Time-lapse FWI with distributed Compressive Sensing

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contributions from Rajiv Kumar and Ernie Esser

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Objective

New approach to FWI of time-lapse seismic data

Dealing with large acquisition gaps in data

Improved time-lapse inversion results

Full waveform inversion

Problem

$$\underset{\mathbf{m},\alpha}{\text{minimize}}\ \frac{1}{2}\|\mathbf{d}-\alpha\boldsymbol{\mathcal{F}}[\mathbf{m}]\|_2^2$$

$$\begin{aligned}
\mathbf{d} &: && \text{observed data}\\
\boldsymbol{\mathcal{F}} &: && \text{forward modelling kernel}\\
\alpha &: && \text{source wavelet}\\
\mathbf{m} &: && \text{model parameters}
\end{aligned}$$

Assume known source wavelet

Full waveform inversion

Problem

$$\underset{\mathbf{m},\alpha}{\text{minimize}} \ \frac{1}{2}\|\mathbf{d} - \boldsymbol{\mathcal{F}}[\mathbf{m}]\|_2^2$$

$$\begin{aligned} \mathbf{d} &: \quad \text{observed data} \\ \boldsymbol{\mathcal{F}} &: \quad \text{forward modelling kernel} \\ \mathbf{m} &: \quad \text{model parameters} \end{aligned}$$

Standard FWI

$$\underset{\mathbf{m}}{\text{minimize}} \frac{1}{2}\|\mathbf{d} - \boldsymbol{\mathcal{F}}[\mathbf{m}]\|_2^2$$

$$\begin{aligned} \text{Initialization, iteration } k = 0 : & \qquad \mathbf{m}_k \\ \text{Compute gradient} : & \qquad \delta\mathbf{m} \\ \text{Update model- iteration @ } k+1 : & \qquad \mathbf{m}_{k+1} = \mathbf{m}_k + \delta\mathbf{m} \end{aligned}$$

Xiang Li, Aleksandr Y. Aravkin, Tristan van Leeuwen, and Felix J. Herrmann, "Fast randomized full-waveform inversion with compressive sensing", *Geophysics*, vol. 77, p. A13-A17, 2012.

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Linearization + sparsity on update

Modified Gauss-Newton

$$\tilde{\mathbf{x}}^k = \arg\min_{\mathbf{x}} \frac{1}{2}\|\underbrace{\mathbf{d} - \mathcal{F}(\mathbf{m}^k)}_{\mathbf{b}} - \underbrace{\nabla\mathcal{F}(\mathbf{m}^k)\mathbf{C}^T}_{\mathbf{A}}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 < \tau$$

model update: $\mathbf{m}^{k+1} = \mathbf{m}^k + \mathbf{C}^T\tilde{\mathbf{x}}^k$

Xiang Li, Aleksandr Y. Aravkin, Tristan van Leeuwen, and Felix J. Herrmann,
"Fast randomized full-waveform inversion with compressive sensing",
Geophysics, vol. 77, p. A13-A17, 2012.

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Method

(1) Select frequency batch

(2) Initialization, iteration $k = 0$ $:\mathbf{m}_k$

(3) Draw subset (randomly select shots) of data $:\tilde{\mathbf{d}} = \mathbf{RMd}$

(4) Compute gradient via sparsity promotion $:\delta\mathbf{m}$

(5) Update model- iteration @ $k+1$ $:\mathbf{m}_{k+1} = \mathbf{m}_k + \delta\mathbf{m}$

(6) Repeat (3)

(7) Select next frequency batch

(8) Repeat (3) to (5)

(9) Repeat until last frequency batch is reached

Timelapse FWI

Given:

$$\begin{aligned} \text{Baseline data :} \quad & \mathbf{d}_1 \\ \text{A starting model from } \mathbf{d}_1 \text{ :} \quad & \mathbf{m}_0 \\ \text{Monitor data :} \quad & \mathbf{d}_2 \end{aligned}$$

Objective:

$$\begin{aligned} \text{Inversion for baseline model :} \quad & \mathbf{m}_1 \\ \text{Inversion for monitor model :} \quad & \mathbf{m}_2 \\ \text{Estimate/interprete timelapse model :} \quad & d\mathbf{m} = \mathbf{m}_2 - \mathbf{m}_1 \end{aligned}$$

Timelapse FWI approaches

Timelapse FWI approaches

Parallel difference

$$
\begin{aligned}
\text{Start with similar starting model, given observed data :} &\quad \mathbf{m}_0, \mathbf{d}_1, \mathbf{d}_2 \\
\text{Invert for baseline and monitor separately :} &\quad \mathbf{m}_1, \mathbf{m}_2 \\
\text{Estimate timelapse model :} &\quad d\mathbf{m} = \mathbf{m}_2 - \mathbf{m}_1
\end{aligned}
$$

Sequential difference

$$
\begin{aligned}
\text{Start with baseline data and initial model :} &\quad \mathbf{m}_0, \mathbf{d}_1 \\
\text{Invert for baseline :} &\quad \mathbf{m}_1 \\
\text{Inversion of } \mathbf{d}_2 \text{ using } \mathbf{m}_1 \text{ as starting model :} &\quad \mathbf{m}_2 \\
\text{Estimate timelapse model :} &\quad d\mathbf{m} = \mathbf{m}_2 - \mathbf{m}_1
\end{aligned}
$$

Watanabe et al., 2004; Denli and Huang, 2009; Zheng et al., 2011;
Asnaashari et al., 2012; Raknes et al., 2013)

Timelapse FWI approaches

Double difference or Differential FWI

$$\text{minimize } \Delta\mathbf{d} := (\mathbf{d}_2 - \mathbf{d}_1) - (\boldsymbol{\mathcal{F}}[\mathbf{m}_2] - \boldsymbol{\mathcal{F}}[\mathbf{m}_1])$$

$$\begin{aligned}
\text{Start with baseline data and initial model :} & \quad \mathbf{m}_0, \mathbf{d}_1 \\
\text{Invert for baseline :} & \quad \mathbf{m}_1 \\
\text{Construct composite data :} & \quad \widetilde{\mathbf{d}_2} = \mathbf{d}_2 - \mathbf{d}_1 + \boldsymbol{\mathcal{F}}[\mathbf{m}_1] \\
\text{Replace } \mathbf{d}_2 \text{ with } \widetilde{\mathbf{d}_2} \text{ obtain :} & \quad \widetilde{\mathbf{m}_2} \\
\text{Estimate timelapse model :} & \quad d\mathbf{m} = \widetilde{\mathbf{m}_2} - \mathbf{m}_1
\end{aligned}$$

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Timelapse joint FWI approaches

Robust joint FWI with TV regularization

$$\alpha\|\mathbf{M}_1\boldsymbol{\mathcal{F}}[\mathbf{m}_1]-\mathbf{d}_1\|_2^2+\beta\|\mathbf{M}_2\boldsymbol{\mathcal{F}}[\mathbf{m}_2]-\mathbf{d}_2\|_2^2+ \tag{1}$$

$$\gamma\|(\mathbf{M}_2^s\boldsymbol{\mathcal{F}}[\mathbf{m}_2]-\mathbf{M}_1^s\boldsymbol{\mathcal{F}}[\mathbf{m}_1])-(\mathbf{M}_2\mathbf{d}_2-\mathbf{M}_1\mathbf{d}_1)\|_2^2+ \tag{2}$$

$$\alpha_1\|\mathbf{W}_1\mathbf{R}_1(\mathbf{m}_1-\mathbf{m}_1^{prior})\|_1+ \tag{3}$$

$$\beta_1\|\mathbf{W}_2\mathbf{R}_2(\mathbf{m}_2-\mathbf{m}_2^{prior})\|_1+ \tag{4}$$

$$\delta\|\mathbf{W}\mathbf{R}(\mathbf{m}_2-\mathbf{m}_1-\Delta\mathbf{m}^{prior})\|_1+ \tag{5}$$

Our separate versus joint inversion approach

Full waveform inversion in time-lapse

Independent inversion

for $\quad i = 1, 2$

$$\tilde{\mathbf{x}}_i^k = \arg\min_{\mathbf{x}_i} \frac{1}{2} \| \underbrace{\mathbf{d}_i^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_i^k)}_{\mathbf{b}_i} - \underbrace{\nabla \boldsymbol{\mathcal{F}}(\mathbf{m}_i^k)\mathbf{C}^T}_{\mathbf{A}_i} \underset{\mathbf{x}_i}{\mathbf{x}_i} \|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}_i\|_1 < \tau_i^k$$

$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T \tilde{\mathbf{x}}_i^k$$

Objective: Invert for baseline, monitor; difference = baseline-monitor

Dror Baron , Marco F. Duarte , Shriram Sarvotham , Michael B. Wakin , Richard G. Baraniuk. An Information-Theoretic Approach to Distributed Compressed Sensing (2005).

Distributed compressive sensing
– *joint* recovery model (JRM)

vintages

$$\mathbf{x}_1 = \mathbf{z}_0 + \mathbf{z}_1$$
$$\mathbf{x}_2 = \mathbf{z}_0 + \mathbf{z}_2$$

innovations

common part

$$\overbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{A}_2 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}}^{\mathbf{z}} = \overbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}}^{\mathbf{b}}$$

baseline

monitor

- Decompose vintage into common and innovations
- Timelapse vintages share a lot of common information
- DCS exploits the common or shared information
- Invert for common component and innovations

$$\tilde{\mathbf{z}} = \arg\min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{s.t.} \quad \mathbf{b} = \mathbf{A}\mathbf{z}$$

Previous applications

Missing trace interpolation of time-lapse data

NMO Stacking of prestack timelapse data

Recovery of time-lapse data from time-jittered marine acquisition

FWI of time-lapse data with different acquisition geometry

Sparsity promoting least-squares migration of time-lapse data

Joint inversion

with distributed compressed sensing

$$\tilde{\mathbf{z}}_k = \arg\min_{\mathbf{z}_k} \frac{1}{2}\|\mathbf{b}_k - \mathbf{A}_k\mathbf{z}_k\|_2^2 \quad \text{s.t.} \quad \|\mathbf{z}_k\|_1 < \tau^k$$

$$\mathbf{b}_k = \begin{bmatrix} \mathbf{d}_1^k - \mathcal{F}(\mathbf{m}_1^k) \\ \mathbf{d}_2^k - \mathcal{F}(\mathbf{m}_2^k) \end{bmatrix}$$

$$\mathbf{A}_k = \begin{bmatrix} \nabla\mathcal{F}(\mathbf{m}_1^k)\mathbf{C}^T & \nabla\mathcal{F}(\mathbf{m}_1^k)\mathbf{C}^T & \mathbf{0} \\ \nabla\mathcal{F}(\mathbf{m}_2^k)\mathbf{C}^T & \mathbf{0} & \nabla\mathcal{F}(\mathbf{m}_2^k)\mathbf{C}^T \end{bmatrix}$$

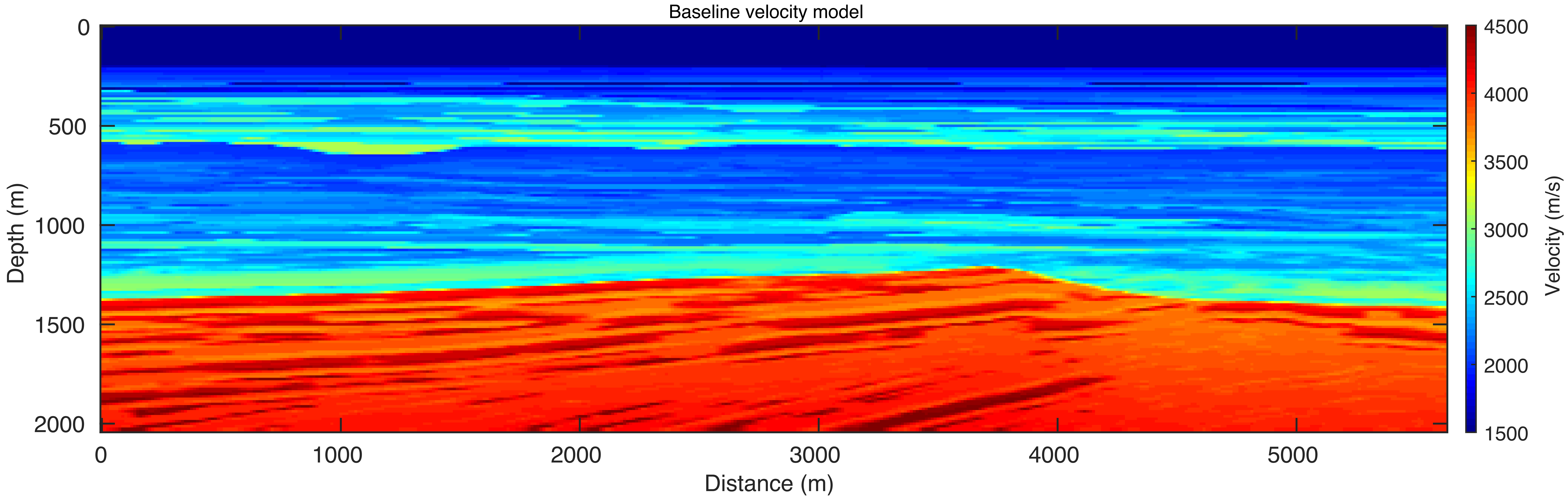
$$\mathbf{z}_k = \begin{bmatrix} \mathbf{z}_0^k \\ \mathbf{z}_1^k \\ \mathbf{z}_2^k \end{bmatrix}$$

$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T(\tilde{\mathbf{z}}_0^k + \tilde{\mathbf{z}}_i^k)$$

