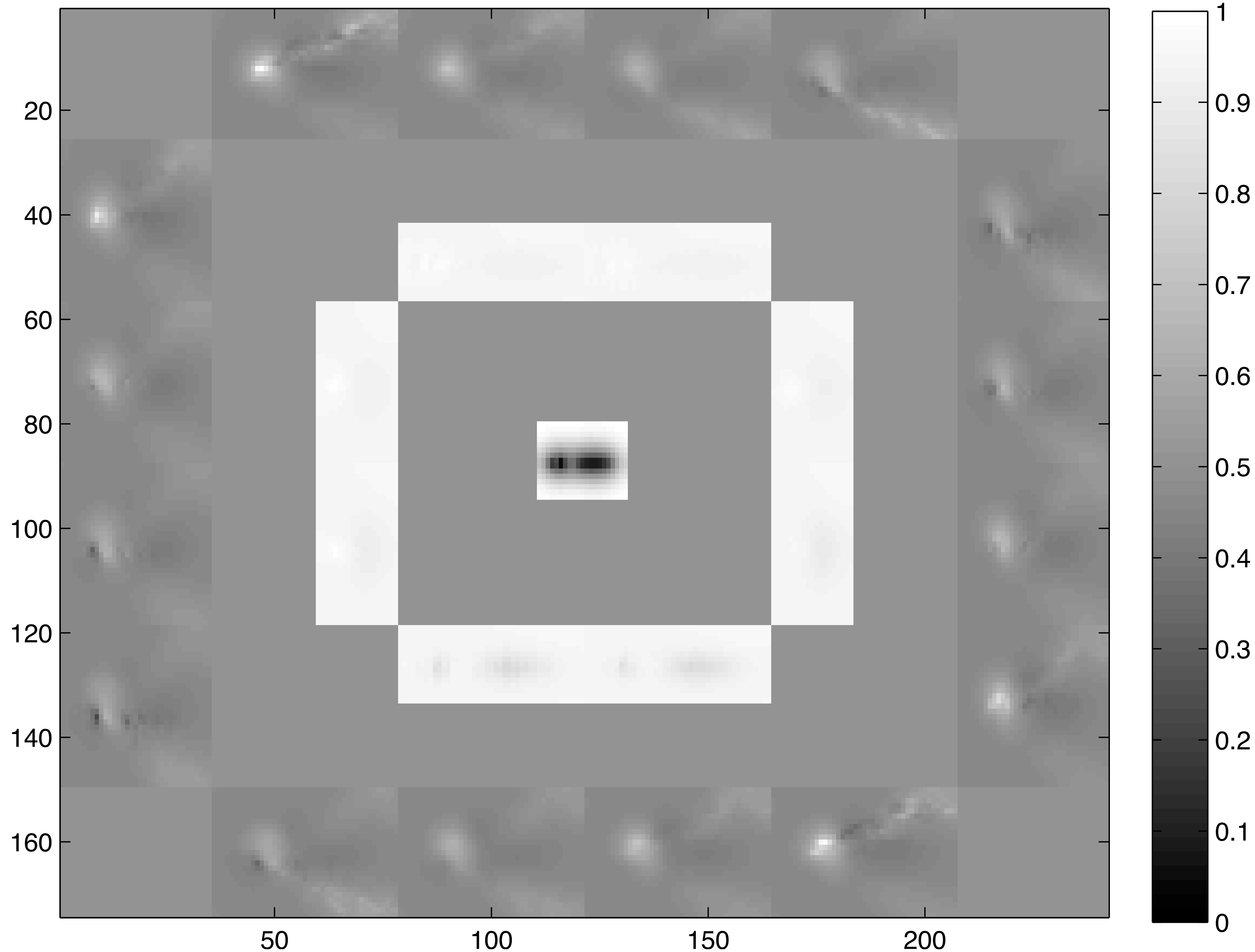


Phase-space regularization



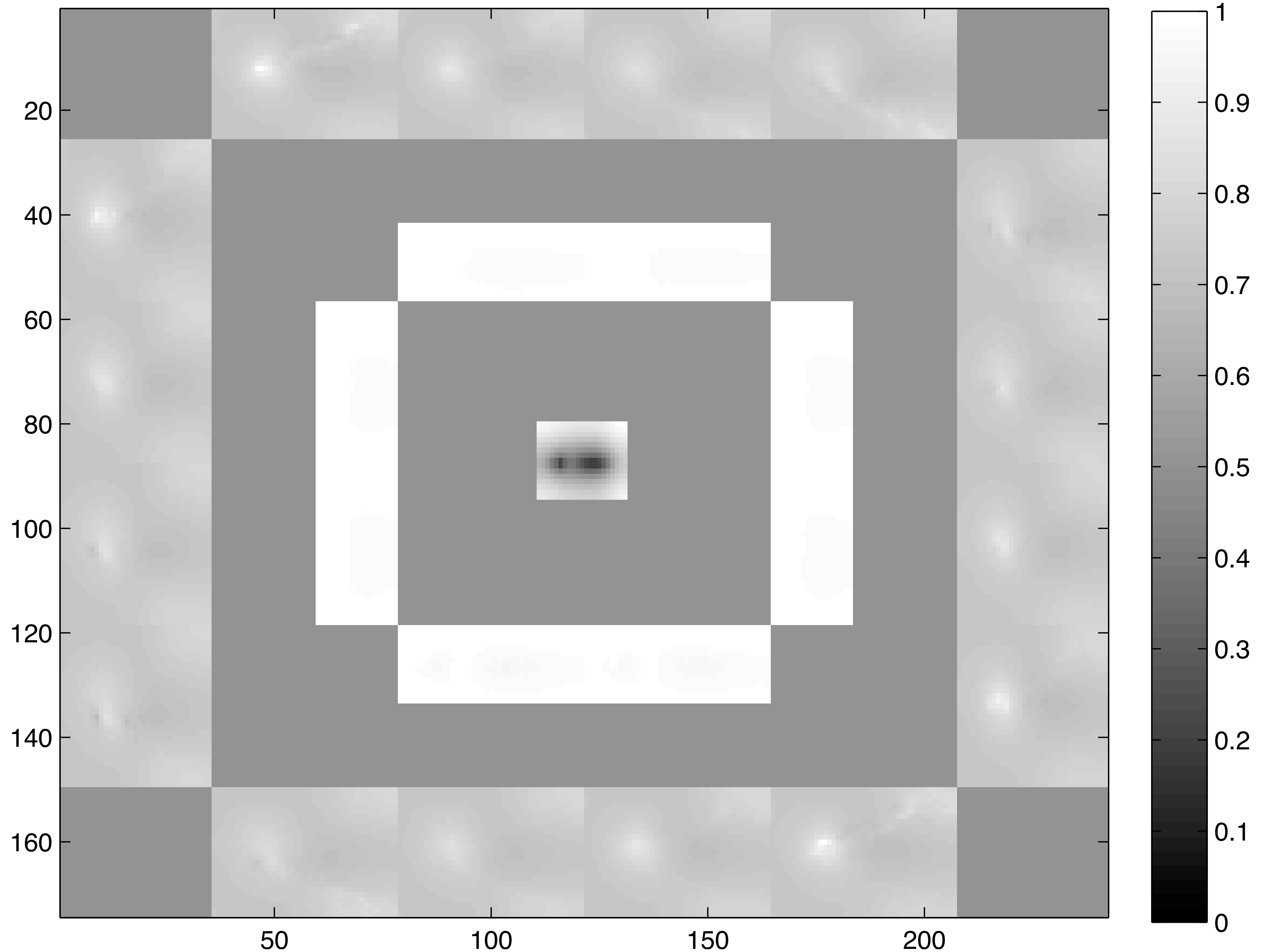
$\gamma = 0.0$

Phase-space regularization



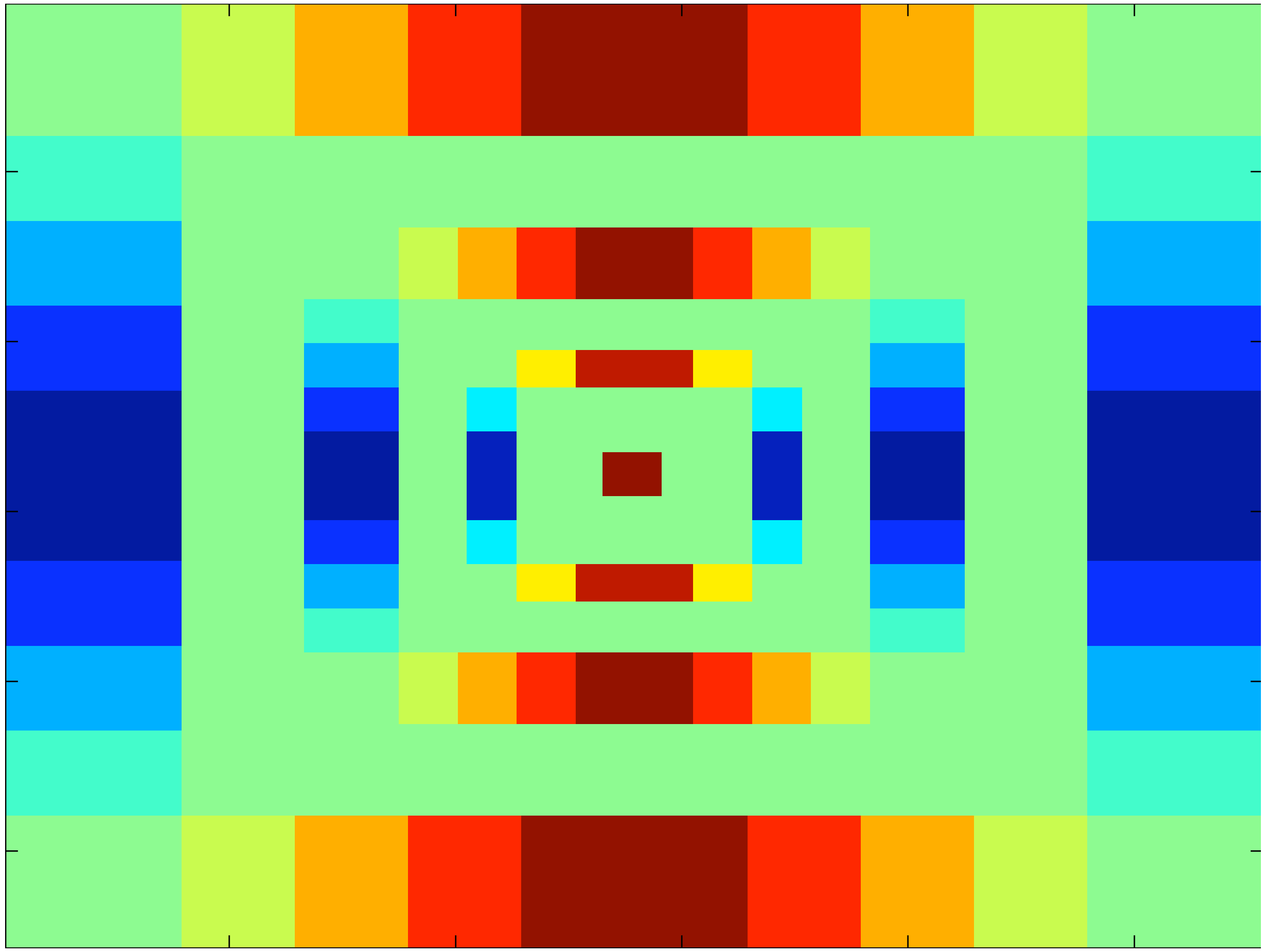
$\gamma = 1.0$

Phase-space regularization



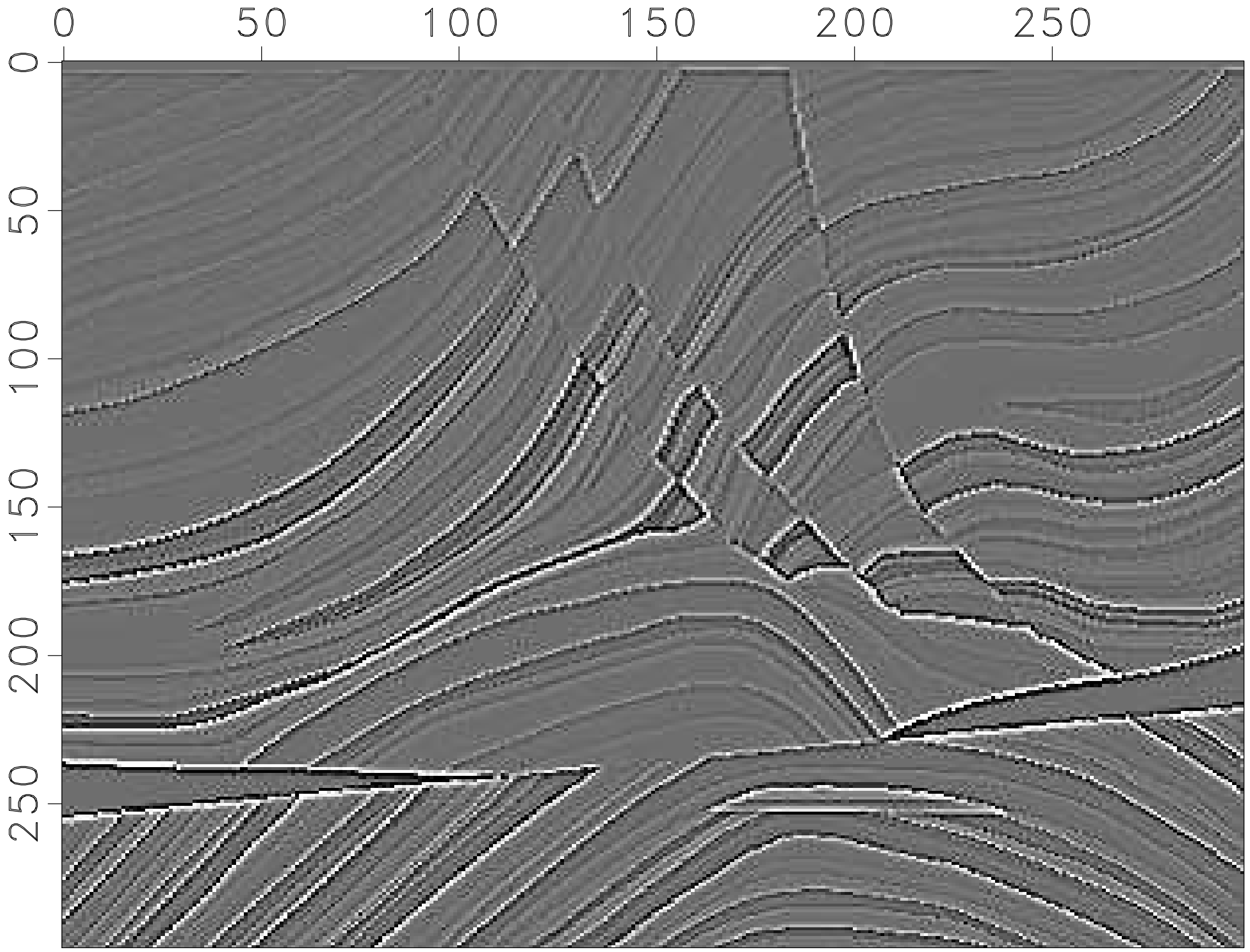
$\gamma = 2.0$

Example



Exact PDO: $\cos^2(\theta)$

Application of pseudodifferential operator



Original Image

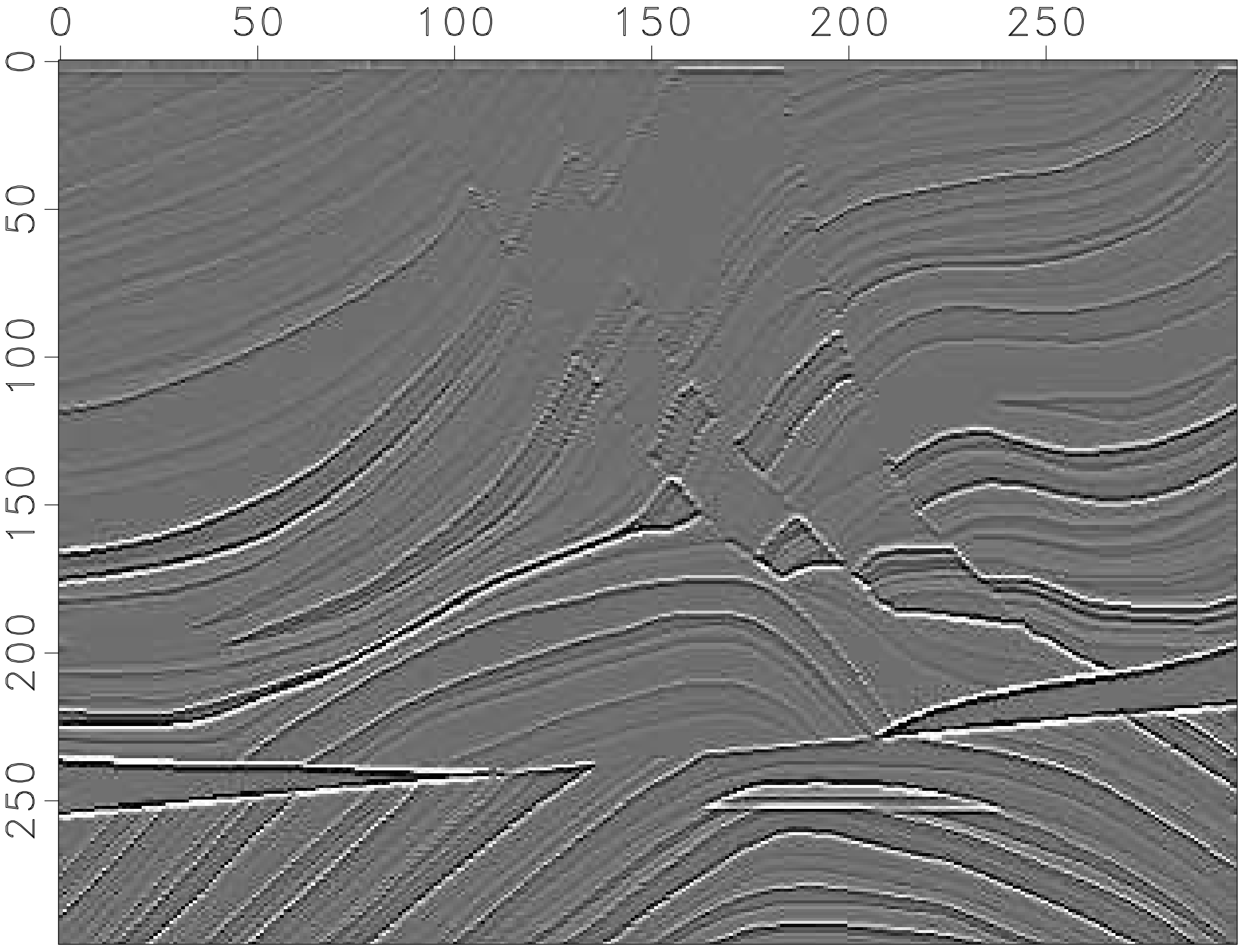
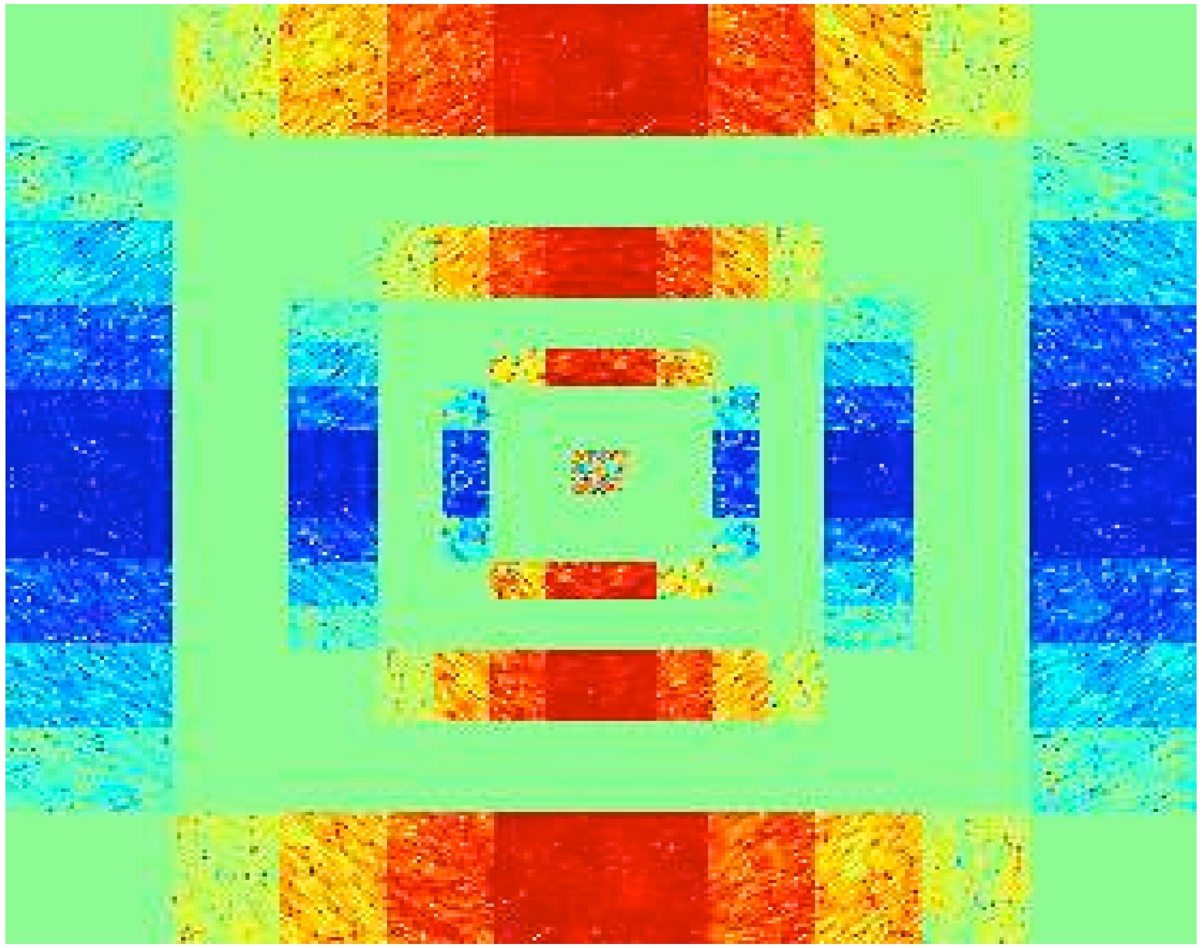
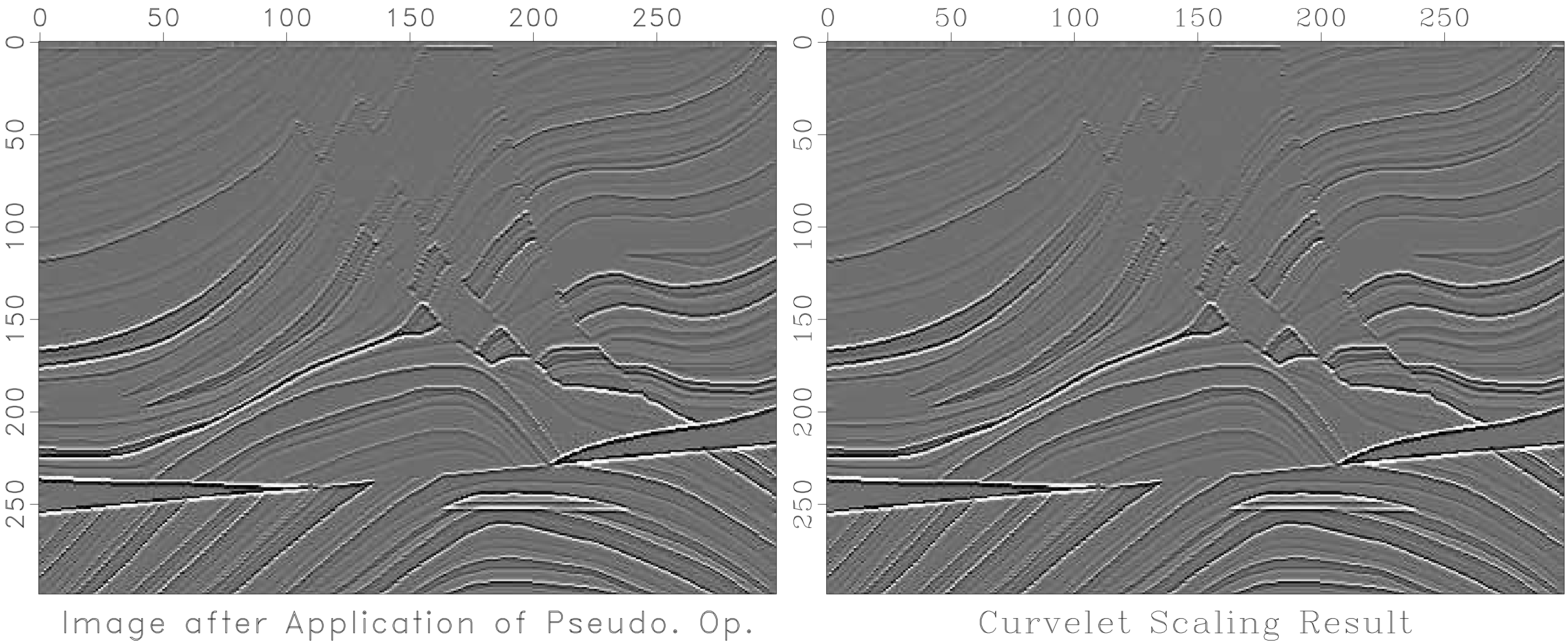


Image after Application of Pseudo. Op.

Estimation of the diagonal



Diagonal approximation



Comparison of exact application of PsDO with estimation of PsDO by diagonal weighting in curvelet domain.

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Application I: migration-amplitude recovery & preconditioning

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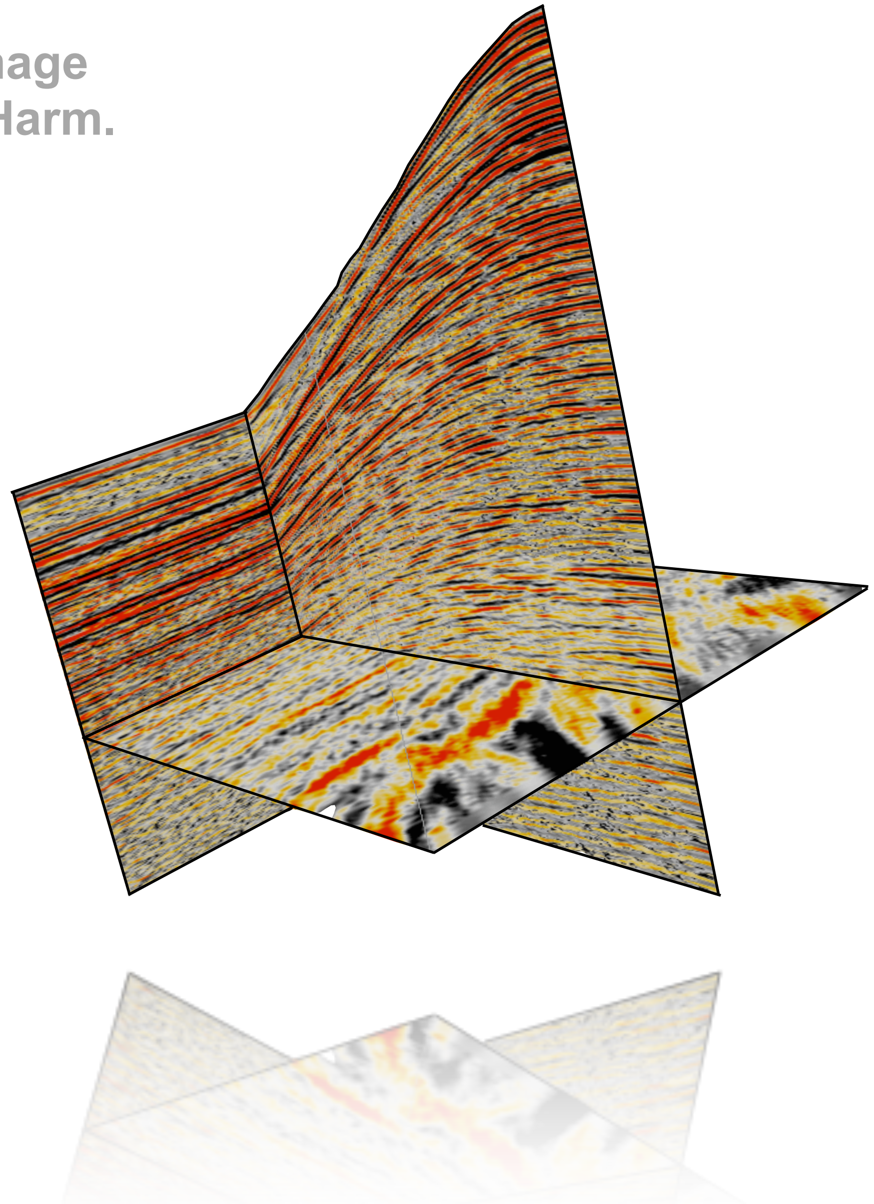

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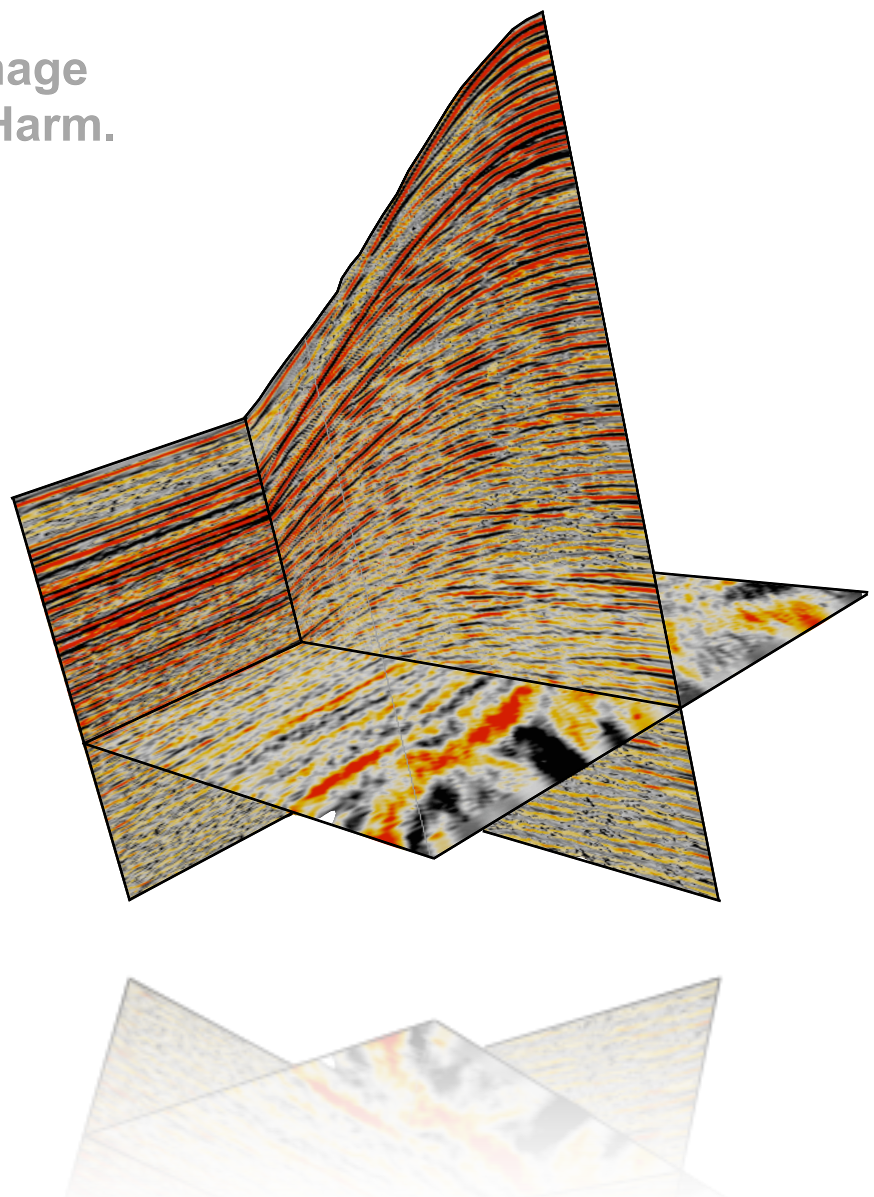
Amplitude recovery by scaling and sparsity promotion

Herrmann, F. J., Moghaddam, P. and Stolk, C.
Sparsity- and continuity-promoting seismic image recovery with curvelet frames. App. & Comp. Harm. Analys., Vol. 24, No. 2, pp. 150-173, 2008.



