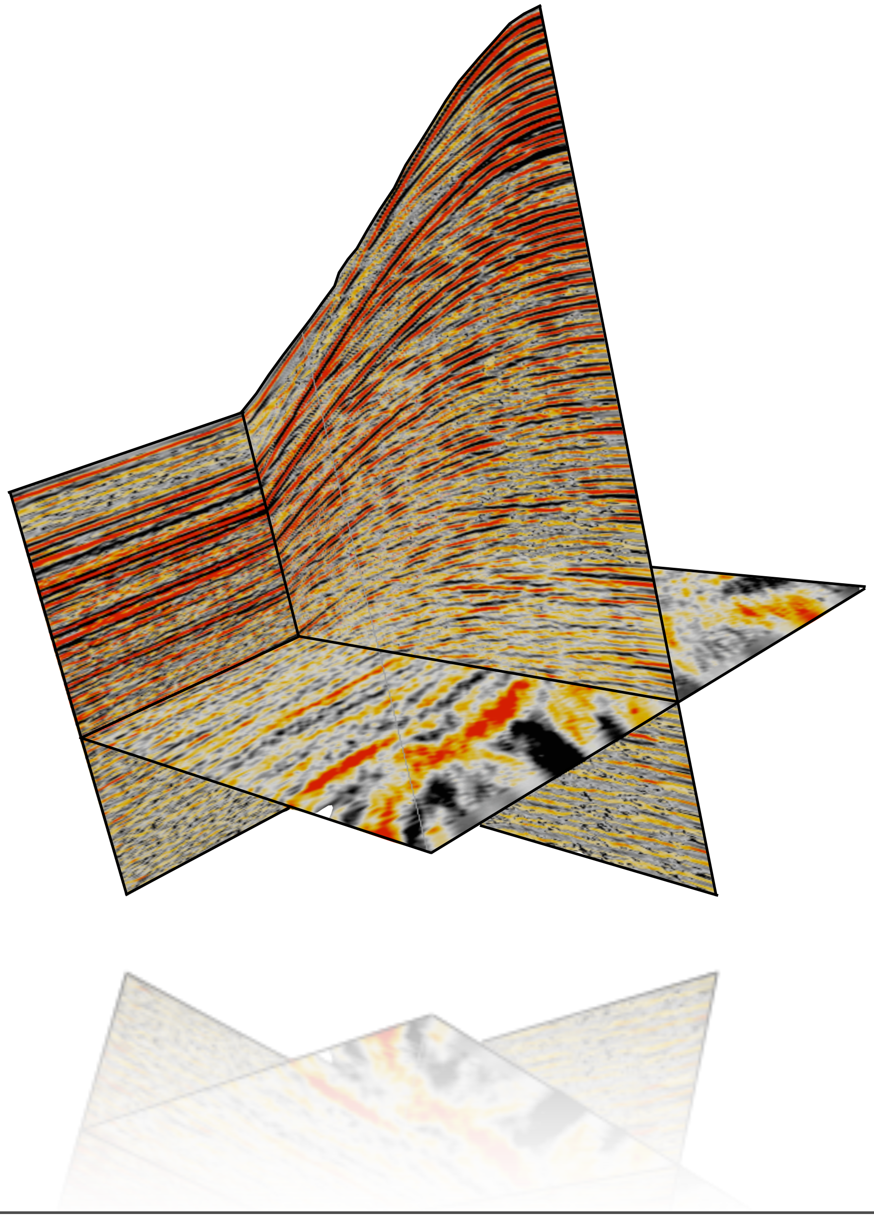
THE UNIVERSITY OF BRITISH COLUMBIA | VANCOUVER

Compressive sampling: a new paradigm for seismic data acquisition and processing?

Felix J. Herrmann
fherrmann@eos.ubc.ca
http://slim.eos.ubc.ca

Gilles Hennenfent
ghennenfent@eos.ubc.ca
http://wigner.eos.ubc.ca/~hegilles

Seismic Laboratory for Imaging & Modeling
Department of Earth & Ocean Sciences
The University of British Columbia



ION Technical forum - Sprowston, UK
Tuesday 15th – Thursday 17th April

Motivation

- Current state of affairs:
  - Seismic data processing firmly rooted in paradigm of regular Nyquist sampling
  - Practitioners go all out to create regularly-sampled data volumes
  - Preferred by Fourier-based processing flows
- Recent theoretical & hardware developments
  - Alternative multiscale, localized & directional transform domains that compress seismic data
  - New nonlinear sampling theory that supersedes the overly pessimistic Nyquist sampling criterion
  - New autonomous data acquisition devices that allow for more flexibility during acquisition
  - New simultaneous & continuous recording
- Recent successful application of directional transforms in seismic
  - wavefield separation
  - wavefield matching
  - image-amplitude recovery

Today’s agenda

- Sparsity-promoting wavefield recovery
  - sparsifying transform
  - favorable (random) acquisition
  - nonlinear recovery by sparsity promotion

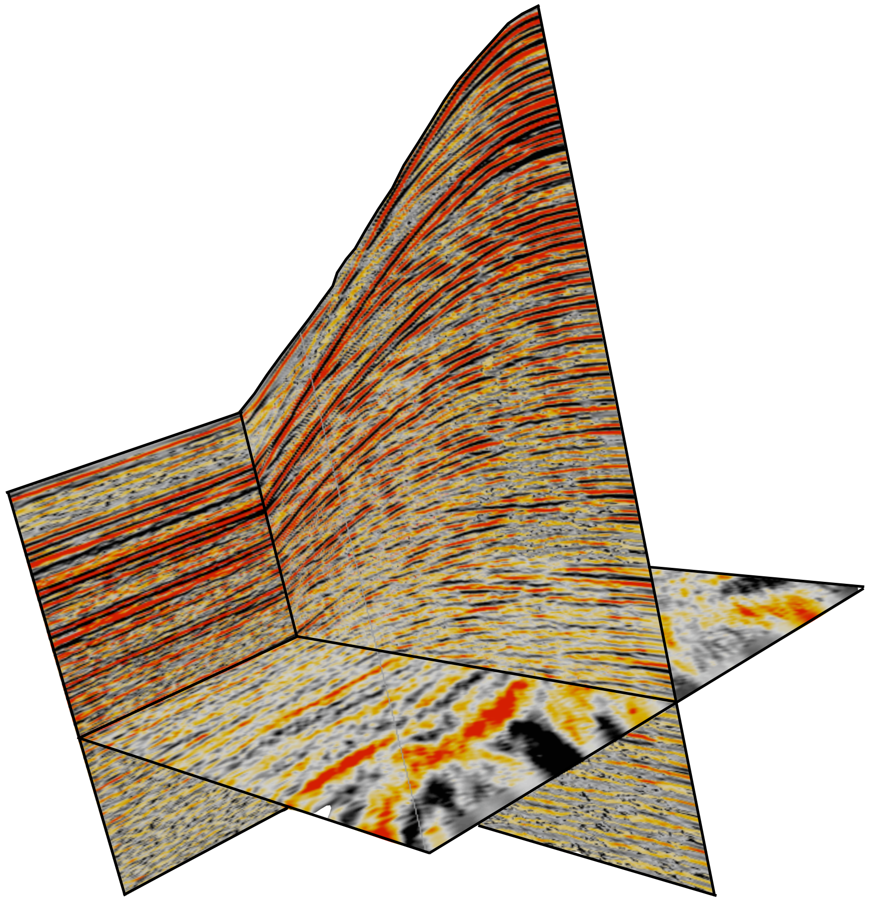
- Seismic data processing with curvelets
  - primary-multiple separation

- A look ahead ...
  - stable wavefield inversion
  - multidimensional acquisition design

Sampling and reconstruction of seismic wavefields in the curvelet domain

Gilles Hennenfent
ghennenfent@eos.ubc.ca
http://wigner.eos.ubc.ca/~hegilles

Seismic Laboratory for Imaging & Modeling
Department of Earth & Ocean Sciences
The University of British Columbia



ION Technical forum - Sprowston, UK
Tuesday 15th – Thursday 17th April

Wavefield reconstruction methods

- filter-based methods [Spitz’91, Fomel’00]
  - convolve the incomplete data with an interpolating filter

- wavefield-operator-based methods [Canning and Gardner’96, Biondi et al.’98, Stolt’02]
  - explicitly include wave propagation
  - require knowledge of velocity model
  - computationally intensive

- transform-based methods [Sacchi et al.’98, Trad et al.’03, Zwartjes and Sacchi’07]
  - fastest approaches
  - no explicit link with wave propagation

Performance of most aforementioned methods deteriorates for data with acquisition irregularities.

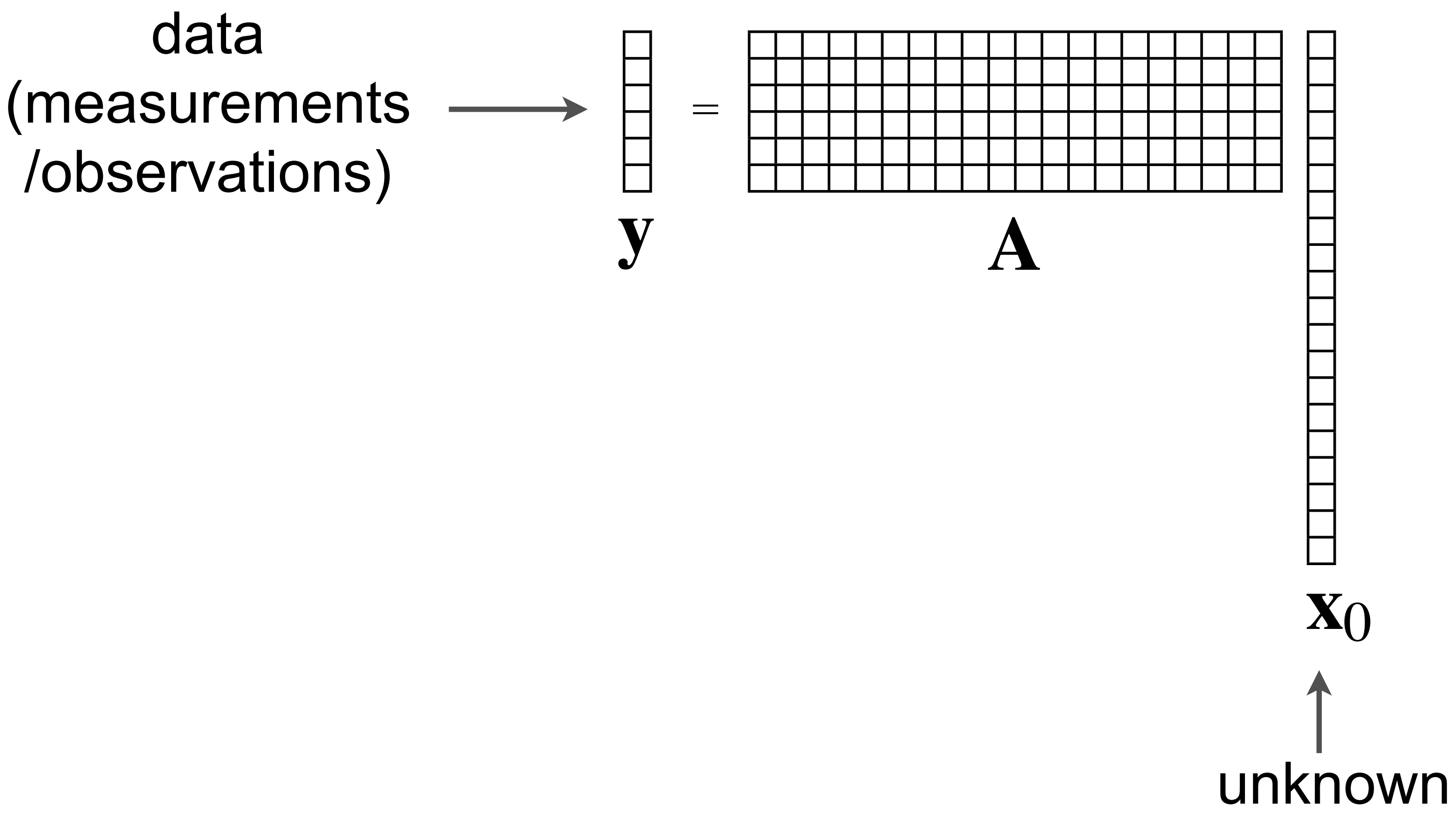
Key ideas

Use recent insights from the field of compressive sensing to

- formulate a new wavefield reconstruction method that handles both regular and irregular acquisition geometries
  - curvelet reconstruction with sparsity-promoting inversion (CRSI) [Herrmann and Hennenfent‘08]
- develop a new random coarse sampling scheme that maximizes the performance of CRSI
  - jittered undersampling scheme [Hennenfent and Herrmann‘08]
- implement a new large-scale, one-norm solver
  - iterative soft thresholding with cooling (ISTc) [Herrmann and Hennenfent’08, Hennenfent et al.‘08]
- formalize nonlinear *ad hoc* methods
  - anti-leakage Fourier transform [Xu et. al. ‘05]

Problem statement

Consider the following (severely) underdetermined system of linear equations

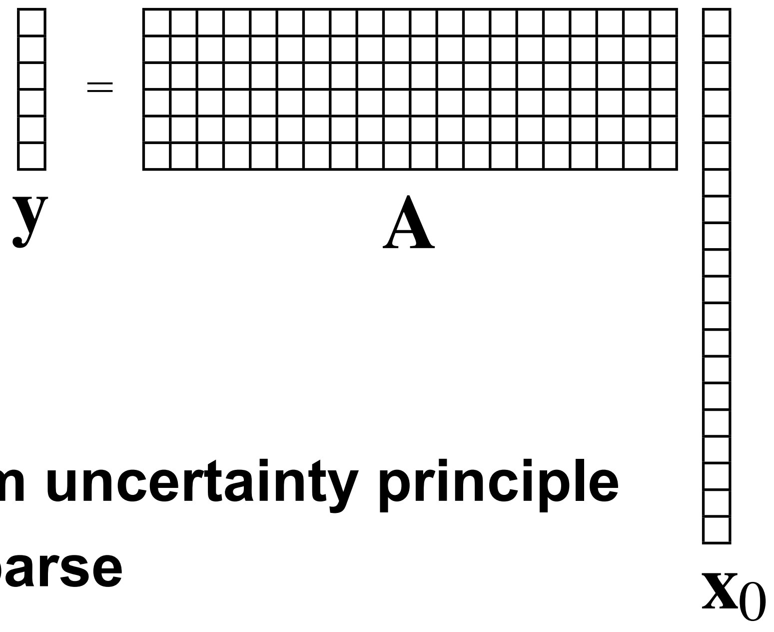


data
(measurements
/observations)

unknown

Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

Perfect recovery



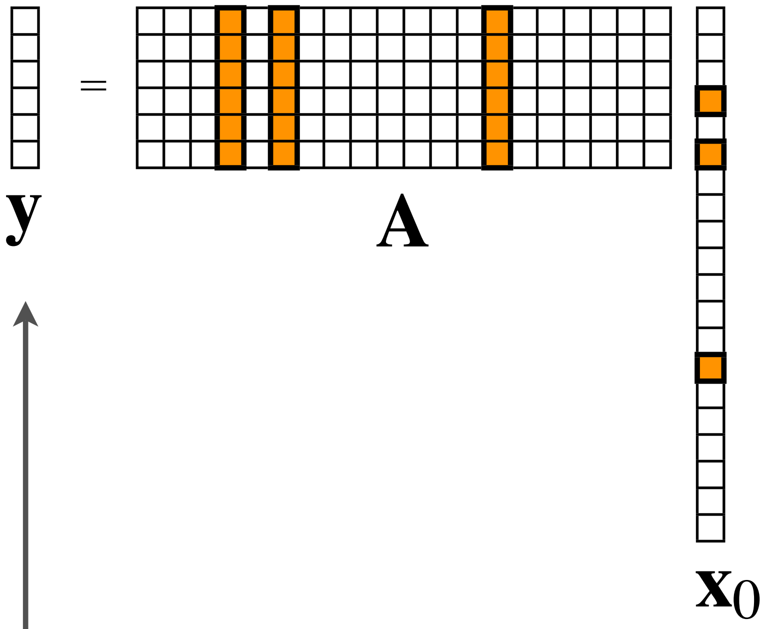
- conditions
  - **A** obeys the **uniform uncertainty principle**
  - **$\mathbf{x}_0$** is **sufficiently sparse**
- procedure

$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

- performance
  - *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

[Candès et al.‘06]
[Donoho‘06]

Simple example



with

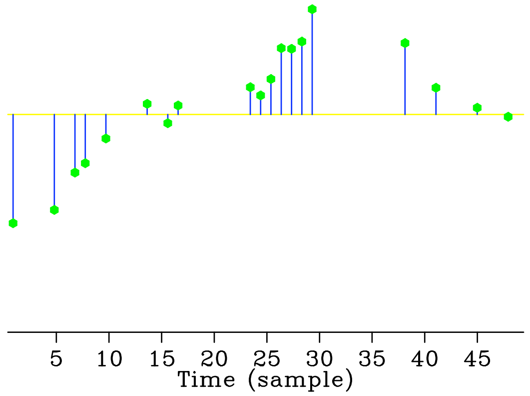
restriction
operator

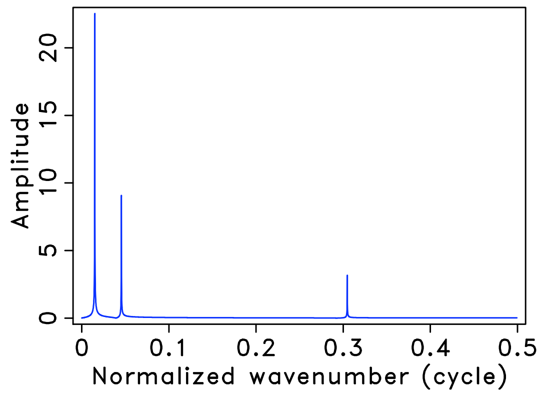
$$\mathbf{A} := \mathbf{R}\mathbf{F}^H$$

Fourier
transform

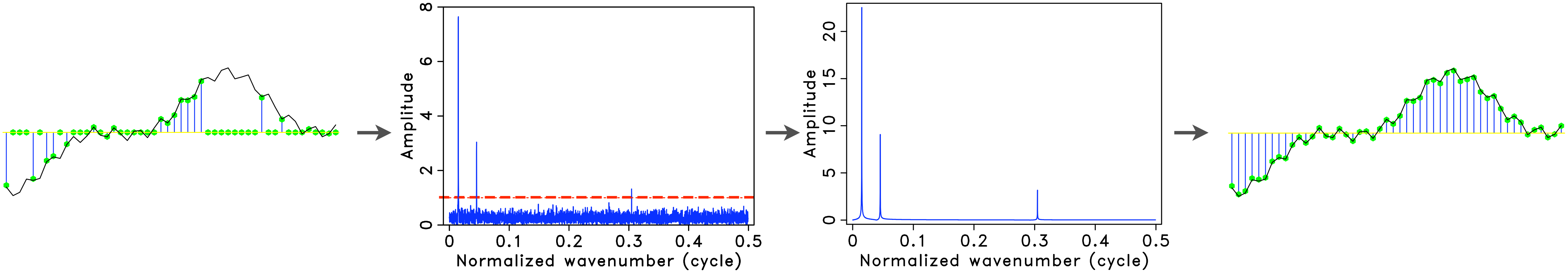
signal

Fourier coefficients
(sparse)