Application

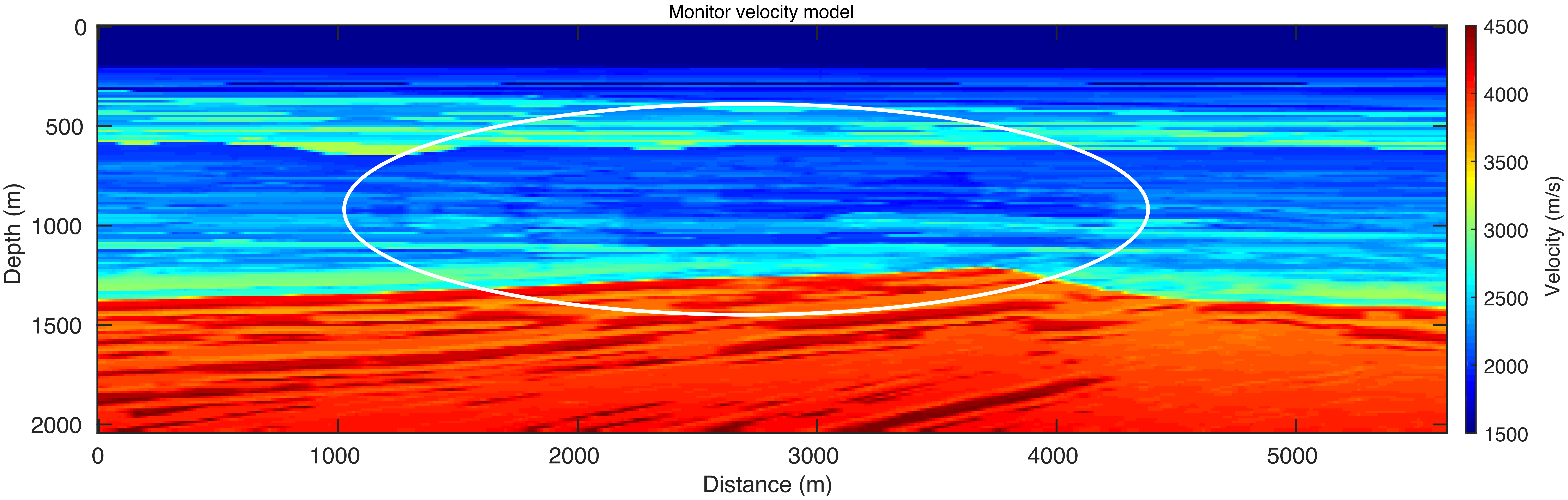
Baseline

BG Compass model

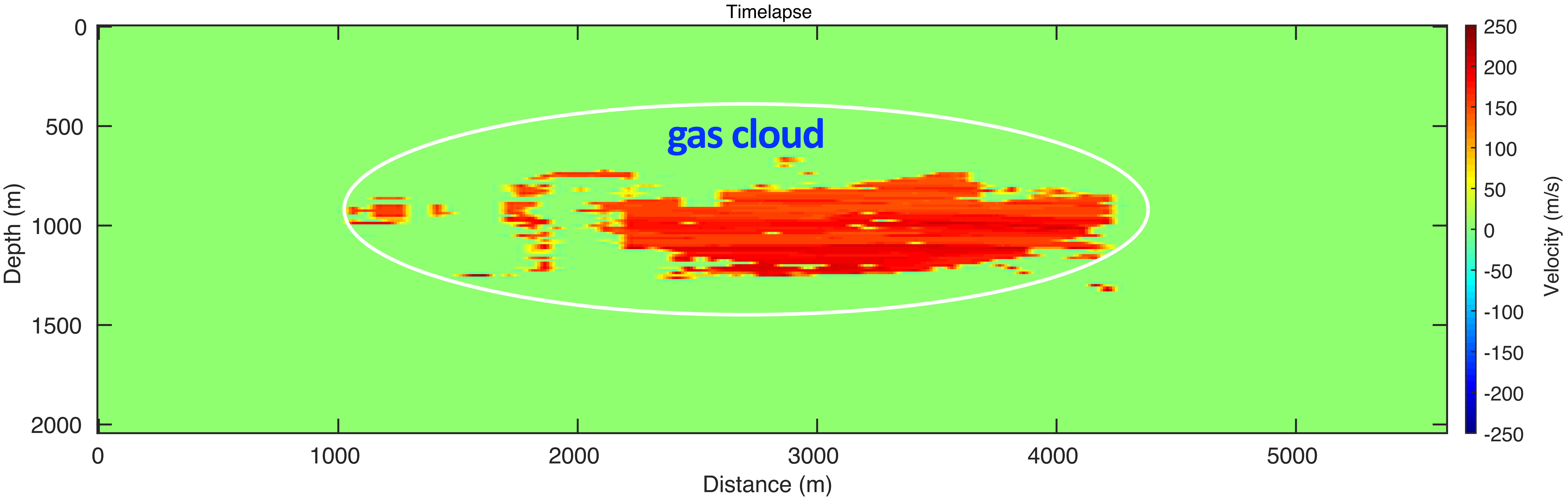


Monitor

BG Compass model

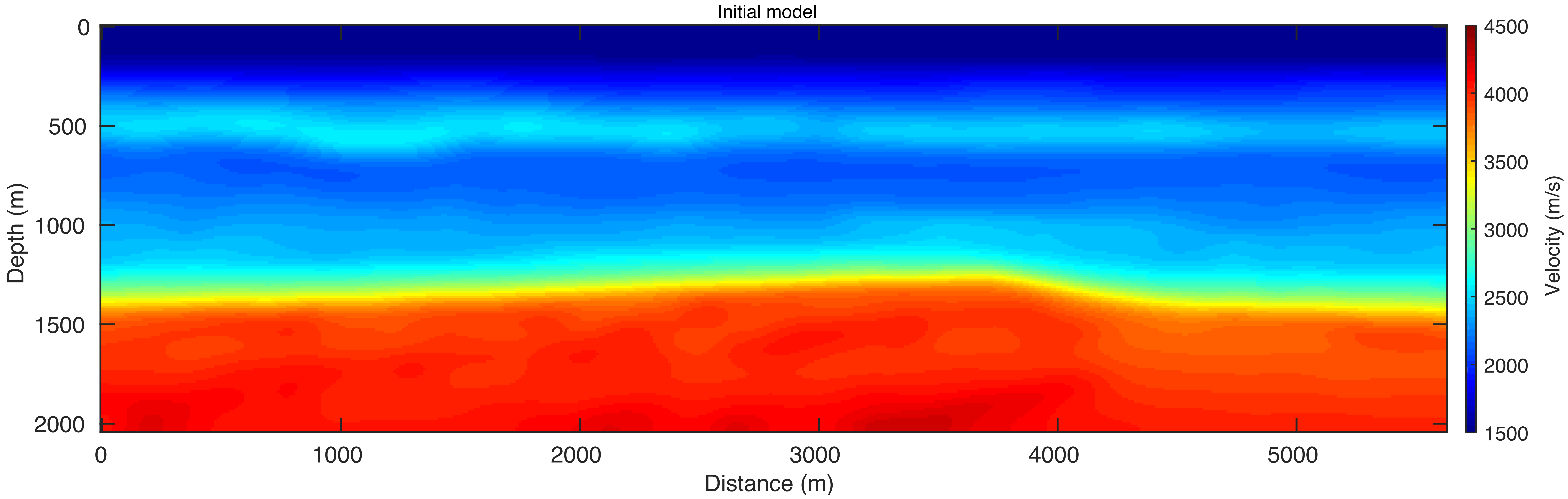


Timelapse



Timelapse
gas cloud
Depth (m)
0
500
1000
1500
2000
Distance (m)
0
1000
2000
3000
4000
5000
Velocity (m/s)
250
200
150
100
50
0
-50
-100
-150
-200
-250

Starting model



Baseline inversion

Modeling parameters

- 113 jittered shots, nominal sampling of 50m
- Co-located sources and receivers
- 80 frequencies from 3 to 22.5Hz

Modified Gauss-Newton

- Assume *good* background velocity model
- *Baseline* : use few randomly selected shots, with renewal
- Started inversion at 3Hz
- 8 frequencies per band
- 10 Gauss-Newton subproblems per band
- Approximately 10 iterations per subproblem

Monitor inversion

Modeling parameters

- 113 jittered shots, nominal sampling of 50m
- Co-located sources and receivers
- 80 frequencies from 3 to 22.5Hz

Modified Gauss-Newton

- Assume *good* background velocity model (same as baseline starting model)
- *Monitor* : use few randomly selected shots, with renewal
- Started inversion at 3Hz
- 8 frequencies per band
- 10 Gauss-Newton subproblems per band
- Approximately 10 iterations per subproblem

Recap

Baseline and monitor acquisition are different in source/receiver positions

Same depth for sources/receivers in the baseline and monitor

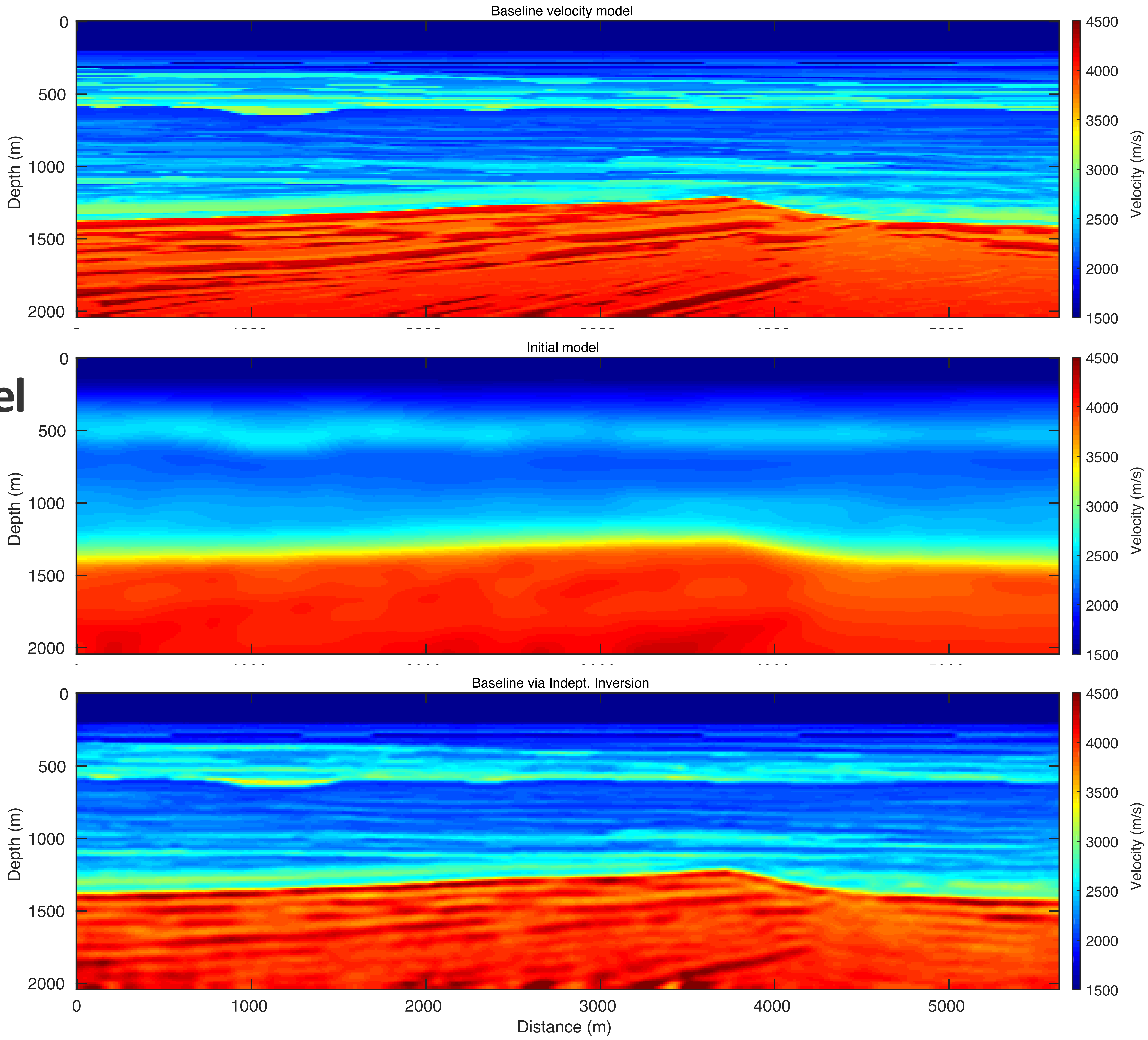
Same starting model used for baseline and monitor inversions

Equal number of iterations for independent inversions for baseline/monitor, and the joint inversion