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Normal equations

Forward model after linearization (Born app.) & noise:

$$\delta d(x_s, x_r, t) = \big(K[\bar{m}]\delta m\big)(x_s, x_r, t) + n(x_s, x_r, t)$$

with

$$\begin{aligned} \delta d &= d - \mathcal{F}[m] \\ m &= \bar{m} + \delta m \\ \bar{m} &= \text{smooth} \end{aligned}$$

Normal equation after migration & high-frequency limit

$$\begin{aligned} \big(K^T d\big)(x) &= \big(K^T K \delta m\big)(x) + \big(K^T n\big)(x) \\ y(x) &= \big(\mathbf{\Psi}\delta m\big)(x) + e(x) \end{aligned}$$

where Ψ can be modeled by a ΨDO.

Normal equations cont'd [Stolk '02, ten Kroode '97, de Hoop '00, '03]

Migrated image contains imprint of normal/Hessian operator.

Least-squares migration

$$\delta m_{LS} = K^{\dagger}\delta d = \arg\min_{\delta m} \frac{1}{2}\|\delta d - K\delta m\|_2^2$$

based on Lanczos (e.g. lsqr) methods may be computationally prohibitive.

In high-frequency limit Ψ is a PsDO (for d=2), i.e.,

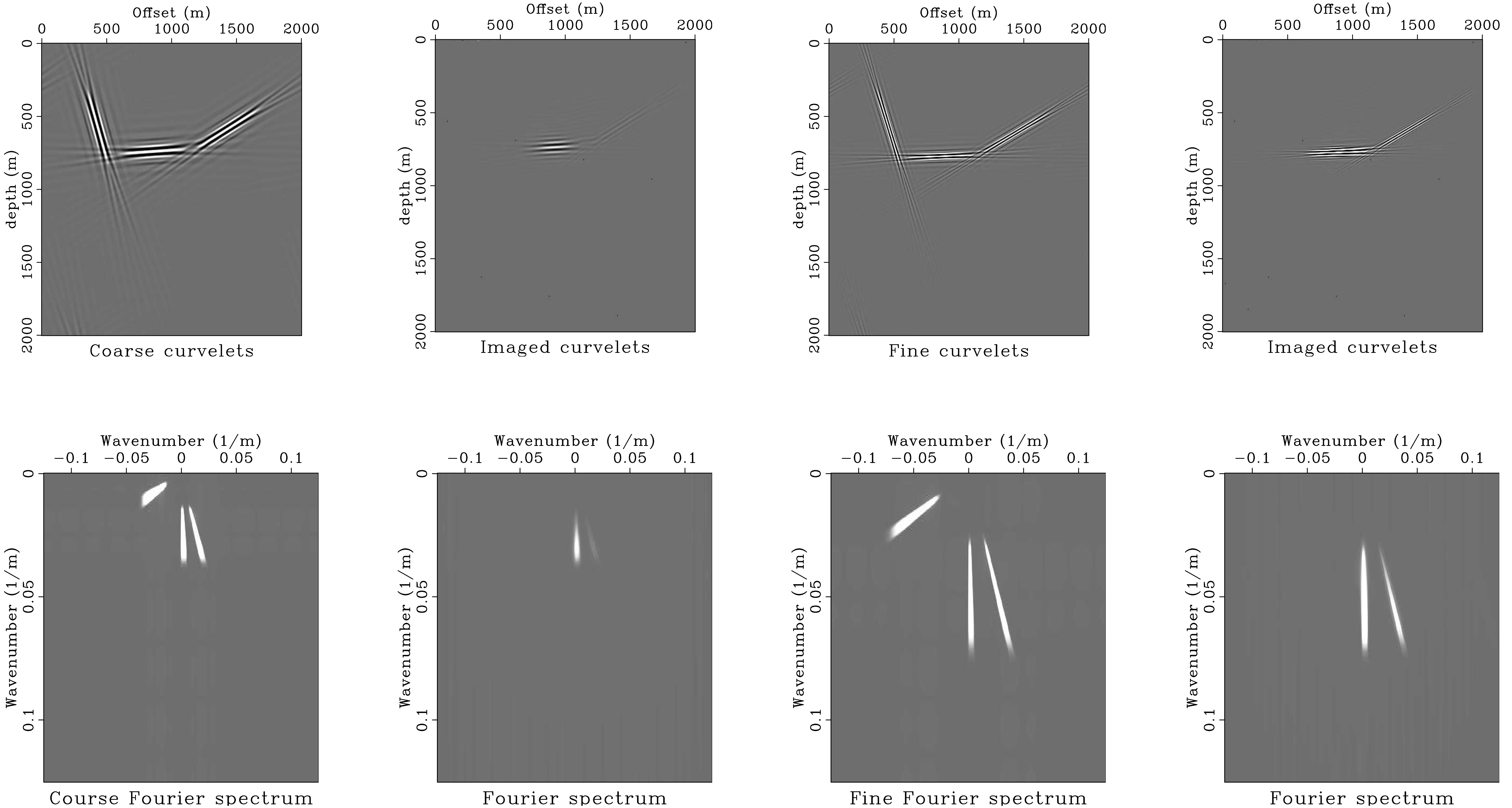
$$(\Psi f)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} a(x,\xi)\hat{f}(\xi)\mathrm{d}\xi$$

- correct background velocity model
- pseudolocal
- singularities are preserved

Corresponds to a spatially-varying dip filter.

Can be approximated using diagonal scaling methods [Symes '08, FJH '08].

Invariance



- curvelets remain invariant (for angles in the range Ψ)
- approximation improves for higher frequencies

Approximate forward model

Make modeling operator zero order by the following transformations:

$$K \mapsto K(-\Delta)^{-1/2} \qquad K \mapsto \partial_{|t|}^{-1/2} K = \mathcal{F}^* |\omega|^{-1/2} \mathcal{F} K$$

$$m \mapsto (-\Delta)^{1/2} m \qquad ((-\Delta)^{\alpha} f)^{\wedge}(\xi) = |\xi|^{2\alpha} \cdot \hat{f}(\xi).$$

Use the decomposition

$$\begin{aligned} \big(\Psi\varphi_\mu\big)(x) &\simeq \big(C^T \mathbf{D}_\Psi C\varphi_\mu\big)(x) \\ &= \big(AA^T\varphi_\mu\big)(x) \end{aligned}$$

with $A := \sqrt{\mathbf{D}_\Psi} C$ and $A^T := C^T \sqrt{\mathbf{D}_\Psi}$.

to define the following approximate forward model:

$$\begin{aligned} y(x) &= \big(\Psi\delta m\big)(x) + e(x) \\ &\approx \big(AA^*\delta m\big)(x) + e(x) \\ &= A\mathbf{x}_0 + e \end{aligned}$$

Amplitude-recovery by sparsity promotion

Forward model:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \boldsymbol{\epsilon}$$

Sparsity-promoting program:

$$\mathbf{P}_\sigma : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma \\ \\ \tilde{\mathbf{m}} = (\mathbf{A}^*)^\dagger \tilde{\mathbf{x}} \end{cases}$$

Solve with iterative thresholding.

Work flow

Select a reference vector that is close to the unknown image

- migrated image after spherical spreading correction

Form the ***normal*** operator by compounding discretized linearized modeling & migration operators $\mathbf{K}^*\mathbf{K}$ and apply.

Estimate the diagonal $(i.e., \mathbf{D}_\mu)$.

Construct the matrix $\mathbf{A} = \mathbf{C}^*\sqrt{\mathbf{D}_\mu}$.

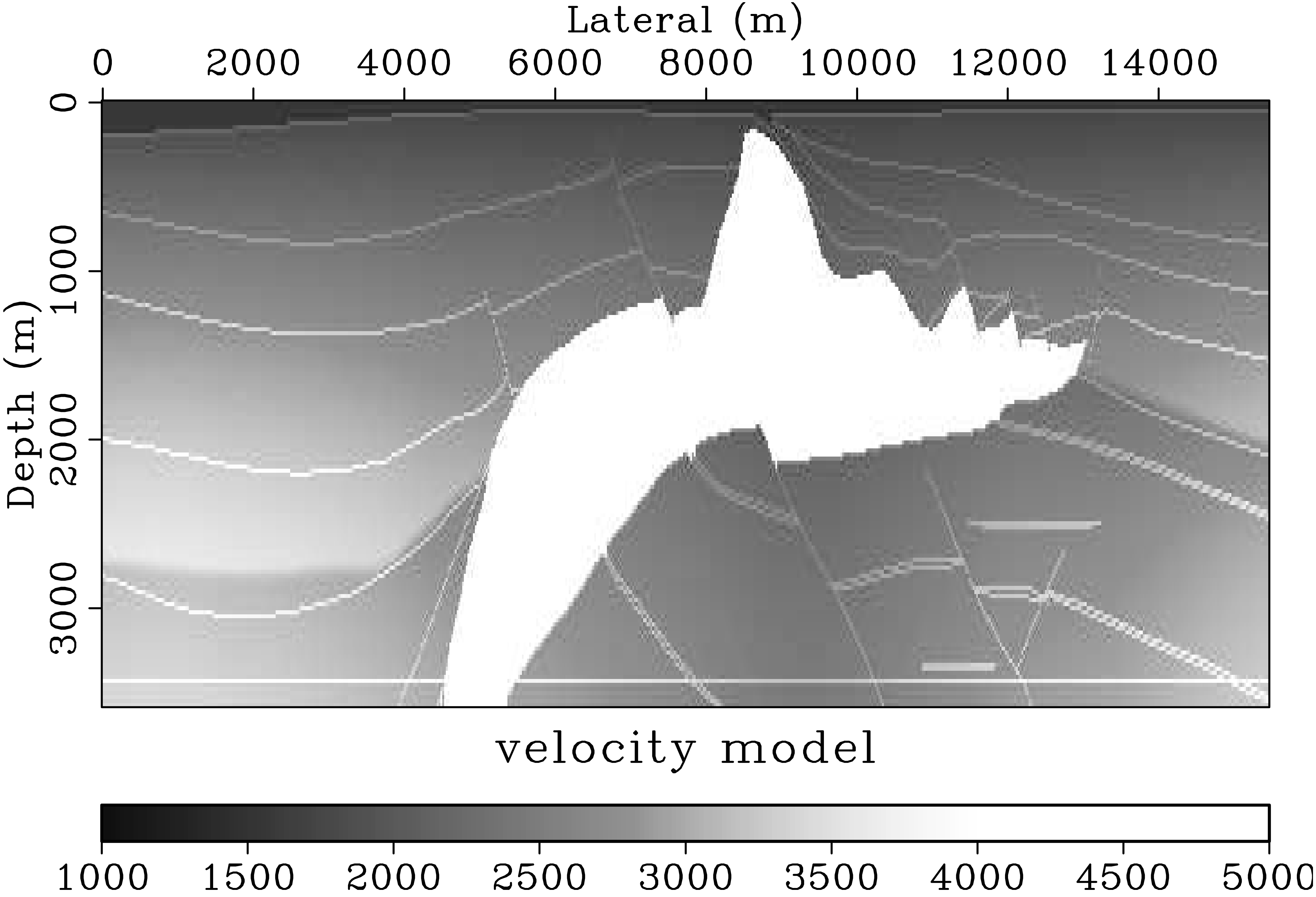
Invert **A** with a sparsity & continuity promotion program

- exploit curvelet-domain sparsity
- remove artifacts

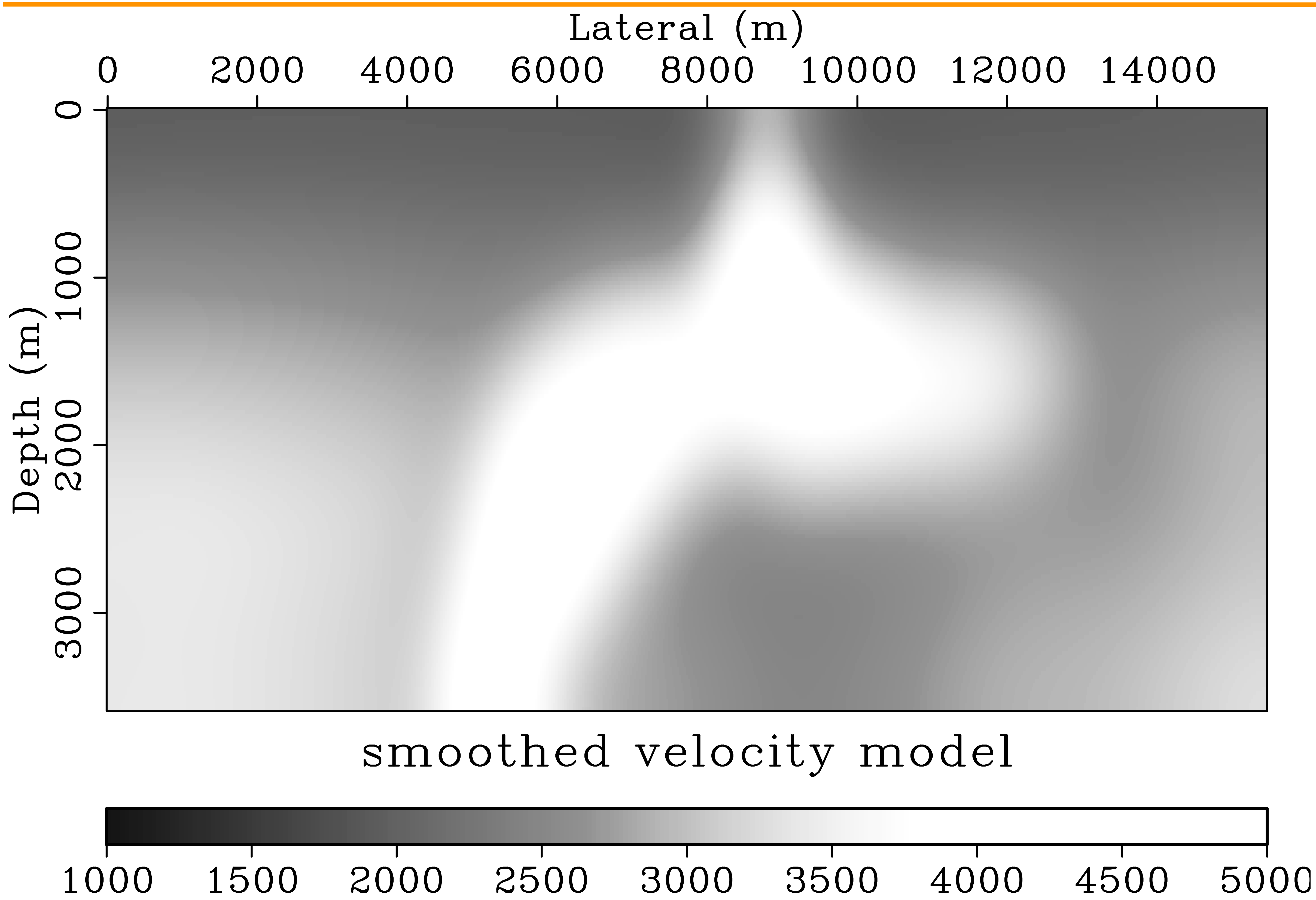
Example

- SEGAA' data:
  - "broad-band" half-integrated wavelet [5-60 Hz]
  - 324 shots, 176 receivers, shot interval of 48 m, yielding a maximal offset of 4224 m
  - 5 s of data
- Modeling operator
  - Reverse-time migration with optimal check pointing (Symes '07)
  - 8000 time steps
  - **linearized** modeling 64, and migration 294 minutes on 68 CPU's
- Estimation of the scaling requires 1 extra migration-demigration.