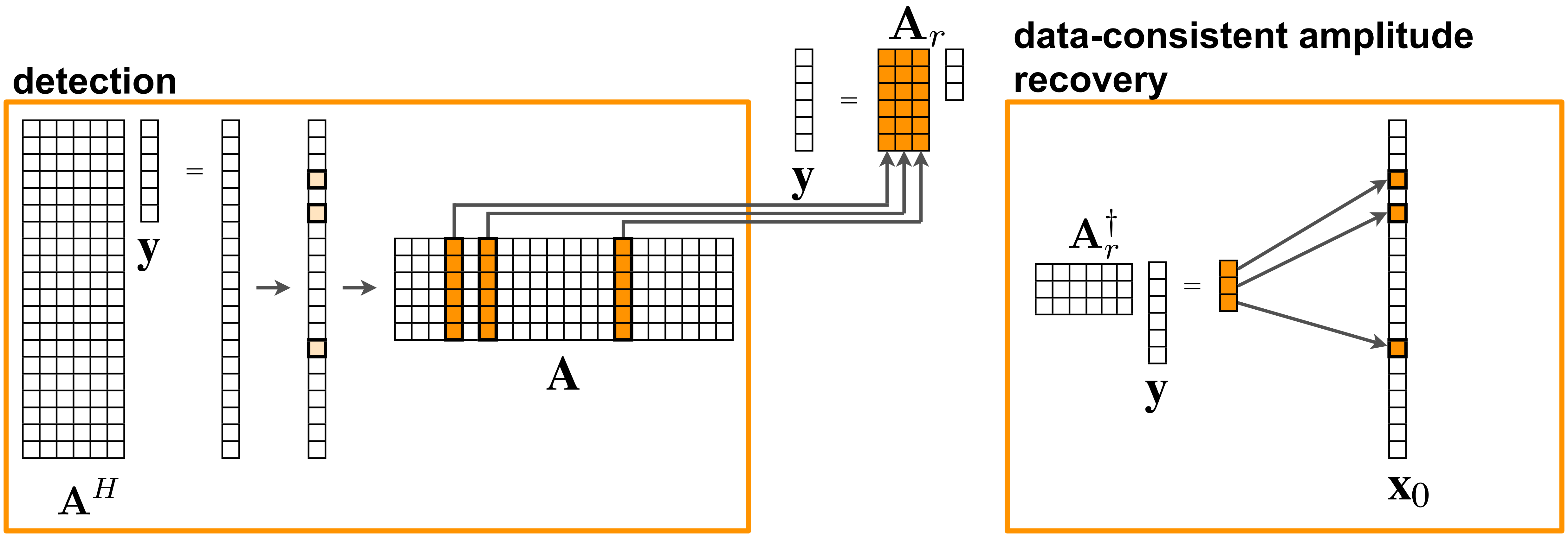
NAIVE sparsity-promoting recovery



inverse
Fourier
transform

detection +
data-consistent
amplitude recovery

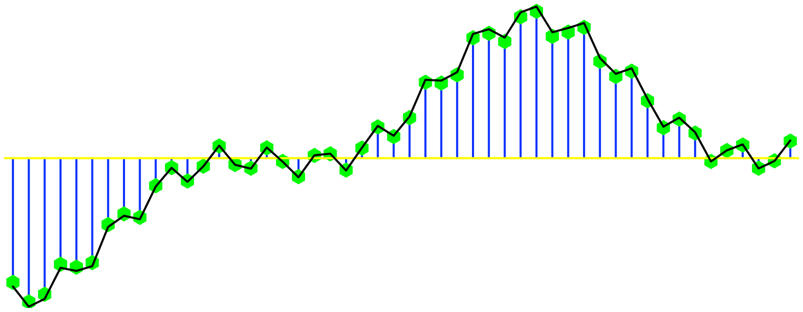
Fourier
transform



detection

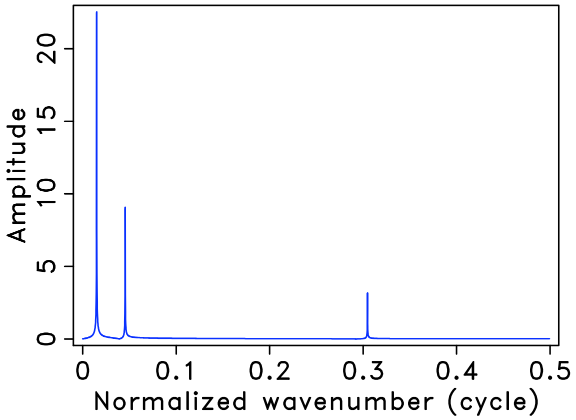
data-consistent amplitude recovery

Coarse sampling schemes



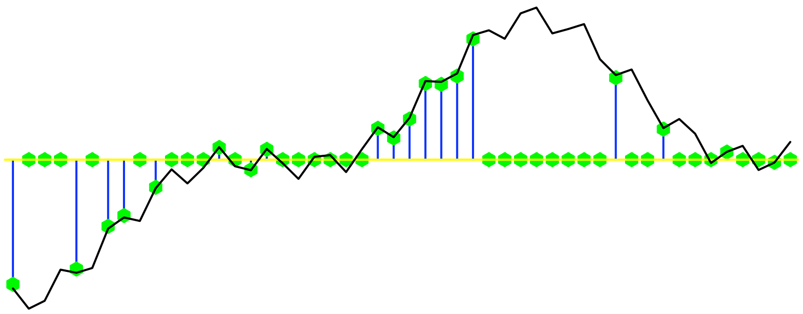
Fourier

transform



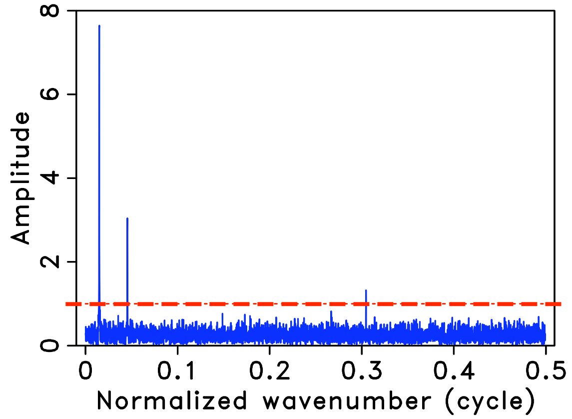
few significant
coefficients

3-fold under-sampling



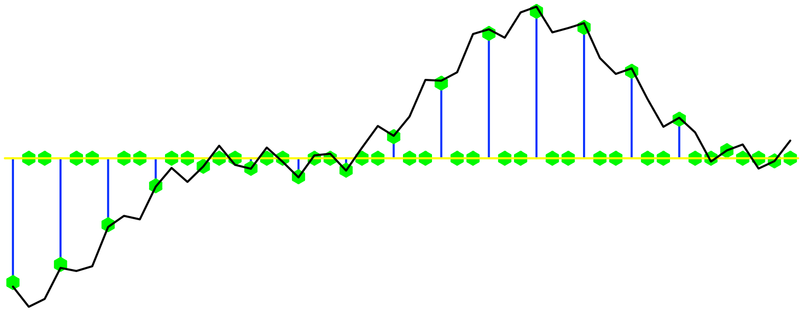
Fourier

transform



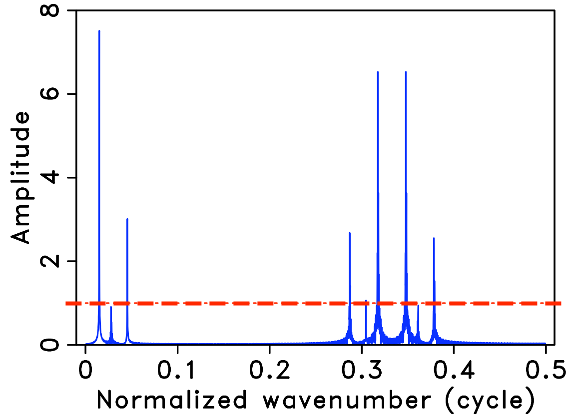
✓

significant
coefficients detected



Fourier

transform



✗

ambiguity

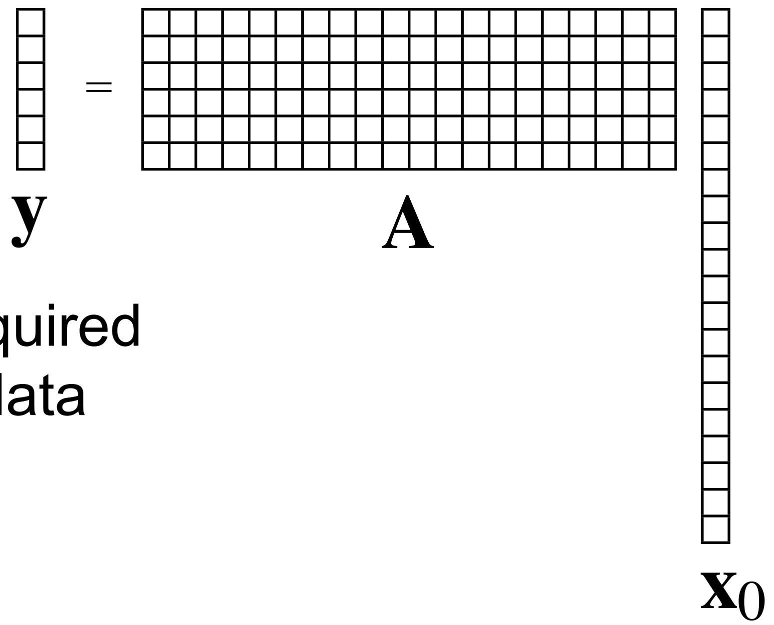
[Hennenfent and Herrmann‘08]

Observations

- Random undersampling breaks the constructive interferences, i.e. *aliases*
- Turns *alias* into incoherent *noise*
- Works by virtue of
  - *incoherence* (correlations) between the *rows* of the *Dirac* measurement basis and the columns of the *Fourier synthesis* basis
  - maximum *spreading* of Diracs in Fourier domain
  - maximum *leakage*
  - *independence* amongst columns of **A**, i.e., there exists a subset of columns of **A** that forms an orthonormal basis

- According to theory of compressive sensing
  - recovery stable w.r.t. noise
  - measurement & sparsity bases can be more general

Sparsity-promoting wavefield reconstruction



acquired
data

complete wavefield
(transform domain)

restriction operator

with $\mathbf{A} := \mathbf{R}\mathbf{S}^H$

sparsifying transform
for seismic data

Interpolated data given by $\tilde{\mathbf{f}} = \mathbf{S}^H \tilde{\mathbf{x}}$ with

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$

[Sacchi et al.‘98]
[Xu et al.‘05]
[Zwartjes and Sacchi‘07]
[Herrmann and Hennenfent‘08]

Key elements

- ☐ *sparsifying transform*
  - typically **localized** in the time-space domain to handle the complexity of seismic data

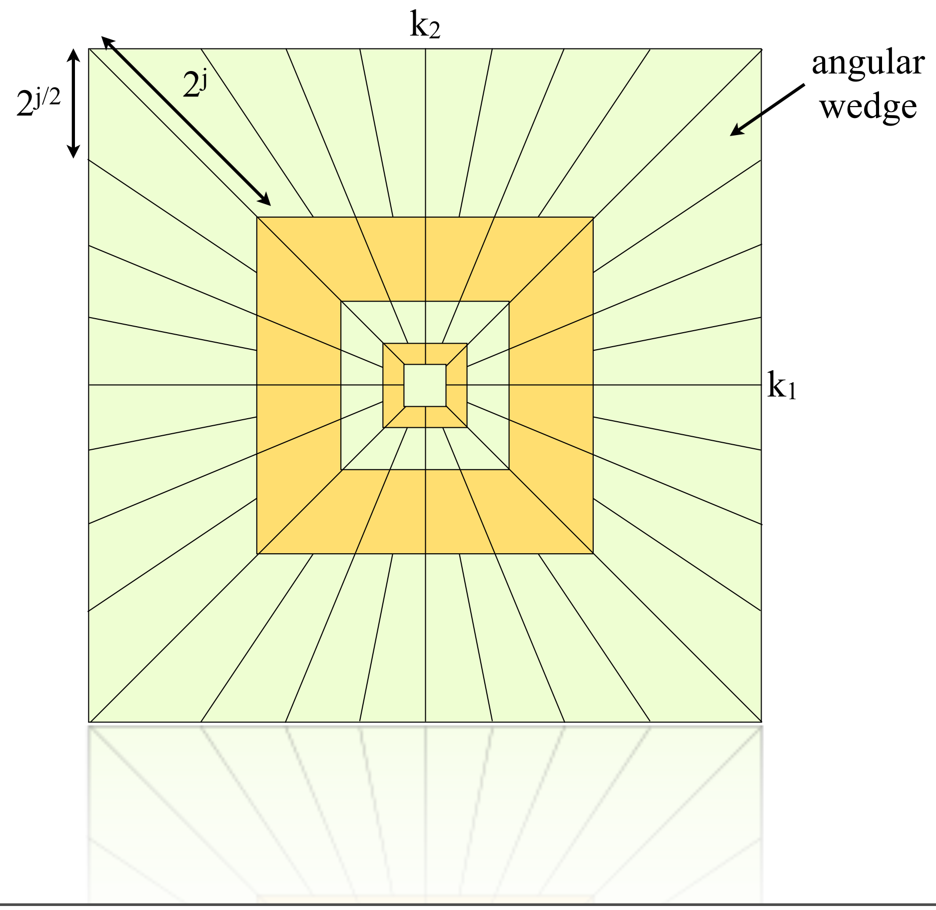
- ☐ *advantageous coarse sampling*
  - generates incoherent random undersampling “noise” in the sparsifying domain
  - does not create large gaps
    - because of the limited spatiotemporal extent of transform elements used for the reconstruction

- ☐ *sparsity-promoting solver*
  - requires few matrix-vector multiplications

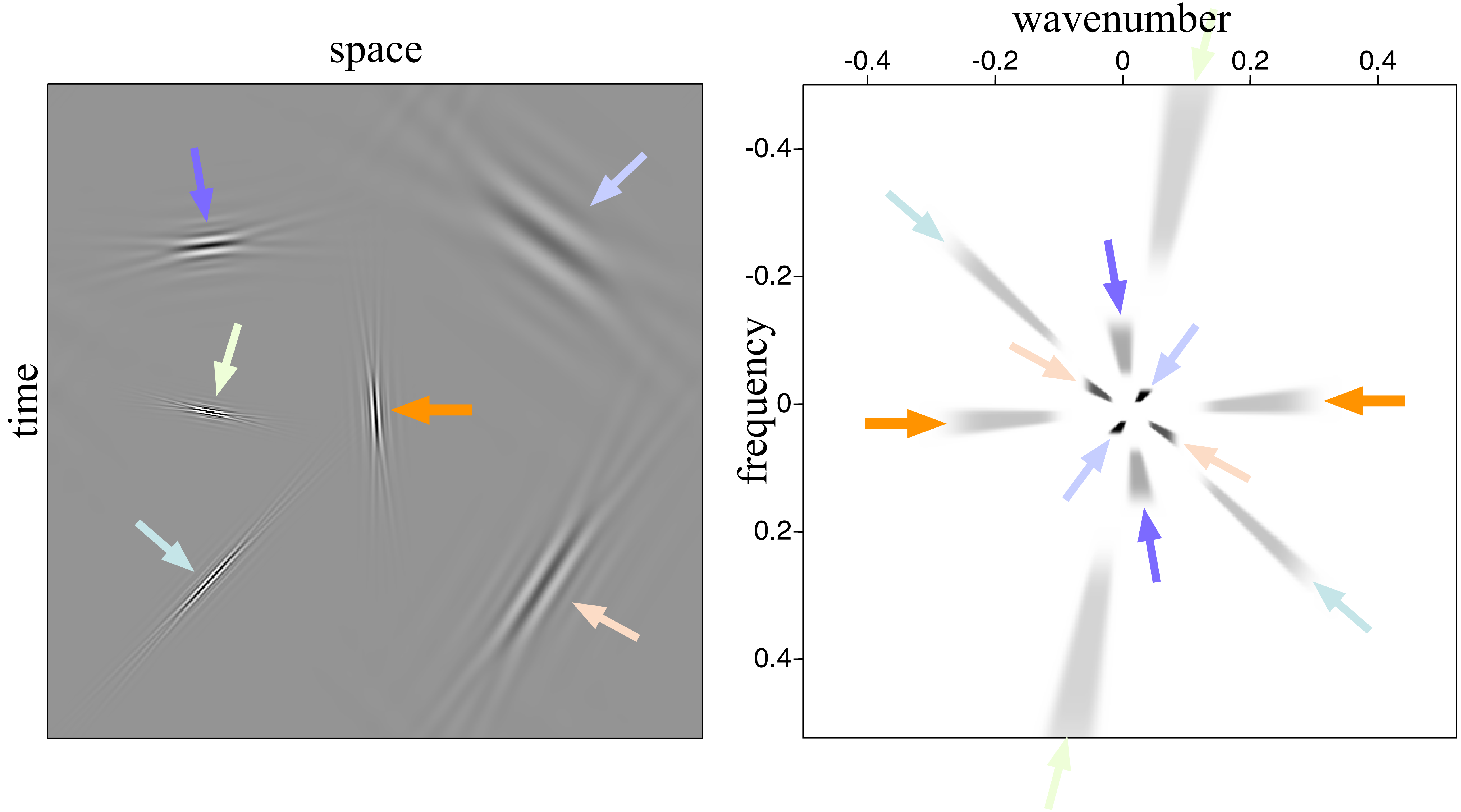
Representations for seismic data

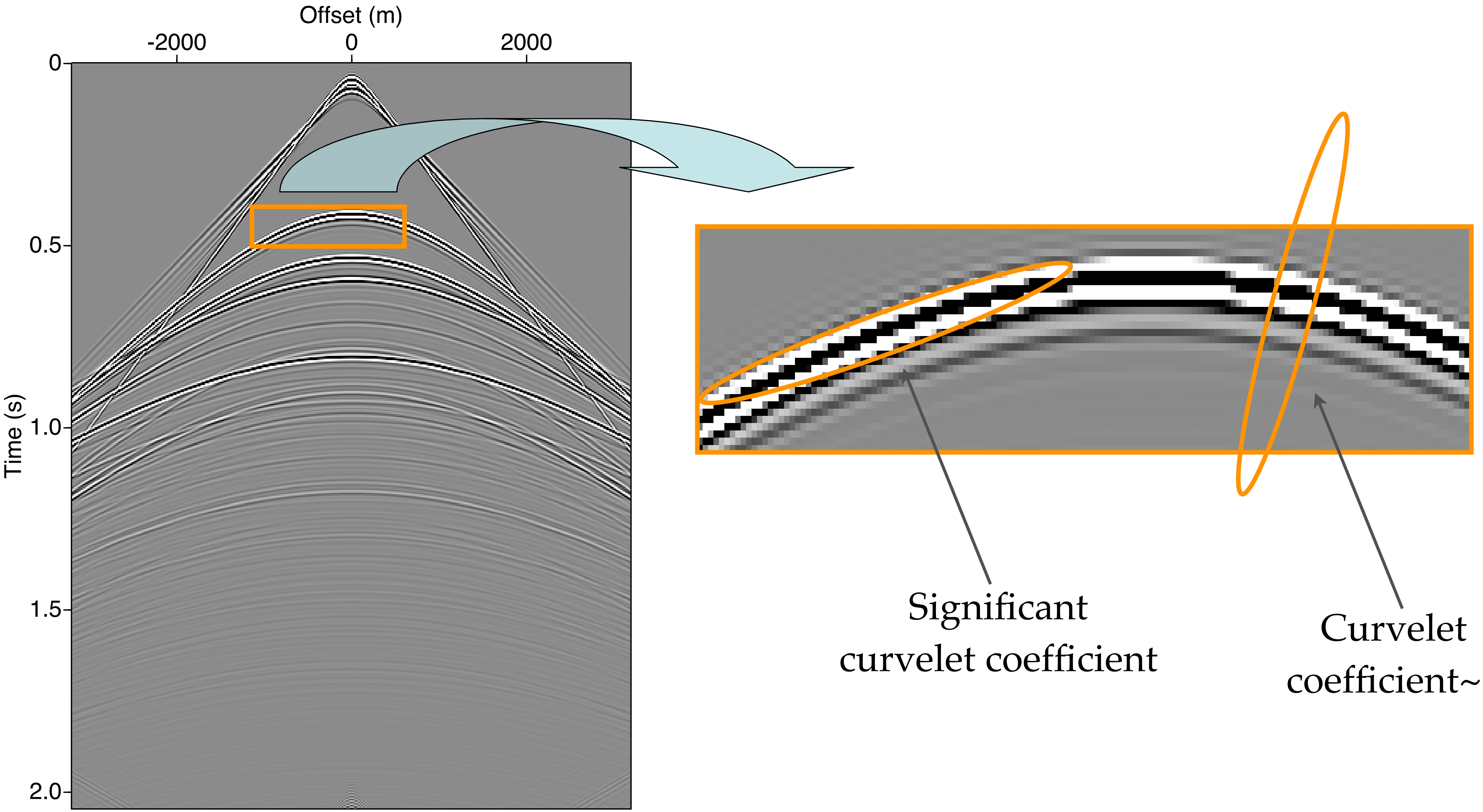
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

- curvelet transform
  - **multiscale**: tiling of the FK domain into dyadic coronae
  - **multidirectional**: coronae sub-partitioned into angular wedges, # of angles doubles every other scale
  - **anisotropic**: parabolic scaling principle
  - **local**



