From seismic data to the composition of rocks: an interdisciplinary and multiscale approach to exploration seismology

*Felix J. Herrmann** *and the SLIM Team**

ABSTRACT

In this essay, a nonlinear and multidisciplinary approach is presented that takes seismic data to the composition of rocks. The presented work has deep roots in the 'gedachtengoed' (philosophy) of Delphi spearheaded by Guus Berkhout. Central themes are multiscale, object-orientation and a multidisciplinary approach.

Introduction

In this essay, I would like to showcase the research that is currently being conducted at the UBC-Seismic Laboratory for Imaging and Modeling (SLIM). It goes without saying, that this line of research has deep roots in the research I conducted under the supervision of Guus Berkhout and Kees Wapenaar, first as a MSc. student, and later as a Ph.D. student as a member of the Delphi project. There are three central themes that I would to share with the readers and that emphasize (i) the benefits of interdisciplinary research; (ii) the link with rock physics and (iii) the importance of abstraction or in Guus' words 'detail hiding'. In one way or another, these themes translate into the three major research areas worked on at SLIM that are best paraphrased by the following three slogans

- *'From seismic data to seismic reflectivity'*, involving the development of a new suite of seismic processing and imaging techniques, integrating recent insights from the physics of seismic exploration with recent developments in the field of information theory known as "compressed/compressive sensing".
- *'From seismic reflectivity to seismic connectivity'*, involving the development of new seismic image characterization and rock physical modeling techniques, integrating scale attributes with critical points for the transport properties of rocks as function of the lithology (volume fractions). These critical points are related to the statistical mechanical behavior of rocks governed by percolation theory;

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- *'Towards an object-oriented approach of exploration seismology'*, involving the development of an educational object-oriented development environment integrating existing pipe-based Unx applications with a python-based abstraction of (non)linear operators.

In the next pages, I would like to give the reader a flavor of these three Delphi-inspired research topics.

SEISMIC IMAGING AND PROCESSING WITH MULTISCALE AND MULTIDIRECTIONAL CURVELET FRAMES

Upon my arrival at UBC, I became convinced of the importance of curvelet frames for seismic data processing and imaging. I came to this insight from the experience I gained while working with the multiscale wavelet transform in Delft. Both Frank Dessing and I worked hard to apply this transform to exploration seismology and I have to say with somewhat mixed success. Frank was able to compress migration operators but we both agreed that wavelets lacked the directionality, so omnipresent in seismic data. In retrospect, we were right. Because of their lack of directivity, wavelets are not able to detect the 'wave front set', i.e., the collection of normals to the wavefronts and consequently lacked a certain invariance under the imaging operators. This explains their lackluster performance in exploration seismology.

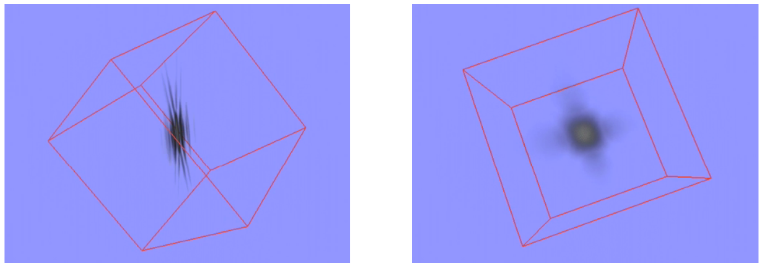
Recently developed curvelet frames (see e.g. Candes et al., 2006; Hennenfent and Herrmann, 2006b), on the other hand, address these issues and allowed for the development of a new suit of seismic data and processing techniques that bank on

- **detection of wave-fronts** without requiring *prior* information on the dips or on the velocity model;
- **invariance** of curvelets under certain aspects of wave propagation.

These two properties make this transform suitable for a robust formulation of problems, such as seismic data regularization (Hennenfent and Herrmann, 2006a; Herrmann and Hennenfent, 2007), primary-multiple separation (Herrmann et al., 2006a), ground-roll removal (Yarham et al., 2006) and stable migration-amplitude recovery (Herrmann et al., 2006b). All these methods exploit sparsity in the curvelet domain that is a direct consequence of the above two properties and corresponds to a rapid decay for the magnitude-sorted coefficients. This sparsity allows for a separation of 'noise' and 'signal' underlying all these problems (see e.g. Hennenfent and Herrmann, 2006b; Herrmann et al., 2006a).

