Compressive sampling meets seismic imaging

Felix J. Herrmann

joint work with

Tim Lin*, Peyman Moghaddam*, Gilles Hennenfent*,

Deli Wang* & Chris Stolk (Universiteit Twente)

*Seismic Laboratory for Imaging and Modeling

slim.eos.ubc.ca

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Monterrey, October 19

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Today’s challenges

Aside from *spurious local minima* seismic waveform inversion is difficult because of

- lack of control on the image amplitudes
- missing data and noise
- computational cost to form the operators

Today’s agenda is to leverage recent insights from applied harmonic analysis and information theory to

- restore amplitudes => affordable q-Newton updates
- stably reconstruct wavefields
- compress wavefield-extrapolation operators

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Motivation

Exploit *two* aspects of curvelets, namely their

- parsimoniousness
- invariance under certain operators

Formulate

- *data-adaptive* scaling algorithms
- *non-adaptive* wavefield reconstruction algorithms

Applications

- *nonlinear* migration-amplitude recovery
- *nonlinear* sampling for wavefields
- *nonlinear* sampling for operators

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Today's topics

Sparsity-promoting seismic-image amplitude recovery

- curvelet-domain diagonal approximation of PsDO's
- stable sparsity-promoting inversion

Directional frame-based wavefield reconstruction by sparsity promotion

- curvelet parsimoniousness
- jitter sampling

Compression of FIO's through compressive sampling

- measurement basis diagonalizes operator

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The problem

Minimization:

$$\widetilde{c} = \arg\min_{c} \|d - F[c]\|_2^2$$

After linearization (Born app.) forward model with noise:

$$d(x_s, x_r, t) = \big(K[\bar{c}]m\big)(x_s, x_r, t) + n(x_s, x_r, t)$$

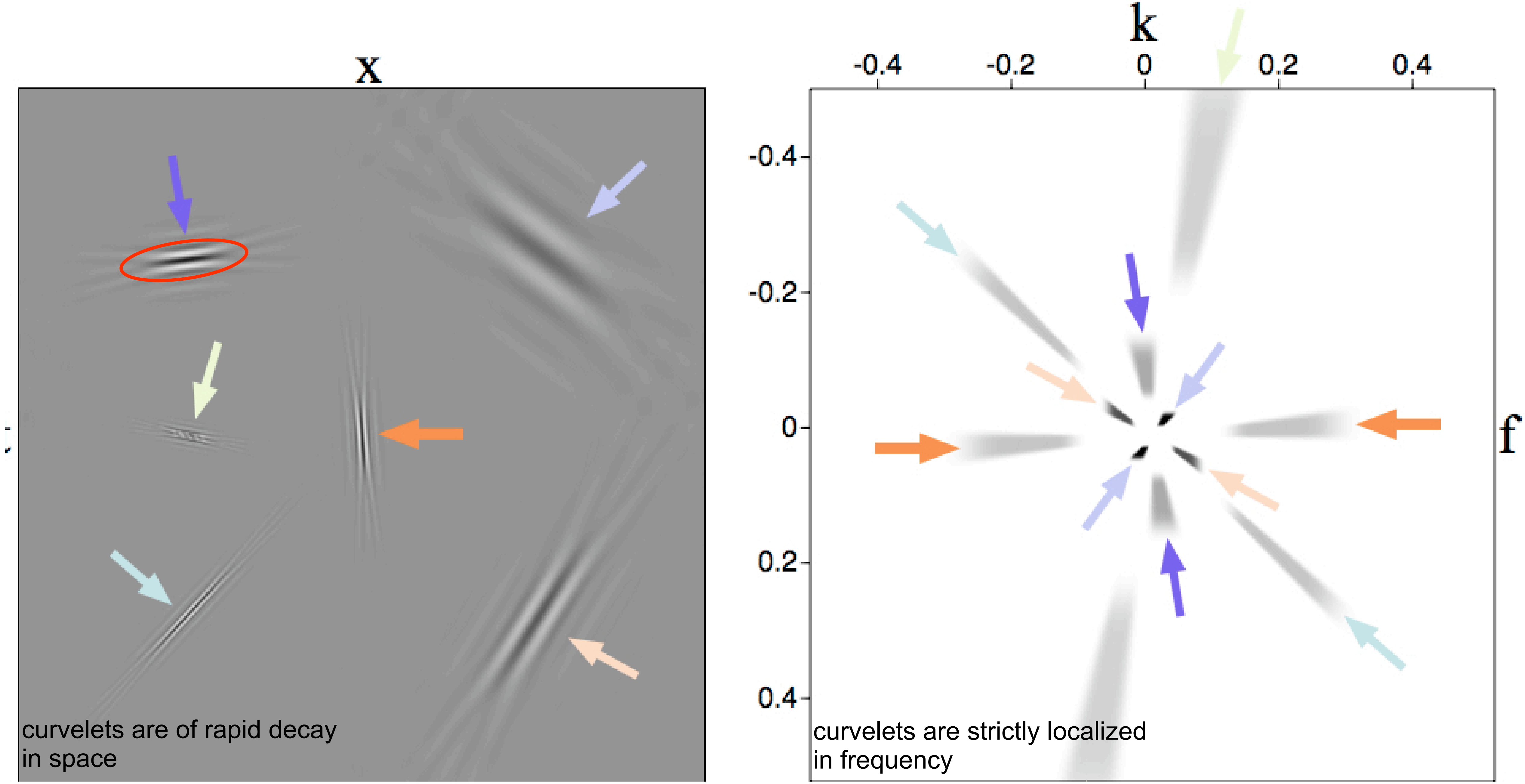
Conventional imaging:

$$\begin{aligned} \big(K^T d\big)(x) &= \big(K^T K m\big)(x) + \big(K^T n\big)(x) \\ y(x) &= \big(\mathbf{\Psi} m\big)(x) + e(x) \end{aligned}$$

Ψ is prohibitively expensive to invert

evaluation of $K[\bar{c}]$ involves expensive wavefield extrapolators

2-D curvelets



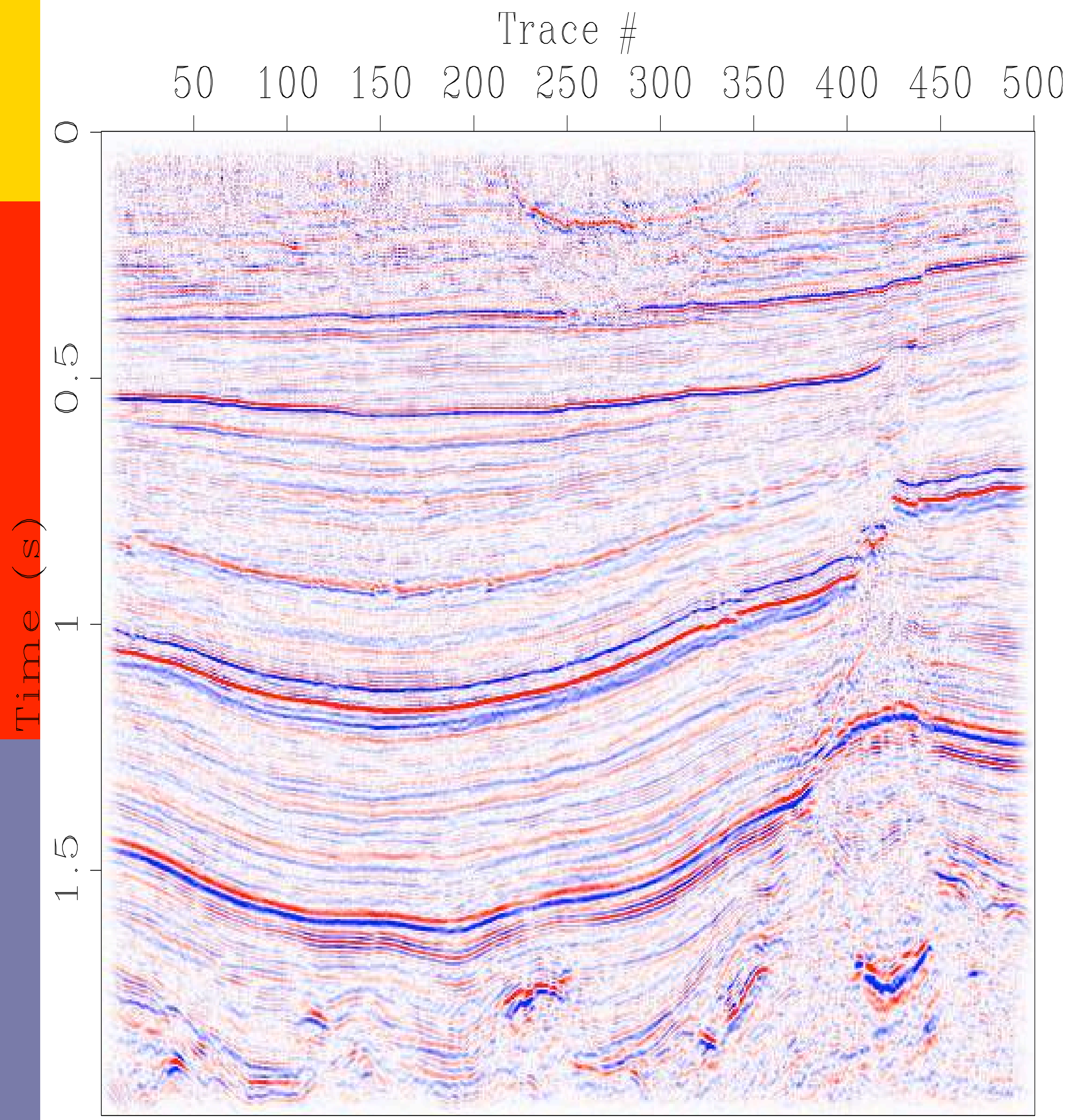
x-t

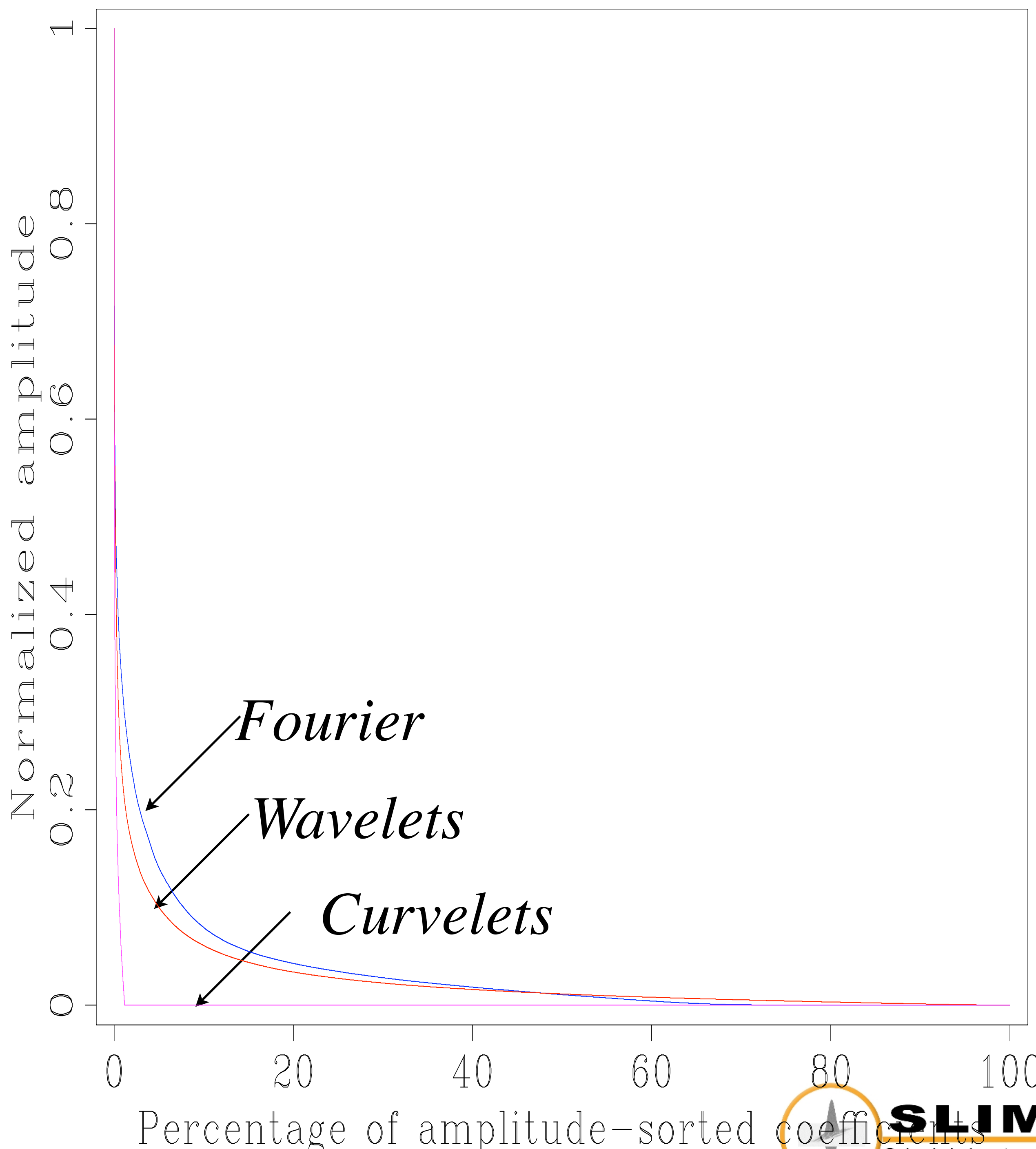
f-k

Oscillatory in one direction and smooth in the others!

Obey *parabolic* scaling relation $\text{length} \approx \text{width}^2$

Coefficients Amplitude Decay In Transform Domains





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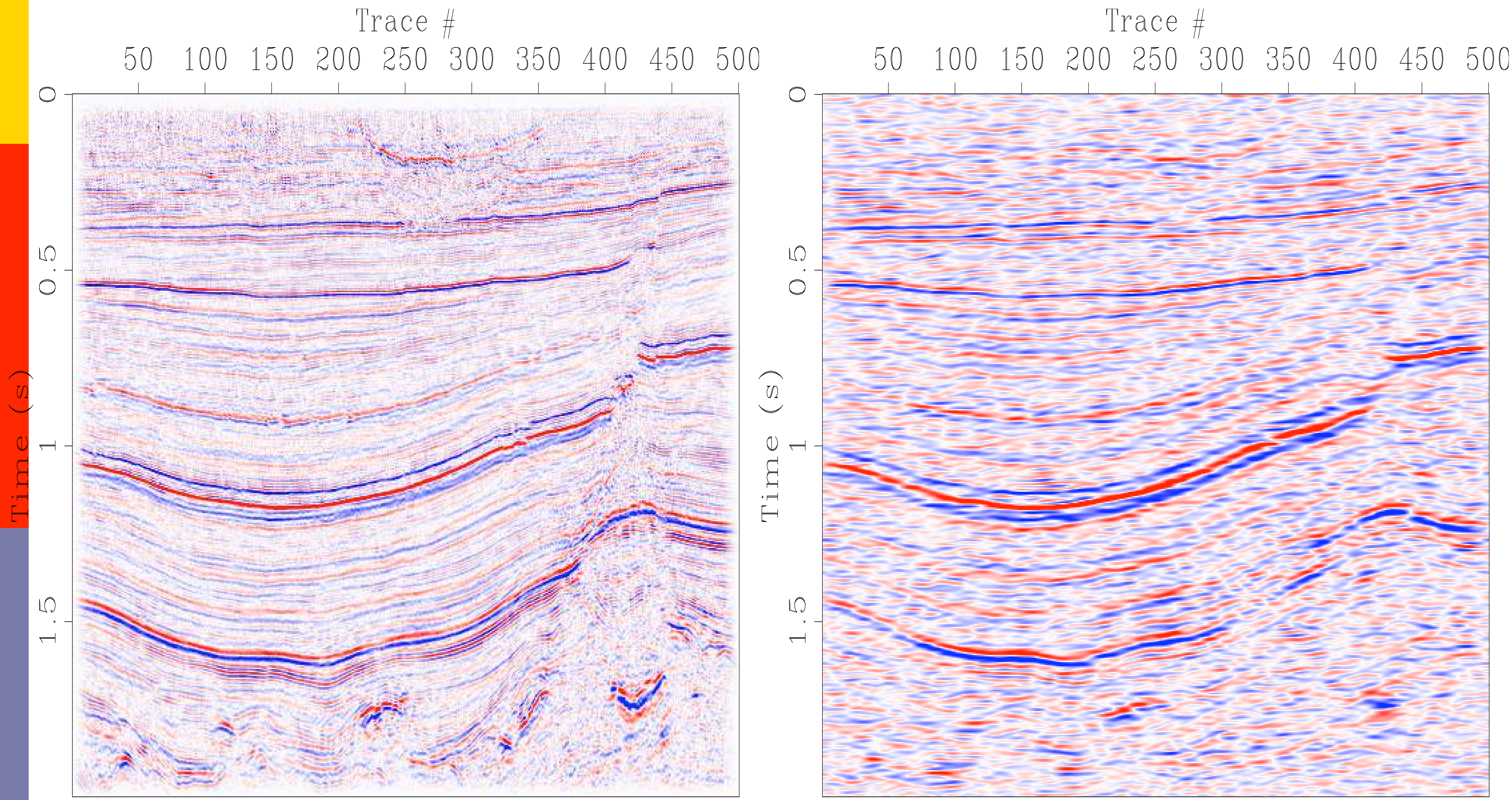
Partial Reconstruction
Fourier (1% largest coefficients)



SNR = 2.1 dB

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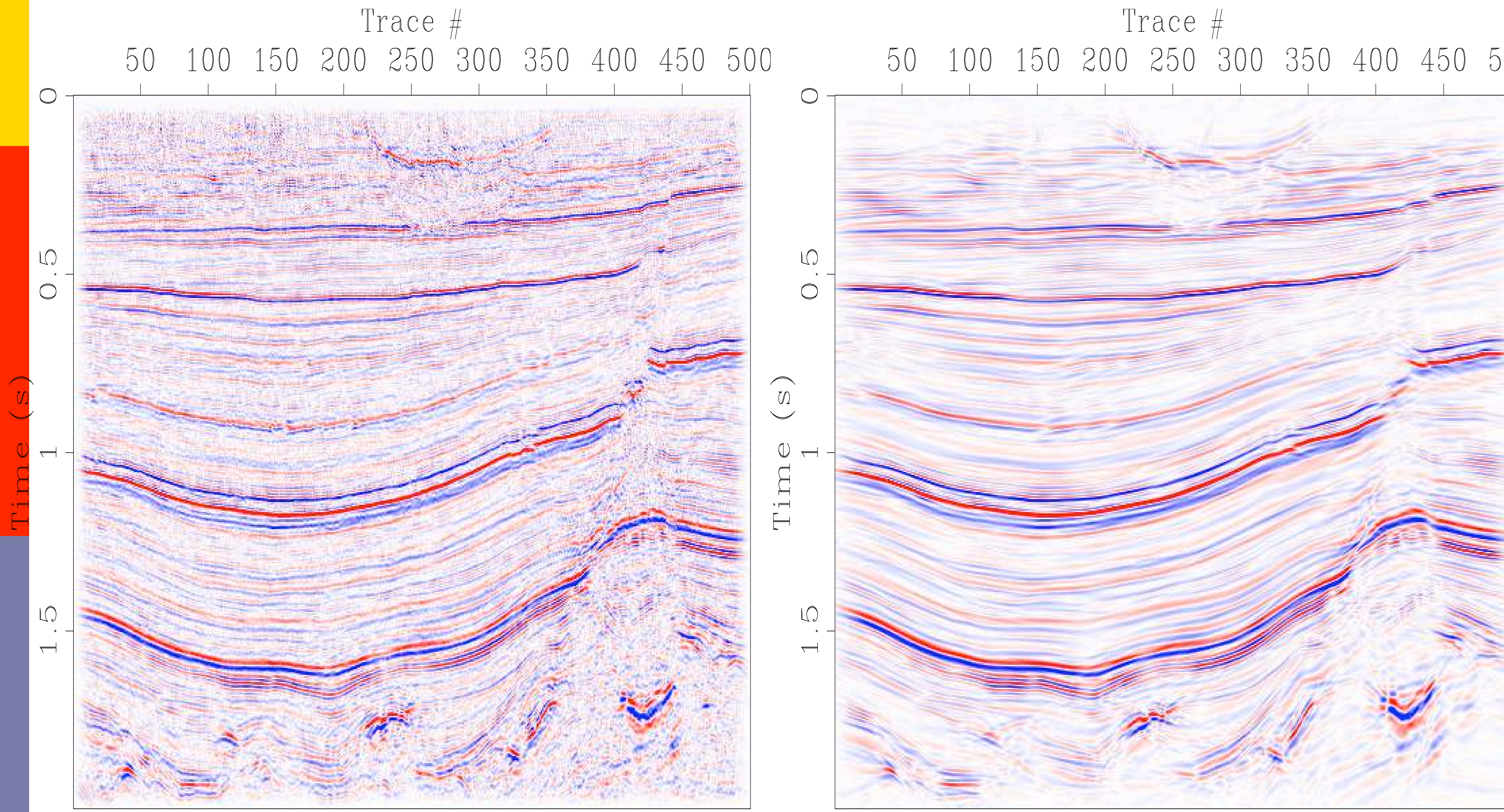
Partial Reconstruction
Curvelets (1% largest coefficients)



SNR = 6.0 dB

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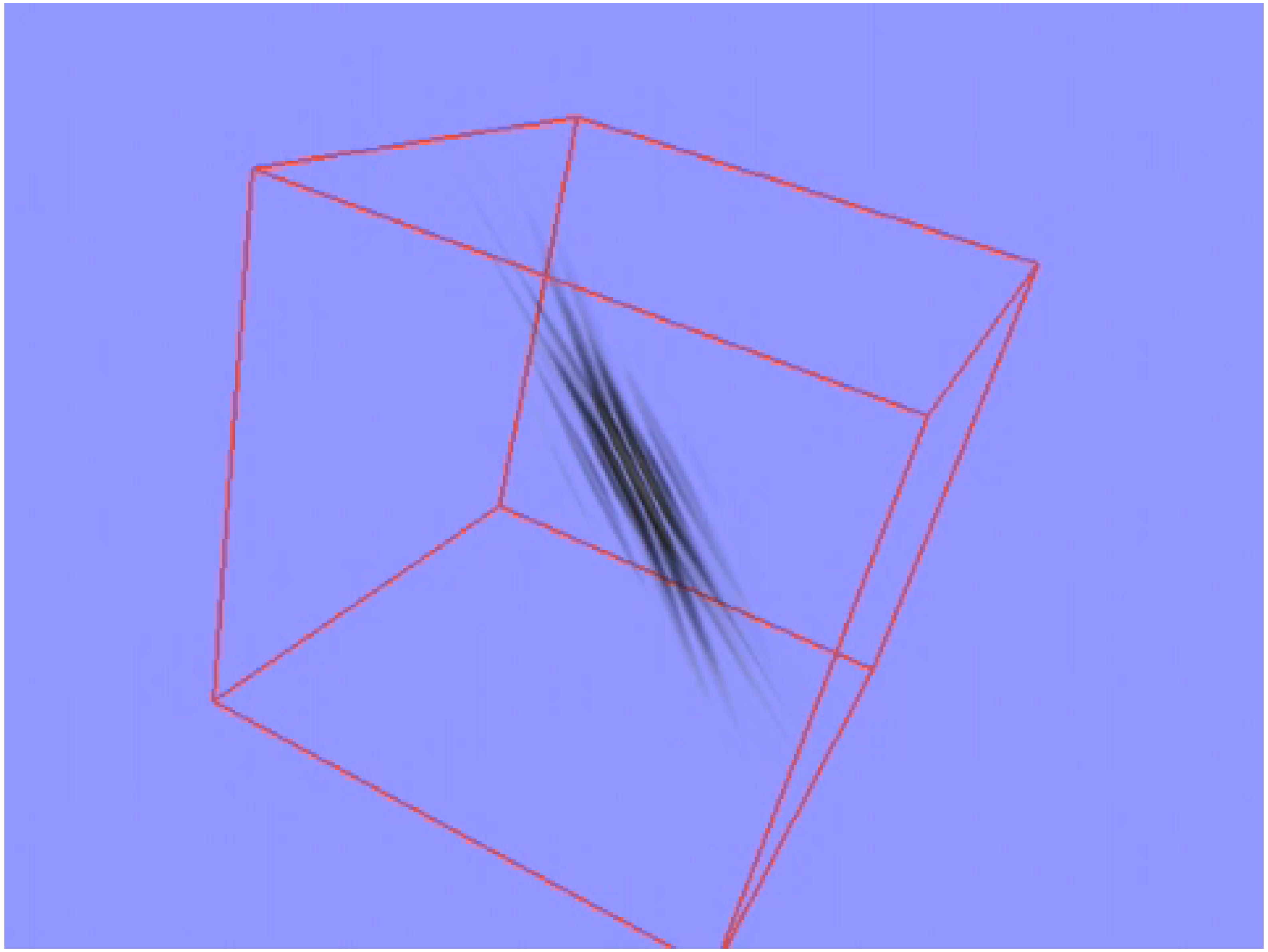
3-D curvelets



Curvelets are oscillatory in one direction and smooth in the others.

Approximate linearized inversion by curvelet scaling & sparsity promotion

Joint work with Chris Stolk* and Peyman Moghaddam

Mathematics Department,

Twente University, the Netherlands

"Sparsity- and continuity-promoting seismic imaging with curvelet frames" to appear in ACHA

Related work

Wavelet-Vaguelette/Quasi-SVD methods based on

- homogeneous operators
- absorb "square-root" of the Gramm matrix in WVD's
- Wavelets/curvelets near diagonalize the operator and are sparse on the model
  - Nonlinear solution of linear inverse problems by wavelet-vaguelette decomposition (Donoho '95)
  - Recovering Edges in Ill-posed Problems: Optimality of curvelet Frames (Candes & Donoho '00)

Scaling methods based on a diagonal approximation of Ψ, assuming

- smoothness on the symbol and conormality reflectors
  - Illumination-based normalization (Rickett '02)
  - Amplitude preserved migration (Plessix & Mulder '04)
  - Amplitude corrections (Guitton '04)
  - Amplitude scaling (Symes '07)

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Hessian/Normal operator

[Stolk 2002, ten Kroode 1997, de Hoop 2000, 2003]

Alternative to expensive least-squares migration.

In high-frequency limit Ψ is a pseudo-differential operator

$$\big(\Psi f\big)(x) := \big(K^T K f\big)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} a(x,\xi)\hat{f}(\xi)\mathrm{d}\xi$$

- composition of two Fourier integral operators
- pseudolocal (near unitary)
- singularities are preserved
- symbol is smooth for smooth velocity models $\bar{c}$

Corresponds to a spatially-varying dip filter after appropriate preconditioning (=> zero-order PsDO).

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Approximation

So let $\mathbf{\Psi} = \mathbf{\Psi}(x, D)$ be a pseudodifferential operator of order 0, with homogeneous principal symbol $a(x, \xi)$.