2D discrete curvelets

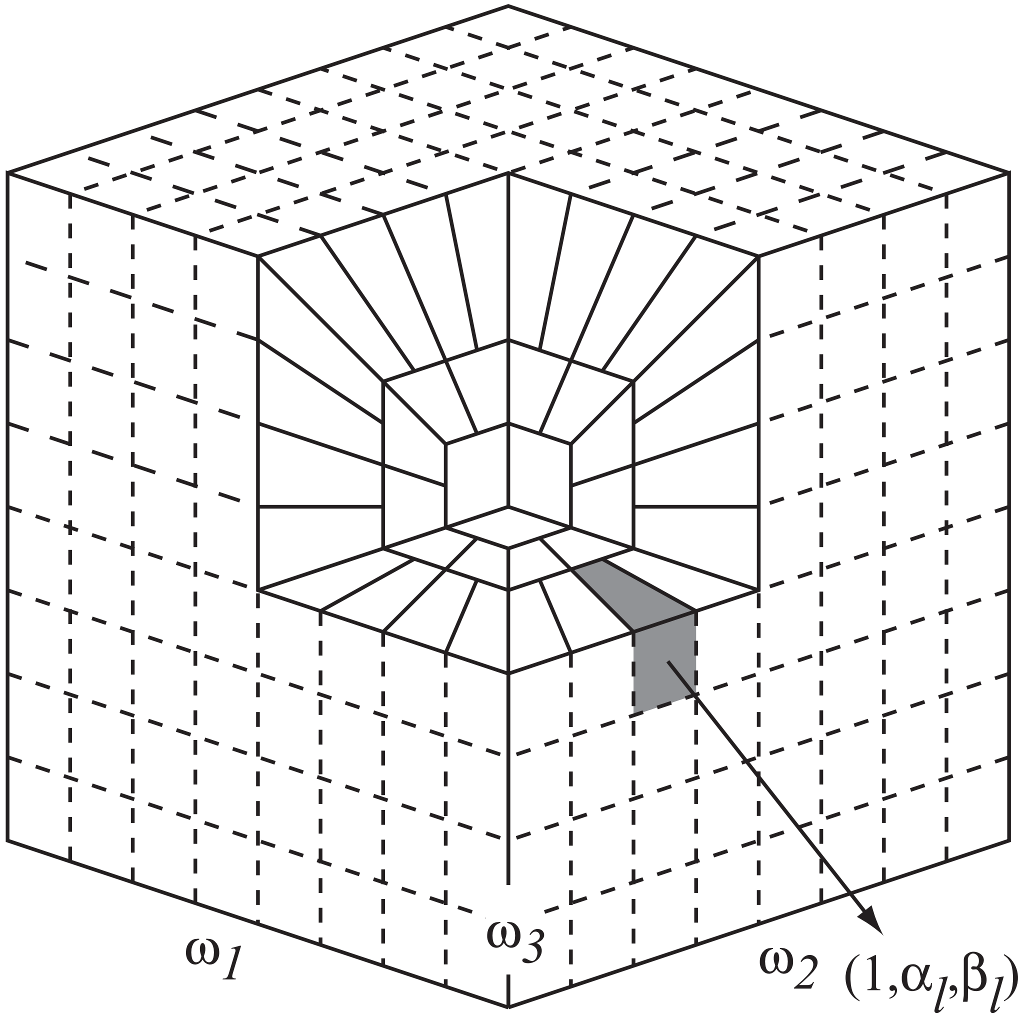


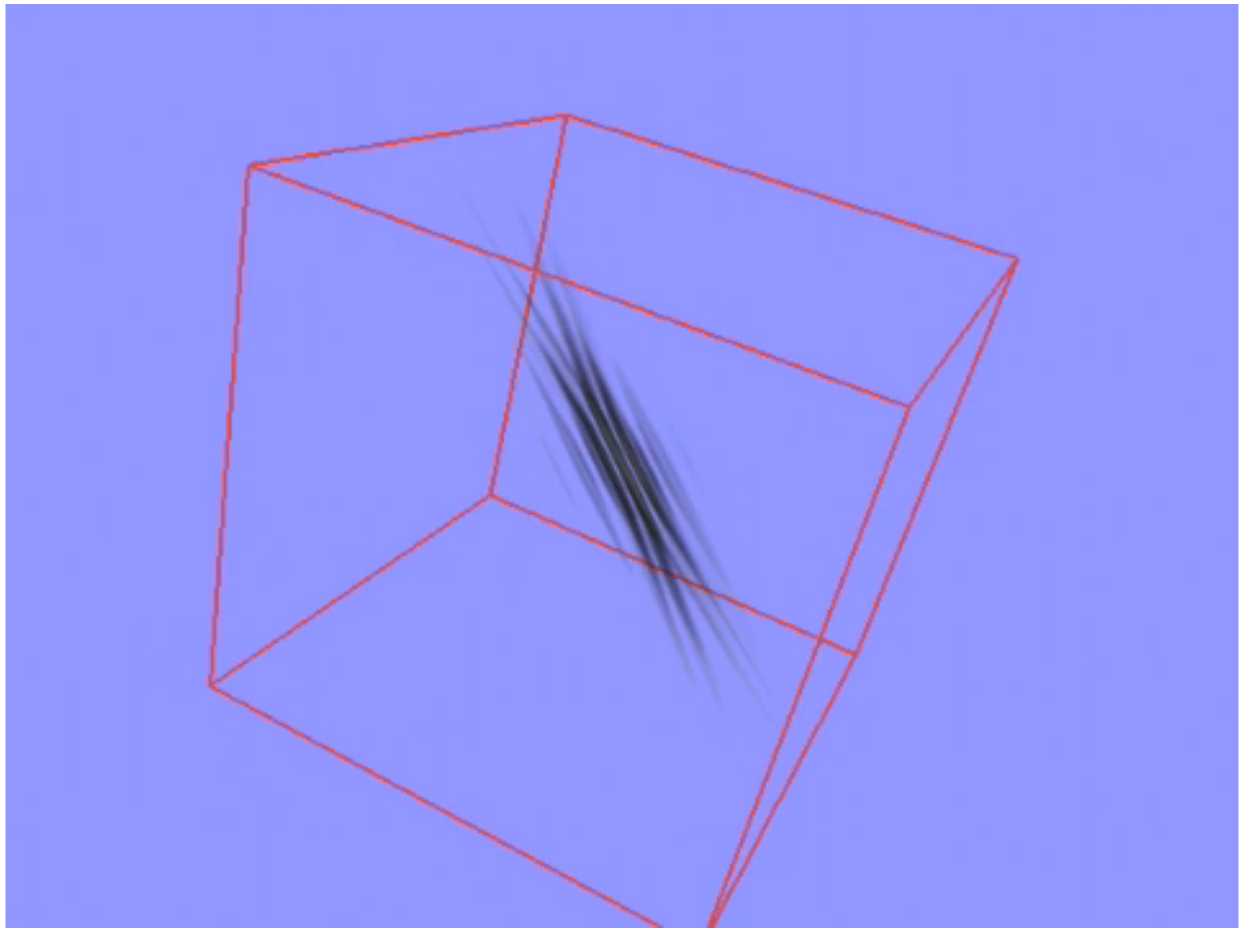


Significant curvelet coefficient

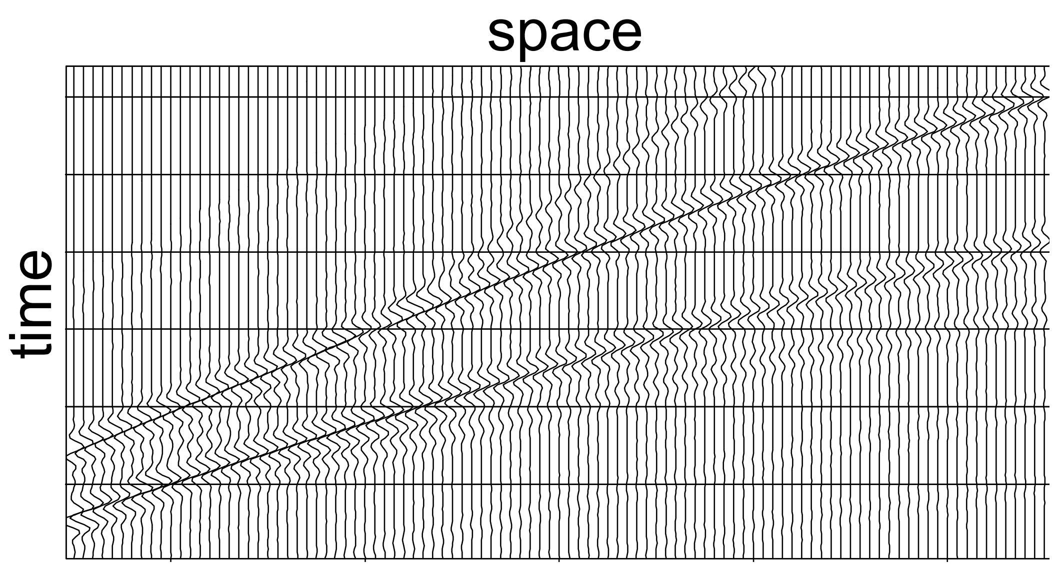
Curvelet coefficient~0

3D discrete curvelets



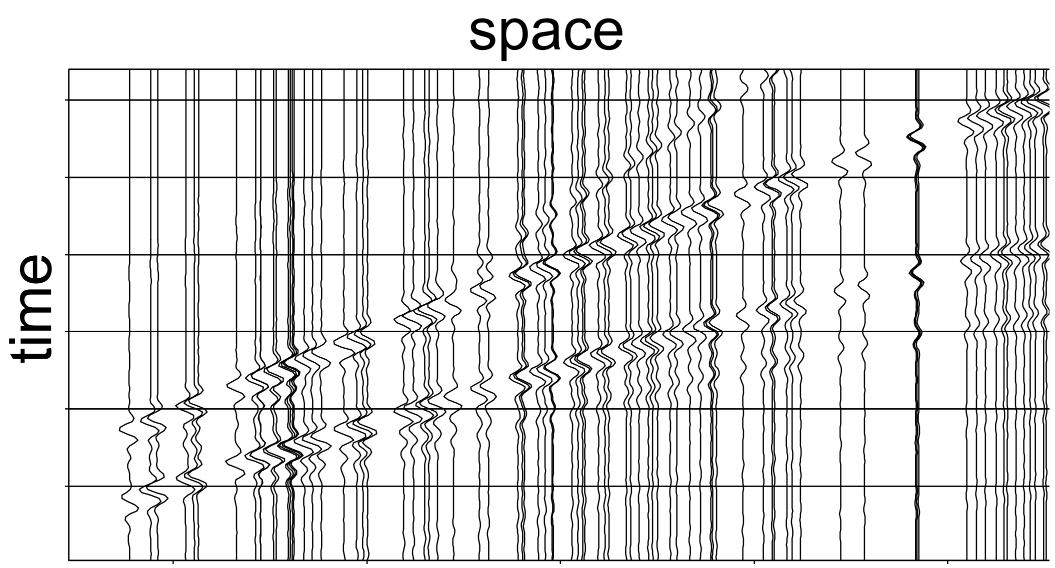


2D nonequispaced fast discrete curvelets



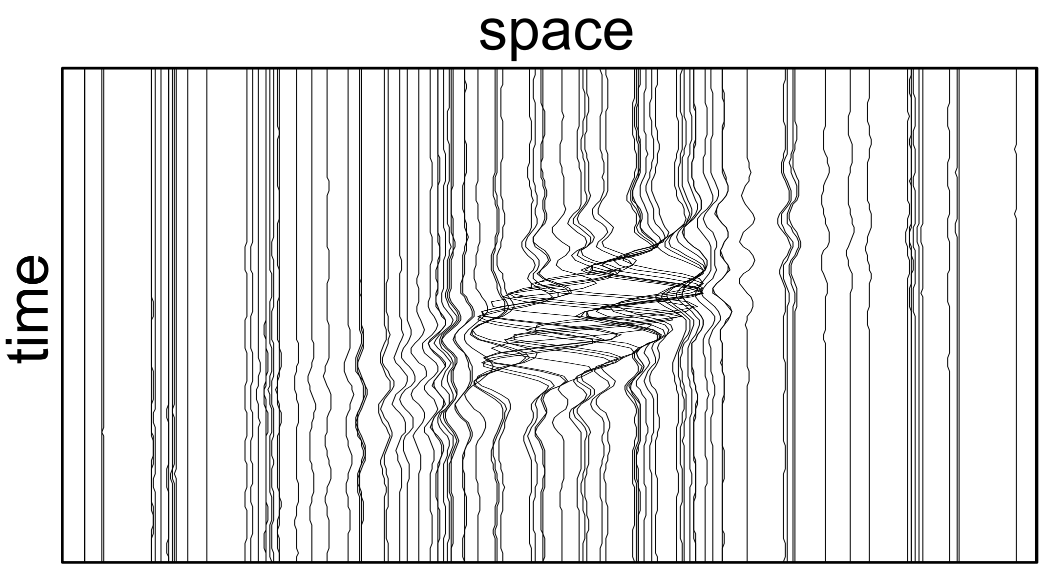
processed → using

fast discrete curvelet transform



data with acquisition irregularities

processed → using



"seismic" curvelet

[Hennenfent and Herrmann'06]

Key elements

☑ *sparsifying transform*

- typically **localized** in the time-space domain to handle the complexity of seismic data

☐ *advantageous coarse sampling*

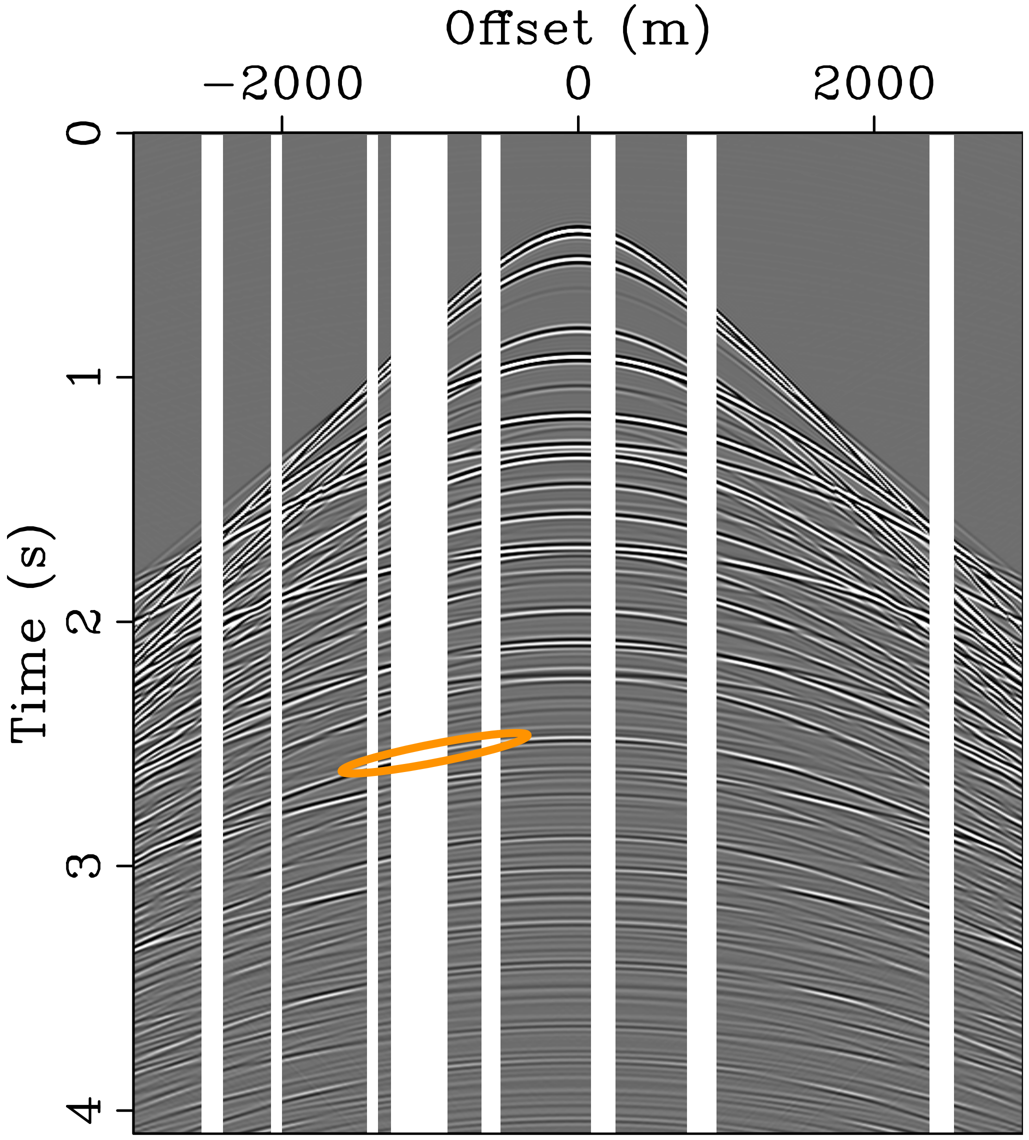
- generates incoherent random undersampling “noise” in the sparsifying domain
- does not create large gaps
  - because of the limited spatiotemporal extent of transform elements used for the reconstruction

☐ *sparsity-promoting solver*

- requires few matrix-vector multiplications

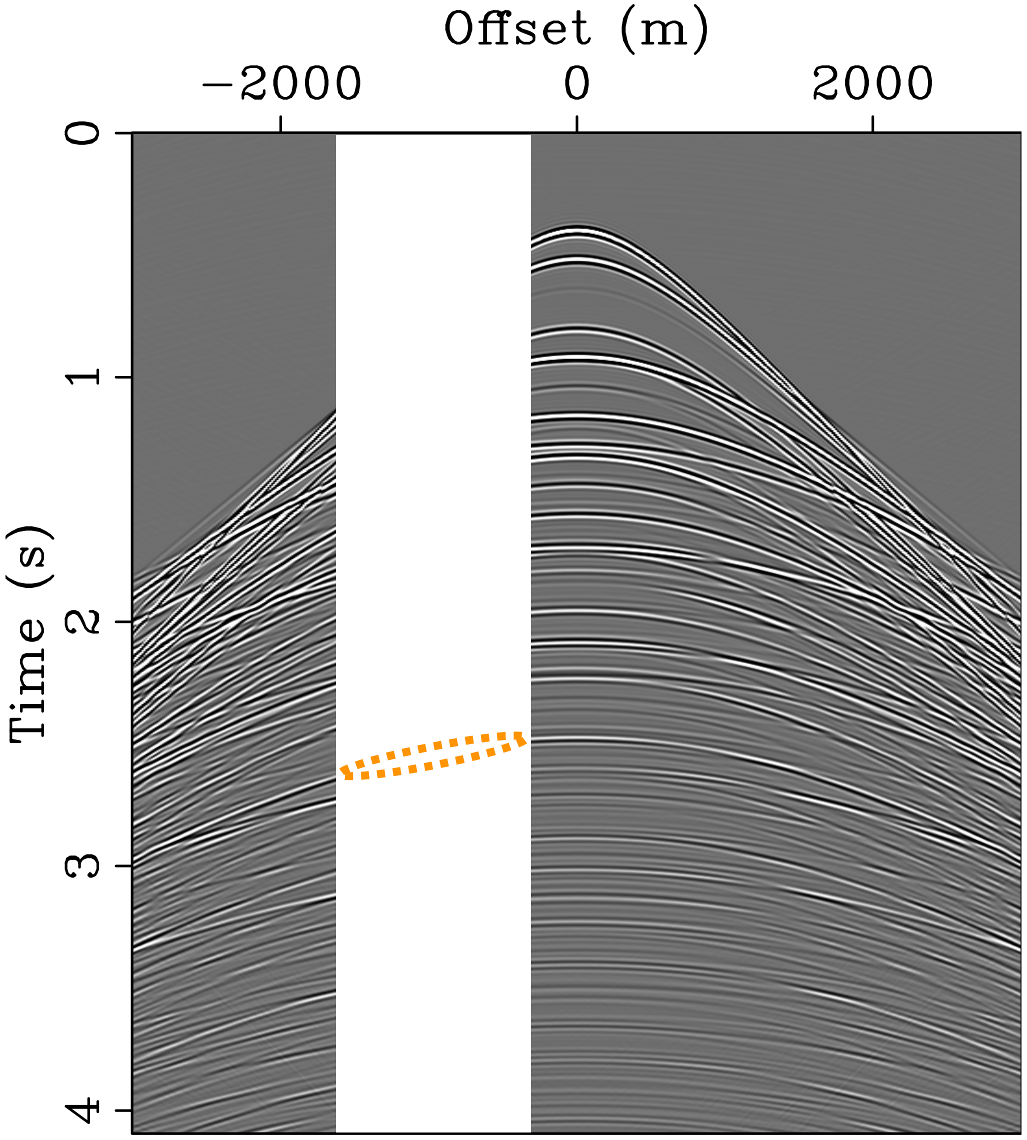
Localized transform elements & gap size

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$



Data

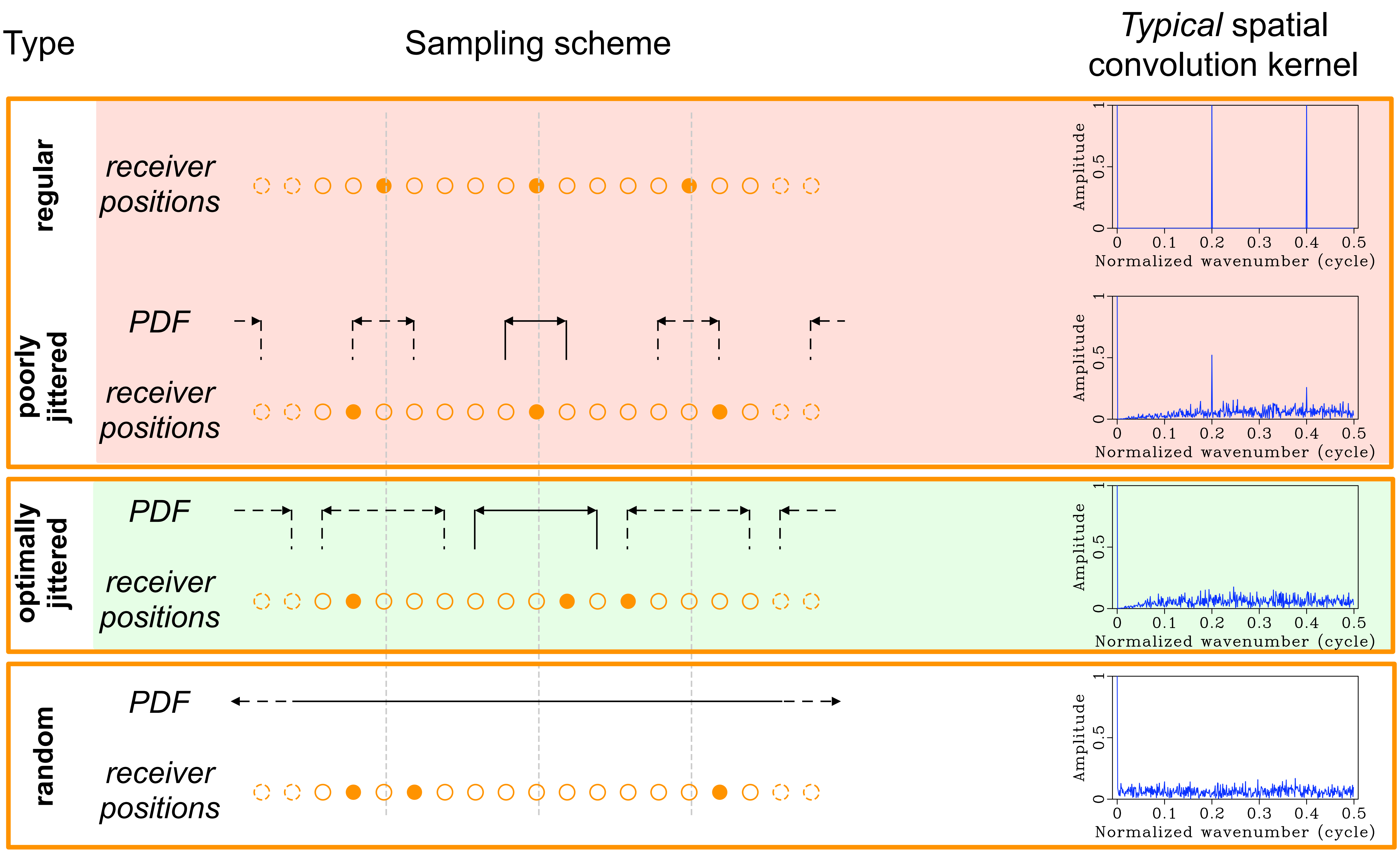
✓



Data

✗

Discrete random jittered undersampling



Type

Sampling scheme

Typical spatial convolution kernel

regular

receiver positions

poorly jittered

PDF

receiver positions

optimally jittered

PDF

receiver positions

random

PDF

receiver positions

[Hennenfent and Herrmann‘08]

Key elements

☑ *sparsifying transform*

- typically **localized** in the time-space domain to handle the complexity of seismic data

☑ *advantageous coarse sampling*

- generates incoherent random undersampling “noise” in the sparsifying domain
- does not create large gaps
    - because of the limited spatiotemporal extent of transform elements used for the reconstruction

☐ *sparsity-promoting solver*

- requires few matrix-vector multiplications

Approaches

- quadratic programming [many references!]

$$\mathrm{QP}_\lambda : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1$$

- basis pursuit denoise [Chen et al.'95]

$$\mathrm{BP}_\sigma : \quad \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2 \leq \sigma$$

- LASSO [Tibshirani'96]

$$\mathrm{LS}_\tau : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \tau$$

Approaches

- quadratic programming [many references!]

$$\mathrm{QP}_\lambda : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 + \boxed{\lambda}\|\mathbf{x}\|_1$$

- basis pursuit denoise [Chen et al.'95]

$$\mathrm{BP}_\sigma : \quad \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2 \leq \sigma$$

- LASSO [Tibshirani'96]

$$\mathrm{LS}_\tau : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \tau$$

Approaches

- quadratic programming [many references!]

$$\mathrm{QP}_\lambda : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \boxed{\lambda}\|\mathbf{x}\|_1$$

- basis pursuit denoise [Chen et al.'95]

$$\mathrm{BP}_\sigma : \quad \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \boxed{\sigma}$$

- LASSO [Tibshirani'96]

$$\mathrm{LS}_\tau : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \tau$$

Approaches

- quadratic programming [many references!]

$$\mathrm{QP}_\lambda : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 + \boxed{\lambda}\|\mathbf{x}\|_1$$

- basis pursuit denoise [Chen et al.'95]

$$\mathrm{BP}_\sigma : \quad \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2 \leq \boxed{\sigma}$$

- LASSO [Tibshirani'96]

$$\mathrm{LS}_\tau : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \boxed{\tau}$$

Approaches

- quadratic programming [many references!]

$$\mathrm{QP}_\lambda : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 + \boxed{\lambda}\|\mathbf{x}\|_1$$

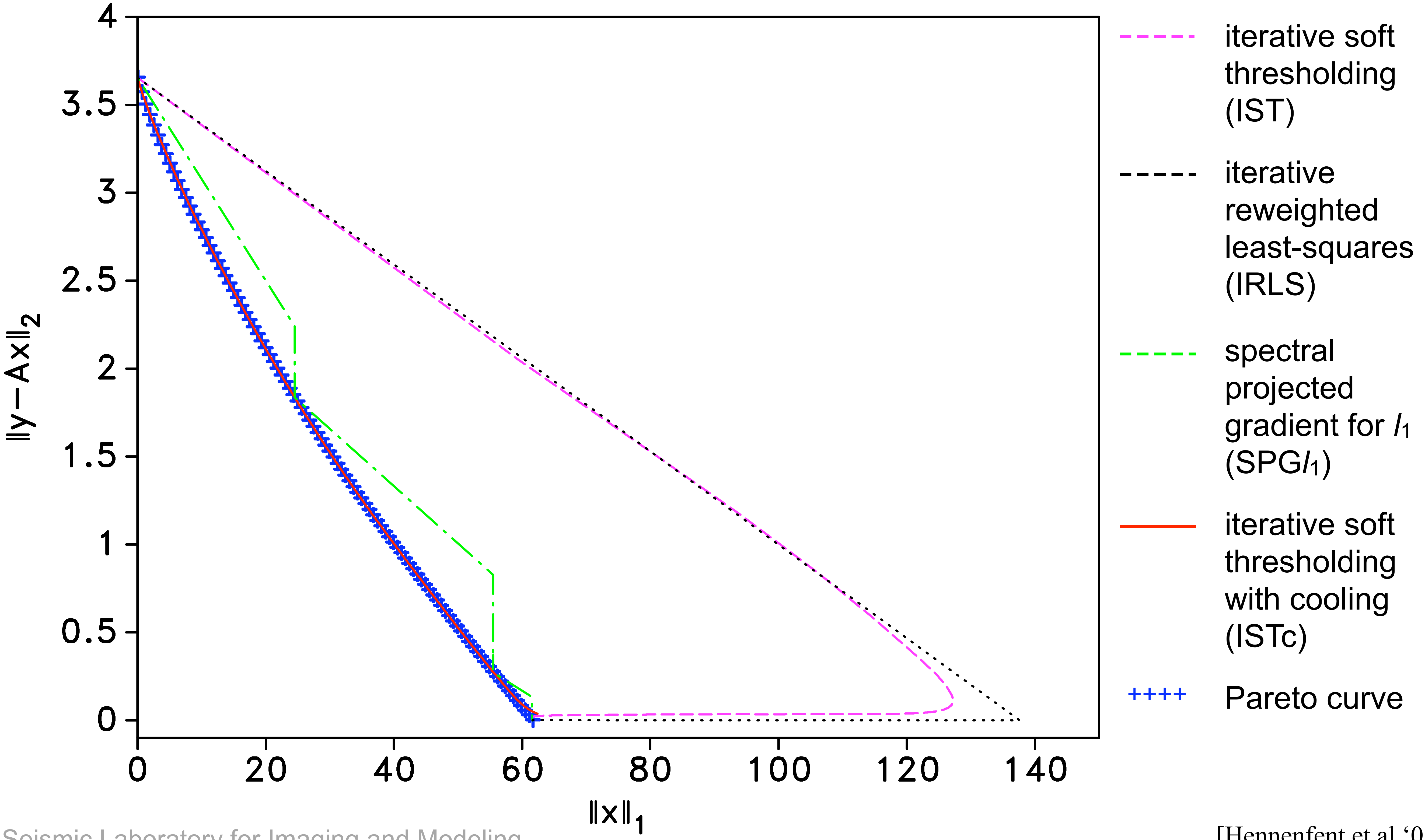
- basis pursuit denoise [Chen et al.'95]

$$\mathrm{BP}_\sigma : \quad \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2 \leq \boxed{\sigma}$$

- LASSO [Tibshirani'96]

