Improved seismic survey design by maximizing the spectral gap with global optimization

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Motivations

Seismic data

- expensive to acquire

Random subsampling

- increasingly employed in seismic data acquisition
- reduce cost

Matrix completion (MC)

- reconstruct fully sampled wavefields from sparsely sampled seismic data
- computationally efficient method

Goal: Automatically generate sampling schemes that favor recovery by MC.
Motivations

Simulation-based acquisition design

- expensive
- time consuming

Uniform & jittered sampling scheme

- not optimal
- is not flexible - i.e., cannot add constraints

Spectral gap (SG) of sampling mask

- the gap between the first & second singular value
- a cheap metric to predict a performance of an acquisition design
- should be large to ensure success of matrix completion

Come up with a quantity to predict wavefield reconstruction w/ MC
Motivation
relationship between reconstruction quality & sampled matrix

$M$ binary sampling matrix

$\sigma_1(\cdot)$ the first singular value

$\sigma_2(\cdot)$ the second singular value

$\frac{\sigma_2(M)}{\sigma_1(M)}$ SG ratio

an average of 100 experiments

Large spectral gap corresponds to small SG ratio
Problem

Generate sampling scheme w/o expensive wavefield reconstruction

Take advantage of spectral gap, a cheap metric to evaluate an acquisition mask

Start with 2D acquisition on a grid…

Hands-on tutorial:
Breakout 2. Acquisition design and wavefield reconstruction (code)
2D acquisition

Sampling mask in source-receiver domain
2D acquisition

Sampled mask in source-receiver domain

```
  1 0
  1 0
  1 0
  1 0
  1 0
  1 0
```

Sources

Receivers

Source

Receivers
### 2D acquisition

#### Sampled mask in source-receiver domain

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
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**Receivers**

![Diagram of Receivers and Source](image)

**Source**

![Image of Source](image)
2D acquisition

Receivers

Sampled mask in source-receiver domain

<table>
<thead>
<tr>
<th>Sources</th>
<th>1 0 1 0 0 1 1</th>
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<tbody>
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</table>
2D acquisition

Sampled mask in source-receiver domain

<table>
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<tr>
<th>Sources</th>
<th>Receivers</th>
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<tbody>
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Sampled mask in midpoint-offset domain

<table>
<thead>
<tr>
<th>Midpoints</th>
<th>Offsets</th>
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<tbody>
<tr>
<td>0 0 0 0 0 1 1 1 0 0 0 0 0 0 0</td>
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<td>0 0 0 0 0 0 1 1 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 1 0 0 0 0 0 0 0 0</td>
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</table>
2D acquisition

Sampled mask in source-receiver domain

Sampled mask in midpoint-offset domain

SG ratio = 0.804
Optimized problem
2D acquisition

Given \( n_s \) source locations & subsampling ratio \( r \), find \( m = \lfloor n_s \times r \rfloor \times n_r \) subsampling locations \( X \)

\[
\text{minimize} \quad \frac{\sigma_2(\mathcal{S}X)}{\sigma_1(\mathcal{S}X)} \quad \text{subject to} \quad \|X\|_0 = m, \quad X \in [0,1]^{n_s \times n_r}.
\]

- \( \mathcal{S} \): an operator that transfers the data from SR domain to midpoint-offset domain
- \( \sigma_1(\cdot) \) and \( \sigma_2(\cdot) \): the first & second singular values
- \( n_s \), the number of sources & \( n_r \), the number of receivers.
- \( n_m \), the number of midpoints & \( n_o \), the number of offsets.
Simulated annealing (SA) 
Combinatorial search technique

Outline:

► Select a neighbor at random
  
  - If better than current state, update the state (improving move)
  
  - Otherwise, update current state w/ some probability (worsening move)

► Probability goes down with time

High probability means diversify (many worsening moves)

Low probability means intensify (focus on improving moves)


Select a neighbor at random

Previous state
Fix 80% subsampled positions, move 20% subsampled positions
Set move range
New neighbor state

- ○: all possible locations;
- ●: initial subsampled locations;
- •: to be moved location;
- ●: the new neighbor subsampled location
Stylized example
w/ equal source/receiver dimension (300 × 300)
Experiment 1
Optimal SG ratio w/ simulated annealing

Mask dimension: 300 x 300

Subsampling ratio: 33 %

Source and receiver sampling interval: 12.5 m

Decrease the SG ratio of given initial subsampling masks:

- uniform random subsampling
- optimal jittered subsampling
Uniform random mask
Initial state (input) of the optimal method

SG ratio = 0.341
Improved random mask
Output of the optimal method w/ SA algorithm

SG ratio = 0.196
Optimal jittered mask
Initial state (input) of the optimal method

SG ratio = 0.334
Improved jittered mask
Output of the optimal method w/ SA algorithm
SG ratio comparison
W/ 10 independent tests

Subsampling ratio = 33%
Synthetic example
Experiment 2
Test masks by using wavefield reconstruction (LR matrix completion)

Data dimension: 300 x 300 x 1024 (nr x ns x nt)

Dimension of each frequency slice: 300 x 300

Source sampling interval: 12.5 m

Receiver sampling interval: 12.5 m

Time sampling interval: 0.002 s
2D synthetic Compass dataset

Ground truth

Observed data
Scenarios

Use wavefield reconstruction to recover the subsampled data w/ 4 masks

- Uniform random
- Improved random mask w/ proposed optimized method
- Optimal jittered (w/ max gap control)
- Improved jittered subsampling w/ proposed optimized method
Recovery & difference
Uniform random mask

Recovery (w/ SNR = 13.25 dB)

Difference = ground truth - recovery
Recovery & difference
Improved uniform random mask
Recovery & difference
Optimal jittered mask

Recovery (w/ SNR = 12.39 dB)

Difference

Times [s]

Sources [Km]
Recovery & difference
Improved jittered mask w/ proposed method
SNR comparison
W/ 10 independent tests

Subsampling ratio = 33%
SG ratio comparison vs. SNR comparison
W/ 10 independent tests

Subsampling ratio = 33%

SG ratio

Uniform random
Improved random
Optimal jittered
Improved jittered

SNR [dB]

Uniform random
Improved random
Optimal jittered
Improved jittered
Stylized example
w/ unequal source/receiver dimension (300×150)
Experiment 3
Optimal SG ratio w/ simulated annealing

Mask dimension: 300 × 150

Subsampling ratio: 20%

Source & receiver sampling interval: 12.5 m

Decrease SG ratio of given initial subsampling masks:

- uniform random subsampling
- optimal jittered subsampling
Subsampled mask in source-receiver domain
Subsampling ratio = 20%
Subsampled mask in source-receiver domain

Subsampling ratio = 20%
Uniform random mask
Initial state (input) of the optimal method
Improved random mask
Output of the optimal method w/ SA algorithm

SG ratio = 0.249
Optimal jittered mask
Initial state (input) of the optimal method
Improved jittered mask
Output of the optimal method w/ SA algorithm
SG ratio comparison
W/ 10 independent tests

Subsampling ratio = 20%

- Uniform random
- Improved random
- Optimal jittered
- Improved jittered
Conclusion & future works

Improved seismic survey design w/o expensive wavefield reconstruction

Proposed optimization scheme that finds subsampling masks w/ small SG ratio

Optimized masks improve wavefield reconstruction

Test reconstruction quality w/ unequal source/receiver dimension

3D seismic data survey design
Thank you for your attention!