Curvelets: As can be observed from Fig. 1, curvelets are localized functions that oscillate in one direction and that are smooth in the other directions. They are multiscale and multi-directional and because of their anisotropic shape (they obey the so-called parabolic scaling relationship, yielding a width $\propto 2^{j/2}$ and a length $\propto 2^j$

with j the scale), curvelets are optimal for detecting wavefronts. This explains their high compression rates for seismic data and images as reported in the literature. (Candes et al., 2006; Hennenfent and Herrmann, 2006c; Herrmann et al., 2006a,b).



(a) (b)

Figure 1: Example of a 3-D curvelet. Notice the oscillations in one direction and the smoothness in the other two directions.

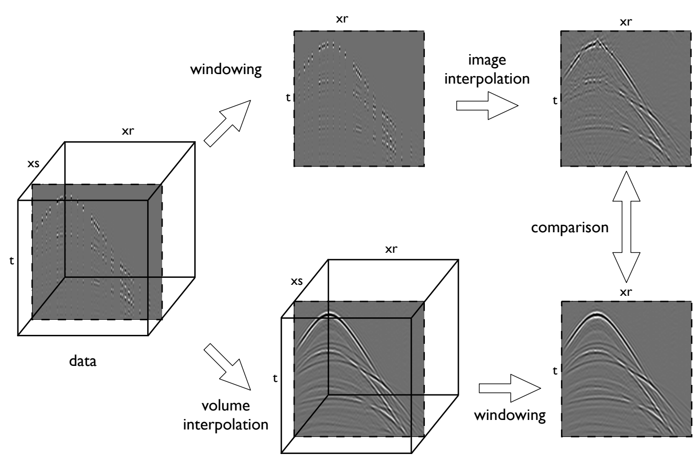
Sparsity promoting inversion High compression rates for signal representations are a prerequisite for the robust formulation of stable signal recovery problems and other inverse problems. These compression rates allow for a nonlinear sparsity promoting solution. As such sparsity-promoting norm-one penalty functionals are not new to the geosciences (see for instance the seminal work of Claerbout and Muir (1973), followed by many others), where sparsity is promoted on the model. What is different in the current surge of interest in sparsity-promoting inversion, known as 'compressed sensing' (Candes et al., 2005; Donoho et al., 2006), is (i) the existence of sparsity promoting transforms such as the curvelet transform; (ii) the deep theoretical understanding on what the conditions are for a successful solution. This work can be seen as the application of these recent ideas to the seismic situation and involves the solution of the following norm-one nonlinear program,

$$\mathbf{P}_\epsilon : \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{d}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases} \tag{1}$$

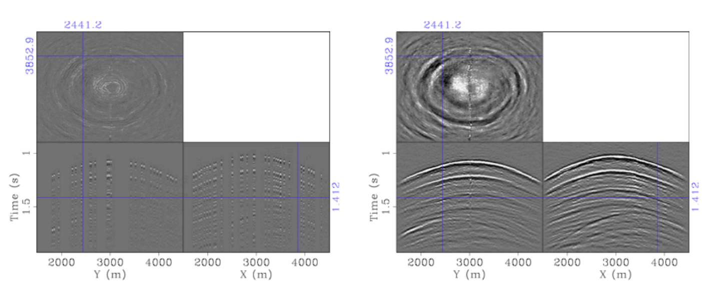
in which $\mathbf{y}$ is the (incomplete) data, $\mathbf{A}$ the synthesis matrix and $\mathbf{S}^T$ the inverse sparsity transform. Both these matrices consist of the inverse curvelet transform matrix, $\mathbf{S}^T$ (the symbol ^{T} denoting the transpose) compounded with other operators depending on the problem. The arg in the above equation refers assigning the vector that minimizes the ℓ_1-norm while satisfying the data to within ϵ to $\widetilde{\mathbf{x}}$. The above constrained optimization problem is solved to an accuracy of ϵ that depends on the noise level. Finally, $\widetilde{\mathbf{d}}$ stands for the recovered vector with the symbol $\widetilde{}$ reserved for optimized quantities.