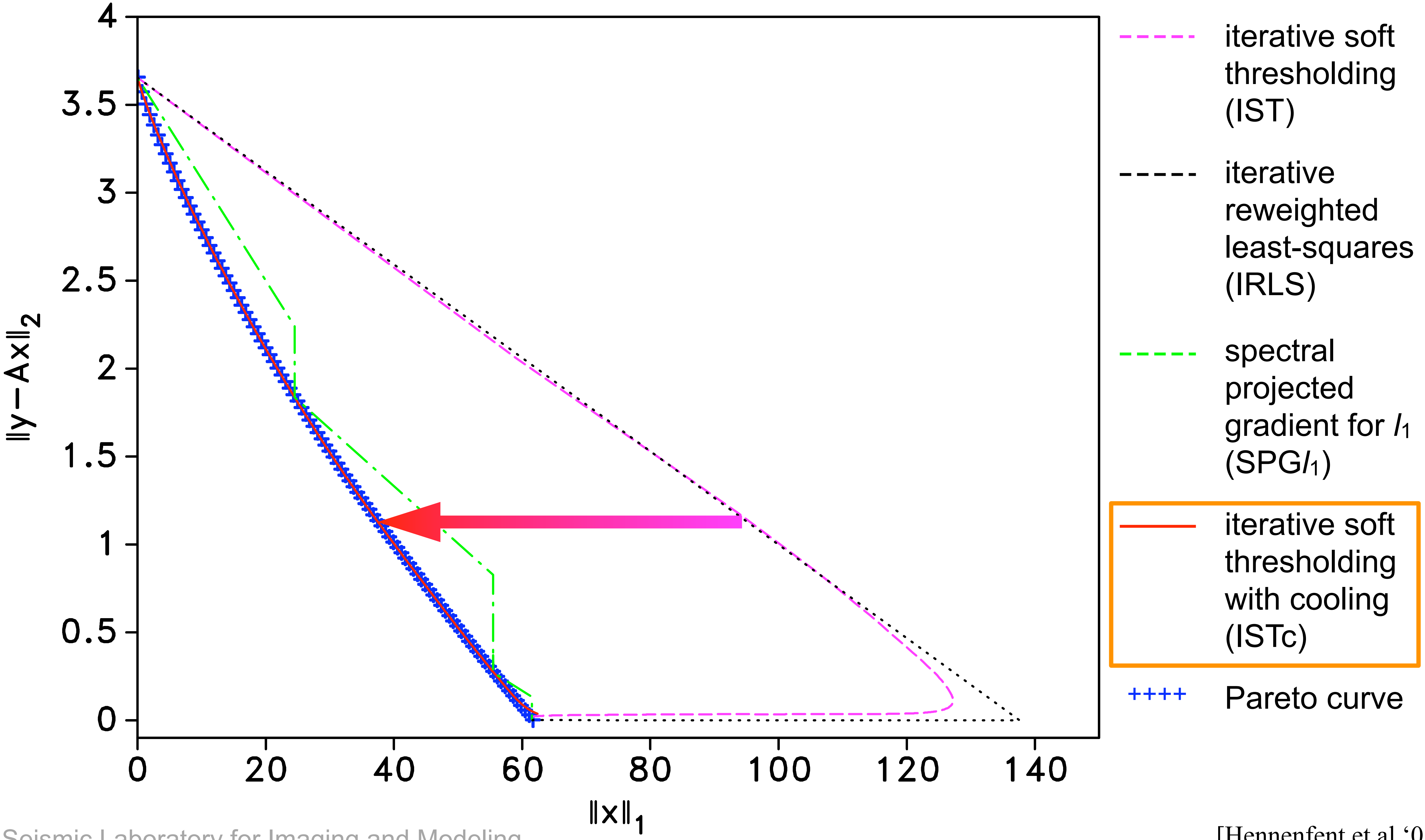
$$\mathrm{LS}_\tau : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \boxed{\tau}$$

One-norm solvers



[Hennenfent et al.'08]

One-norm solvers



[Hennenfent et al.‘08]

Key elements

☑ *sparsifying transform*

- typically **localized** in the time-space domain to handle the complexity of seismic data

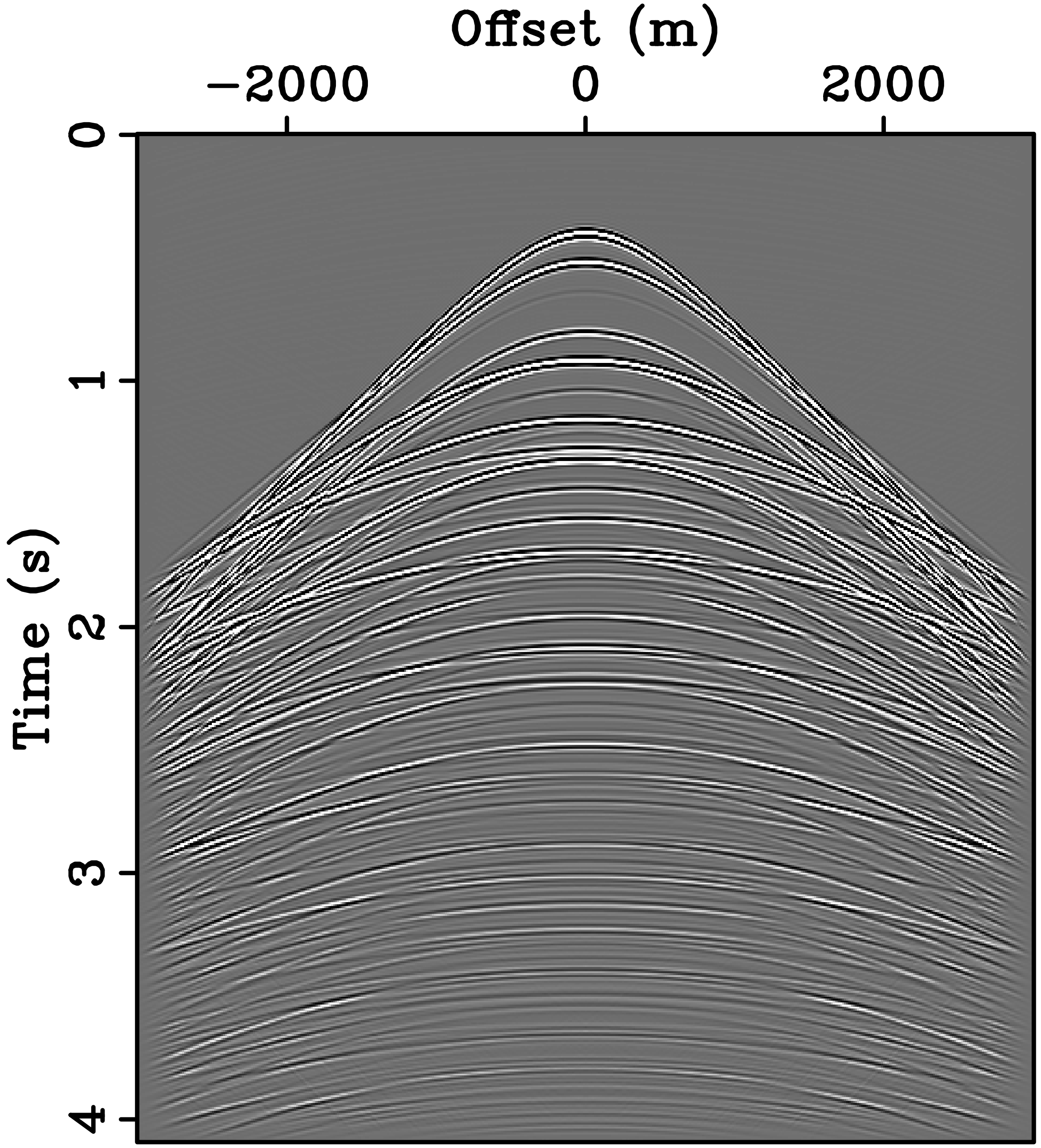
☑ *advantageous coarse sampling*

- generates incoherent random undersampling “noise” in the sparsifying domain
- does not create large gaps
  - because of the limited spatiotemporal extent of transform elements used for the reconstruction

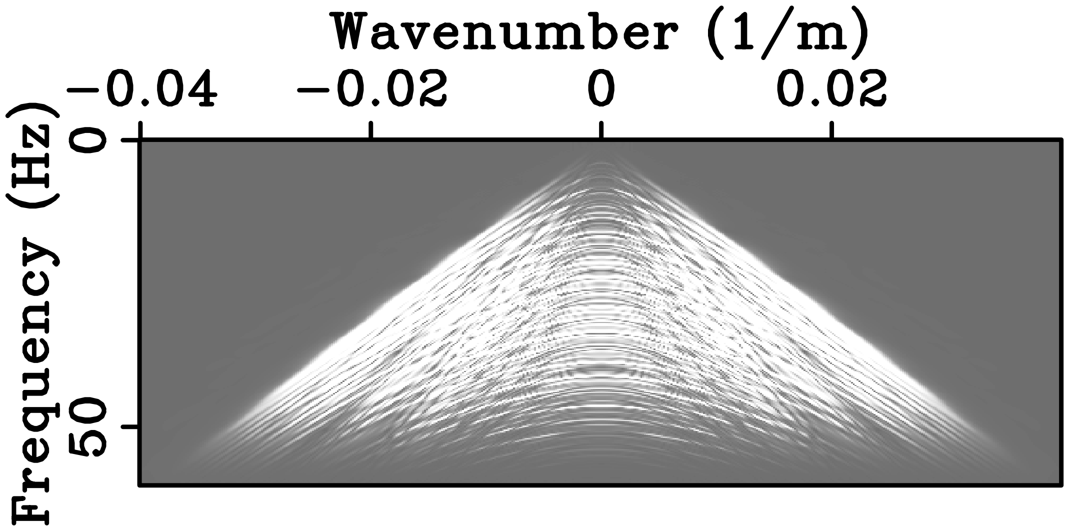
☑ *sparsity-promoting solver*

- requires few matrix-vector multiplications

Model

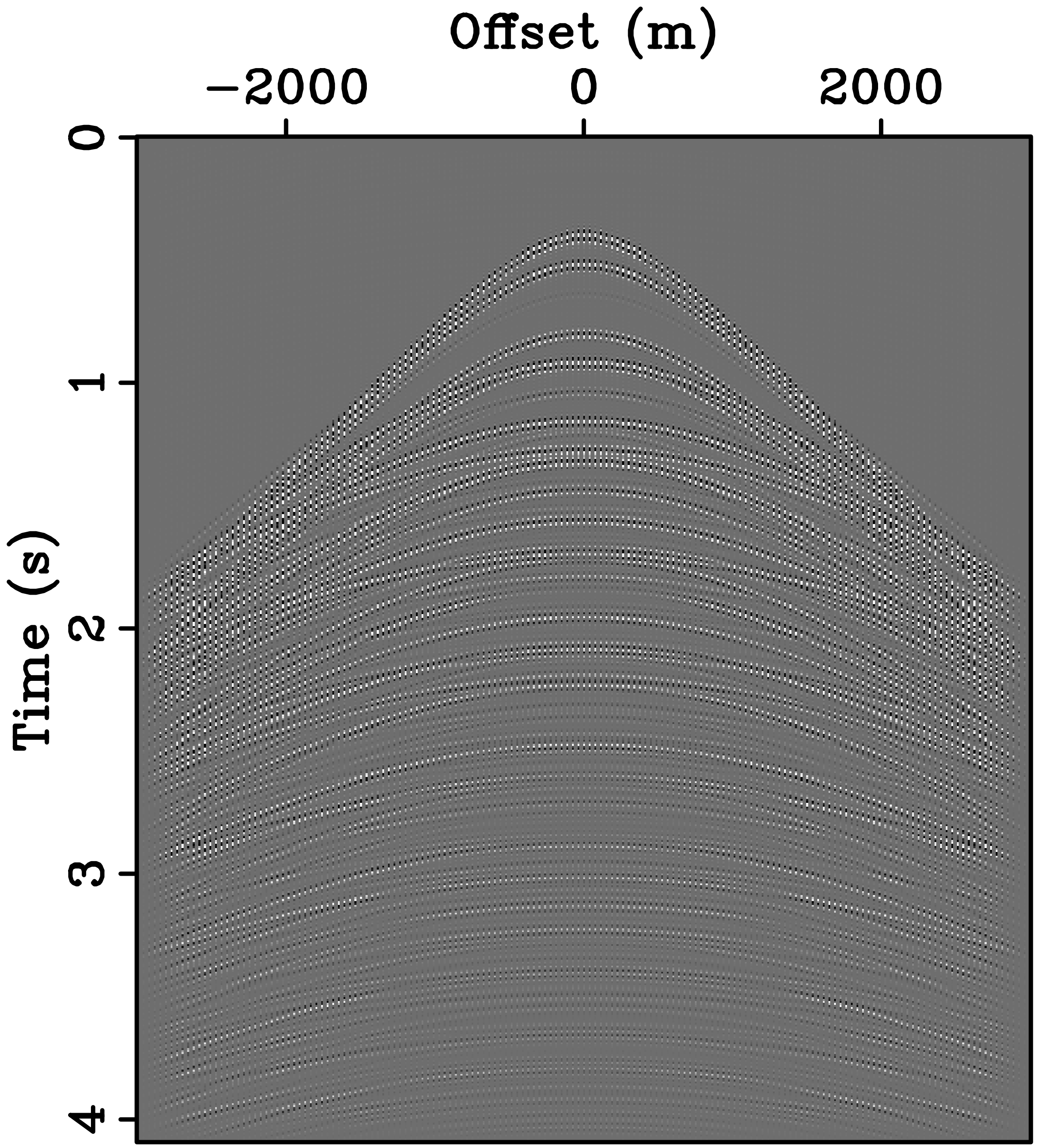


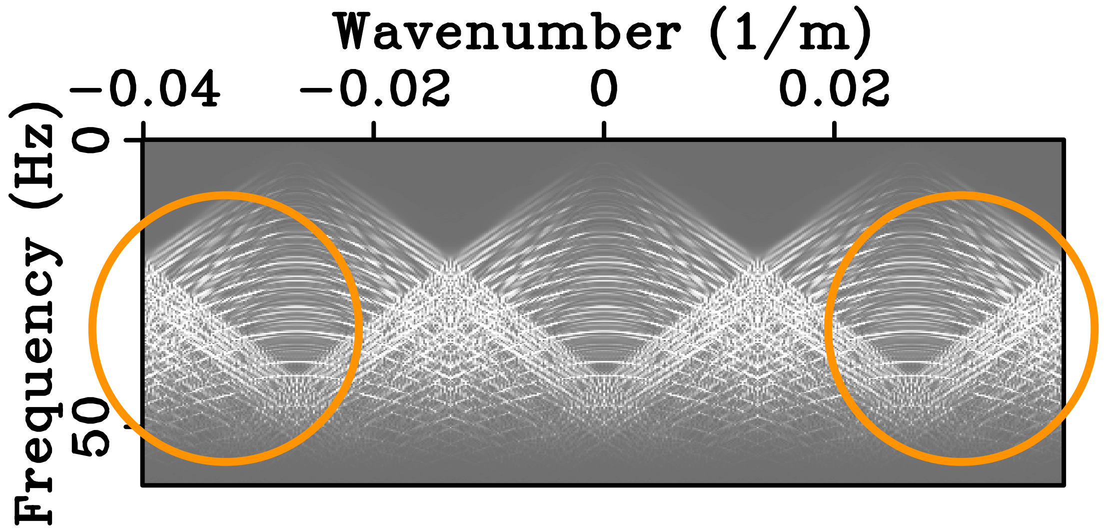
Offset (m)
−2000
0
2000
Time (s)
0
1
2
3
4



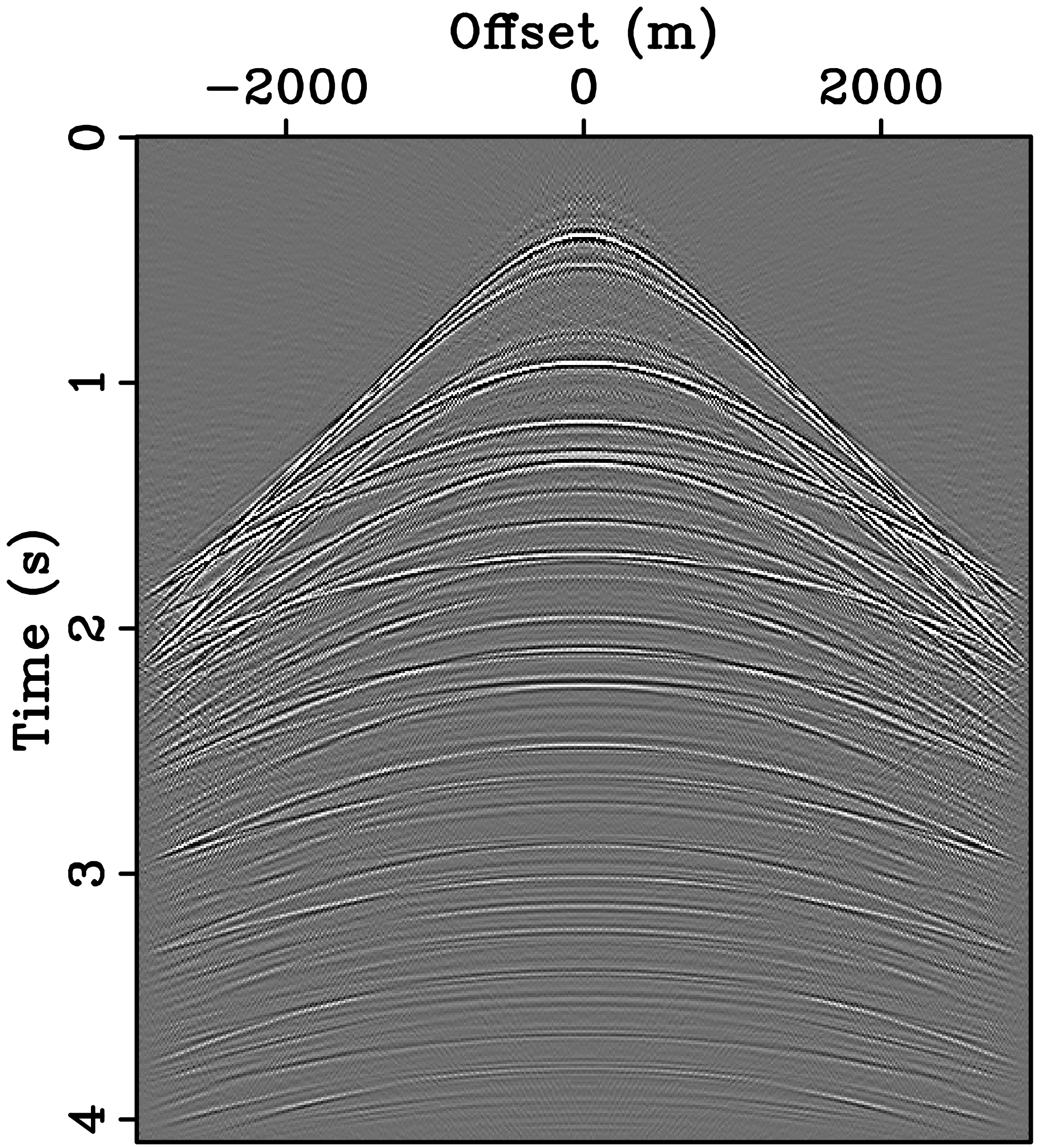
Wavenumber (1/m)
−0.04
−0.02
0
0.02
Frequency (Hz)
0
50