Substitutions:

$$K \mapsto K\,(-\Delta)^{-1/2} \quad \textbf{or} \quad K \mapsto \partial_t^{-1/2} K$$

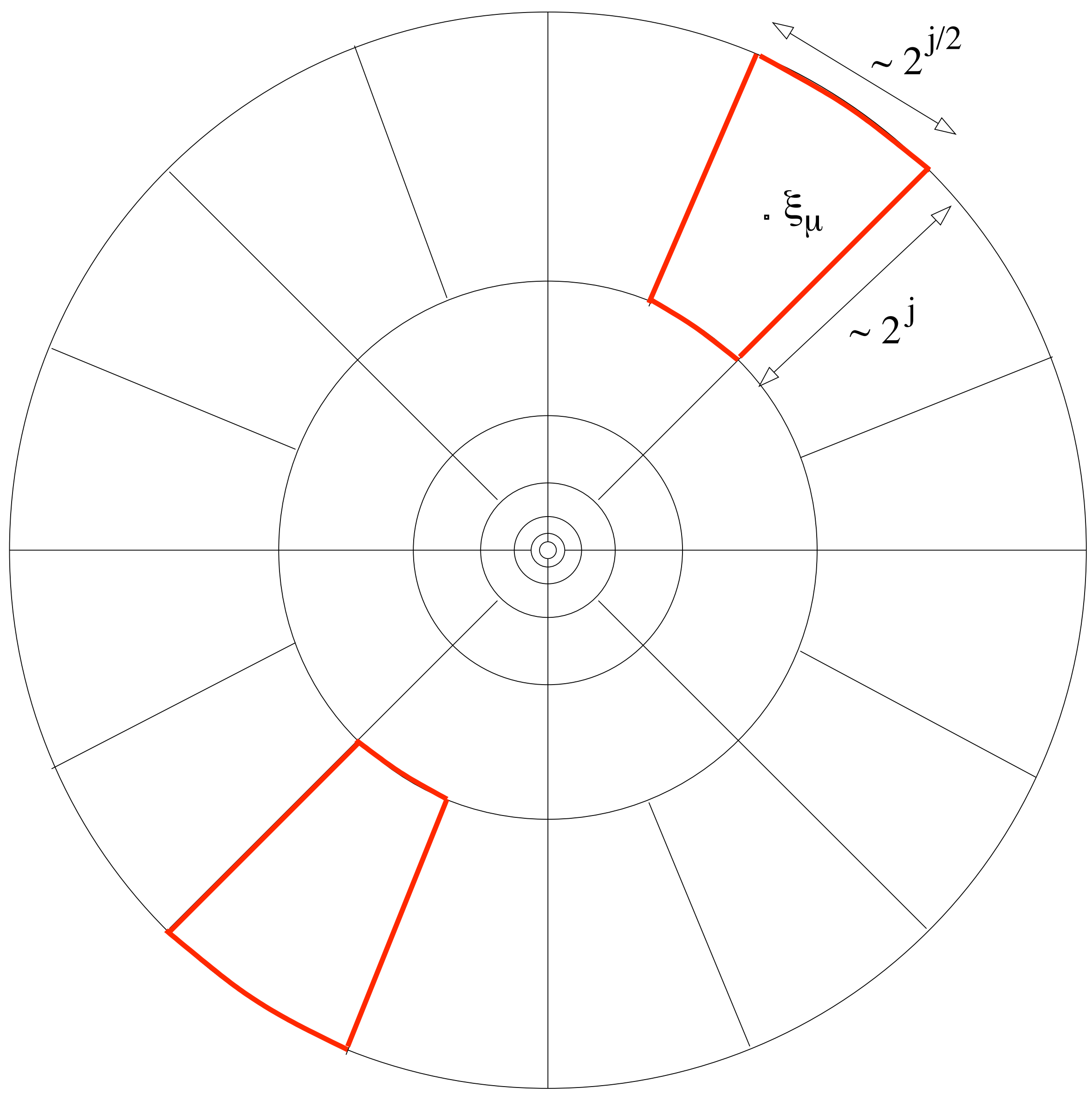
$$m \mapsto (-\Delta)^{1/2}\, m \quad \textbf{with} \quad ((-\Delta)^{\alpha} f)^{\wedge}(\xi) = |\xi|^{2\alpha} \cdot \hat{f}(\xi).$$

Lemma 1. *With C' some constant, the following holds*

$$\|(\Psi(x, D) - a(x_\nu, \xi_\nu))\varphi_\nu\|_{L^2(\mathbb{R}^n)} \leq C' 2^{-|\nu|/2}. \tag{14}$$

To approximate Ψ, we define the sequence $\mathbf{u} := (u_\mu)_{\mu \in \mathcal{M}} = a(x_\mu, \xi_\mu)$. Let $\mathbf{D}_\Psi$ be the diagonal matrix with entries given by $\mathbf{u}$. Next we state our result on the approximation of Ψ by $C^T \mathbf{D}_\Psi C$.

Tiling the ξ space



Scaling

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T \mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' is a constant depending on Ψ.

Allows for decomposition of the normal operator

$$\begin{aligned}\big(\Psi\varphi_\mu\big)(x) &\simeq \big(C^T \mathbf{D}_\Psi C \varphi_\mu\big)(x)\\ &= \big(AA^T\varphi_\mu\big)(x)\end{aligned}$$

with $A := \sqrt{\mathbf{D}_\Psi} C$ and $A^T := C^T \sqrt{\mathbf{D}_\Psi}$.

Matching procedure

Compute *reference* vector <=> defines **g**

- migrate data
- apply spherical-divergence correction

Create “data” <=> defines **f**

- demigrate
- migrate

Estimate *scaling* by inversion procedure

Define *scaled* curvelet transform

Recover migration amplitudes by *sparsity* promotion.

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Key idea

Estimation curvelet-domain **scaling**

- inversion of an *underdetermined* system
- *over* fitting
- *positivity* and reasonable *scaling*

Solution:

- use *smoothness* of the symbol
- formulate *nonlinear* estimation problem that minimizes

$$J_\gamma(\mathbf{z}) = \frac{1}{2}\|\mathbf{d} - \mathbf{F}_\gamma e^{\mathbf{z}}\|_2^2,$$

with

$$\mathrm{grad} J(\mathbf{z}) = \mathrm{diag}\{e^{\mathbf{z}}\}\left[\mathbf{F}^T\left(\mathbf{F}e^{\mathbf{z}} - \mathbf{d}\right)\right]$$

- solve with l-BFGS [Noccedal, Symes '07]

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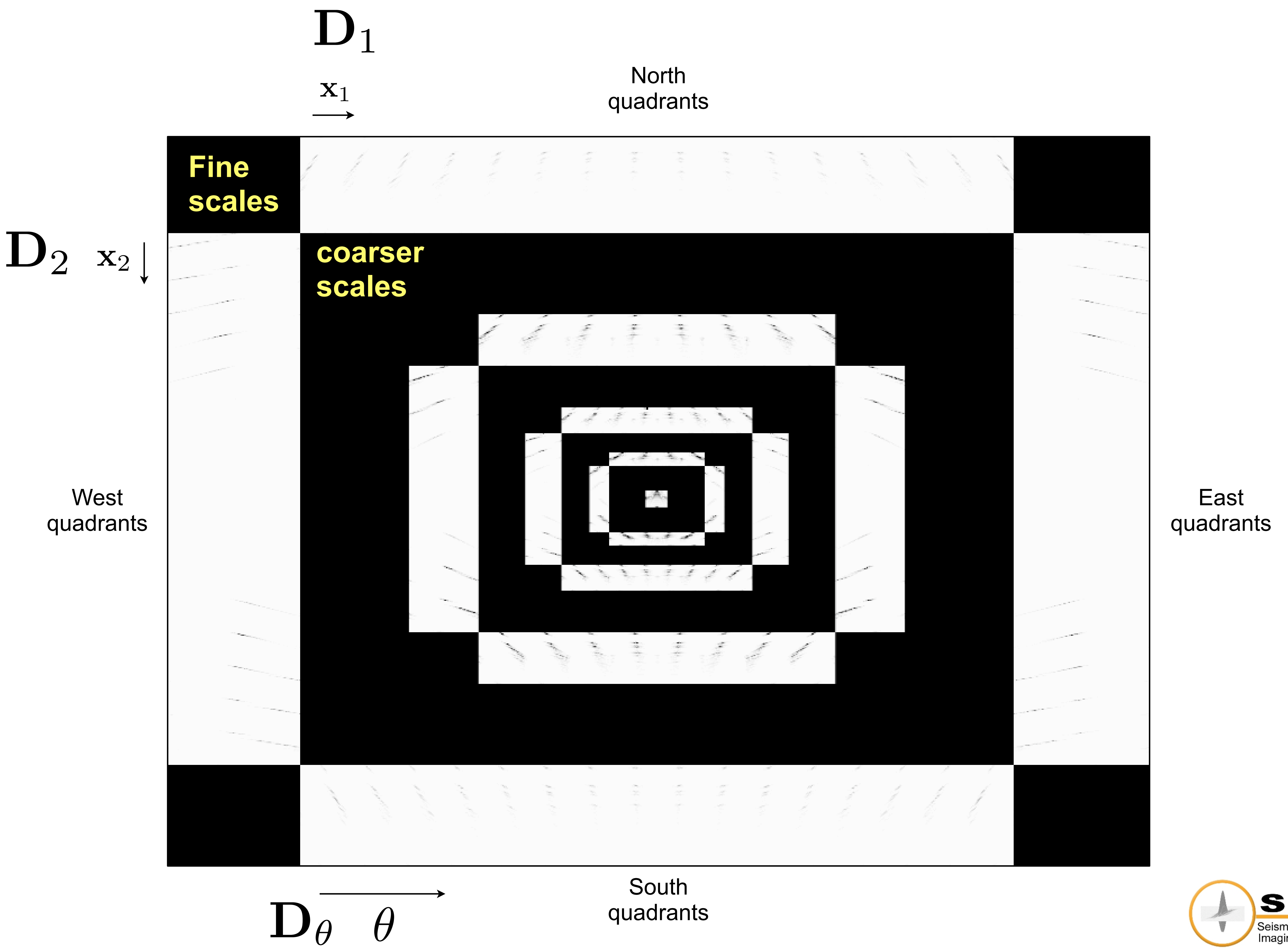
Key idea



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Key idea

Impose *smoothness* via following system of equations

$$\begin{aligned} \mathbf{f} &= \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{g}\}\mathbf{w} \\ \mathbf{0} &= \gamma \mathbf{L}\mathbf{w} \end{aligned}$$

with

$$\mathbf{L} = \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \mathbf{D}_\theta^T \end{bmatrix}^T$$

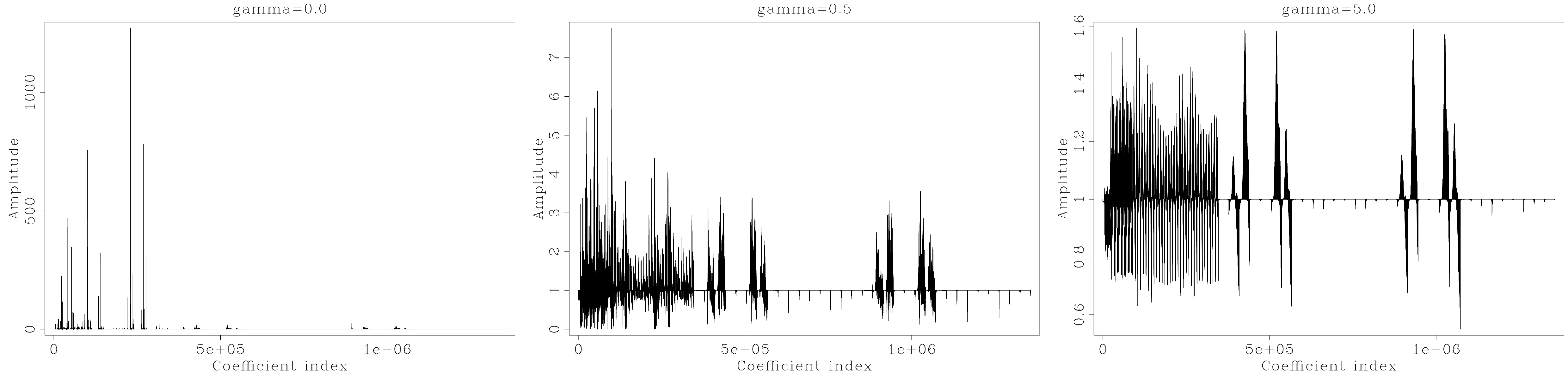
first-order differences in *space* and *angle* directions for each *scale*. Equivalent to

$$\tilde{\mathbf{w}} = \arg\min_{\mathbf{w}} \frac{1}{2}\|\mathbf{b} - \mathbf{P}[\mathbf{w}]\|_2^2 + \gamma^2 \|\mathbf{L}\mathbf{w}\|_2^2$$

with

$$\mathbf{P} = \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{g}\}$$

Smoothness penalty



- reduces overfitting
- scaling is positive and reasonable

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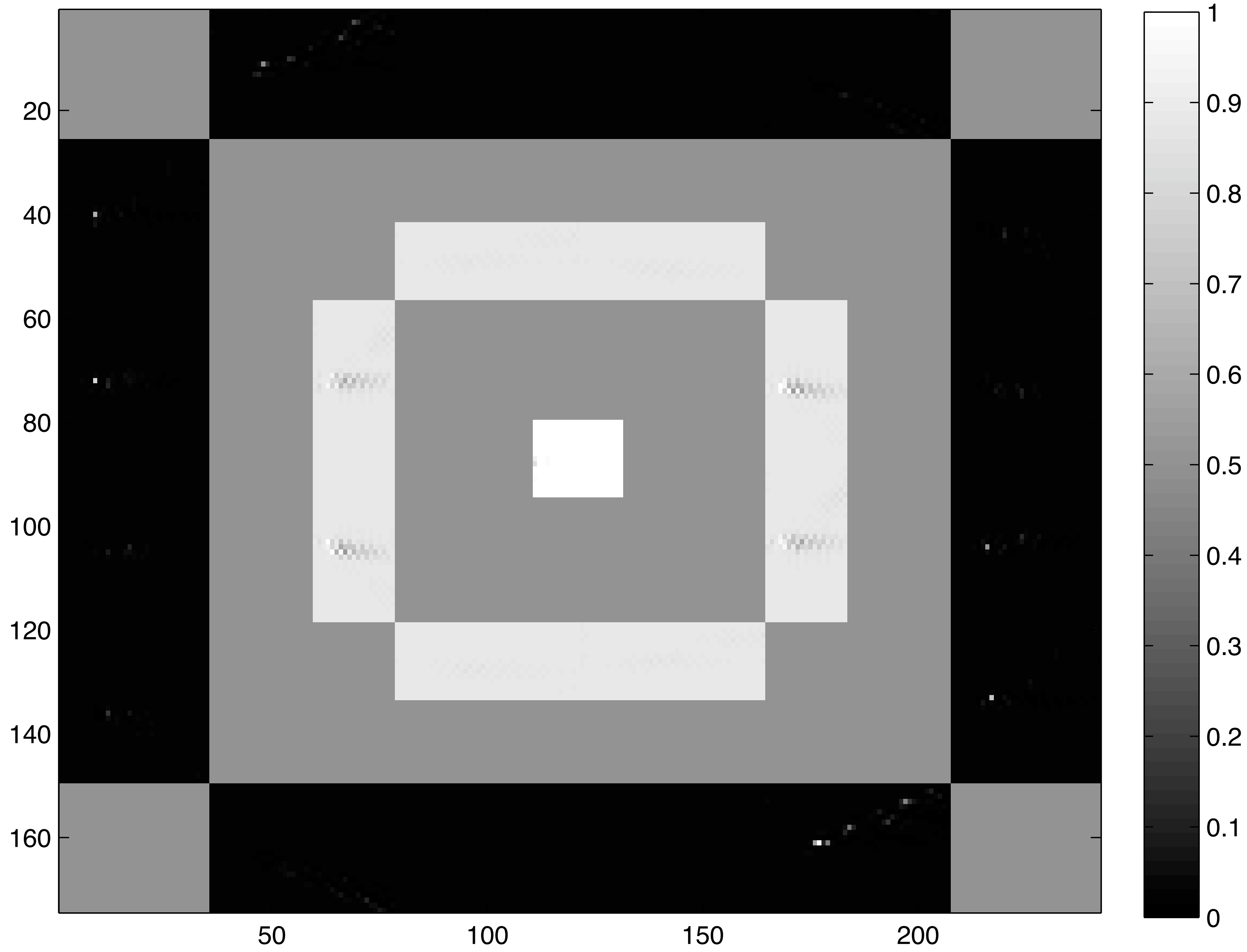
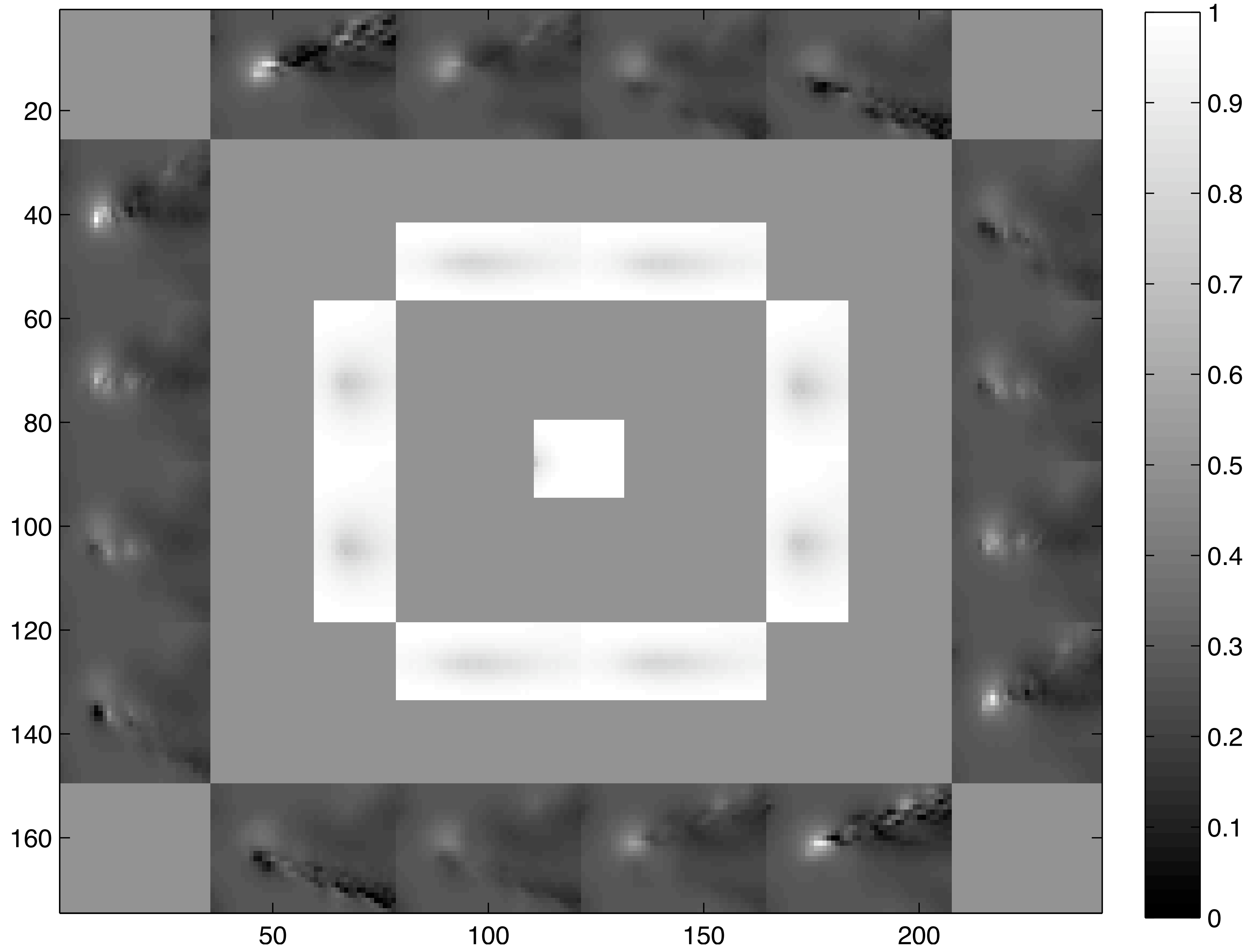
Smoothness penalty



$\gamma = 0$

Smoothness penalty



$$\gamma = 1/2$$

Our approach

"Forward" model:

$$\begin{aligned}\mathbf{y} &= \mathbf{K}^T\mathbf{K}\mathbf{m} + \varepsilon \\ &\approx \mathbf{A}\mathbf{x}_0 + \varepsilon\end{aligned}$$

with

$$\begin{aligned}\mathbf{y} &= \text{migrated data} \\ \mathbf{A} &:= \mathbf{C}^T\mathbf{\Gamma} \\ \mathbf{A}\mathbf{A}^T\mathbf{r} &\approx \mathbf{K}^T\mathbf{K}\mathbf{r} \\ \mathbf{K} &= \text{the demigration operator} \\ \boldsymbol{\epsilon} &= \text{migrated noise.}\end{aligned}$$

- diagonal approximation of the demigration-migration operator
- costs one demigration-migration to estimate the diagonal weighting

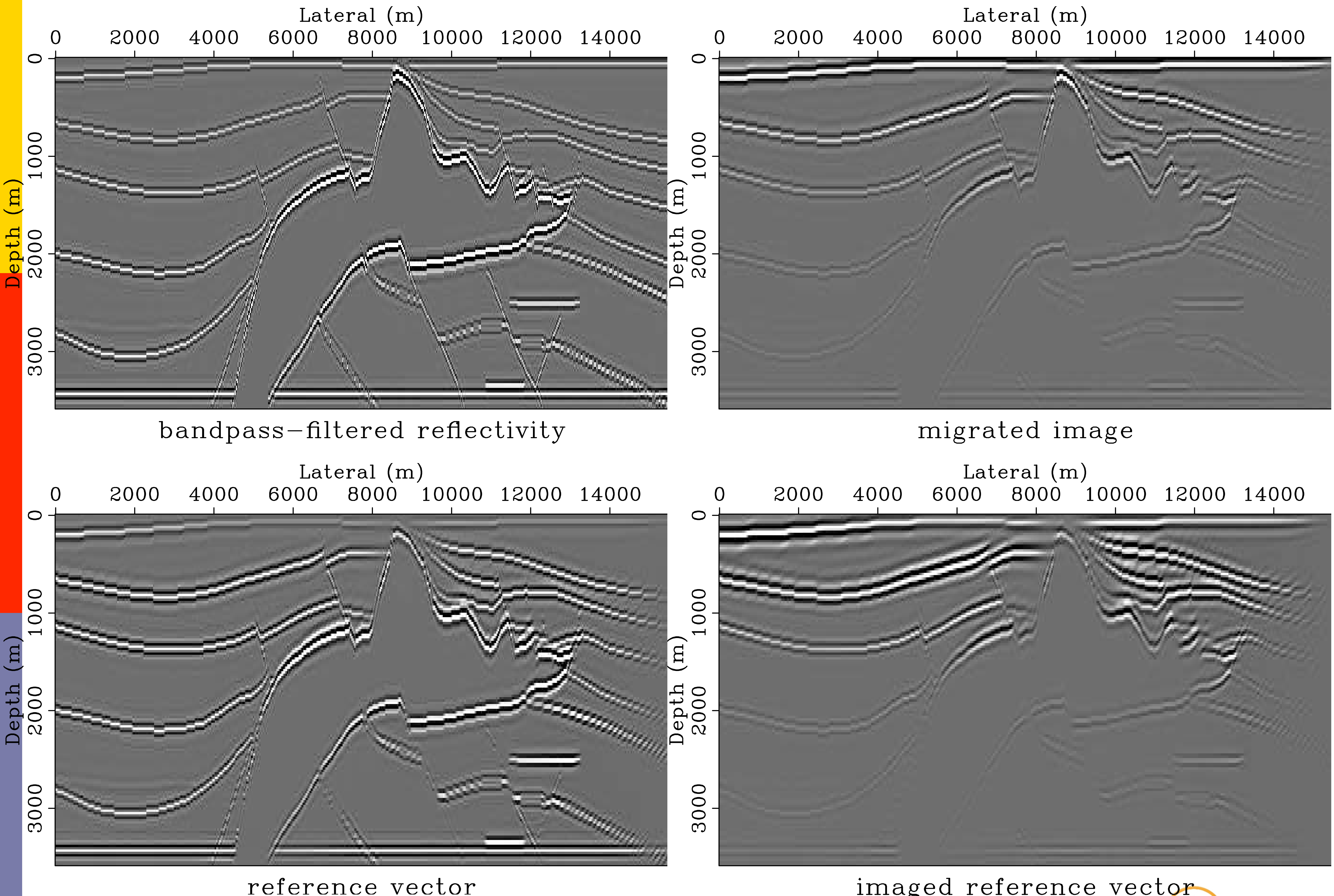
Solution

Solve

$$\mathbf{P}: \quad \begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = (\mathbf{A}^{\mathbf{H}})^{\dagger}\tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha\|\mathbf{x}\|_1}^{\text{sparsity}} + \beta \underbrace{\|\mathbf{\Lambda}^{1/2}\left(\mathbf{A}^{H}\right)^{\dagger}\mathbf{x}\|_p}_{\text{continuity}} .$$



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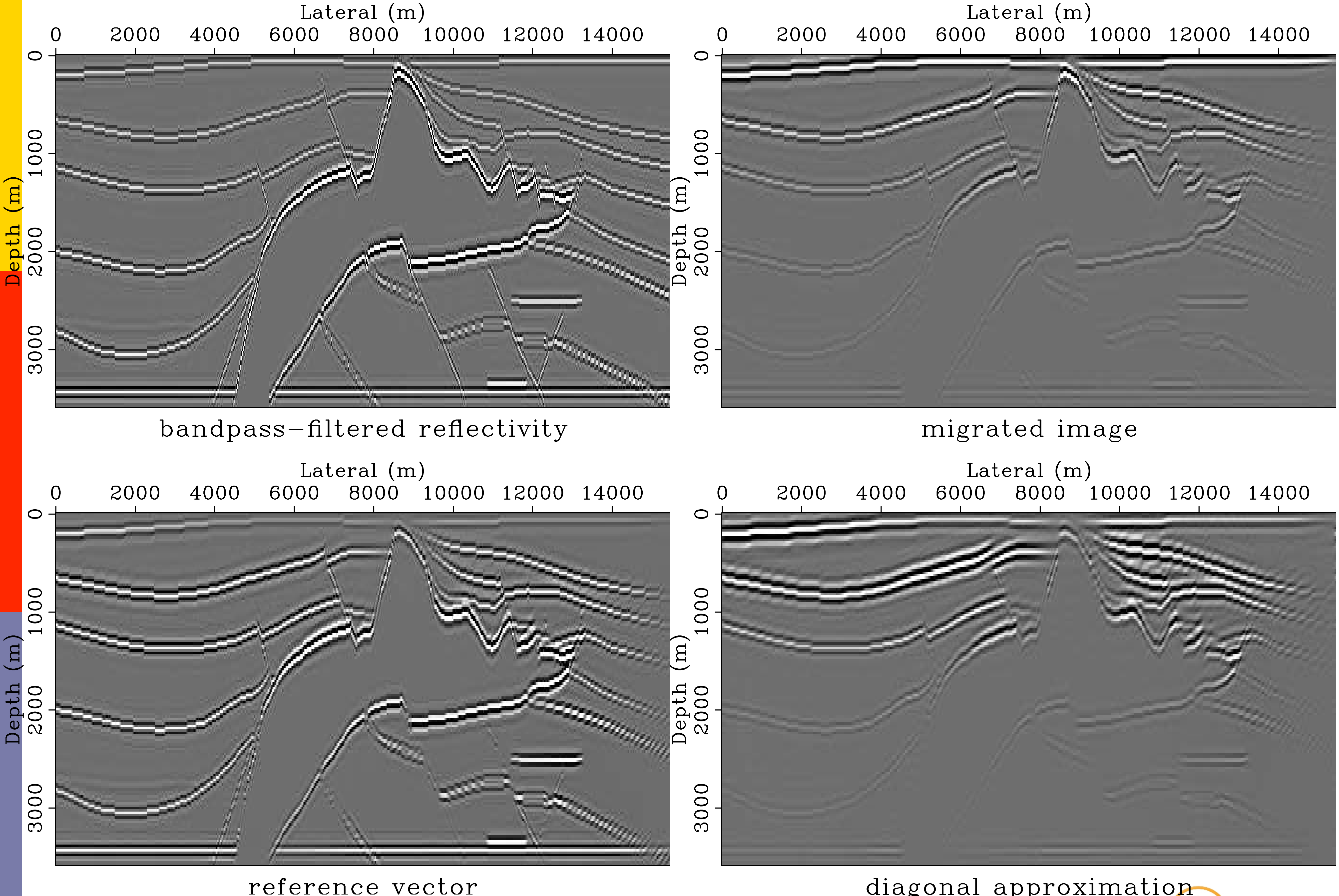

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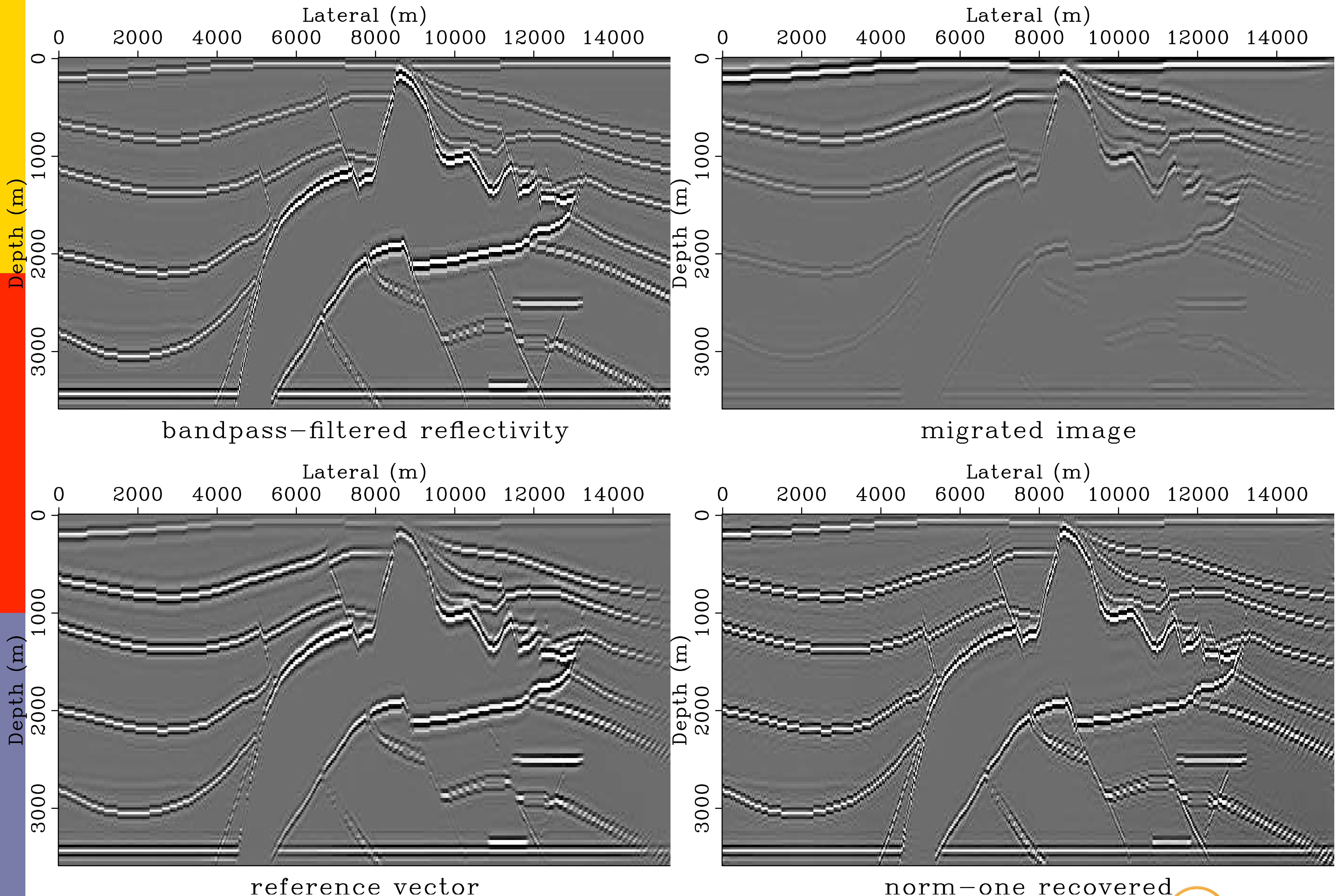