True baseline

Starting model

Inversion result



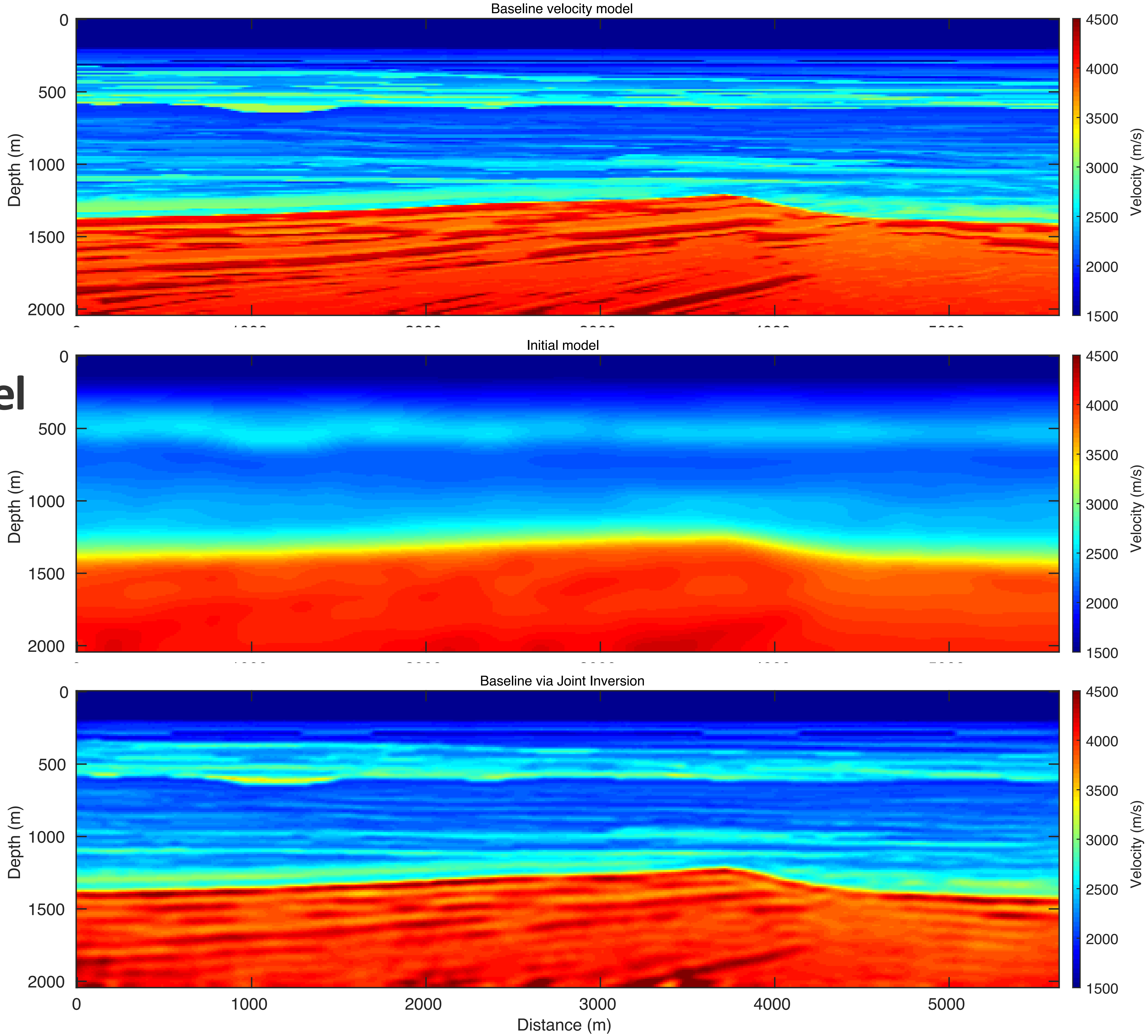
Independent inversion

True baseline

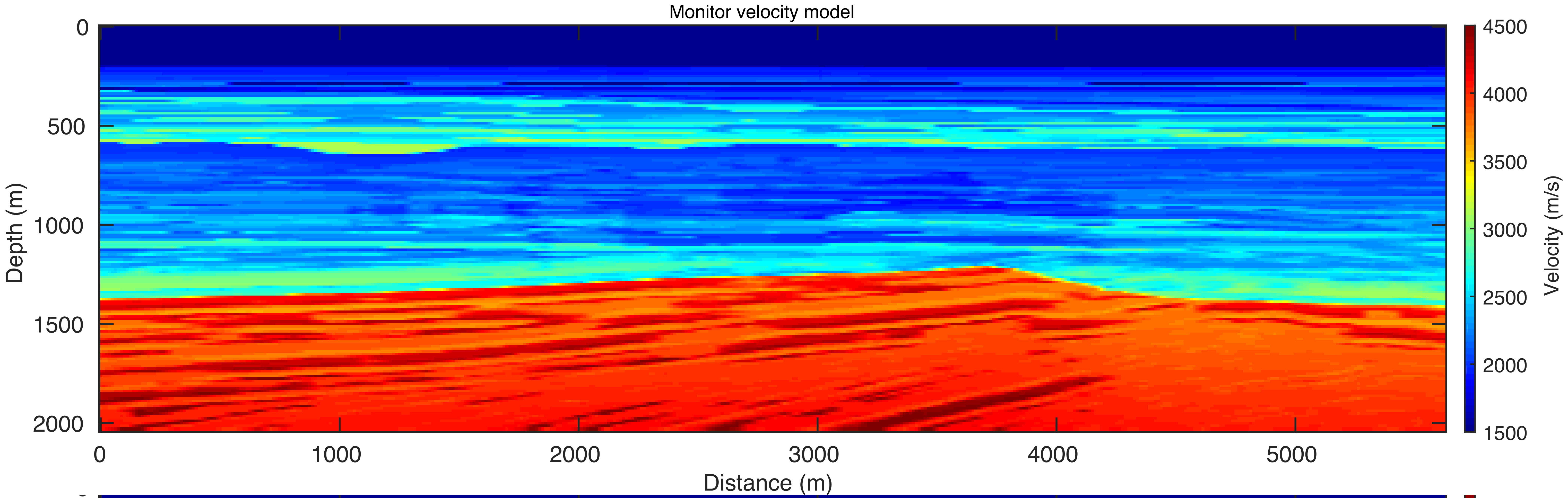
Starting model

Inversion result

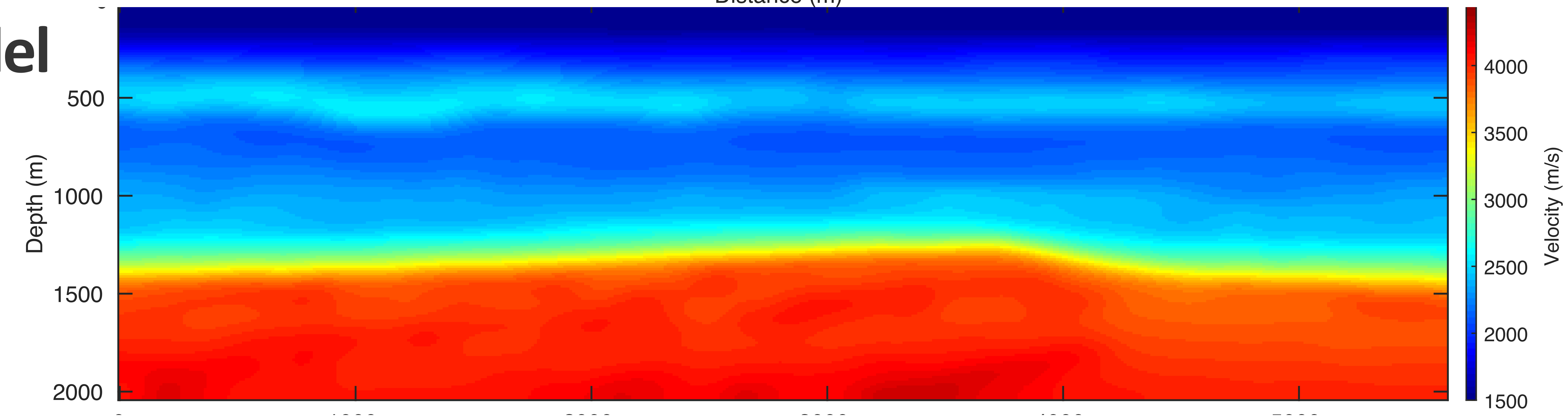
Joint inversion



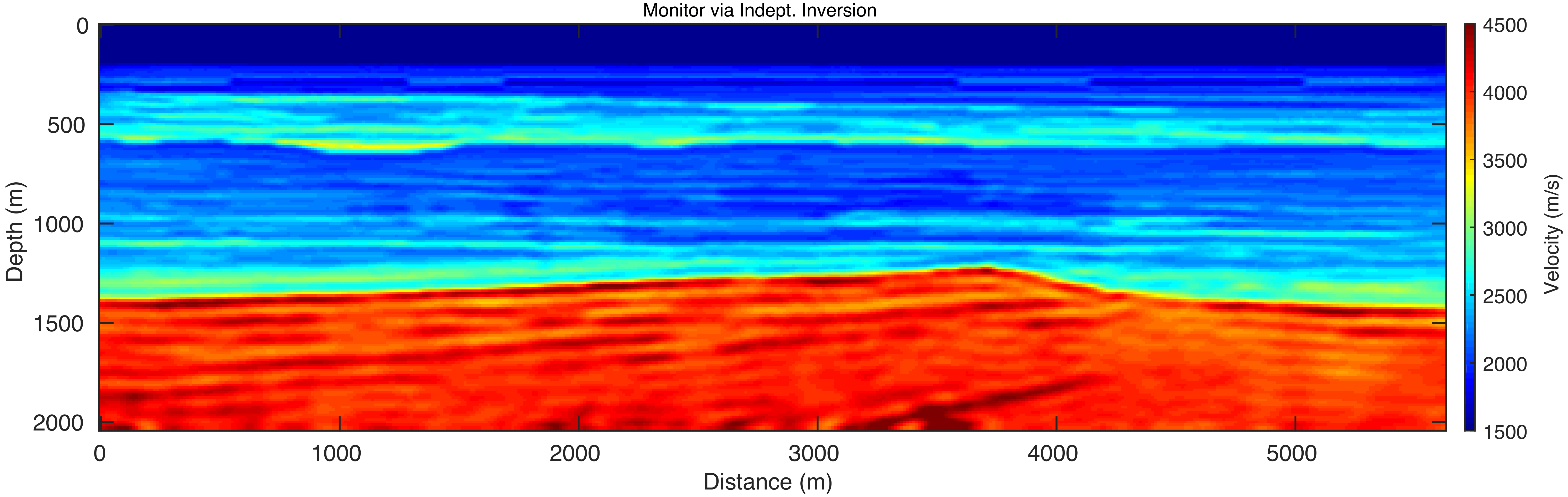
True monitor



Starting model

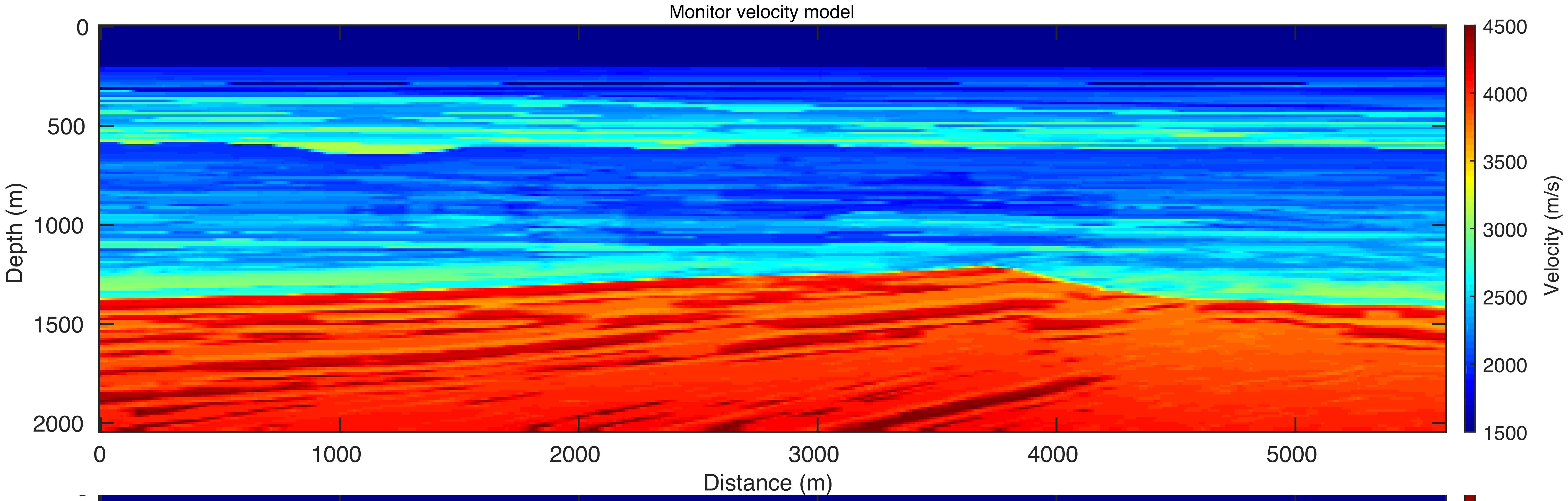


Inversion result

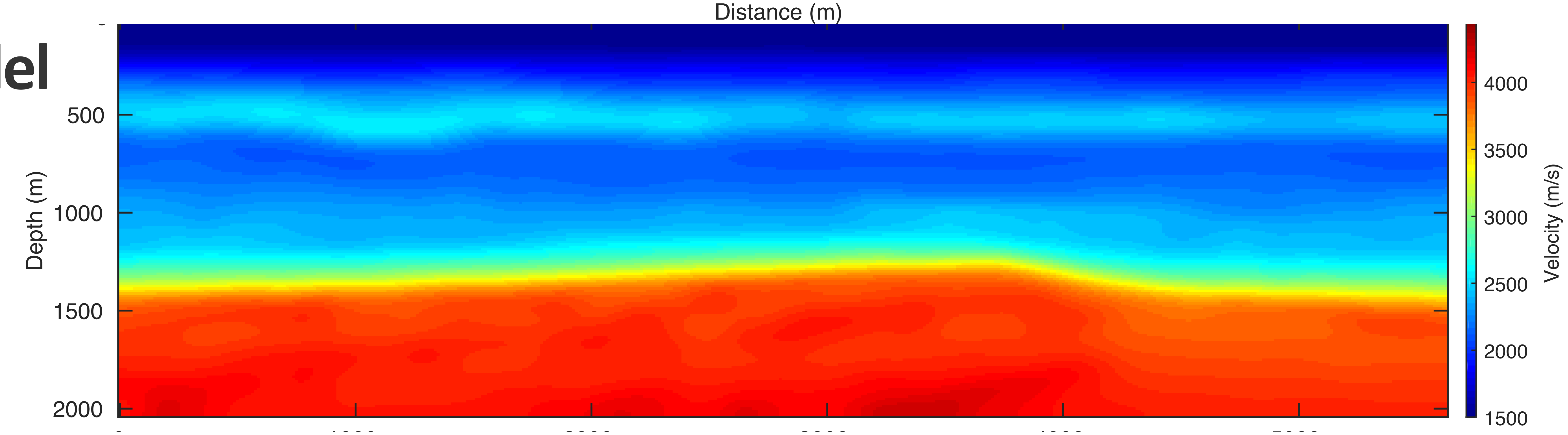


Independent inversion

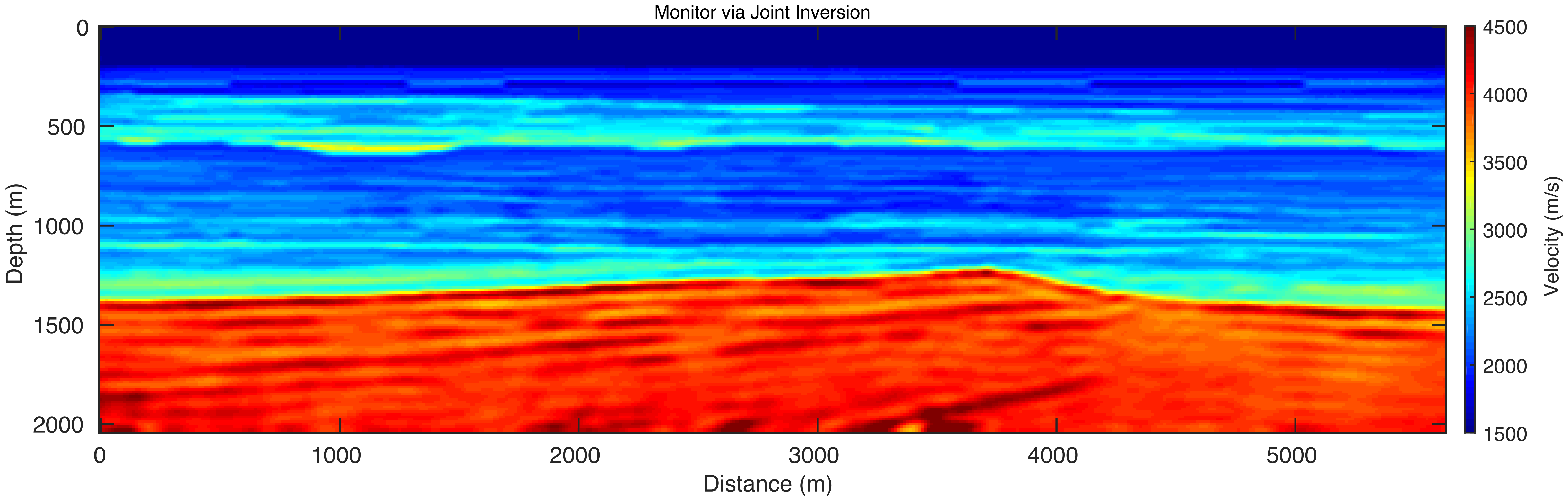
True monitor



Starting model

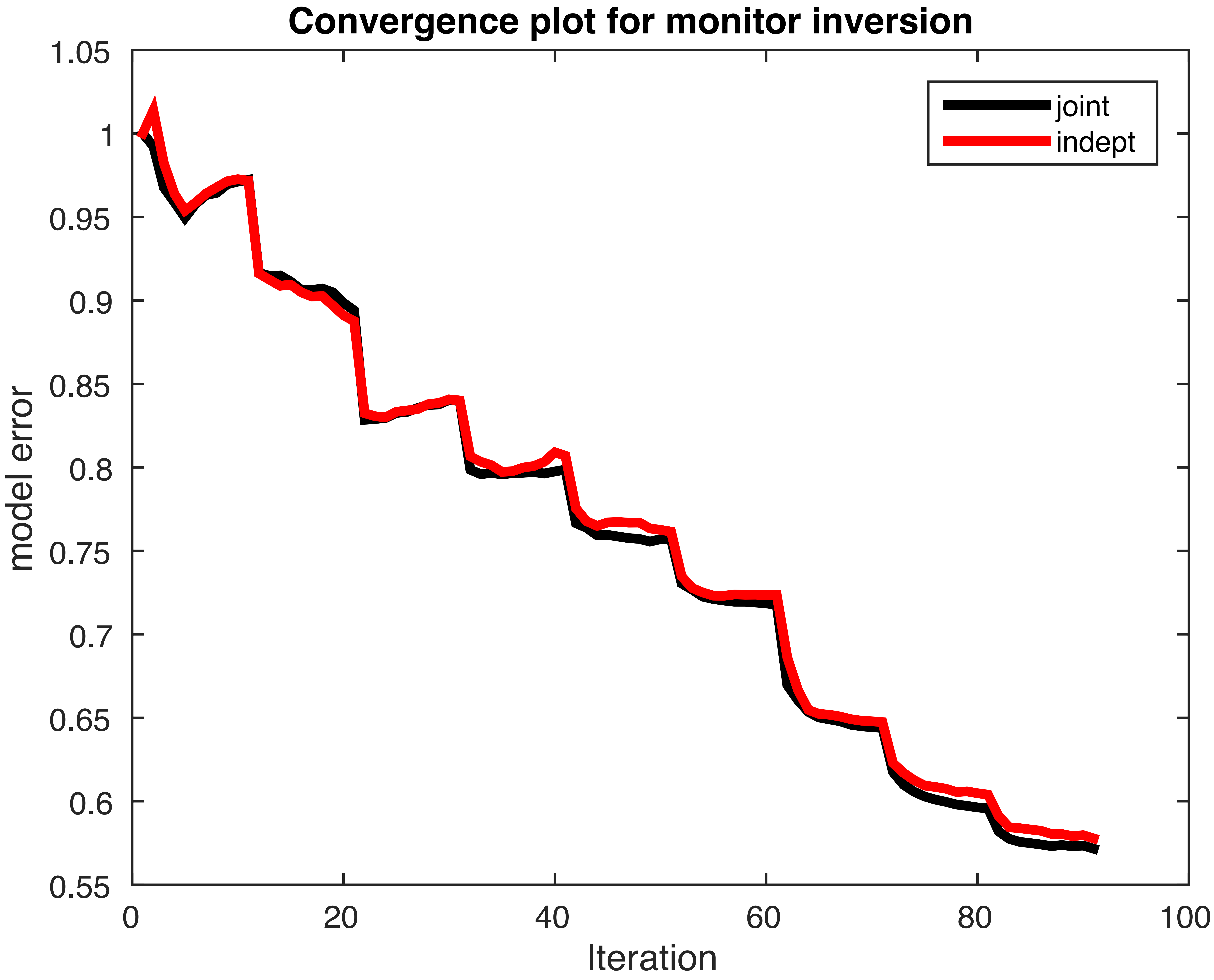


Inversion result



Joint inversion

Model error

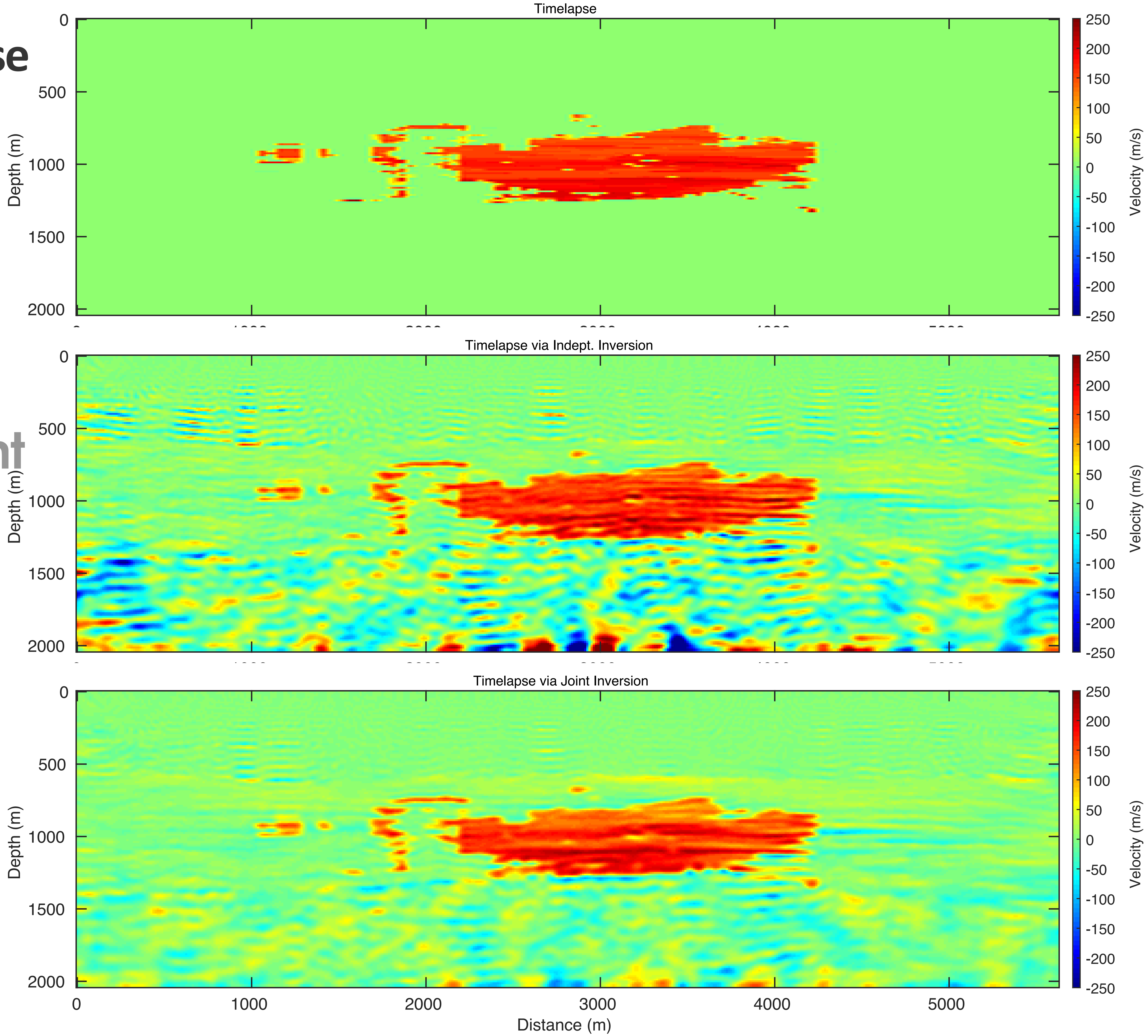


Convergence plot for monitor inversion
joint
indept
model error
Iteration
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0
20
40
60
80
100

True timelapse

Independent inversion

Joint inversion



Observation

Differences in acquisition geometry (source/receiver locations) impact the quality of the time-lapse difference