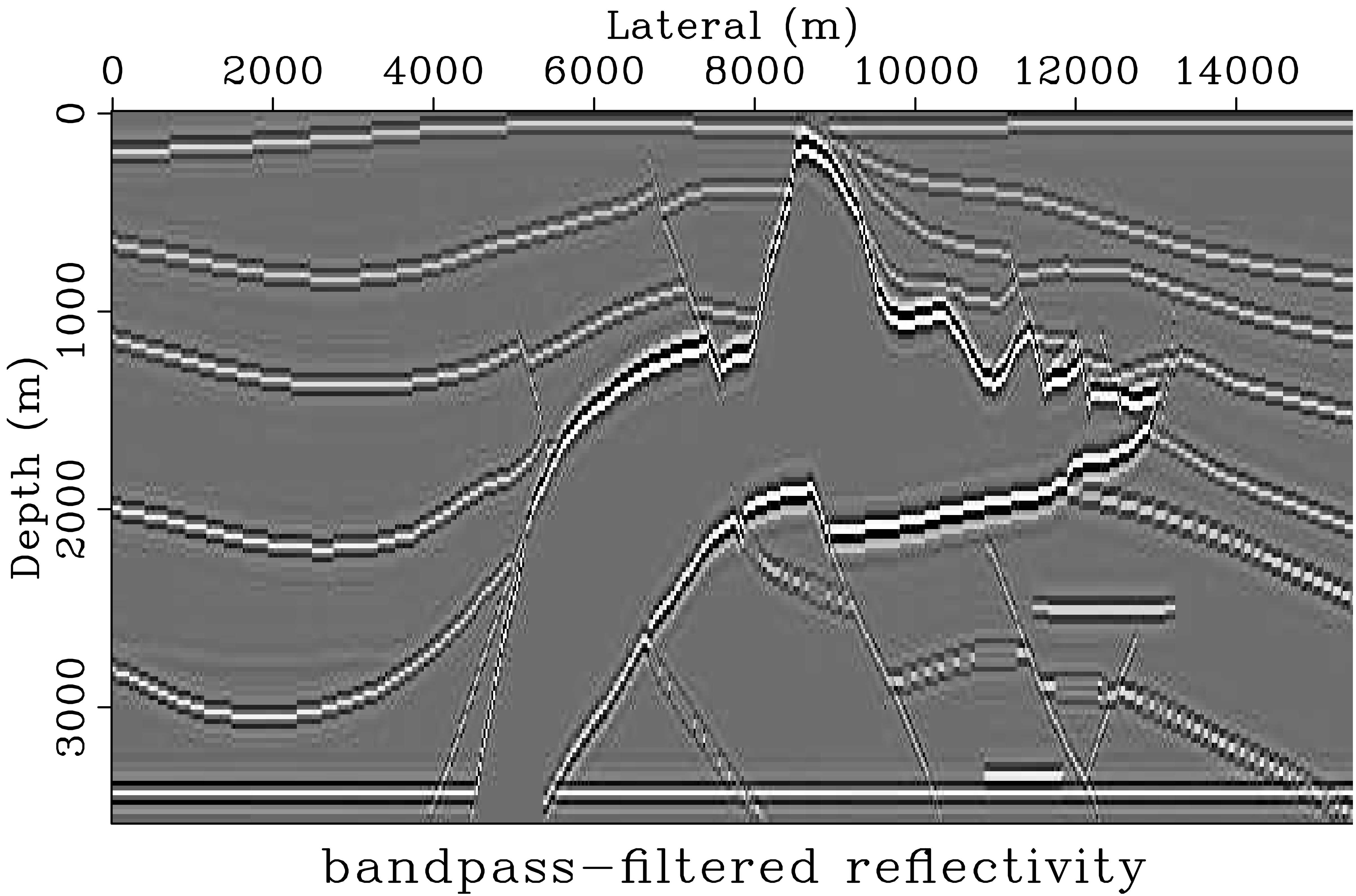
SEG AA' Salt Model



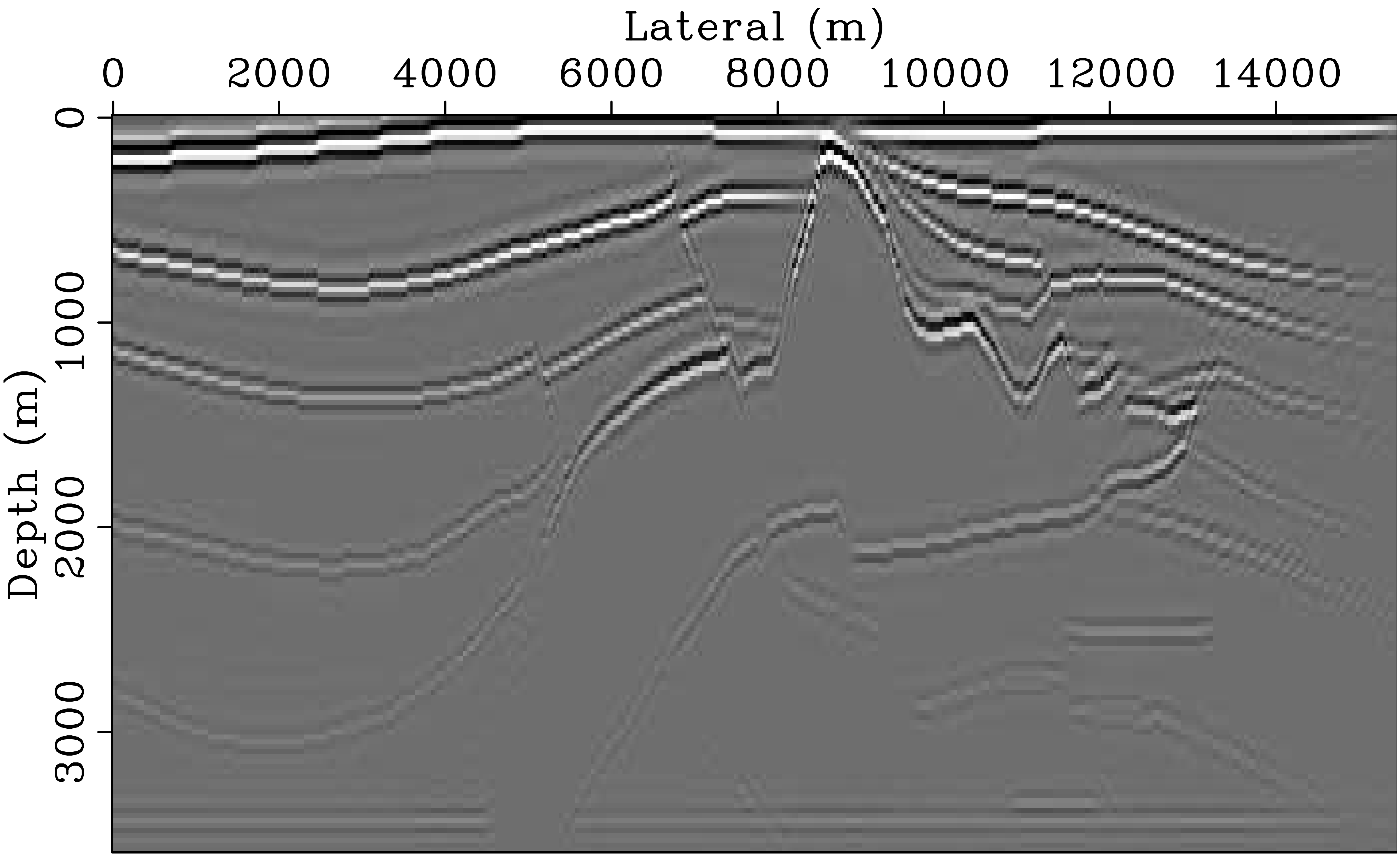
Smoothed SEG AA’ Salt Model



Bandpass-filtered SEG AA' Salt Model

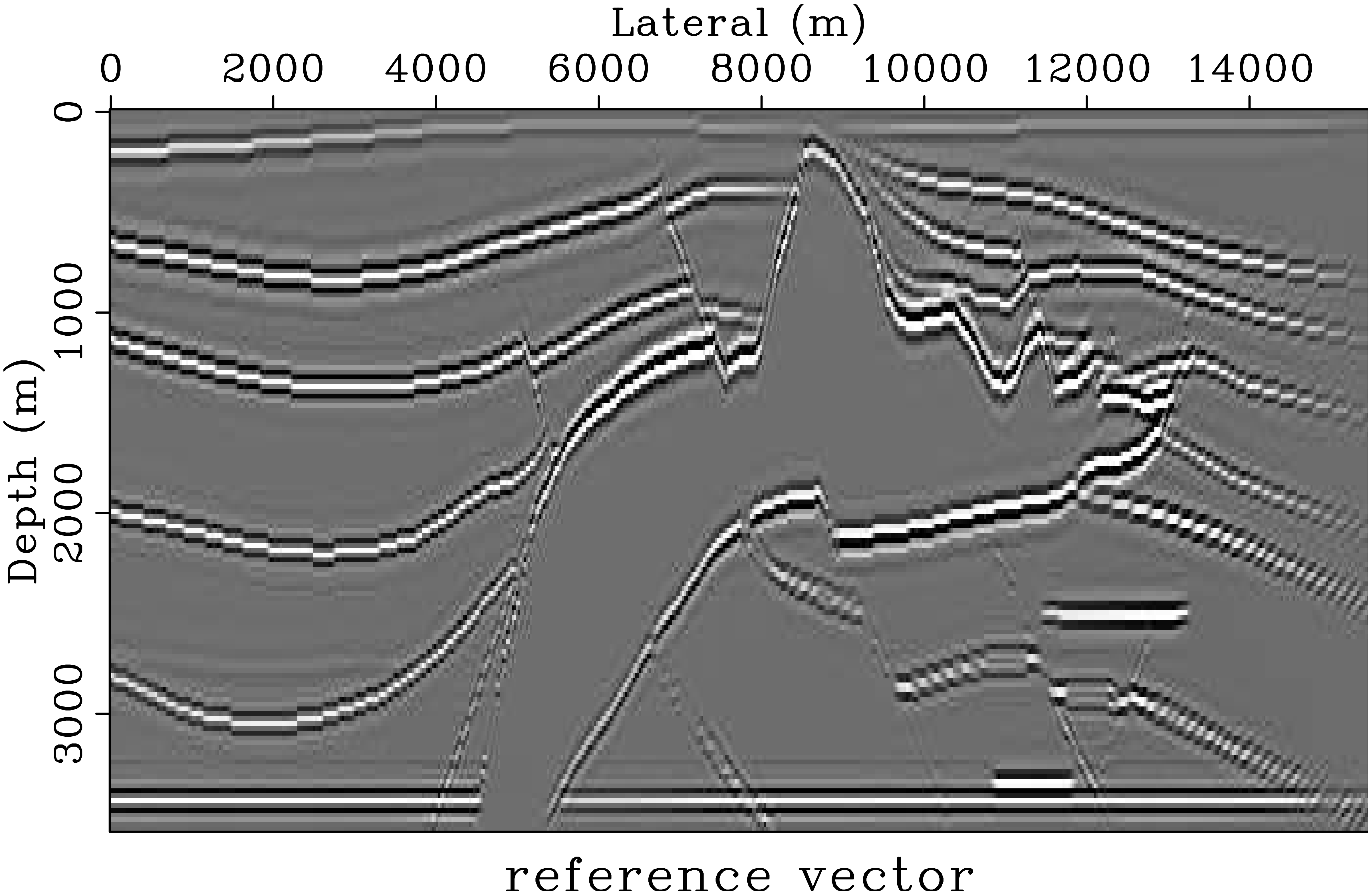


Migrated image

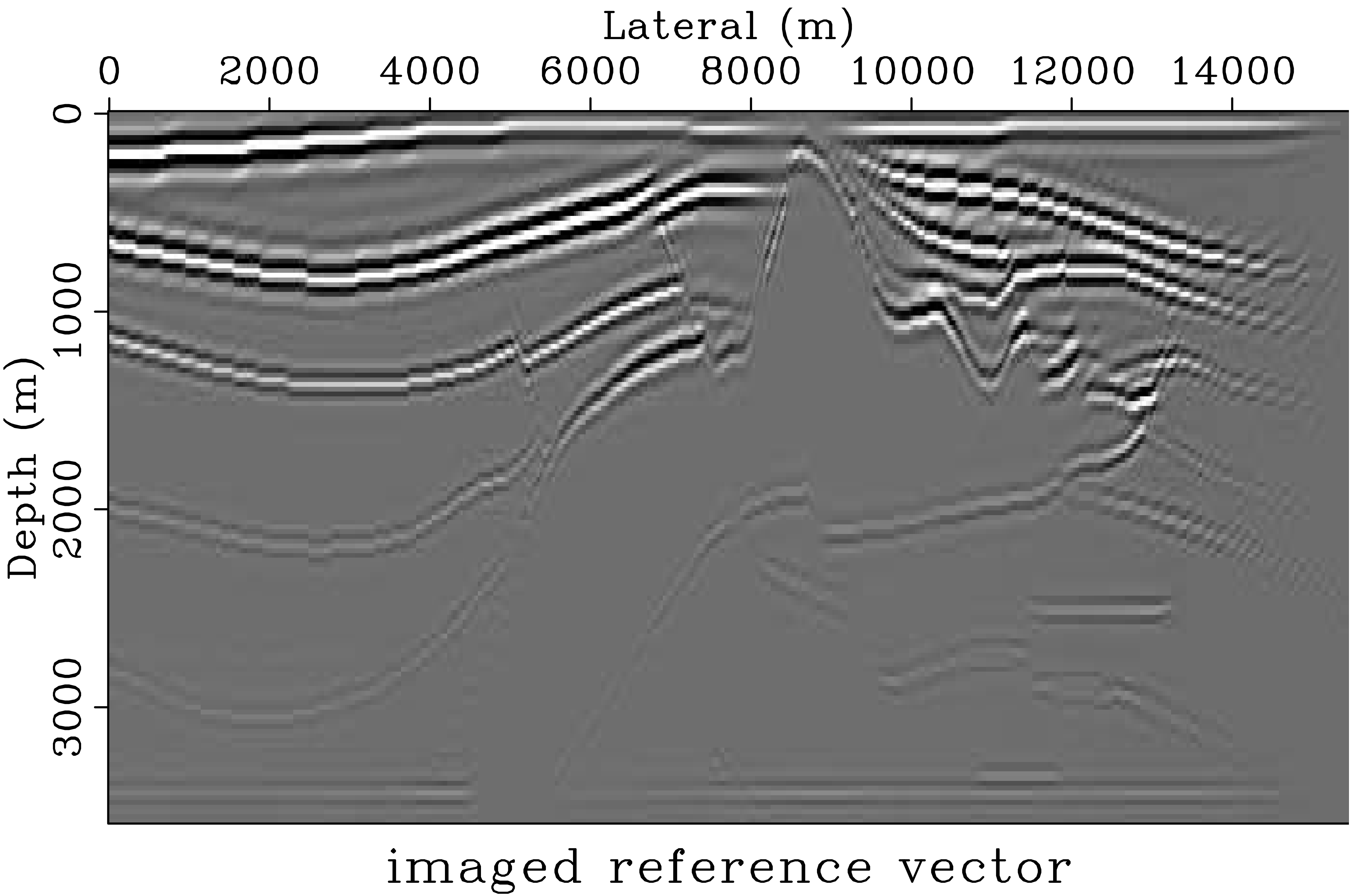


migrated image

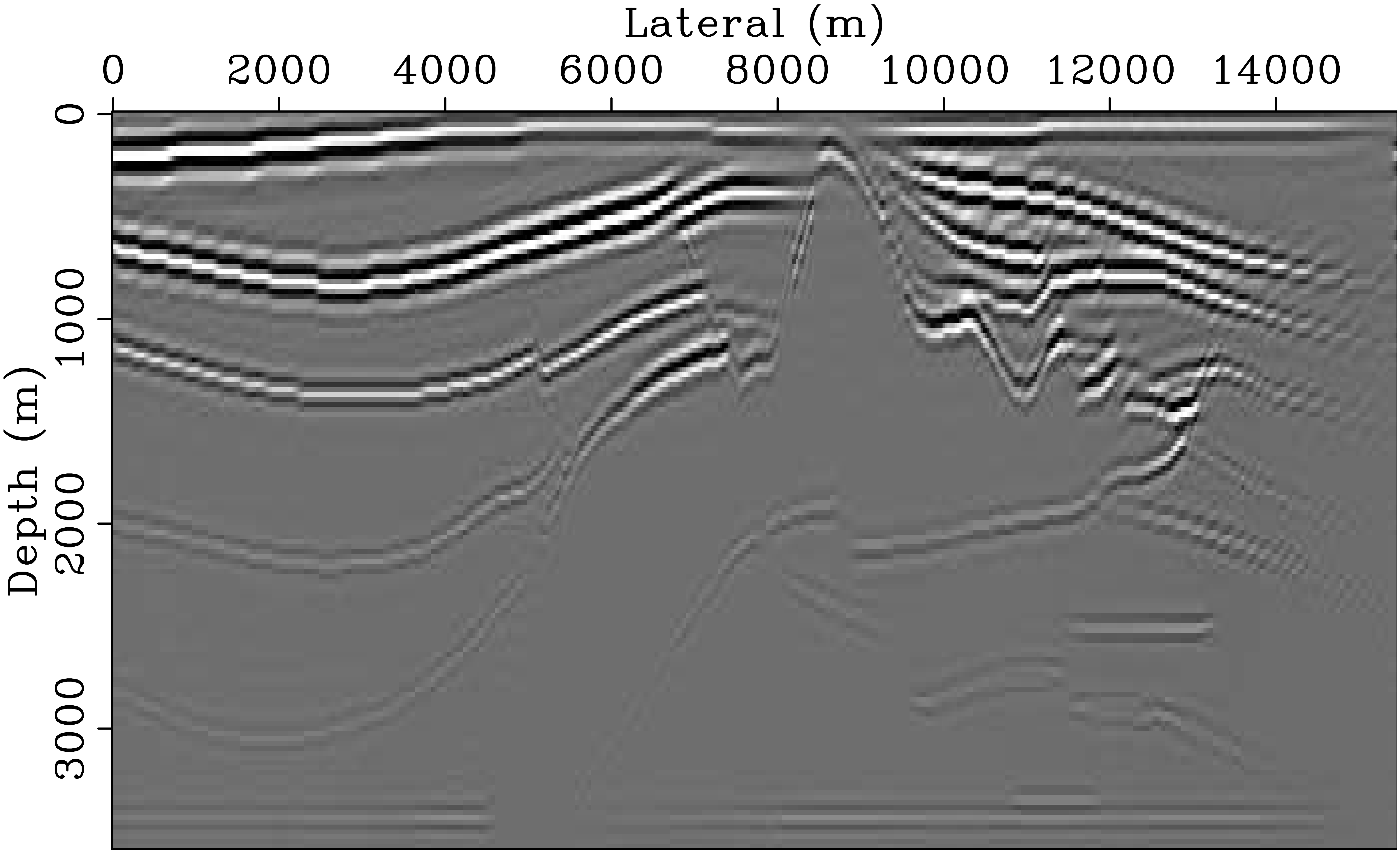
Depth-corrected image = reference image



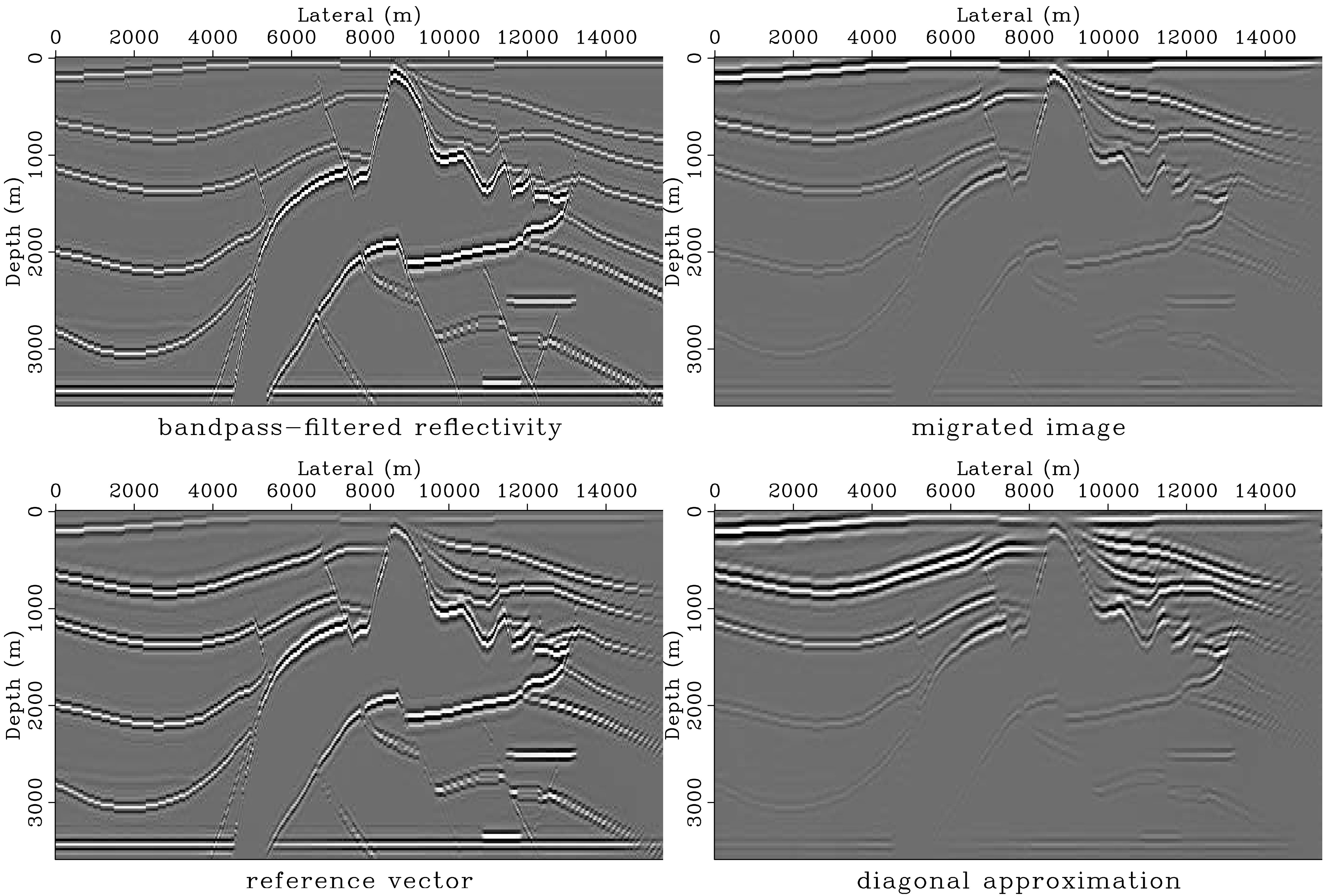
Remigrated reference image



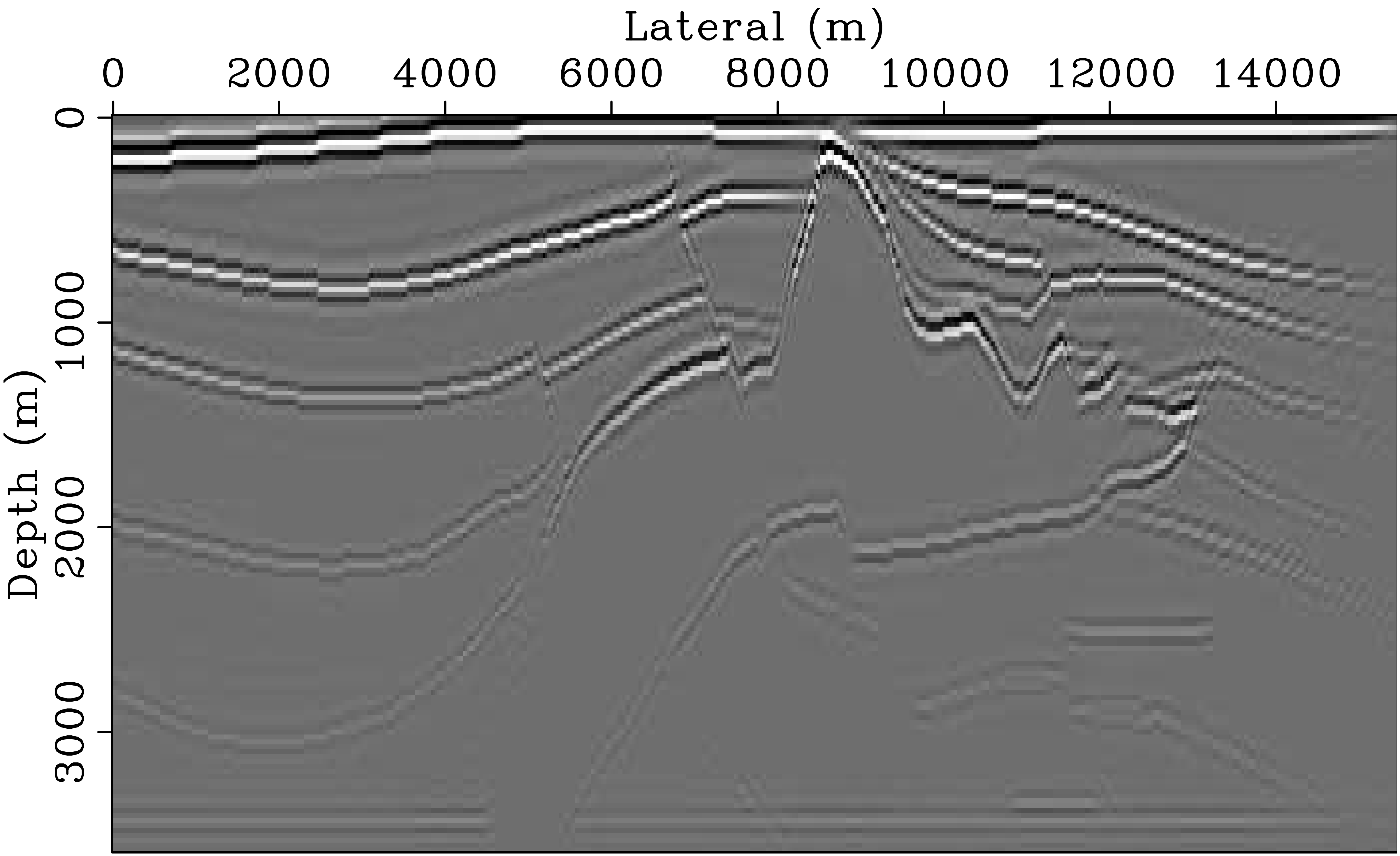
Approximated remigrated reference vector



diagonal approximation


Lateral (m)
0 2000 4000 6000 8000 10000 12000 14000
Depth (m)
0 1000 2000 3000
bandpass−filtered reflectivity
migrated image
reference vector
diagonal approximation

Migrated image



migrated image

Amplitude-recovery by sparsity promotion

Forward model:

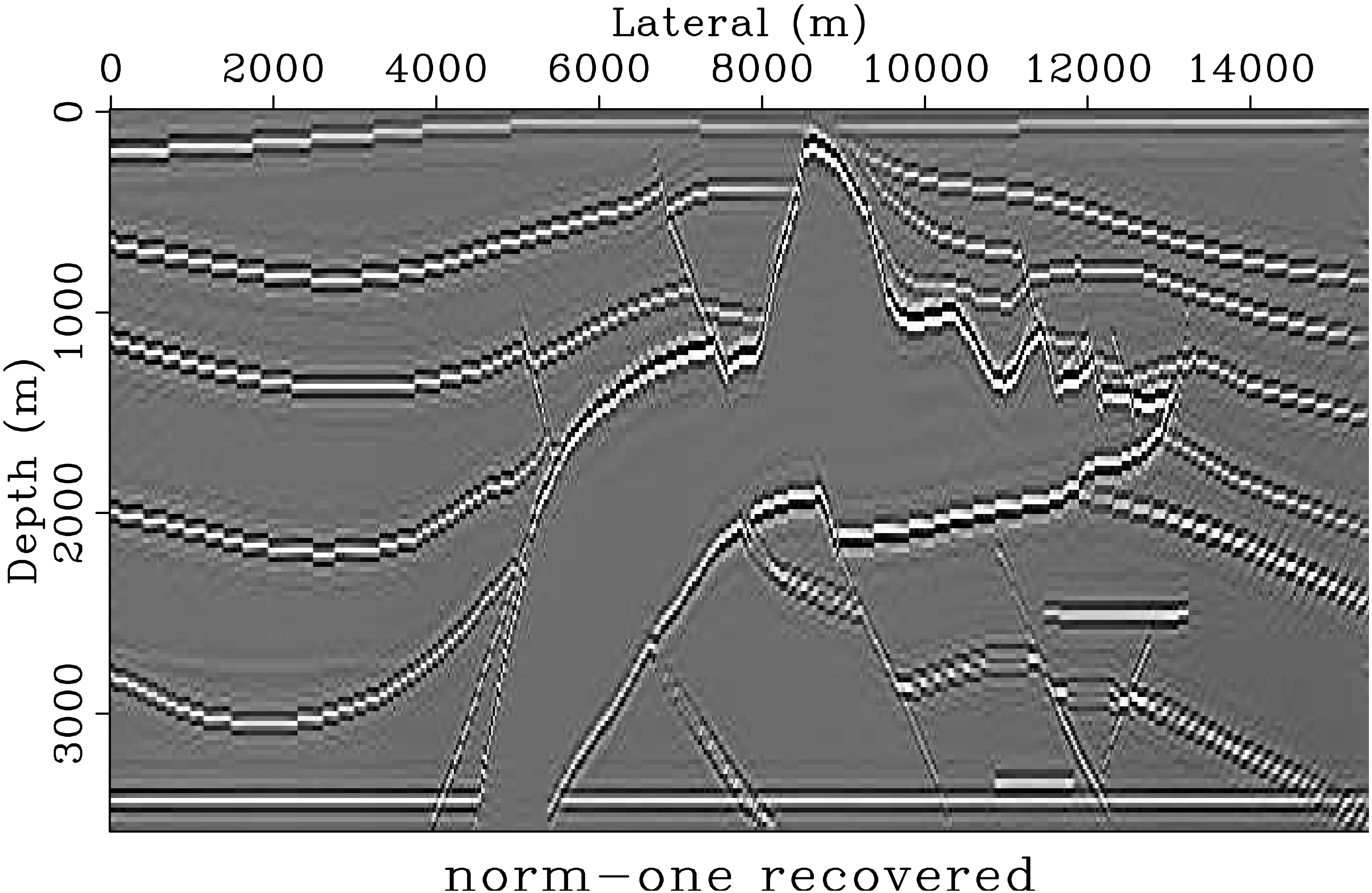
$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \boldsymbol{\epsilon}$$

Sparsity-promoting program:

$$\mathbf{P}_\sigma : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma \\ \\ \tilde{\mathbf{m}} = (\mathbf{A}^*)^\dagger \tilde{\mathbf{x}} \end{cases}$$

Solve with iterative thresholding.

Amplitude recovery by sparsity promotion



Anisotropic diffusion [Black et. al ’98, Fehmers et. al. ’03 and Shertzer ‘03]

Remove spurious artifacts by continuity promotion by minimizing

$$\mathbf{P}_\sigma : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma \\ \\ \tilde{\mathbf{m}} = (\mathbf{A}^*)^\dagger \tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha\|\mathbf{x}\|_1}^{\text{sparsity}} + \beta \underbrace{\left\|\mathbf{\Lambda}^{1/2}\left(\mathbf{A}^H\right)^\dagger \mathbf{x}\right\|_2}_{\text{continuity}} .$$

and the anisotropic diffusion term

$$\mathbf{\Lambda}[\mathbf{r}] = \frac{1}{\|\mathbf{\nabla}\mathbf{r}\|_2^2 + 2\upsilon} \left\{ \begin{pmatrix} +\mathbf{D}_2\mathbf{r} \\ \\ -\mathbf{D}_1\mathbf{r} \end{pmatrix} \begin{pmatrix} +\mathbf{D}_2\mathbf{r} & -\mathbf{D}_1\mathbf{r} \end{pmatrix} + \upsilon\mathbf{Id} \right\}$$

Algorithm

Result: Estimate for $\mathbf{x}$

initialization;

$m \leftarrow 0$; $\mathbf{x}^0 \leftarrow \mathbf{0}$; $\mathbf{y} \leftarrow \mathbf{K}^T \mathbf{d}$;

Set M, L, and $\|\mathbf{A}^T \mathbf{y}\|_\infty > \lambda_1 > \lambda_2 > \cdots$;

while $\|\mathbf{y} - \mathbf{A}\tilde{\mathbf{x}}\|_2 > \epsilon$ *and* $m < M$ **do**

   $m = m + 1$; $\mathbf{x}^m = \mathbf{x}^{m-1}$;

   for $l = 1$ *to* L **do**

      Iterative thresholding;

      $\mathbf{x}^m \leftarrow S_{\lambda_m}\left(\mathbf{x}^m + \mathbf{A}^T \left(\mathbf{y} - \mathbf{x}^m\right)\right)$

   end

   Anisotropic descent update;

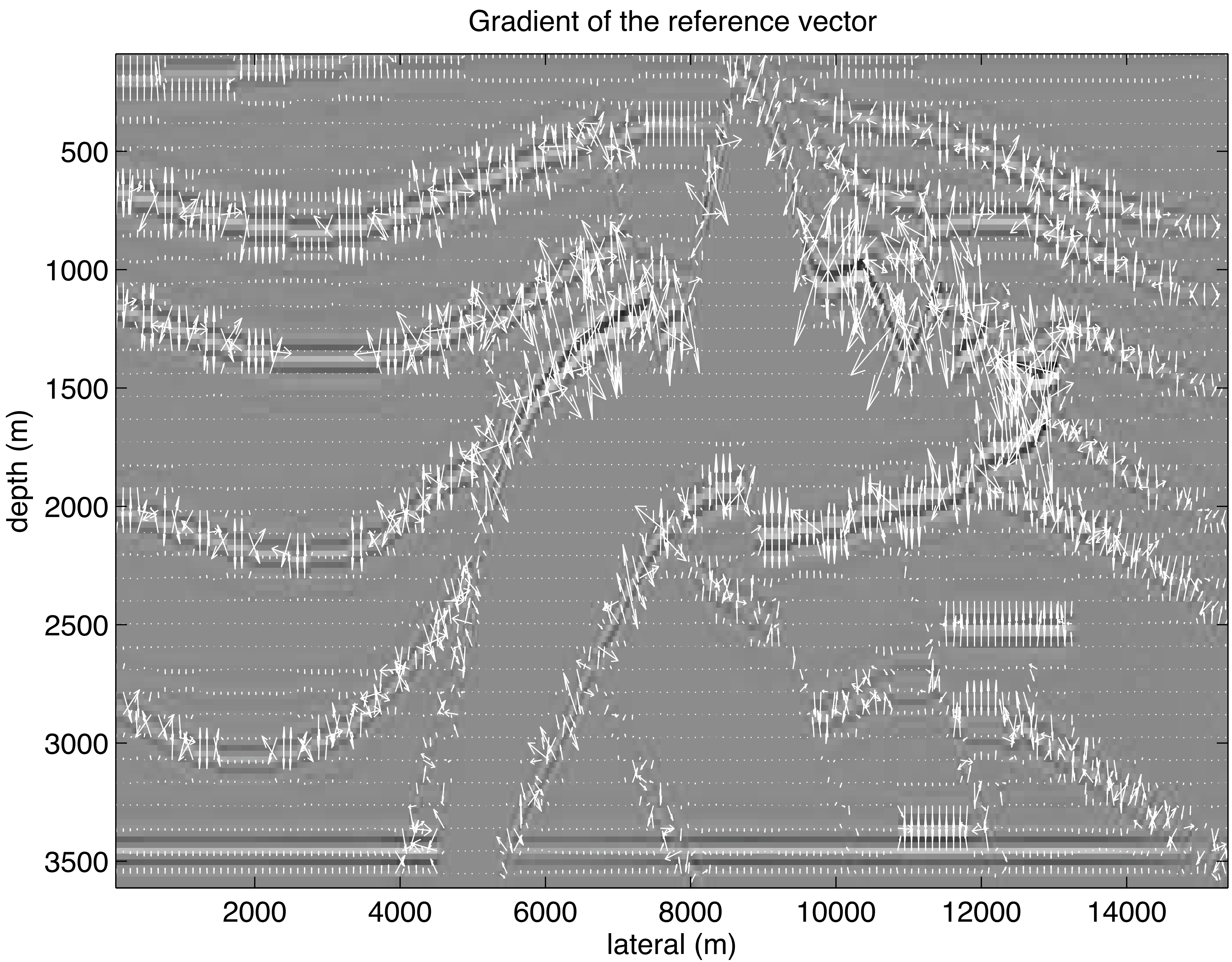
   $\mathbf{x}^m = \mathbf{x}^m - \beta \boldsymbol{\nabla}_{\mathbf{x}^m} J_c(\mathbf{x}^m)$ width

   $$\boldsymbol{\nabla}_{\mathbf{x}} J_c(\mathbf{x}^m) = 2\mathbf{A}^\dagger \boldsymbol{\nabla} \cdot \left(\boldsymbol{\Lambda} \boldsymbol{\nabla} \left(\left(\mathbf{A}^T\right)^\dagger \mathbf{x}^m\right)\right);$$

end

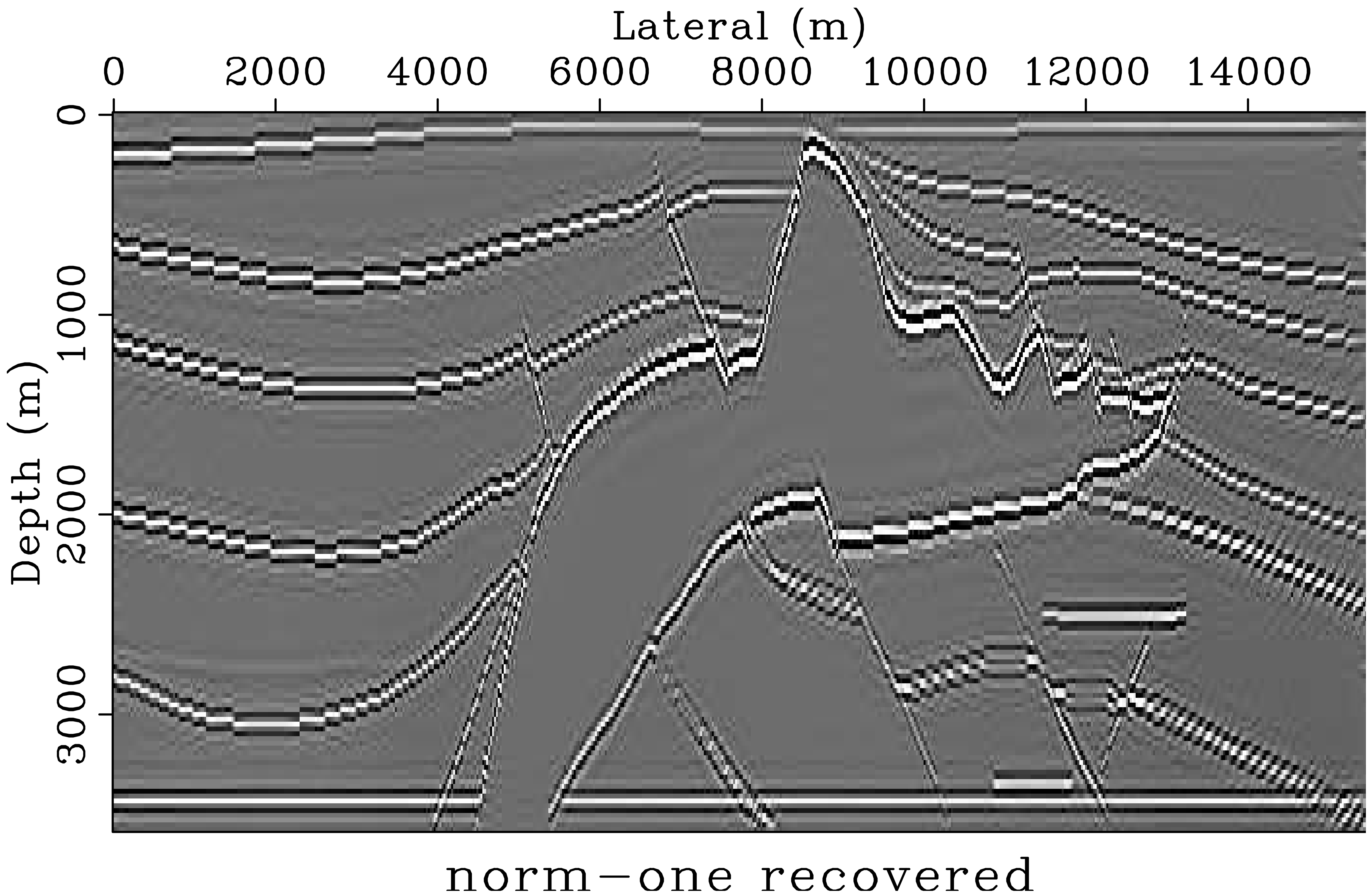
$\tilde{\mathbf{x}} \leftarrow \mathbf{x}_m$;

$\tilde{\mathbf{m}} = \left(\mathbf{A}^*\right)^\dagger \tilde{\mathbf{x}}$.

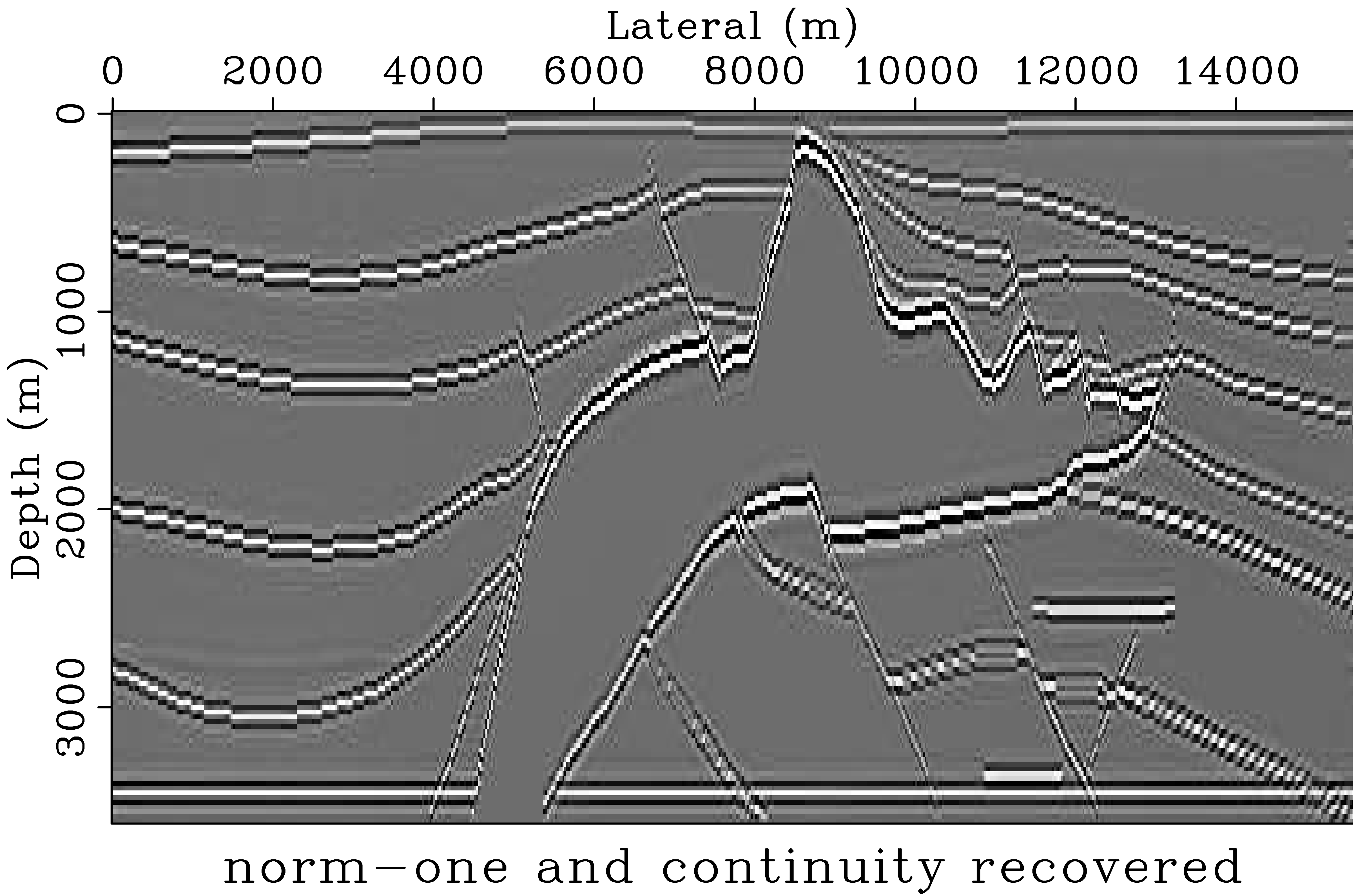


Gradient of the reference vector
500
1000
1500
2000
2500
3000
3500
depth (m)
2000
4000
6000
8000
10000
12000
14000
lateral (m)

Amplitude recovery by sparsity promotion



Recovery by sparsity & continuity promotion



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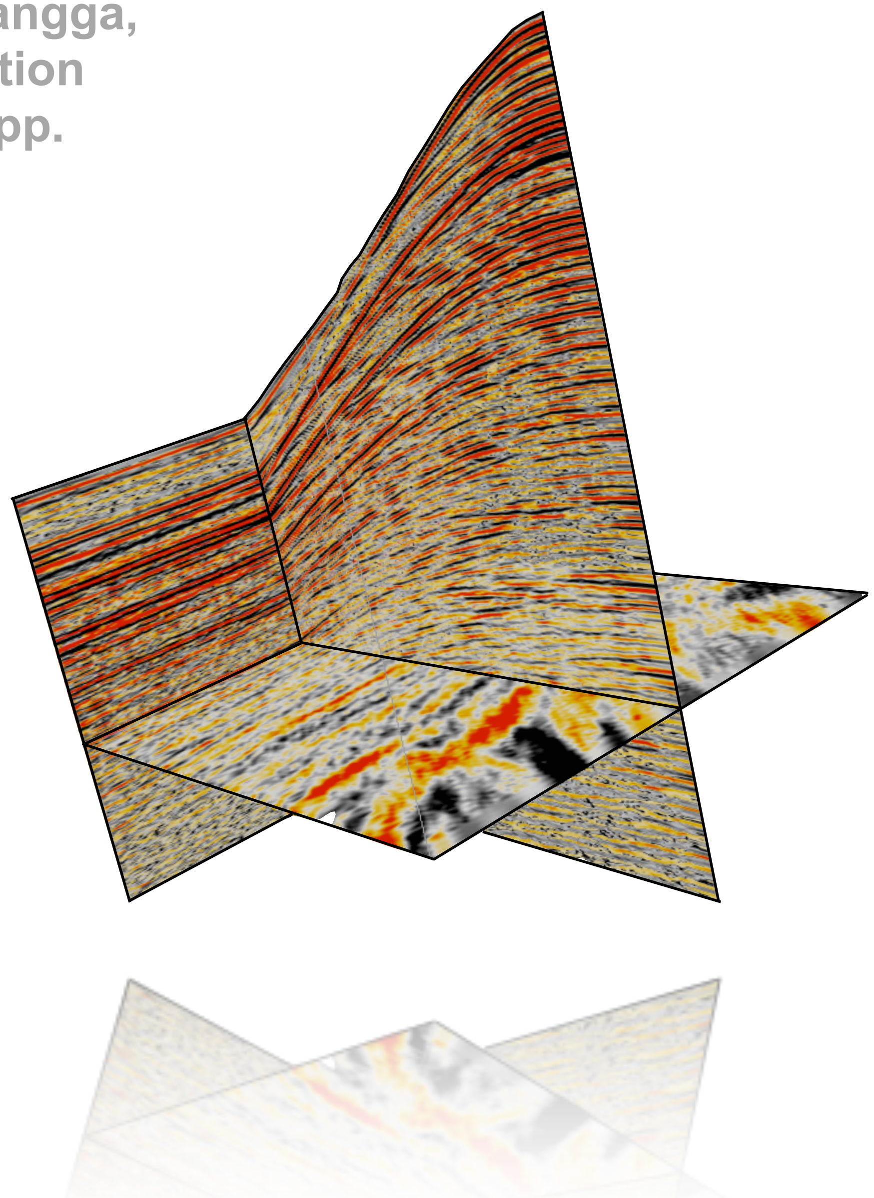
Curvelet-based migration preconditioning and scaling

Felix J. Herrmann, Cody R. Brown, Yogi A. Erlangga, Peyman P. Moghaddam. Curvelet-based migration preconditioning and scaling. Geophysics, 74, pp. A41, 2009.



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Our Problem

- Inverting this is not so trivial because of the size:

$$\widetilde{\mathbf{x}}_{LS} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2$$

- We want to **condition** this as well as possible.

- With accurate background velocity the normal operator is *near unitary*
  - iterative solution is known to converge.
  - sheer size of the problem however makes this a very time consuming problem.

- *A reduction in the number of iterations will be necessary!*

[Paige and Saunders, 1982]

Our Solution

- We propose to do this by replacing our initial system with a series of preconditioning levels:

$$\mathbf{M}_L^{-1}\mathbf{A}\mathbf{M}_R^{-1}\mathbf{u} \approx \mathbf{M}_L^{-1}\mathbf{b}, \qquad \mathbf{x} := \mathbf{M}_R^{-1}\mathbf{u}$$

- This involves a series of *right* and *left* preconditioning matrices.
  - matrix free

- The cost for applying these preconditioners correspond to a matrix-free matrix-vector multiplies with a complexity of at most $\mathcal{O}(n \log n)$ opposed to a cost of for the evaluation of the normal operator, which is $\mathcal{O}(n^4)$ in dimension two.