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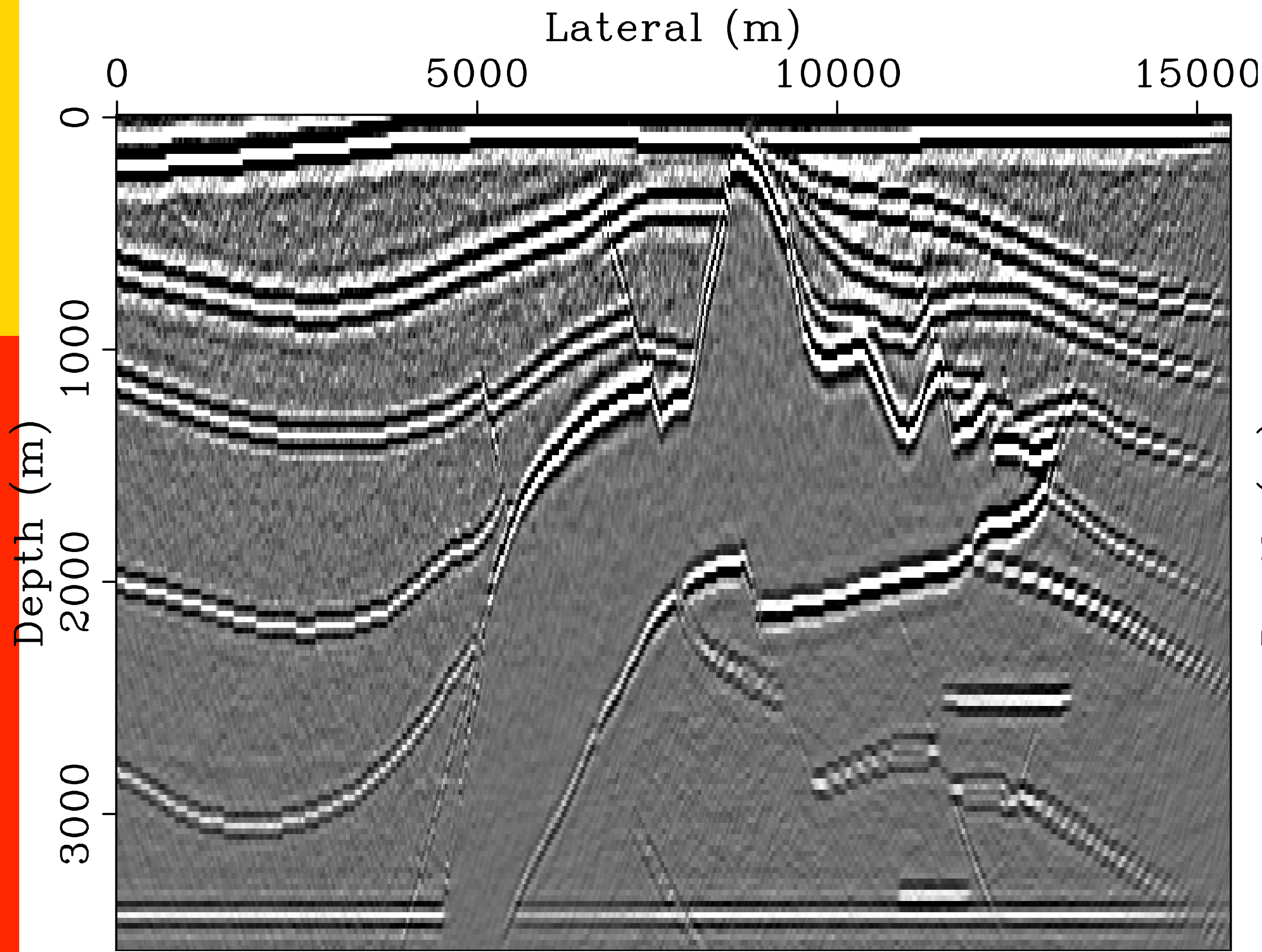
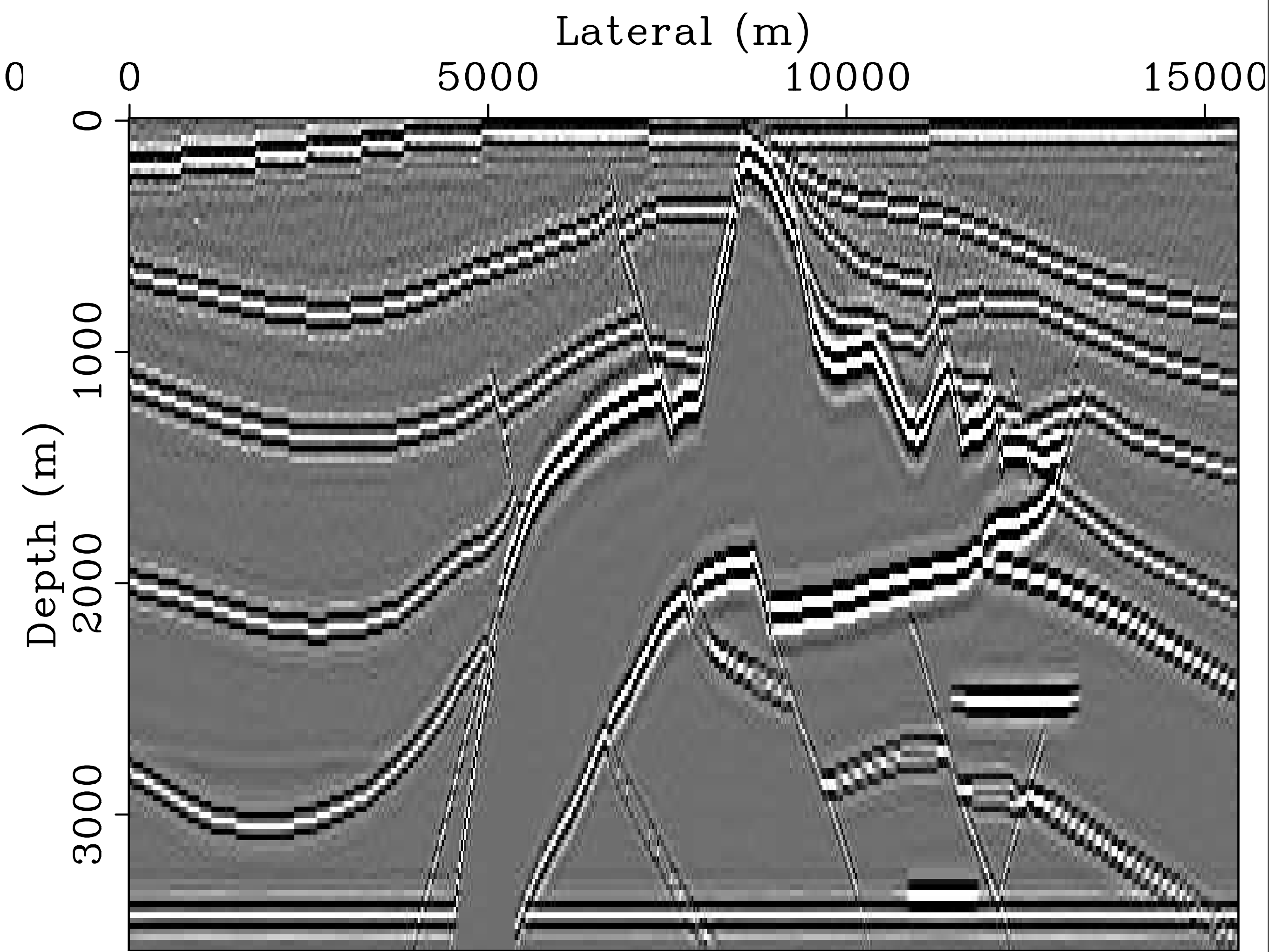
Imaging example



Migrated data



Amplitude-corrected & denoised migrated data

- two-way reverse time wave-equation migration with checkpointing [Symes '07]
- adjoint state method with 8000 time steps
- evaluation $\mathbf{K}^T$ takes 6 h on 60 CPU's

Observations

Curvelet-domain scaling

- handles conflicting dips (conormality assumption)
- exploits invariance under the PsDO

Diagonal approximation

- exploits smoothness of the symbol
- uses "neighbor" structure of curvelets

Results on the SEG AA' show

- recovery of amplitudes beneath the Salt
- successful recovery from clutter
- improvement of the continuity
- robust w.r.t. noise

Curvelet-domain matched filter ...

A primer on compressive sampling

Compressive sensing

[Candes, Romberg & Tao, Donoho, many others]

Three key ingredients

- existence of a sparsifying transform
  - handle wavefronts & reflectors with conflicting dips
- existence of a sub-Nyquist sampling strategy that reduces coherent aliases
  - incoherence
  - random sampling scheme
- existence of a large-scale (norm-one) solver
  - sparsity promotion by iterative thresholding and cooling

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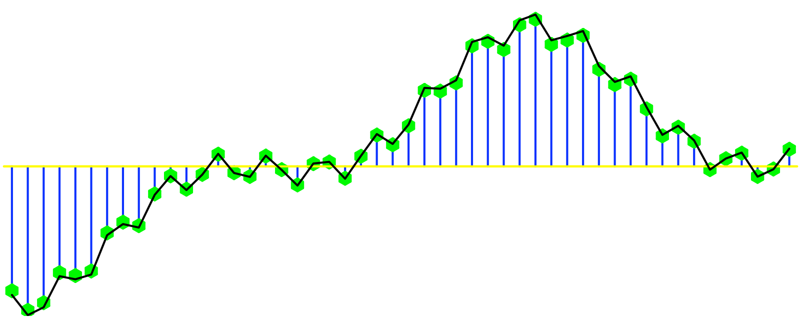
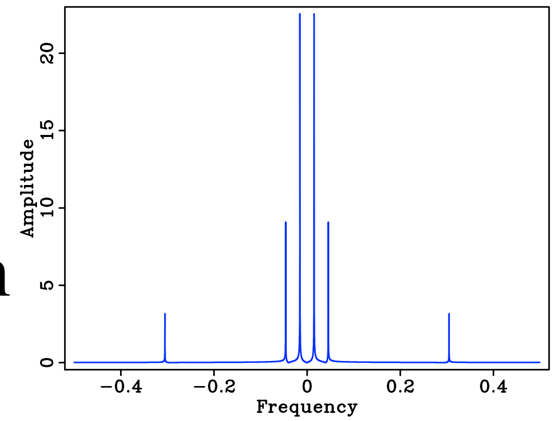
Simple example

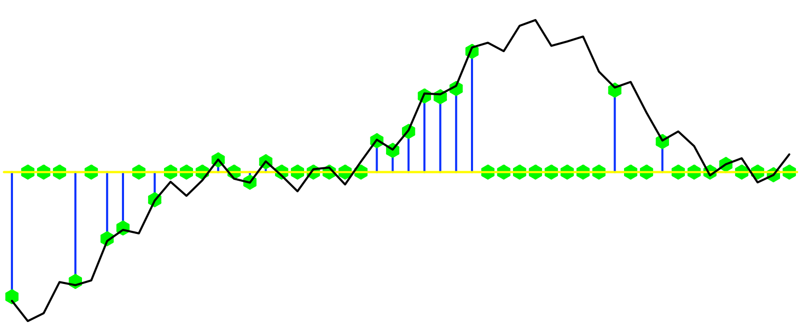


Fourier
transform

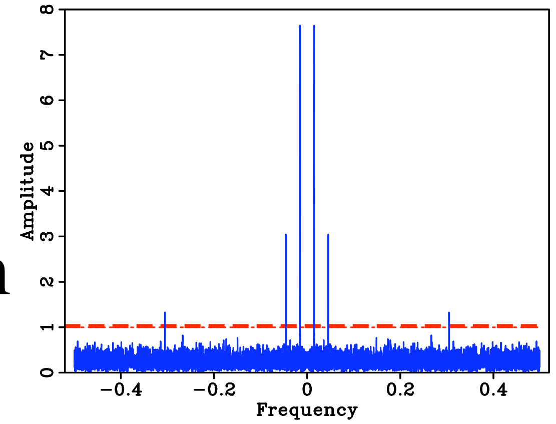


few significant coefficients

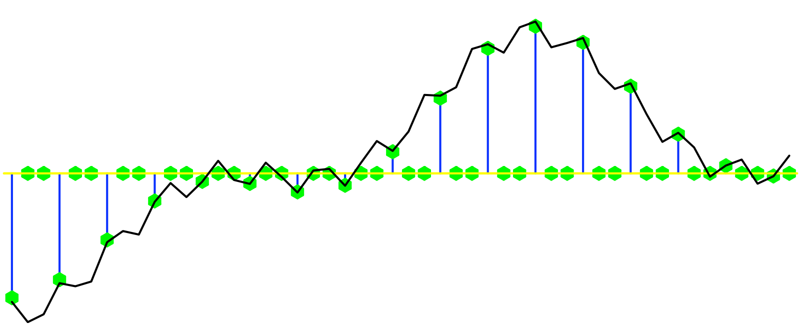
3-fold under-sampling



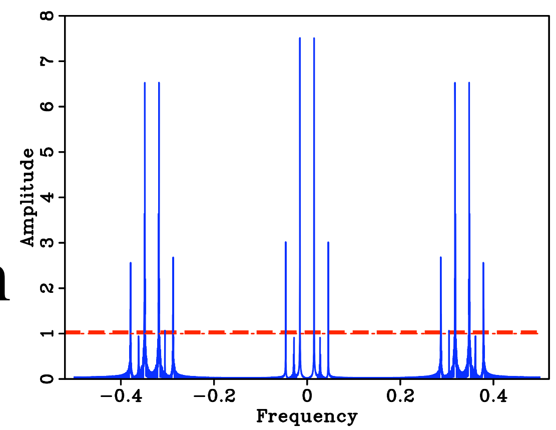
Fourier
transform



✓ significant coefficients detected

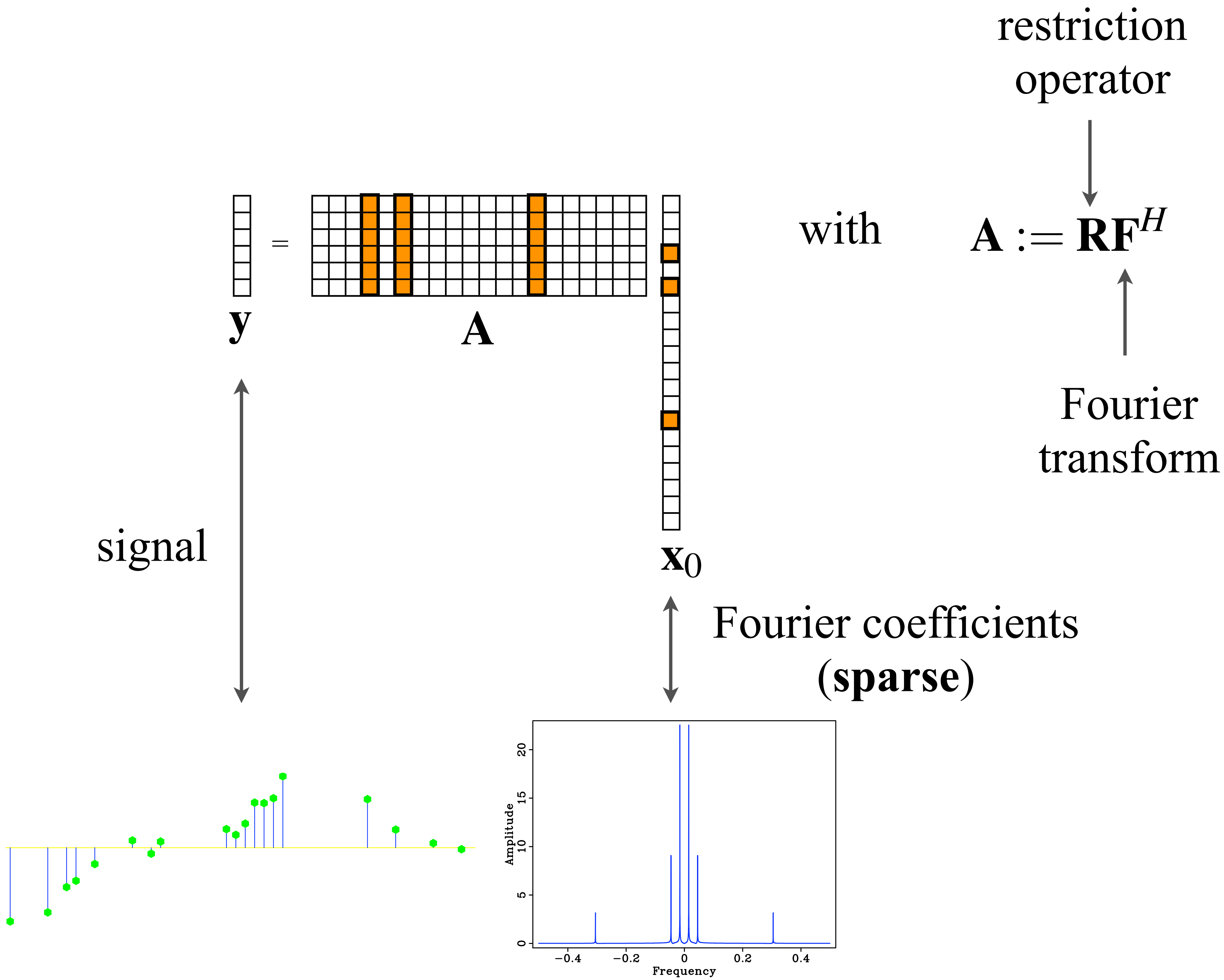


Fourier
transform

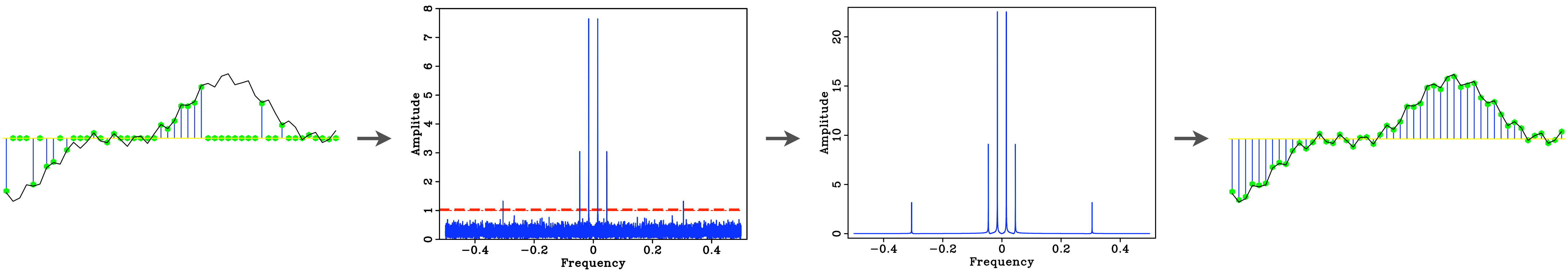


✗ ambiguity

Forward problem



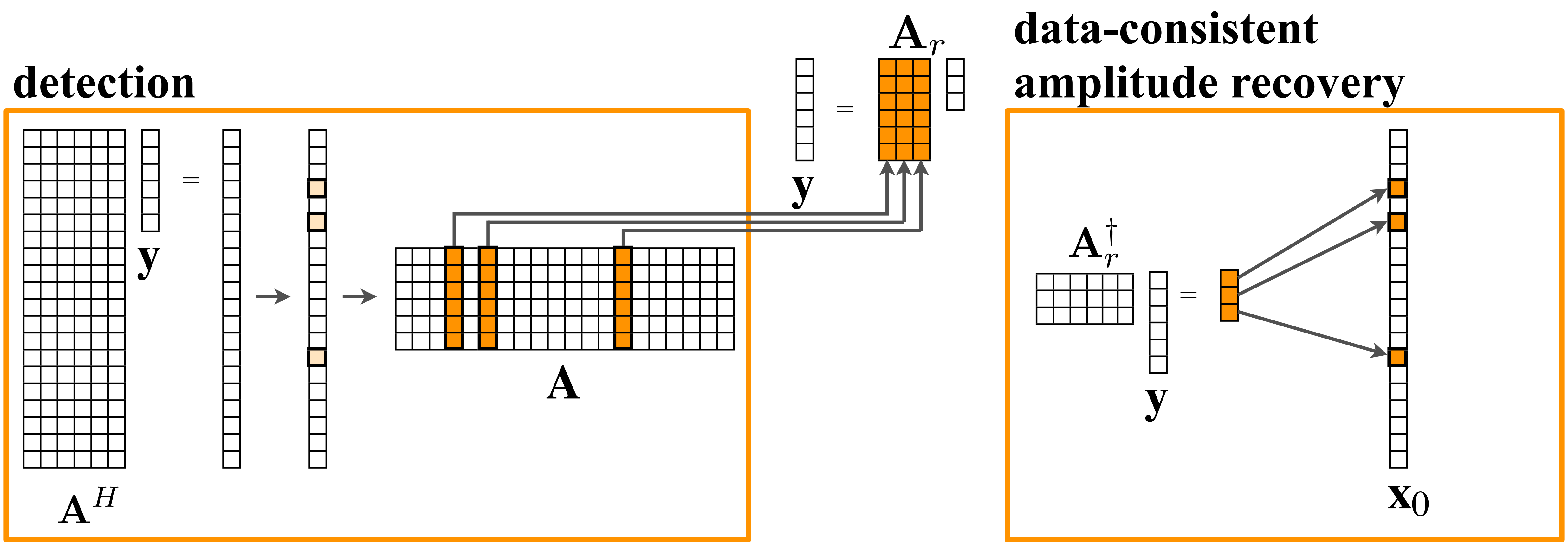
Naive sparsity-promoting recovery



inverse Fourier transform

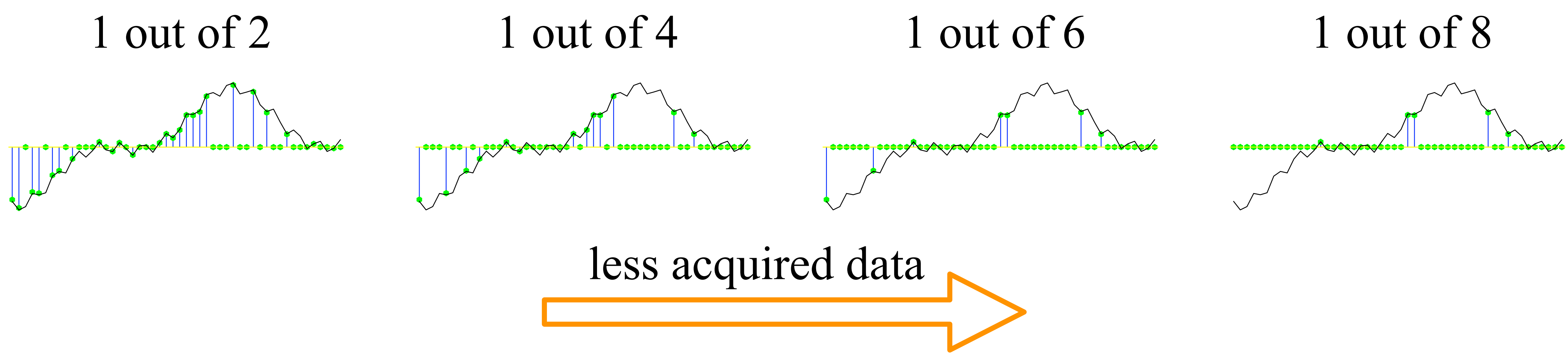
detection + data-consistent amplitude recovery

Fourier transform



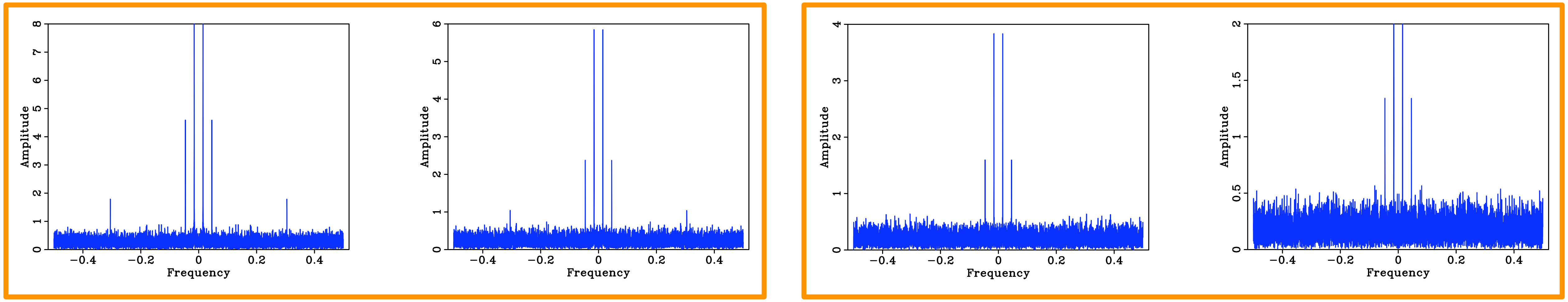
Undersampling “noise”

- “noise”
  - due to $\mathbf{A}^H\mathbf{A} \neq \mathbf{I}$
  - defined by $\mathbf{A}^H\mathbf{A}\mathbf{x}_0 - \alpha\mathbf{x}_0 = \mathbf{A}^H\mathbf{y} - \alpha\mathbf{x}_0$



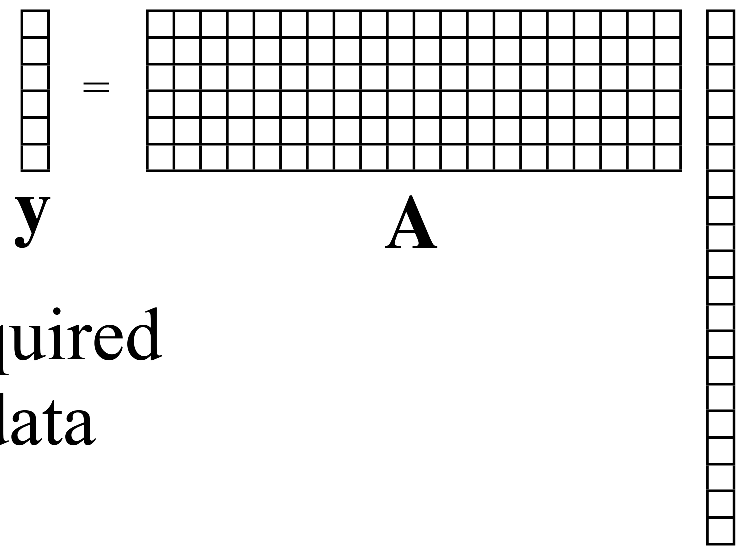
3 detectable Fourier modes

2 detectable Fourier modes



Sparsity-promoting wavefield reconstruction

restriction operator



$\mathbf{y}$ $\mathbf{A}$ $\mathbf{x}_0$

acquired data

complete wavefield (transform domain)

with $\mathbf{A} := \mathbf{R}\mathbf{S}^H$

sparsifying transform for seismic data

Interpolated data given by $\tilde{\mathbf{f}} = \mathbf{S}^H \tilde{\mathbf{x}}$ with

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$

[Sacchi et al ‘98]
[Xu et al ‘05]
[Zwartjes and Sacchi ‘07]
[Herrmann and Hennenfent ‘07]

Observations

- bla bla
- generalized to A=RMS^H
- depends on solver, sampling strategy and sparsity transform

Compressive sampling of wavefields

joint work with Deli Wang (visitor from Jilin university) and Gilles Hennenfent

“Curvelet-based seismic data processing: a multiscale and nonlinear approach” & to appear in Geophysics, “Non-parametric seismic data recovery with curvelet frames” and “Simply denoise: wavefield reconstruction via jittered undersampling”

General form compressive sampling

Solution of

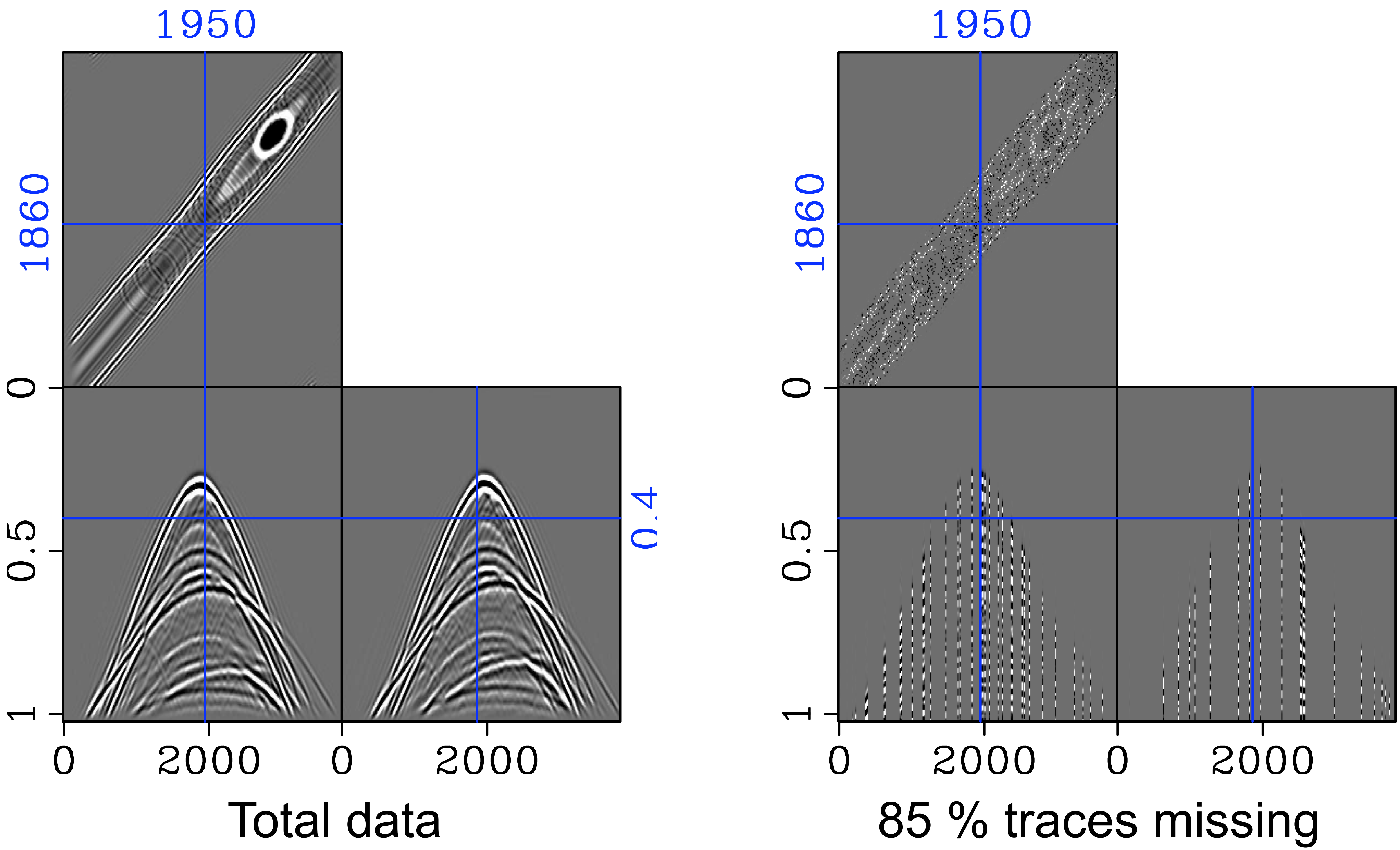
$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &= \mathbf{RMS}^T \\ \mathbf{R} &= \text{restriction matrix} \\ \mathbf{M} &= \text{measurement matrix} \\ \mathbf{S}^T &= \text{sparsity synthesis matrix} \\ \mathbf{y} &= \mathbf{RMf} \end{aligned}$$

recovers the function **f**.