CRSI:

An important topic in seismic processing is the seismic regularization problem, where attempts are made to recover fully-sampled seismic data volumes from incomplete data, i.e., data with large percentages ($> 80\,\%$) of traces missing. By choosing $\mathbf{A} := \mathbf{RC}^T$, $\mathbf{S} := \mathbf{C}$ and $\mathbf{y} = \mathbf{Rd}$ for the incomplete data, one arrives at the formulation for curvelet recovery by sparsity-promoting inversion (CRSI), which has successfully been applied to the recovery of incomplete seismic data (see e.g. Hennenfent and Herrmann, 2006a). In this formulation, $\mathbf{R}$ is the restriction matrix, selecting the rows from the curvelet transform matrix that correspond to active traces. As opposed to other recovery methods, such as sparse Fourier recovery and plane wave destruction, curvelet-based methods have the advantage of working in situations where there are conflicting dips without stationarity assumptions. The method exploits the high-dimensional continuity of wavefronts and as Fig. 2 demonstrates, recovery results improve when using the 3-D curvelet transform compared to the 2-D transform. *This project is joint work with my PhD. student Gilles Hennenfent (See also our contribution(s) G. Hennenfent and Herrmann, 2007, to next's week EAGE meeting).*



(a)

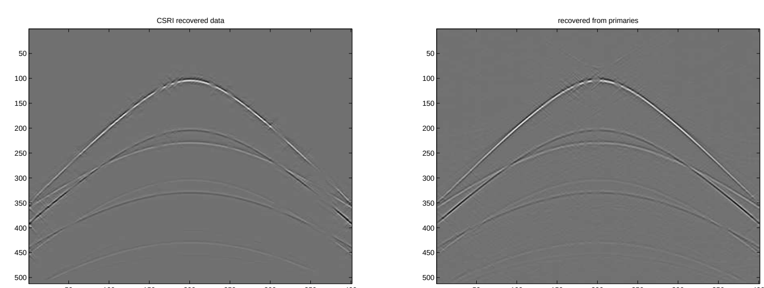


(b)

Figure 2: **(a)** Illustration of sliced versus volumetric interpolation. **(b)** Oseberg 3D walkaway VSP survey and its reconstruction using 3D CRSI.

CRSI with focusing:

While the CRSI recovery method is able to recover seismic data volumes with large percentages of traces missing, it misses some of the physics of wave propagation. The recently developed *focal* transform (Berkhout and Verschuur, 2006) provides an interesting alternative to CRSI by focusing the data with the primary operator (weighted multidimensional correlation). While curvelets make no assumptions on the locations of the wavefronts, i.e. the phase of the propagated wavefield, the focal transform does and is in that way able to strip the data from one path interacting with the surface. This leads to a focusing of the energy as first-order multiples are mapped to primaries. How about combining the best of two worlds by composing the sparsity-promoting curvelet transform with the focal transform? In this way, the focal transform will make the data sparser while the curvelet transform adds robustness by virtue of its sparsity on general wavefields. This combination can accomplished by setting the synthesis matrix to $\mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$ and $\mathbf{S}^T = \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$ with $\boldsymbol{\Delta}\mathbf{P}$ standing for a multidimensional convolution with the primaries. In words, the solution of $\mathbf{P}_\epsilon$ involves finding the sparsest set of curvelet coefficients that matches the incomplete data when convolved with the primaries. The data, in this case, includes primaries and multiples (see Fig. 3). As such, the estimated coefficients represent an estimate for the *focused* data since they are converted back into data by the 'primary' operator during the optimization. The result of the sparse recovery from the incomplete data using standard-CRSI (Herrmann and Hennenfent, 2007) and CRSI + focusing are summarized in Fig 3. Expectedly, the curvelet transform compounded with the primary operator improves the recovery (See also our contribution(s) Herrmann, 2007, to next's week EAGE meeting).



(a) (b)

Figure 3: Comparison between CRSI **(a)** and CRSI + focusing **(b)** for data with 80 % of the traces missing. Notice the significant uplift from compounding the inverse curvelet transform with the focusing 'primary' operator.

