A Correlation-based Misfit Criterion for Wave-equation Traveltime Tomography

Tristan van Leeuwen

Felix J. Herrmann, Wim Mulder

SLIM

University of British Columbia

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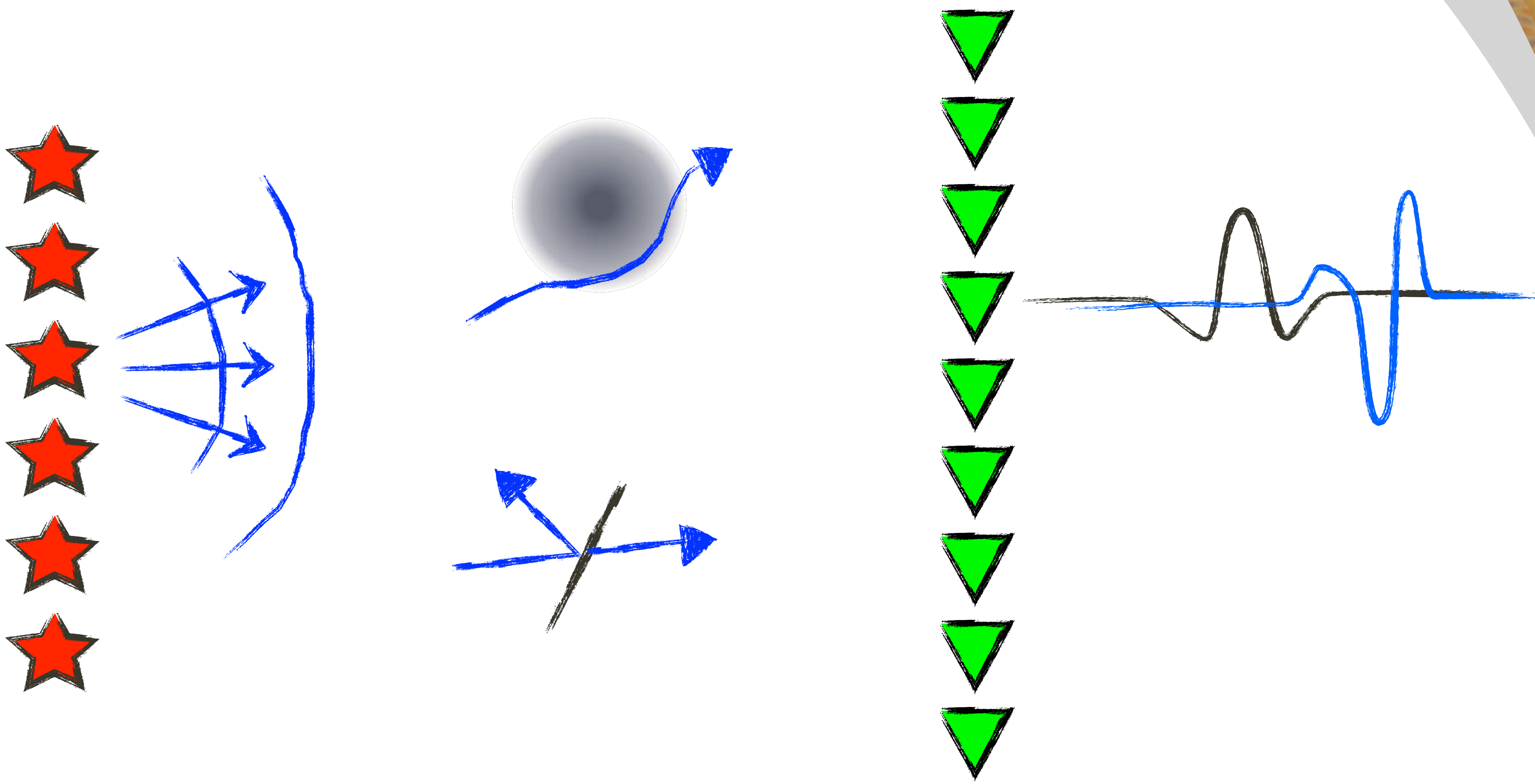
Contemporary Issues in Geophysical Inversion:
Imaging and Velocity

Waveform imaging



Overview

- Waveform tomography
- Wavefrontset detection
- Misfit criteria
- Numerical example
- Future work & Conclusions