The problem


1950
1860
0
0.5
1
0.4
0
2000
0
2000
Total data
1950
1860
0
0.5
1
0.4
0
2000
0
2000
85 % traces missing

Requirements

Sparsifying transform (**S**)

- curvelet
- focussed curvelets

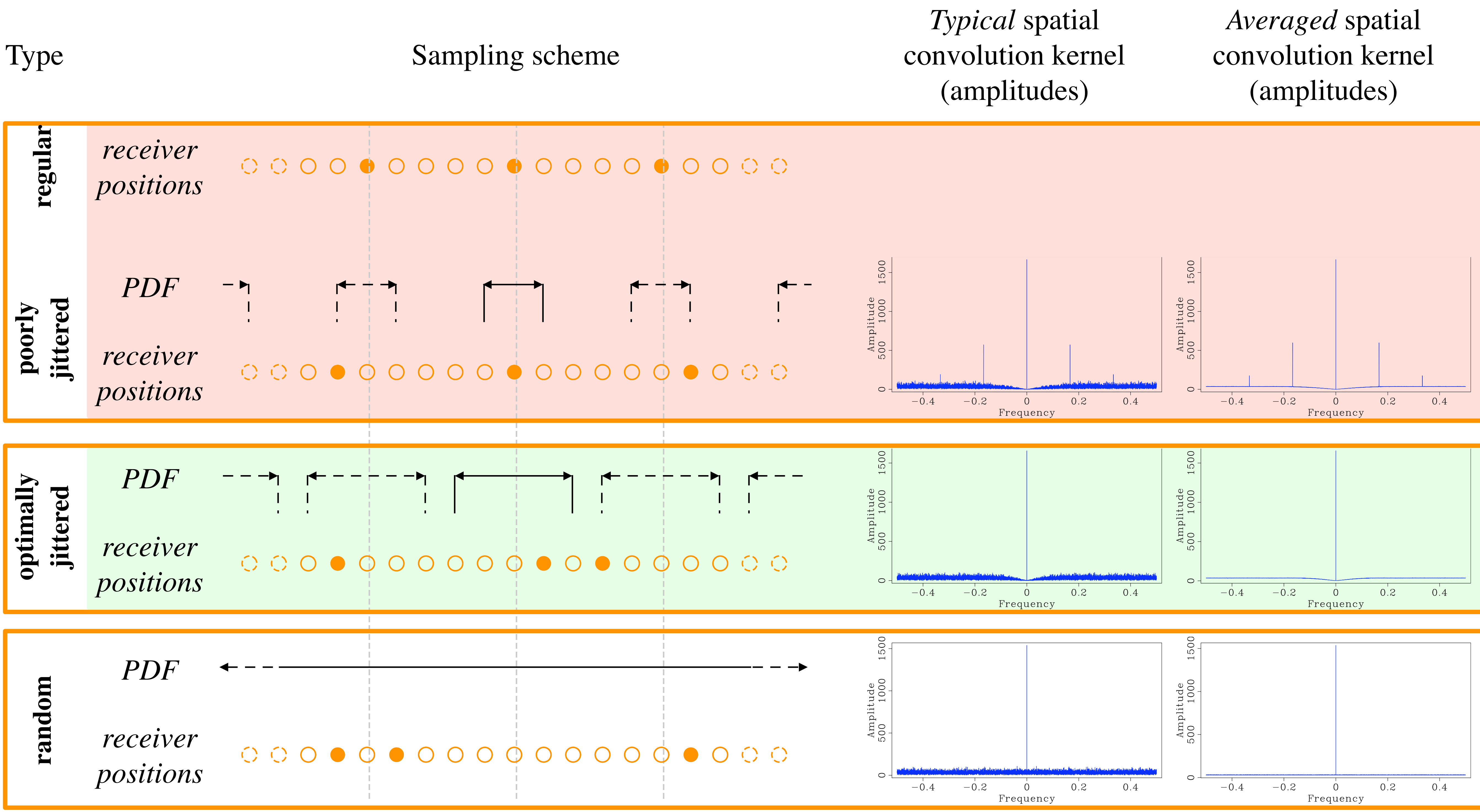
Sampling scheme (**RM**)

- random sampling
- random jittered sampling => control largest gaps

Sparsity promoting solver (**P**)

- Iterative thresholding (Landweber + soft threshold)

Discrete random jittered undersampling



[Hennenfent and Herrmann ‘07]

Curvelet-based recovery

Solution of

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &= \mathbf{R}\mathbf{I}\mathbf{C}^T \\ \mathbf{R} &= \text{jitter sampling} \\ \mathbf{I} &= \text{Dirac basis} \\ \mathbf{C}^T &= \text{curvelet synthesis} \\ \mathbf{y} &= \mathbf{R}\mathbf{f} \end{aligned}$$

recovers the wavefield f.

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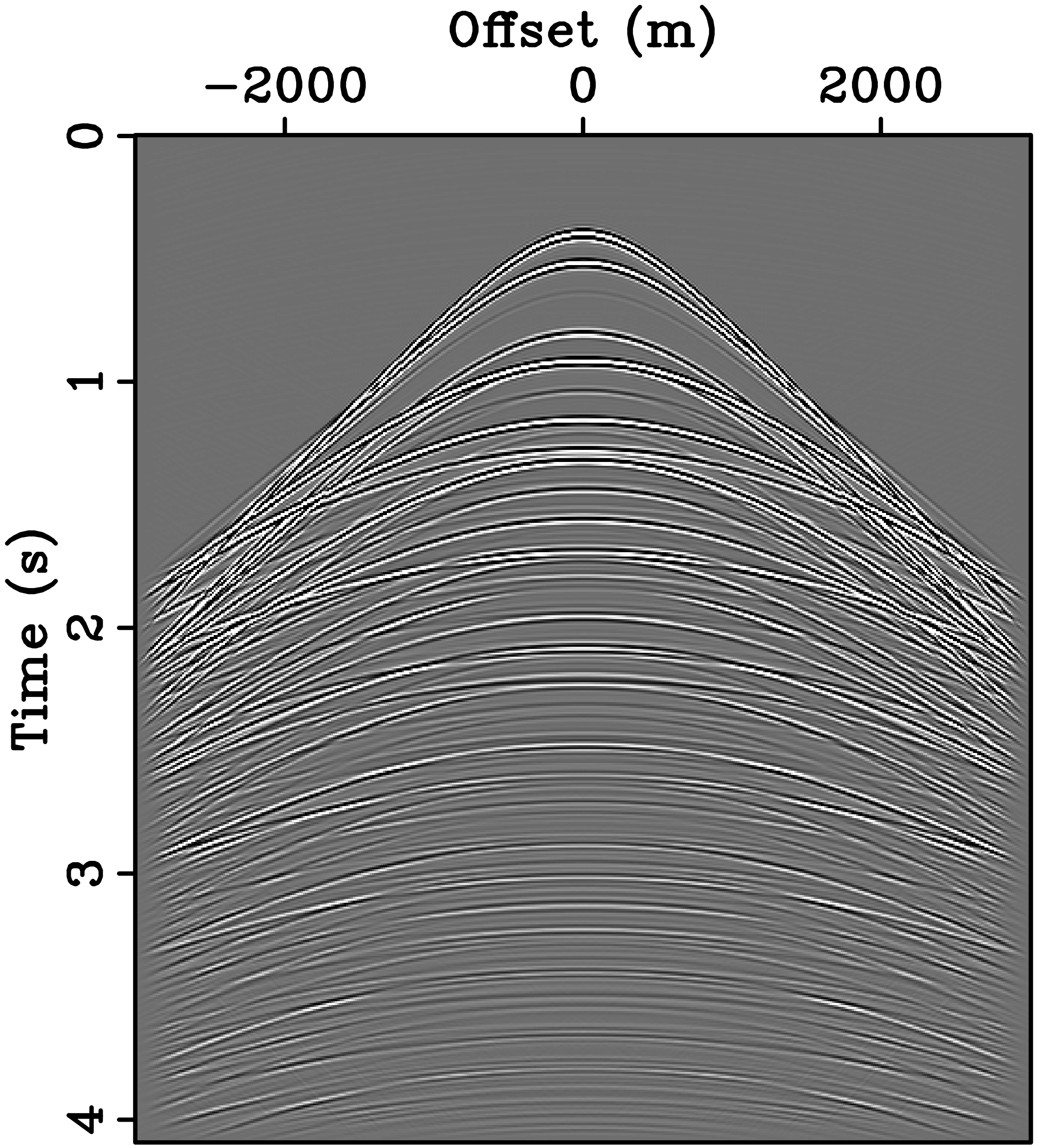
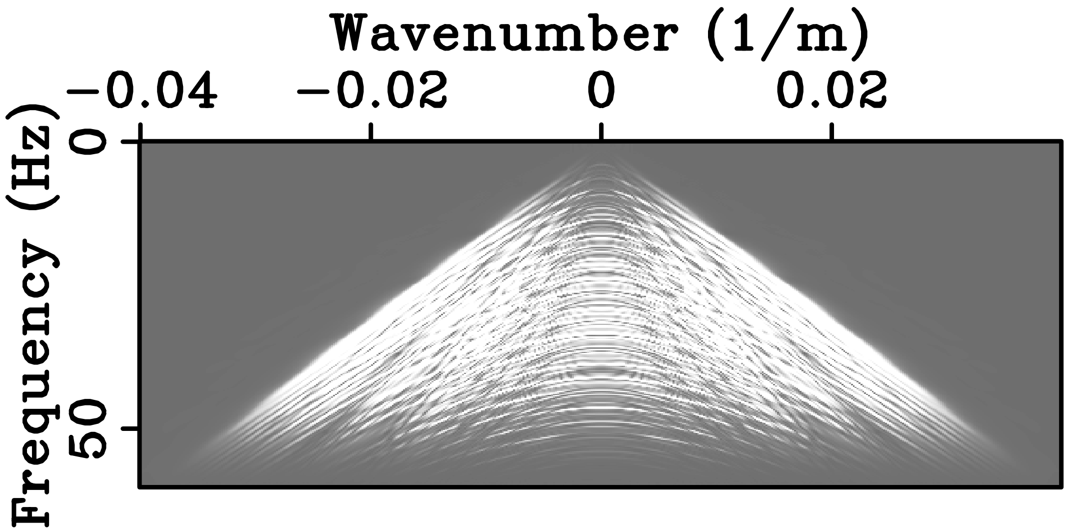
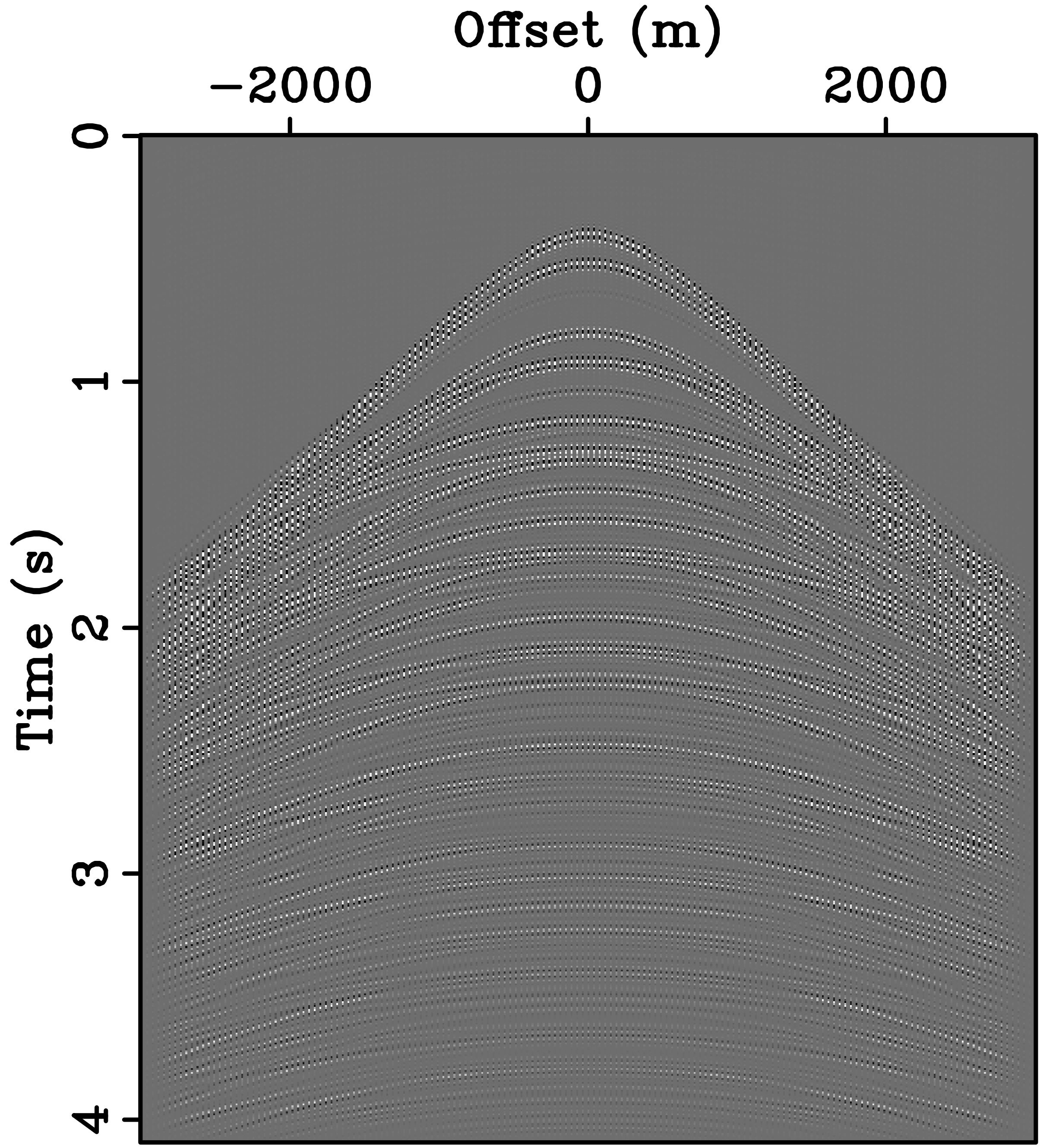
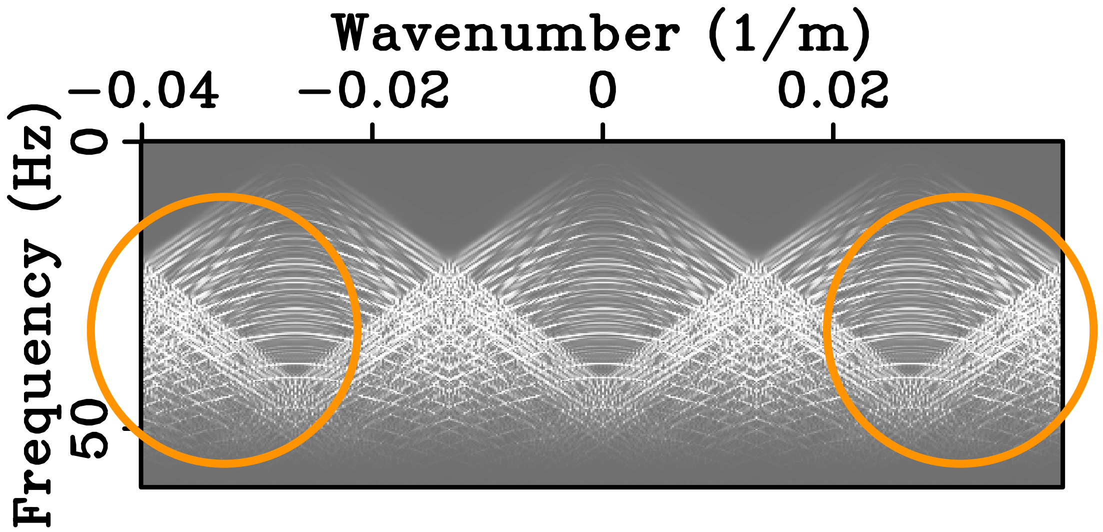
Model