Migration amplitude recovery:

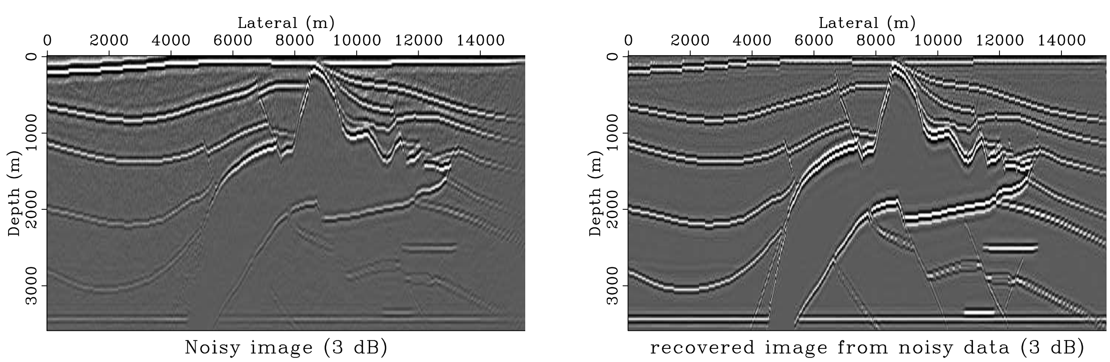
Because of the 'alleged' invariance of curvelets under wave propagation, there has been a substantial interest in deriving migration operators in the curvelet domain

(Douma and de Hoop, 2006; Chauris, 2006). In these approaches, one comes to benefit when strict sparsity is preserved under (de-)mirgation. Strict sparsity is a significant stronger assumption than the preservation of high decay rates for the sorted coefficients. Curvelets are discrete and hence move around on grids and this makes it a challenge to define fast migration operators in the curvelet domain. Curvelets, however, prove to be very useful for solving the seismic amplitude recovery problem, during which curvelets are being imaged. On theoretical grounds (Herrmann et al., 2006b), one can expect the following identity to approximately hold

$$\mathbf{A}\mathbf{A}^T\mathbf{r} \simeq \mathbf{\Psi}\mathbf{r} \tag{2}$$

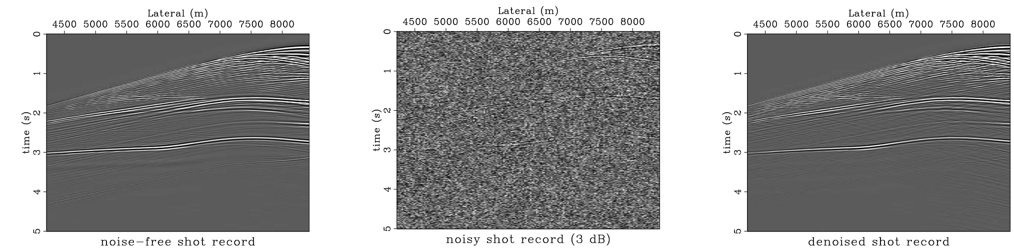
with $\mathbf{r}$ an appropriately chosen discrete reference vector and $\mathbf{\Psi}$ the discrete normal operator, formed by compounding the discrete scattering and its transpose, the migration operator. The synthesis operator in this case is defined as $\mathbf{A} := \mathbf{C}^T\mathbf{\Gamma}$ with $\mathbf{\Gamma}$ a diagonal weighting matrix. This identity diagonalizes the normal operator and allows for a stable recovery of the migration amplitudes by setting $\mathbf{y} = \mathbf{K}^T\mathbf{d}$, with $\mathbf{K}^T$ the migration operator and $\mathbf{d}$ the seismic data, and $\mathbf{S}^T := \left(\mathbf{A}^T\right)^\dagger$ with † the pseudo inverse. Results of this procedure on the SEG AA' dataset with a reverse-time migration operator, are summarized in Fig. 4. The resulting image shows a nice recovery of the amplitudes. Data generated from the estimated image, $\widetilde{\mathbf{d}} = \mathbf{K}\widetilde{\mathbf{m}}$ shows a significant removal of the noise (cf. Fig. 5(b)-5(c)), with reflection events that match the noise-free data plotted in Fig. 5(a). This visual improvements leads to an improvement of SNR for the data of 19.2 dB.

This project is a nice example were the physics of migration, in particular the demigration-migration operator, is married with recent insights from applied and computational harmonic analysis. *This is a joint project with my PhD. student Peyman Moghaddam and Chris Stolk of the University of Twente. (See also our contribution(s) Peyman P. Moghaddam and Stolk, 2007, to next's week EAGE meeting)*



(a) (b)

Figure 4: Reverse-time migration on the SEG AA' salt model. **(a)** Conventional migrated image $\mathbf{y} = \mathbf{K}^T\mathbf{d}$ from noisy data (3 dB). **(b)** Image after nonlinear recovery from noisy data ($\mathbf{P}_\epsilon$). The clearly visible non stationary noise in **(a)** is removed during the recovery while the amplitudes are also restored.