Artifacts due to acquisition difference are attenuated with the joint inversion

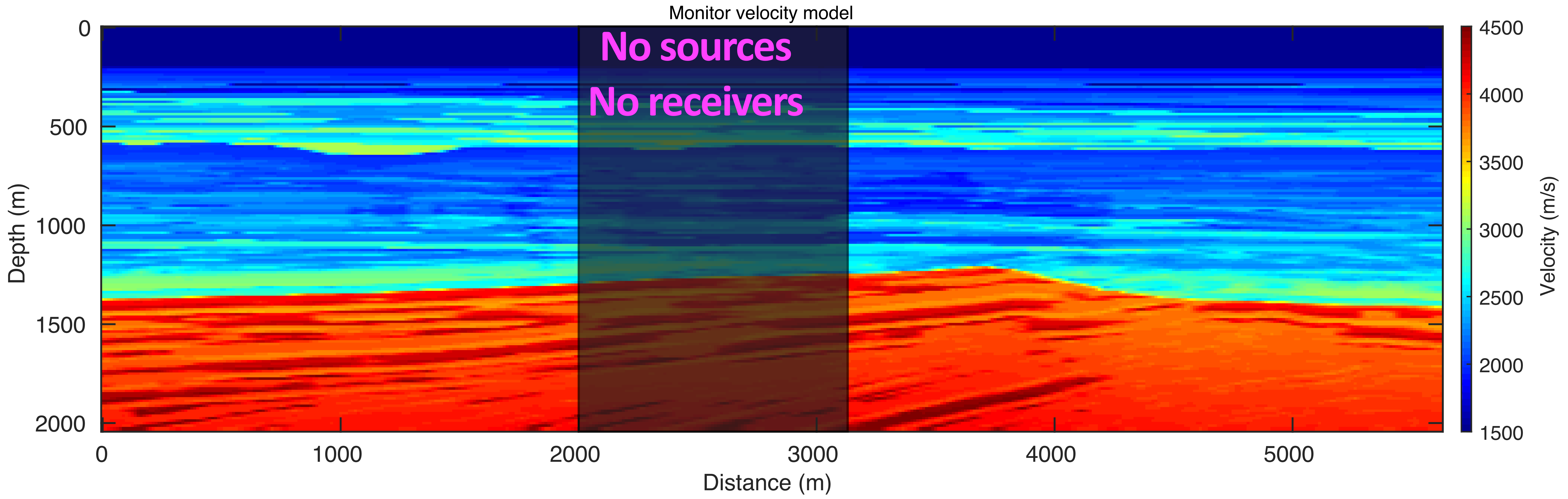
Better image below and above the gas cloud

More realistic scenario

Monitor inversion
– *with acquisition gap*

Modeling parameters

- Presence of **acquisition gap**, nominal sampling of 50m for sources outside the gap



Monitor inversion
– *with acquisition gap*

Modeling parameters

- Presence of **acquisition gap**, nominal sampling of 50m for sources outside the gap
- Fewer sources/receivers relative to baseline
- Co-located sources and receivers
- 80 frequencies from 3 to 22.5Hz

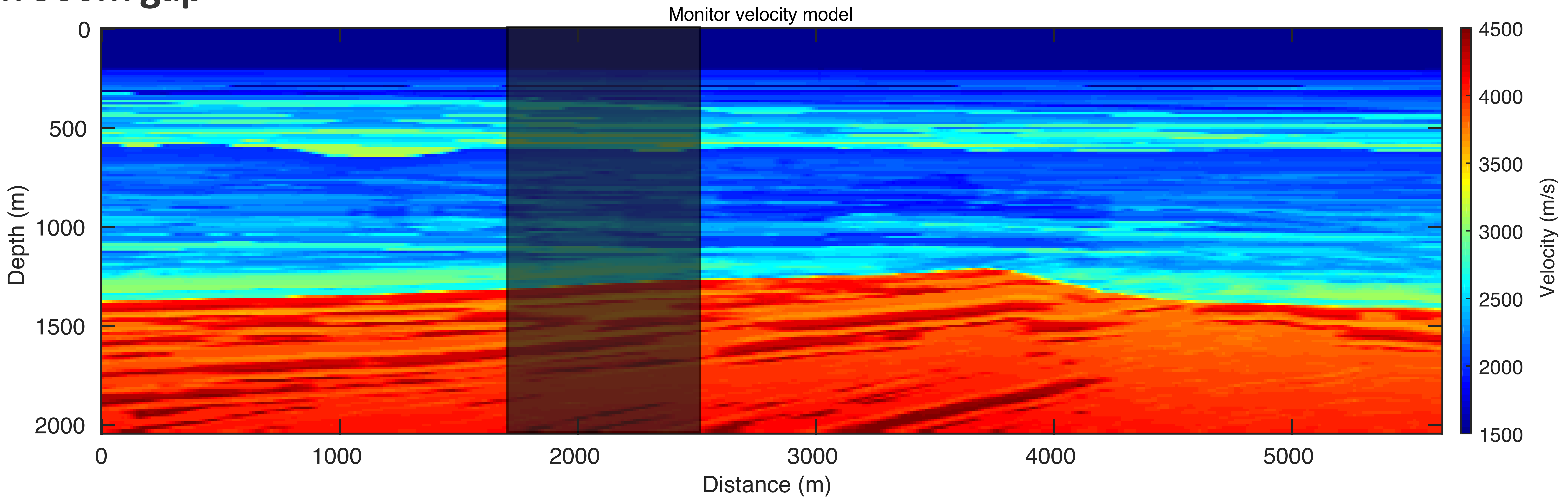
Modified Gauss-Newton

- Assume *good* background velocity model (same as baseline starting model)
- *Monitor* : use few randomly selected shots, with renewal
- Started inversion at 3Hz, 8 frequencies per band
- 10 Gauss-Newton subproblems per band
- Approximately 10 iterations per subproblem