Offset (m)
−2000
0
2000
Time (s)
0
1
2
3
4

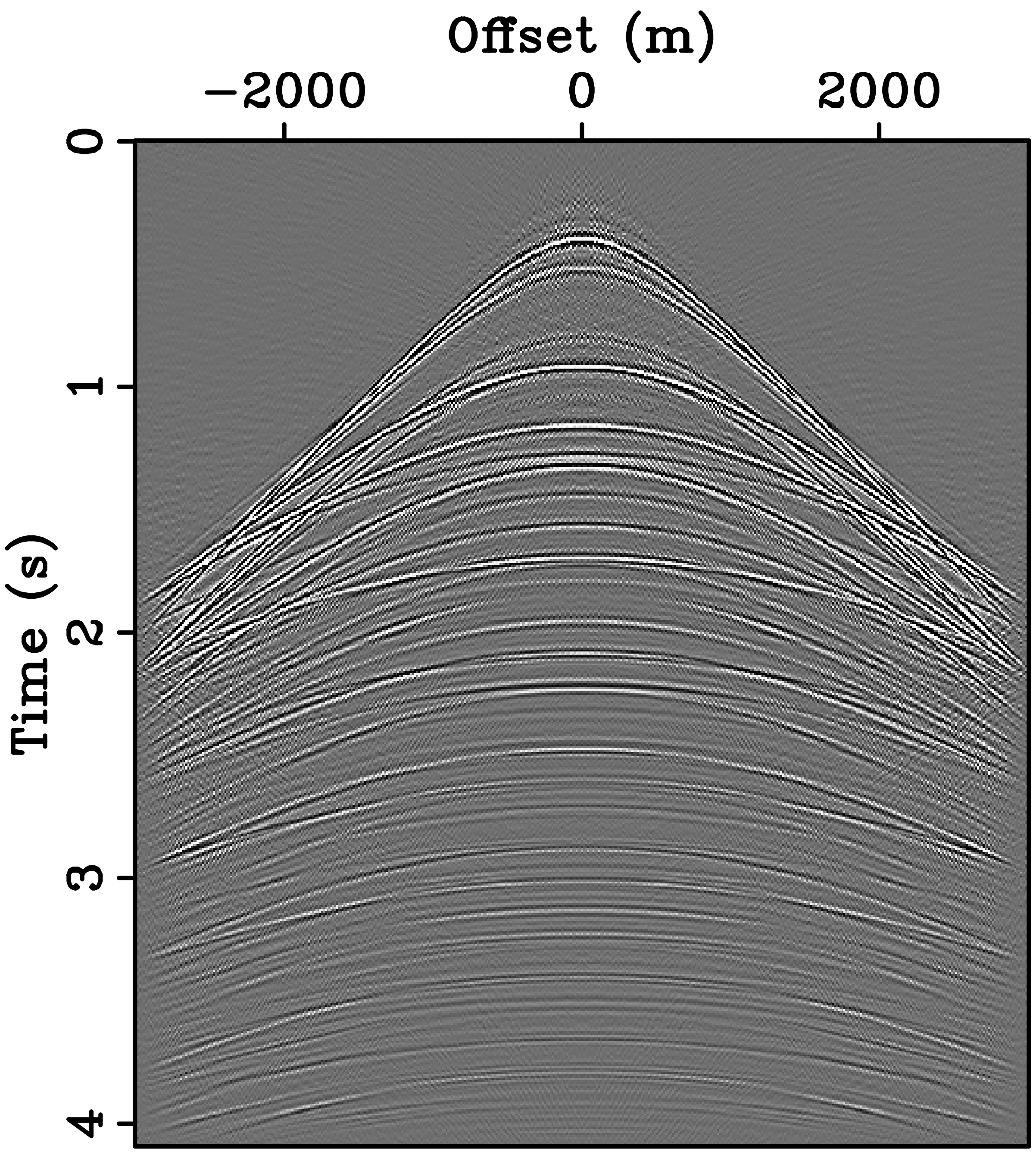

Wavenumber (1/m)
−0.04
−0.02
0
0.02
Frequency (Hz)
0
50

Regular 3-fold undersampling

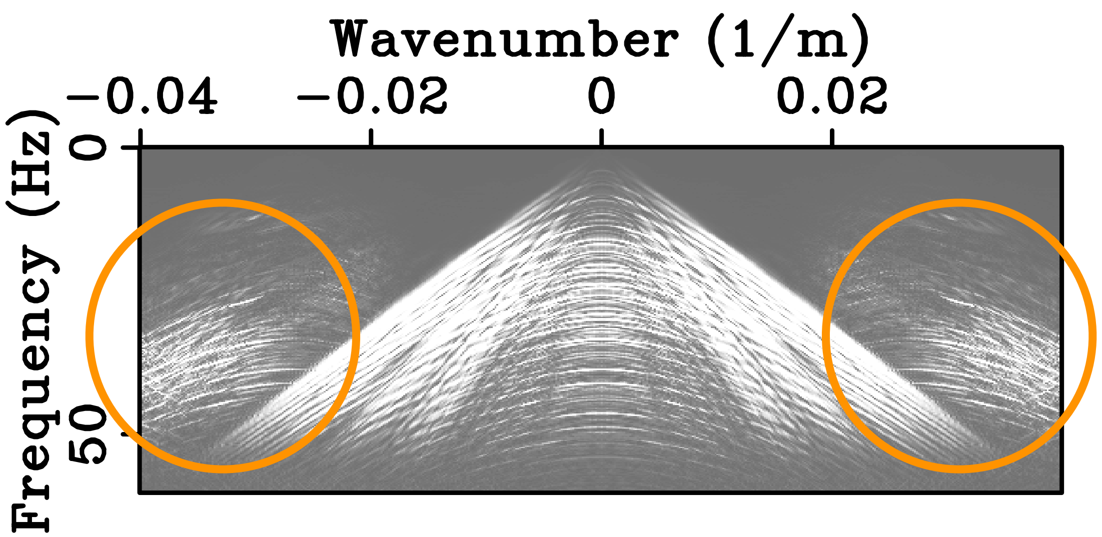




CRSI from regular 3-fold undersampling

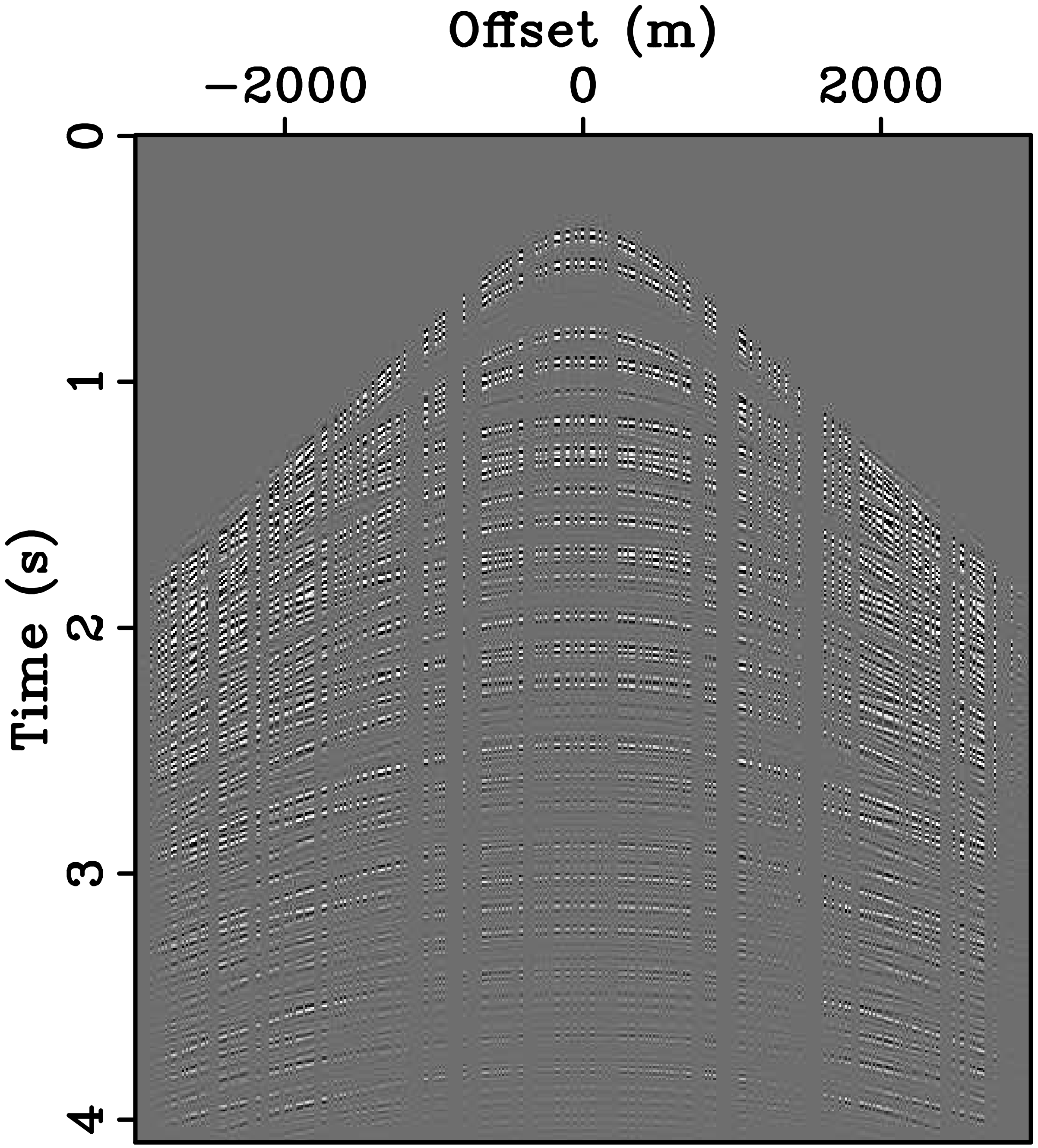


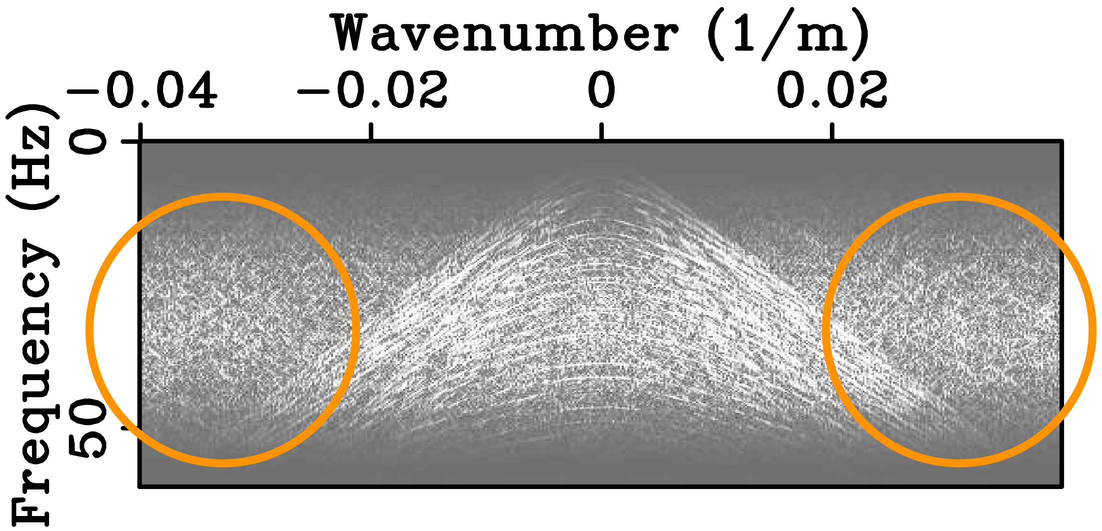
SNR = 6.92 dB



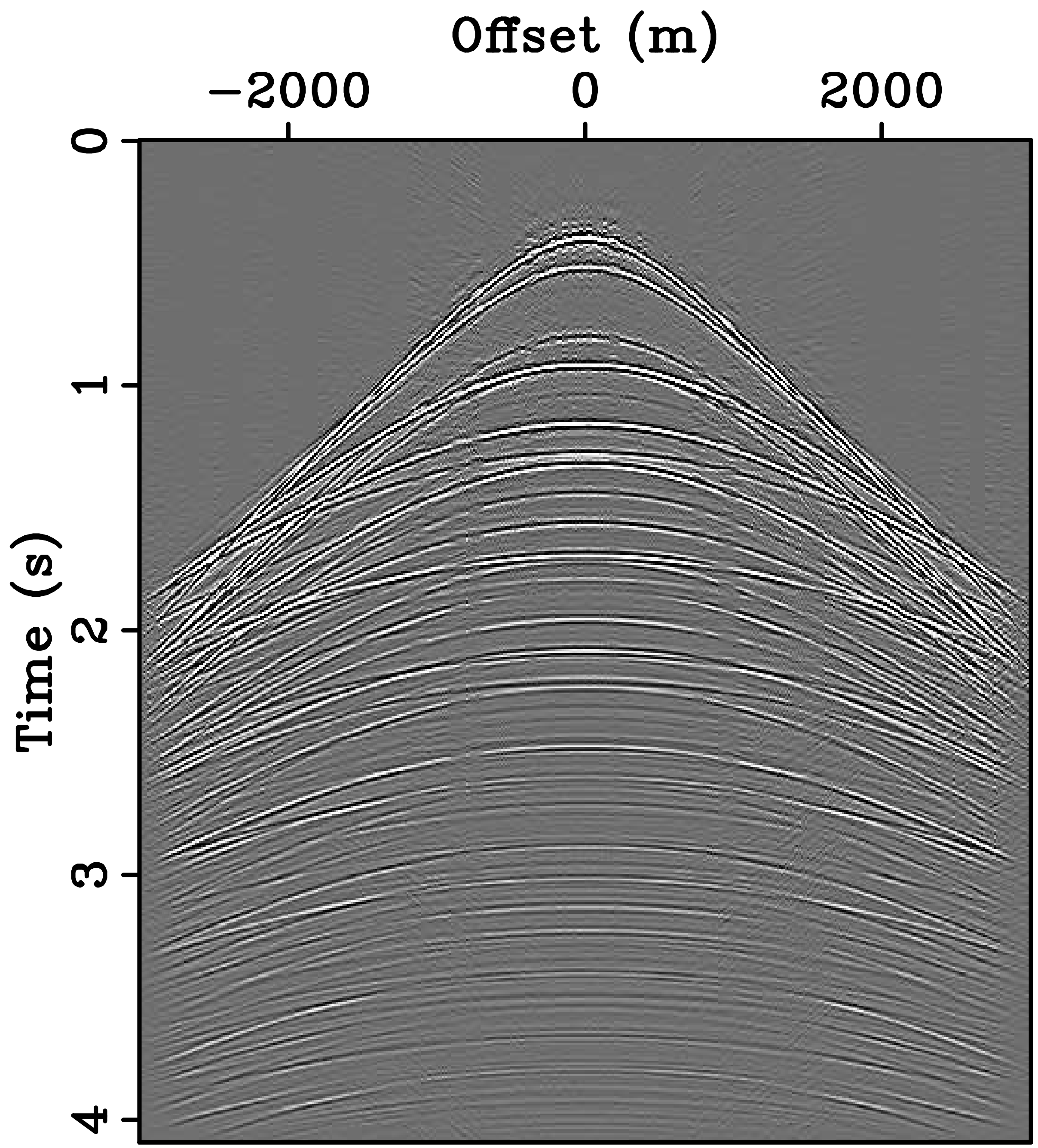
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

Random 3-fold undersampling

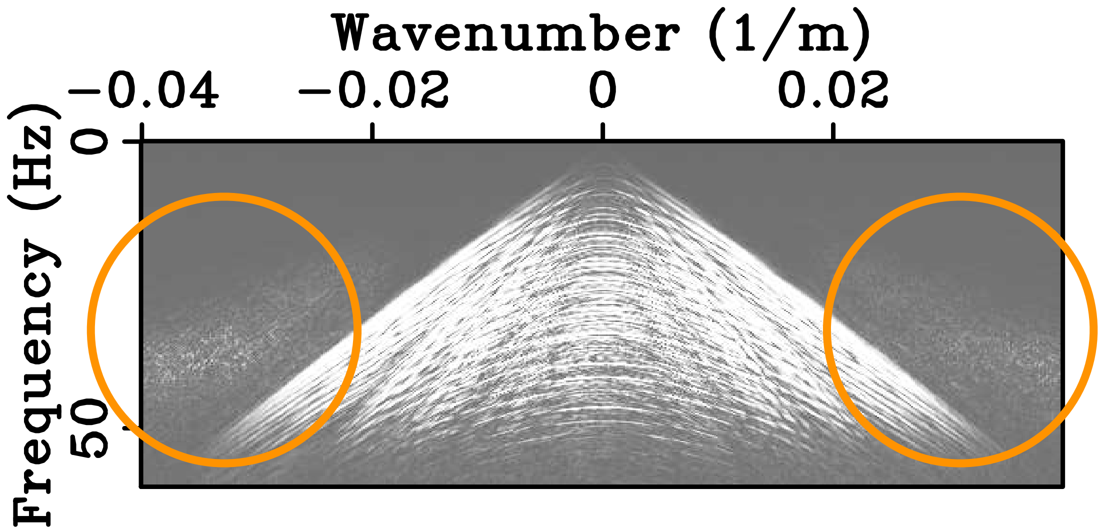




CRSI from random 3-fold undersampling

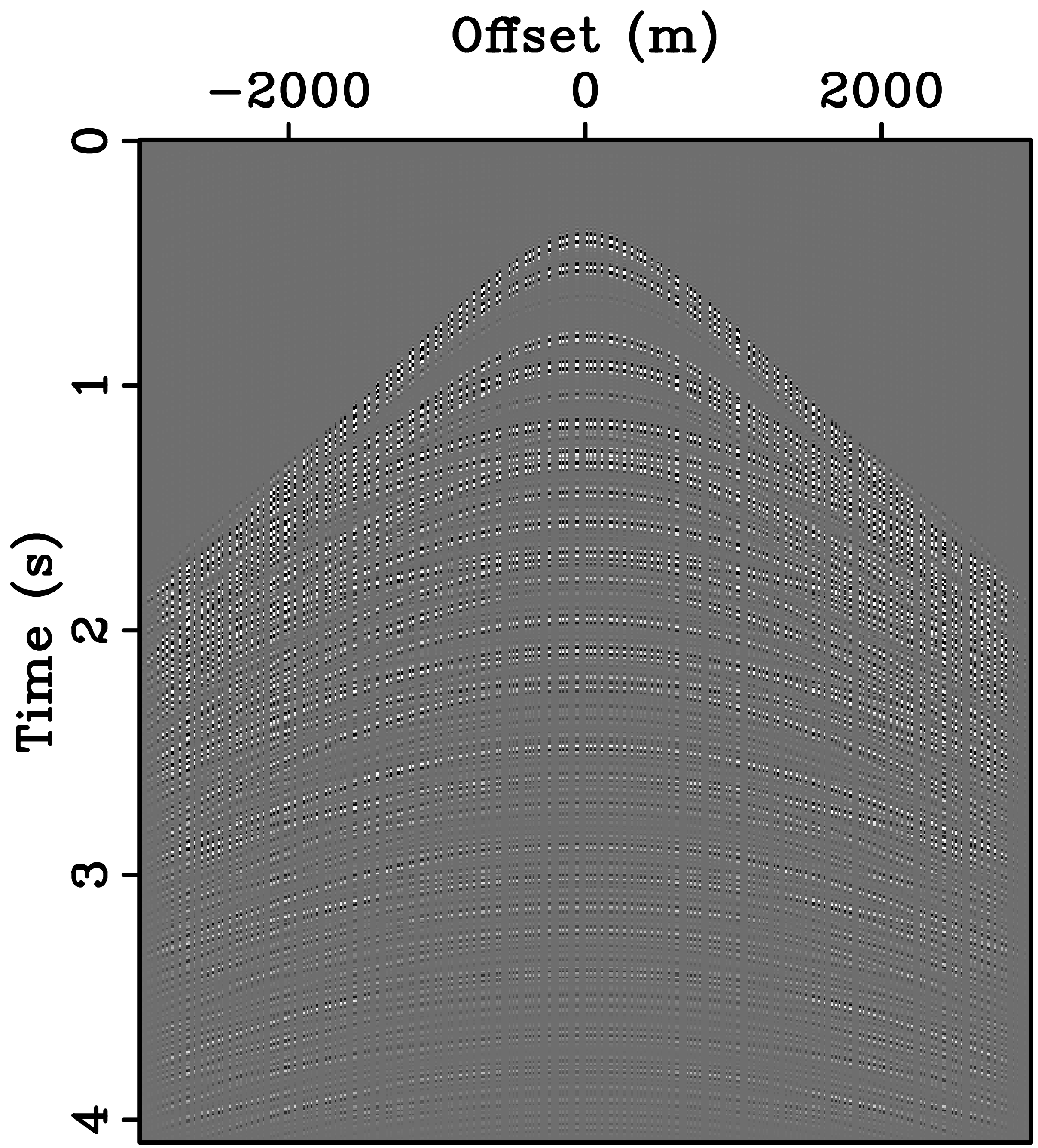


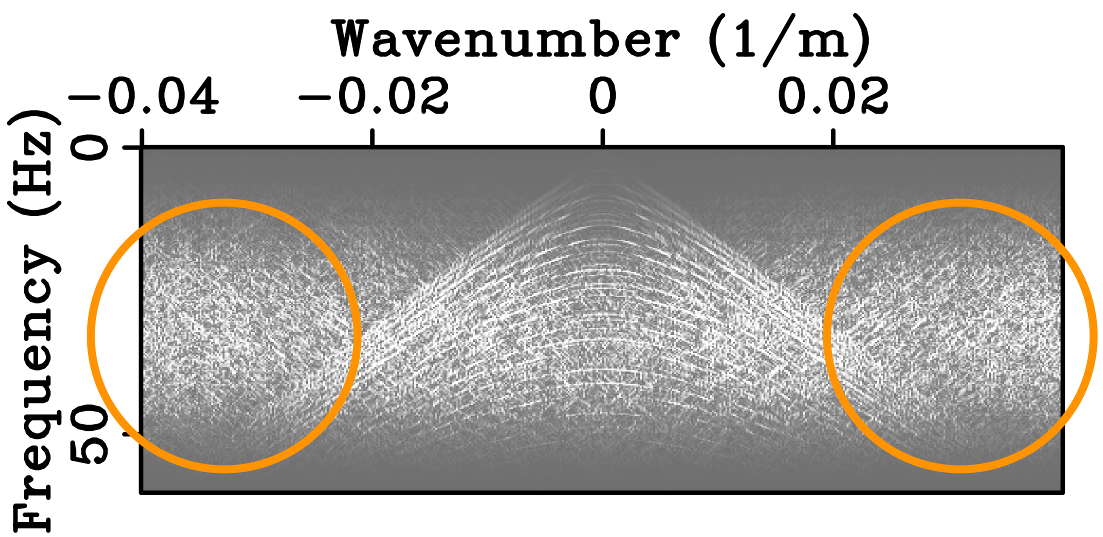
SNR = 9.72 dB



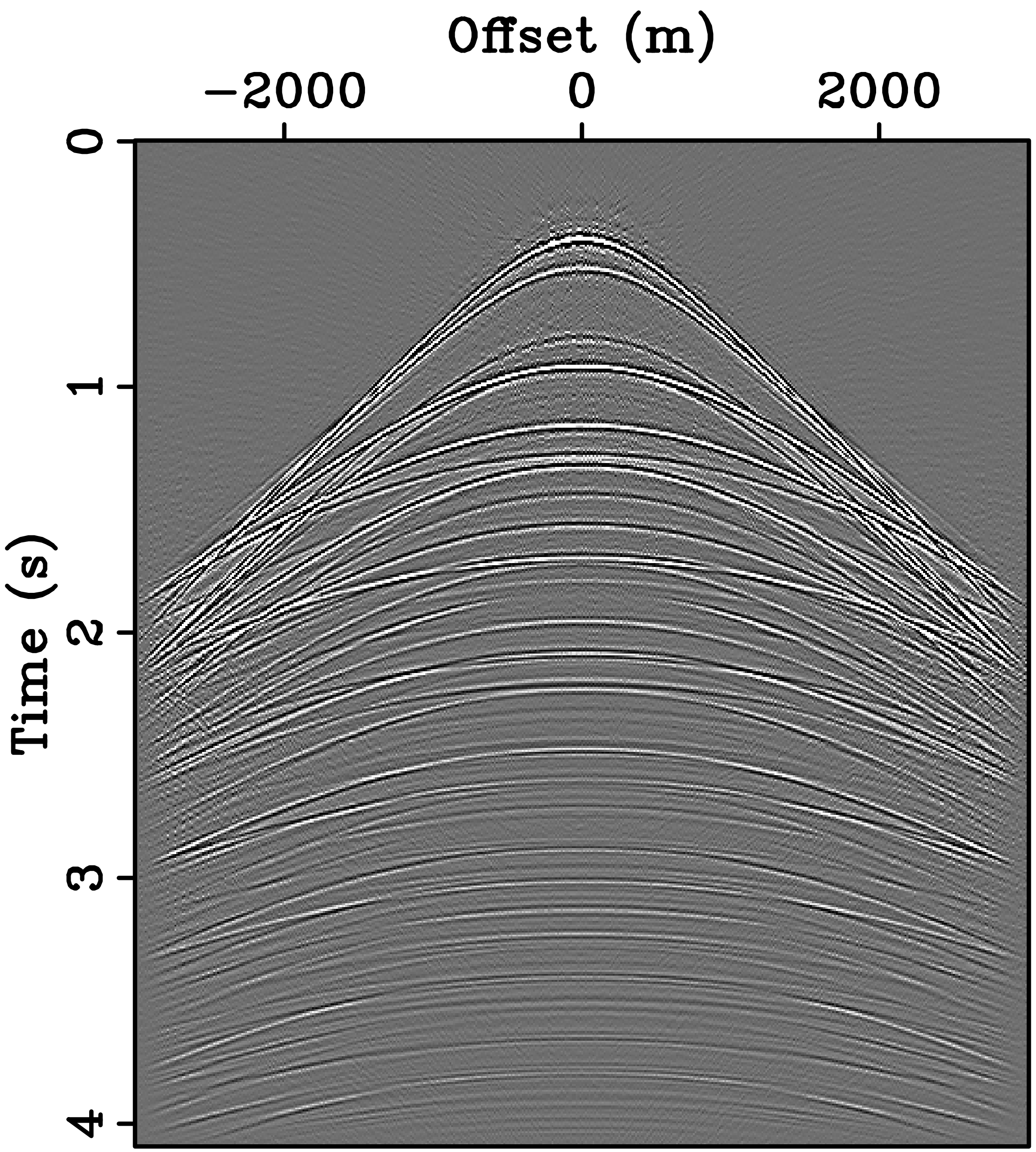
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

Optimally-jittered 3-fold undersampling

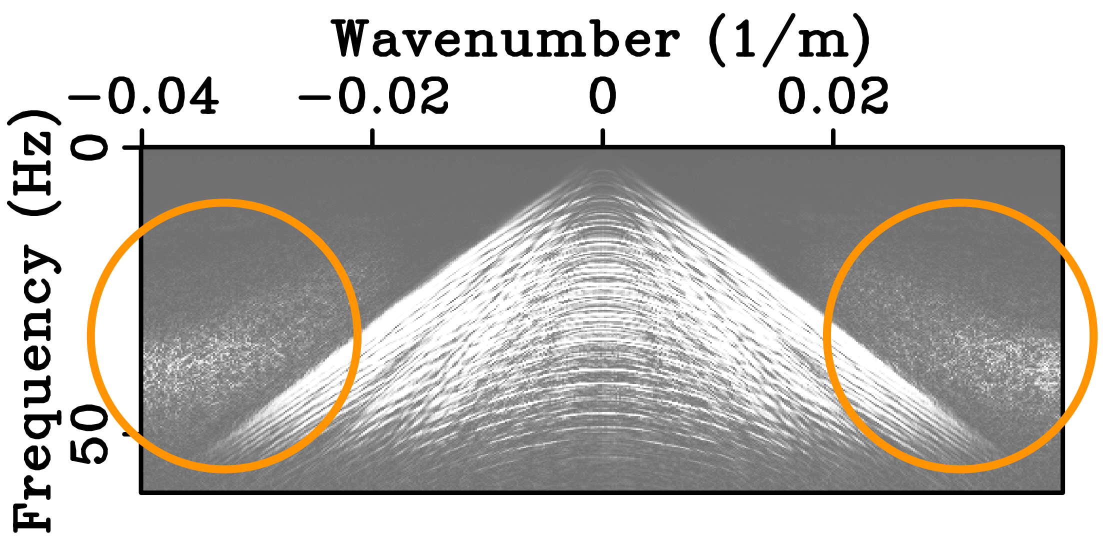




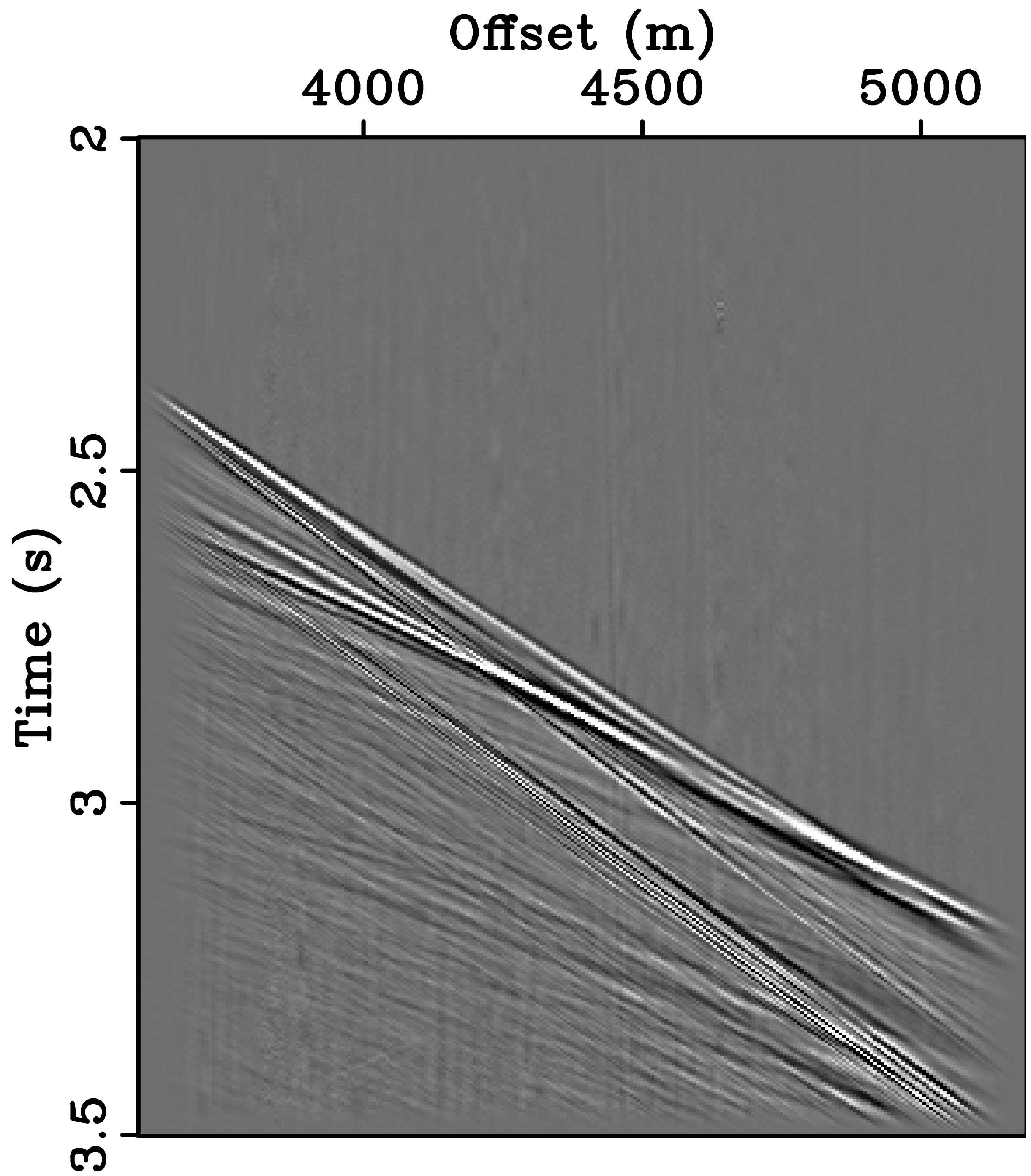
CRSI from opt.-jittered 3-fold undersampling