(a) (b) (c)

Figure 5: 'Denoising' of a shot record. **(a)** the noise-free data received by the receiver array. **(b)** Noisy data with a SNR of 3 dB. **(c)** forward modeled data after amplitude recovery. Observe the significant improvement in the data quality, reflected in an increase for the SNR of 19.2 dB.

Primary-multiple separation:

So far, we looked at exploiting the sparsity of curvelets in the data and image domains for the purpose of recovery. The ability of curvelets to detect wavefronts with conflicting dips, allows for a formulation of a coherent signal separation method that uses inaccurate predictions as weightings. By defining the synthesis matrix as $\mathbf{A} := \left[\mathbf{C}^T\mathbf{W}_1 \;\; \mathbf{C}^T\mathbf{W}_2\right]$, $\mathbf{x} = \left[\mathbf{x}_1 \;\; \mathbf{x}_2\right]^T$ and $\mathbf{y} = \mathbf{d}$ and by setting the diagonal weighting matrices $\mathbf{W}_{1,2}$ in terms of predictions for the primaries and multiples, the solution of $\mathbf{P}_\epsilon$ separates primaries and multiples (Herrmann et al., 2006a) even for inaccurate predictions for which least-squares subtractions fails (see Fig. 6). *This is a joint project with Eric Verschuur, Delphi.*

MULTISCALE SEISMIC CHARACTERIZATION AND MODELING

This topic is closer in line with the PhD. research (Herrmann, 1997) I conducted while at Delphi. The basic premise of this research is that well- and seismic data show evidence of a wider class of transitions than zero-order discontinuities alone (see e.g. Herrmann et al., 2001; Herrmann, 2005). This latter assumption not only underlies spiky deconvolution but it also underlies well-log upscaling through blocking. There are two main questions that interest me, namely

- how to characterize the fine structure of transitions from seismic images?
- how to model these type of transitions from bi-compositional mixtures through a nonlinear 'switch model'.

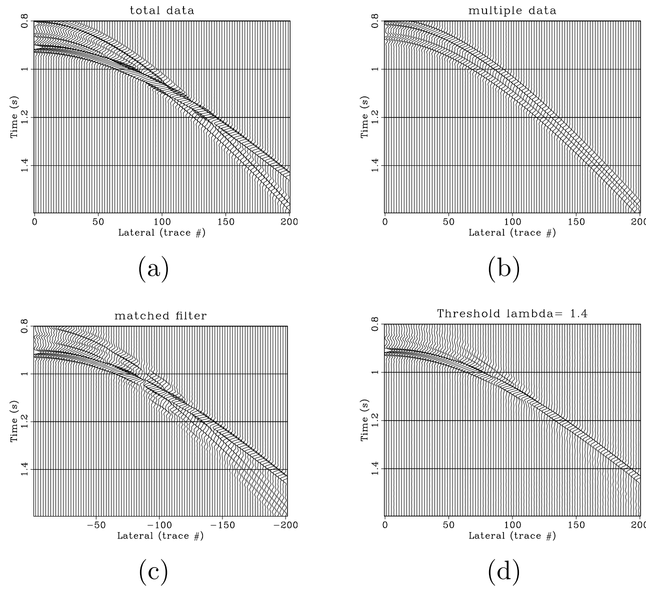


Figure 6: Example of primary-multiple separation through $\mathbf{P}_\epsilon$ for predicted multiples with moveout errors. **(a)** the total data with primaries and multiples. **(b)** the true multiples used for the prediction. **(c)** the result obtained with least-squares adaptive subtraction with localized windows. **(d)** the result obtained with a single curvelet-domain soft thresholding with $\lambda = 1.4$. Notice that least-squares subtractions fails.

Seismic characterization by a detection-estimation method:

The first questions concerns having a new look at seismic deconvolution while the second topic provides a new way to do lithological upscaling. 'Deconvolution' is this case concerns the detection of seismic events, using a multiscale technique based on the wavelet transform modulus maxima (WTMM), followed by a nonlinear inversion of a parametric representation for the waveforms associated with non-zero order transitions (see M. Maysami and Herrmann, 2007, for detail). This 'detection-estimation' method has the advantage over WTMM methods that the scale exponents that describe the fine-scale structure of the reflectors, are no longer estimated by examining the asymptotic scaling behavior of the wavelet transform. Since seismic data is bandwidth-limited, these latter asymptotic arguments are somewhat of stretch, which may lead to inaccurate estimates. As Fig. 7 indicates, our estimation, based on a quasi-Newton method, is able to accurately estimate the scale exponents from a bandwith-limited seismic trace. *This is a joint project with Mohammad Maysami one of my MSc students (See also our contribution(s) M. Maysami and Herrmann, 2007, to next's week EAGE meeting).*