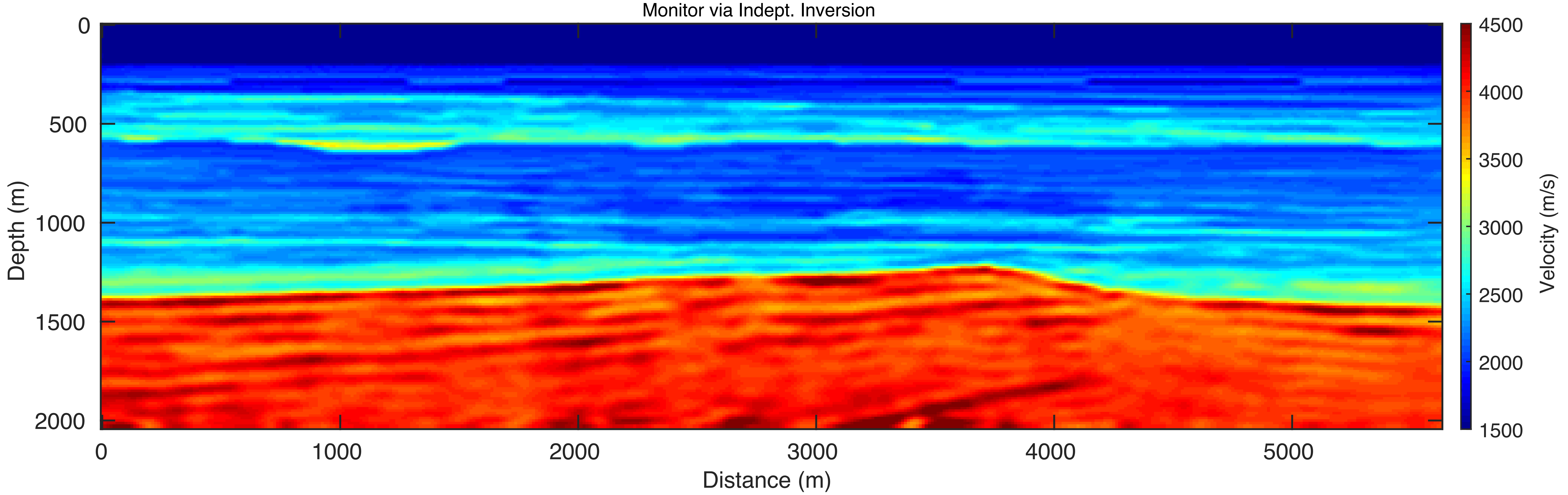
Acquisition gap of 500m

True monitor

-with 500m gap

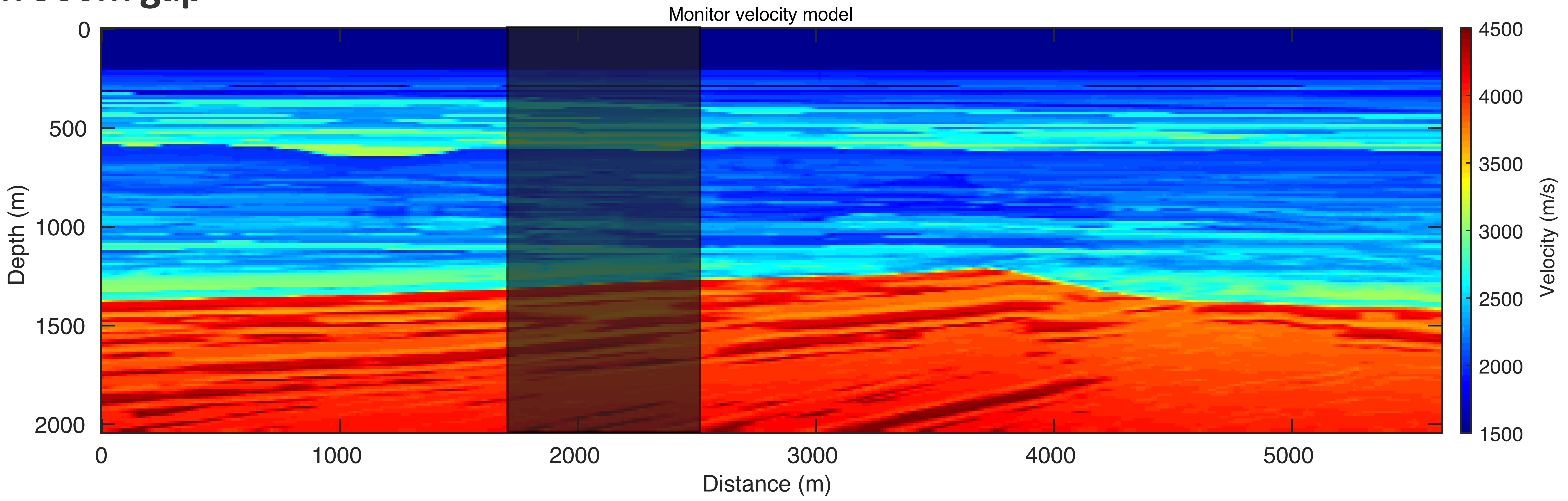


Independent inversion

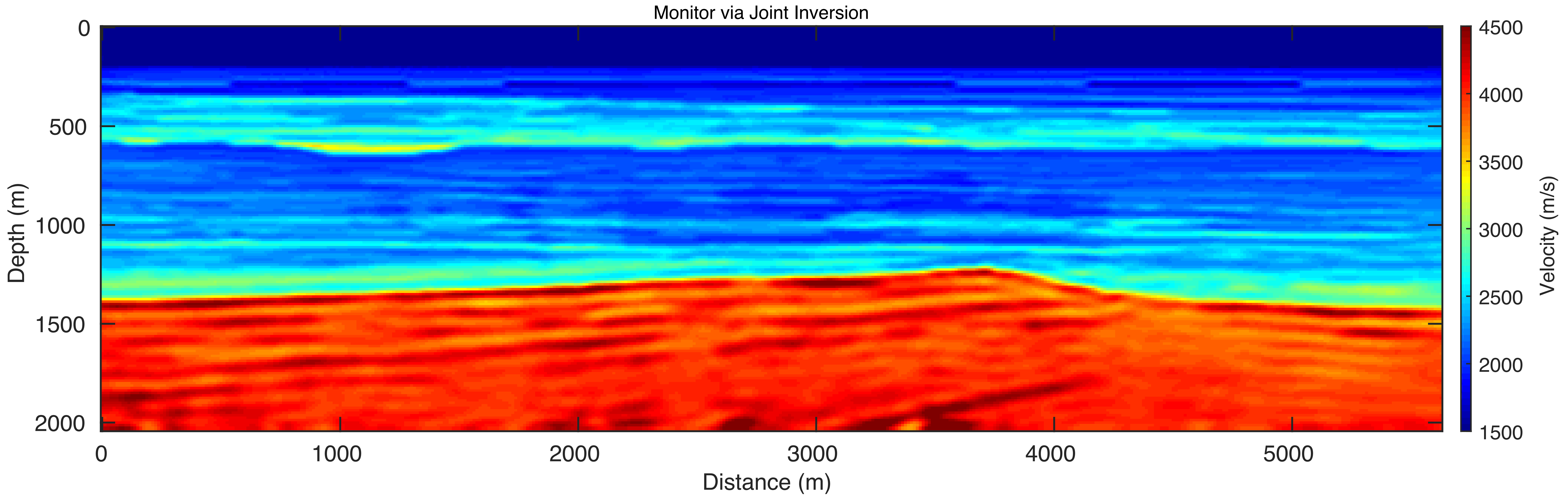


True monitor

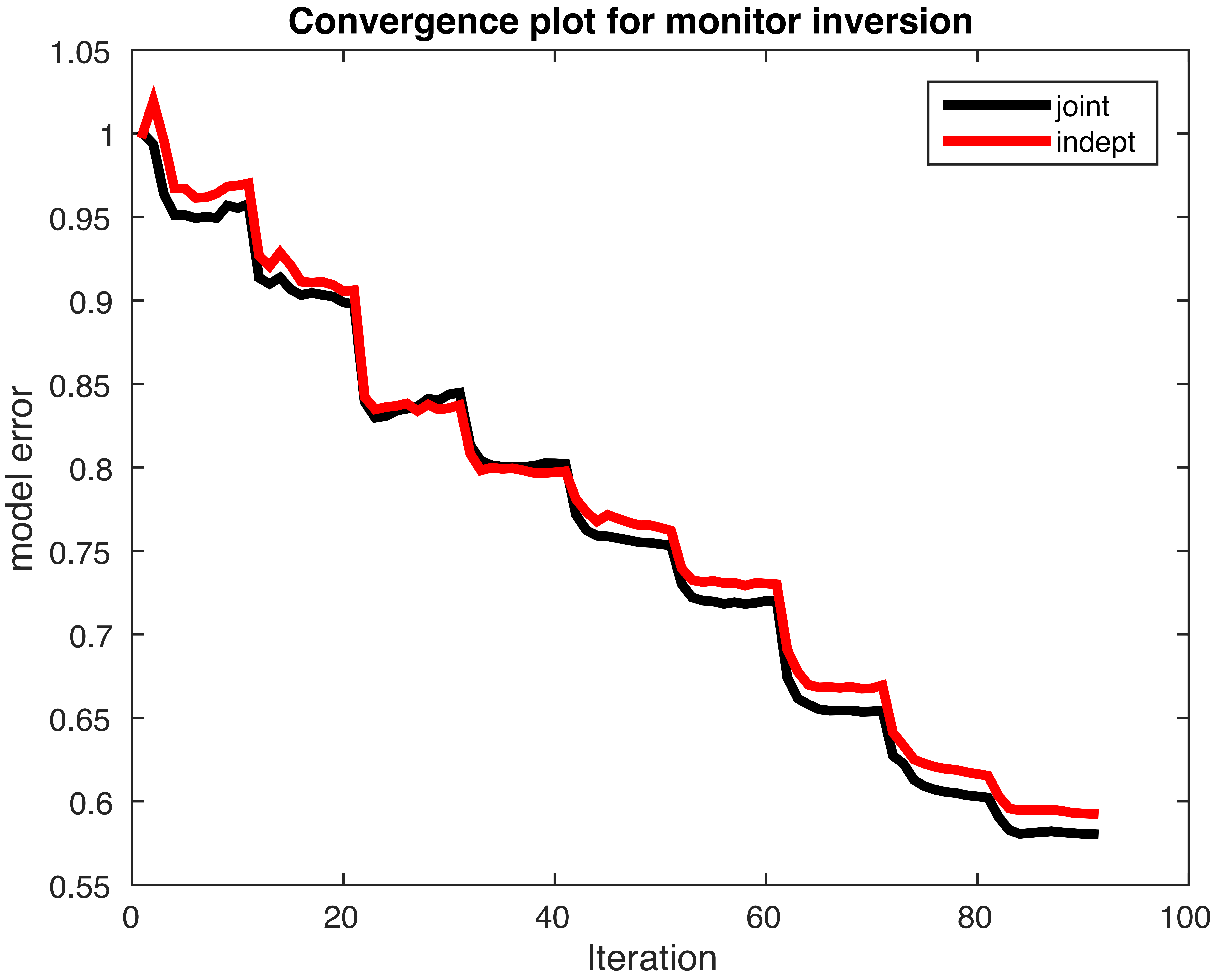
-with 500m gap



Joint inversion

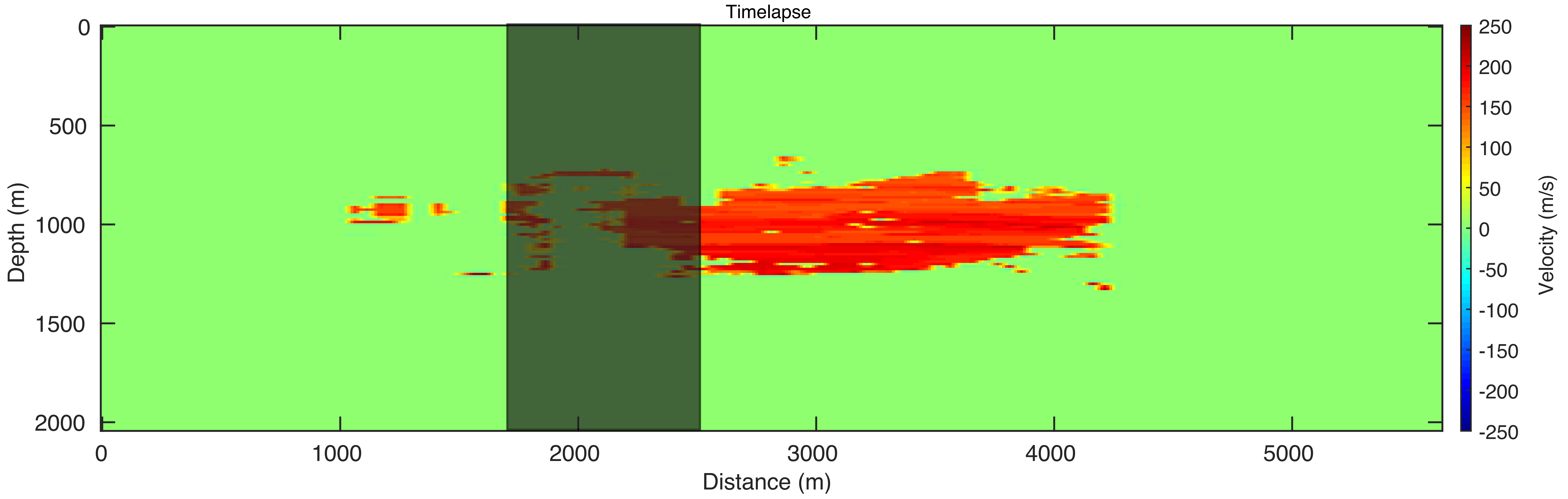


Model error

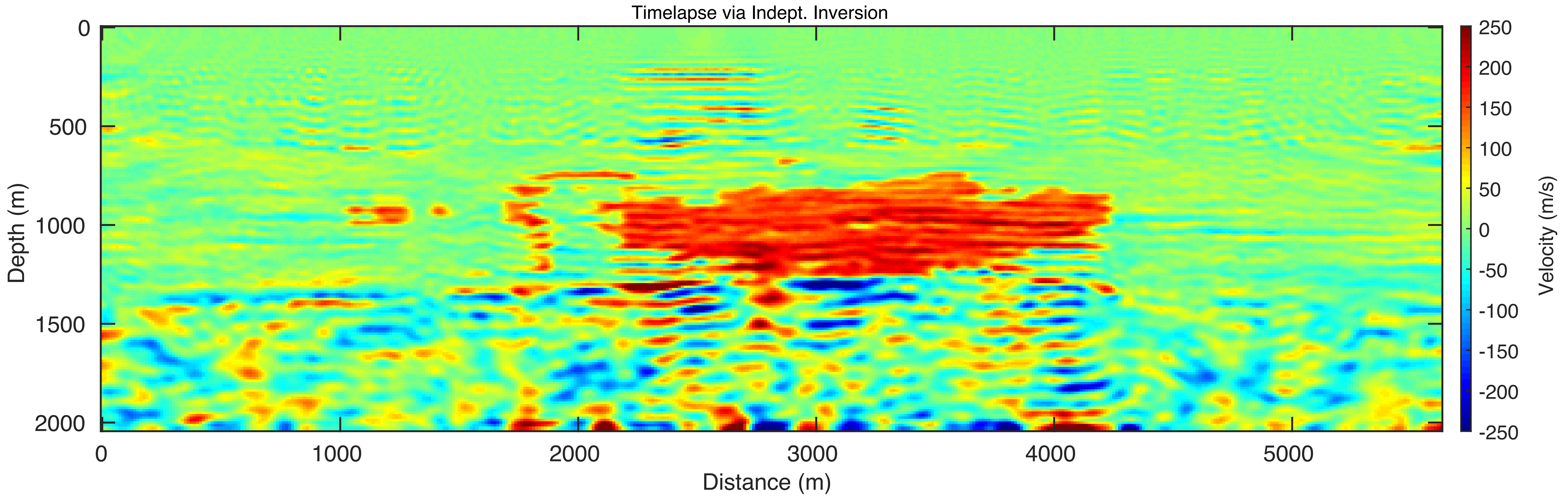


Convergence plot for monitor inversion
joint
indept
model error
Iteration
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0
20
40
60
80
100