Regular 3-fold undersampling

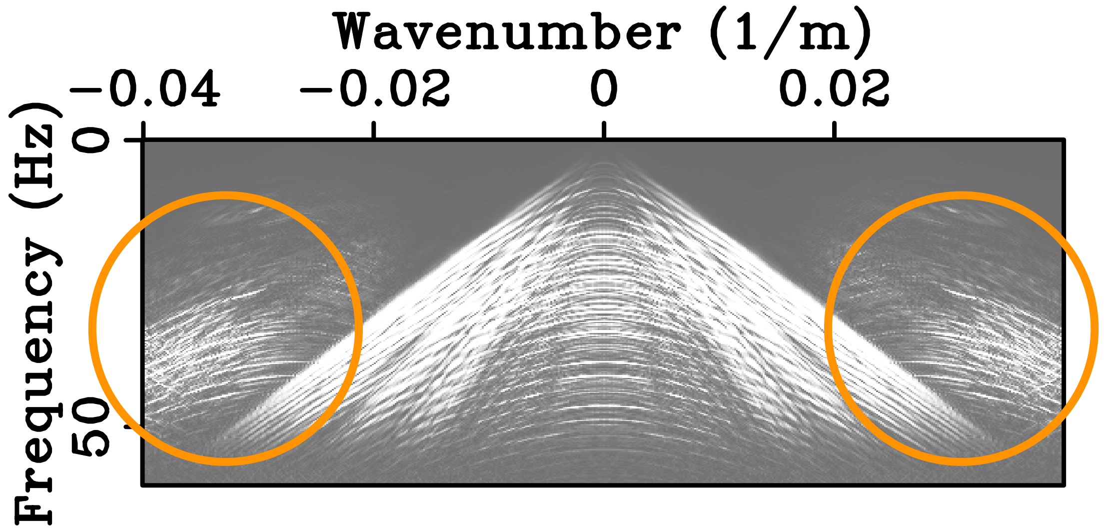




CRSI from regular 3-fold undersampling

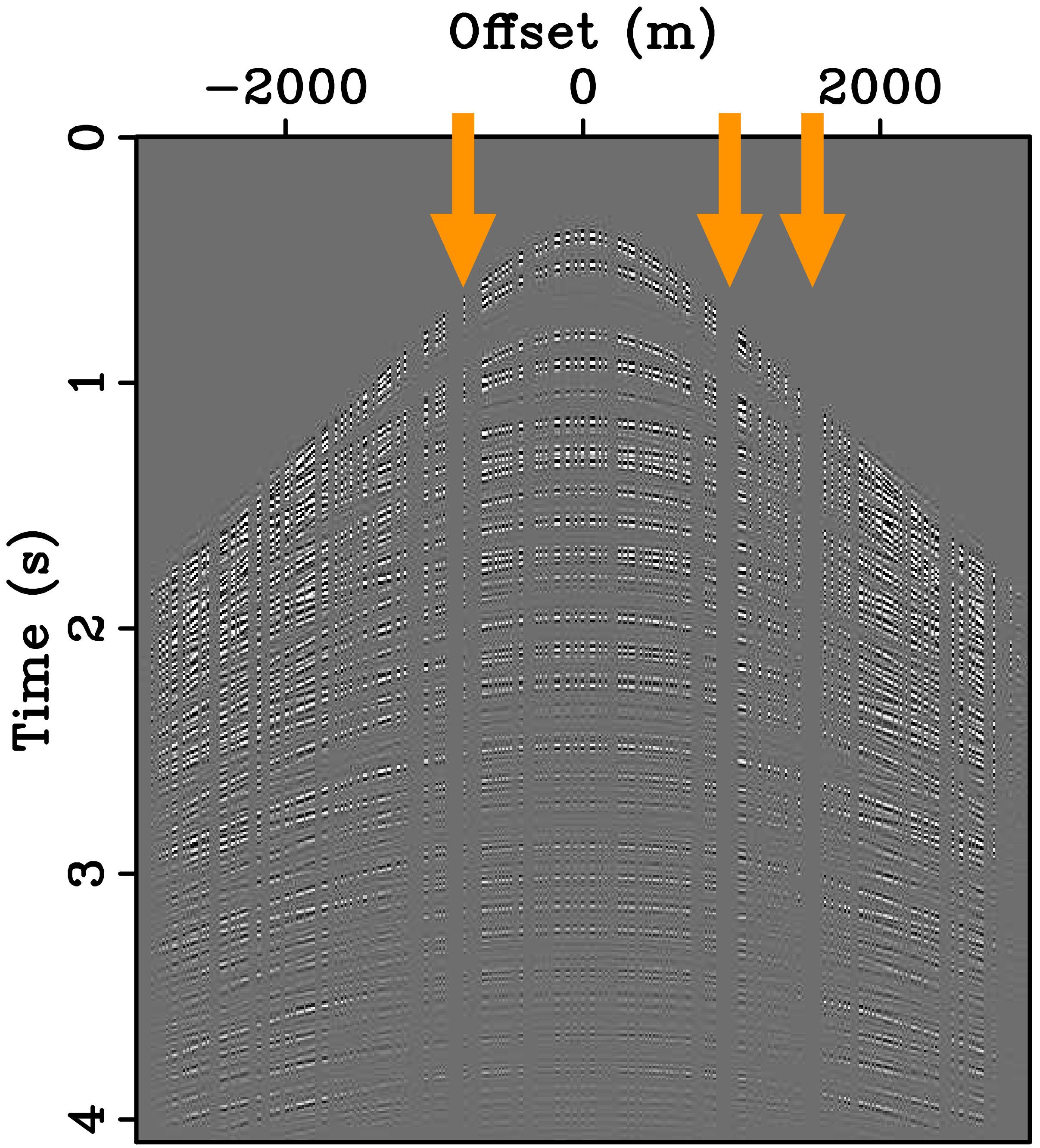


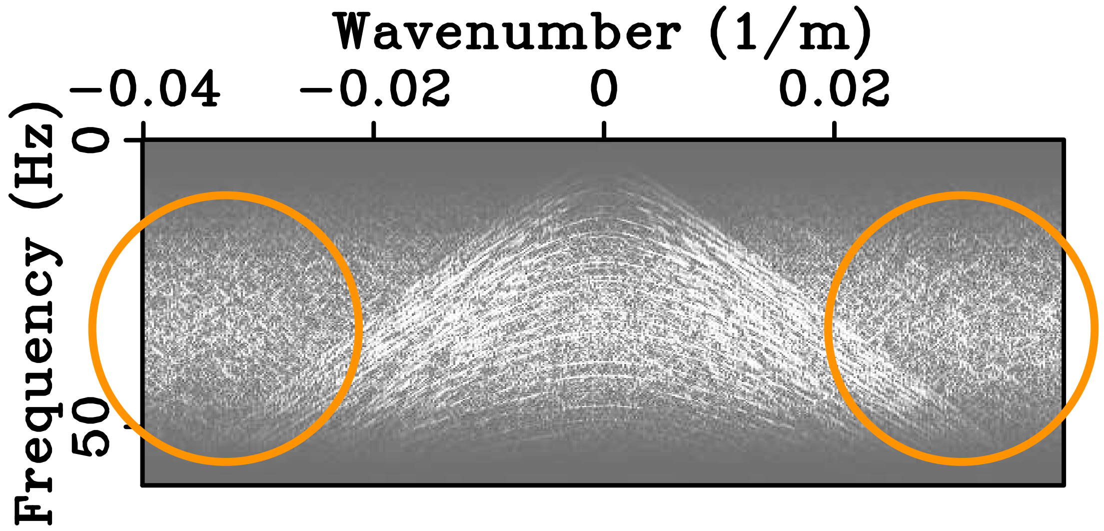
SNR = 6.92 dB



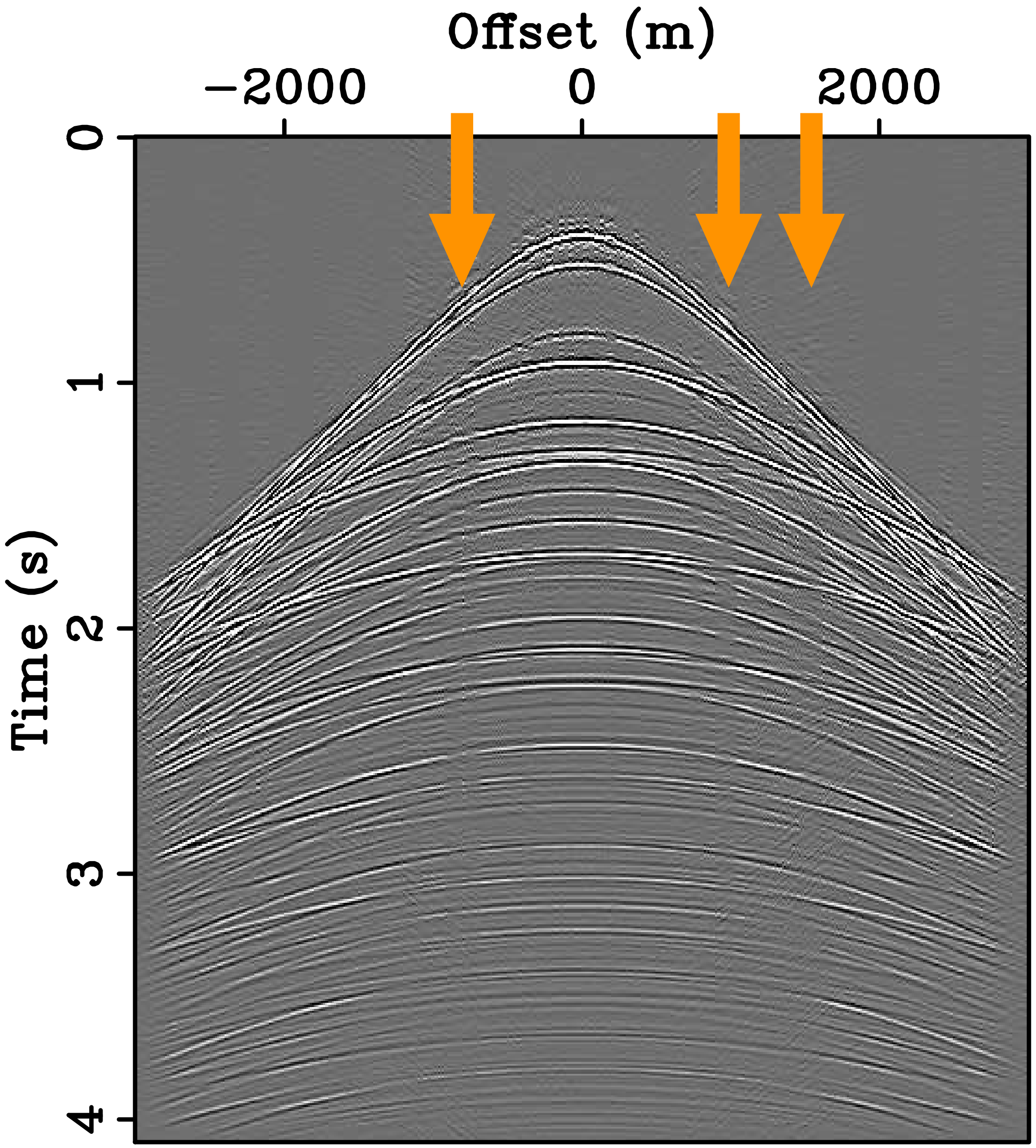
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

Random 3-fold undersampling

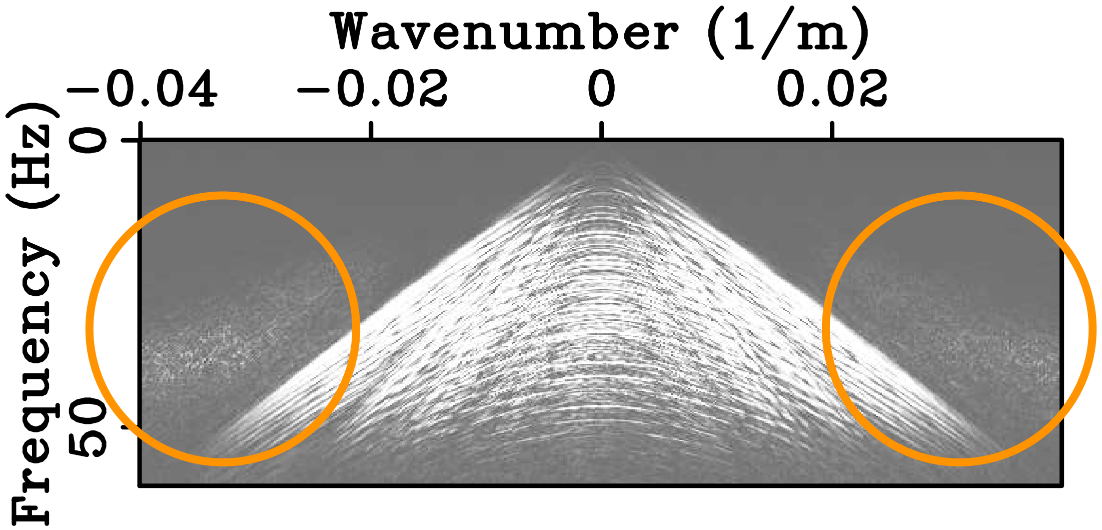




CRSI from random 3-fold undersampling

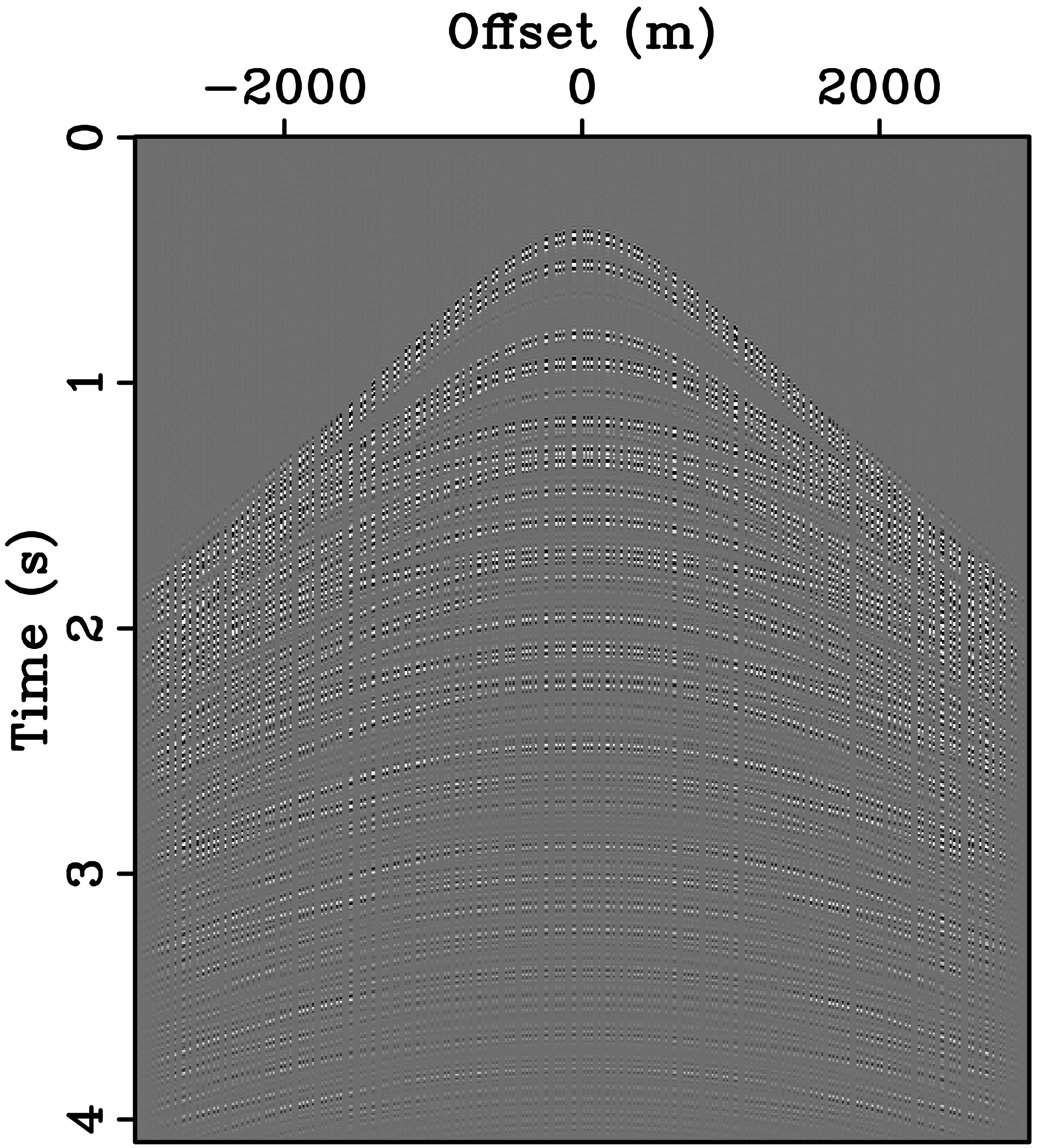


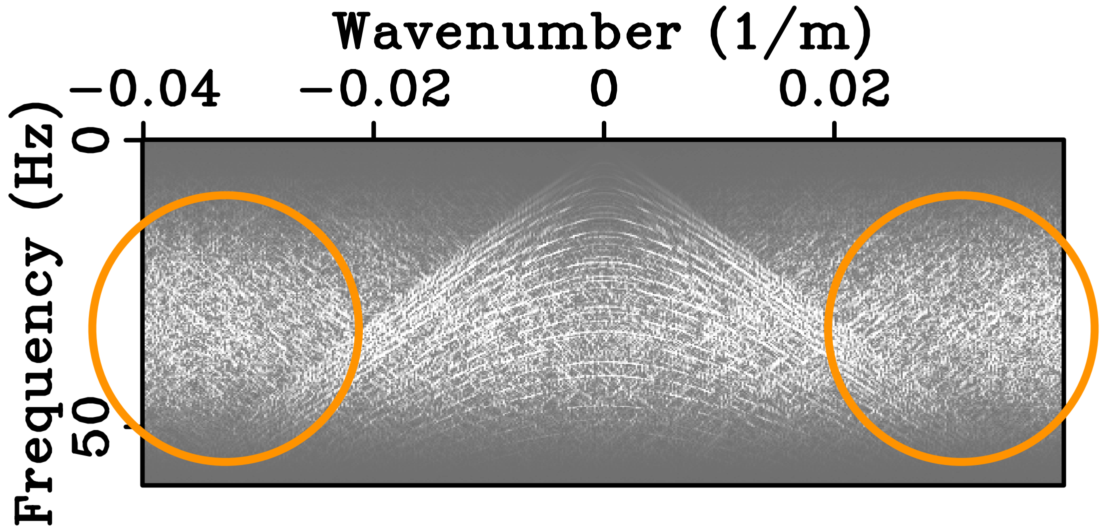
SNR = 9.72 dB



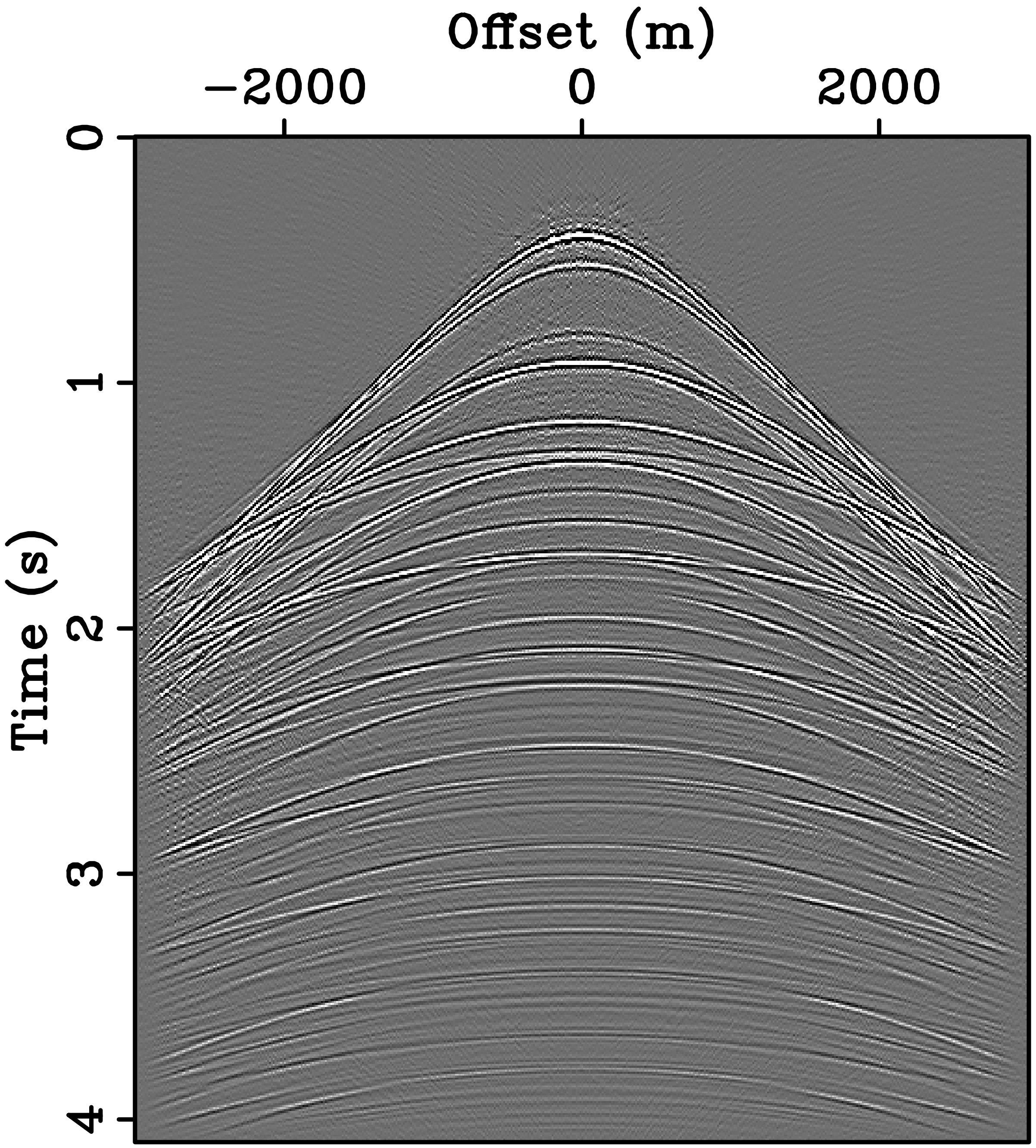
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