Lithological upscaling with a critical switch model

Even though the above characterization by scale exponents has provided some interesting insights, e.g. the estimated locations reveal the stratigraphy while the scale

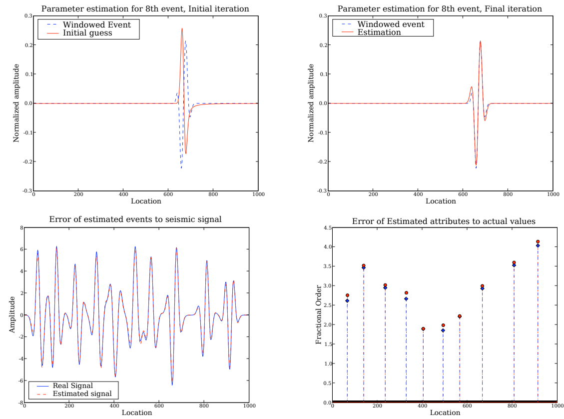
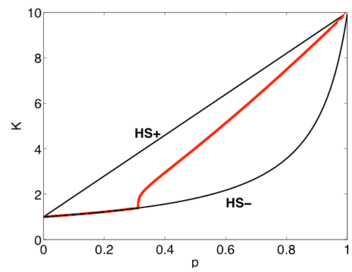


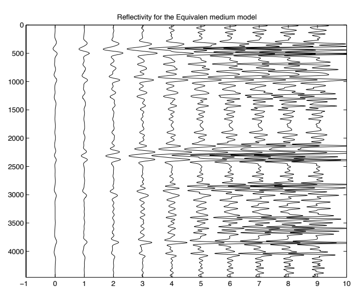
Figure 7: Example of characterization with 10 reflectivity events.**(top)** Initial and final iteration of parameter estimation for one isolated event, where the actaul values, initial guess and estimation are $\theta = (12.2, 667, 2.93)$, $\theta_{\mathbf{init}} = (7.81, 668, 0.7)$, and $\widetilde{\theta} = (12.72, 667, 3.01)$ respectively. Dashed line shows actual component and solid line the estimation. **(bottom left)** Estimated seismic signal is formed by superposition of all characterized events and compared with the original seismic trace. **(bottom right)** The estimated attributes of events (τ, α) are compared to their actual values. Blue diamonds show actual parametes whereas red circles estimated values.

exponents give some information of the sorting of the sands (Herrmann et al., 2001; Herrmann, 2005), a rock-physical model seems to be missing that predicts the existence of fractional order transitions from a solid rock physical arguments.

In recent work, Yves Bernabé and I have been able to derive a model that predicts a rapid change in transport properties as a function of the volume fractions of bi-compositional mixtures. Consider, for instance, a mixture of two materials, one hard one soft. The behavior of such a mixture is well understood. Less well-known is the fact that the mixture undergoes an abrupt change when the concentration of the stronger material reaches a point where the inclusions *connect*. At that critical point, a fractional-order discontinuity is created in the elastic properties of the mixture. This critical percolation phenomenon (Herrmann and Bernabé, 2004; Bernabé et al., 2004) has profound implications on the interpretation of seismic discontinuities, which in this case can no longer be attributed to steep gradients in the composition. Instead the discontinuities are due to an intricate mechanism which, when well understood, provides (i) complementary information on the composition of the subsurface and (ii) a method to do lithological upscaling. Because of the switch at the critical point, upscaling by smoothing the lithology, e.g. smoothing of the volume fractions of shale in sand-shale mixtures, no longer washes out the reflectivity, an unwanted site effect of many equivalent-medium based upscaling techniques. Instead, reflectors will be preserved. To understand this let's have a look at the compliance (k) as function of the volume fraction (p). From mixing theory it is known that the compliance for any rock mixture lies within the Hashin-Shtrikman (HS) bounds (See Fig. 8(a)). According to our model, the rock mixture follows for low volume fractions the lower-HS bound and as it reaches the critical point, the compliance displays a 'cusp-like' behavior departing the lower-bound, followed by a ramping up towards the upper bound (the red line in Fig. 8(a)). This 'switch-like' behavior has a distinct impact on the upscaled reflection traces. While conventional upscaling smoothes the reflection events, the switch model preserves reflection events due to the nonlinear switch. An example of upscaling from the lithology of a synthetic well (kindly provided by Chevron) is included in Fig. 8. This example clearly shows the difference between conventional upscaling and upscaling with our switch model. *This is joint work with Yves Bernabé MIT.*