Waveform tomography

Model the data as

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right] u = w(t)\delta(x - s),$$

$$d(t, s, r) = u(t, x, s)|_{x=r} \equiv F[c].$$

Goal is to find the velocity given the data and source signature

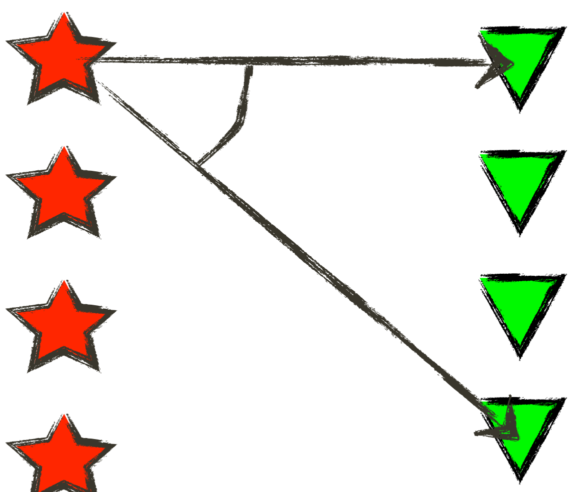
Waveform tomography

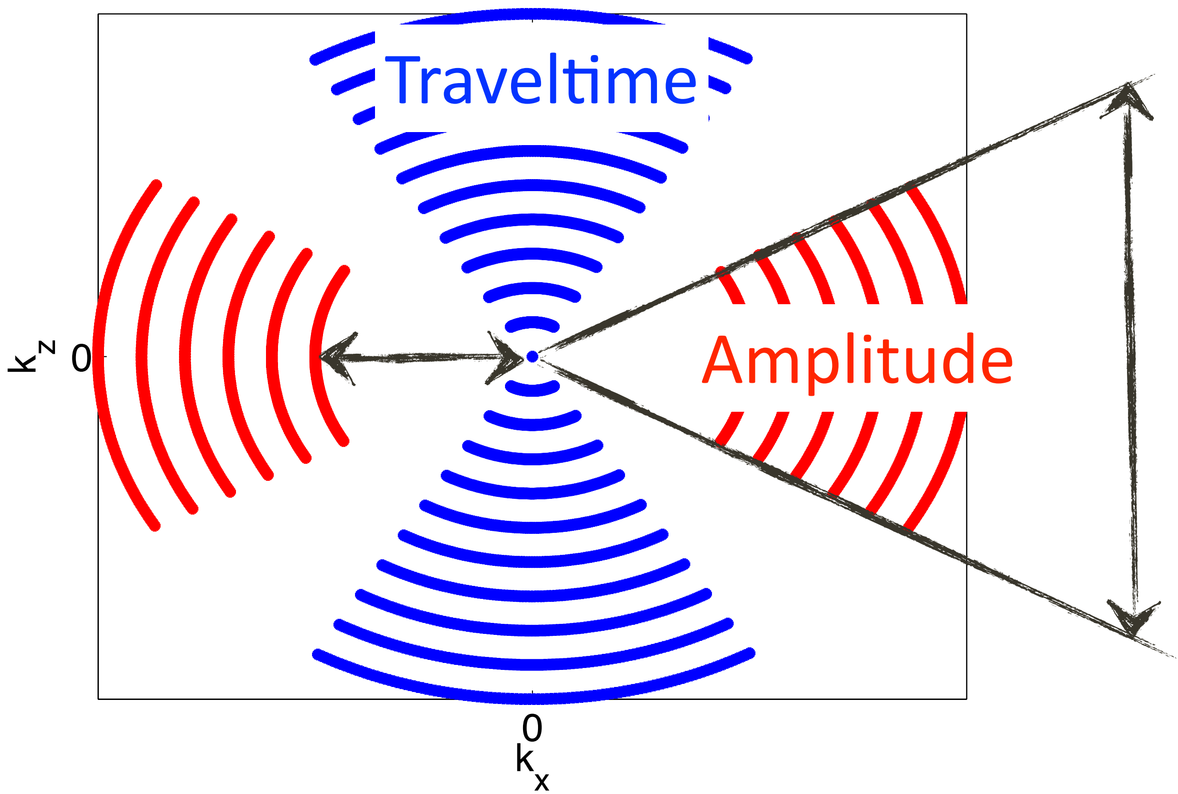
Such inverse problems have been extensively studied. Major findings:

- recovery via LS is problematic for bandlimited data
- some form of traveltime fitting needed for `complete' reconstruction