Levels of Preconditioning

- In data space, we apply a multiplication in the temporal Fourier domain.
- This can be thought as a *left* preconditioning through fractional integration:

$$\mathbf{M}_L^{-1} := \partial_{|t|}^{-1/2} \cdot = \mathcal{F}^* |\omega|^{-1/2} \mathcal{F} \cdot$$

- Makes the normal operator zero order.
  - normal operators will act as a zero-order pseudo-differential operator
  - approximated by curvelet-domain scaling

Levels of Preconditioning

L1 L2 L3

- *Right* preconditioning by scaling in the physical domain:

$$\mathbf{M}_R^{-1} = \mathbf{D}_z := \mathrm{diag}\left(\mathbf{z}\right)^{\frac{1}{2}}$$

- Reflected waves travel from the source at the surface down to the reflector and back.

- This gives a quadratic depth dependence.

Levels of Preconditioning

L1 L2 L3

- *Right* preconditioning by scaling in the curvelet domain:

$$\Psi \mathbf{r} \approx \mathbf{C}^* \mathbf{D}_\Psi^2 \mathbf{C} \mathbf{r}, \quad \mathbf{D}_\Psi^2 := \mathrm{diag}\left(e^{2\widetilde{\mathbf{z}}}\right)$$

$$\mathbf{M}_R^{-1} = \mathbf{D}_z \mathbf{C}^* \mathbf{D}_\Psi^{-1}$$

- Estimation of the diagonal in the curvelet domain, i.e.,

$$\tilde{\mathbf{z}} = \arg\min_{\mathbf{z}} \frac{1}{2} \|\Psi \mathbf{r} - \mathbf{C}^T \mathrm{diag}\left(\mathbf{C}\mathbf{r}\right) e^{2\mathbf{z}}\|_2^2 + \gamma \|\mathbf{L}_{\mathcal{C}} e^{2\mathbf{z}}\|_2^2$$

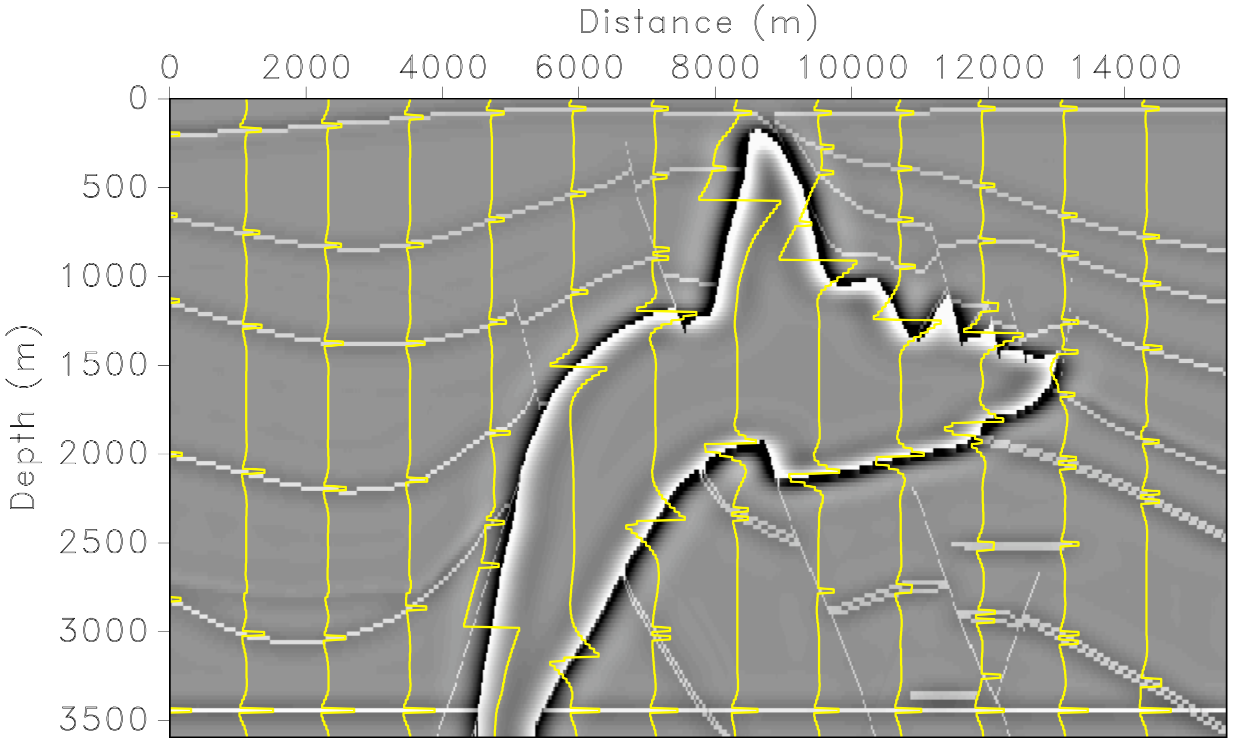
  – solved with l-BFGS [Nocedal '95]

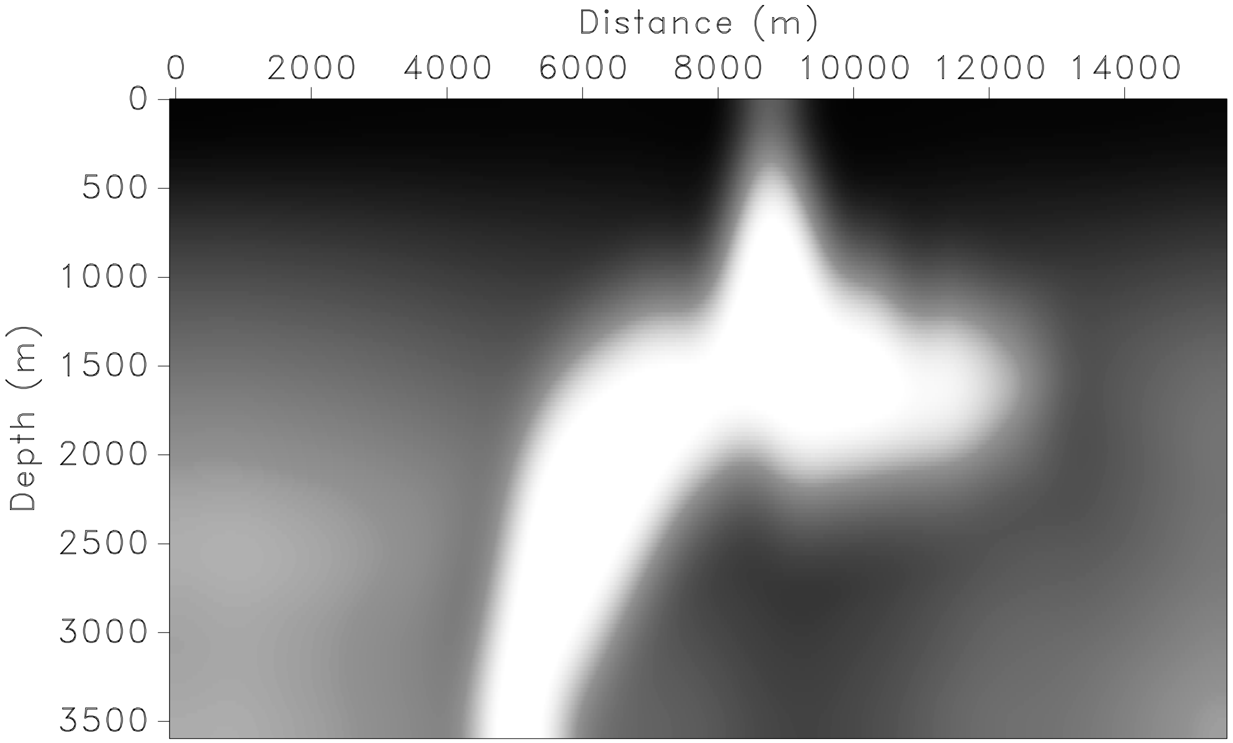
- The cost to compute this diagonal is *one migration and one remigration*.
  – This is equivalent to one iteration of LSQR.

- **"True-amplitude" correction.**

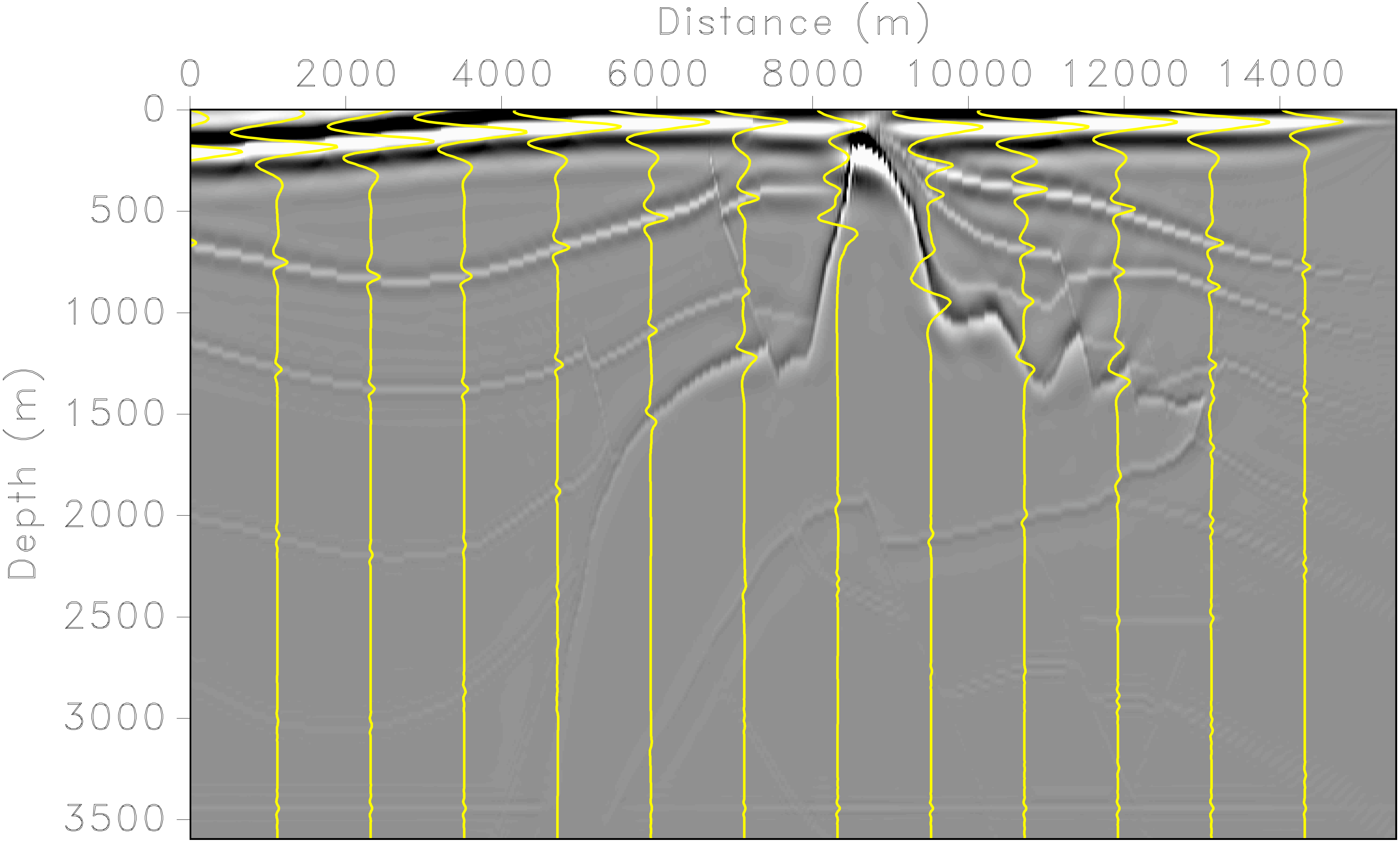
SEG AA' Model w/ Smooth Velocity

- SEG AA' salt model.
- 324 shots.
- Each shot 176 traces of 6.4s with a trace interval of 24m.
- Maximum offset of the data is 4224m.



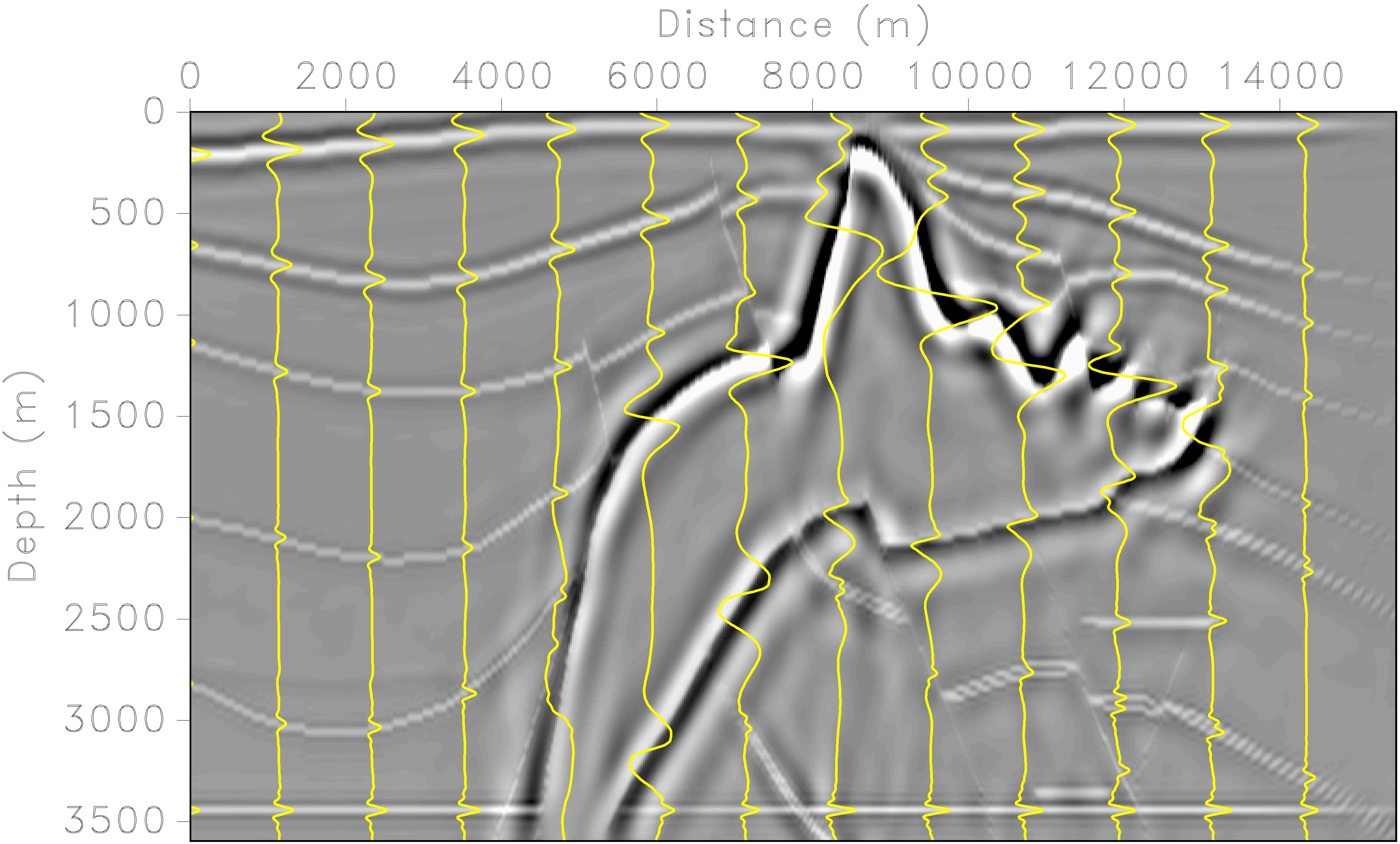


SEG AA' Model w/ Smooth Velocity



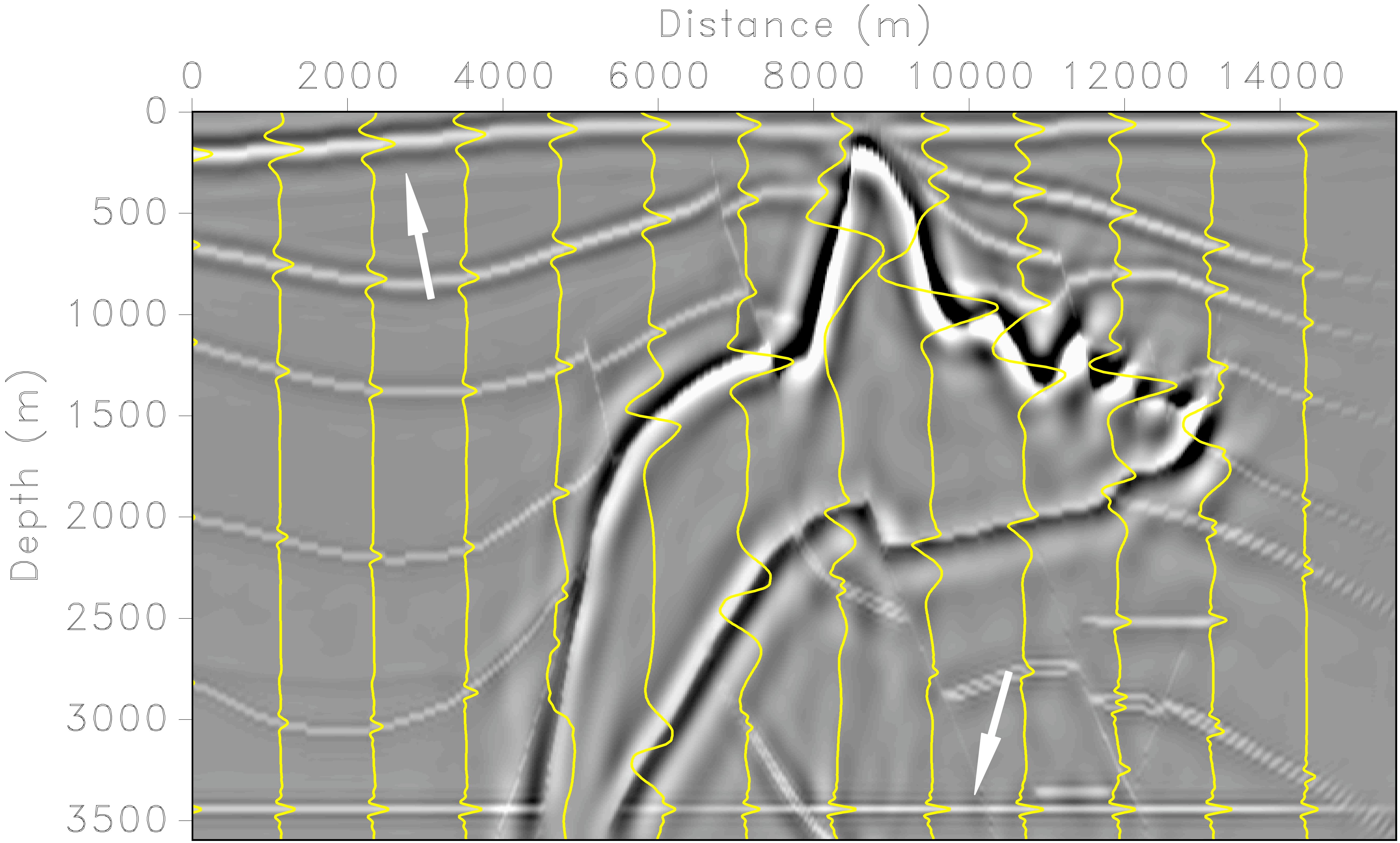
No Preconditioning

SEG AA' Model w/ Smooth Velocity



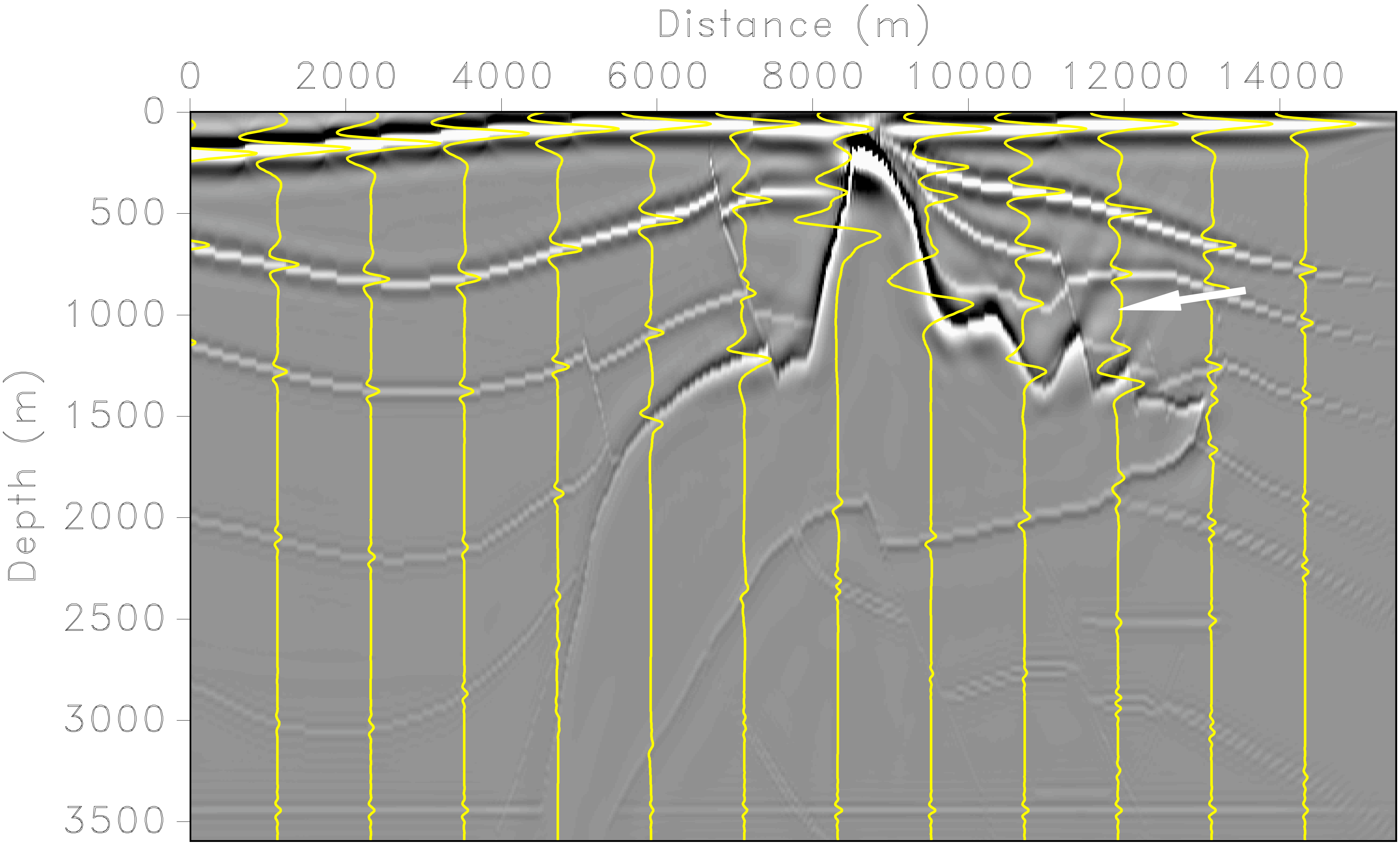
Level III

SEG AA' Model w/ Smooth Velocity



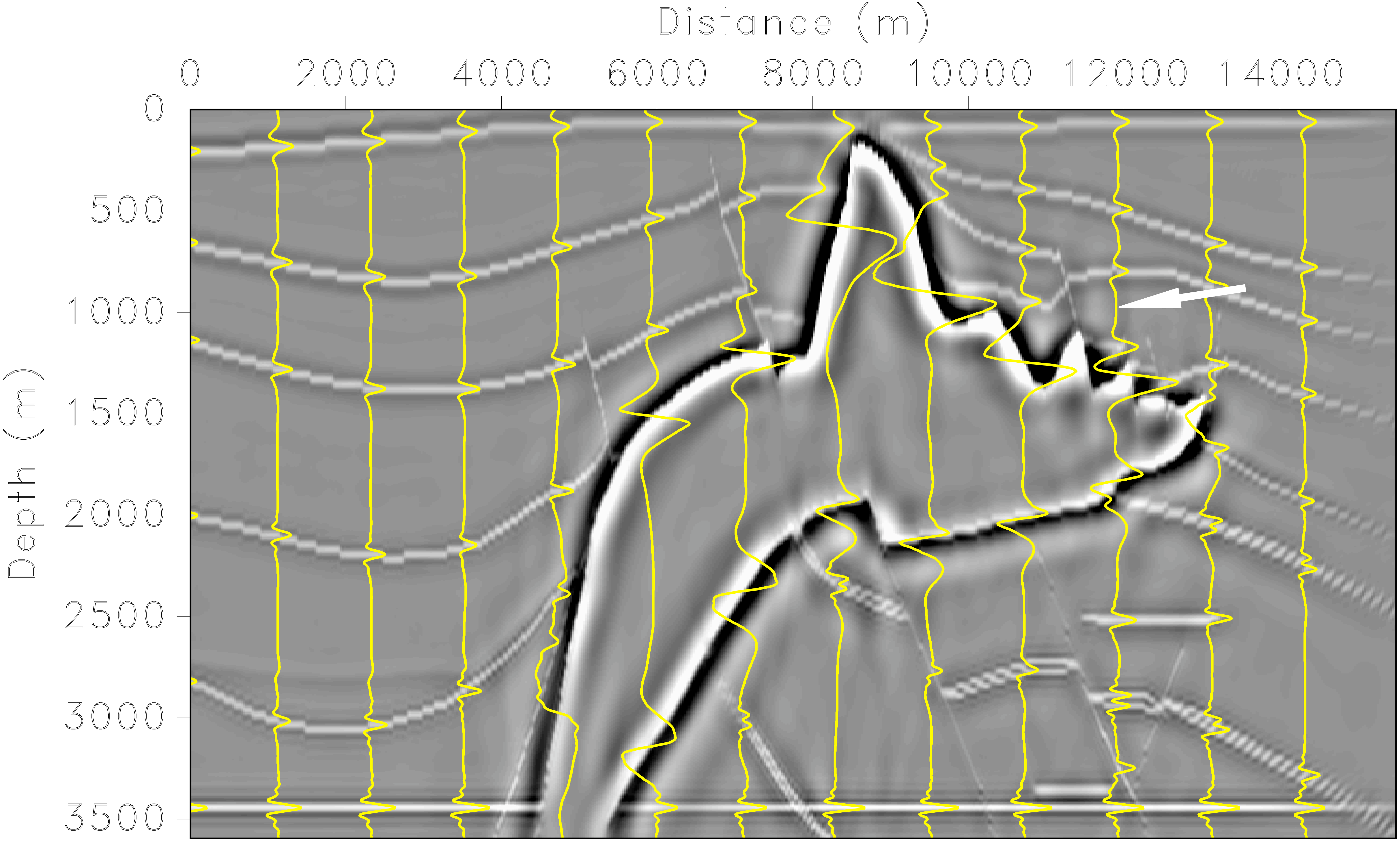
Level III

SEG AA' Model w/ Smooth Velocity - LSQR Results



LSQR 10 iterations - No Preconditioning

SEG AA' Model w/ Smooth Velocity - LSQR Results



LSQR 10 iterations - Level III

SEG AA’ Model w/ Smooth Velocity

- Signal-to-Noise Ratio (SNR) to original reflectivity.
- Defined as follows, with L2 values normalized to one:

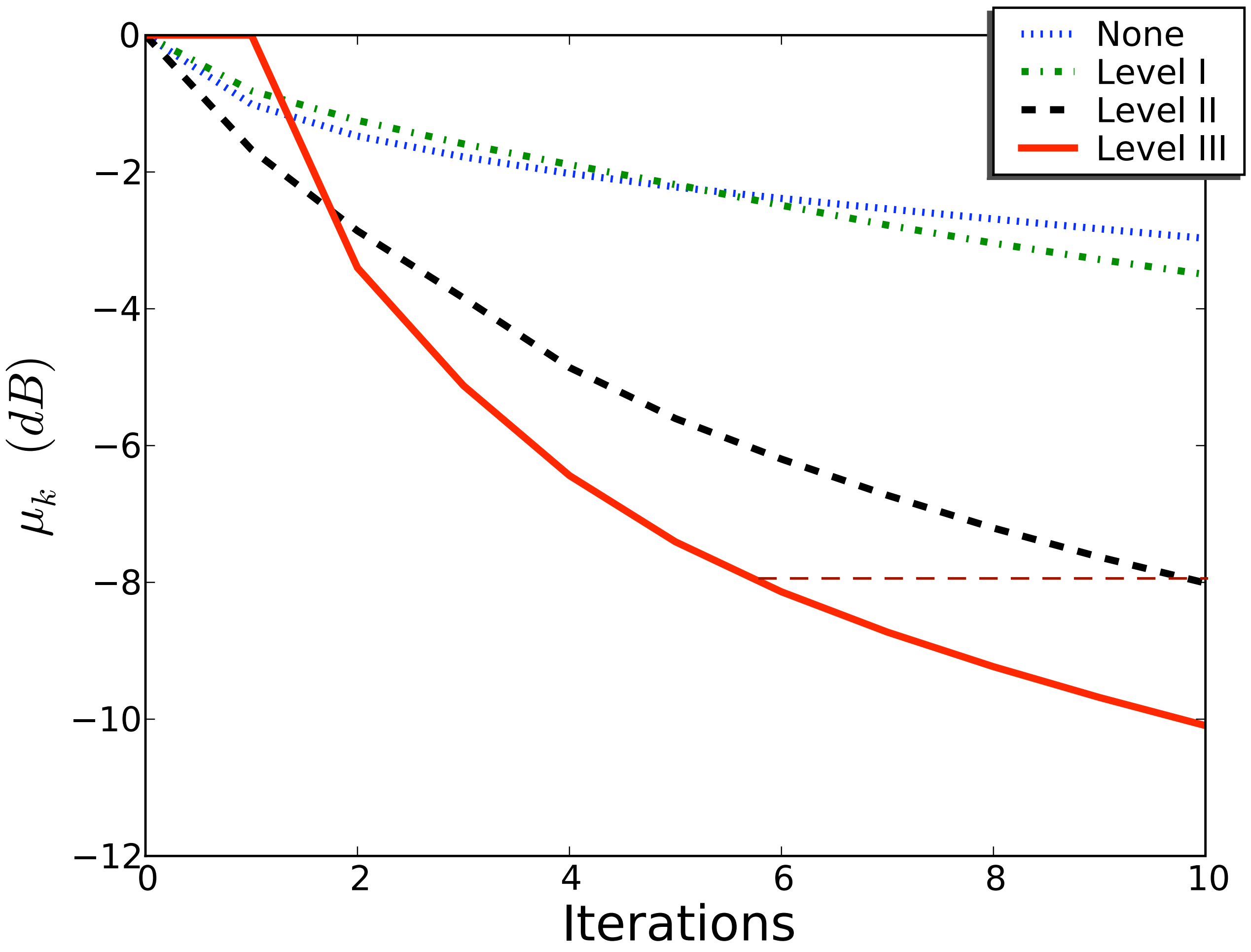
$$SNR = 20\log\|\mathbf{x}_s\|_2/\|\mathbf{x}_n - \mathbf{x}_s\|_2$$

	One iteration SNR	LSQR results* SNR
No Preconditioning	-1.9803	-0.9939
Level I	-1.4147	0.3312
Level II	0.4030	3.2690
Level III	1.3122	3.3230