(a)



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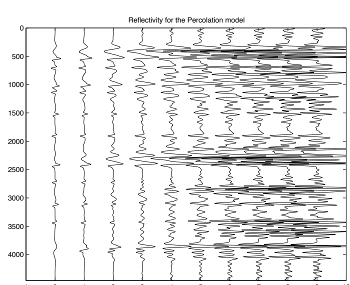

(c)

Figure 8: Reflector- preserved lithological upscaling. **(a)** HS bounds for the elastic compliance (black) as a function of the lithology (volume fraction p) and the cusp-like behavior according to our critical percolation-based model (red). **(b)** Upscaled reflection traces according to the classical upscaling based on equivalent medium theory. **(c)** Upscaled reflectivity according to the switch model. Notice the vanishing of the reflectivity for coarser scales with the classical upscaling and the preservation of the events related to our nonlinear upscaling model.

Object-oriented scientific computing at SLIM

One of topics we as scientists do not often talk about is the scientific computing environment in which we work. Good environments can lead to drastic improvements in development time of scientific ideas. Unfortunately, fundamental developments in this area are rather slow or said in other words we were just ahead of the game back when I was a Delphi-team member. Let me give a couple of examples. The MAC OS X operating system came out of the NextStep operating system, we used to work with in Delphi during the nineties. Our ideas to develop a 'Seismic Workbench', strongly supported by Guus, were also ahead of their time. For instance, we were thinking to develop a matlab-like interactive system that would allow us to do dynamic typing in a distributed computing environment by encapsulating (non)linear operators in distributed objects. We are talking early nineties now, and more than a decade later these ideas are 'finally' taking shape.

At SLIM, we aim to incorporate this 'detail hiding' approach of the 'Seismic Workbench' by combining the object-oriented language of Python with the low-level pipe-based seismic processing programs part of existing libraries such as SU, SEPLib, Delphi and most recently Madagascar (`rsf.sourceforge.net/`). Following ideas by Symes (the Rice Vector Library) and Bartlett (Thyra) these suits of unix-pipe based programs can be categorized into (i) (nonlinear) element-wise operations, known as reduction-transformation operations, including the calculation of norms, scalar multiplication and other element-wise operations; (ii) linear (matrix-free) matrix-vector multiplications and (iii) nonlinear operators, including their Jacobians and Hessians (these are again matrices). This categorization is very useful, since it allows us to develop a coordinate-free framework, suitable for implementing solvers for the nonlinear optimization problems discussed above. Our efforts are designed to add a coordinate-free layer to these libraries, where the 'details' of the operators are hidden from the solver. Arguably such an abstraction would not only greatly facilitate the development of solvers that are transparent and reusable but it will also lead to easy-to-disseminate codes.

At SLIM, we are making progress towards an environment that meets above goals by allowing for

- **in-core** development in *native* Python using the Numpy and SciPy packages;
- **out-of-core** development using Madagascar through overloading of the python operators;
- **parallelization** using MPI and the parallel curvelet transform (Thomson et al., 2006).

By virtue of the abstraction, the solvers are the same irrespective whether the vectors an operators are in-core, out-of-core or parallel. This leads to code reuse and to a rapid development environment for the algorithms developed in my group. I am planning to release this software to the general public in the near future. See Fig. 9 for SLIMPy code examples defining the operators, coding up a solver and shifting to a parallel implementation without changing the solvers.

Observations

It goes without saying that my current research program has significantly been influenced by the time I spend in Delft under Guus. If I am asked to summarize what I considered as most important during these years, I will have to say that one main theme sticks to my mind, namely an multidisciplinary approach, where a thorough understanding of the physics is combined with advanced techniques from mathematics and computer science. It was a privilege to have been a student under Guus.