Optimally-jittered 3-fold undersampling

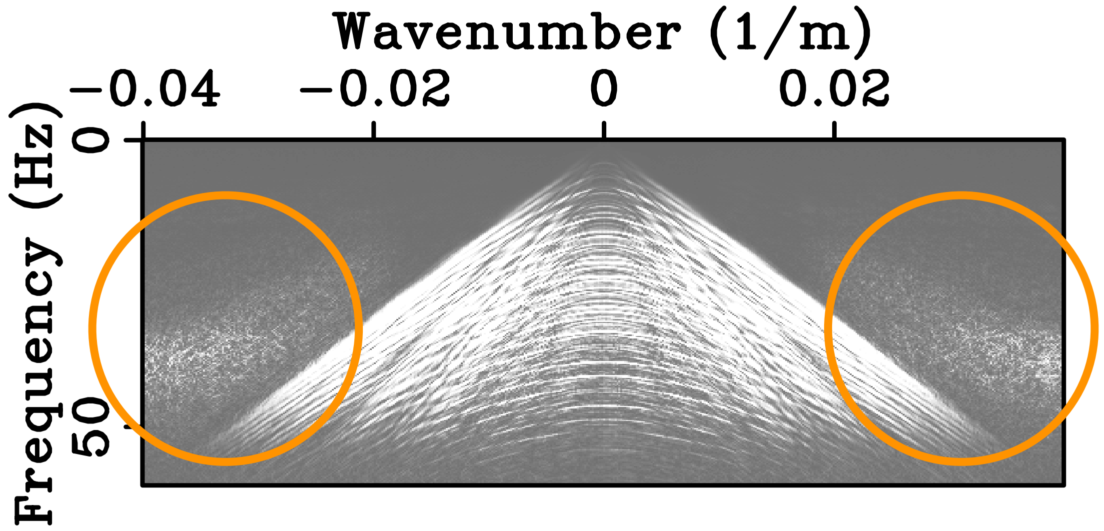


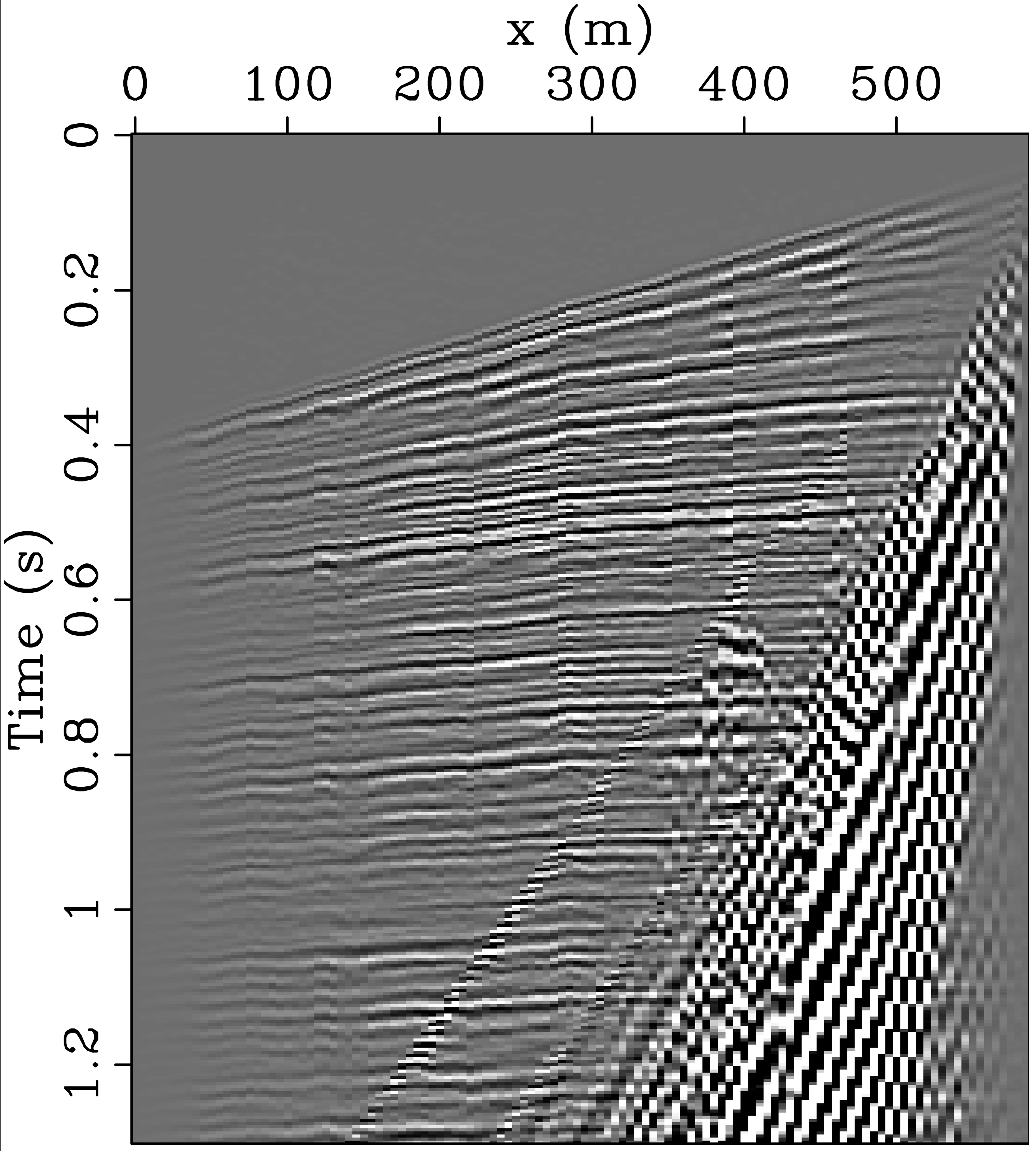


CRSI from opt.-jittered 3-fold undersampling

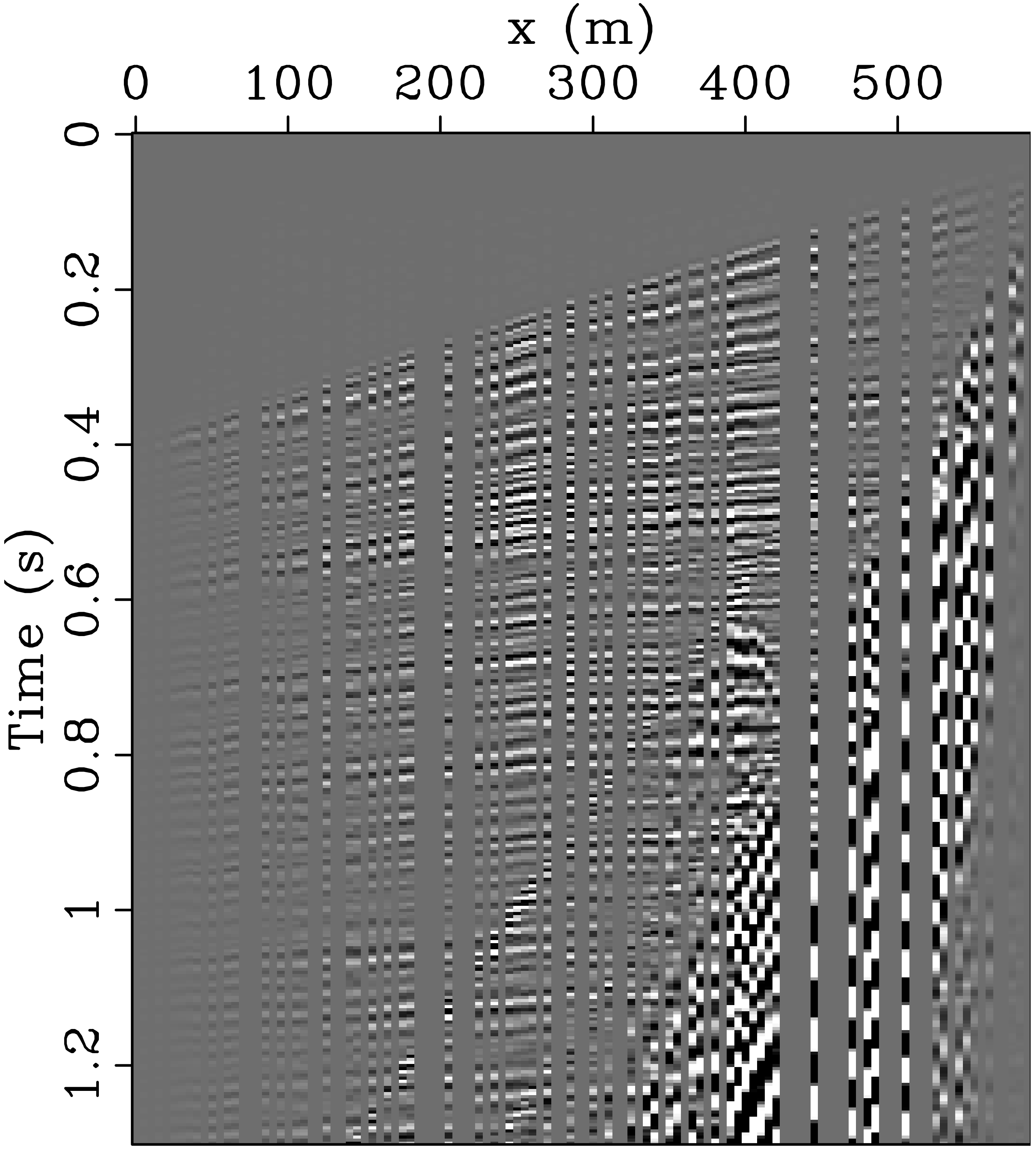


SNR = 10.42 dB



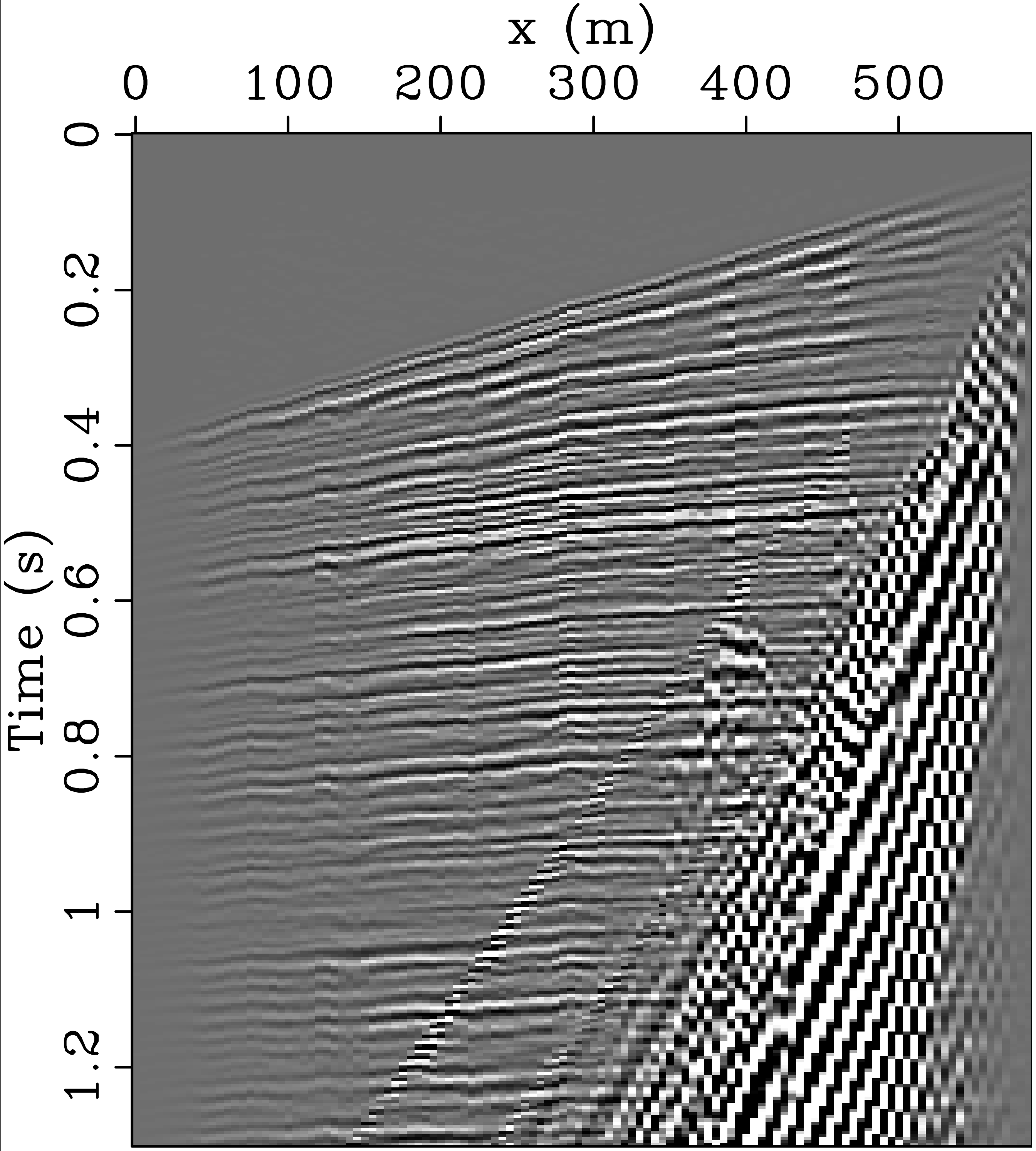


Model (5m)

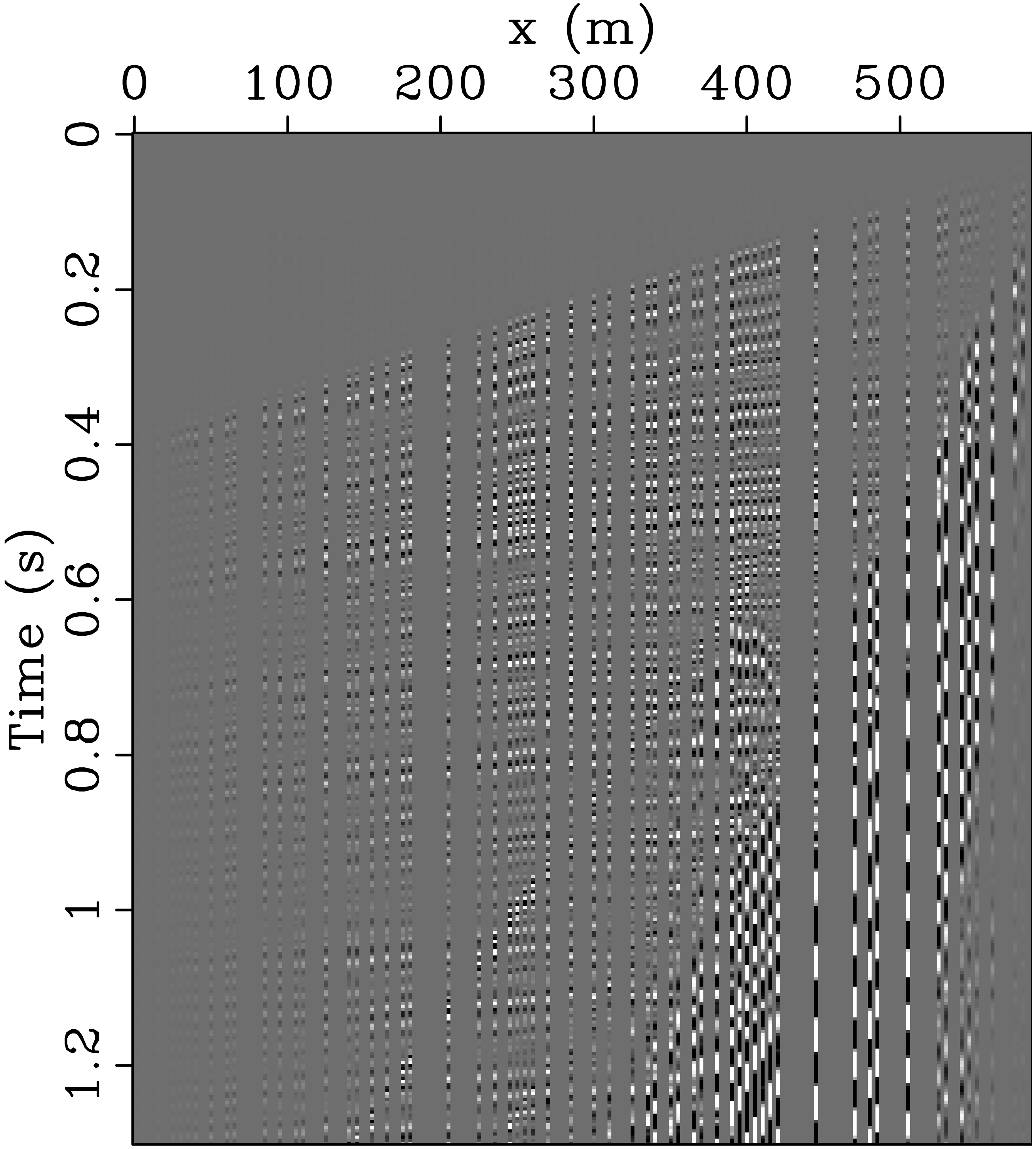


Data (5m)

avg. spatial sampling: 10 m

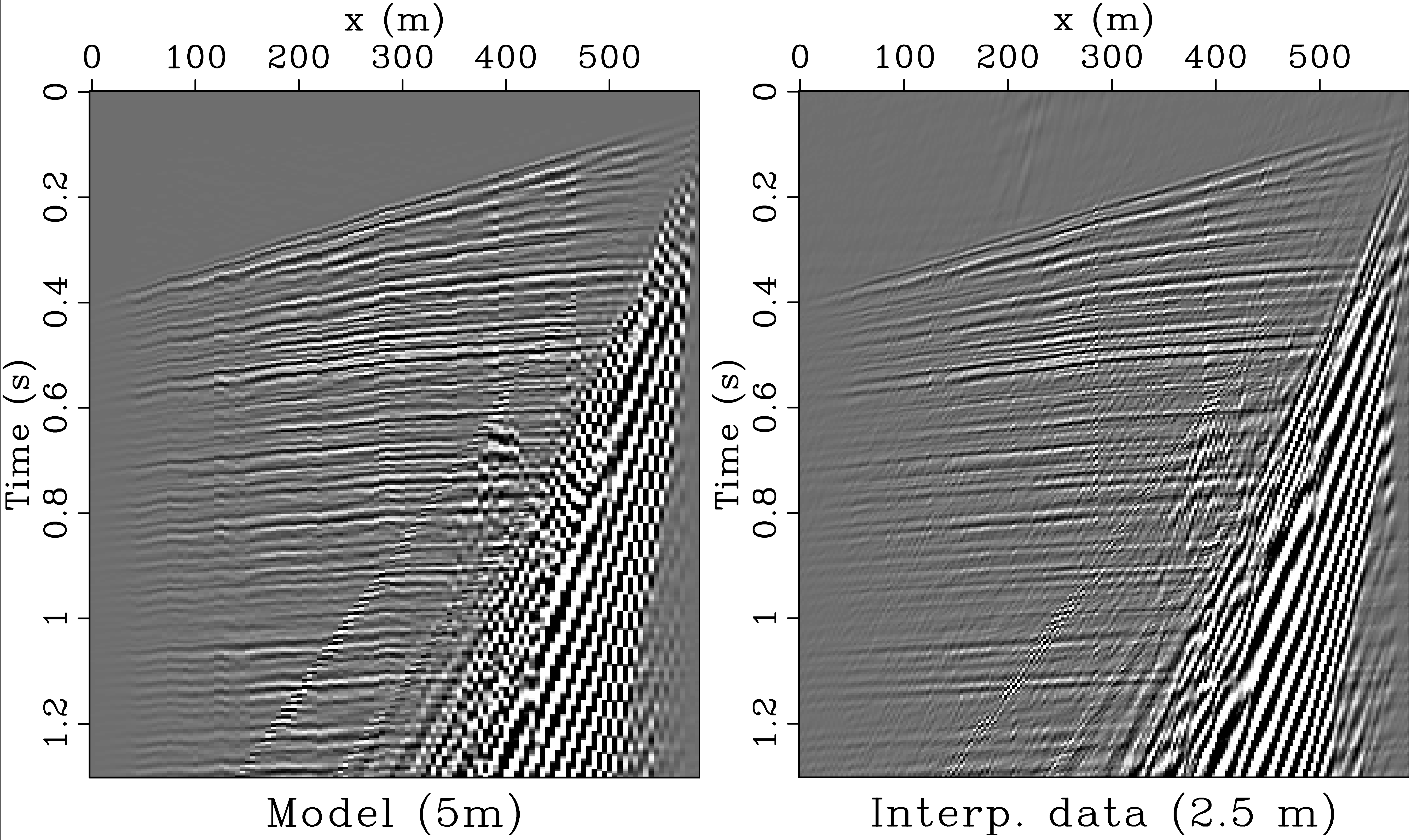


Model (5m)



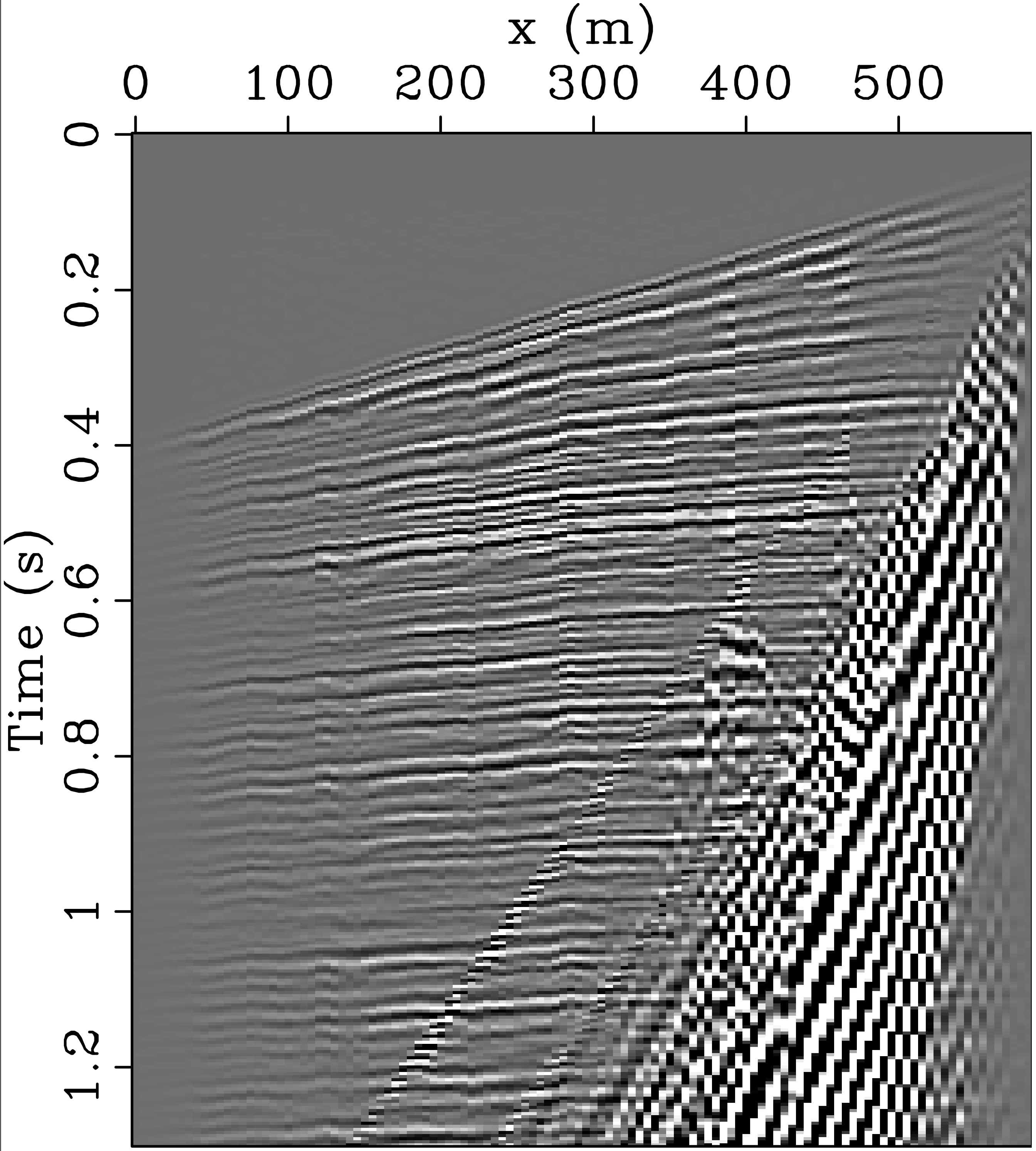
Data (2.5m)

avg. spatial sampling: 10 m

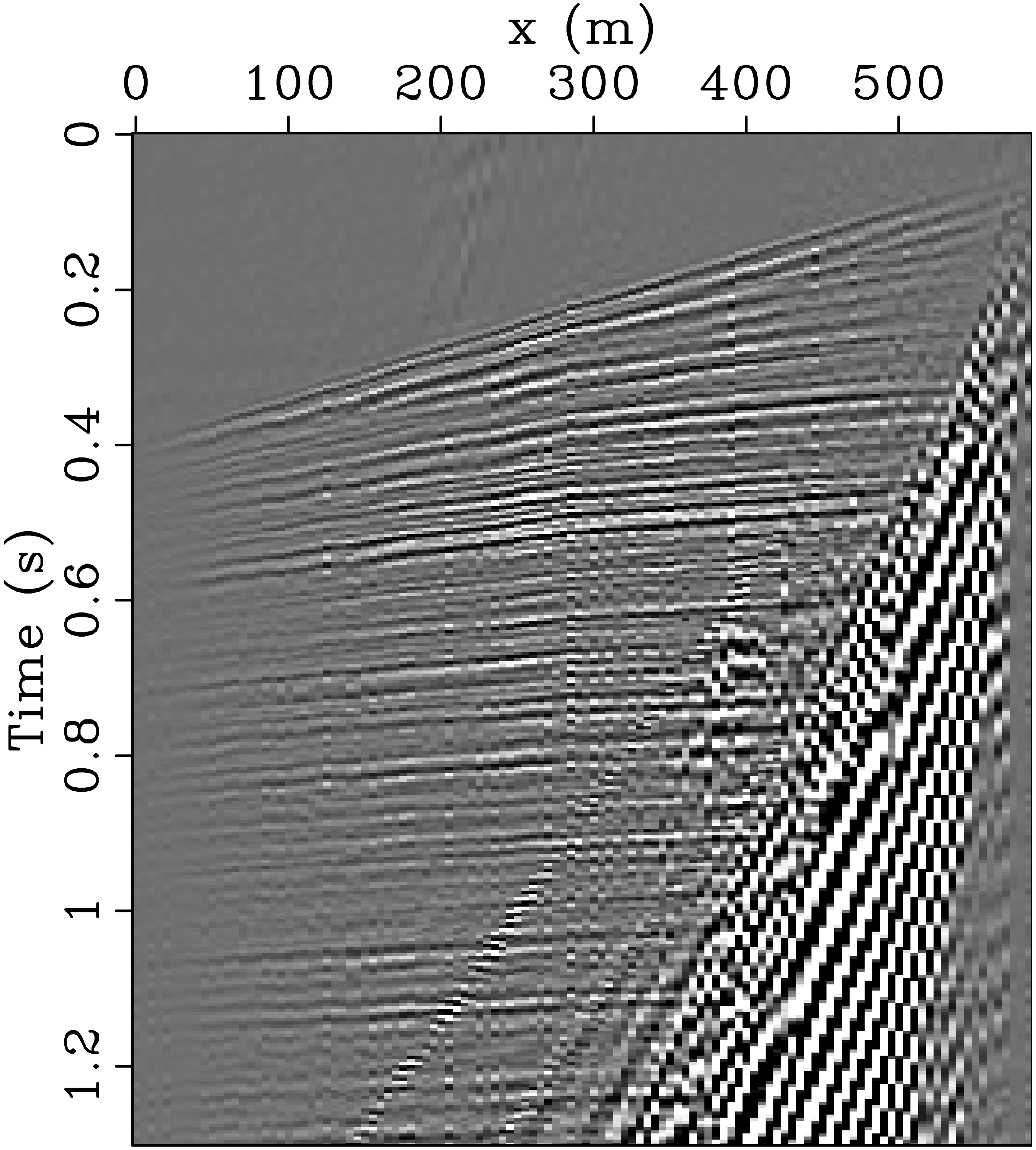


Model (5m)

Interp. data (2.5 m)