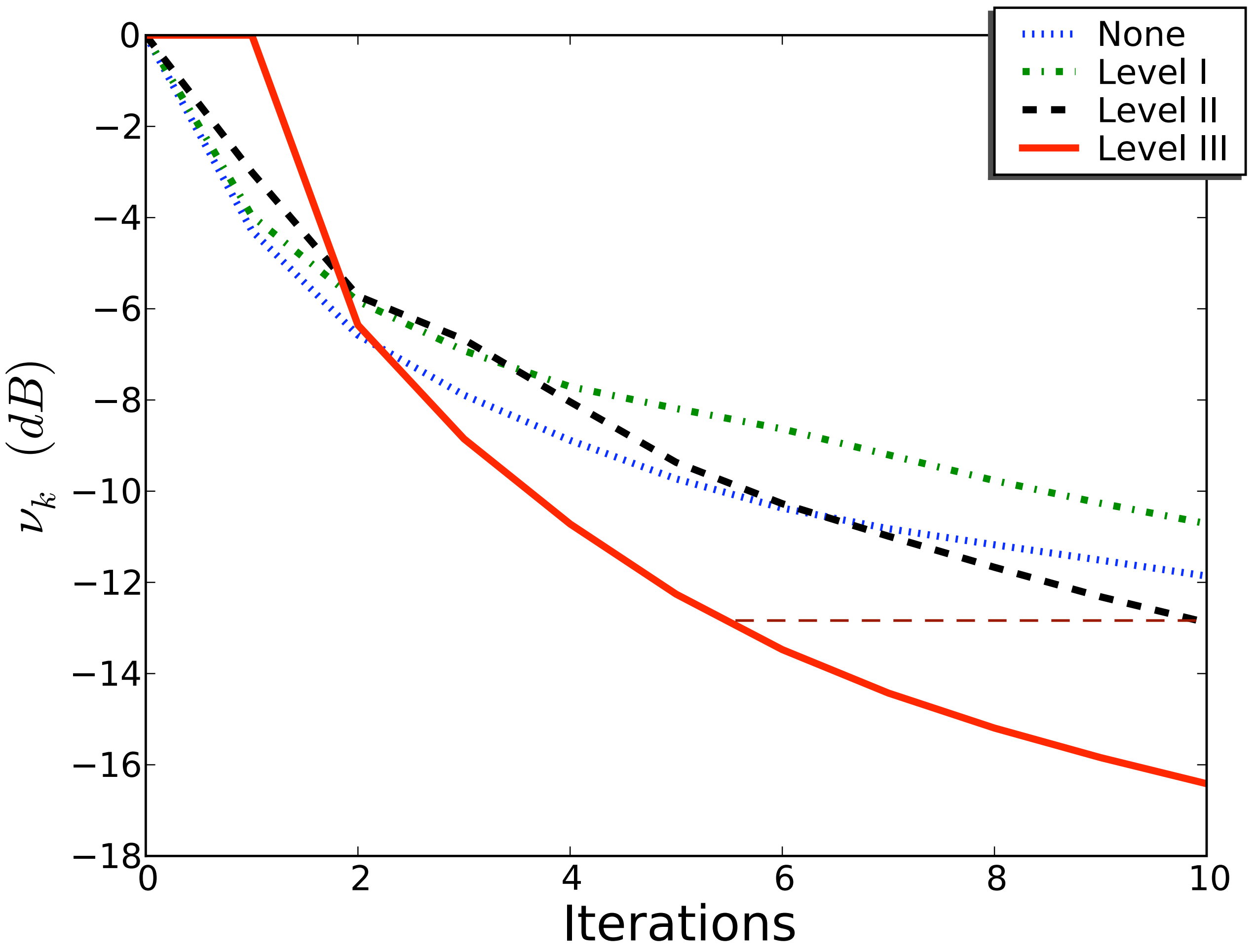
*LSQR to 10 iterations

SEG AA' Model w/ Smooth Velocity

- Residual decay for the data-space and model-space residuals.
- Even after our first few iterations of level III preconditioning, we quickly improve upon the other levels in each figure.
- The red line has already seen one migration-remigration due to the curvelet diagonal estimation process.



$$\mu_k = 20 \log \|\mathbf{A}\mathbf{u}_k - \mathbf{b}\|_2 / \|\mathbf{b}\|_2$$

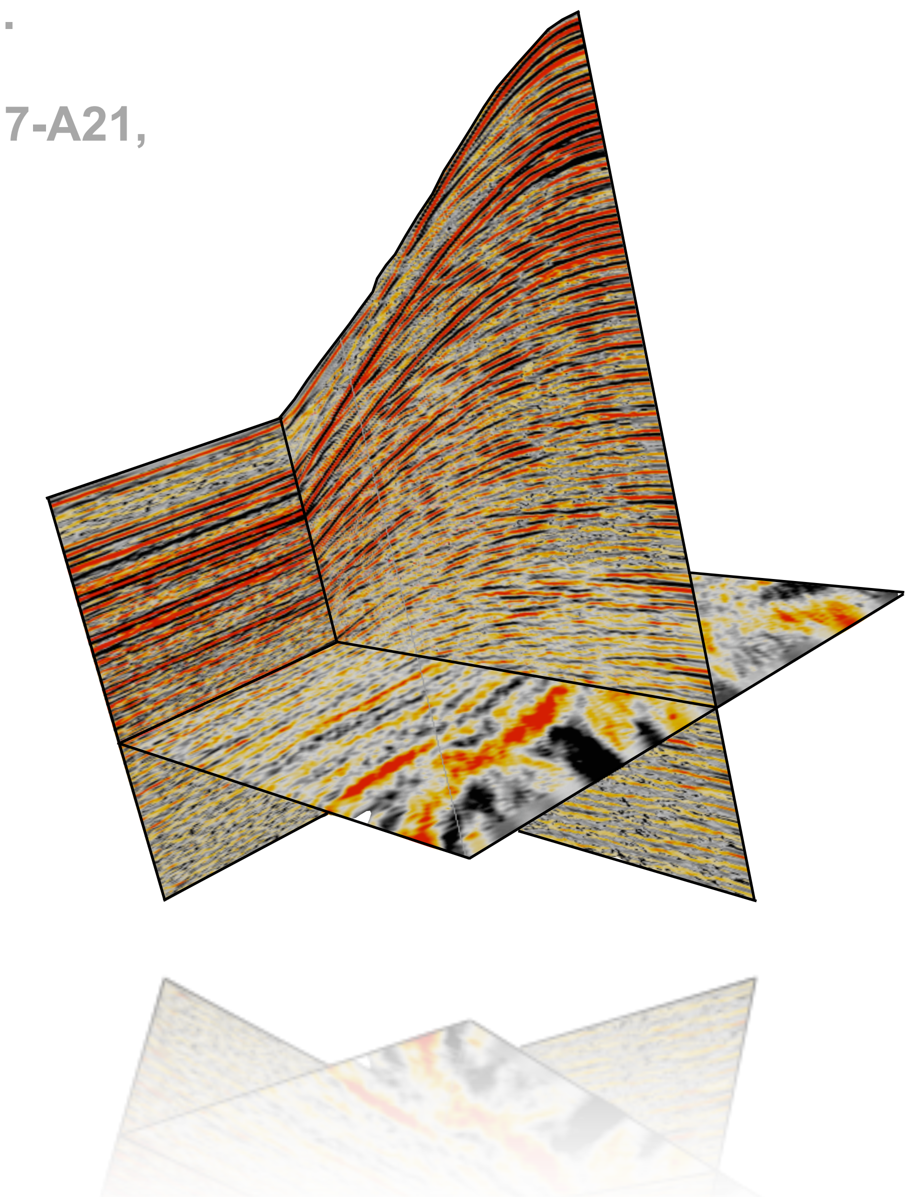


$$\nu_k = 20 \log \|\mathbf{A}^*(\mathbf{A}\mathbf{u}_k - \mathbf{b})\|_2 / \|\mathbf{A}^*\mathbf{b}\|_2$$

[De Roeck, 2002]

Application II: primary-multiple separation

Herrmann, F. J., Wang, D and Verschuur, D. J. Adaptive curvelet-domain primary-multiple separation. Geophysics, Vol 73, No. 3, pp. A17-A21, 2008.



Motivation

- *Transform-domain* matched-filtering forms the basis of
  - *adaptive* subtraction during surface-related multiple elimination [Verschuur ‘97]
  - *idem* during surface-wave removal with interferometry [Vasconcelos ’08, Wapenaar ‘08]
  - scaling during migration “preconditioning” based on migrated-remigrated image matching [Symes ’08,F.J.H. et. al, ‘08]

- *Fourier-based* matching
  - accounts for amplitude-spectra mismatches & global kinematic errors
  - fails for errors that vary spatially & as function of the local dip

- *Spatial & windowed Fourier* matching
  - run risk of over fitting (loss of primary energy)

- *Curvelet-domain* matching in phase space
  - corrects for *amplitude* errors that vary *smoothly* as a function of position & dip

History

- *Fourier*-based matched filtering was built on the premise that

$$\mathbf{m}_{\text{true}} \approx \mathbf{F}\mathbf{m}_{\text{predicted}} \text{ with } \mathbf{F} = \mathcal{F}^H \text{diag}\left(\hat{\mathbf{f}}\right)\mathcal{F}$$

- Estimated during a *global* least-squares estimation procedure

$$\hat{\mathbf{f}} = \arg\min_{\hat{\mathbf{g}}} \frac{1}{2}\|\hat{\mathbf{d}} - \hat{\mathbf{g}}\hat{\mathbf{m}}_{\text{predicted}}\|_2^2 + \lambda\|\mathbf{L}_{\mathcal{F}}\hat{\mathbf{g}}\|_2^2$$

  - $\mathbf{L}_{\mathcal{F}}$ *Fourier-space* sharpening operator that promotes smoothness
  - for each offset separately
- Estimated primaries:

$$\tilde{\mathbf{p}} = \mathbf{d} - \mathbf{F}\mathbf{m}_{\text{predicted}}$$

Workflow

input data

$$\mathbf{m}_{\text{predicted}} \;\; (\text{multi-D convolution})$$

conservative Fourier matching

$$\mathbf{m}_0 = \mathbf{F}\mathbf{m}_{\text{predicted}} \text{ with } \mathbf{F} = \mathcal{F}^H \text{diag}\left(\hat{\mathbf{f}}\right)\mathcal{F}$$

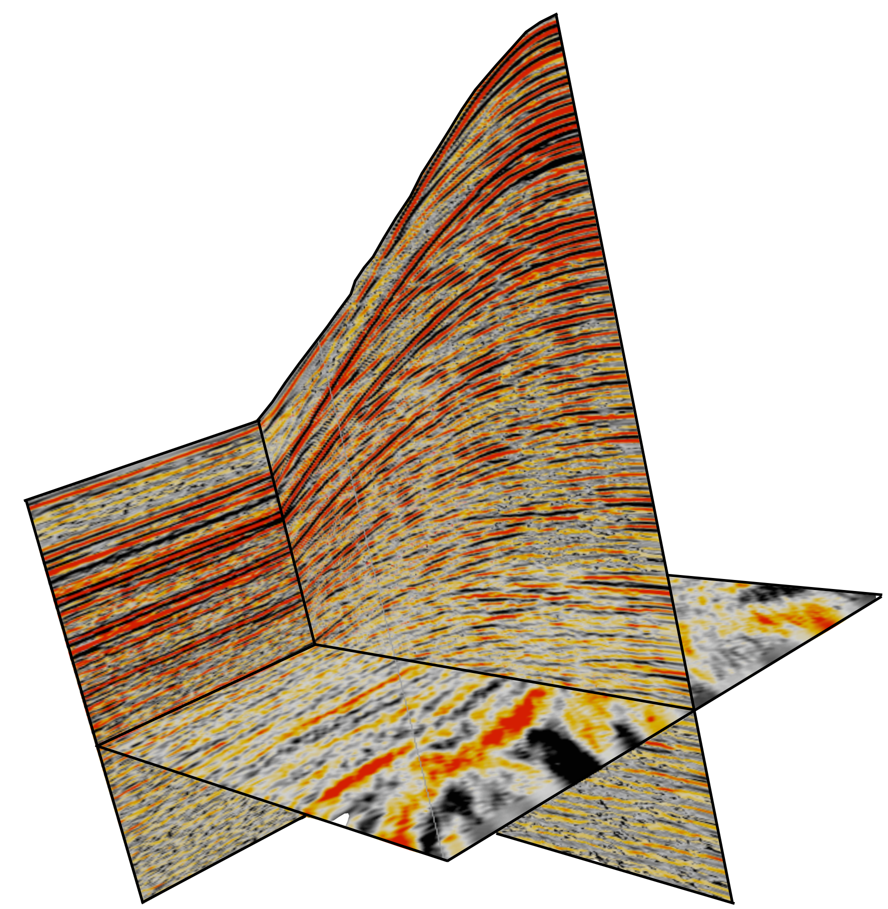
curvelet-domain matching

$$\begin{aligned}\mathbf{b}_2 &= \mathbf{B}\mathbf{m}_0 \\ \text{with } \mathbf{B} &= \mathbf{C}^T \text{diag}\left(e^{\mathbf{z}}\right)\mathbf{C}\mathbf{m}_0 \\ &\approx \mathcal{F}^H b(x,k)\mathcal{F}\mathbf{m}_0\end{aligned}$$

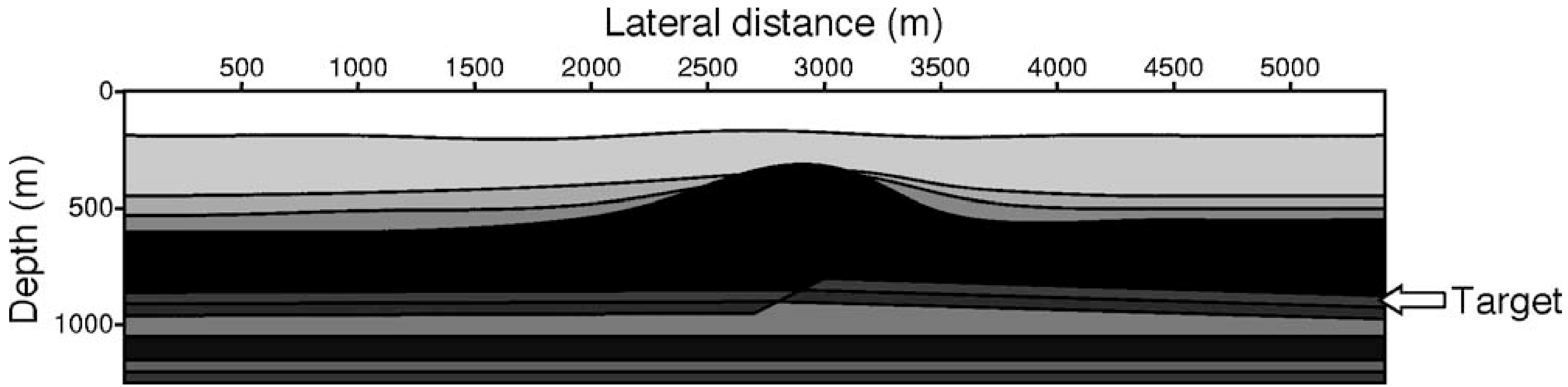
Bayesian separation

$$\mathbf{P}_{\mathbf{w}}: \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda_1 \|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2 \|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2\|_2^2 + \eta\|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