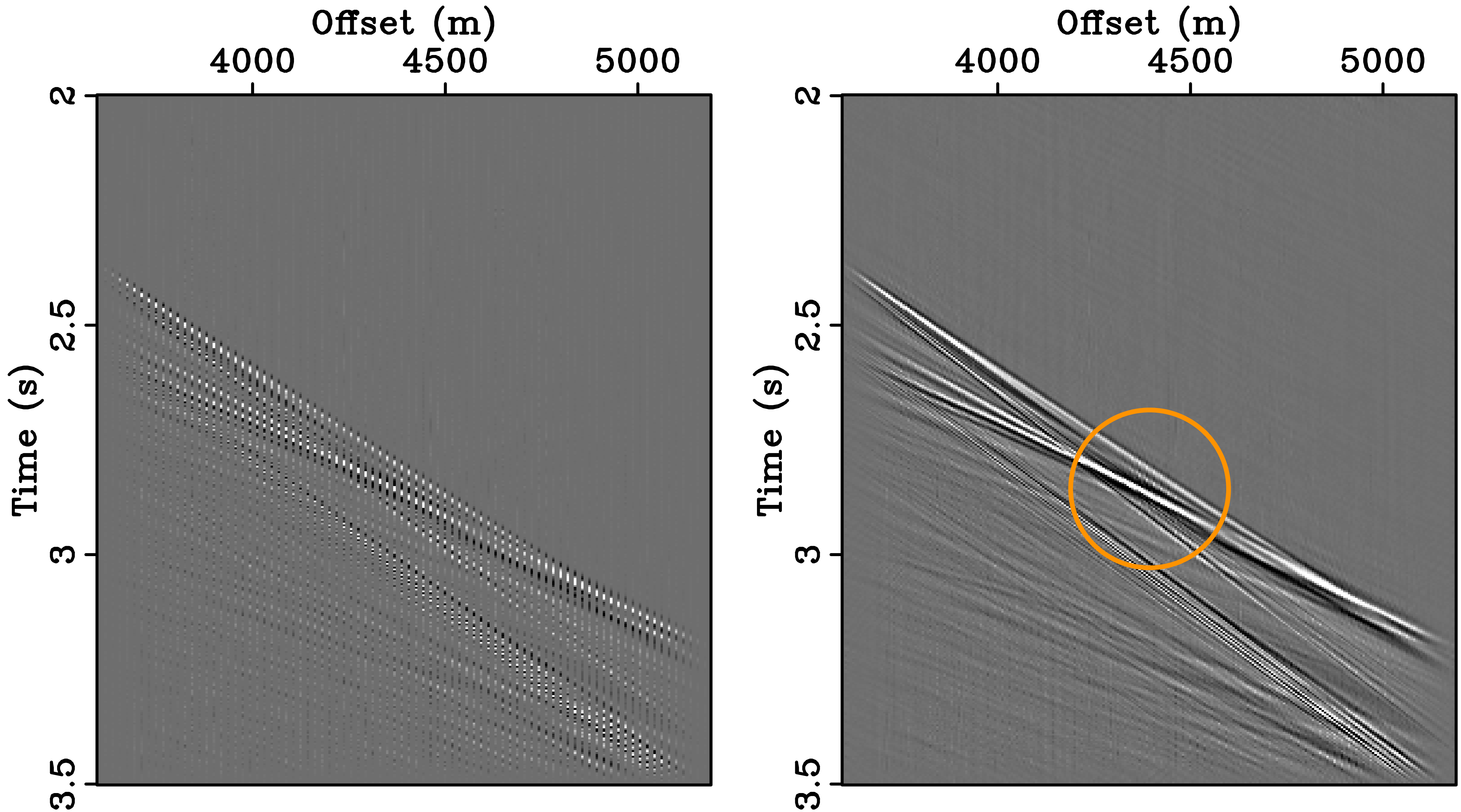
SNR = 10.42 dB



Model

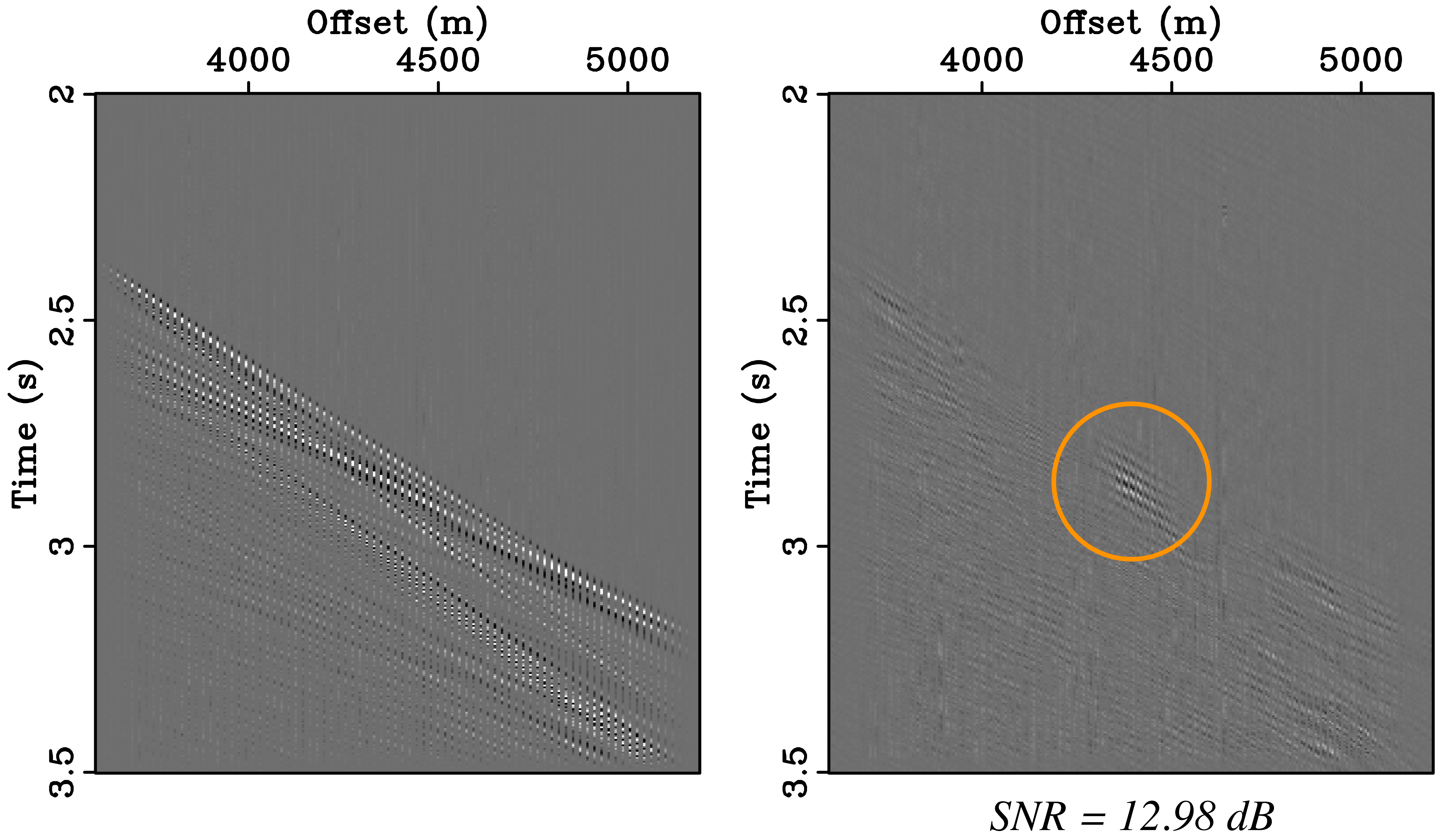


Regular 3-fold undersampling

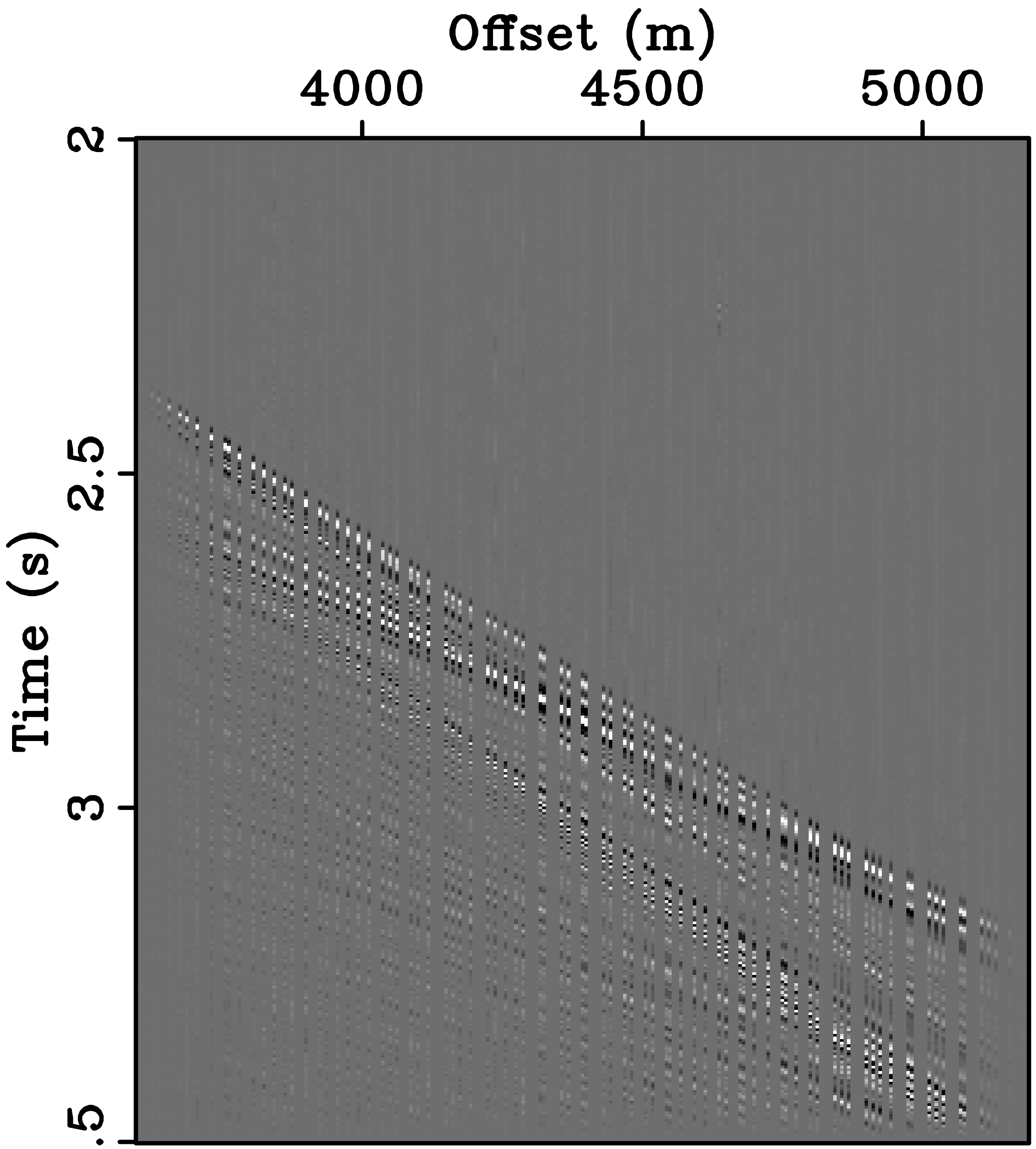


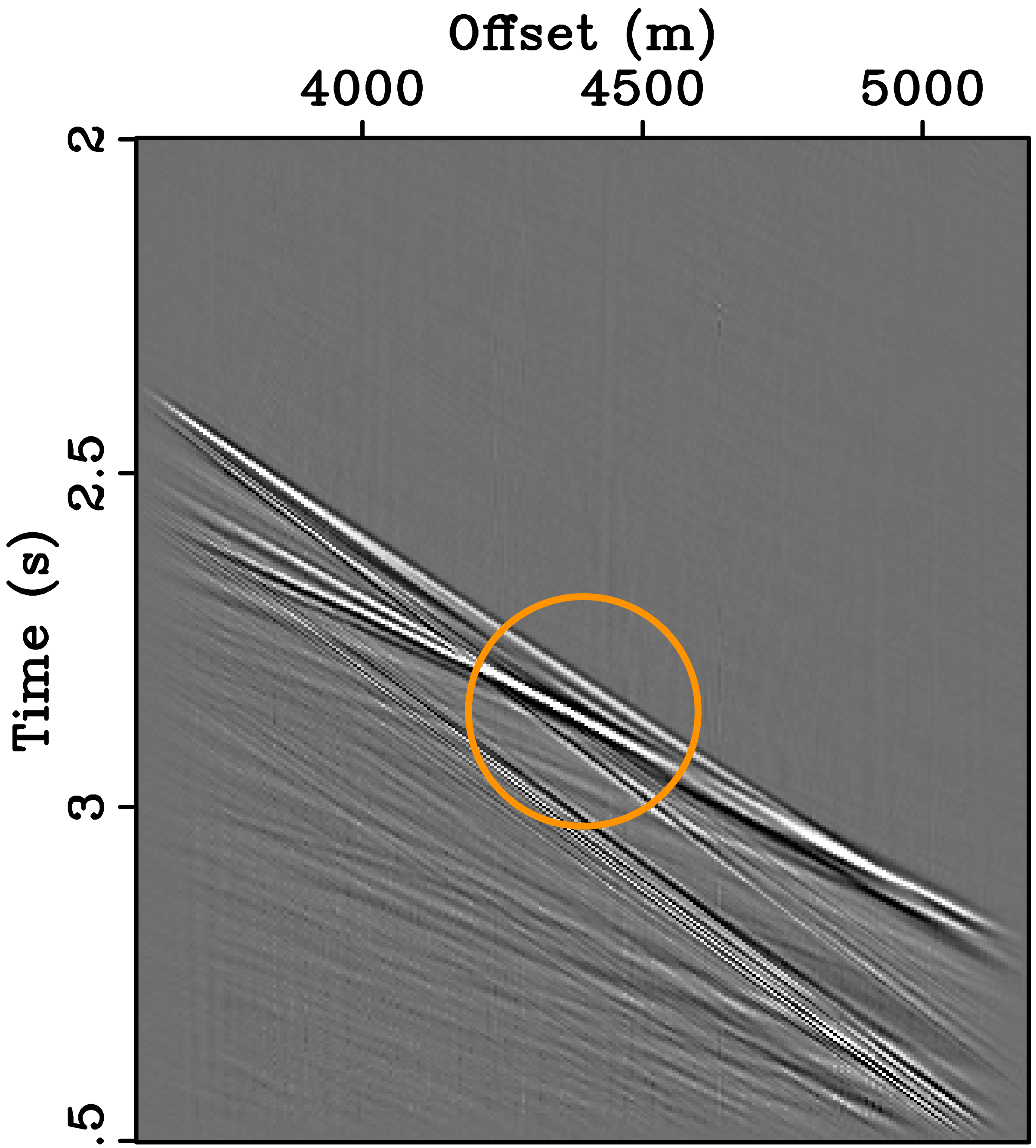
SNR = 12.98 dB

Regular 3-fold undersampling



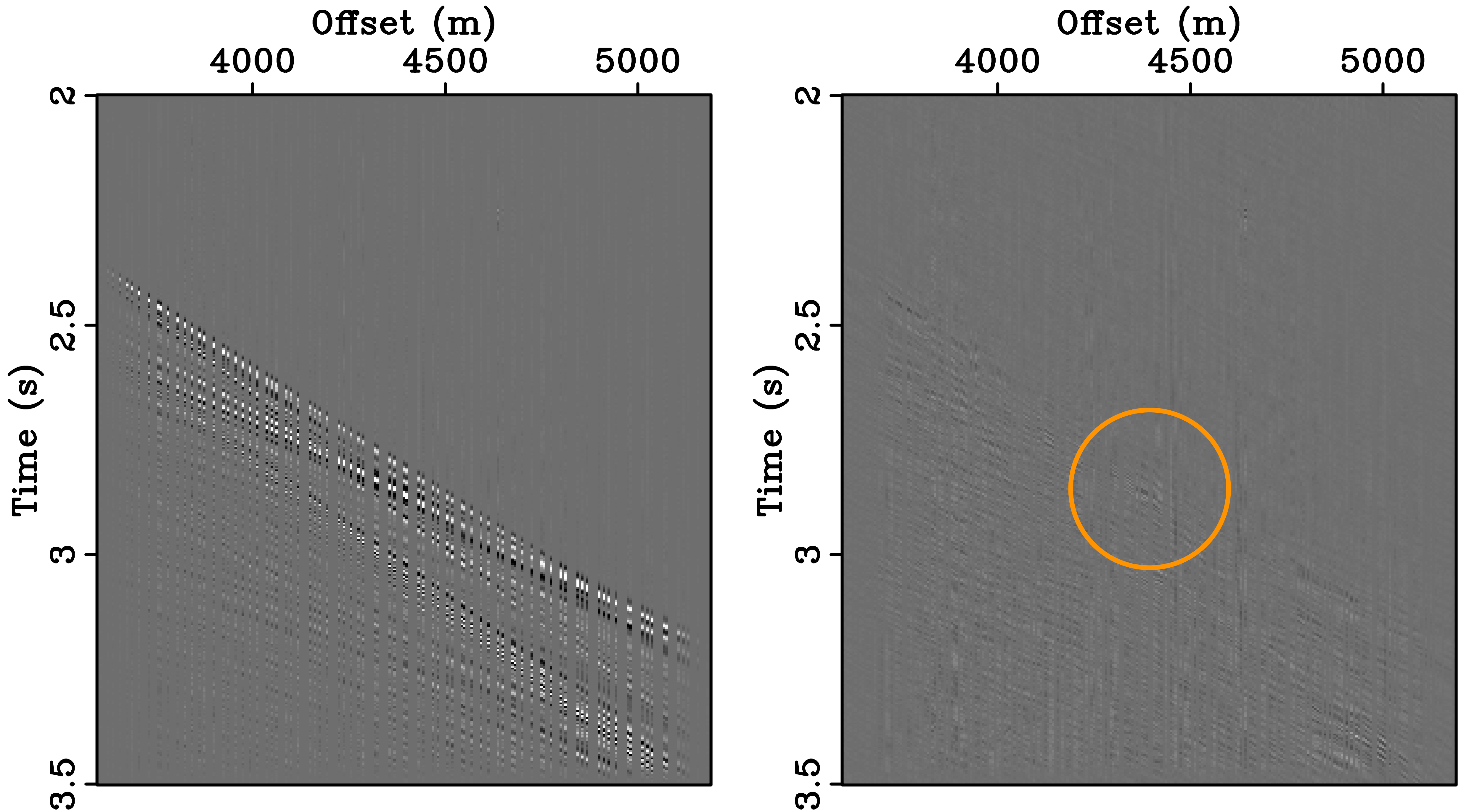
Optimally-jittered 3-fold undersampling





SNR = 15.22 dB

Optimally-jittered 3-fold undersampling



SNR = 15.22 dB

Focussed recovery

Solution of

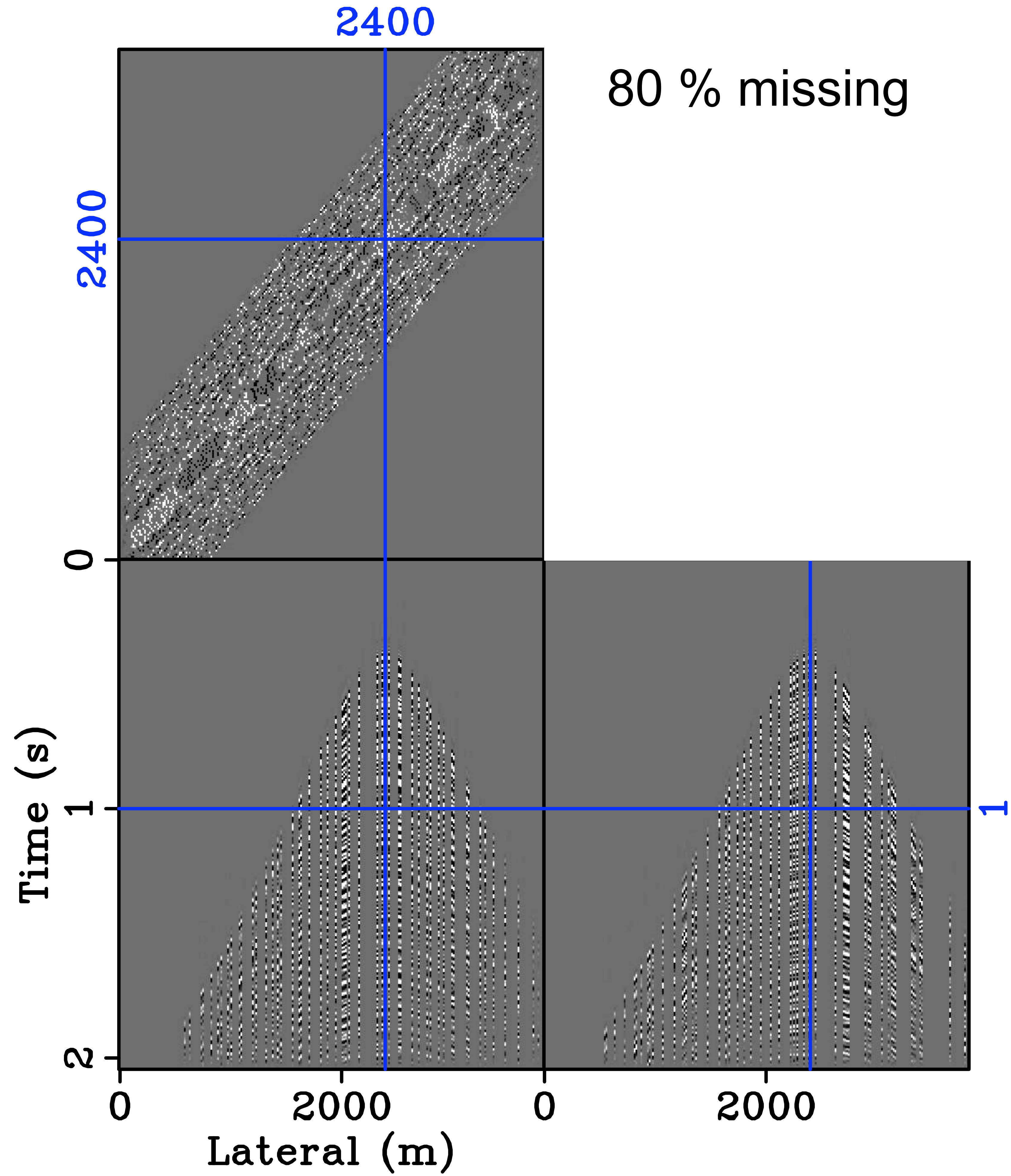
$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{\Delta P C}^T \tilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &= \mathbf{R \Delta P C}^T \\ \mathbf{\Delta P} &= \text{main primaries} \\ \mathbf{y} &= \mathbf{R f} \end{aligned}$$

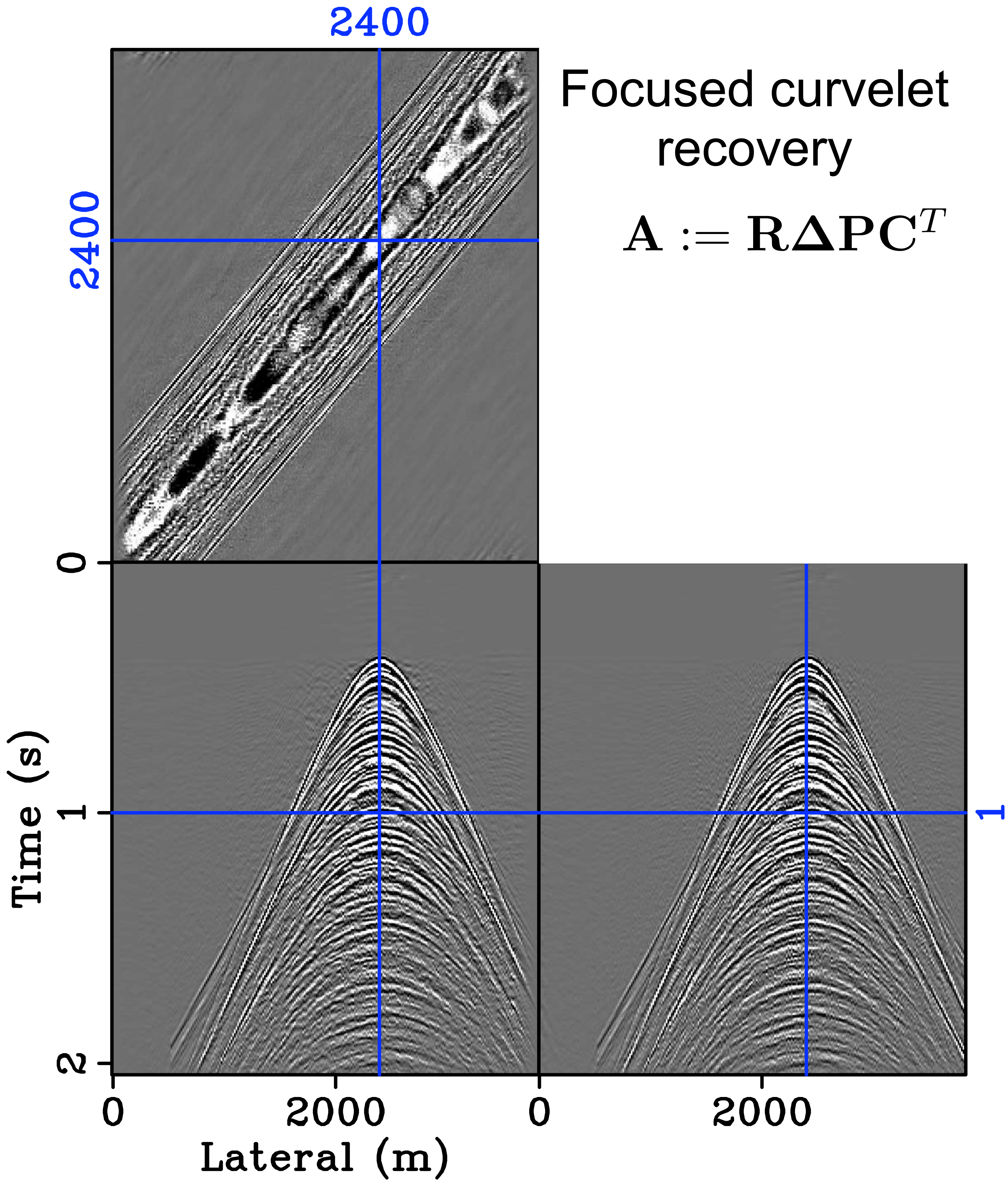
recovers the wavefield f.

80 % missing



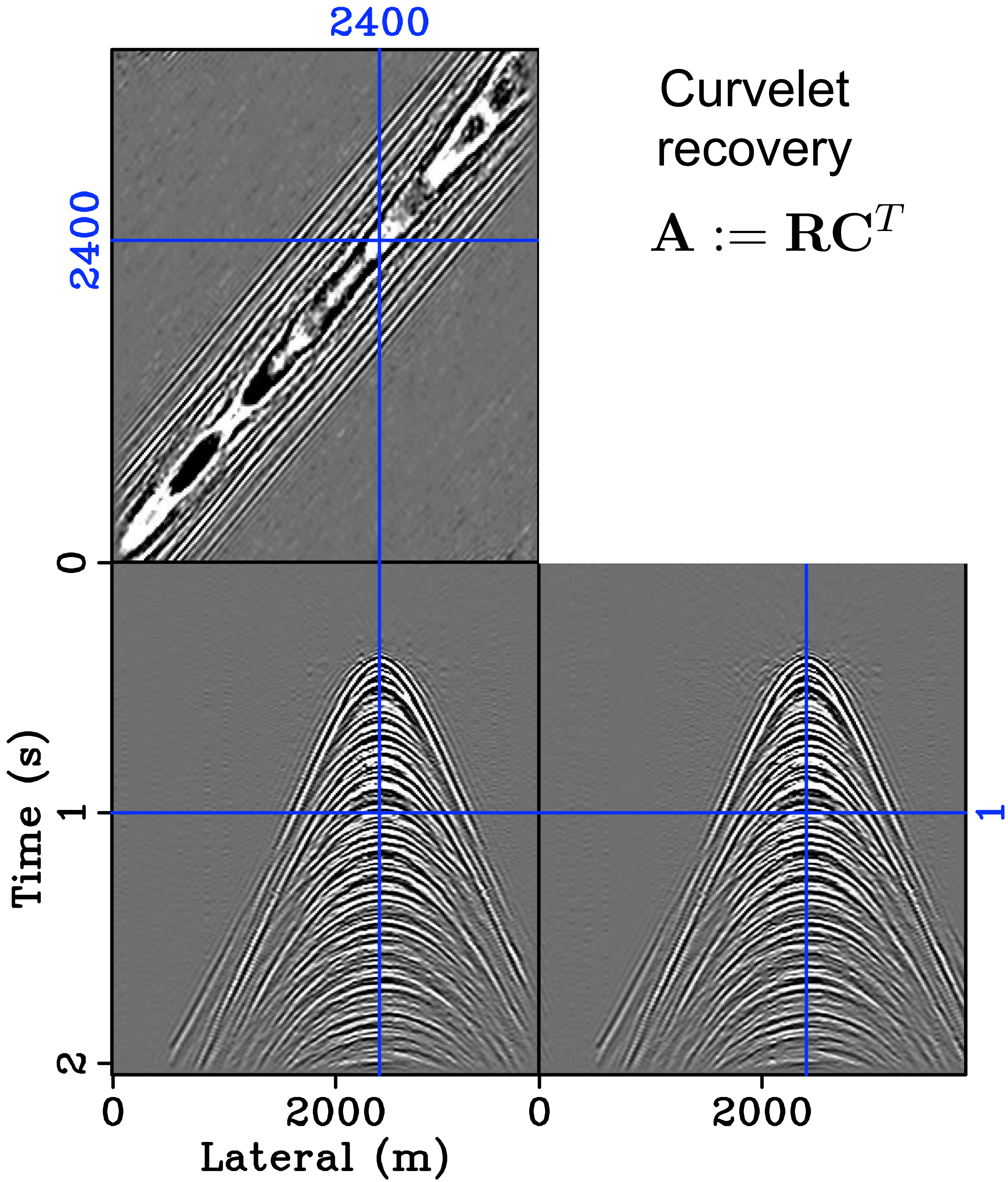
Focused curvelet recovery

$$\mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$$

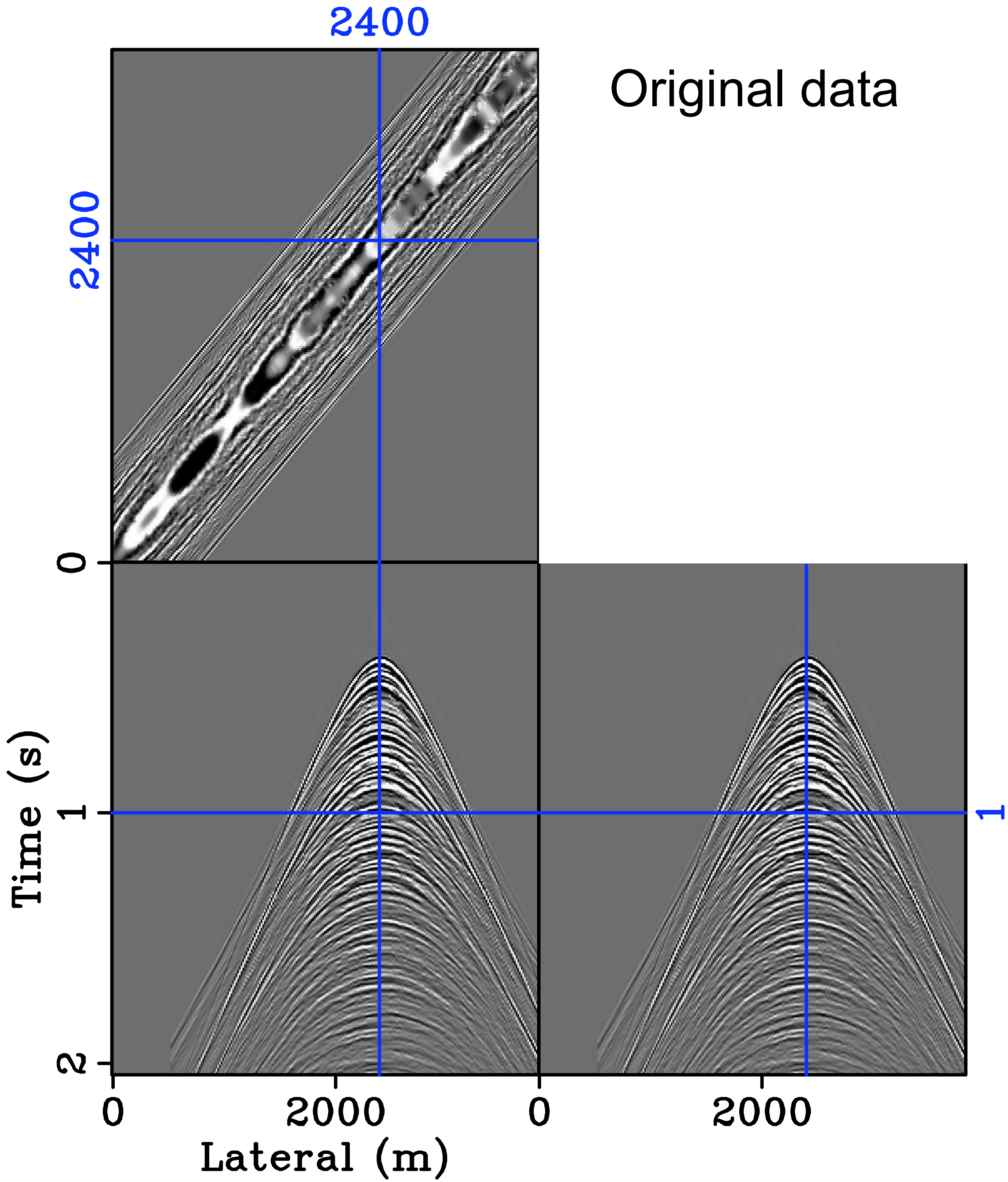


Curvelet recovery

$$\mathbf{A} := \mathbf{R}\mathbf{C}^T$$



Original data



Observations

Regular subsampling is unfavorable

- random sampling favorable but suffers from gaps
- jitter sampling favorable and controls gaps

Focal transform

- is reminiscent of an imaging operator
- improves recovery <=> additional compression

Solver

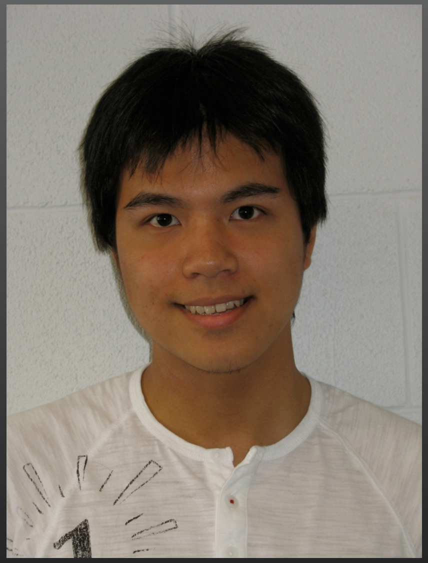
- solves norm one problem for 200-300 matrix-vector multiplications for 2^{30} unknowns ...

Outlook

- Migration-based wavefield reconstruction
  - sparsity on the image
  - focussing of the image (extra constraint)
- or a more “blue sky” approach of compressive one-way wavefield extrapolation

Compressed wavefield extrapolation

joint work with Tim Lin



“Compressed wavefield extrapolation” in Geophysics

Motivation

Synthesis of the discretized operators form bottle neck of imaging

Operators have to be applied to multiple right-hand sides

Explicit operators are feasible in 2-D and lead to an order-of-magnitude performance increase

Extension towards 3-D problematic

- storage of the explicit operators
- convergence of implicit time-harmonic approaches

First go at the problem using CS techniques to compress the operator ...