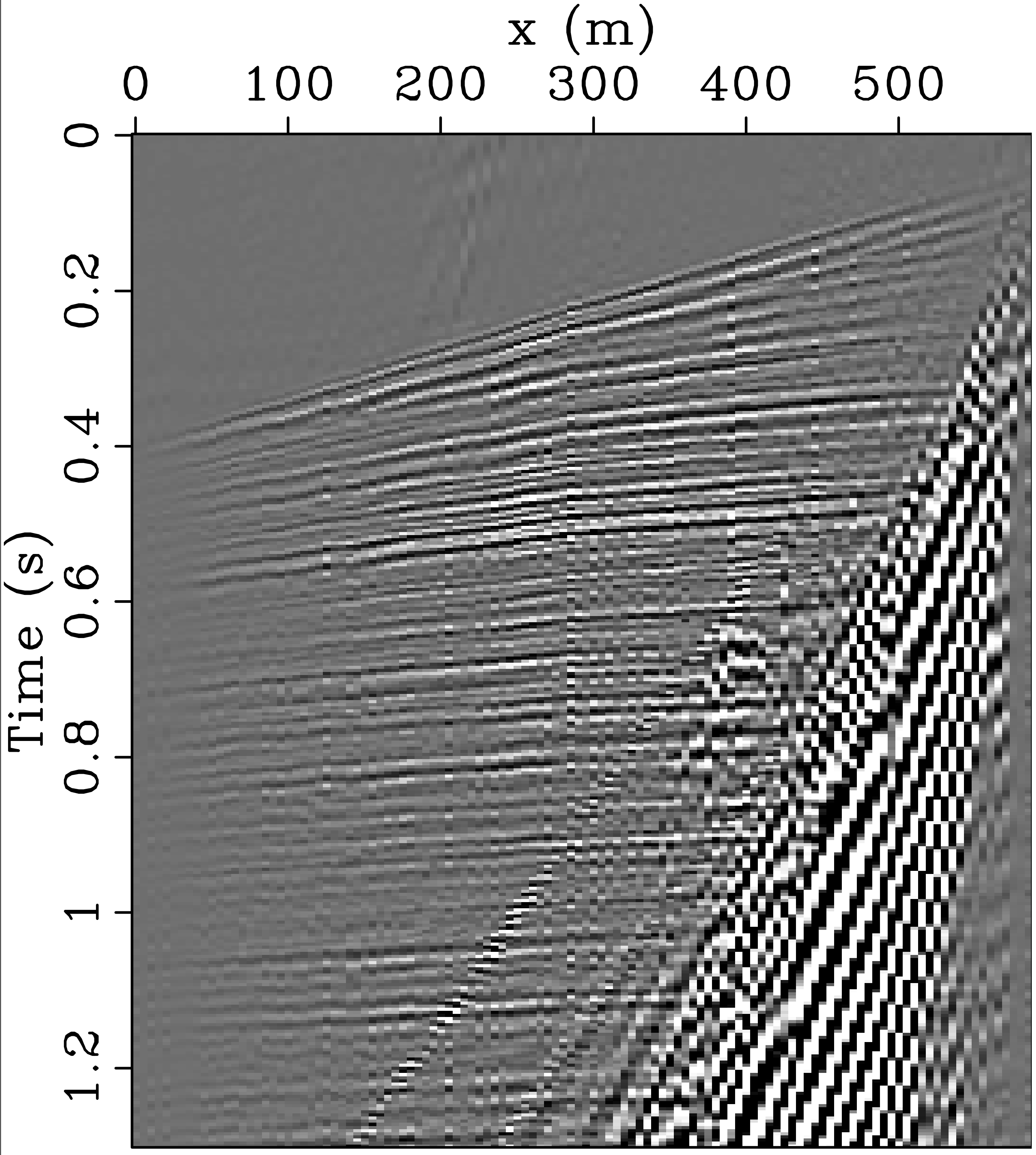


Model (5m)

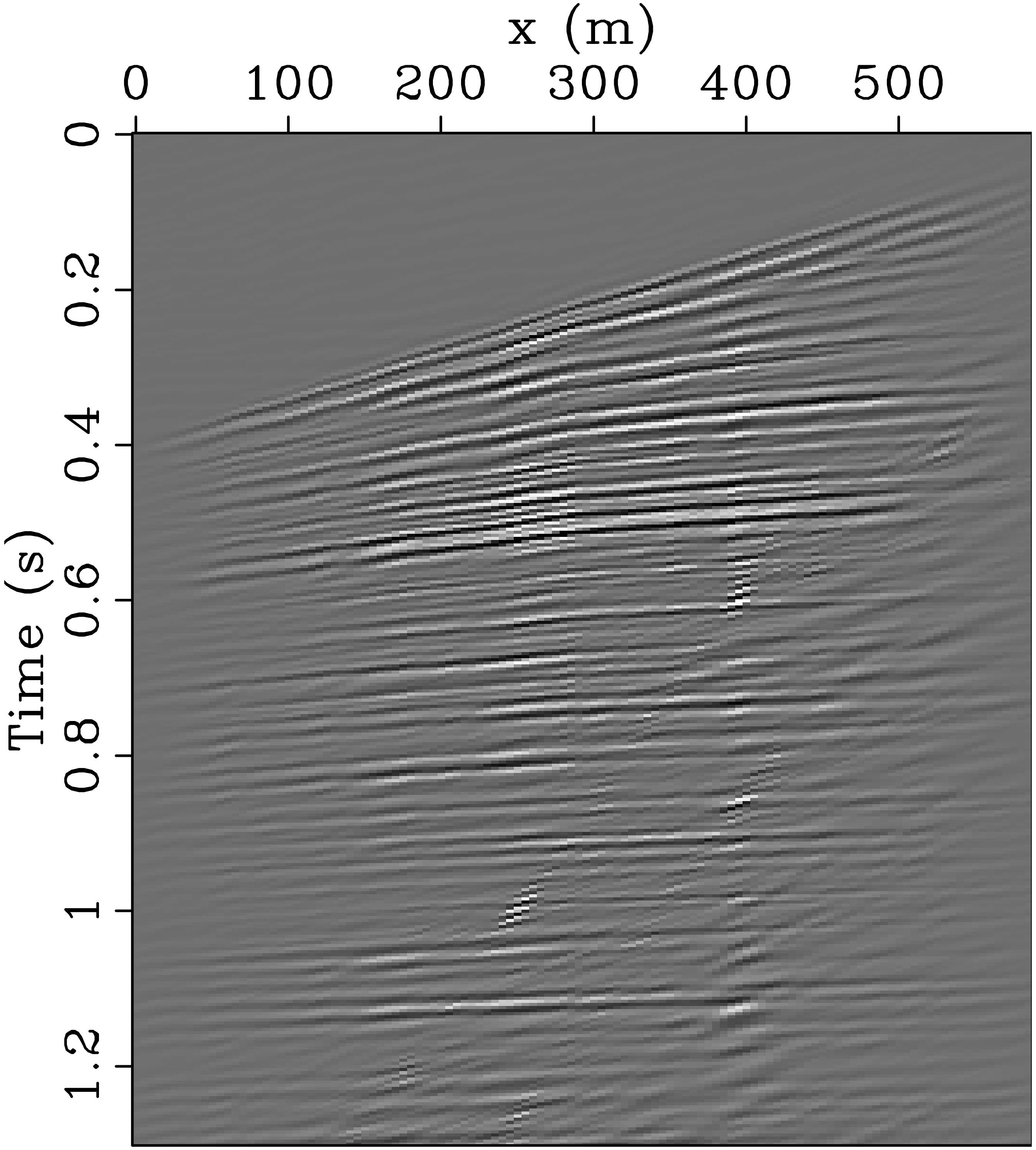


Interp. data (5 m)

SNR = 7.79 dB



Interp. data (5 m)



GR−rm interp. data (5 m)

(basic FK filtering)

Conclusions

- new wavefield reconstruction method that handles both regular and irregular acquisition geometries
  - curvelet reconstruction with sparsity-promoting inversion (CRSI) [Herrmann and Hennenfent‘08]
- extension of the fast discrete curvelet transform to handle irregular seismic data
  - nonequispaced fast discrete curvelet transform (NFDCT) [Hennenfent and Herrmann‘06]
- new coarse sampling schemes that maximize performance of CRSI
  - jittered undersampling schemes [Hennenfent and Herrmann‘08]
- new large-scale, one-norm solver
  - iterative soft thresholding with cooling (ISTc) [Herrmann and Hennenfent’08, Hennenfent et al.‘08]

Opportunities

- paradigm shift
  - from an assumption of band-limited to **sparse representation for seismic data**
  - from linear to **nonlinear wavefield sampling theory**

- design of advantageous coarse sampling schemes
  - same image quality at a **lower acquisition cost**
  - **better image quality** at a given acquisition cost

Other applications of curvelet-domain processing

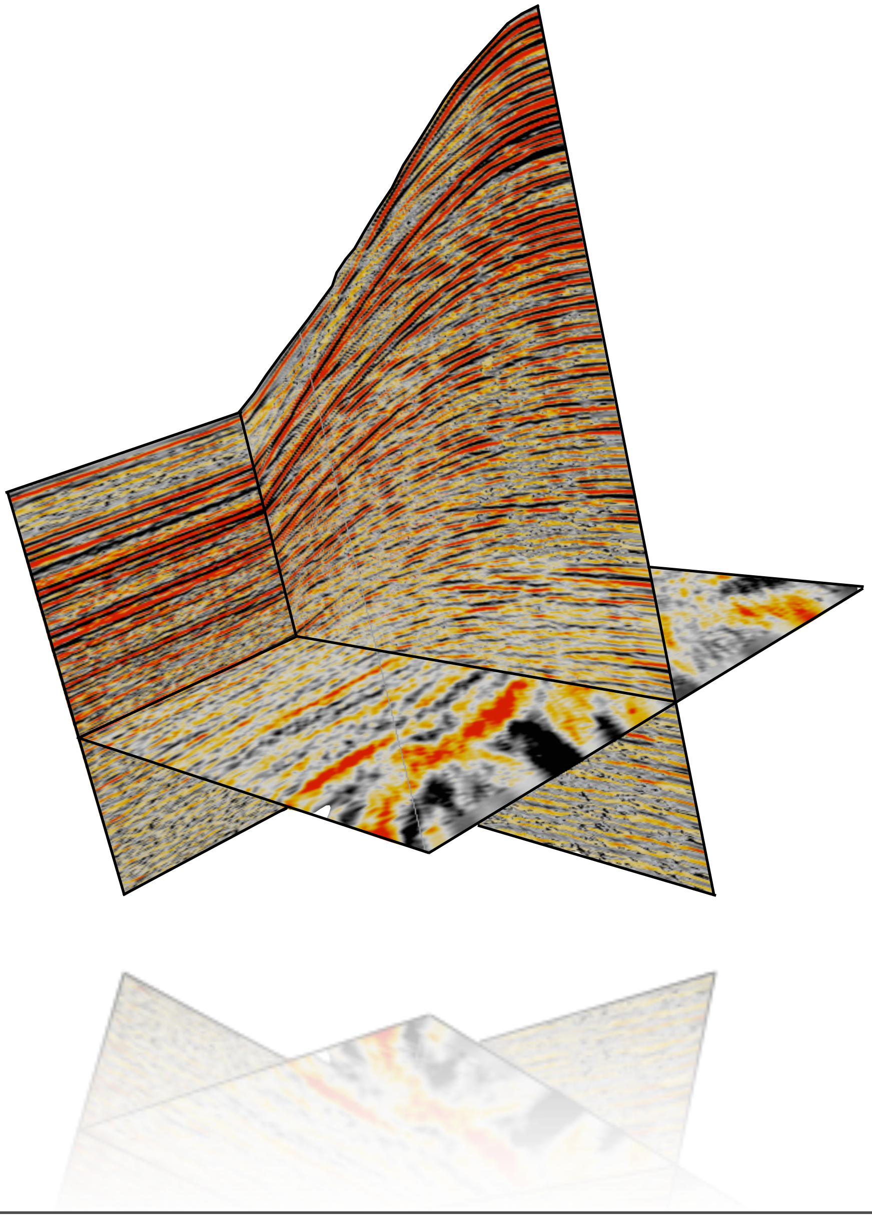
Felix J. Herrmann
fherrmann@eos.ubc.ca
http://slim.eos.ubc.ca

P. Moghaddam, D. Wang, R. Saab, O. Yilmaz

Seismic Laboratory for Imaging & Modeling
Department of Earth & Ocean Sciences
The University of British Columbia

D. J. Verschuur, Delphi

C. C. Stolk, Twente University



ION Technical forum - Sprowston, UK
Tuesday 15th – Thursday 17th April

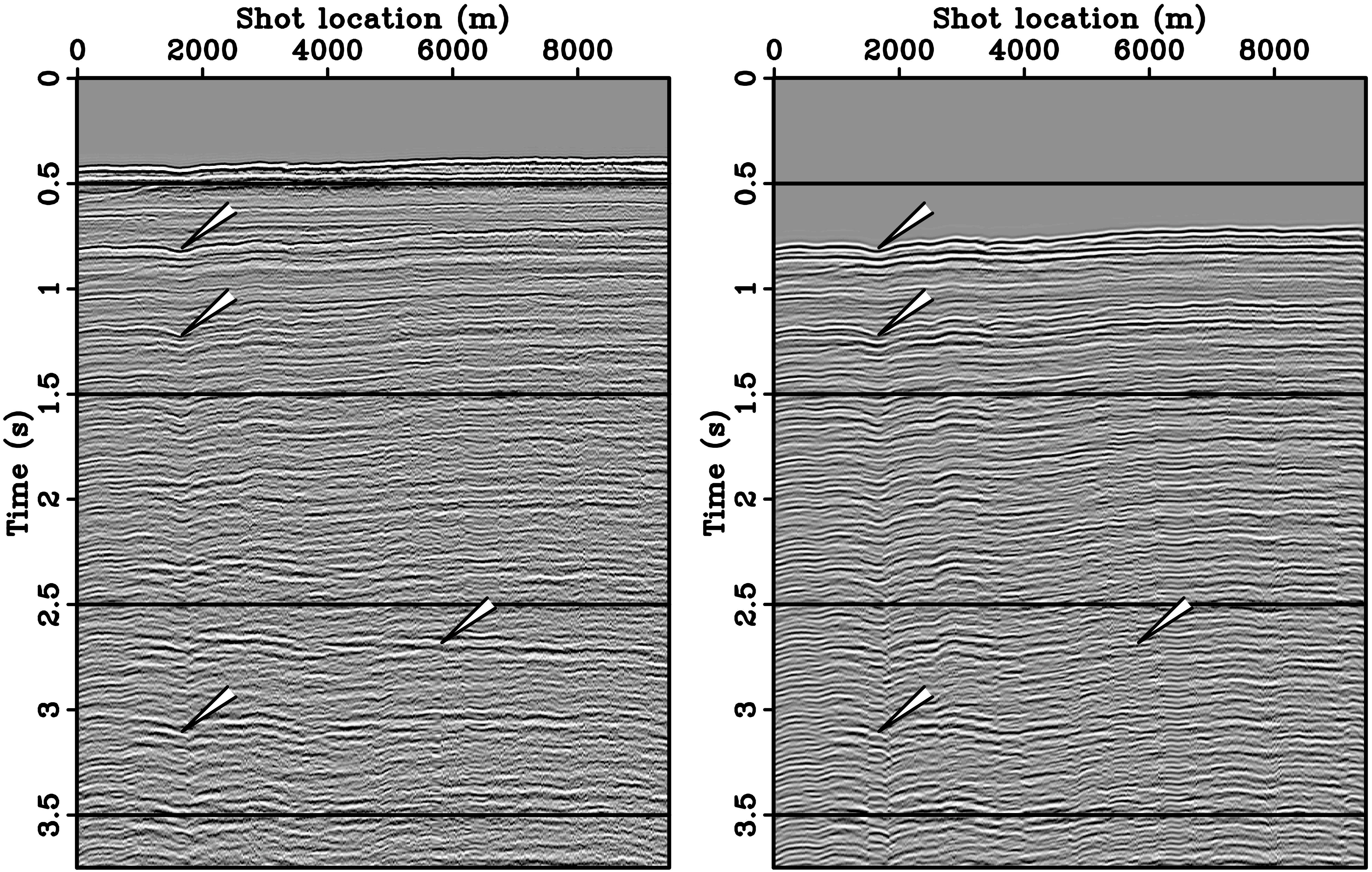
Other applications

- Curvelet-domain primary-multiple separation
  - sparsity promotion [Herrmann et. al. ’07, Saab ’07, Wang ‘08]
  - primary-multiple matching [Herrmann et. al. ’08]

- Curvelet-domain migration amplitude recovery
  - sparsity-promotion [Herrmann et. al. ’08b]
  - image-remigrated-image matching [Herrmann et. al. ’08b]
  - migration preconditioning

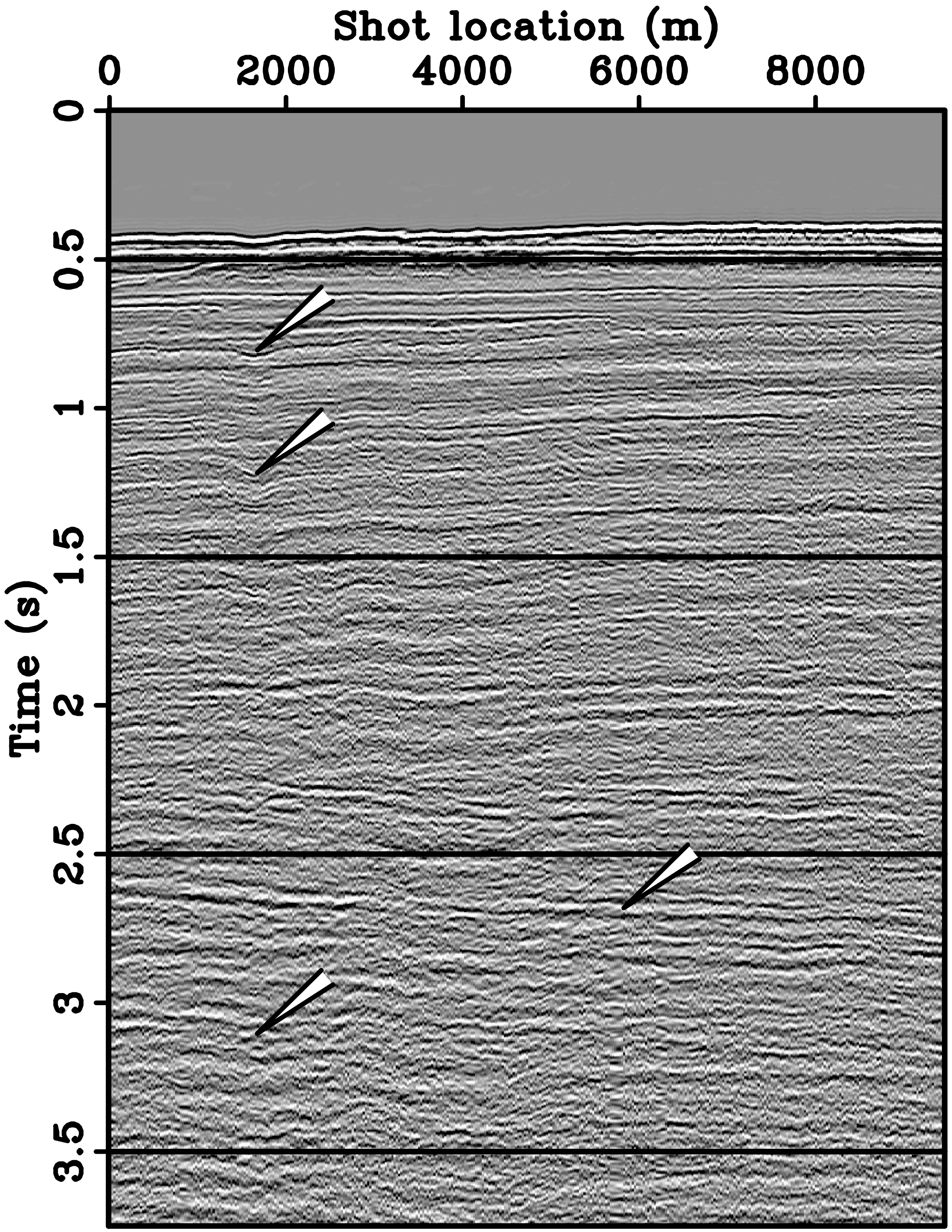
Primary-multiple separation

- Motivation
  - residual multiple energy and inadvertent removal primaries are problematic
  - Achilles' heel is adaptive separation after prediction
  - use curvelet-domain sparsimony and adaptivity
- New curvelet-domain technology
  - uses non-agressive (SRME) prediction as input
  - produces improved separation for primaries and multiples
- Three stages
  - Single-term optimized SRME prediction for the multiples
  - Curvelet-domain matching of predicted multiples with multiples in data
  - Bayesian separation of matched multiples and primaries based on sparsity promotion