Synthetic-data example

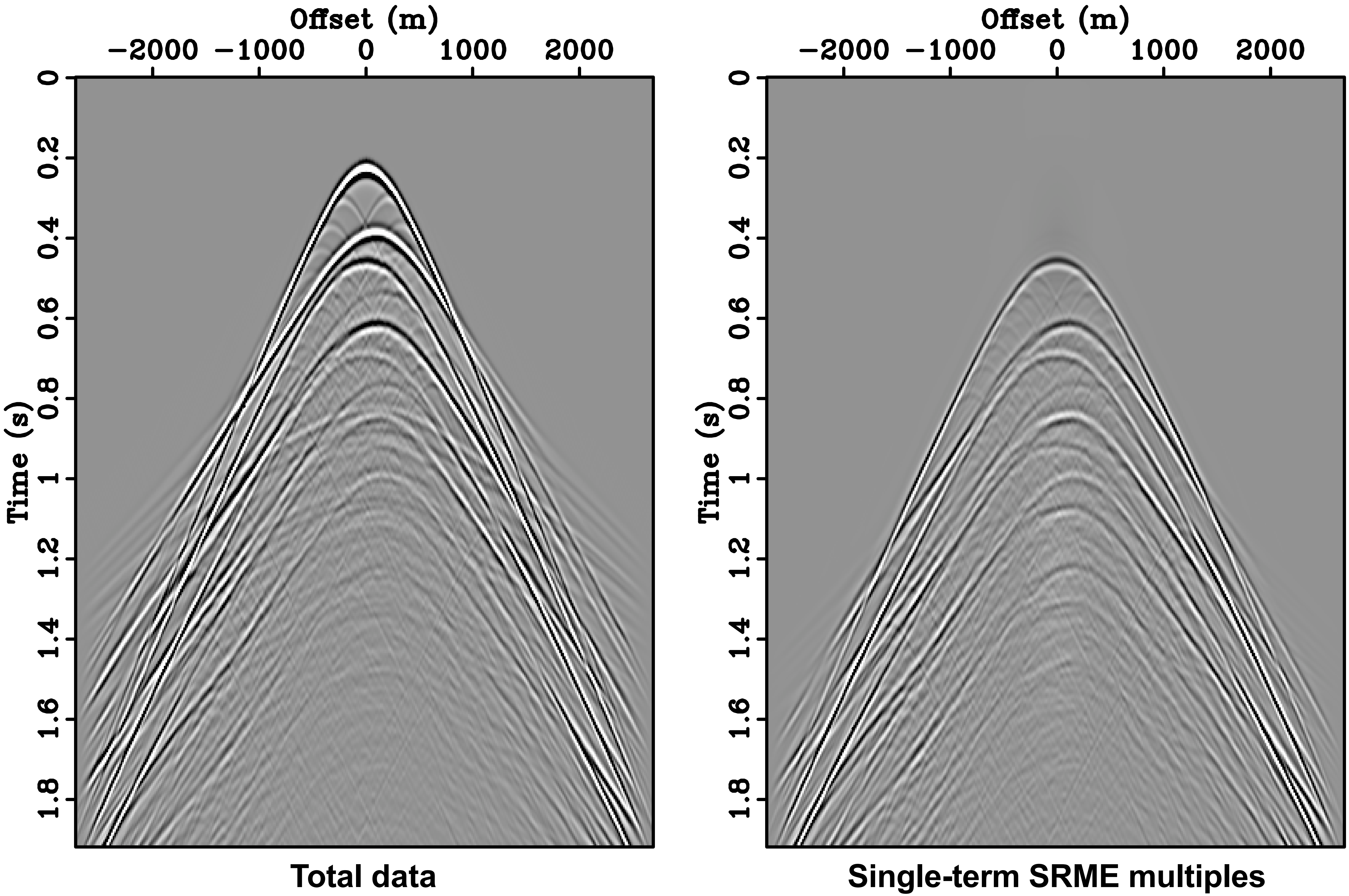


Synthetic-data example

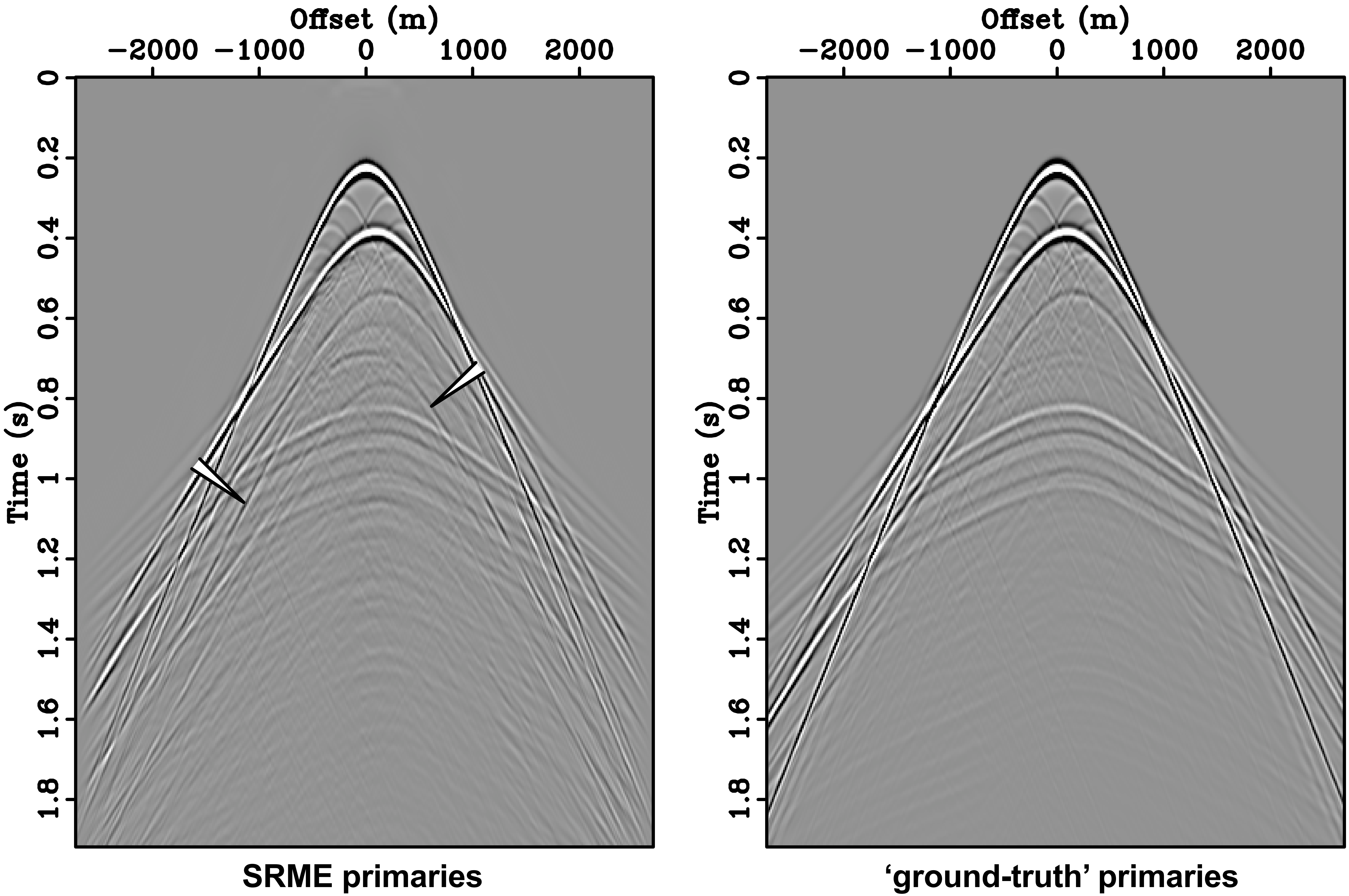


Velocity model used in the synthetic data examples

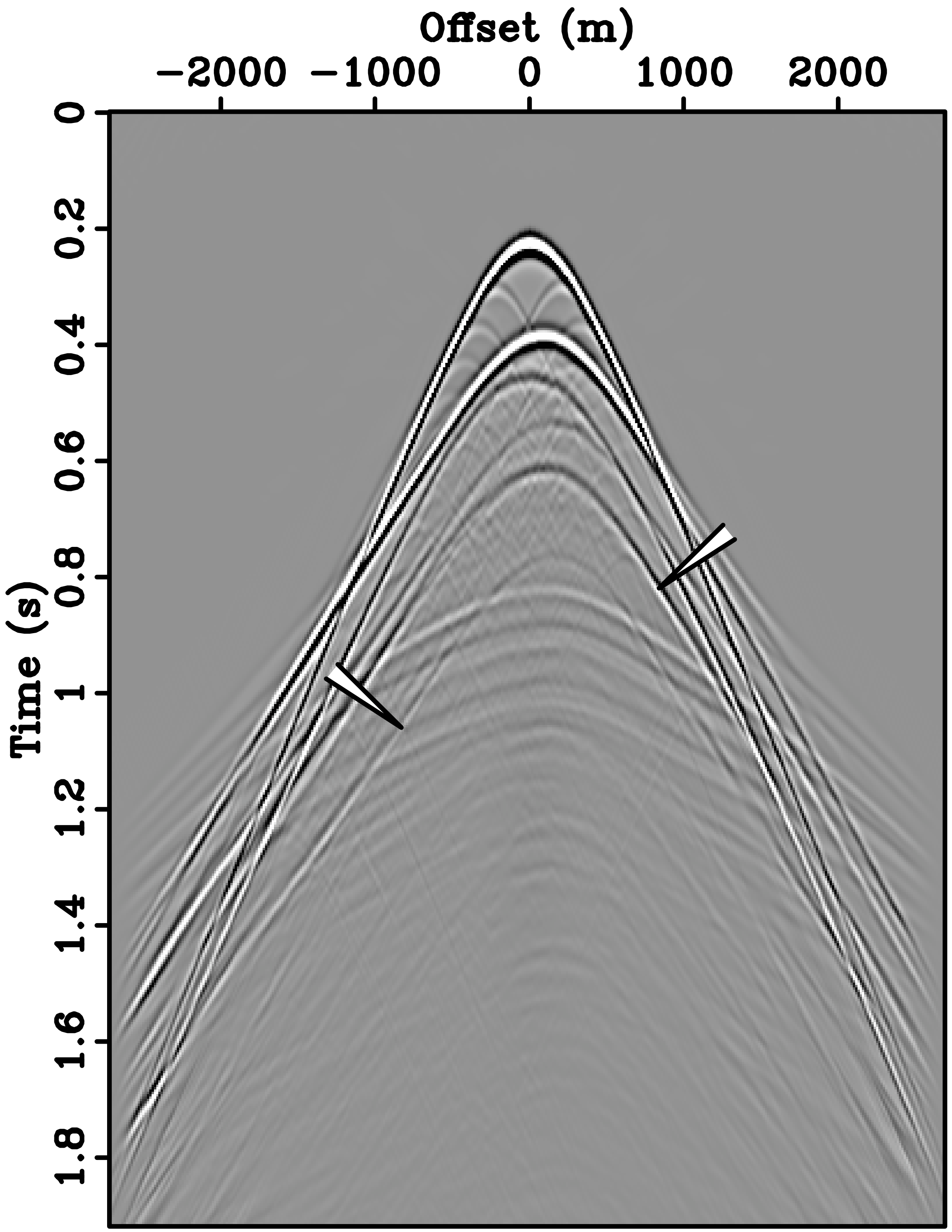
Synthetic-data example



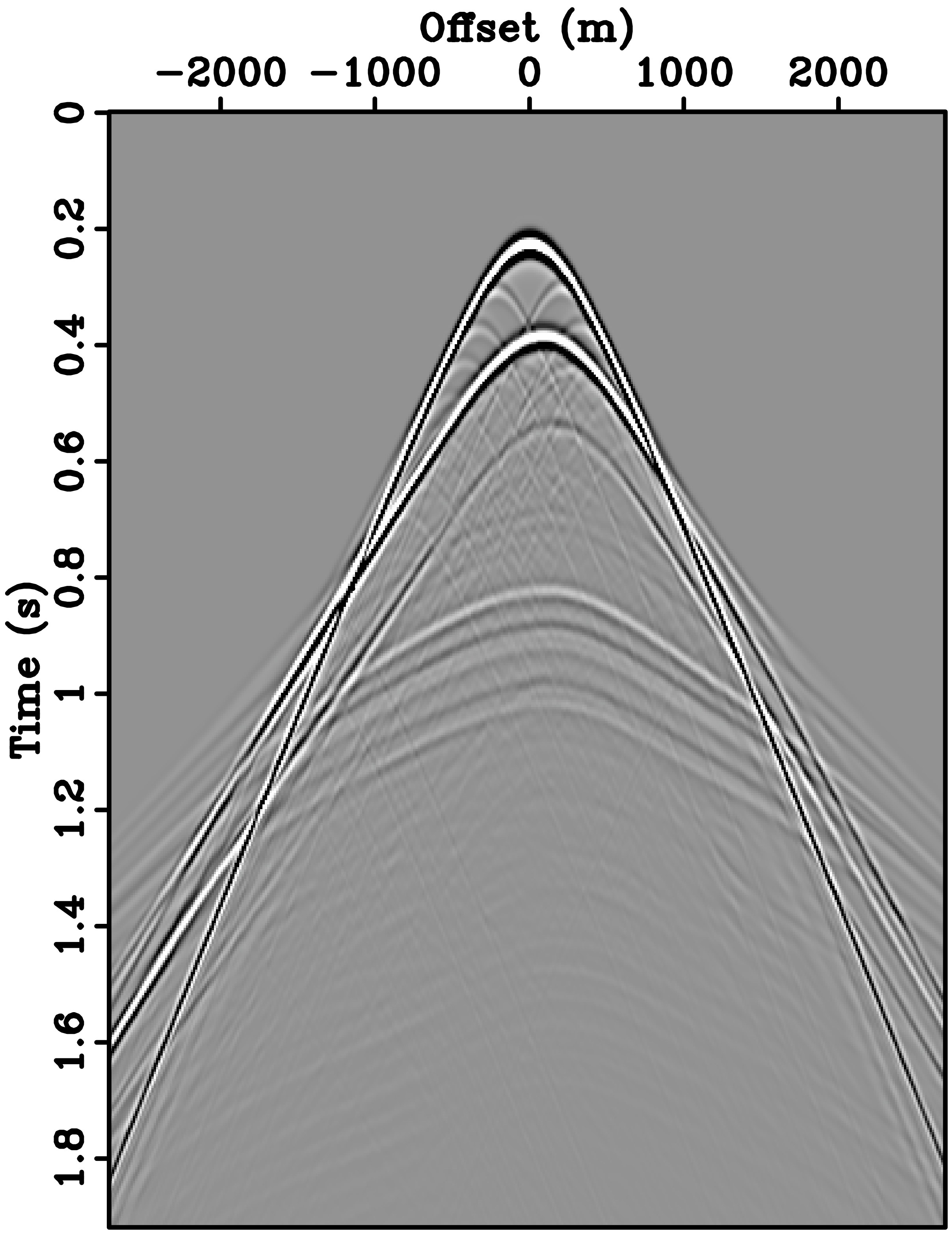
Synthetic-data example



Synthetic-data example

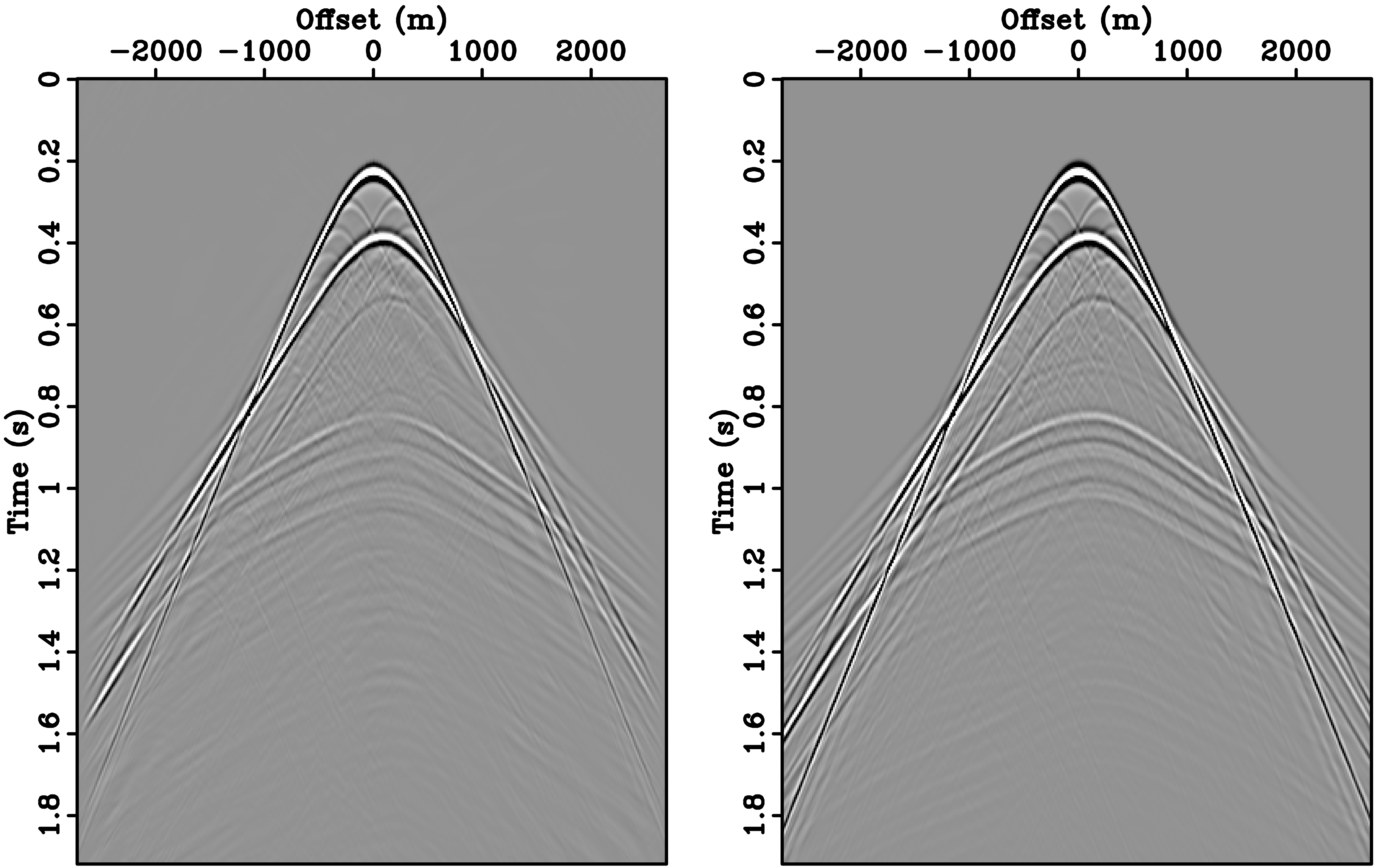


No matching Bayesian



‘ground-truth’ primaries

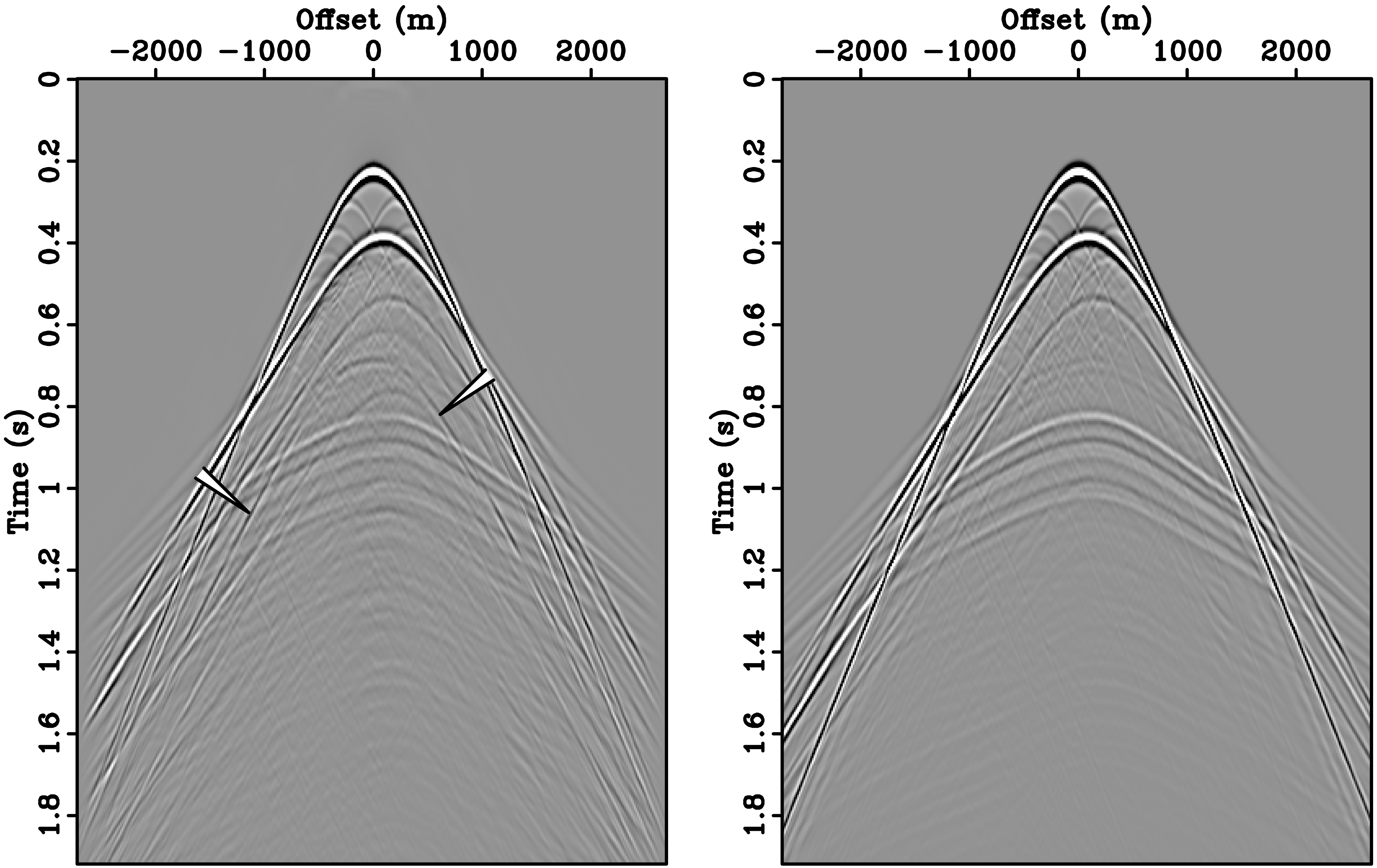
Synthetic-data example



Matched + Bayesian

'ground-truth' primaries

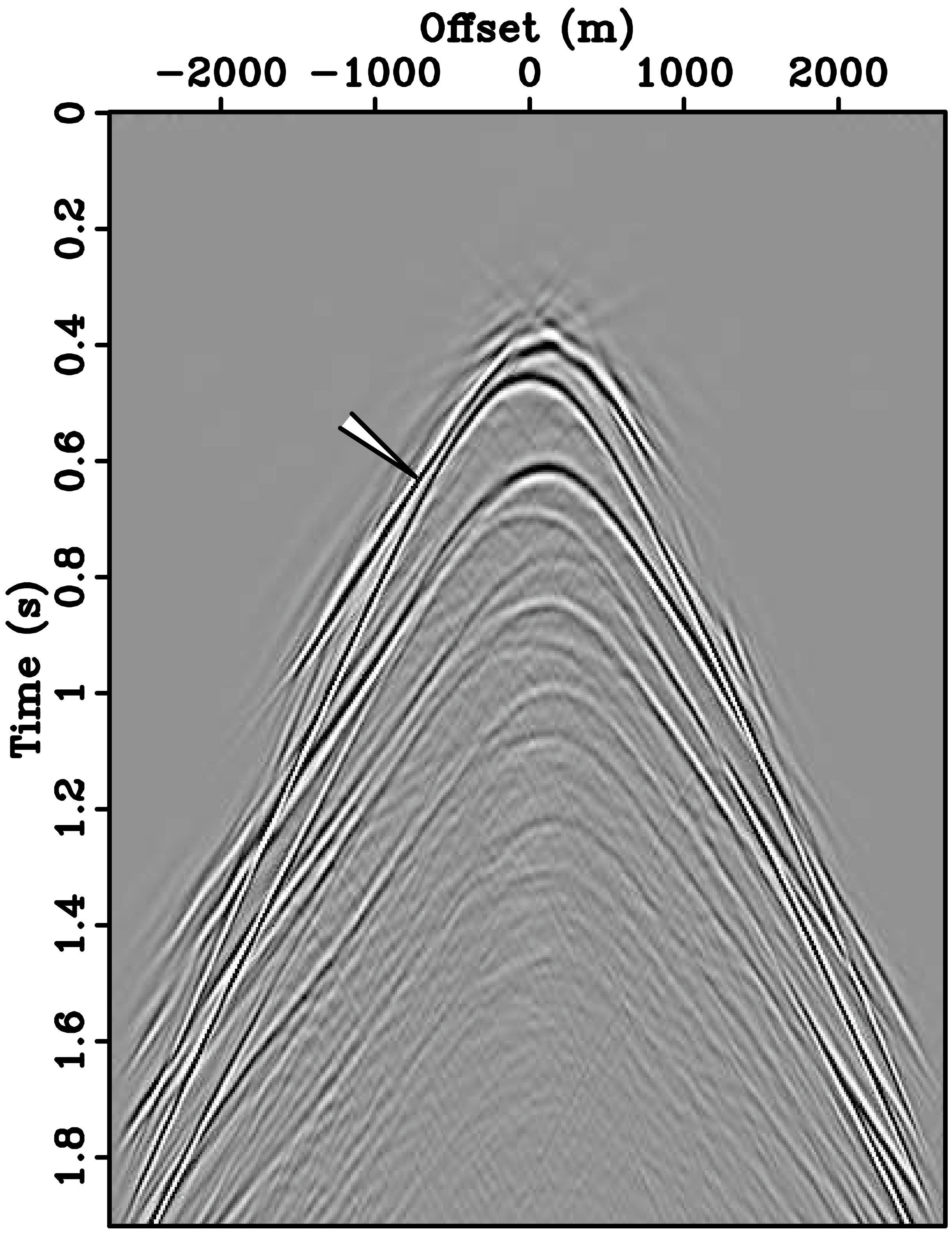
Synthetic-data example



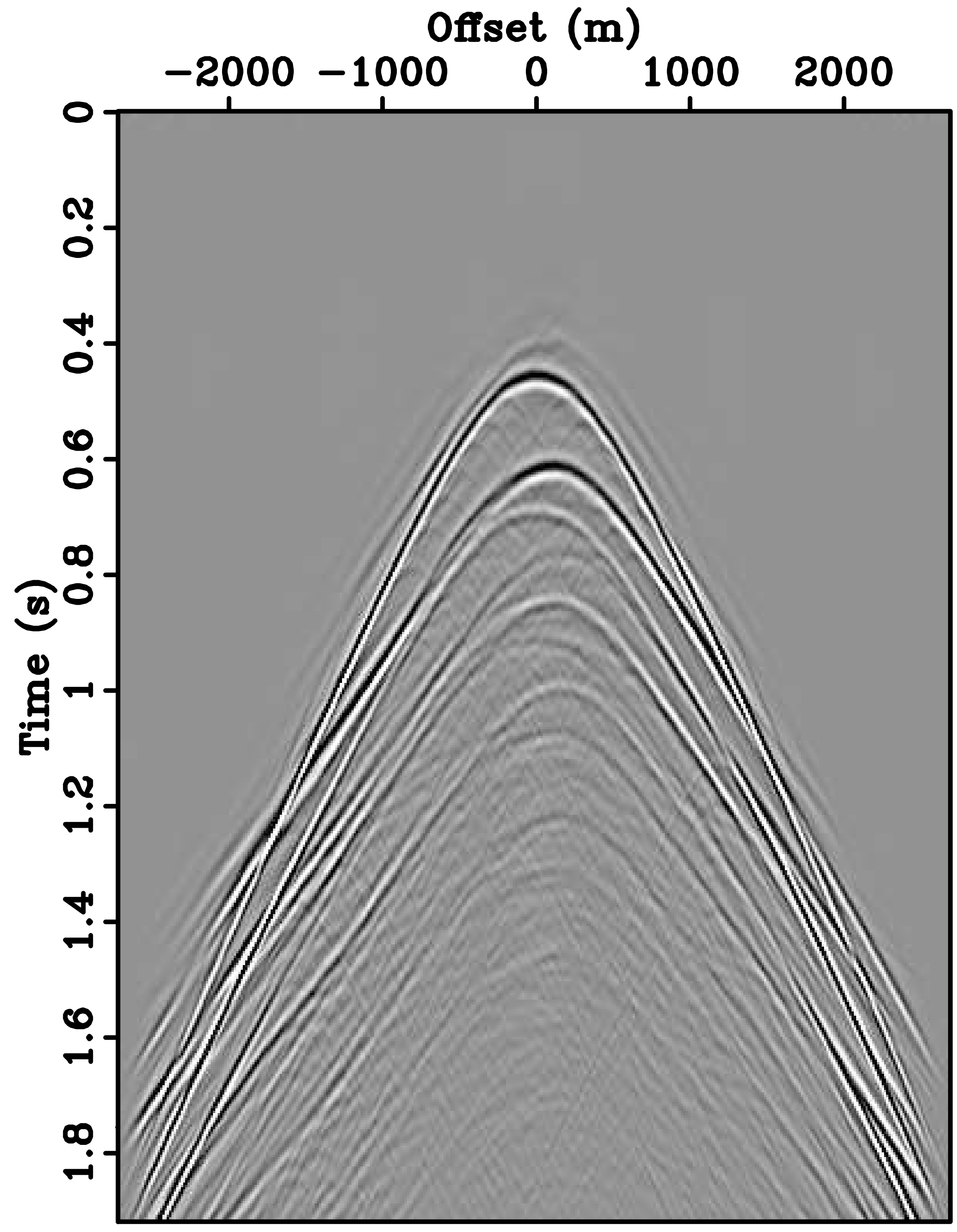
SRME primaries

‘ground-truth’ primaries

Synthetic-data example

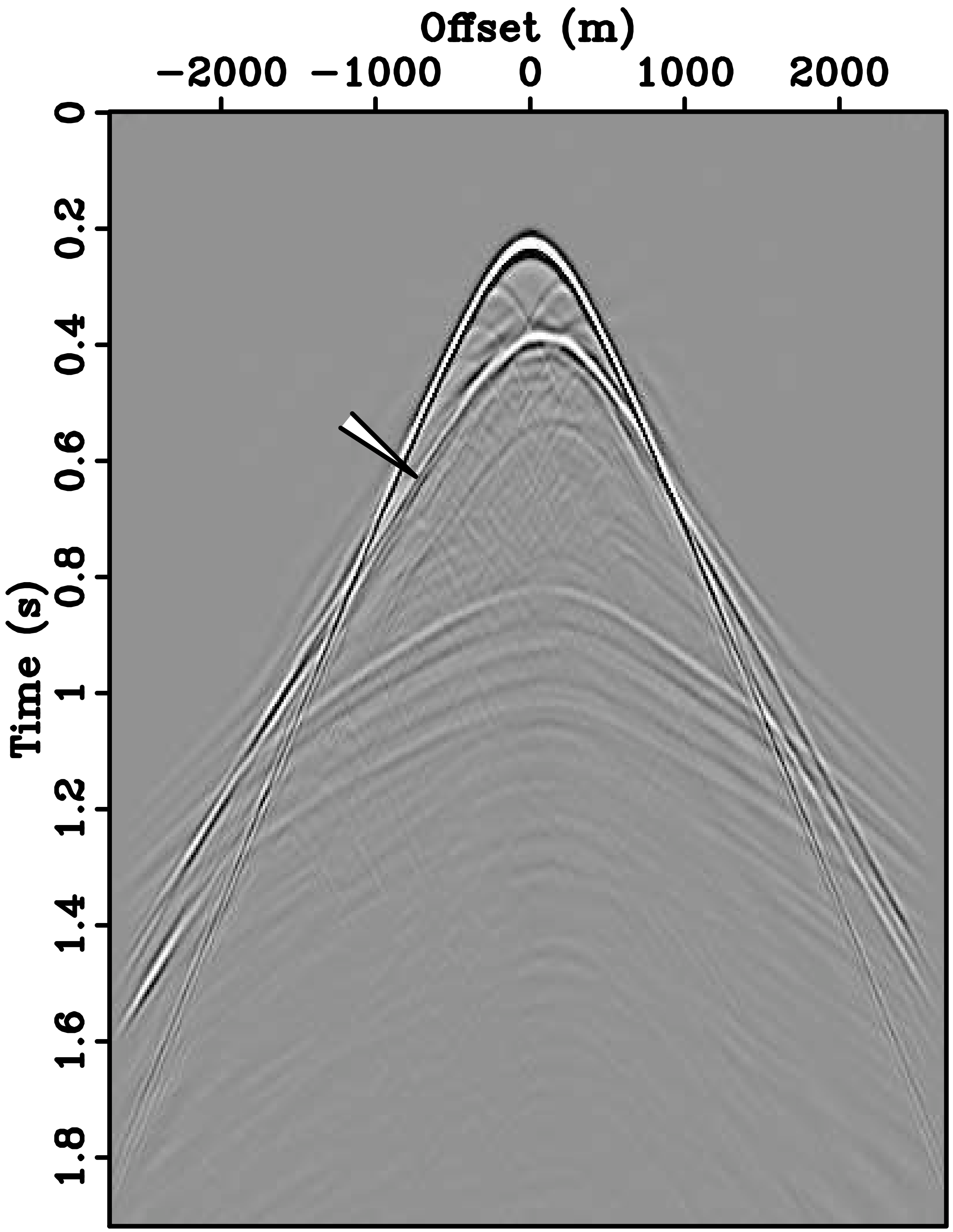


Over matched multiples

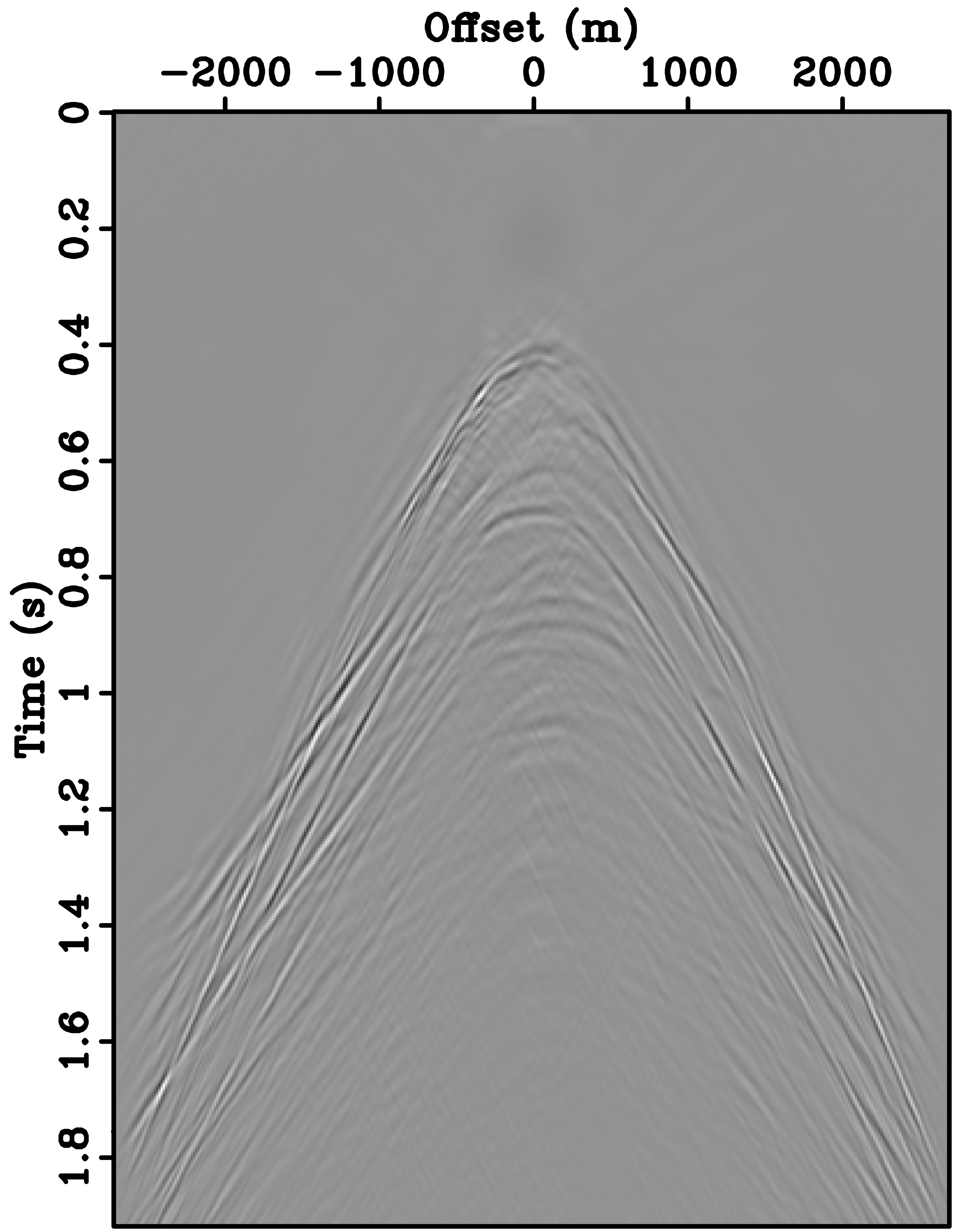


Correctly matched multiples

Synthetic-data example



Estimate for the primaries with over matched multiples



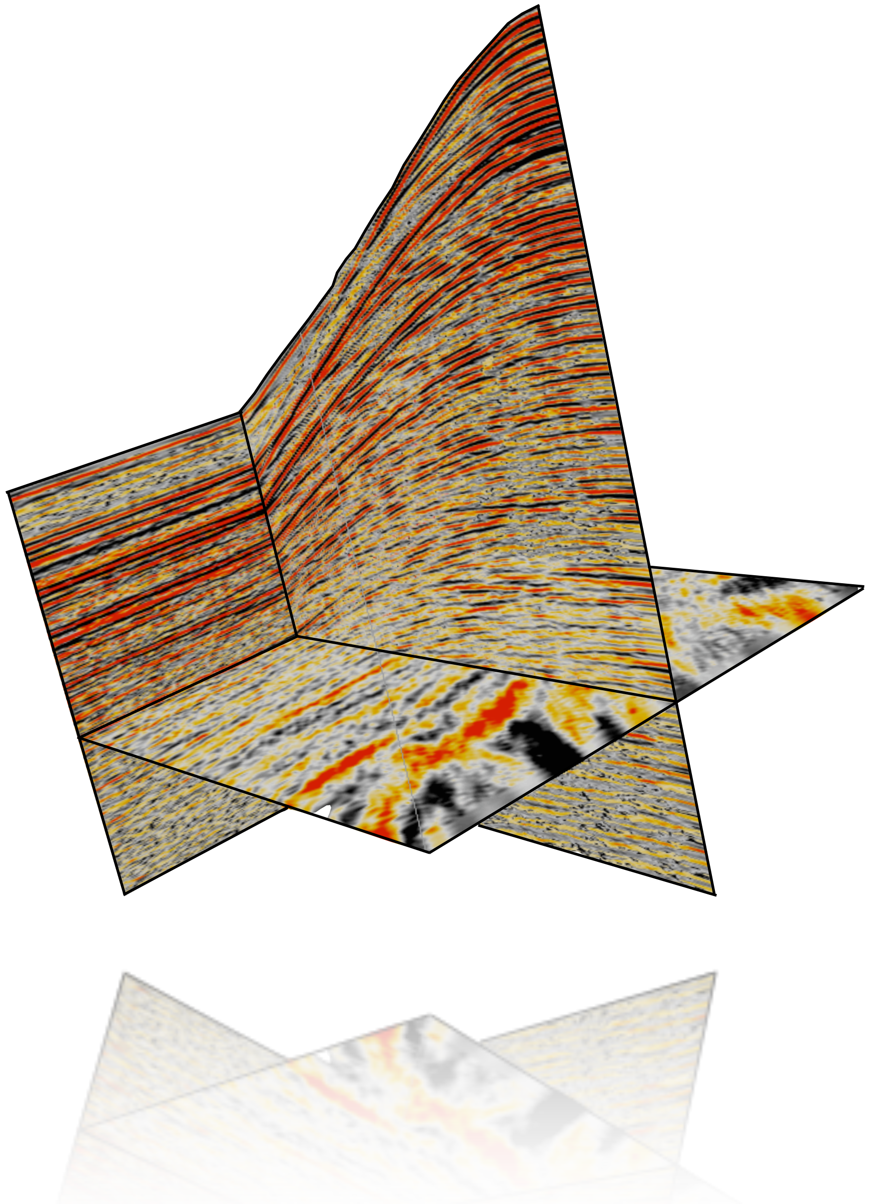
Difference between SRME and curvelet matching

SNRs

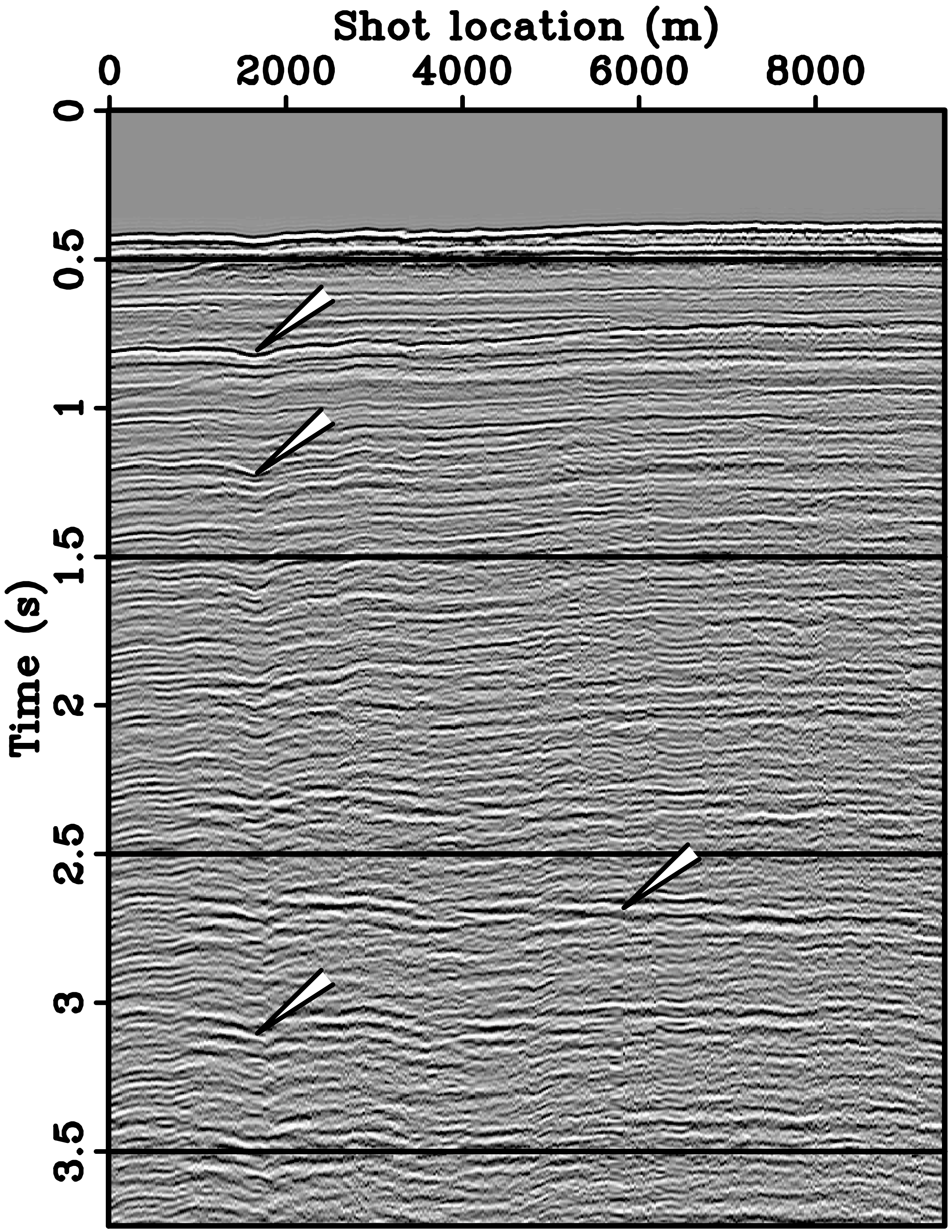
- Comparison with “ground truth”