Related work

Curvelet-domain diagonalization of FIO's

- The Curvelet Representation of Wave Propagators is Optimally Sparse (Candes & Demanet '05)
- Seismic imaging in the curvelet domain and its implications for the curvelet design (Chauris '06)
- Leading-order seismic imaging using curvelets (Douma & de Hoop '06)

Explicit time harmonic methods

- Modal expansion of one-way operators in laterally varying media (Grimbergen et. al. '98)
- A new iterative solver for the time-harmonic wave equation (Riyanti '06)

Fourier restriction

- How to choose a subset of frequencies in frequency-domain finite-difference migration (Mulder & Plessix '04)

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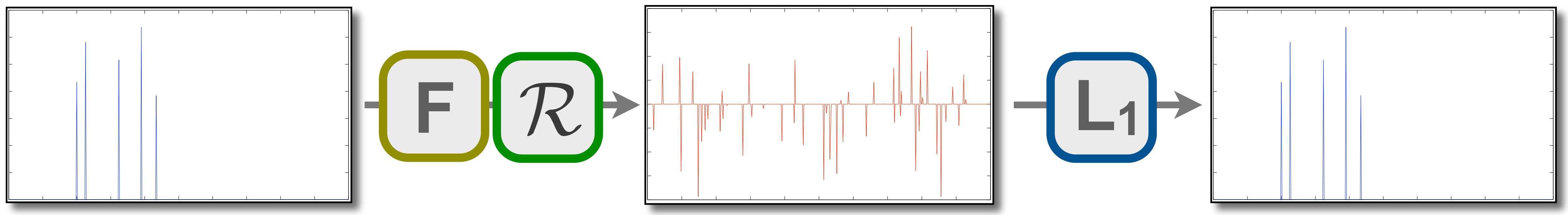
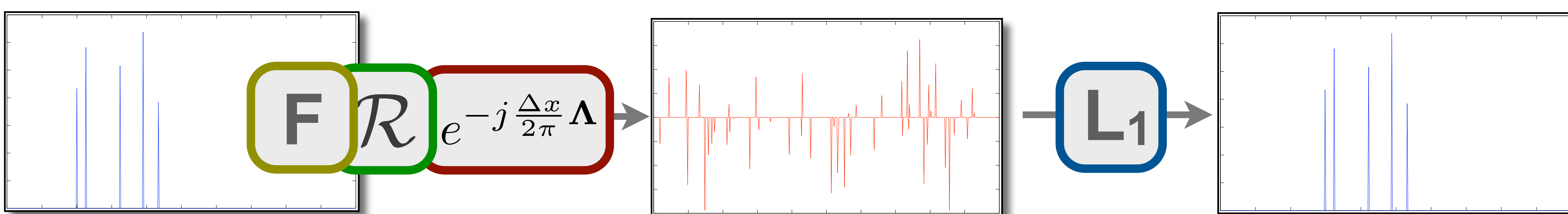
Compressed Sensing



signal in space domain | incomplete signal in Fourier domain | signal in space domain

Compressed Processing



signal in space domain | incomplete and *shifted* signal in Fourier domain | *shifted* signal in space domain

Inspiration

Suppose we want to shift a sparse spike train, i.e.,

$$\begin{aligned} \mathbf{u} &= \mathbf{T}_\tau \mathbf{v} \\ &= e^{-\tau \mathbf{D}} \mathbf{v} \\ &= \mathbf{L} e^{-j\tau \mathbf{\Omega}} \mathbf{L}^H \mathbf{v} \end{aligned}$$

where

$$\begin{aligned} \mathbf{D} &= \mathbf{L}\mathbf{\Omega}\mathbf{L}^H \\ \mathbf{L} &= \text{The Fourier Transform} \end{aligned}$$

- Eigen modes <=> Fourier transform.
- Can this operation be compressed by compressive sampling?

Operators on spikes

[Candes et. al, Donoho]

Calculate instead

$$\begin{cases} \mathbf{y}' & = \mathbf{R} e^{j\boldsymbol{\Omega}\tau} \boldsymbol{\mathcal{F}} \mathbf{v} \\ \mathbf{A} & = \mathbf{R} \boldsymbol{\mathcal{F}} \\ \tilde{\mathbf{u}} & = \arg\min_{\mathbf{u}} \|\mathbf{u}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{u} = \mathbf{y}' \end{cases}$$

- Take compressed measurements in Fourier space.
- Recover with sparsity promotion
- Shift operator is compressed by the restriction

$$\mathbf{R} \in \mathbb{R}^{m \times N} \text{ with } m \ll N$$

yielding compressed rectangular operators.

- **Extend this idea to wavefield extrapolation?**

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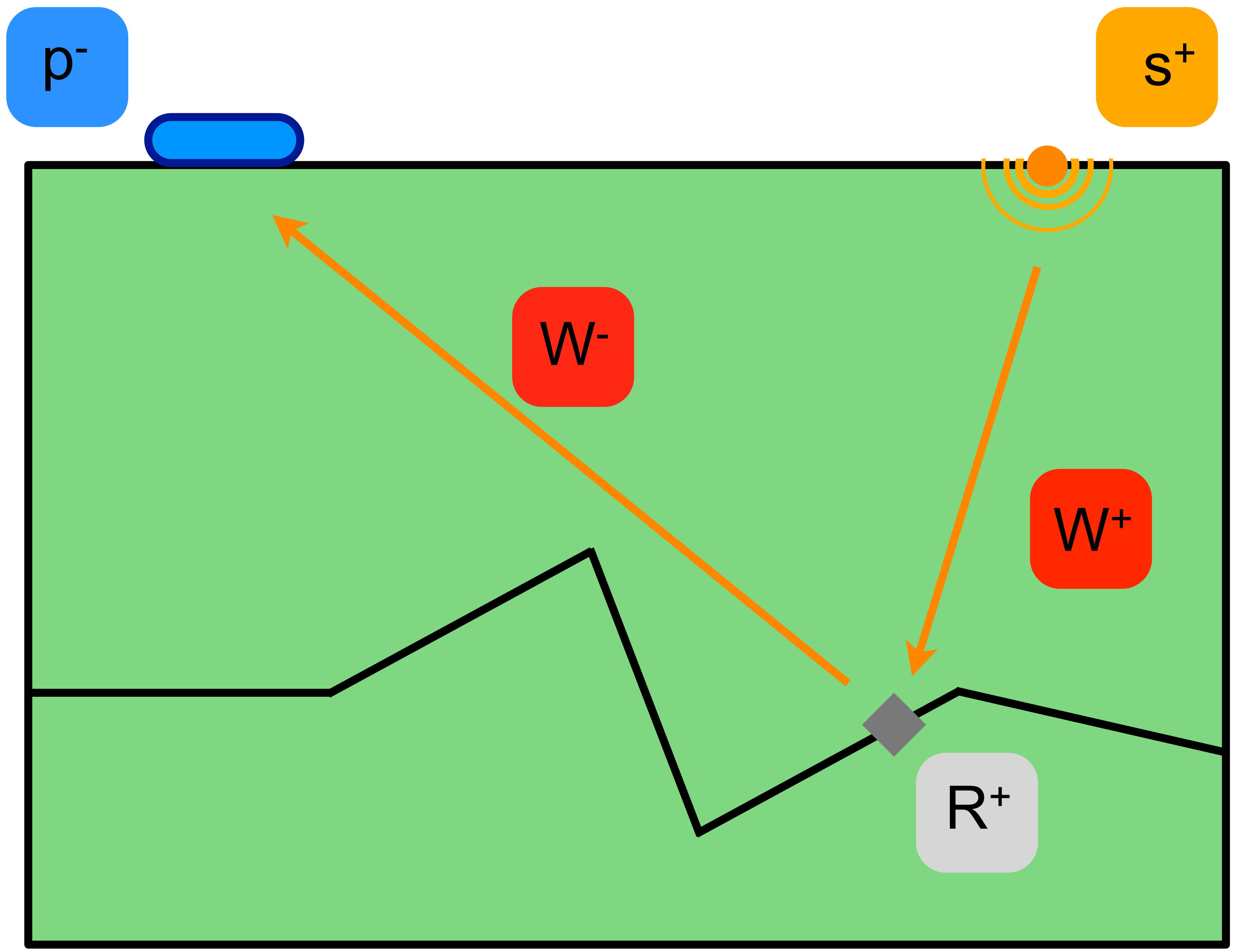
Representation for seismic data

[Berkhout]



Different representations

	diagonalization operator	parsimony wavefield
SVD/Lanczos/ modal	✓	×
curvelets	×	✓

Different representations

	diagonalization operator	parsimony wavefield
SVD/Lanczos/ modal	✓	×
curvelets	×	✓

If incoherent this may actually work ....

Sparsity promoting formulation

Buys us stability w.r.t. missing data

- provided measurement and sparsity representations are mutually incoherent
- sufficient mixing <=> random restriction

Different strategy:

- Let the physics define the measurement basis
- Use the modal domain (domain of eigenfunctions) to define the measurement basis
- See what you can recover

Study eigenfunctions:

- mutual coherence with sparsity representation
- modal spectrum on the to-be-extrapolated wavefield

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One-Way Wave Operator

- Structure of $\mathcal{A}$ confounds the meaning of its exponentiation, due to it being an operator

(Simon & Reed; Dessing '97; Grimbergen '98)

$$\mathcal{A} = \begin{pmatrix} 0 & \omega\rho \\ \frac{1}{\omega\rho^{1/2}}(\mathcal{H}_2\rho^{-1/2}) & 0 \end{pmatrix}$$

Two-way
Wave Operator

$$\mathcal{H}_2 = k^2(\boldsymbol{x}, \omega) + \partial_\mu \partial_\mu$$

- $\mathcal{H}_2$ contains information about medium velocity

One-Way Wave Operator

- Solution of the one-way wave equation

$$\mathcal{W}(x_3; x_3') = \exp(-j(x_3 - x_3')\mathcal{H}_1)$$

- After discretization solve eigenproblem on $\mathbf{H}_2$

$$\mathbf{H}_2 = \begin{bmatrix} \left(\frac{\omega}{\bar{c}_1}\right)^2 & 0 & \cdots & 0 \\ 0 & \left(\frac{\omega}{\bar{c}_2}\right)^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \left(\frac{\omega}{\bar{c}_{n_1}}\right)^2 \end{bmatrix} + \mathbf{D}_2$$

  - Helmholtz operator is Hermitian
  - monochromatic
  - velocity $\bar{c}$ varies laterally

(Claerbout, 1971; Wapenaar and Berkhout, 1989)

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Modal transform

- Solve eigenproblem & take square root

$$\mathbf{H}_1 = \mathbf{L}\mathbf{\Lambda}^{1/2}\mathbf{L}^H$$

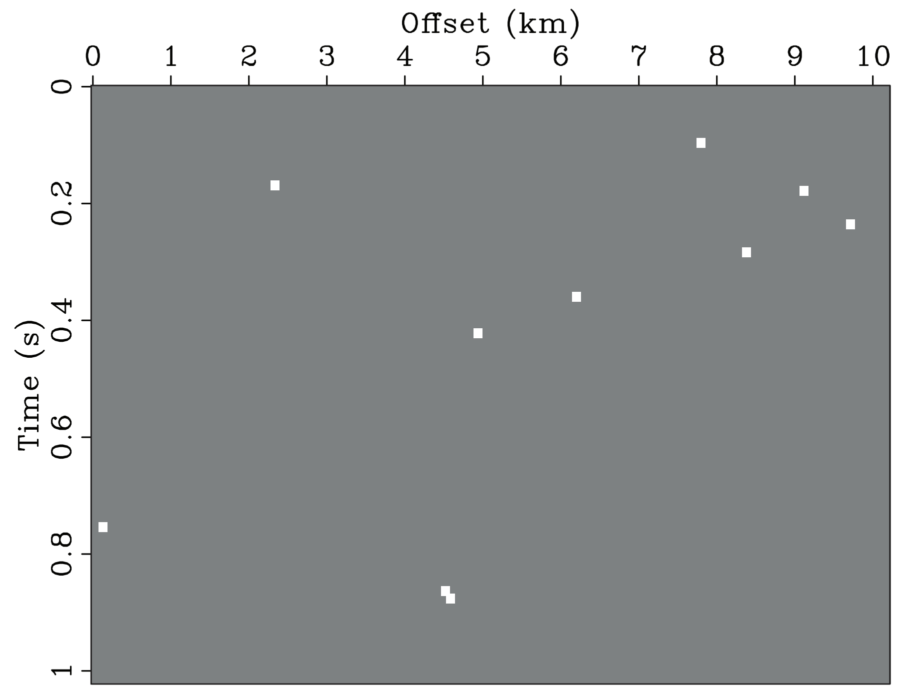
- $\mathbf{L}$ is orthonormal & defines the modal transform that diagonalizes one-way wavefield extrapolation
- Eigenvalues play role of vertical wavenumbers
- Extrapolation operator is diagonalized

$$\mathbf{W} = \boldsymbol{\mathcal{F}}^H \mathbf{L} e^{-j\mathbf{\Lambda}^{1/2}(x_3 - x_3')} \mathbf{L}^H \boldsymbol{\mathcal{F}}$$

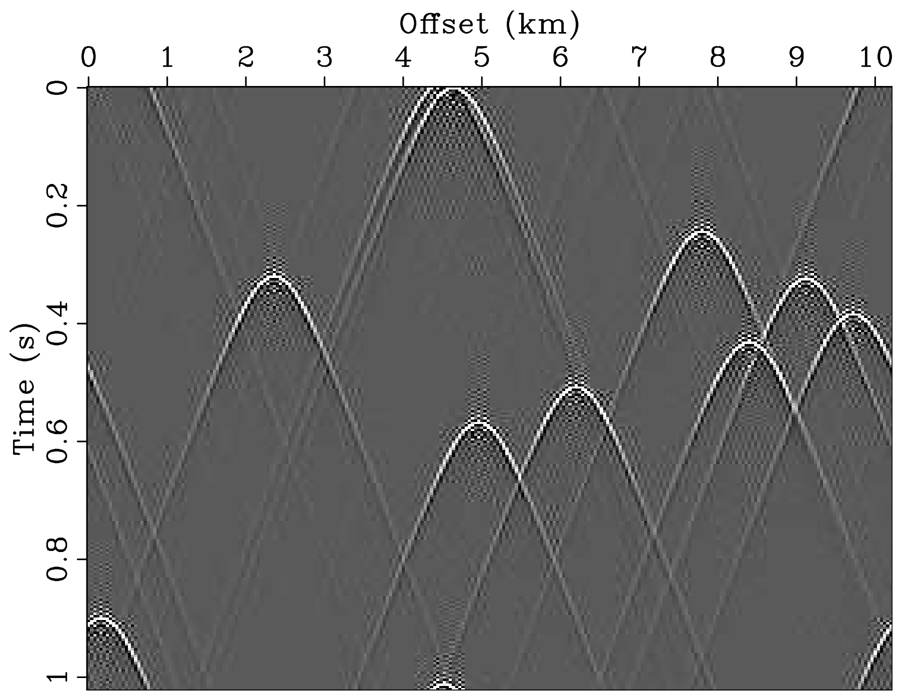
Compressed wavefield extrapolation

Forward model

$$\mathbf{u} = \mathbf{L}e^{-j\mathbf{\Lambda}^{1/2}\Delta x_3}\mathbf{L}^H\mathbf{v}$$



Original events



Recorded Data

Reconstruct point scatterers from recorded data ....

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Compressed wavefield extrapolation

$$\begin{cases} \mathbf{y} & = \mathbf{R}\mathbf{L}^H\mathbf{u} \\ \mathbf{A} & = \mathbf{R}e^{j\mathbf{\Lambda}^{1/2}\Delta x_3}\mathbf{L}^H \\ \tilde{\mathbf{x}} & = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \\ \tilde{\mathbf{v}} & = \tilde{\mathbf{x}} \end{cases}$$

- Randomly subsample & phase rotate in Modal domain
- Recover by norm-one minimization
- Capitalize on
  - the incoherence modal functions and point scatterers
  - reduced explicit matrix size
  - constant velocity <=> Fourier recovery

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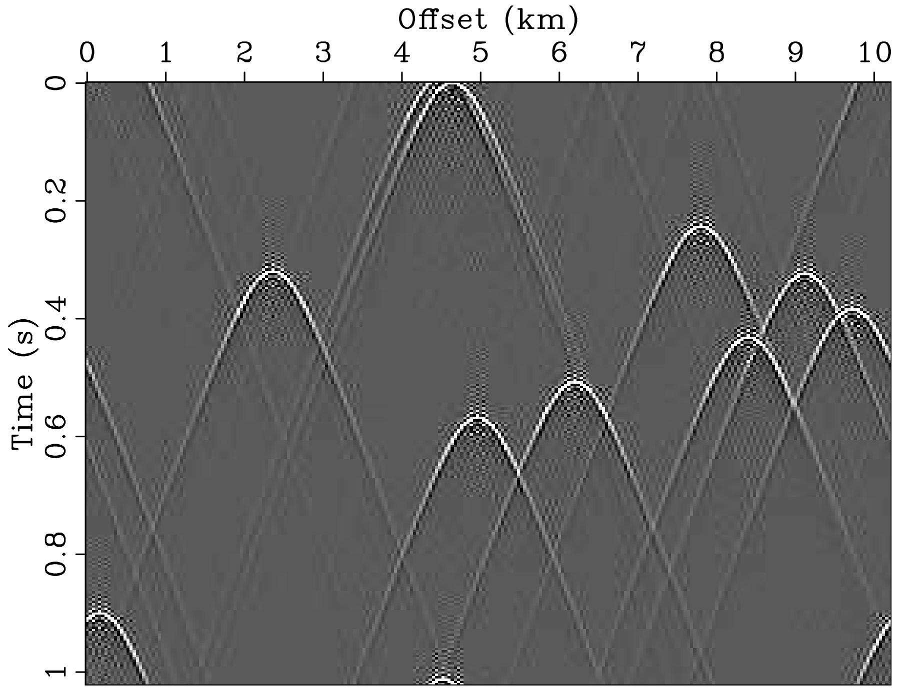
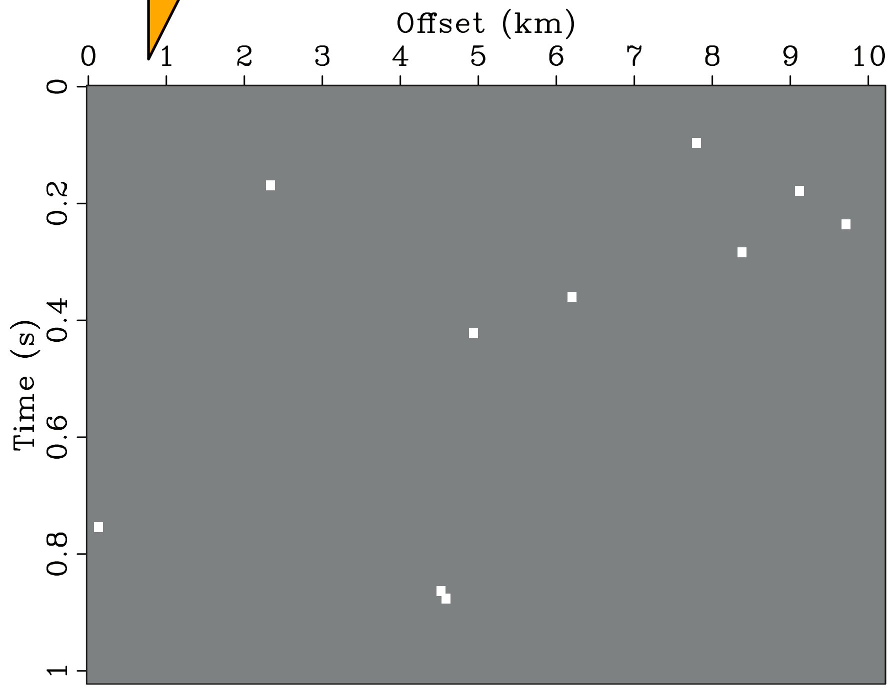
Compressed wavefield extrapolation

Reconstruction



Recorded Data



Reconstructed events

Only 1 % of original modes were used ...

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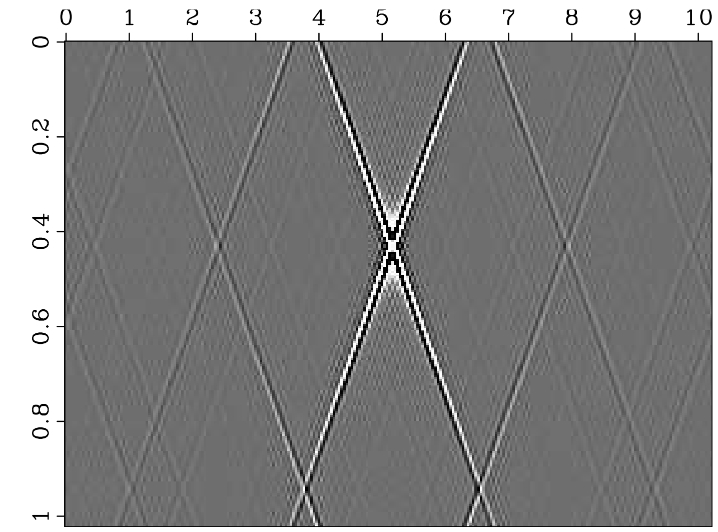
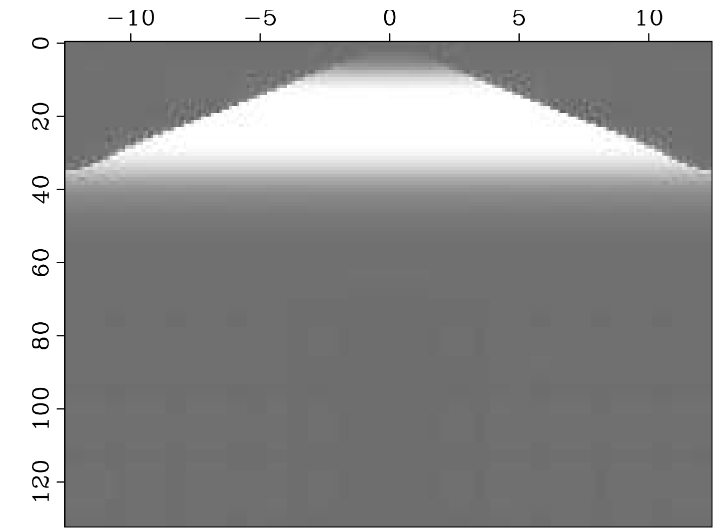
Observations

- Despite the existence of evanescent (exponentially decaying) waves modes recovery is successful
- If you are looking for point-scatterers, we have a proof of concept that is fast
- Earth is more complex ...



(c)



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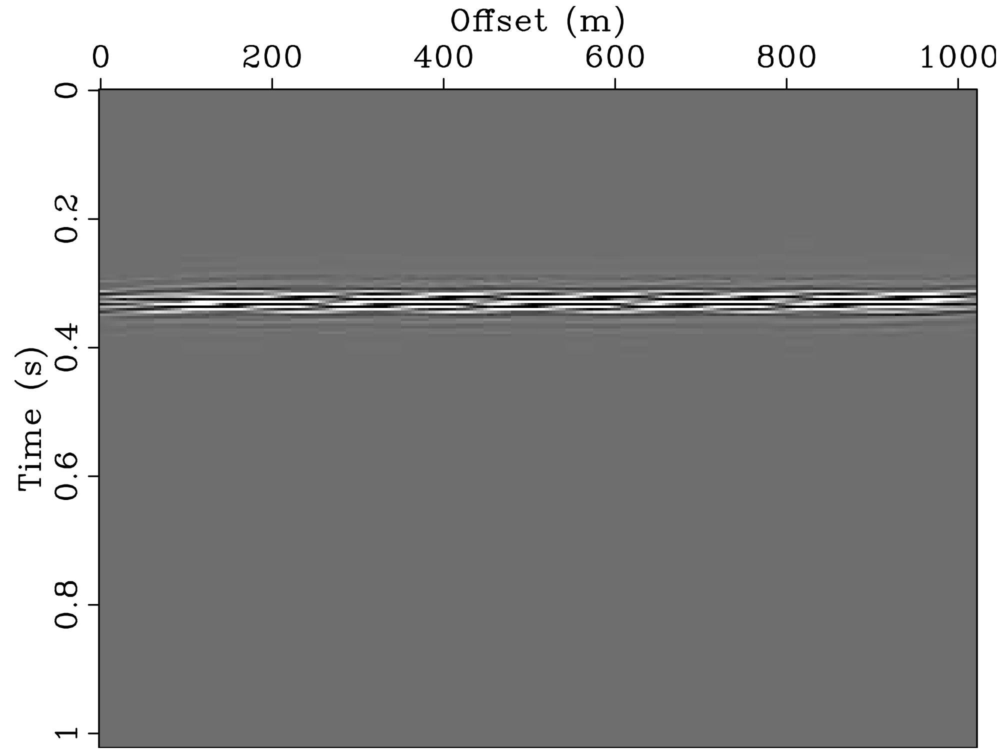
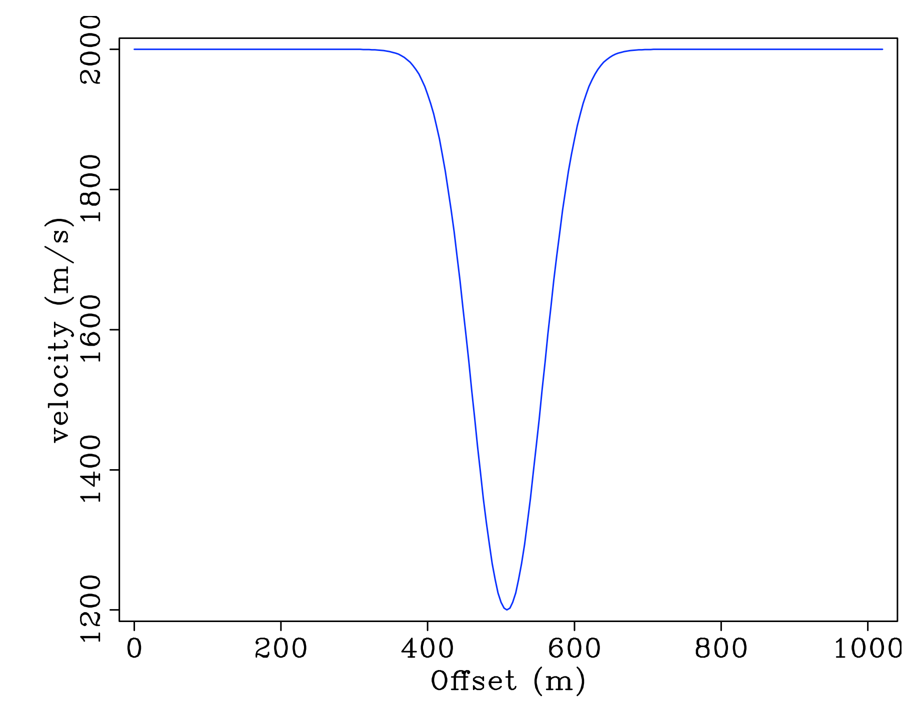
Compressed wavefield extrapolation

- Extend to general wavefields
- Use curvelets as the sparsity representation
- Use the full & compressed forward operator operator
- Compressively extrapolate back 600m to the source





Restriction & sparsity strategies

- **Forward extrapolation:**

$$\mathbf{W_1}: \quad \begin{cases} \mathbf{y}' = \mathbf{R}e^{j\mathbf{\Lambda}^{1/2}\Delta x_3}\mathbf{L}^H \\ \mathbf{A} := \mathbf{R}\mathbf{L}^H\boldsymbol{\mathcal{F}}\mathbf{C}^T \\ \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y}' \\ \tilde{\mathbf{u}} = \mathbf{C}^T\tilde{\mathbf{x}}, \end{cases}$$

- **Inverse extrapolation:**

$$\mathbf{F_1}: \quad \begin{cases} \mathbf{y} = \mathbf{R}\mathbf{L}^H\boldsymbol{\mathcal{F}}\mathbf{u} \\ \mathbf{A}' = \mathbf{R}e^{j\mathbf{\Lambda}^{1/2}\Delta x_3}\mathbf{L}^H\mathbf{C}^H \\ \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}'\mathbf{x} = \mathbf{y} \\ \tilde{\mathbf{v}} = \mathbf{C}^T\tilde{\mathbf{x}}. \end{cases}$$