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———, 2006c, Seismic denoising with non-uniformly sampled curvelets: IEEE Comp. in Sci. and Eng., **8**, 16–25.

```
#define the vector
y = vector( 'data' )
#define transform
A = fdct2( transform params )
#define the solver to use
solver = landweber.GenThreshLandweber( solver params )
#Solve for x from: y = A* x + b
x = solver.solve(A,  y )
```

(a)

```
for lambdaN in step(0.7,0.01,2):
  for j in range(1):
    xTmp = ( Coefs -( A.transp() * ( A * x ) )) + x
    x = xTmp.thr(lambdaN)
    x.flush()
```

```
 < data.rsf /Users/Sean/RSF/bin/sffdct2 sizes=/var/tmp/Sizes.Lwdph5.rsf inv=n nbs=5 nba=32 ac=1 > /var/tmp/fdct2_.DrDSqW.rsf
 < /var/tmp/fdct2_.DrDSqW.rsf /Users/Sean/RSF/bin/sfmath output="0*input"  > /var/tmp/math.TZsFrX.rsf
 < /var/tmp/fdct2_.DrDSqW.rsf /Users/Sean/RSF/bin/sfsort memsize=500 ascmode=n > /var/tmp/sort.xa17Es.rsf
 < /var/tmp/math.TZsFrX.rsf /Users/Sean/RSF/bin/sffdct2 sizes=/var/tmp/Sizes.Lwdph5.rsf inv=y | /Users/Sean/RSF/bin/sfreal | /Users/Sean/RSF/
bin/sffdct2 sizes=/var/tmp/Sizes.Lwdph5.rsf inv=n nbs=5 nba=32 ac=1 > /var/tmp/fdct2_.m0o6na.rsf
 < /var/tmp/fdct2_.DrDSqW.rsf /Users/Sean/RSF/bin/sfmath output="input-a"  a=/var/tmp/fdct2_.m0o6na.rsf | /Users/Sean/RSF/bin/sfmath
output="input+a"  a=/var/tmp/math.TZsFrX.rsf | /Users/Sean/RSF/bin/sfthr thr=0.0114124957472 mode="soft" > /var/tmp/thr.S6QpgY.rsf
 < None /Users/Sean/RSF/bin/sfrm /var/tmp/math.TZsFrX.rsf > None
 < None /Users/Sean/RSF/bin/sfrm /var/tmp/fdct2_.m0o6na.rsf > None
 < /var/tmp/thr.S6QpgY.rsf /Users/Sean/RSF/bin/sffdct2 sizes=/var/tmp/Sizes.Lwdph5.rsf inv=y | /Users/Sean/RSF/bin/sfreal | /Users/Sean/RSF/
bin/sffdct2 sizes=/var/tmp/Sizes.Lwdph5.rsf inv=n nbs=5 nba=32 ac=1 > /var/tmp/fdct2_.js9ZYs.rsf
 < /var/tmp/fdct2_.DrDSqW.rsf /Users/Sean/RSF/bin/sfmath output="input-a"  a=/var/tmp/fdct2_.js9ZYs.rsf | /Users/Sean/RSF/bin/sfmath
output="input+a"  a=/var/tmp/thr.S6QpgY.rsf | /Users/Sean/RSF/bin/sfthr thr=0.00172270357143 mode="soft" > /var/tmp/thr.T01ji5.rsf
 < None /Users/Sean/RSF/bin/sfrm /var/tmp/thr.S6QpgY.rsf > None
 < None /Users/Sean/RSF/bin/sfrm /var/tmp/fdct2_.DrDSqW.rsf > None
 < /var/tmp/thr.T01ji5.rsf /Users/Sean/RSF/bin/sffdct2 sizes=/var/tmp/Sizes.Lwdph5.rsf inv=y | /Users/Sean/RSF/bin/sfreal > cres.rsf
 < None /Users/Sean/RSF/bin/sfrm /var/tmp/thr.T01ji5.rsf > None
 < None /Users/Sean/RSF/bin/sfrm /var/tmp/Sizes.Lwdph5.rsf > None
```

(b)

```
dnoise.py data=data.rsf output=res.rsf [pSLIMpy options]
```

```
mpirun [options]
  dnoise.py data=data.rsf output=res.rsf [pSLIMpy options]
```

(c)

Figure 9: Object-oriented abstraction of linear operators in SLIMPy. **(a)** Example of the definition of linear operators (the curvelet transform in this case). **(b)** Implementation of a nonlinear solver, yielding a series of out-of-core commands on files (below the arrows). **(c)** The built-in parallization.

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