SRME	9.82
Bayesian separation	7.25
Curvele-domain matching & Bayesuan	11.22

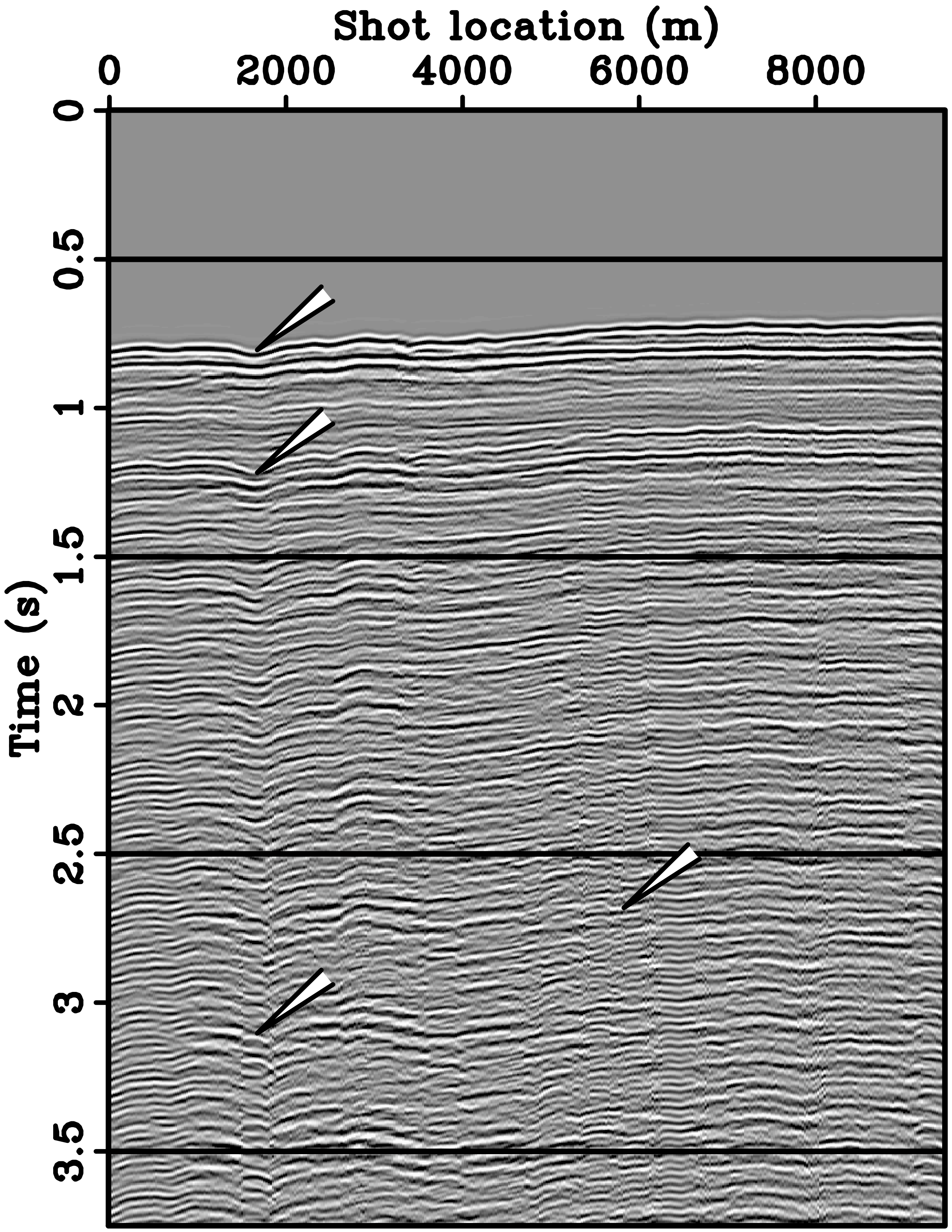
Real-data example



Real-data example

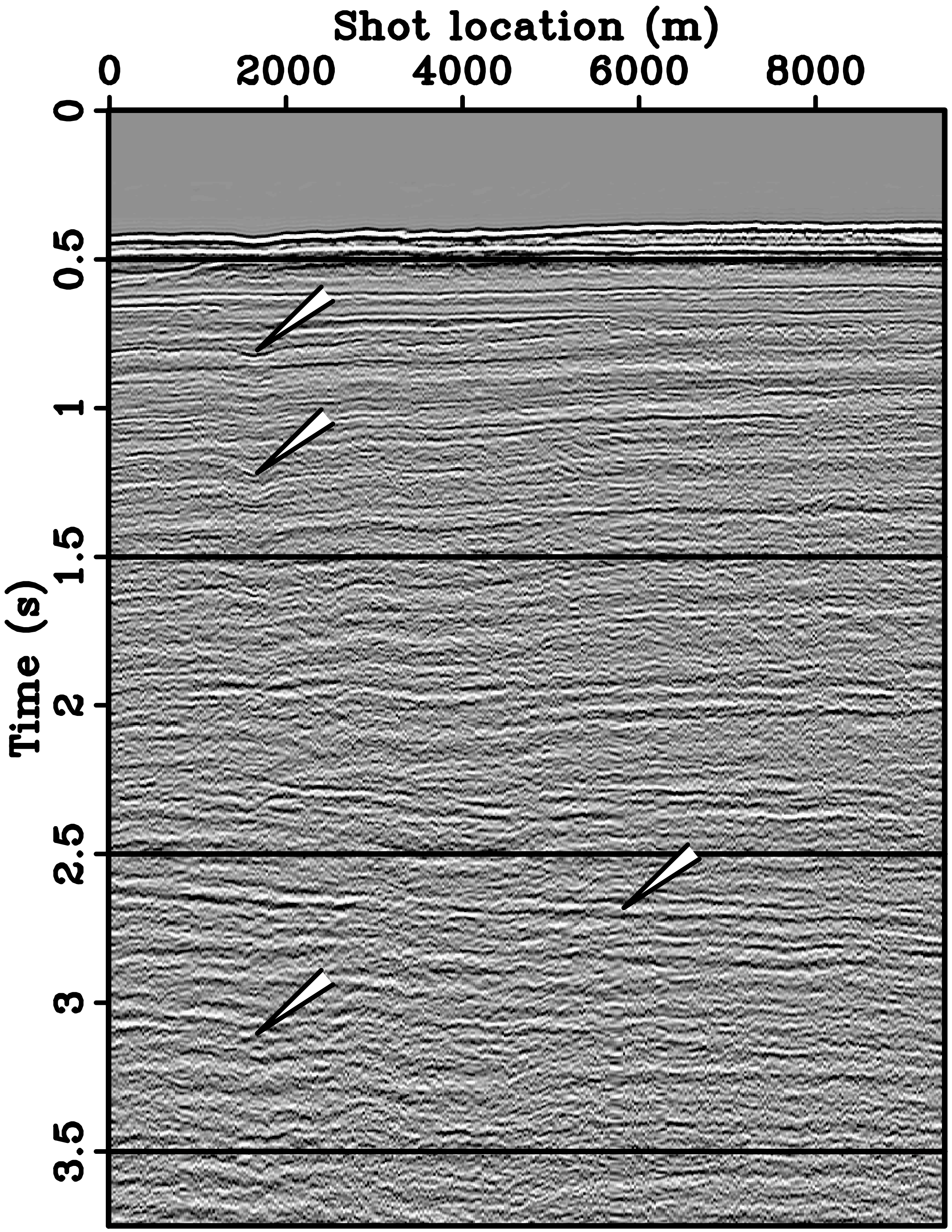


Total data

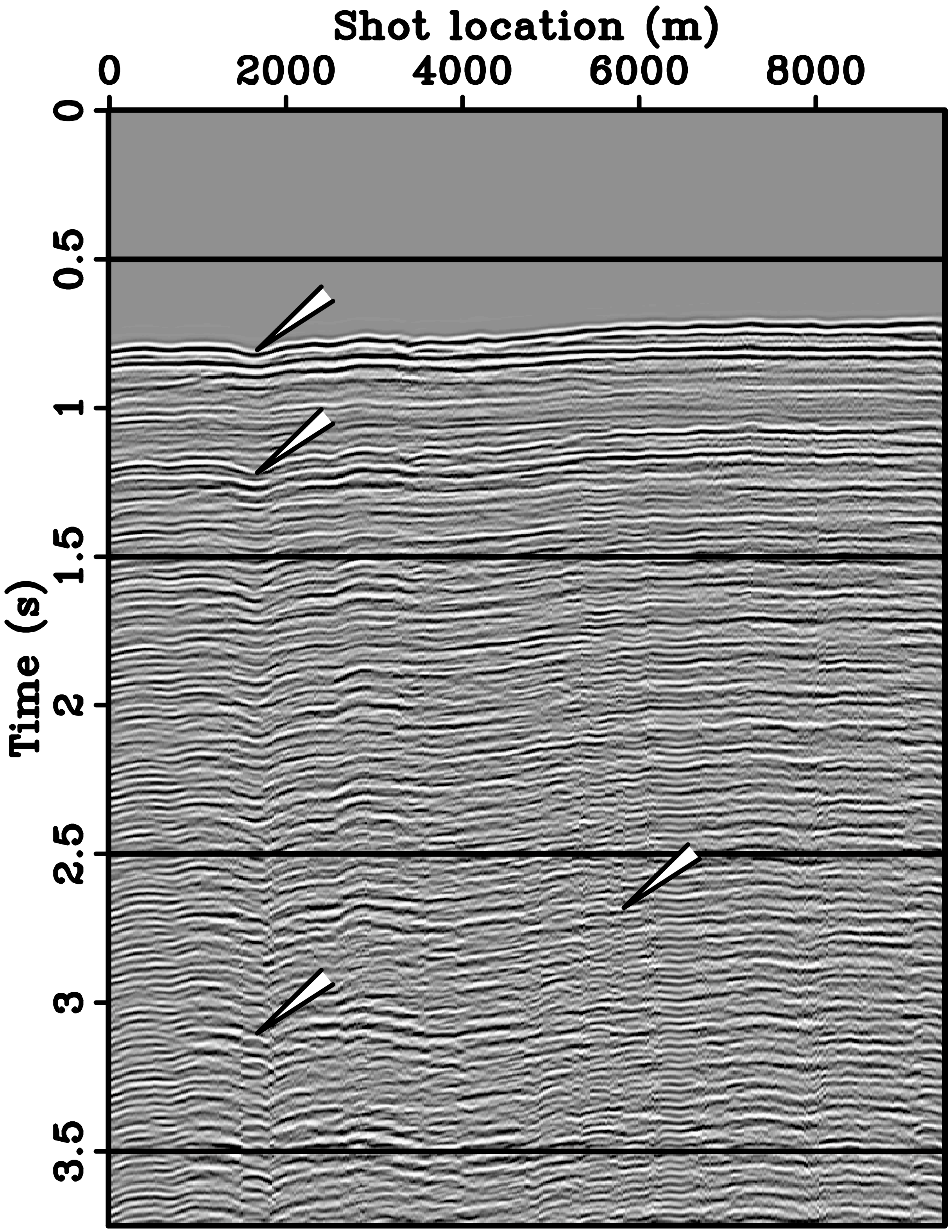


Predicted multiples

Real-data example

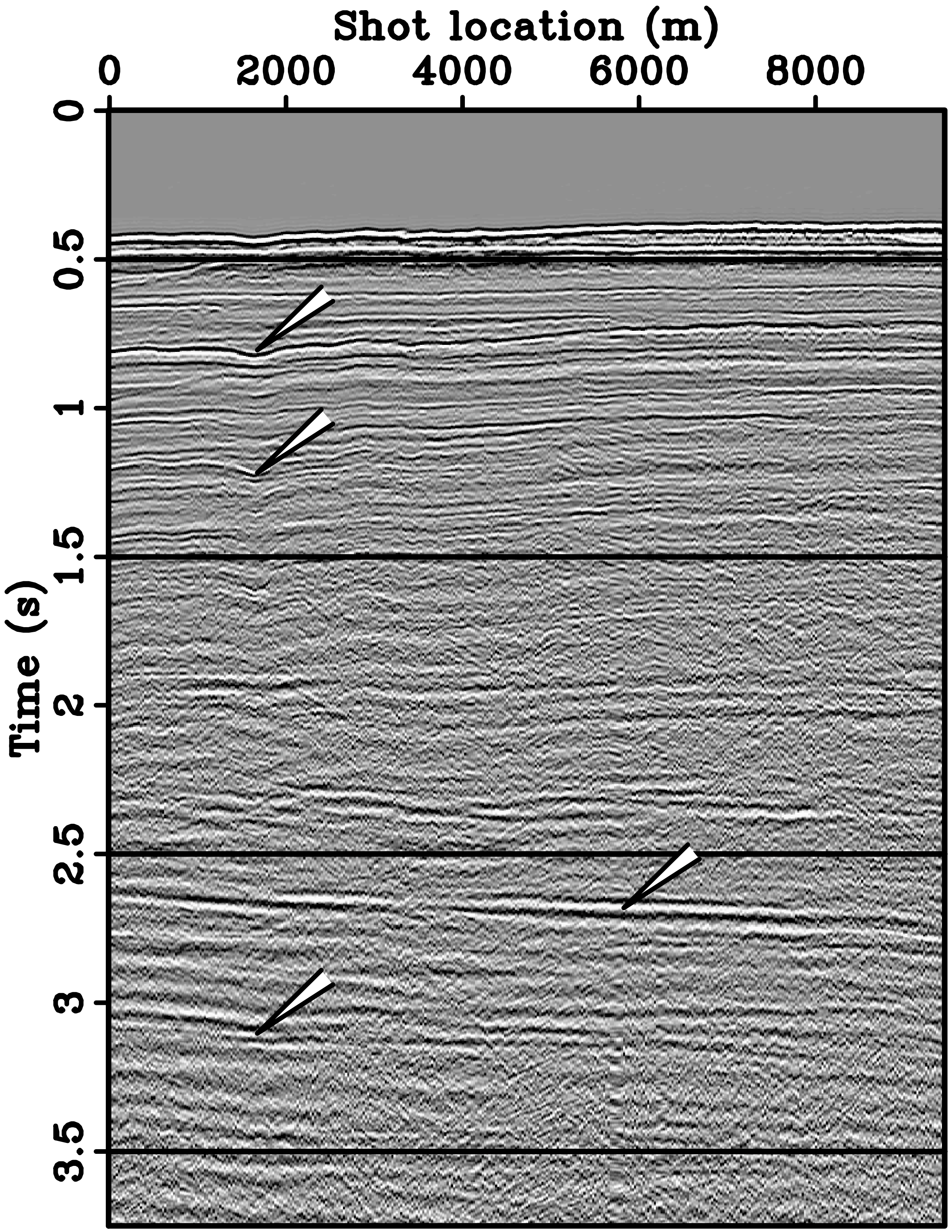


SRME primaries

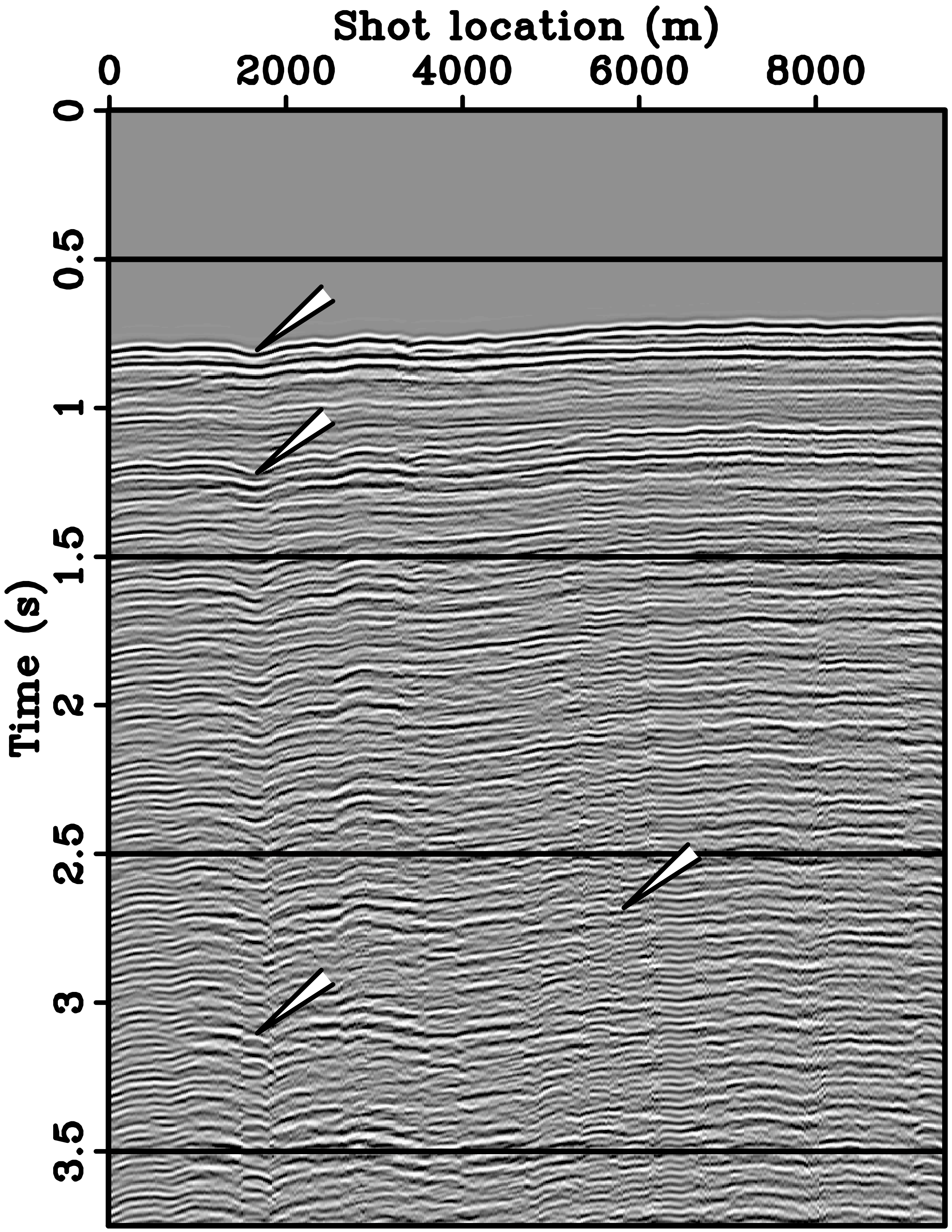


Predicted multiples

Real-data example

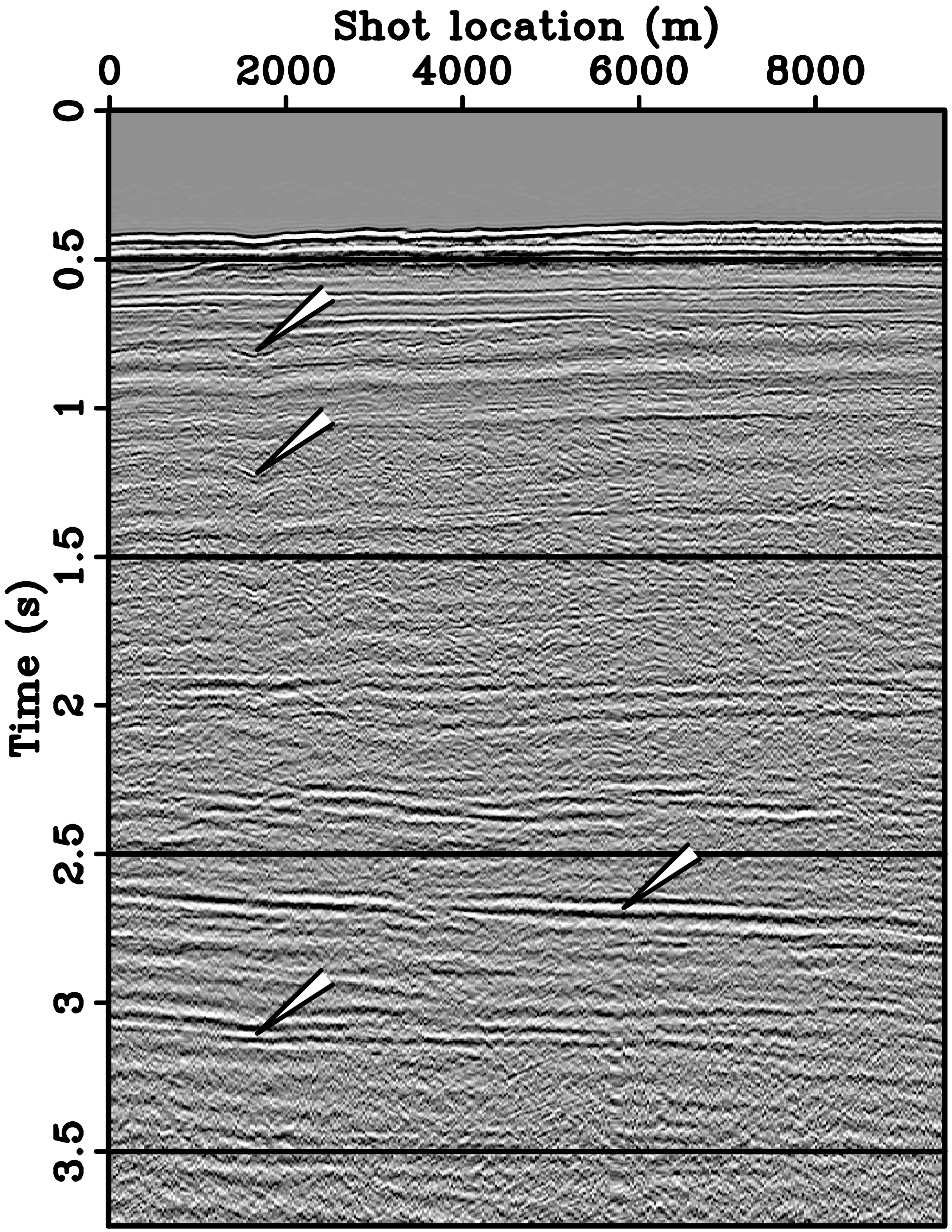


Not scaled Bayesian

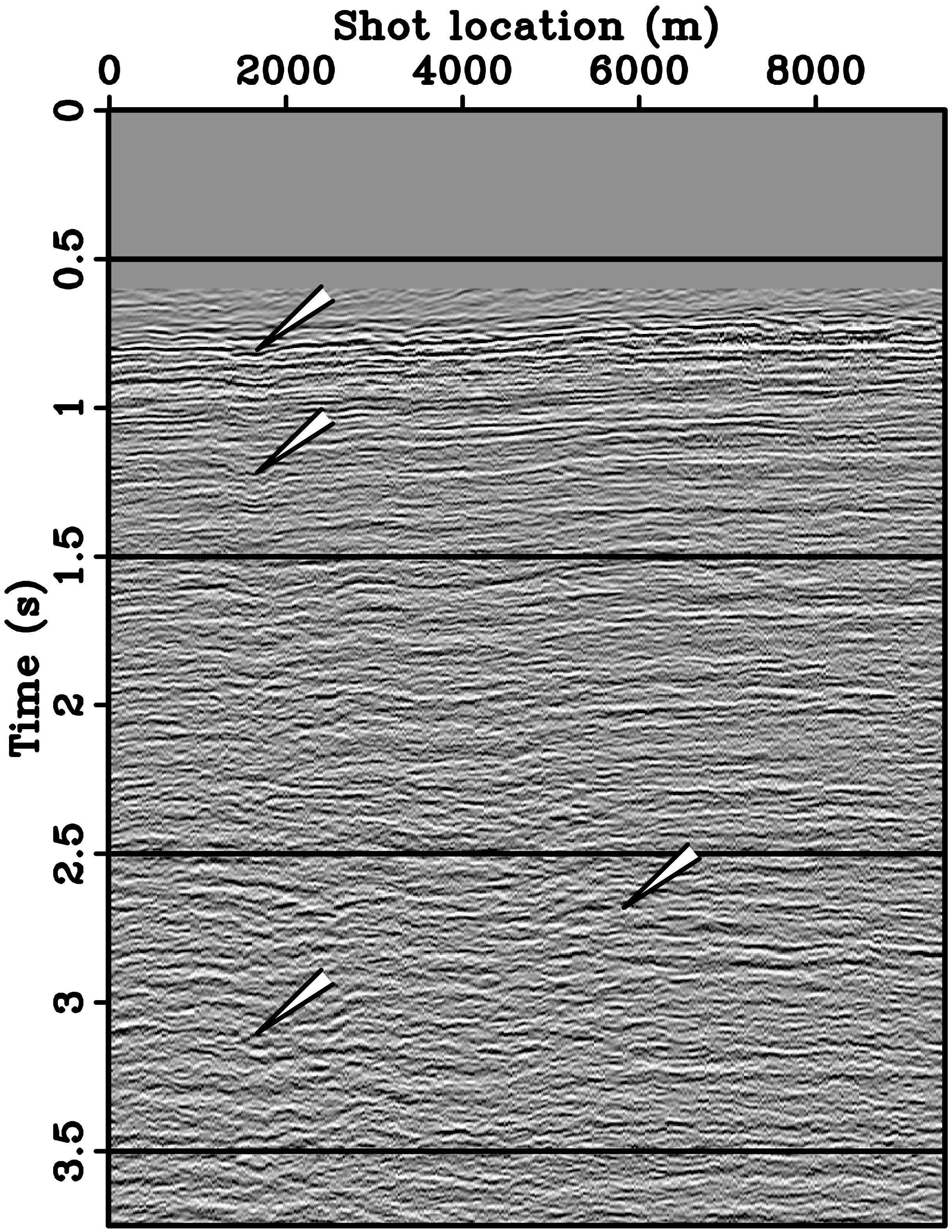


Predicted multiples

Real-data example

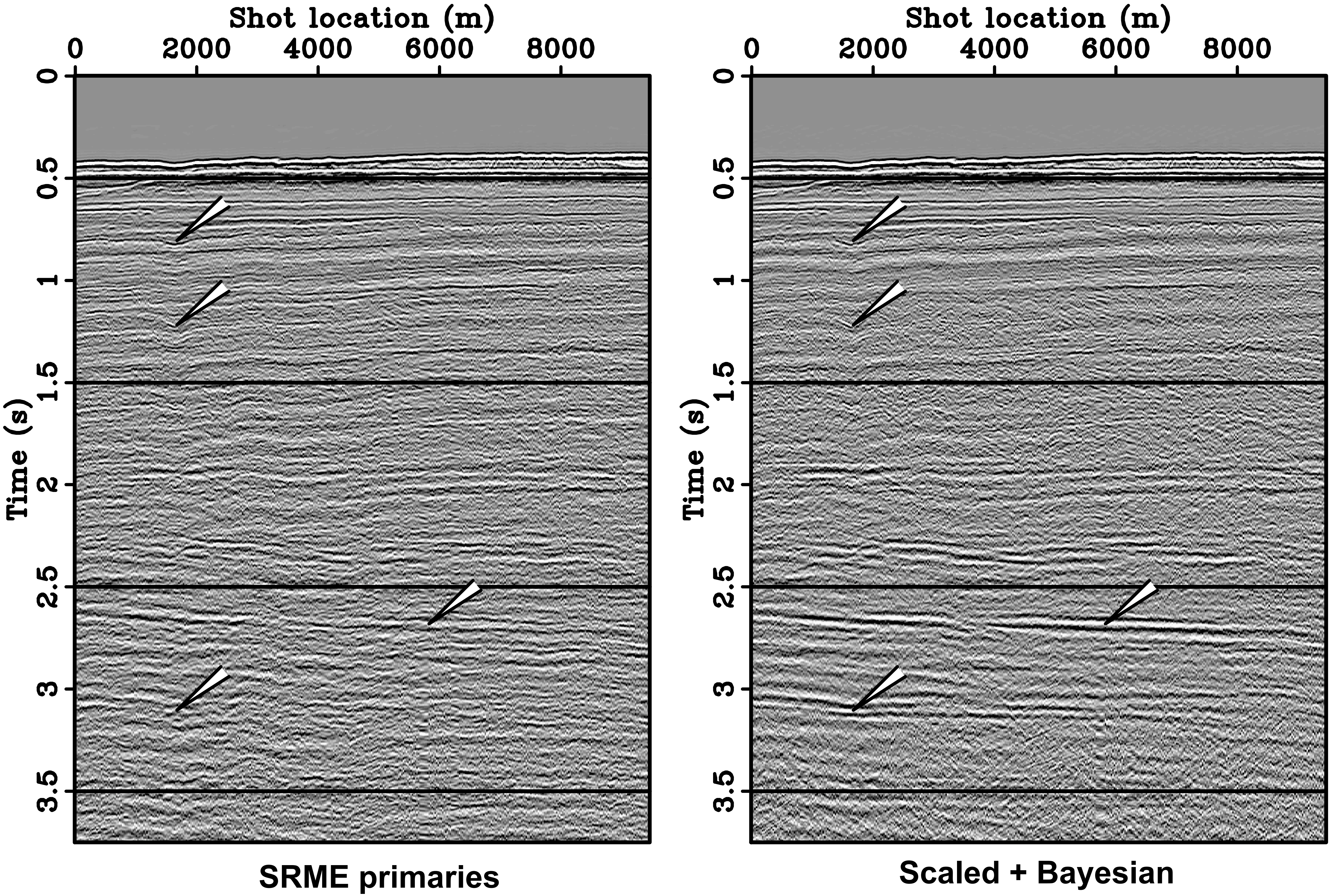


Scaled Bayesian

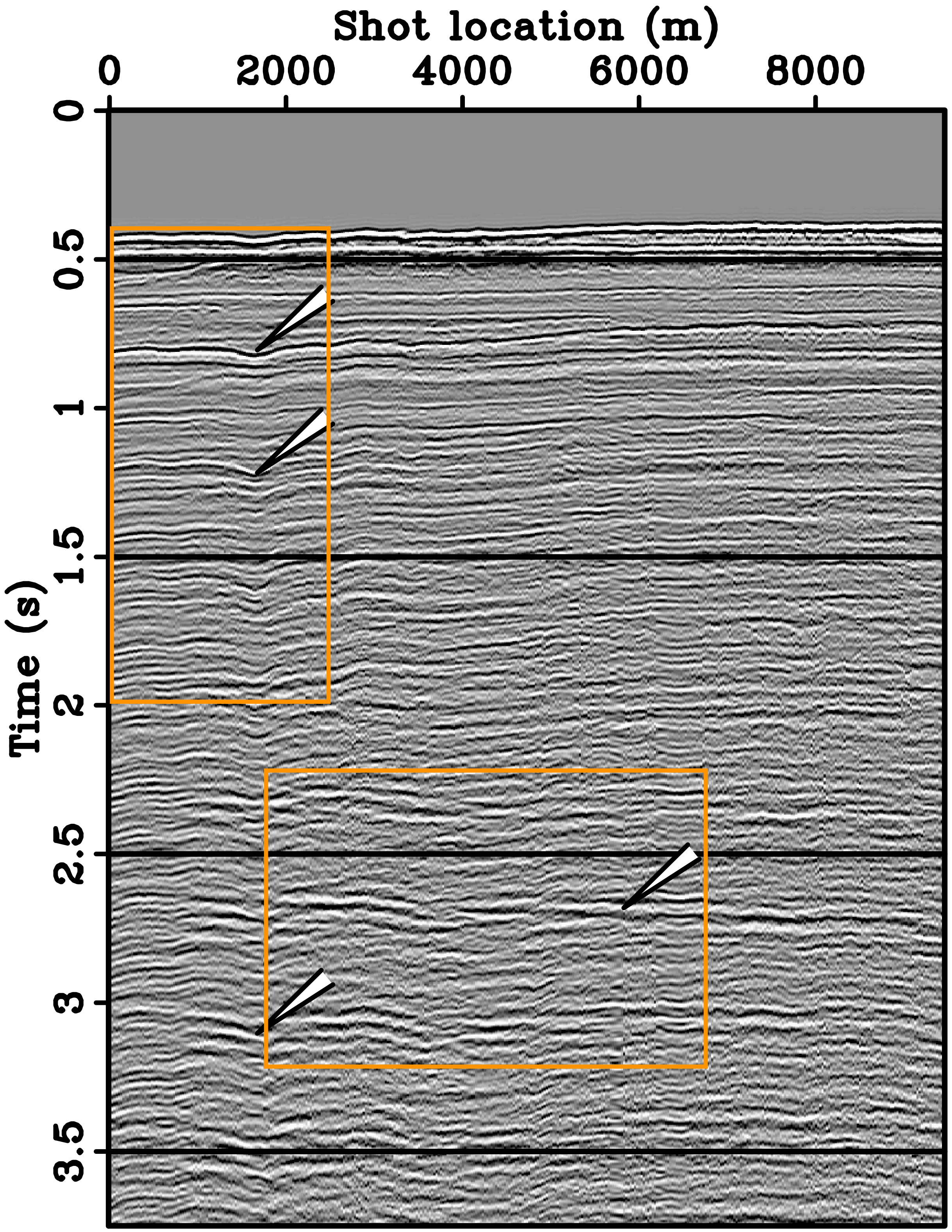


Difference between SRME and scaled Bayesian

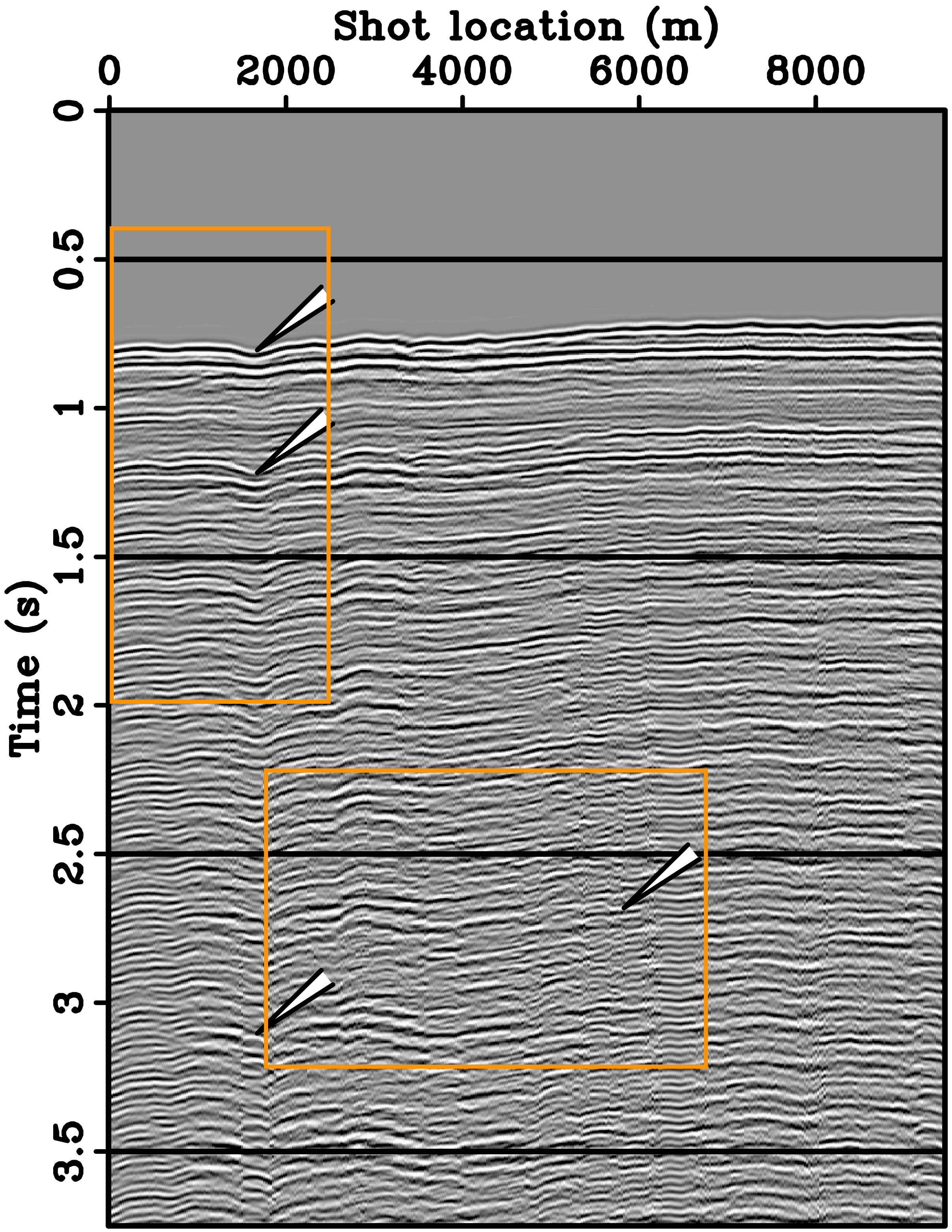
Real-data example



Real-data example

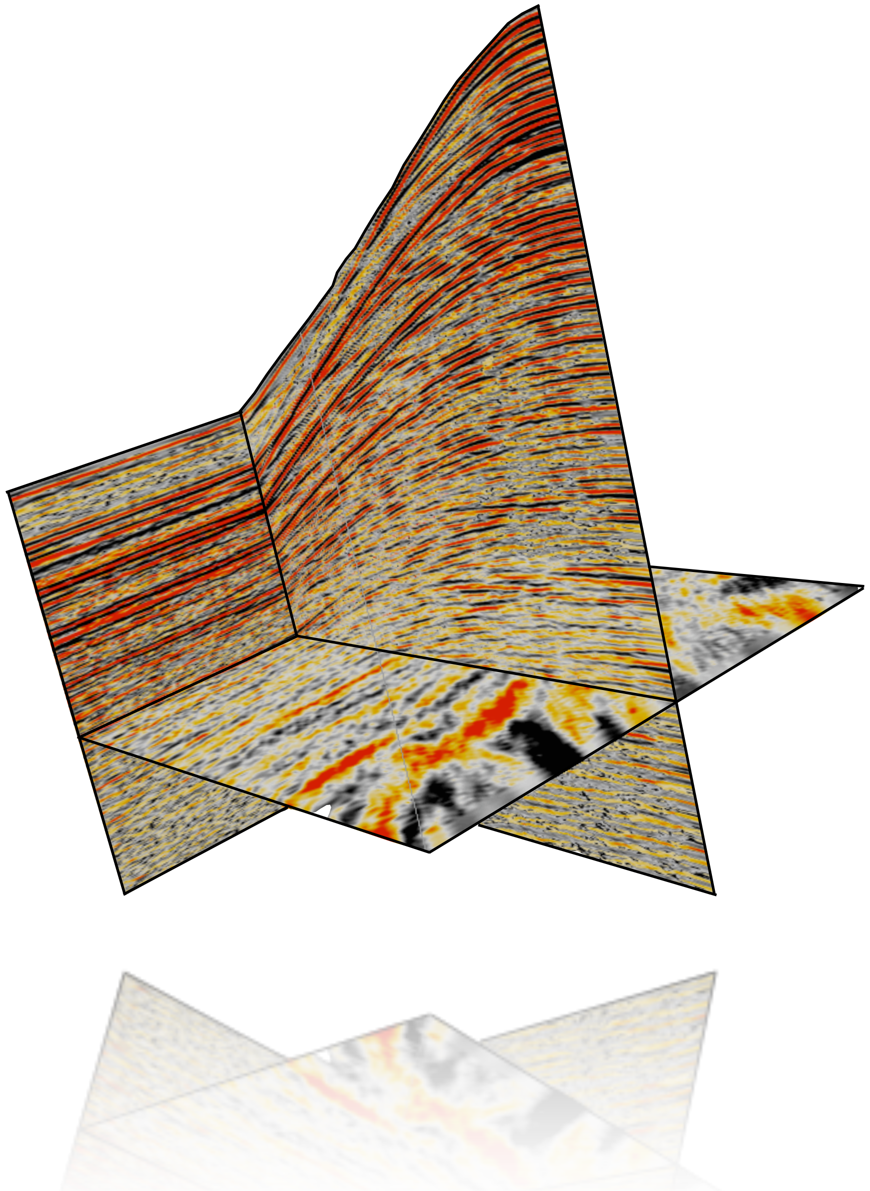


Data



Predicted multiples

Recent developments



Frequency domain smoothness

- It can be shown as in [Symes, Demanet] that symbol is also smooth in frequency variable”

$$\hat{a}(\eta, \zeta) = \sum_{\alpha,\beta} a_{\alpha,\beta} g_\beta(\zeta)$$

- Here $g_\beta(\zeta)$ is a smooth spline function, and thus the Fourier transform of the symbol with respect to the spatial variable α is smooth in the frequency variable ζ.
- This fact can be leveraged in the curvelet matching problem by adding an extra term promoting smoothness in frequency of the spatial Fourier transform of the PsDO symbol.