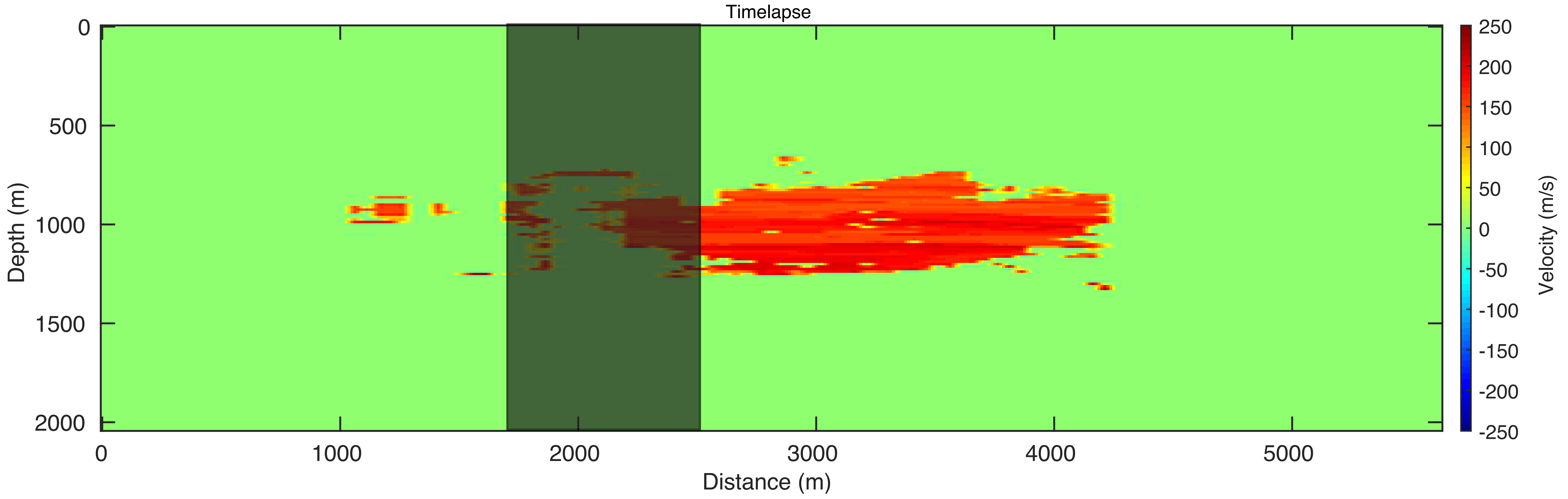
True timelapse



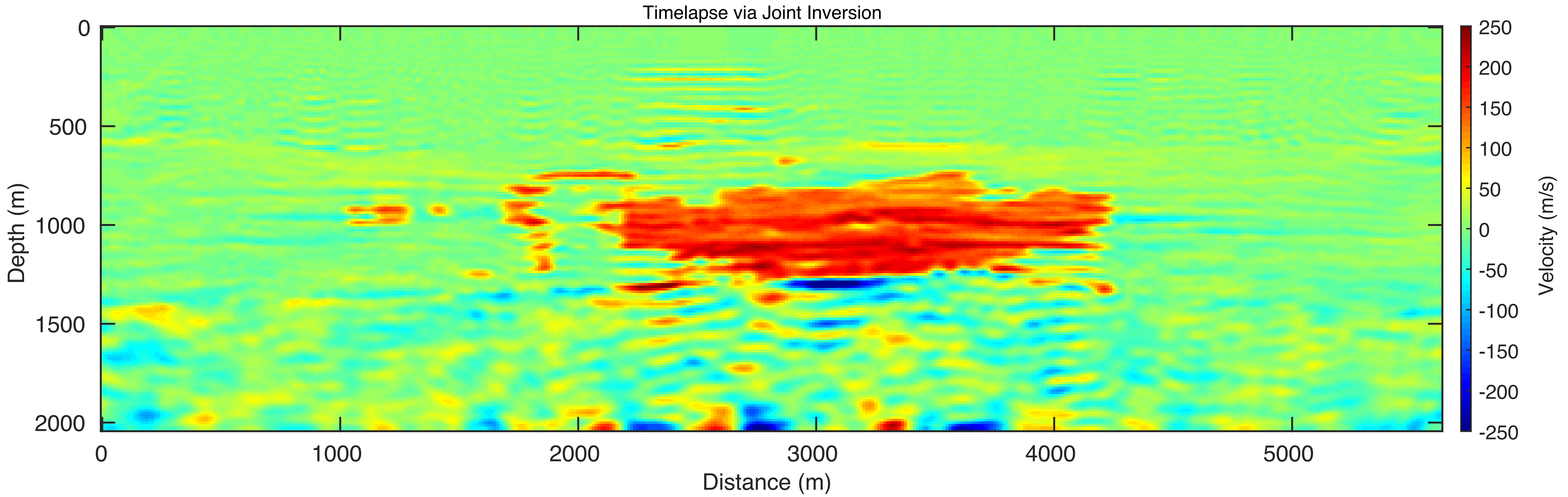
Independent inversion



True timelapse



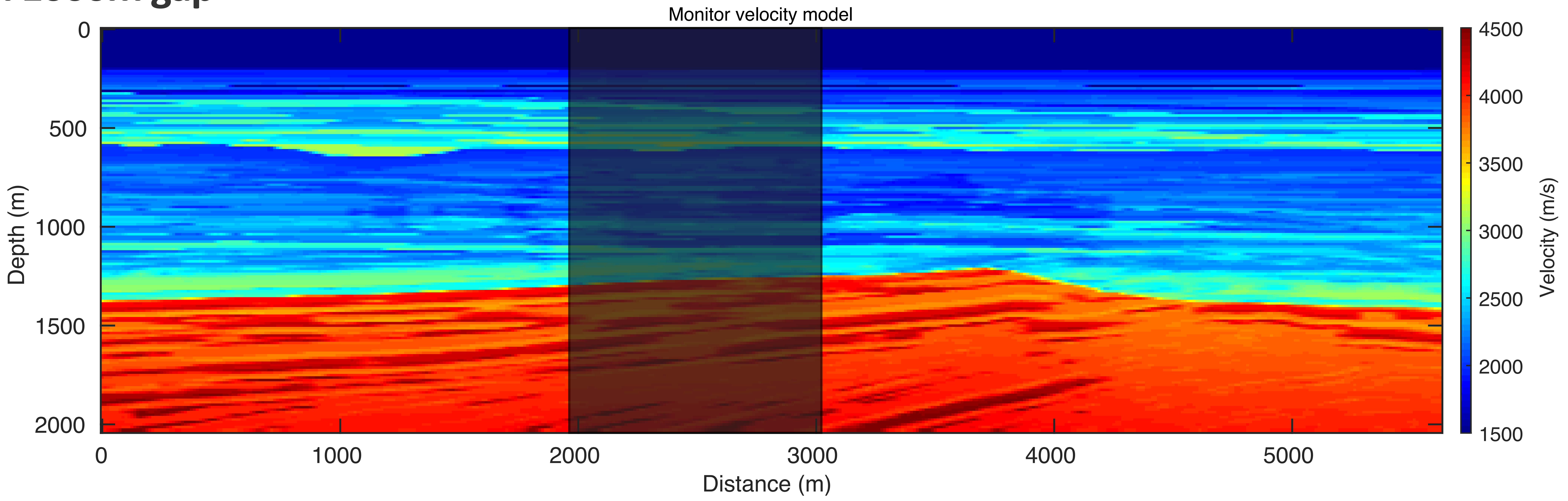
Joint inversion



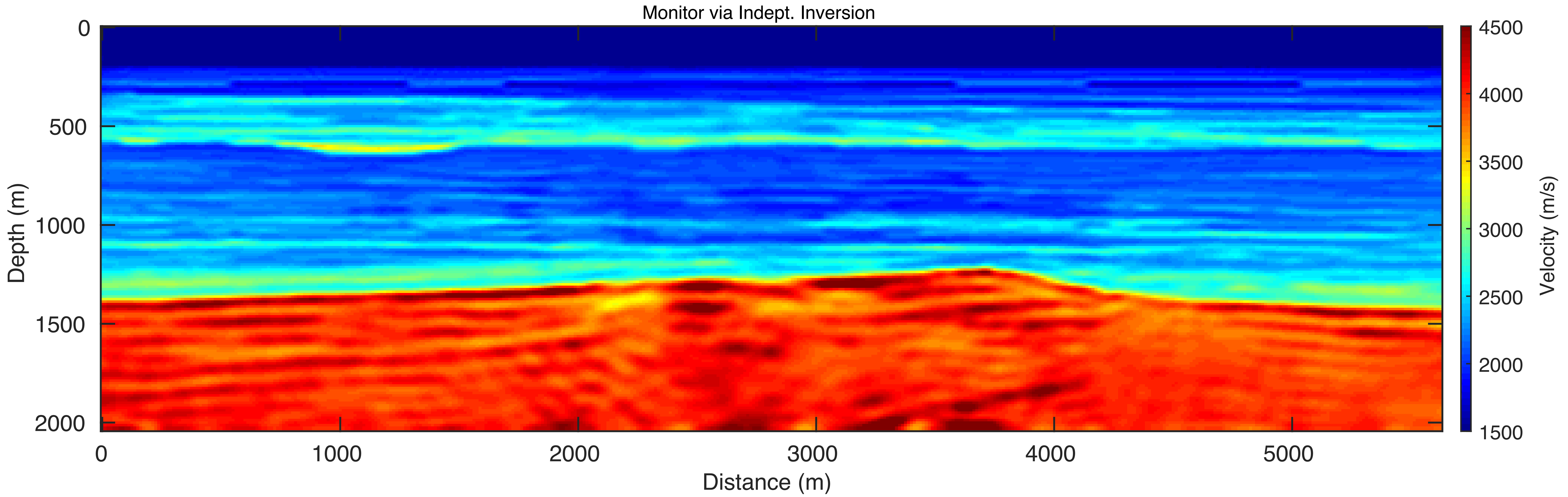
Acquisition gap of 1000m

True monitor

-with 1000m gap

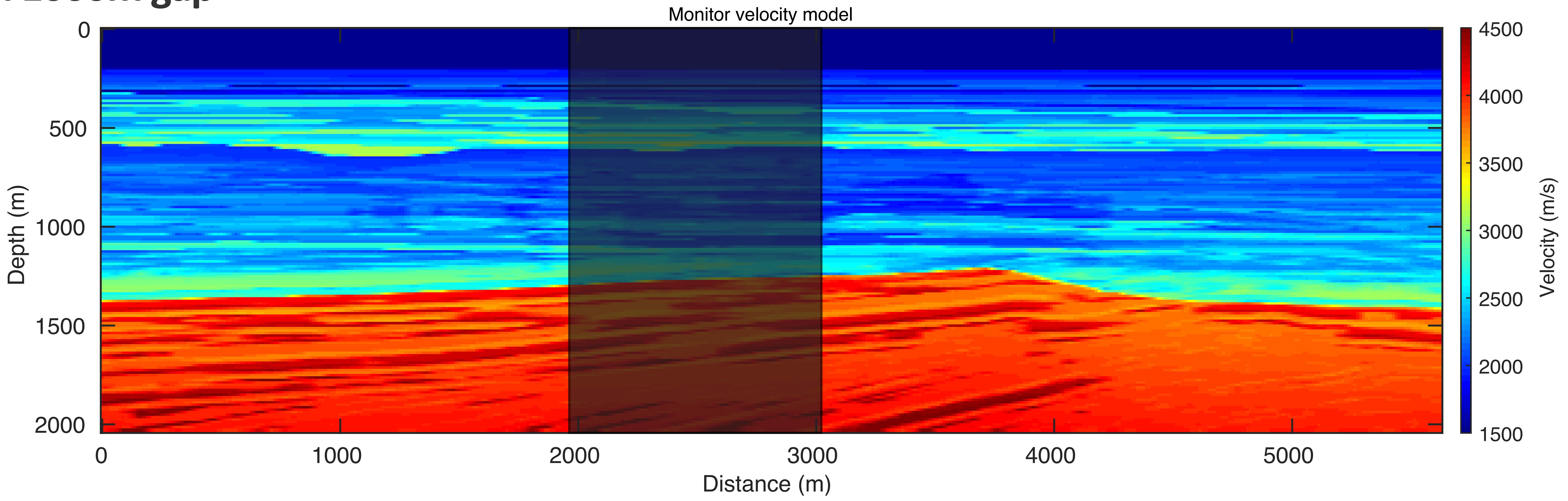


Independent inversion

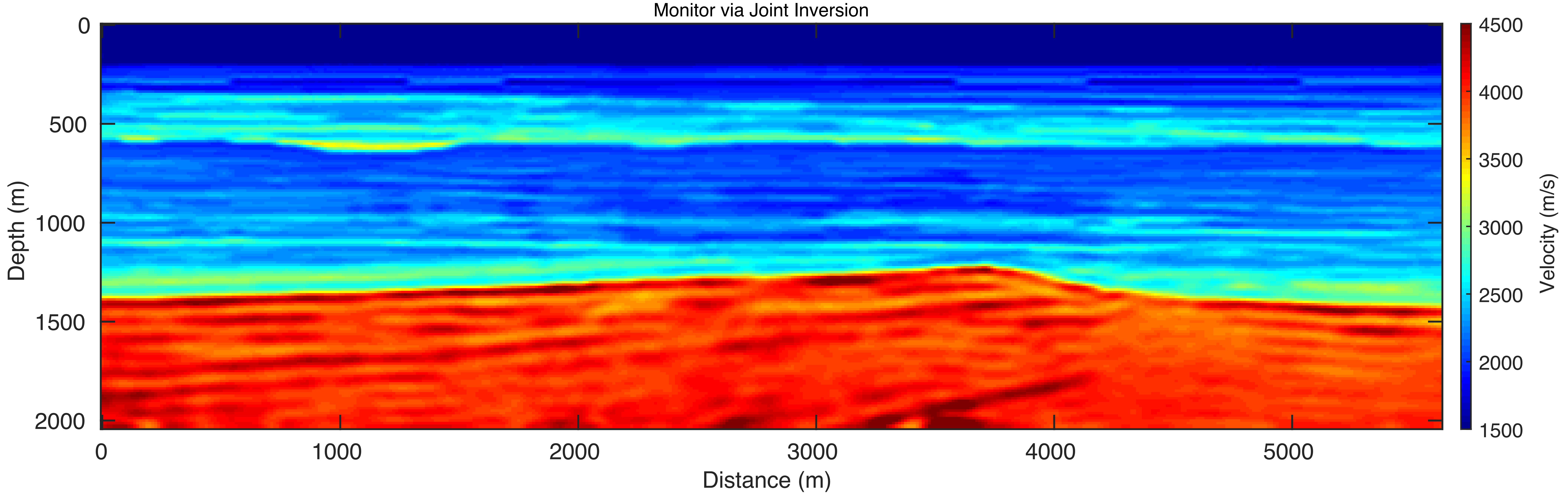


True monitor

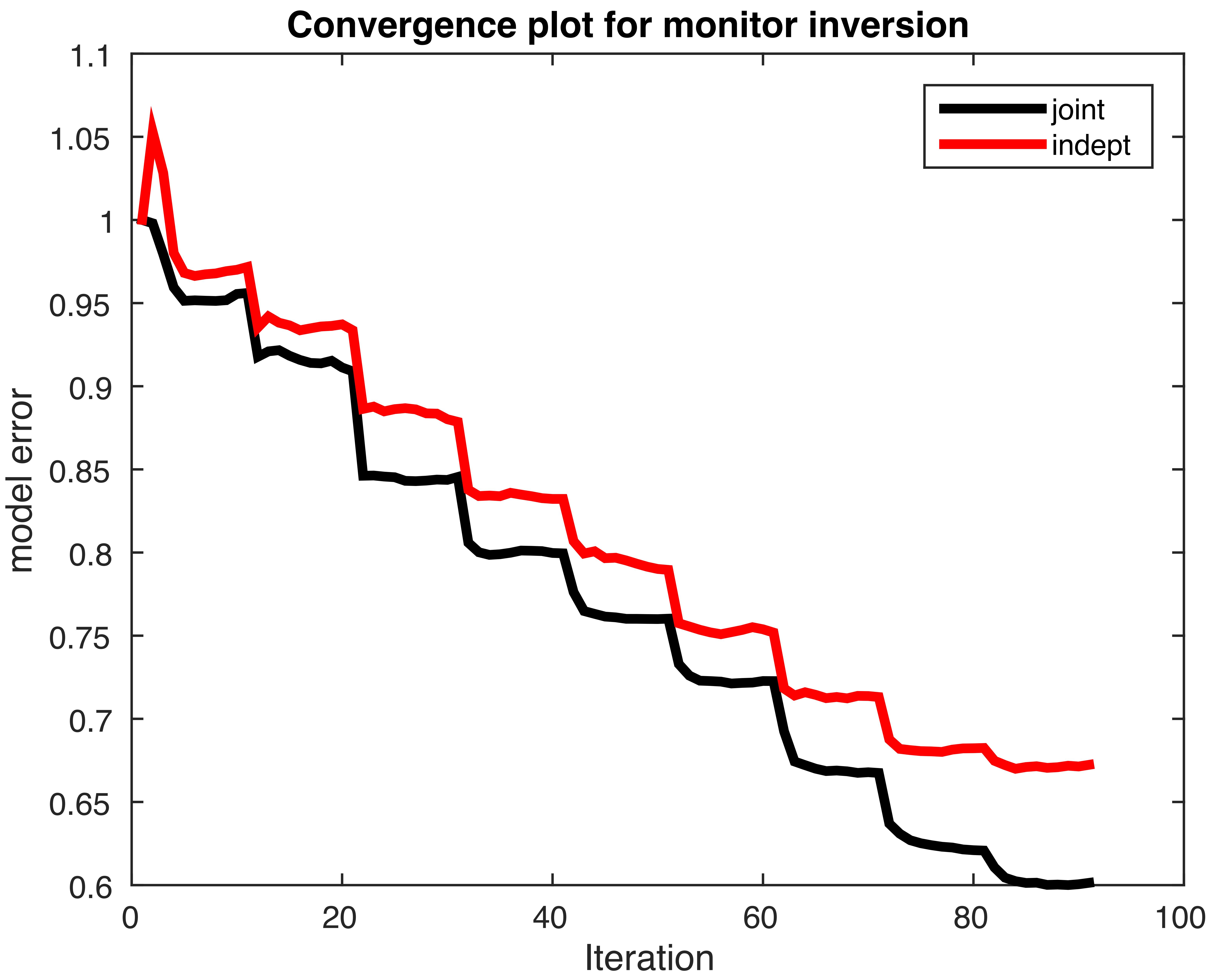
-with 1000m gap



Joint inversion

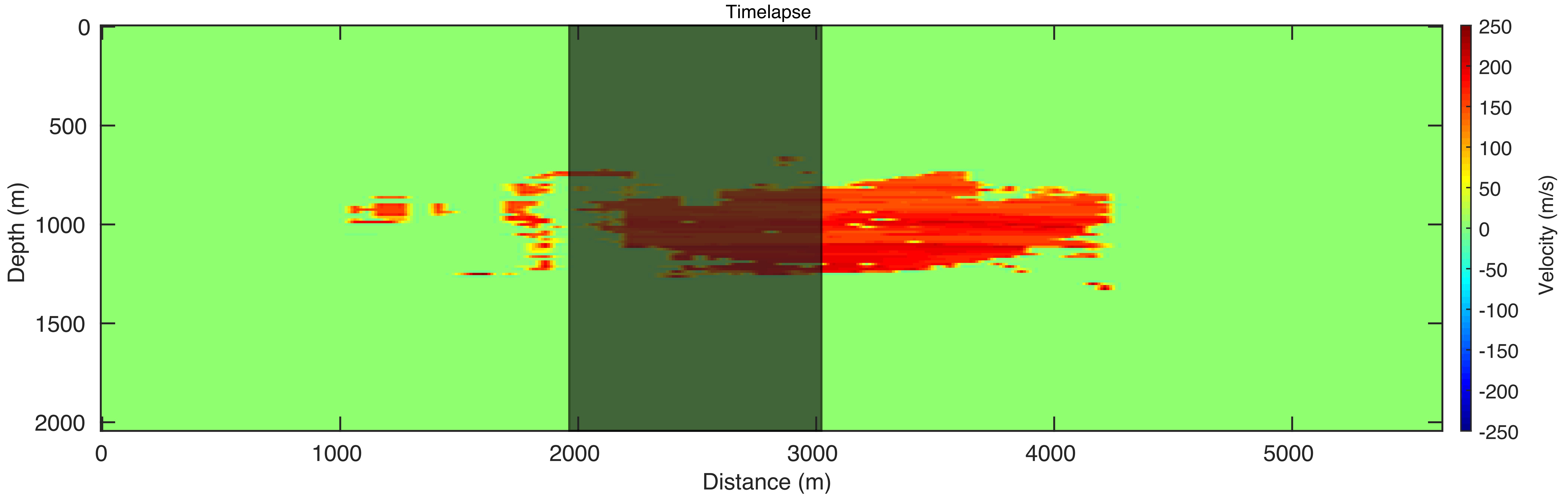


Model error

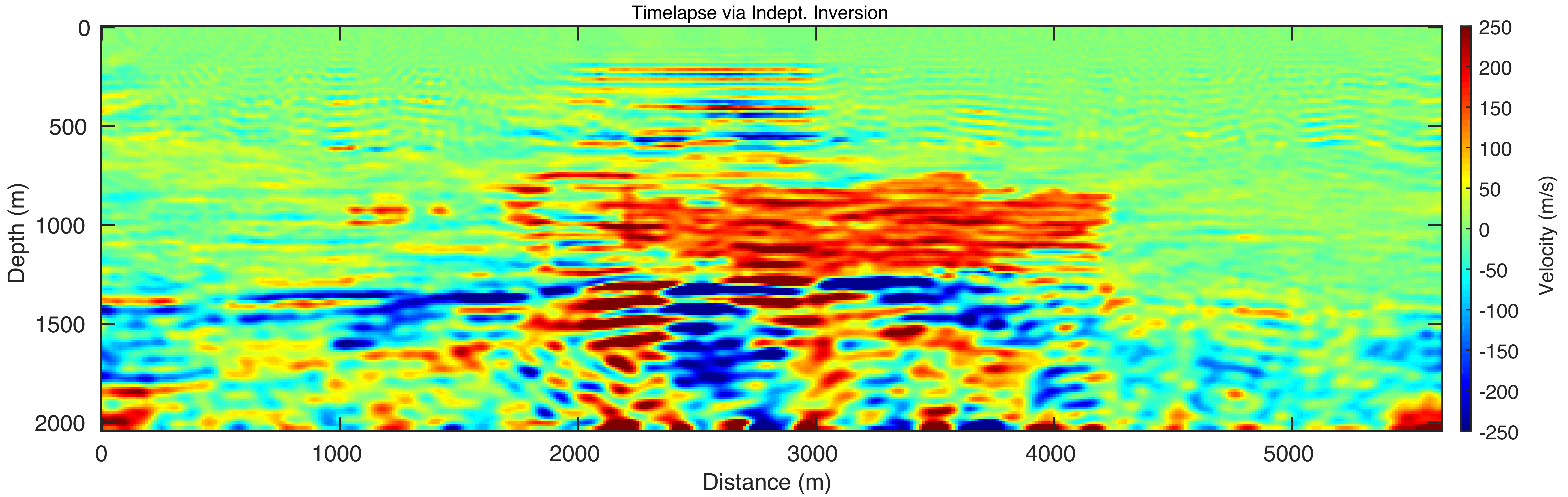


Convergence plot for monitor inversion
joint
indept
model error
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0
20
40
60
80
100
Iteration