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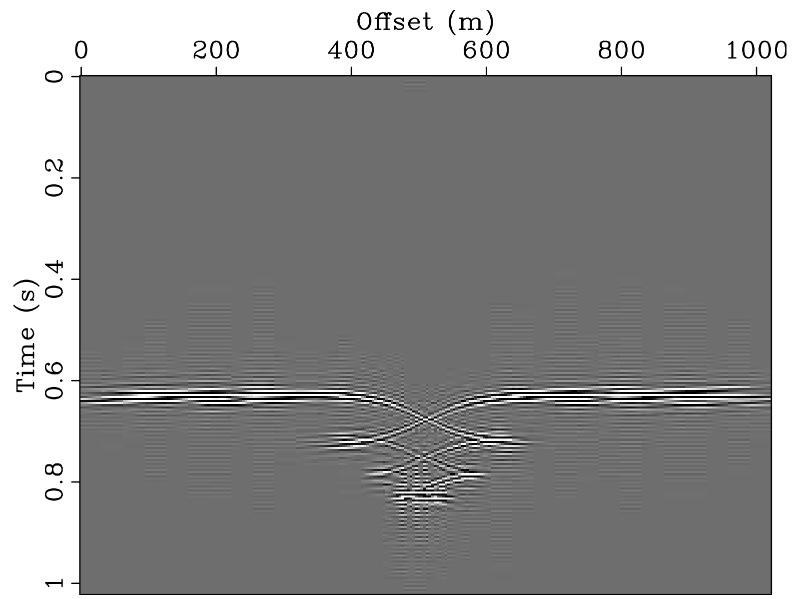
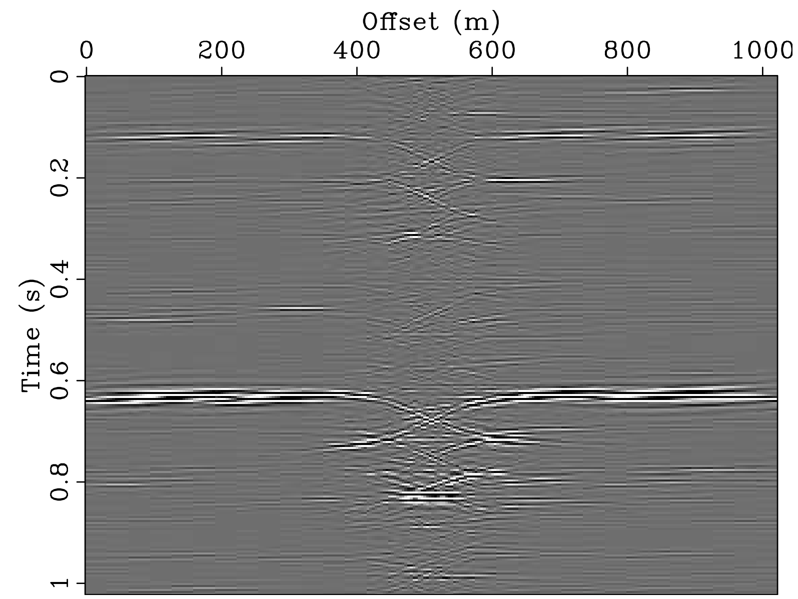
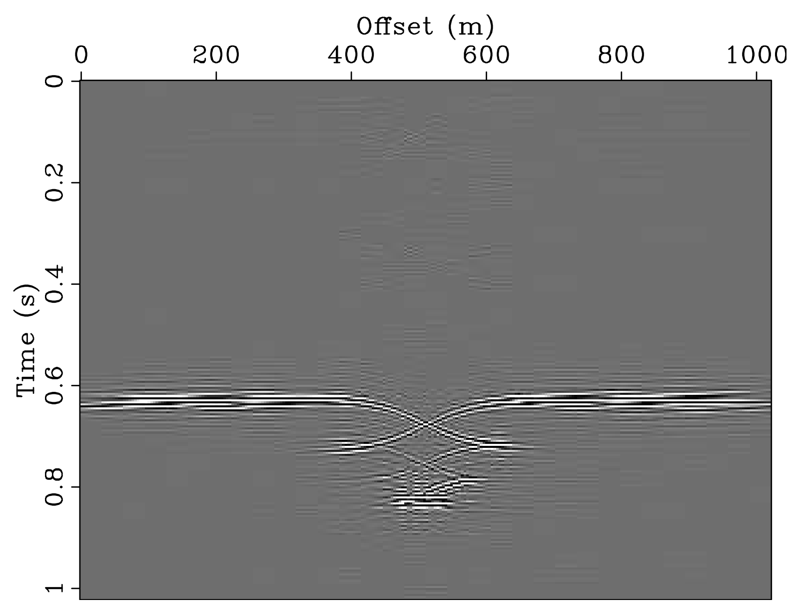
Forward Extrapolation



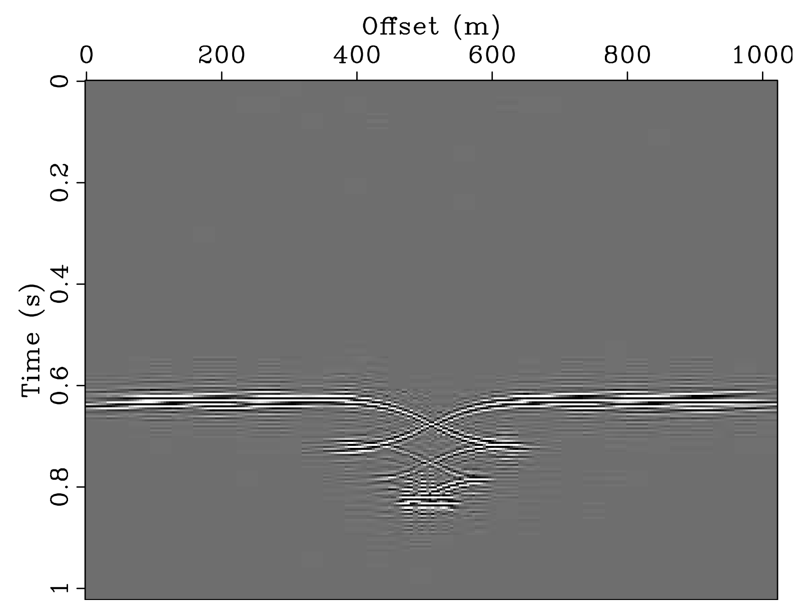
(a)



(b)



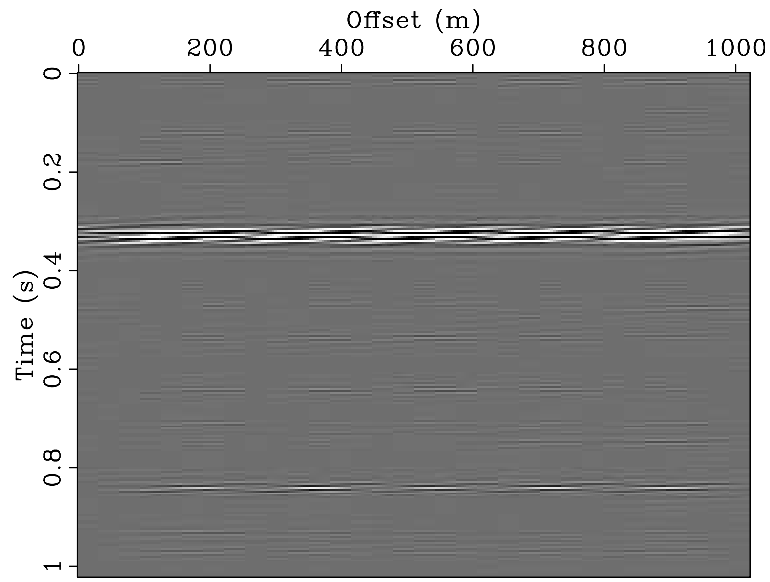
(c)



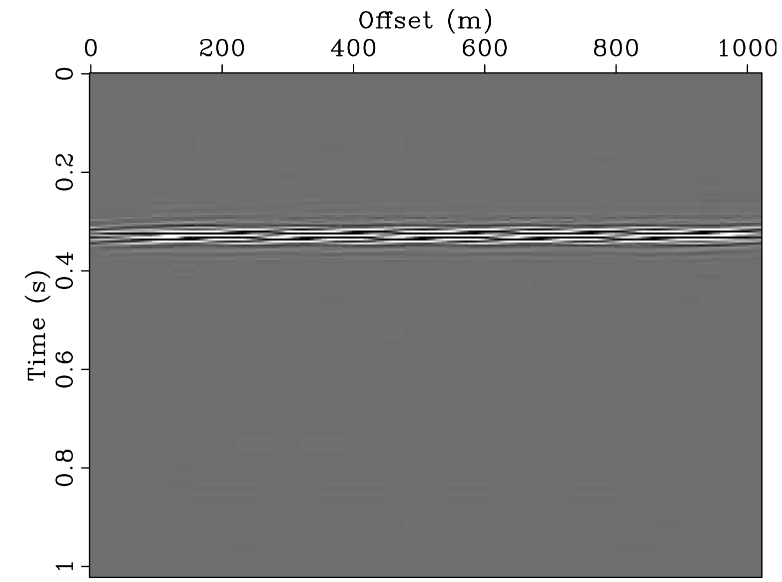
(d)

- (a) is Full extrapolation
- (b)-(d) is compressed extrapolation, (b) p = 0.04, (c) p = 0.16, (d) p = 0.24

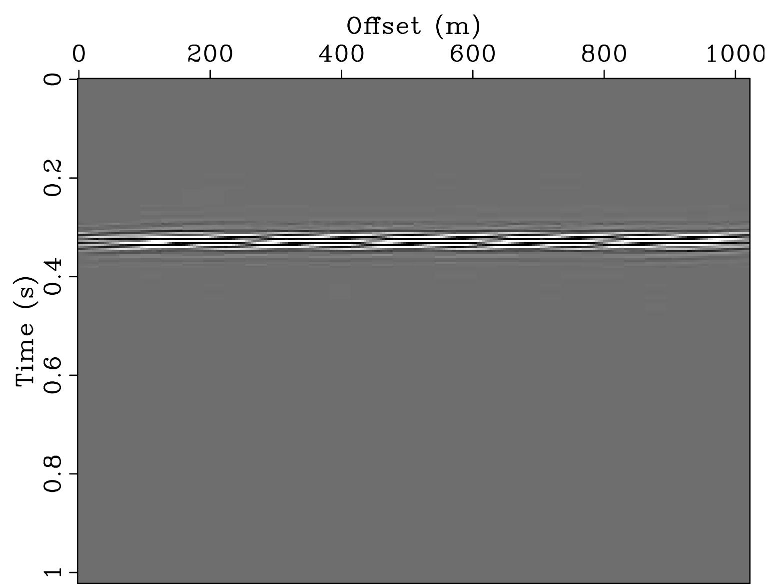
Inverse Extrapolation



(a)



(b)



(c)

- (a) p = 0.04
- (b) p = 0.16, (c) pf=0.4, px=0.4

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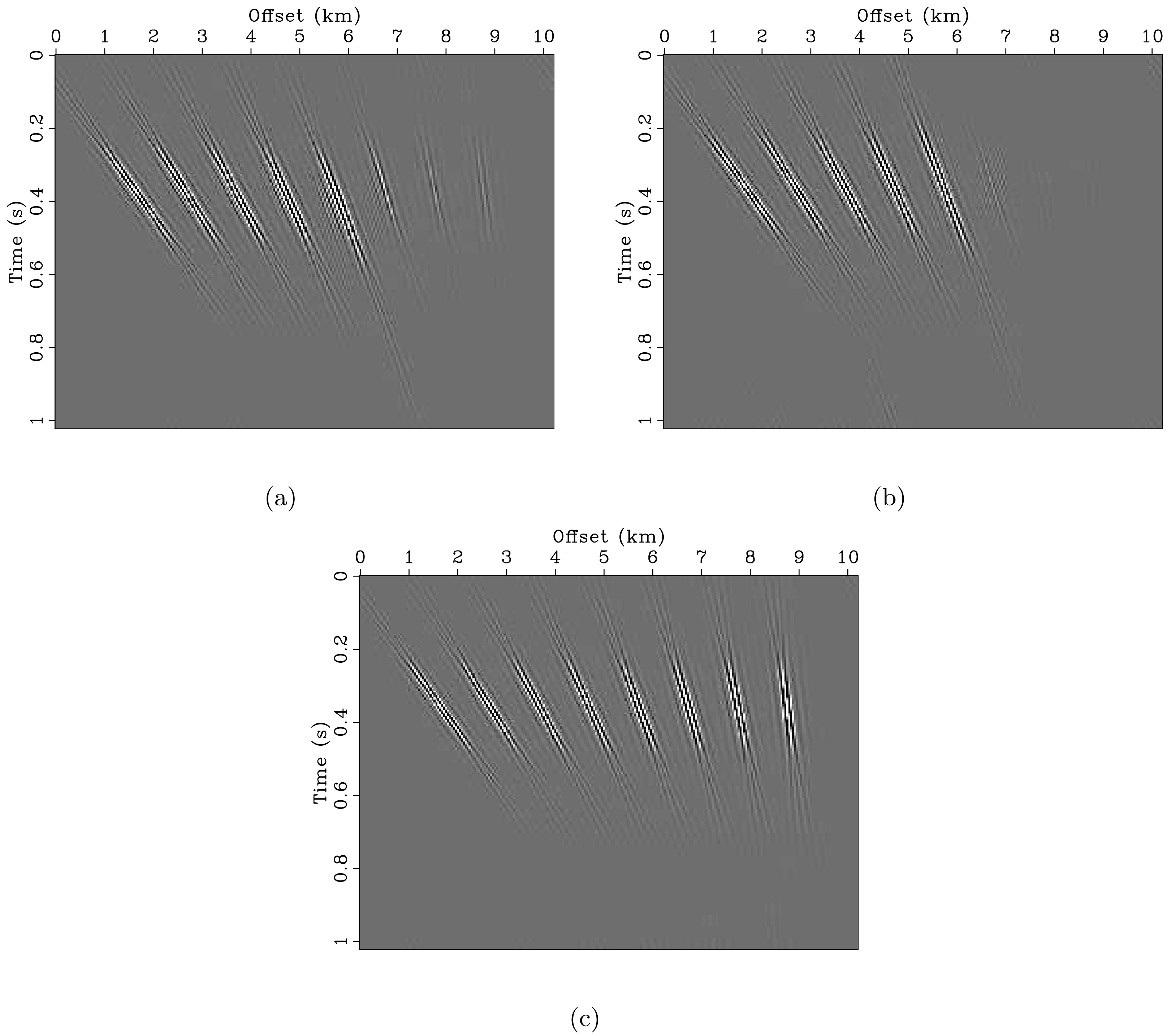
Evanescent Recovery



- (a) is downward extrapolated wavefield
- (b) is matched filter
- (c) is “compressed” inverse extrapolation

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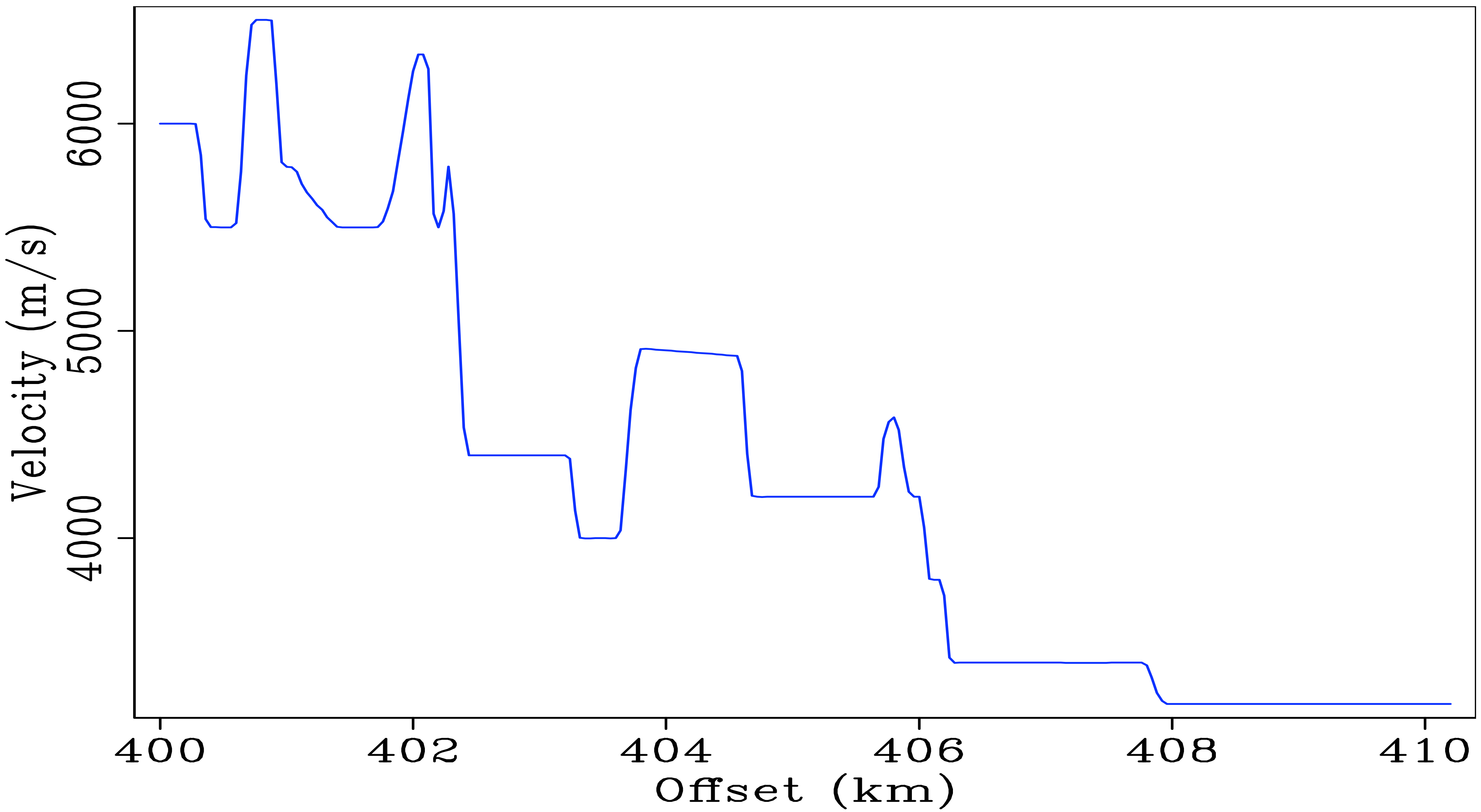
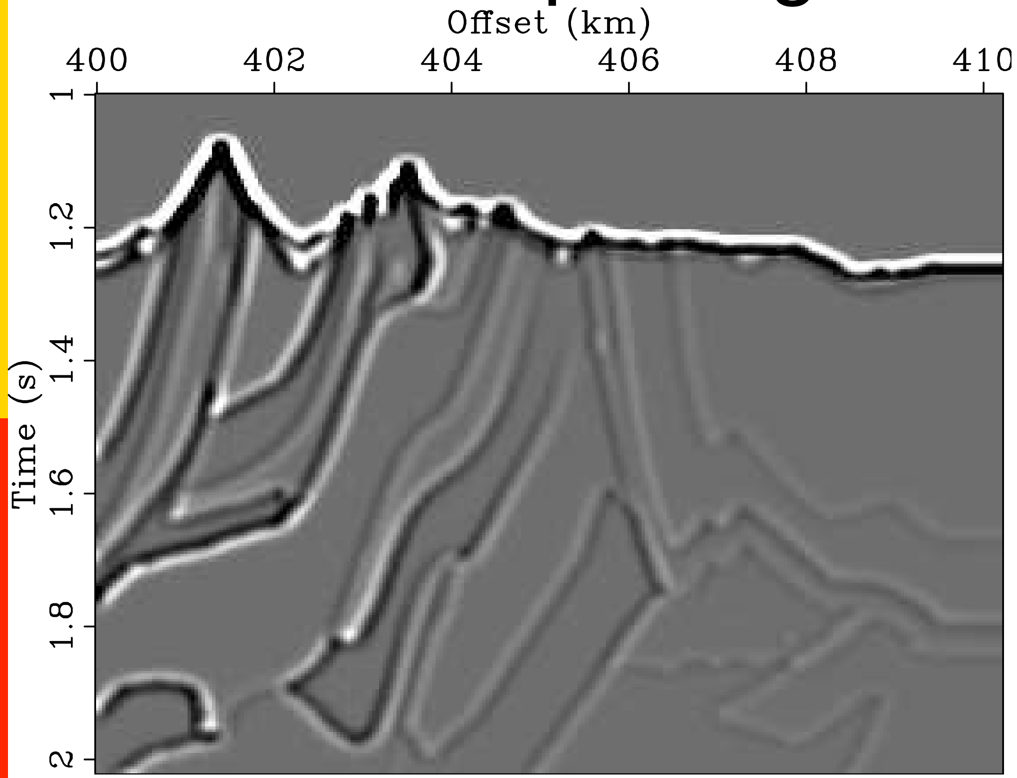
Velocity model

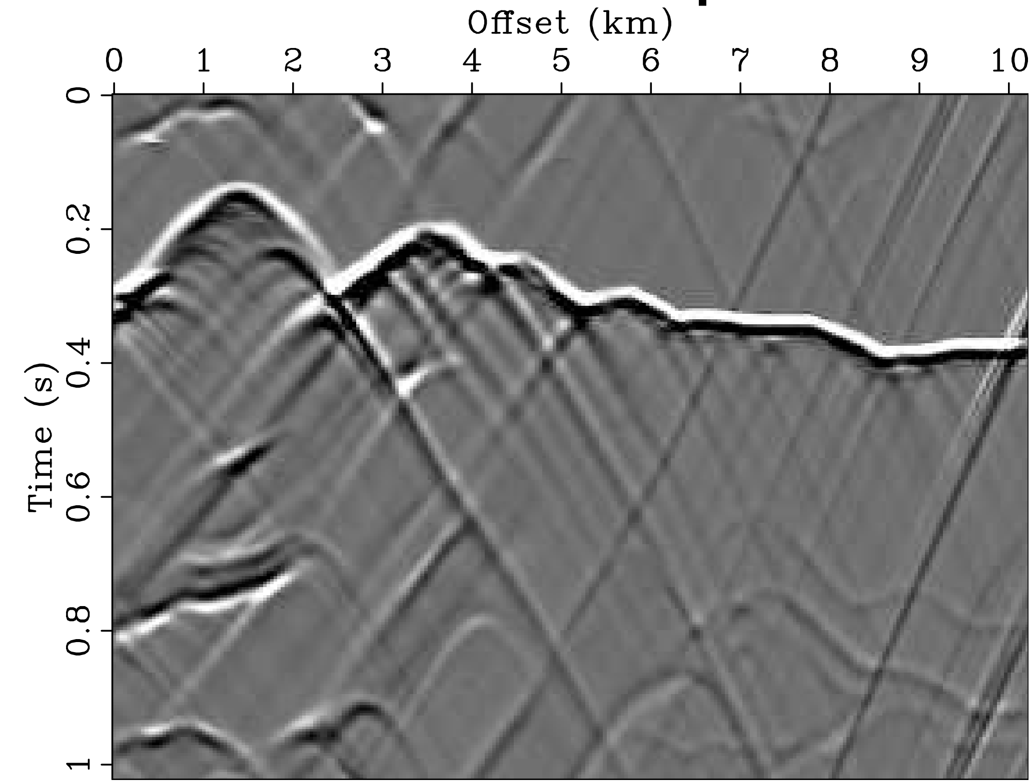


Compressed inverse extrapolation

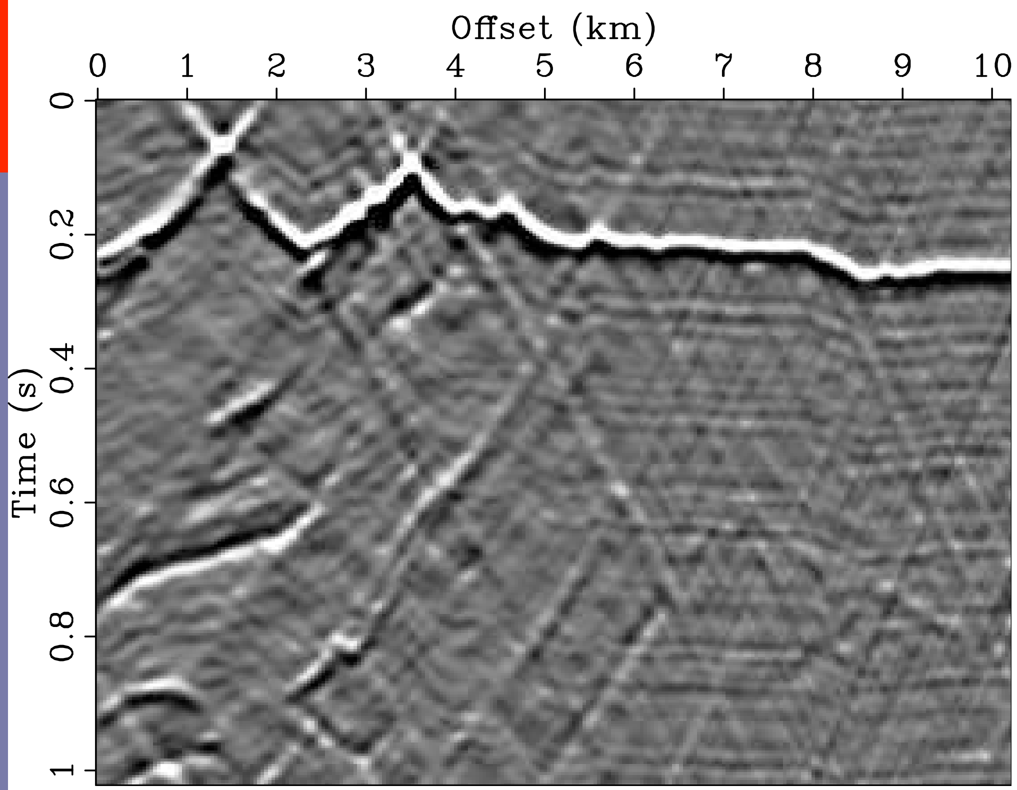
Overthrust exploding reflector



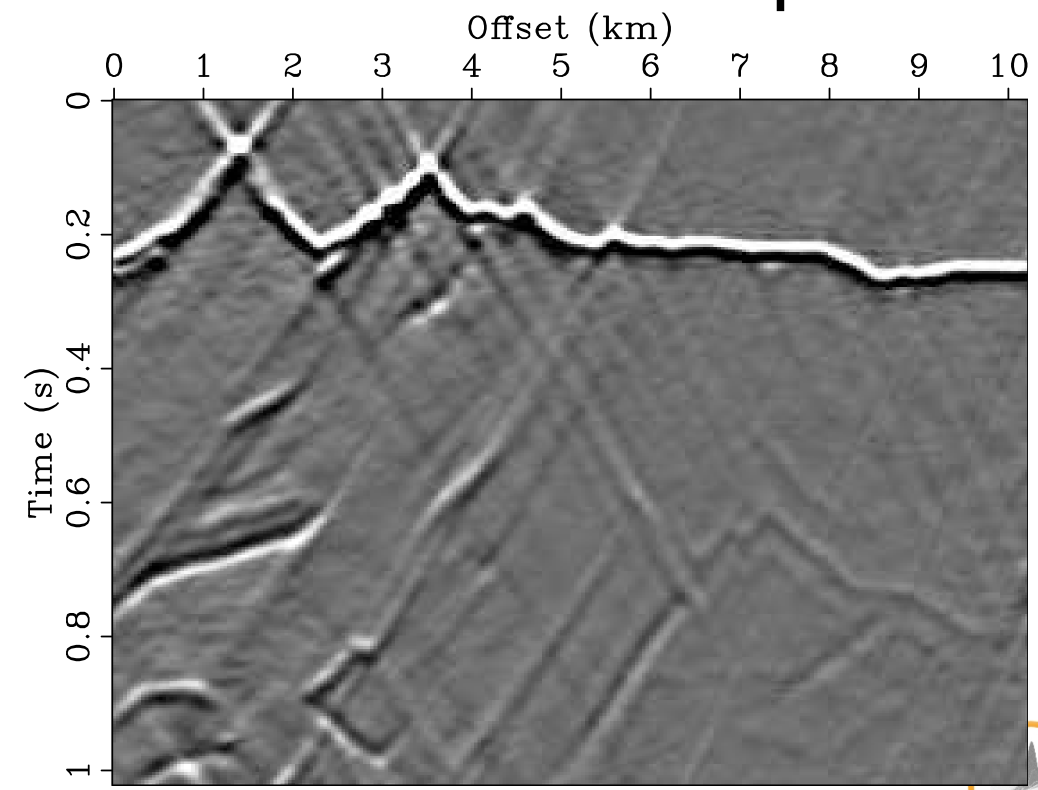
Full forward extrapolation



Matched filter



Recovered from p=0.25



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Multiscale and angular compressed wavefield extrapolation

- Propose a scheme motivated by extensions of CS

(Tsaig and Donoho '06)

$$\mathbf{F}_1^{\mathbf{j}}: \begin{cases} \mathbf{y_j} = \mathbf{R_j M_j u} \\ \mathbf{A'_j} := \mathbf{R_j M'_j C_j^{\mathit{T}}} \\ \tilde{\mathbf{x}}_\mathbf{j} = \arg\min_{\mathbf{x_j}} \|\mathbf{x_j}\|_1 \quad \text{s.t.} \quad \mathbf{A'_j x_j} = \mathbf{y_j} \\ \tilde{\mathbf{v}} = \sum_\mathbf{j} \mathbf{C_j^{\mathit{T}}} \tilde{\mathbf{x}}_\mathbf{j}, \end{cases}$$

with $\mathbf{j} = \{j, l\}$ the scale and angle.

- adapt discretization & restriction
- parallel implementation

Conclusions

- Curvelets sparsity on the model and near diagonalization yields stable inversion Gramm matrix
- Jittered sampling and focussing in combination with curvelets leads to wavefield recovery
- Compressed wavefield extrapolation
  - reduction in synthesis cost
  - inverse extrapolation works well when focussed
  - mutual coherence curvelets and modes
  - performance of norm-one solver
  - keep the ***constants*** under control ...
- Double-role CS matrix is cool ... upscaling to “real-life” is a challenge ....

SLIM
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Open problems

- What deeper insights can CS give?
  - inversion near unitary operators
  - coherency generalized to frames to study
    - cols modeling operator <=> curvelets
    - radiation vs guided modes <=> curvelets
- Norm-one solver for reduced system as fast a LSQR on the full system
- Fast random eigenvalue solver does not exist yet ...
- Extension of CS to waveform inversion & to compressed computations ...
- Many more ...

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