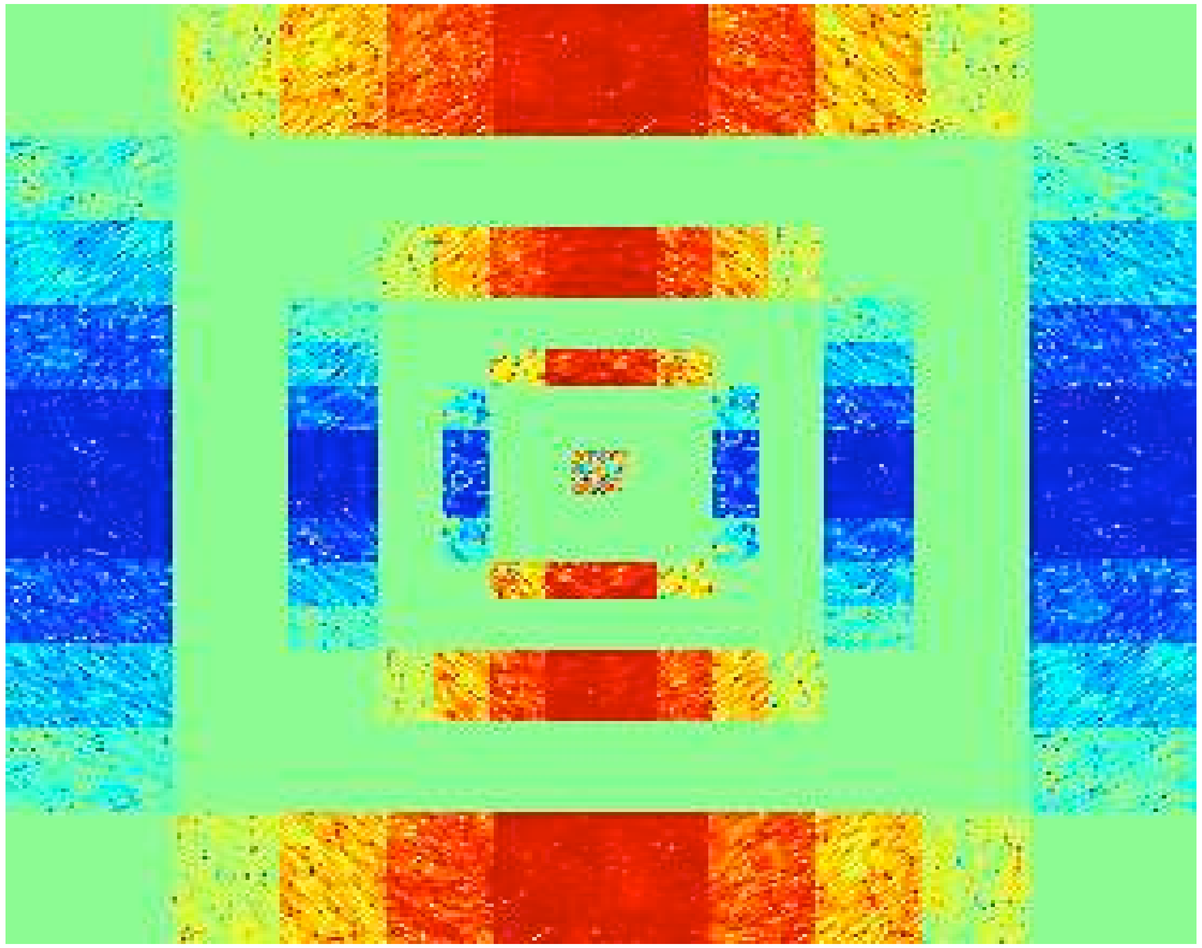
Frequency domain smoothness regularization

- Solve the following optimization problem:

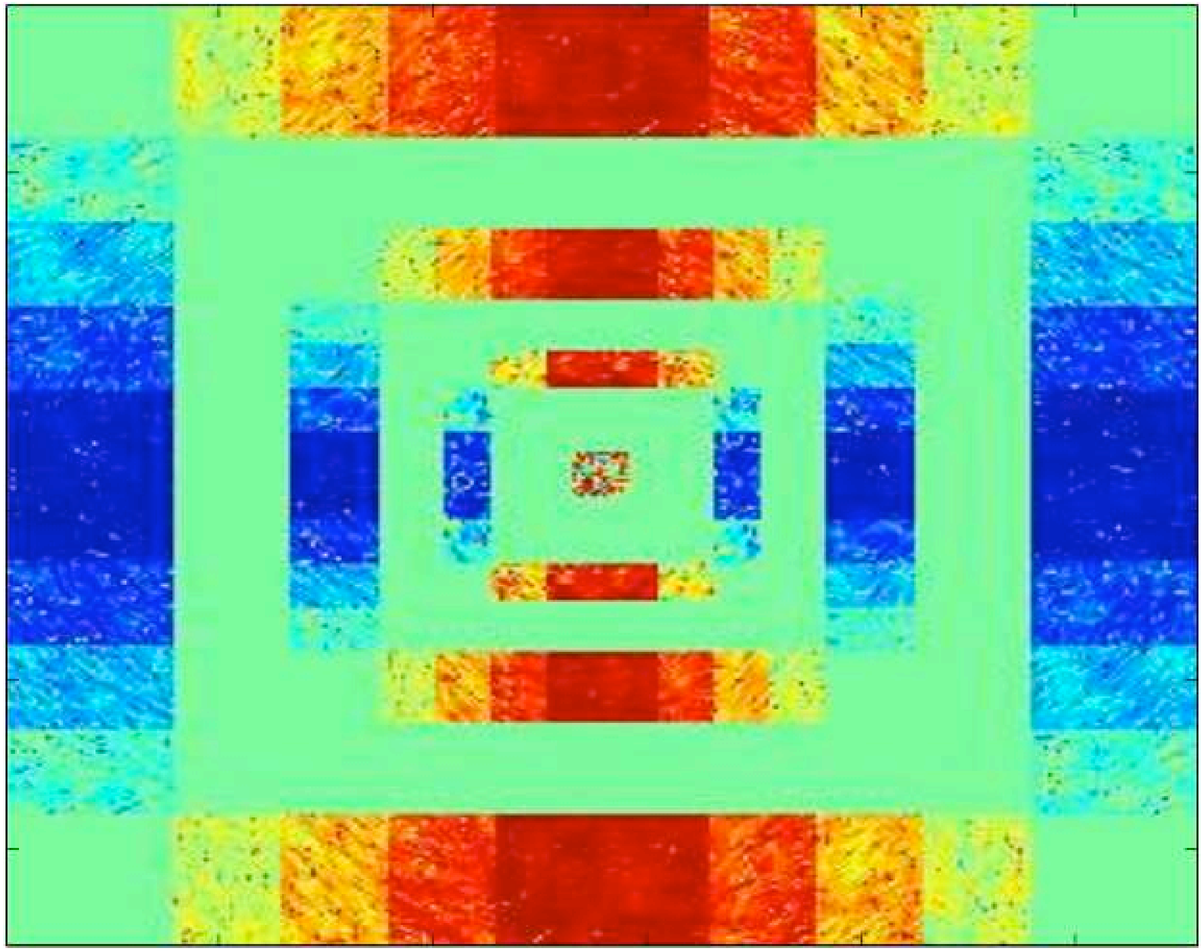
$$\arg\min_{\mathbf{z}} \frac{1}{2}||\mathbf{C}^{\mathbf{T}}\mathrm{diag}(\mathbf{Cf})e^{\mathbf{z}} - \mathbf{g}||_2^2 + \frac{\lambda^2}{2}||\mathbf{L}e^{\mathbf{z}}||_2^2 + \frac{\mu^2}{2}||\mathbf{M}_\zeta \mathbf{R}\mathbf{F}_{\mathbf{x}} e^{\mathbf{z}}||_2^2.$$

- $\mathbf{R}$: Restriction operator (only keep positive frequencies, since we know the symbol is real, and so the Fourier transform will be even).
- $\mathbf{M}_\zeta$: Sharpening operator in ζ (derivative with respect to angle).
- $\mathbf{F}_{\mathbf{x}}$: Fourier transform operator in $\mathbf{x}$ (Fourier transform wedge-by-wedge).

Results on Synthetic Pseudodifferential Operator

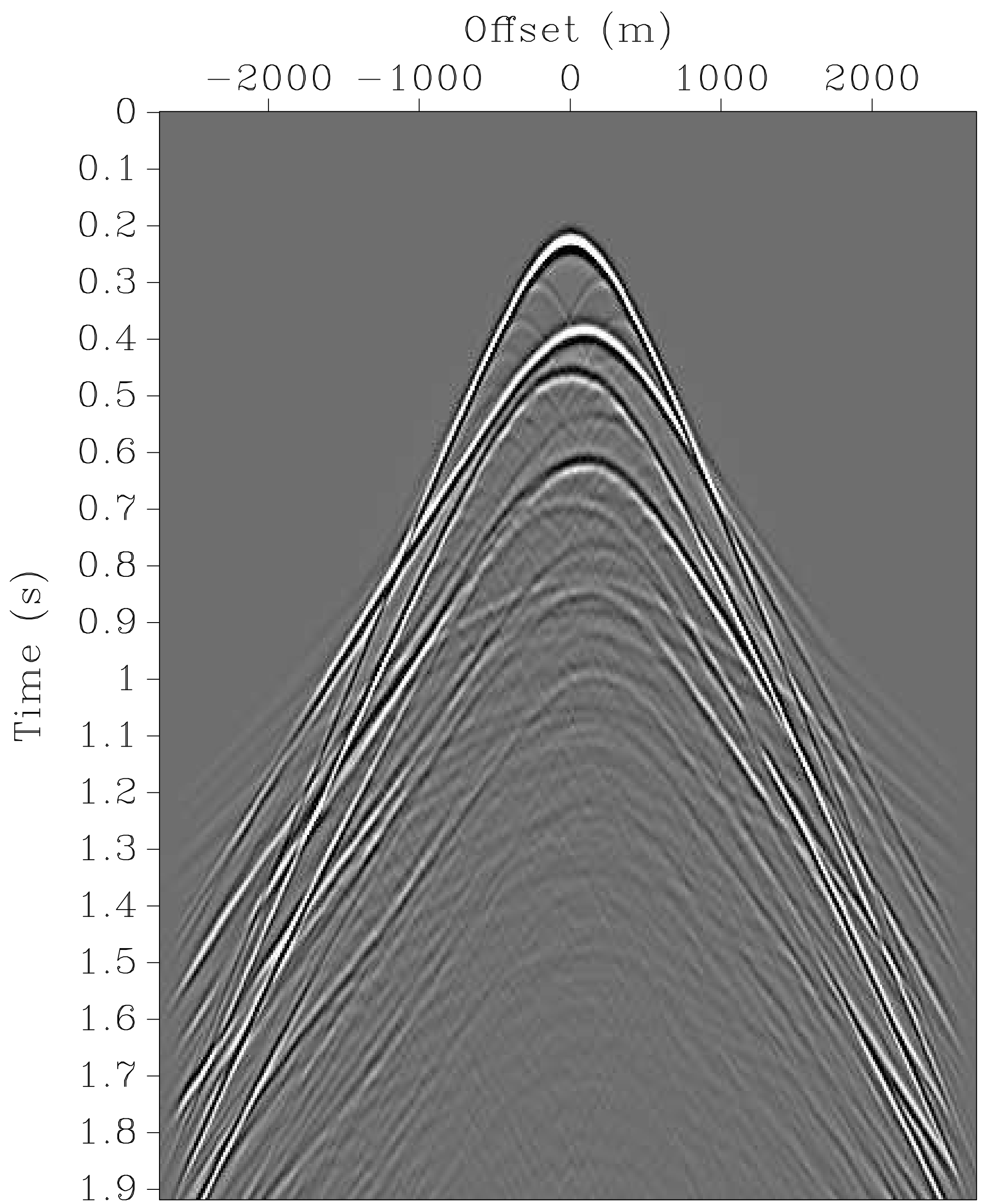


Estimated Symbol from Matched Filter (no freq. regularization, 50 iterations)

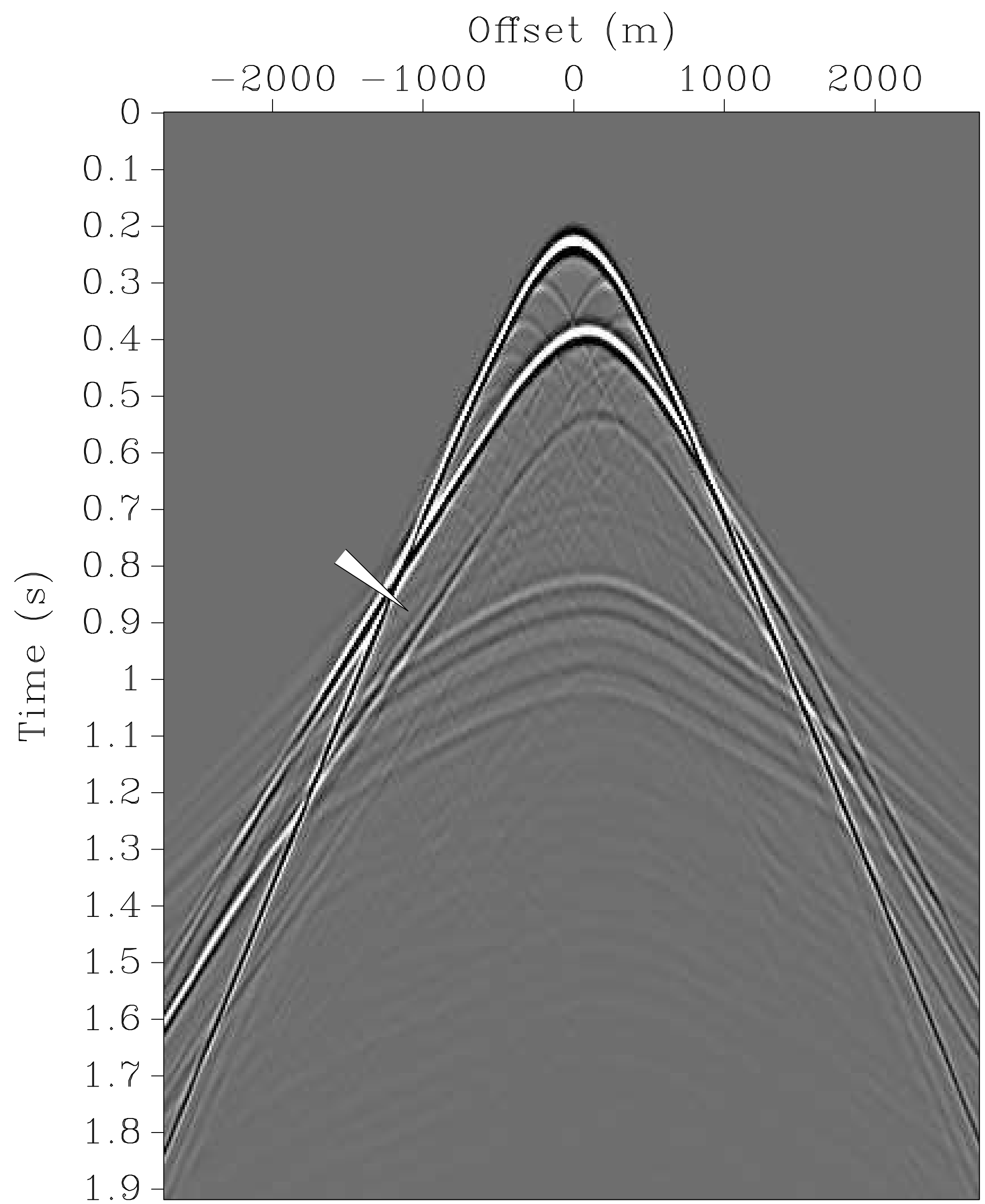


Estimated Symbol from Matched Filter (with frequency regularization, 20 iterations)

Application to Primary-Multiple Separation

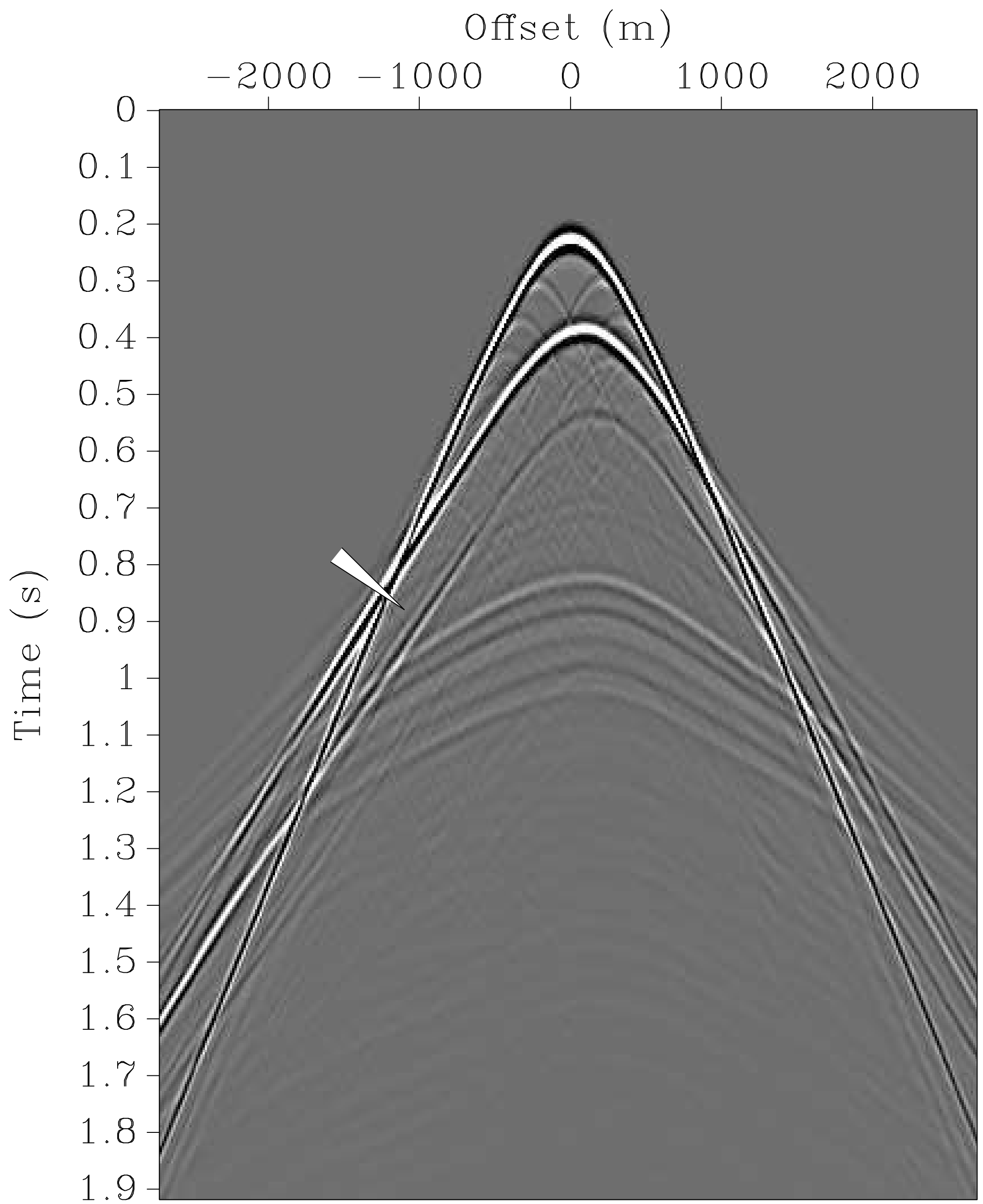


Total Data

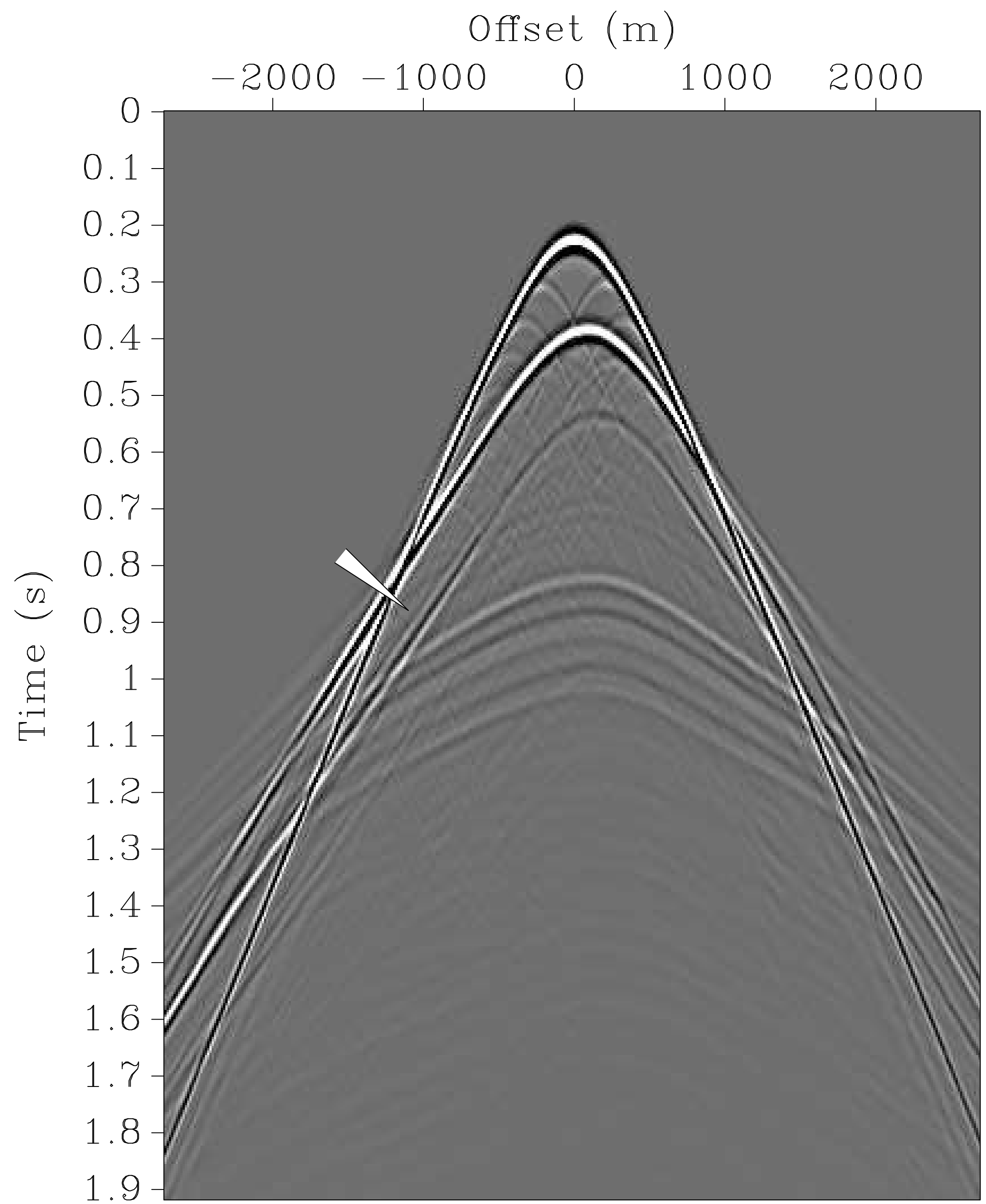


Ground truth Primaries

Application to Primary-Multiple Separation

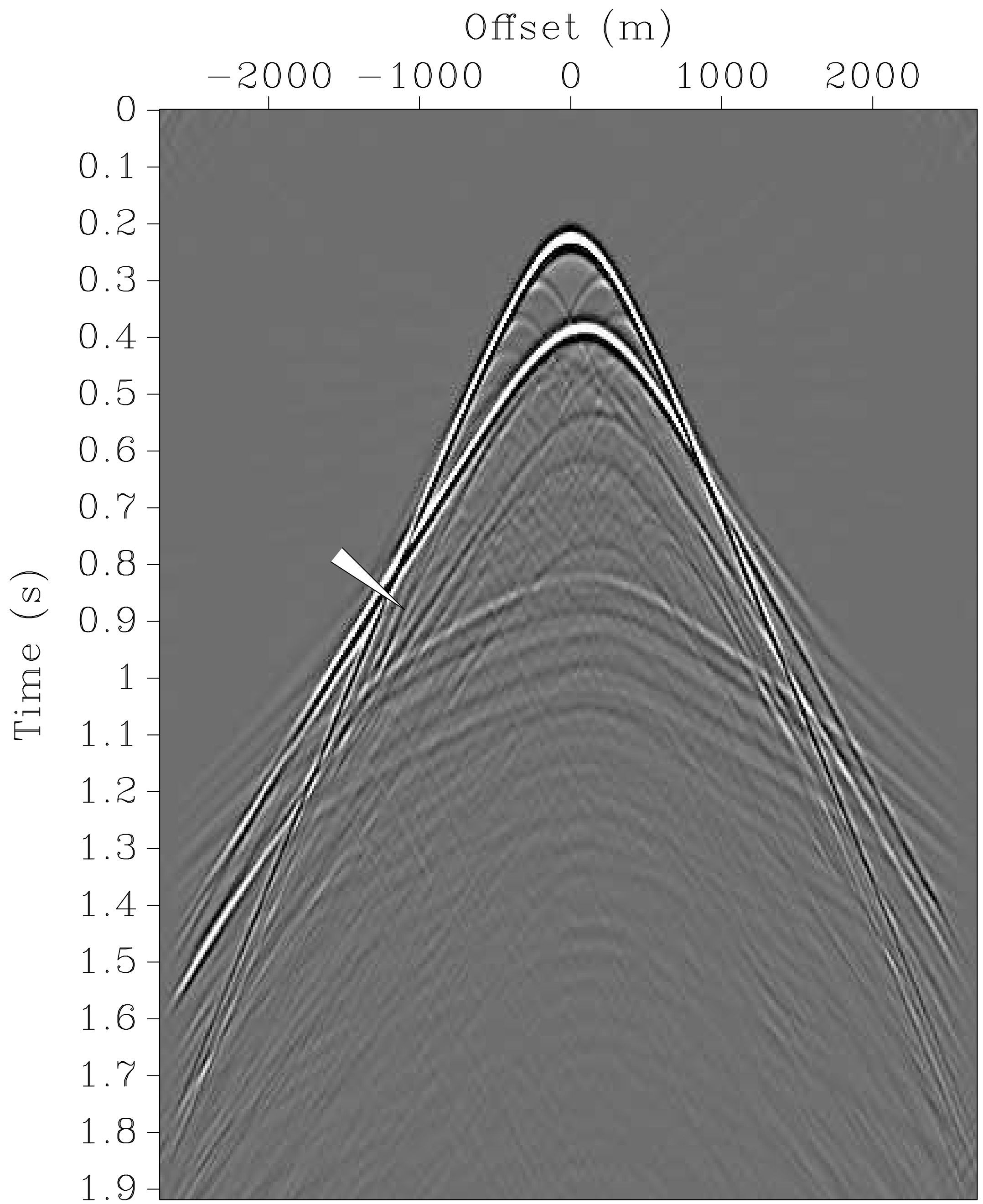


Ground truth Primaries

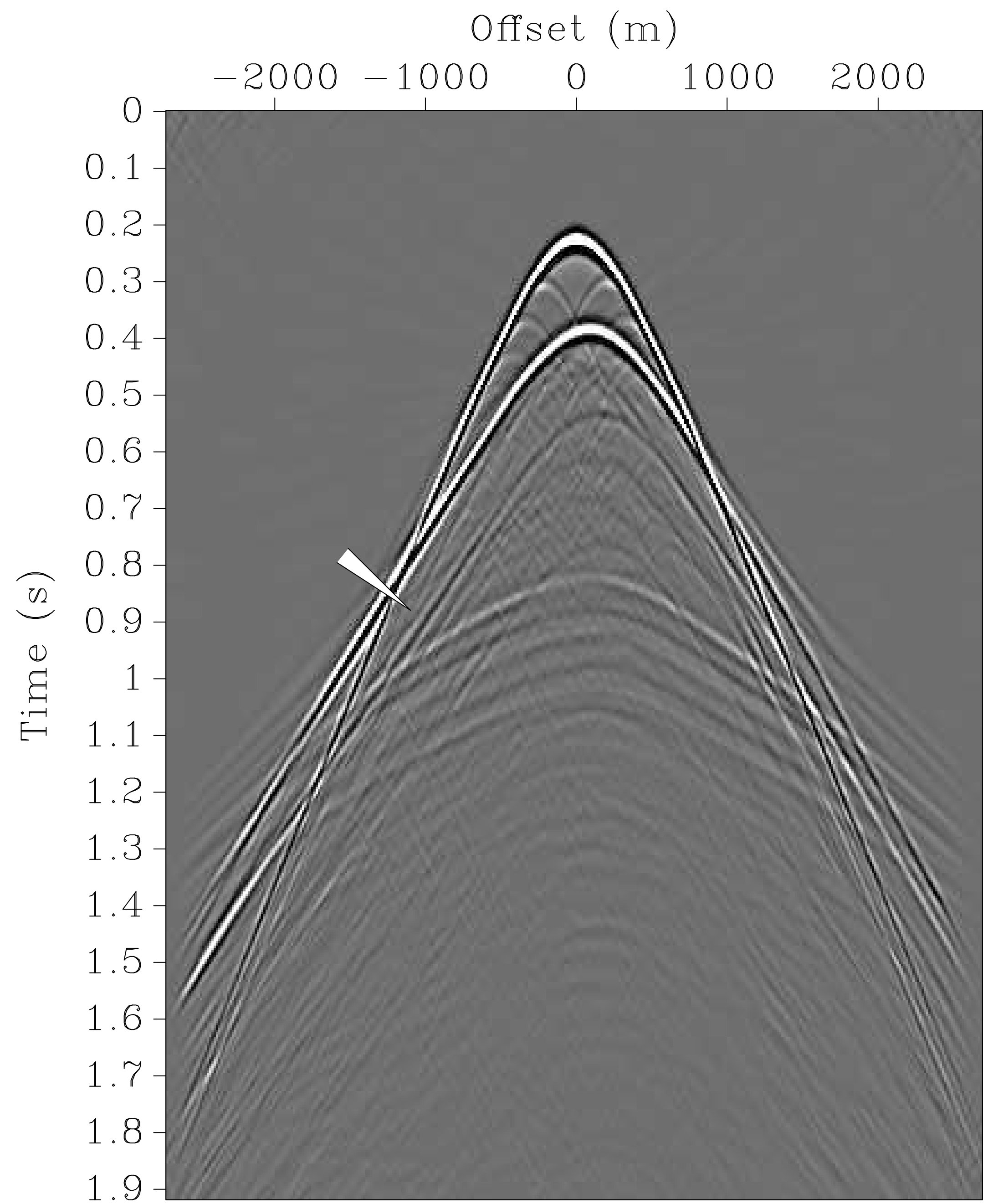


Ground truth Primaries

Results with and without new regularization



Matched Filtering (μ=0)



Matched Filtering (μ=1)

Further reading

- http://slim.eos.ubc.ca