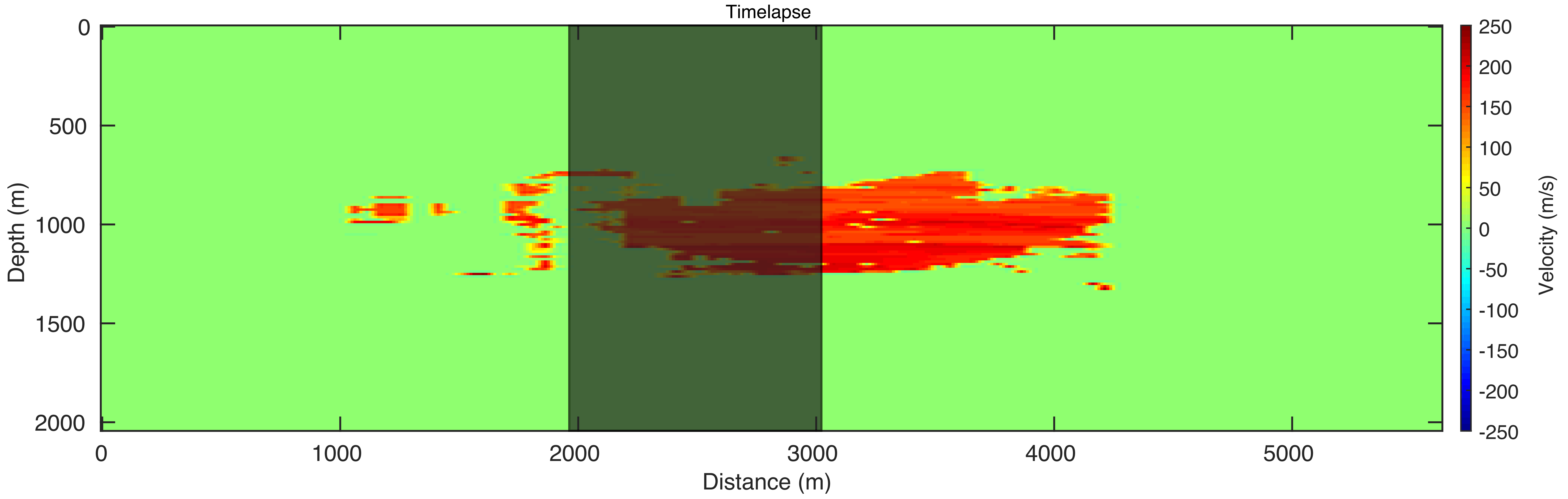
True timelapse



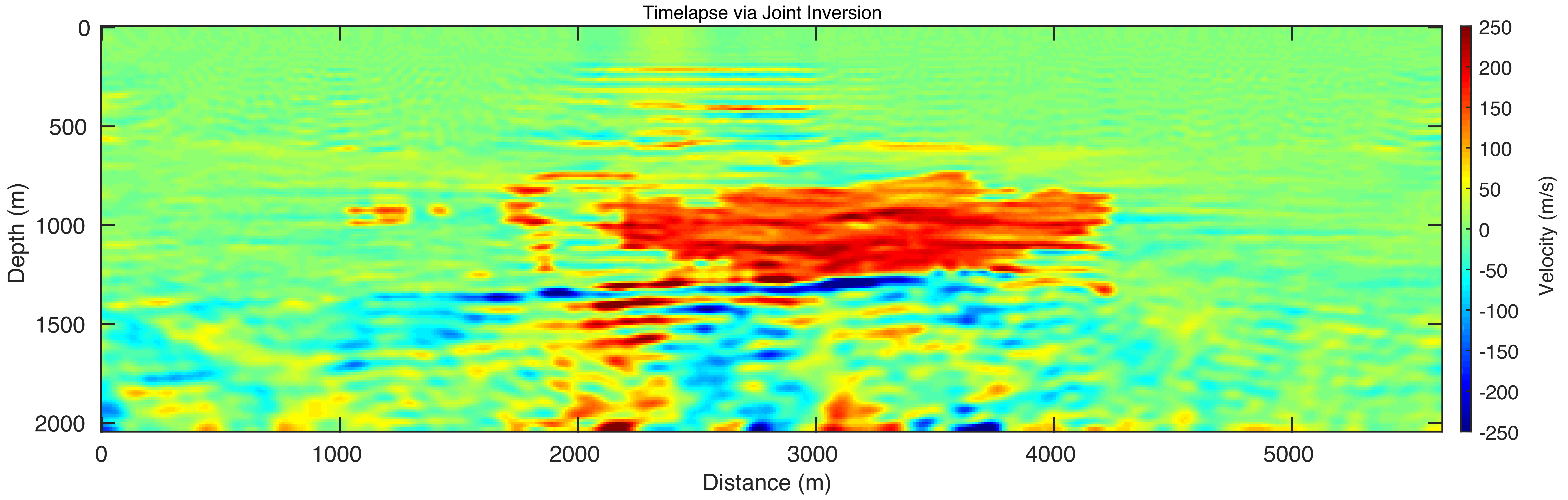
Independent inversion



True timelapse



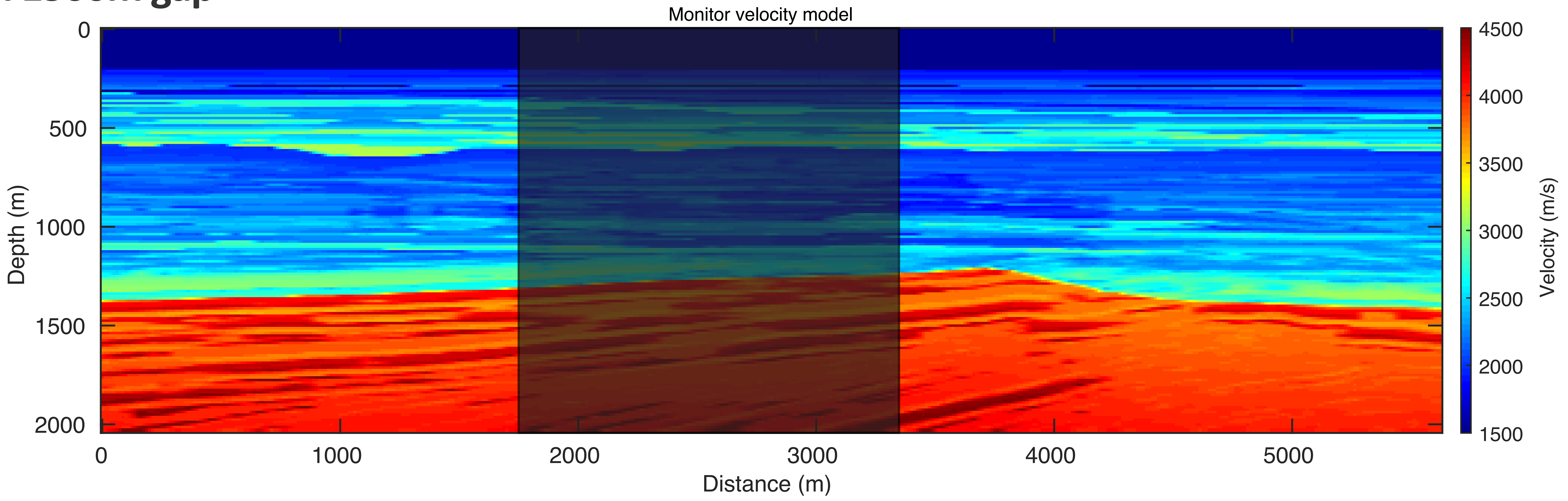
Joint inversion



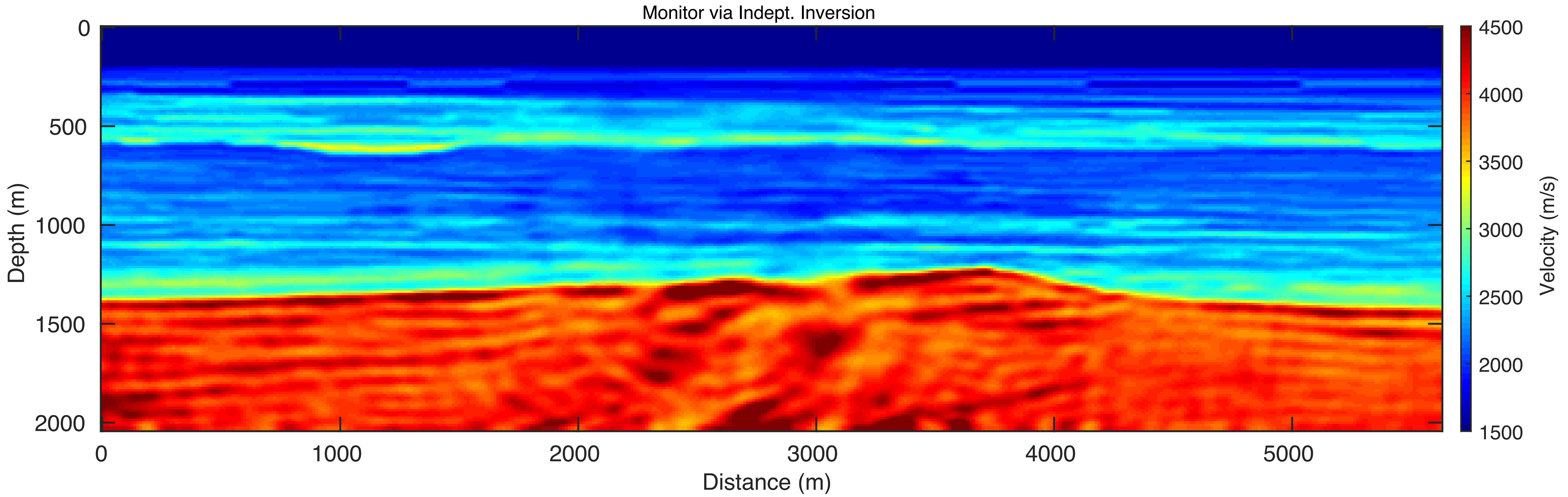
Acquisition gap of 1500m

True monitor

-with 1500m gap

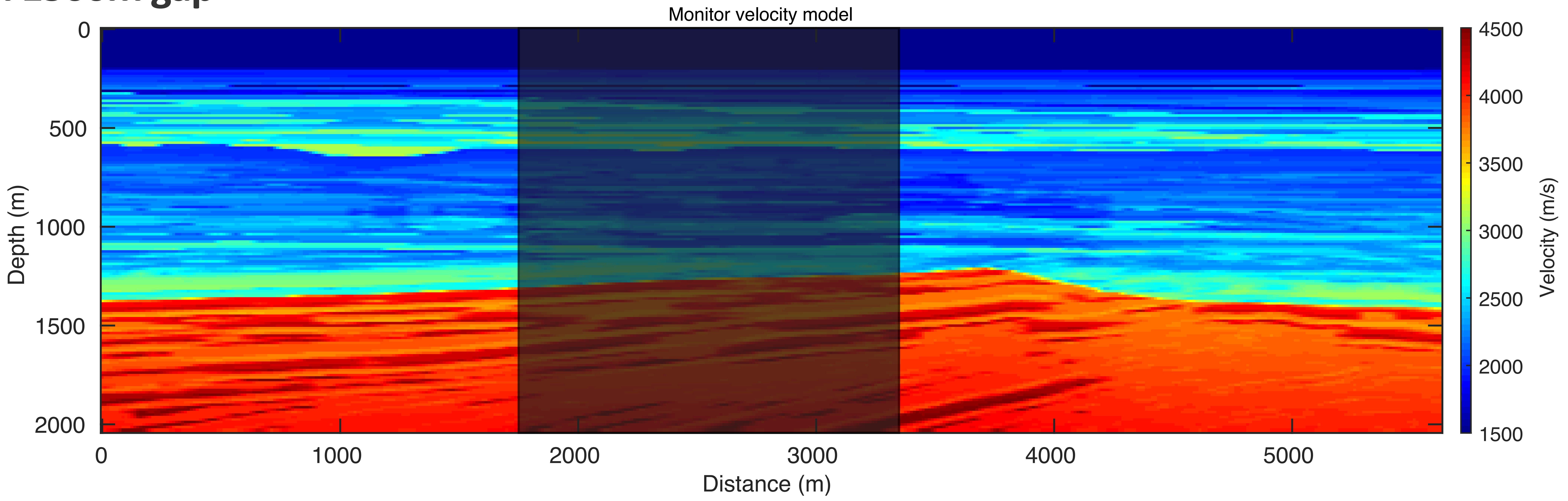


Independent inversion

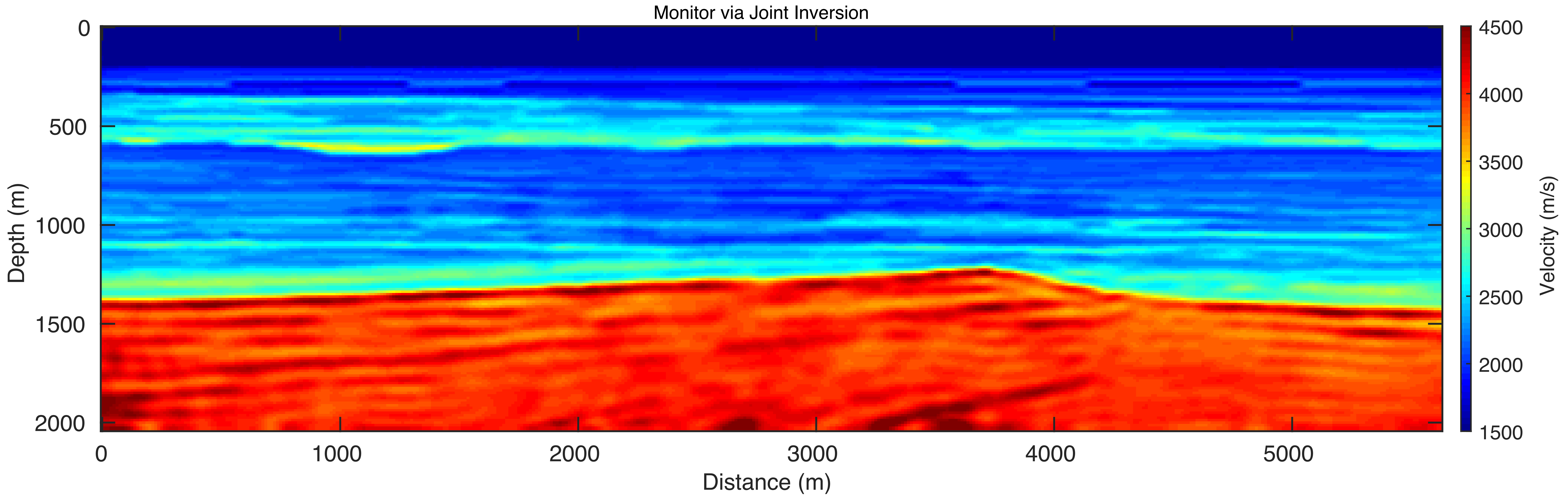


True monitor

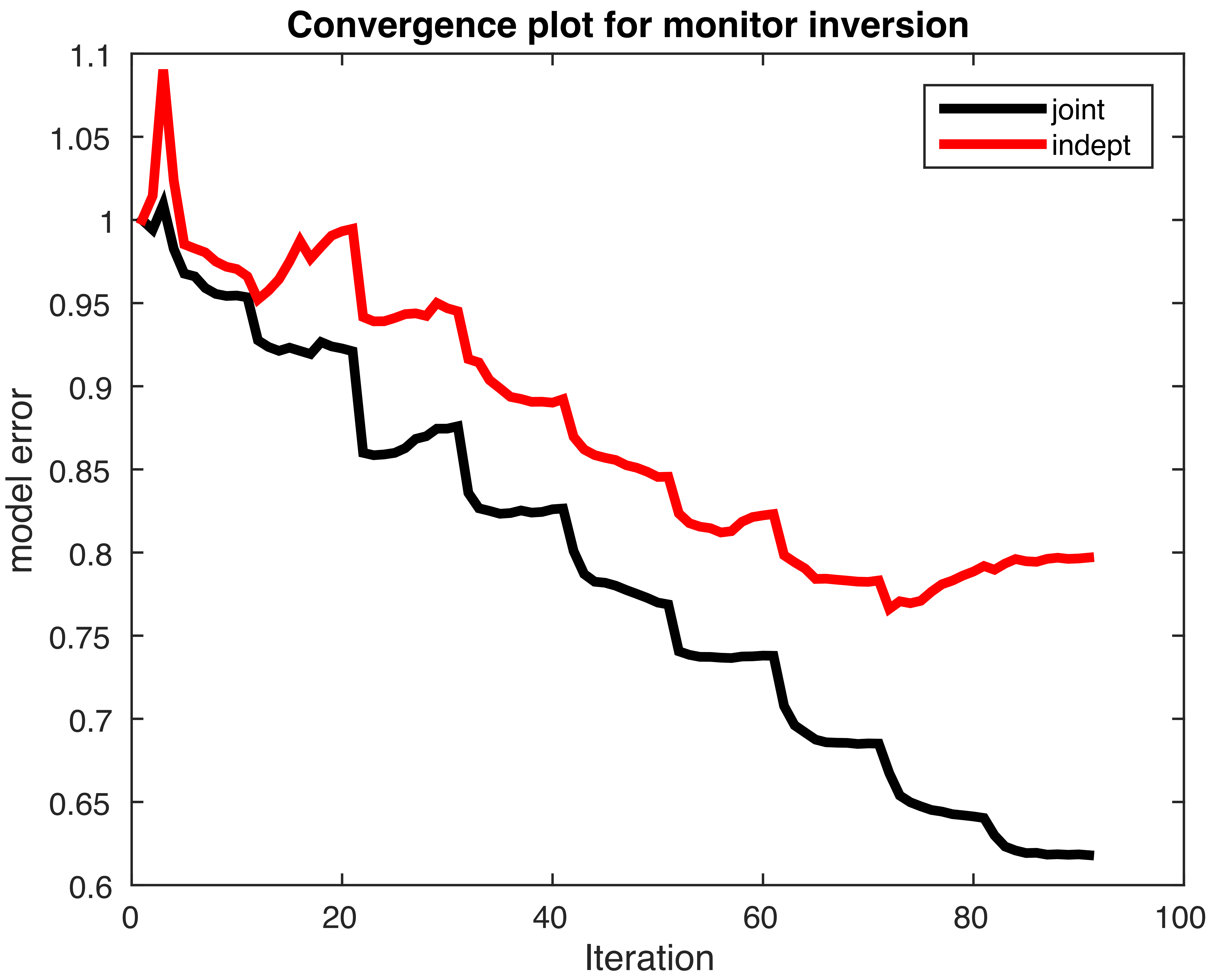
-with 1500m gap



Joint inversion

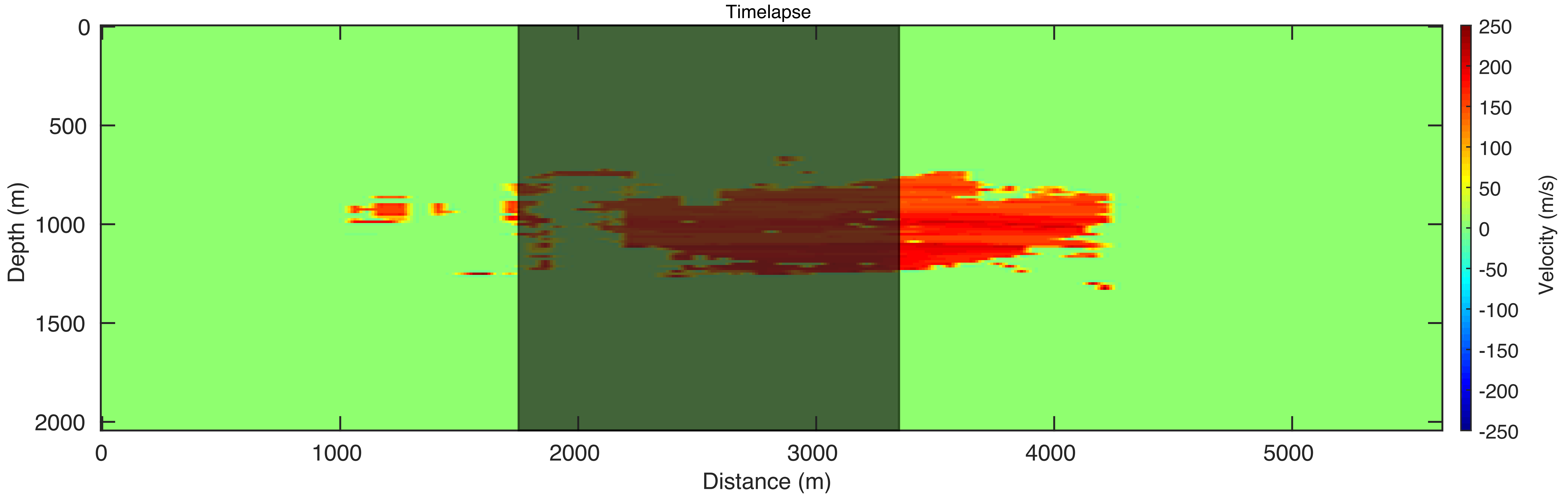


Model error

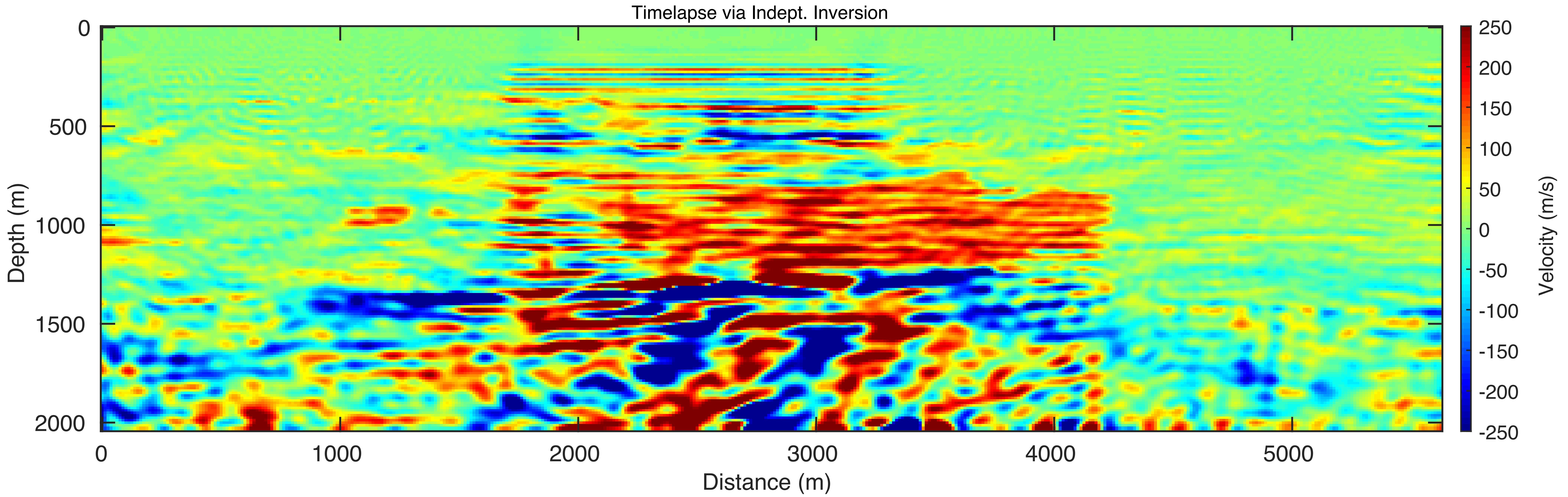


Convergence plot for monitor inversion
joint
indept
model error
Iteration
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0
20
40
60
80
100