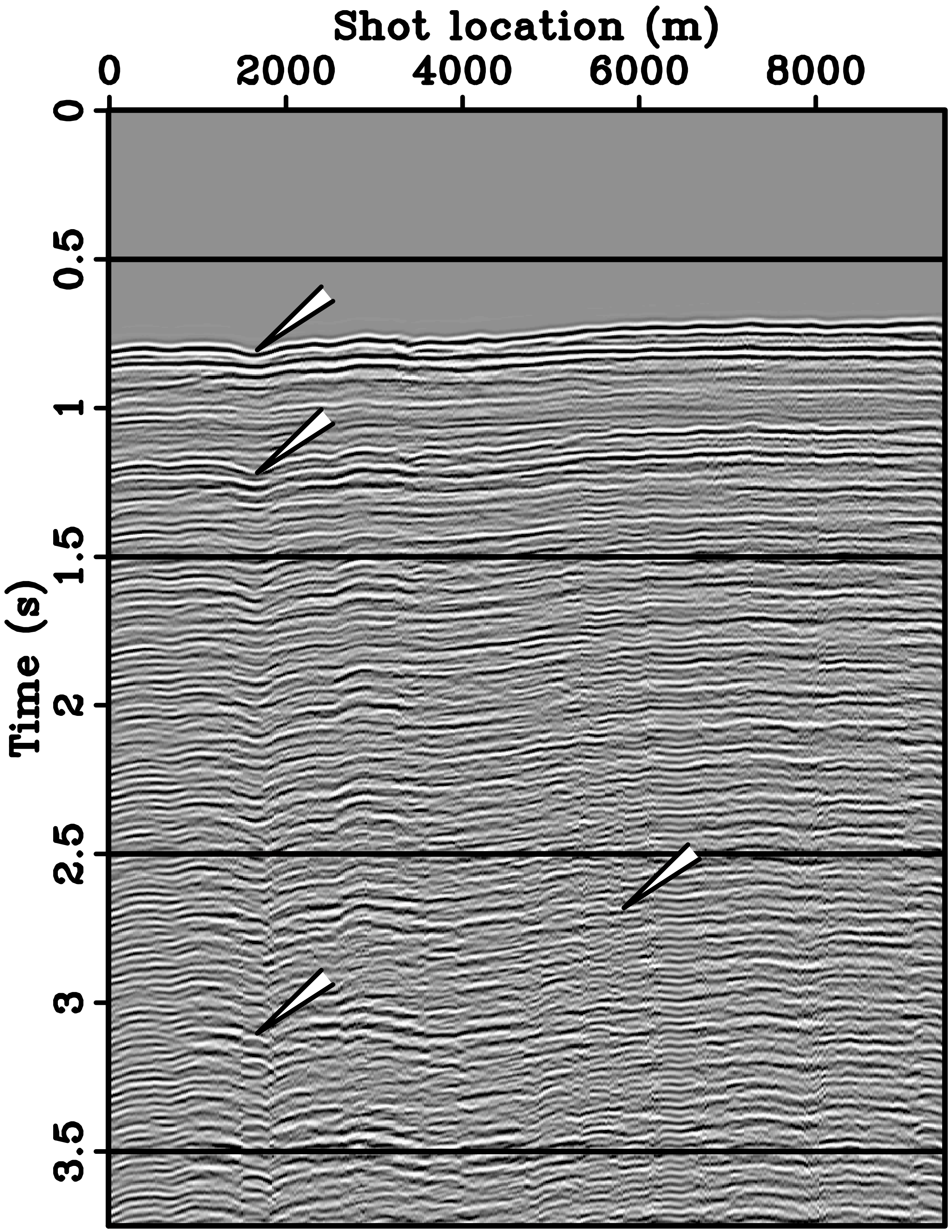


Total data

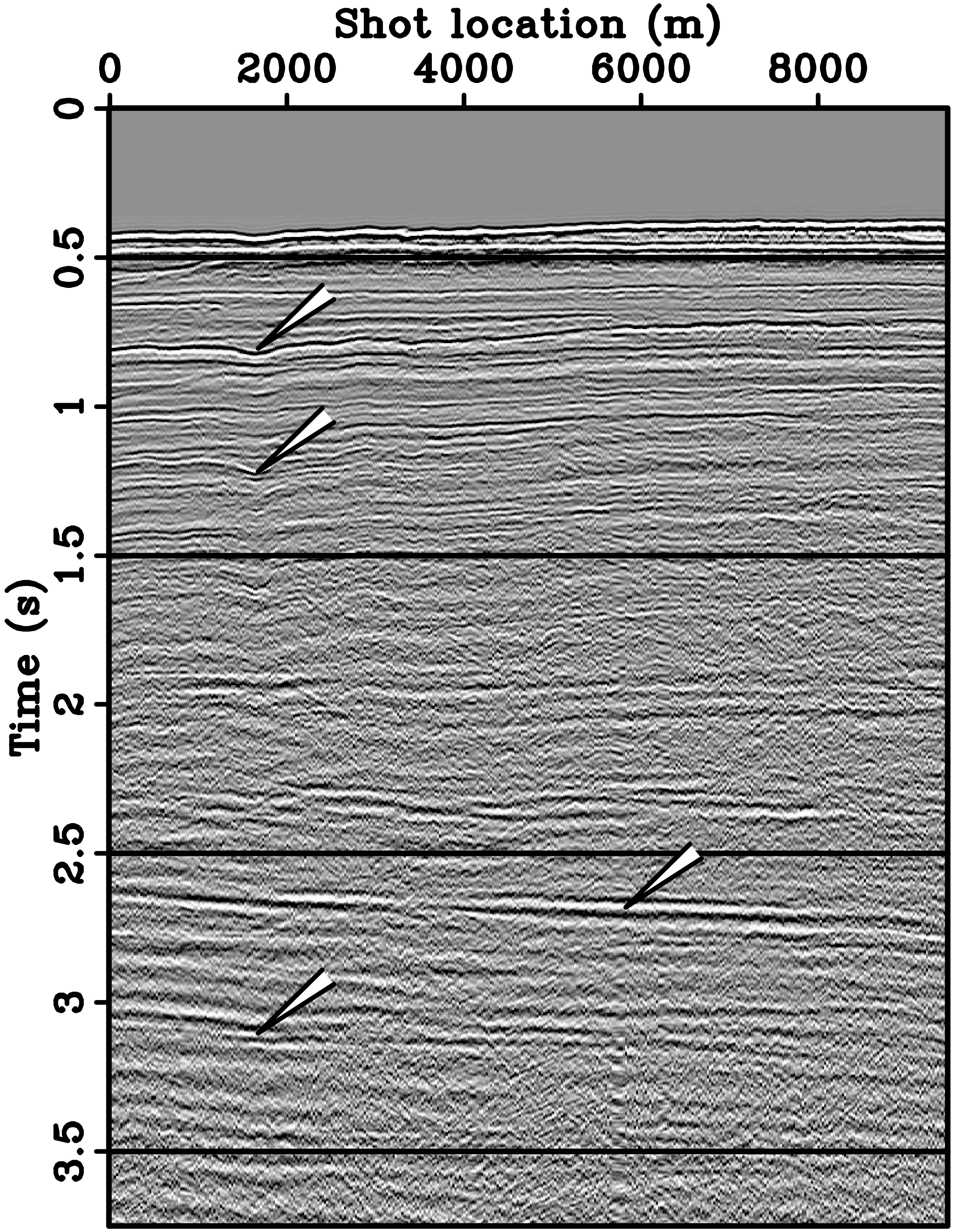
Predicted multiples



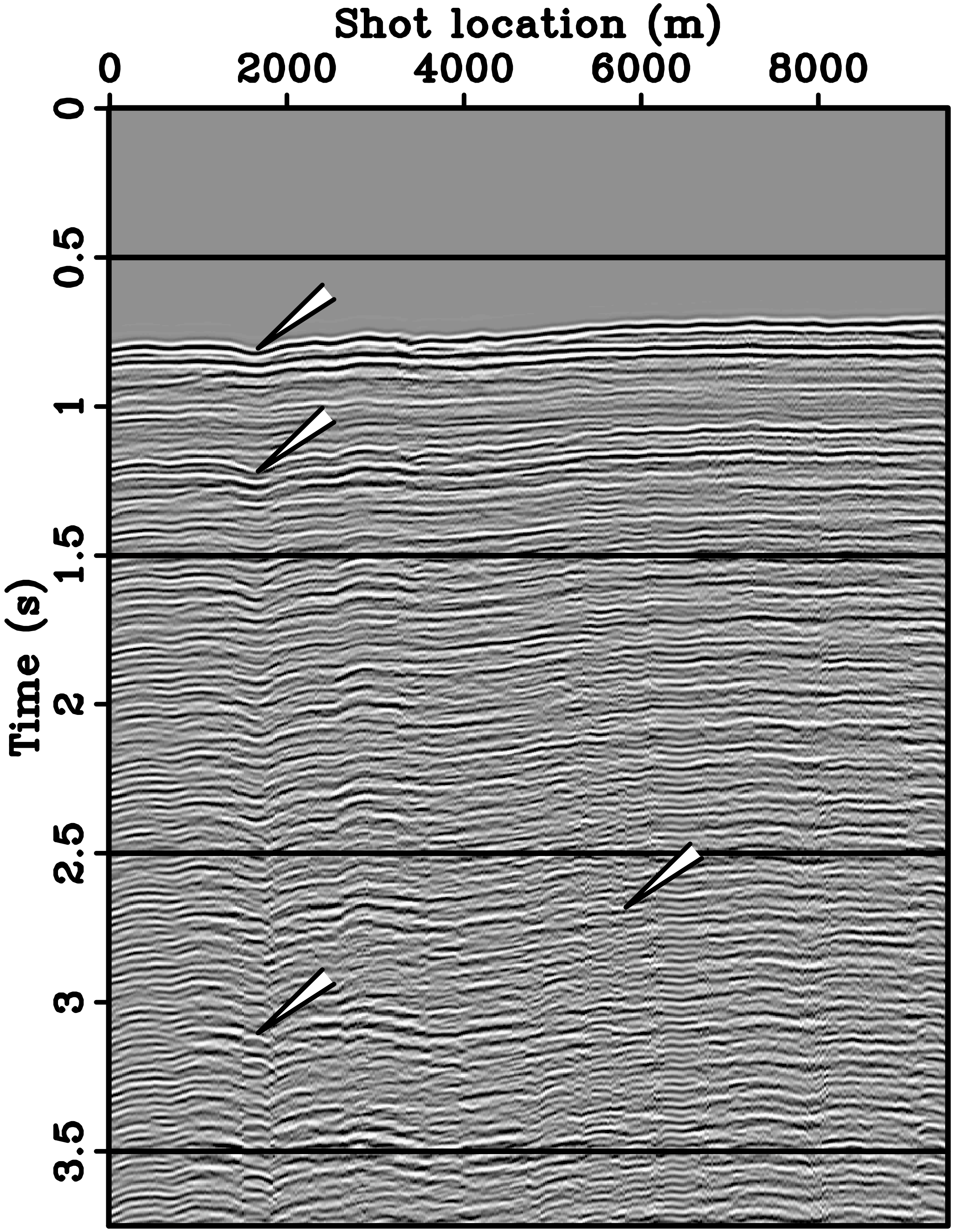
One-term SRME
predicted primaries



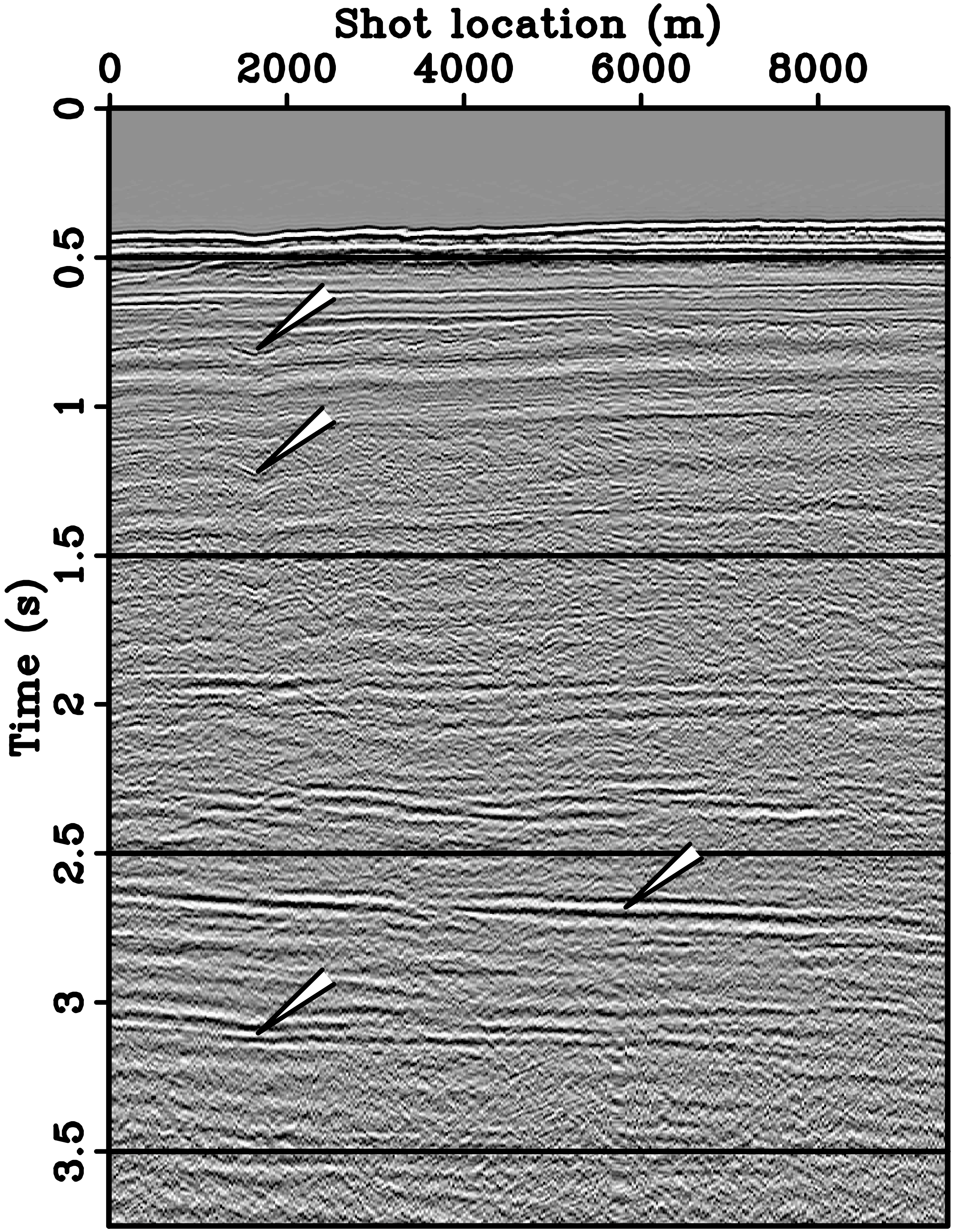
Predicted multiples



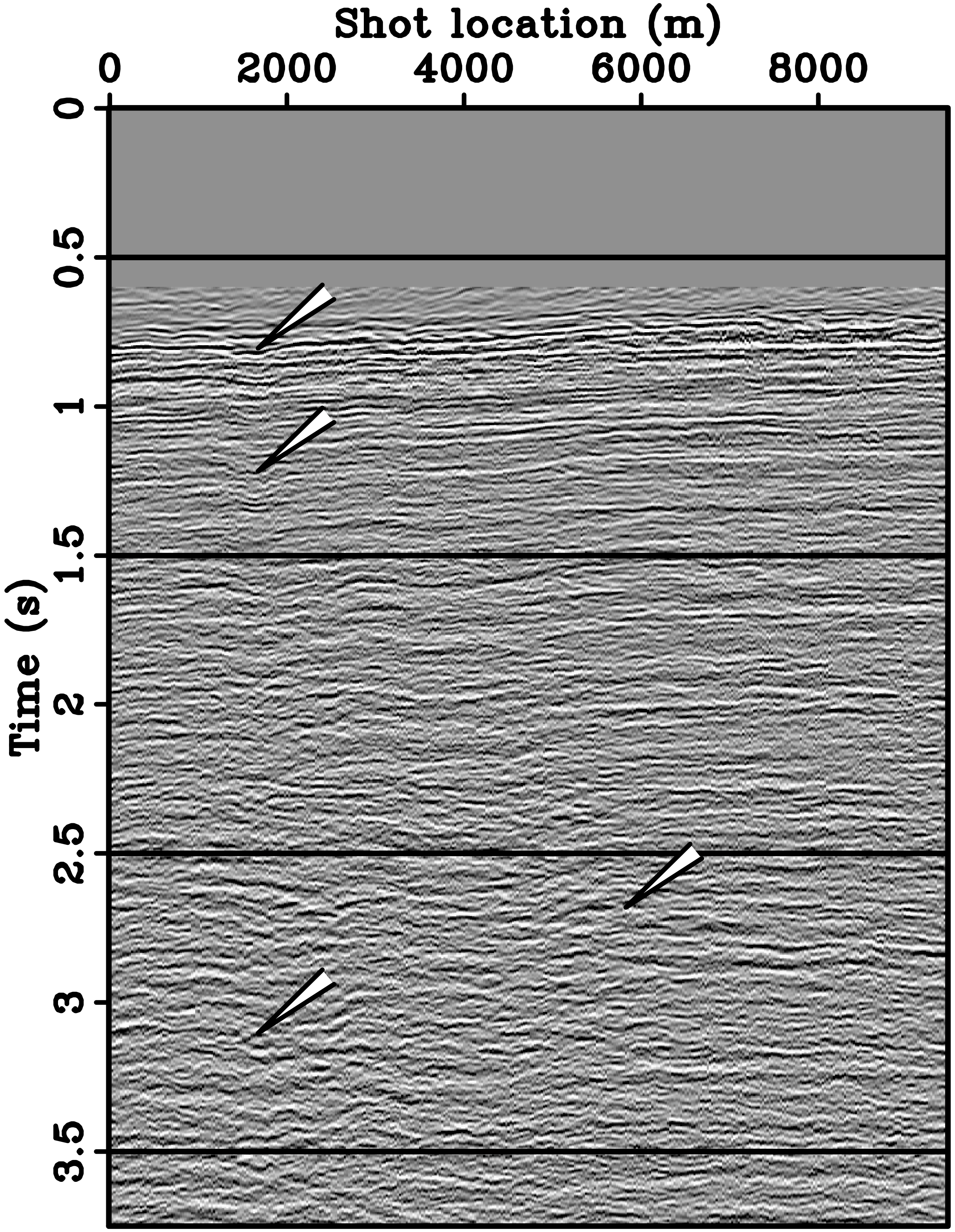
Bayesian estimate without matching



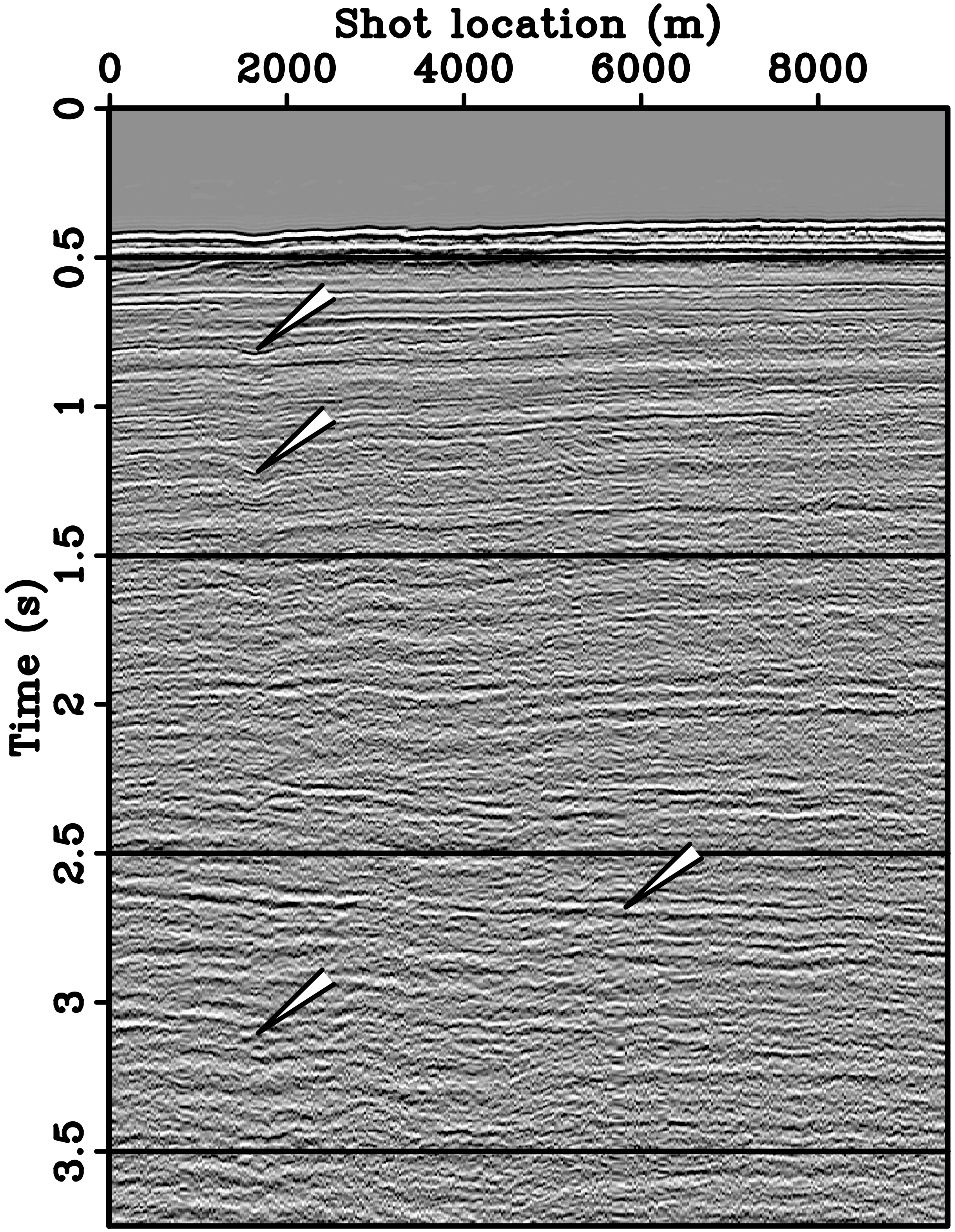
Predicted multiples



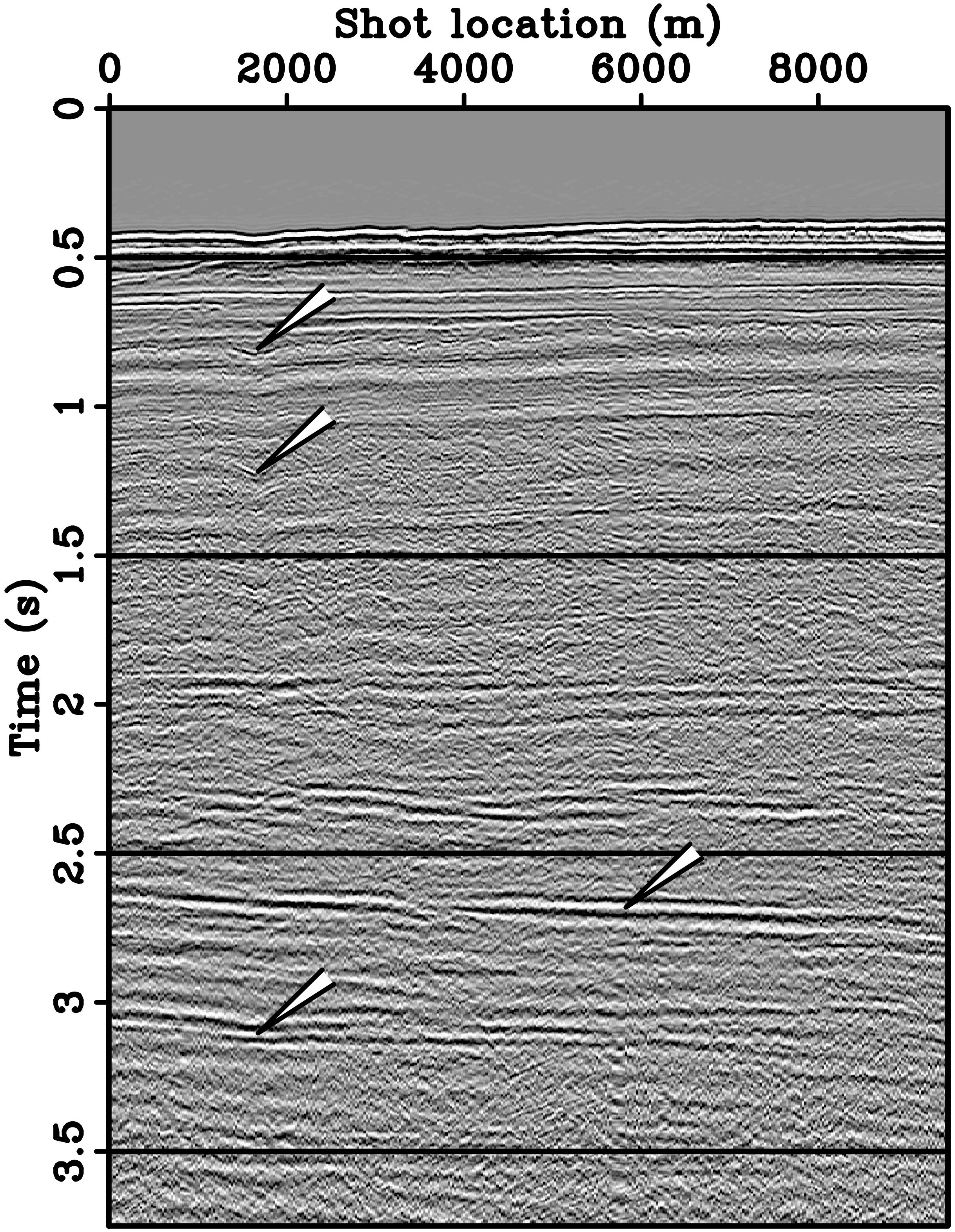
Bayesian estimate with matching



Difference between SRME and Bayesian with matching



One-term SRME predicted primaries



Bayesian estimate with matching

Conclusions

- Nyquist sampling criterion is too pessimistic for seismic data processing
  - new acquisition design based on controlled randomness
  - leverages recent developments in wireless acquisition systems

- Application of curvelet-technology opens a tantalizing perspective of redesigning seismic processing flows via combination of
  - sparsity promotion through norm-one optimization
  - phase-space adaptation through curvelet matching

- By no longer combating sampling irregularity but by embracing it we open the possibility to supersede Nyquist's criterion and further push the envelope ...

Acknowledgments

- SLIM team members
  - C. Brown, H. Modzelewski, and S. Ross-Ross for *SLIMpy* (slim.eos.ubc.ca/SLIMpy)
- D. J. Verschuur for the synthetic dataset
- Norsk Hydro for the real dataset
- E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying for *CurveLab* (www.curvelet.org)
- E. van den Berg and M. P. Friedlander for *SPGL1* (www.cs.ubc.ca/labs/scl/spgl1) & *Sparco* (www.cs.ubc.ca/labs/scl/sparco)
- S. Fomel, P. Sava, and the other developers of *Madagascar* (rsf.sourceforge.net)

This work was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).

References

T. Lin and F. J. Herrmann. Compressed extrapolation. Geophysics, Volume 72, Issue 5, pp. SM77-SM93, September-October 2007

F.J. Herrmann and U. Boeniger and D.J. Verschuur. Nonlinear primary-multiple separation with directional curvelet frames. , Geophysical Journal International, Vol. 170, 781-799, 2007

Felix J. Herrmann, Deli Wang, Gilles Hennenfent and Peyman Moghaddam. Curvelet-based seismic data processing: a multiscale and nonlinear approach. Geophysics, Vol. 73, No. 1, pp. A1–A5, January-February 2008

F.J. Herrman, P.P. Moghaddam and C. C. Stolk. Sparsity- and continuity-promoting seismic image recovery with curvelet frames. Appl. Comput. Harmon. Anal. Vol 24/2, 150-173, 2008

F. J. Herrmann and G. Hennenfent. Non-parametric seismic data recovery with curvelet frames, Geophysical Journal International, 173, 233–248, 2008

F. J. Herrmann, D. Wang and D. J. Verschuur. Adaptive curvelet-domain primary-multiple separation, Geophysics, 73(3), May-June 2008

G. Hennenfent and F. J. Herrmann. Simply denoise: wavefield reconstruction via jittered undersampling. Geophysics, 73(3), May-June 2008

D. Wang, R. Saab, O. Yilmaz and F. J. Herrmann. Bayesian wavefield separation by transform-domain sparsity promotion. To appear in Geophysics. 2008

Thanks

- Check out our website

  slim.eos.ubc.ca