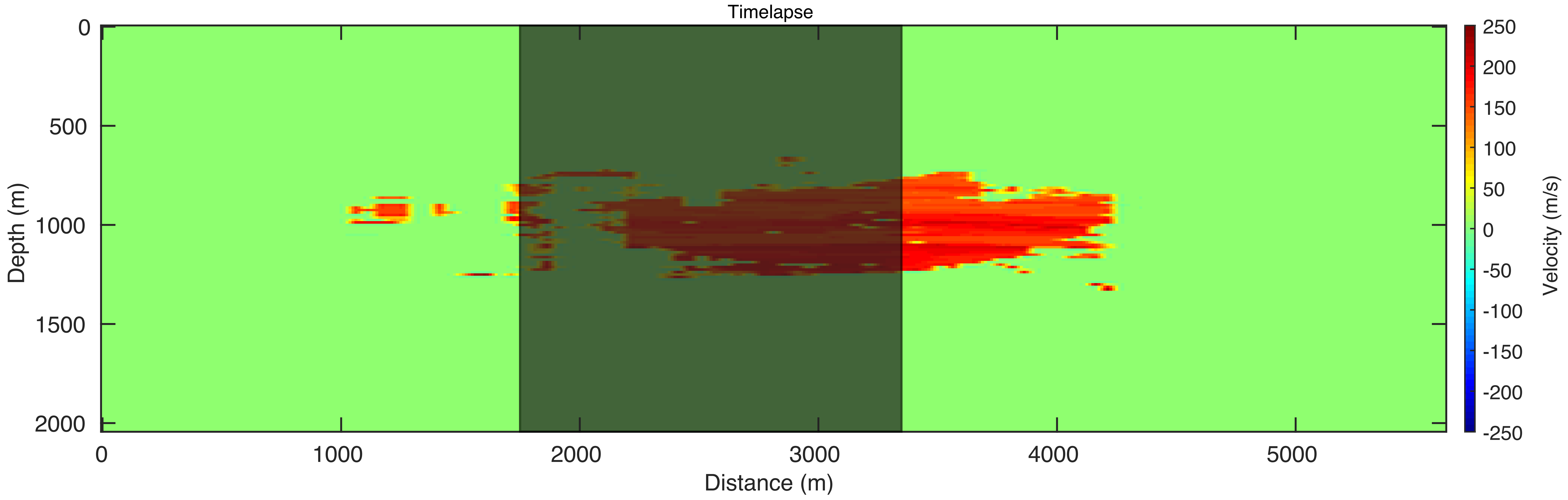
True timelapse



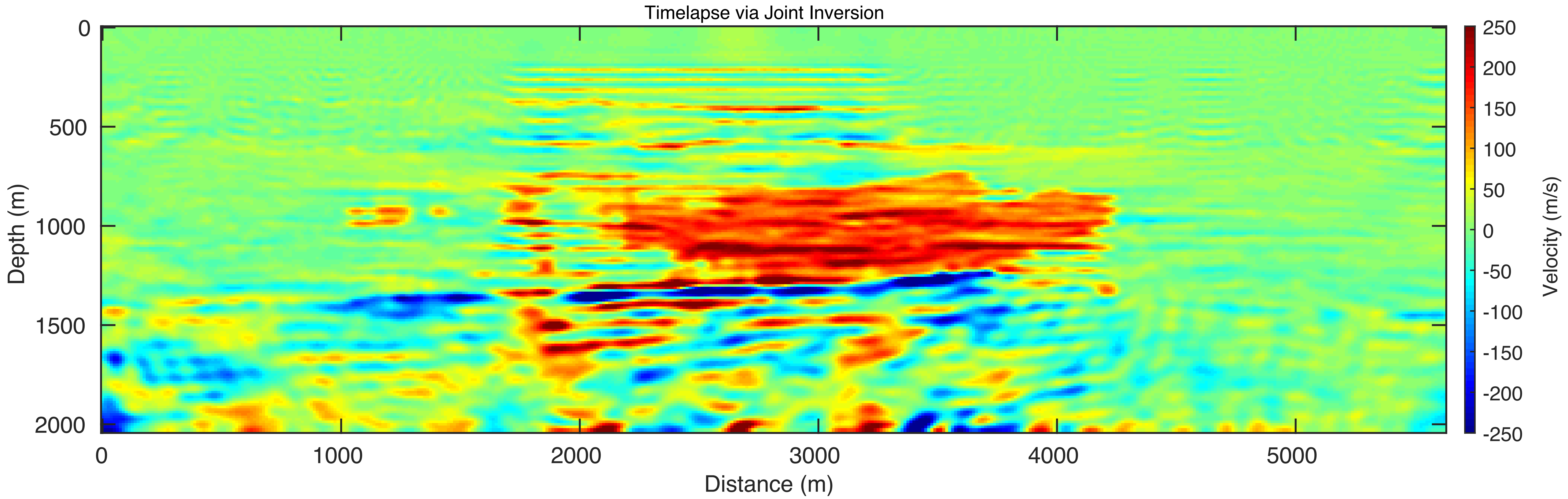
Independent inversion



True timelapse

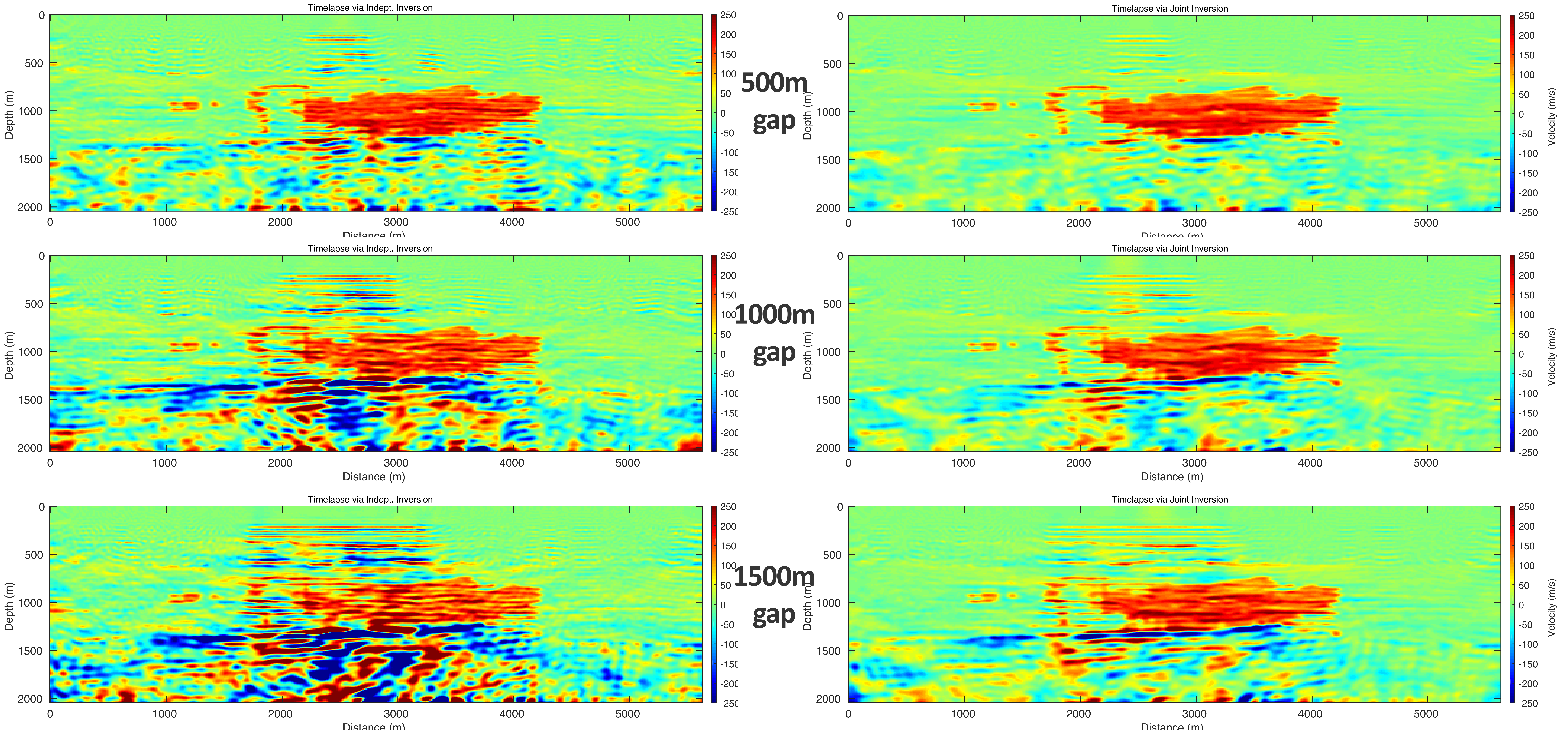


Joint inversion



Independent inversion

Joint inversion



Conclusions

Independent FWI on time-lapse data is more prone to errors in the time-lapse difference.

Larger acquisition gaps adversely affect the time-lapse difference.

Joint inversion with distributed compressed sensing is a more preferable approach, and gives better time-lapse models

Significant attenuation of artifacts in time-lapse difference model with the *joint recovery model*

"The key is in exploiting the shared information".

SLIM

Acknowledgements

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