Waveform tomography

Wavenumber coverage with limited aperture

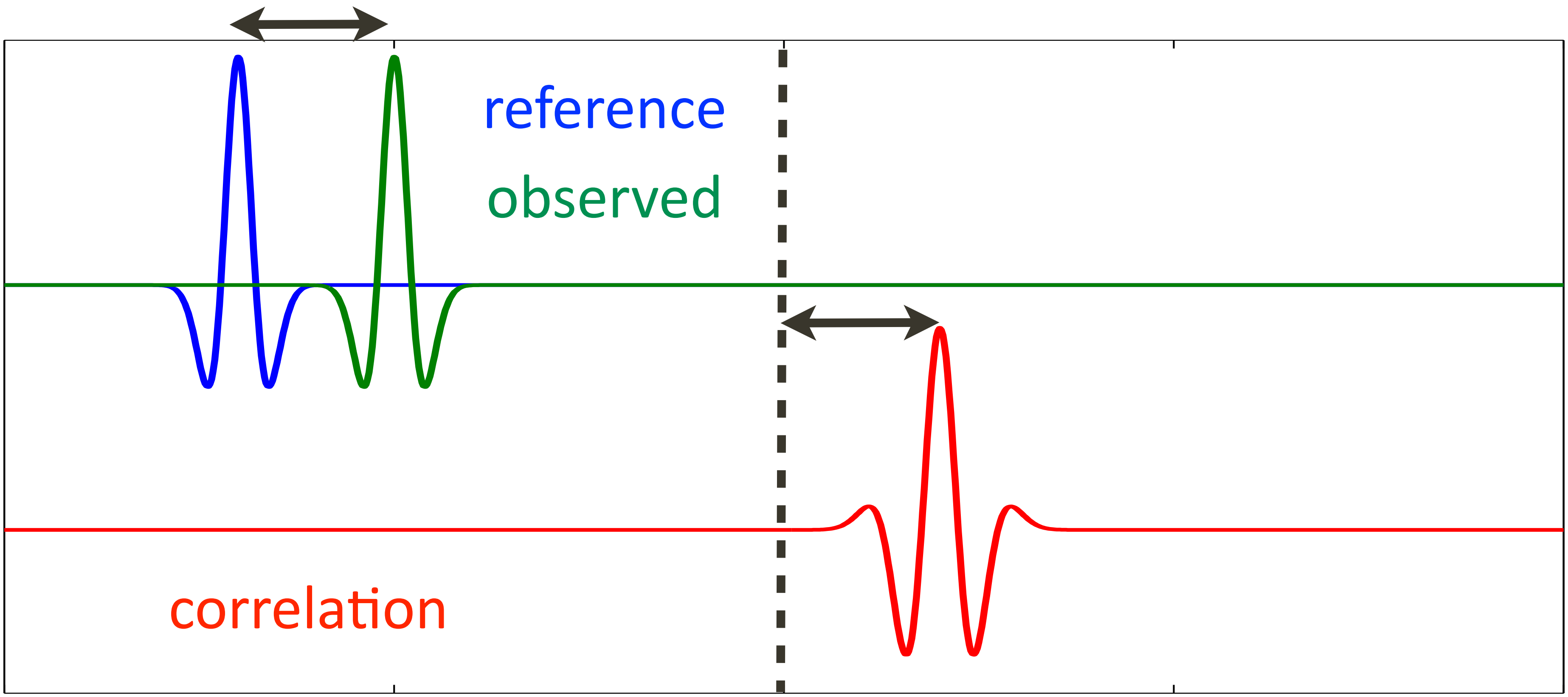




[Stork; Bube; Natterer;]

Waveform tomography

Wave-equation traveltime tomography



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Waveform tomography

WE traveltime tomography:

- relies on detecting shift of singular support
- widely used criterion: maximum of the correlation

$$\min_c ||\tau[c]||_2^2, \quad \tau[c] = \mathrm{argmax}_t (d * \bar{d})(t)$$

[Cara 87; Luo 91; Dahlen 10; Hormann 02; de Hoop 05;Brytik 10]

Waveform tomography

LS may be re-formulated as maximizing the normalized zero-lag correlation

$$||d-\bar{d}||_2^2 = ||d||_2^2 + ||\bar{d}||_2^2 - 2\underbrace{\langle d,\bar{d}\rangle}_{(\bar{d}*d)|_{t=0}}$$

`picking approach' is a clever extension of this

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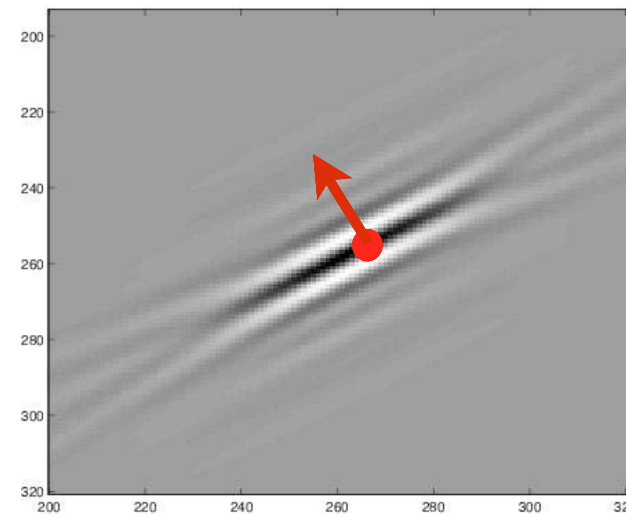
Wavefrontset detection

Given a function of the form

$$f(x,t) = \int \mathrm{d}\omega\, a(\omega,x,t) \exp[\imath\phi(\omega,x,t)]$$

the wavefrontset is given by

$$\mathrm{WF}(f) \subseteq \{x,t; \partial_x\phi, \partial_t\phi \,|\, \partial_\omega\phi = 0\}$$



In particular:

$$\mathrm{WF}(\bar{d} * d) \subseteq \{s, r, \bar{T} - T; \nabla(\bar{T} - T), \imath\omega\}$$

Wavefrontset detection

- Multiscale WF detection via the FBI transform:

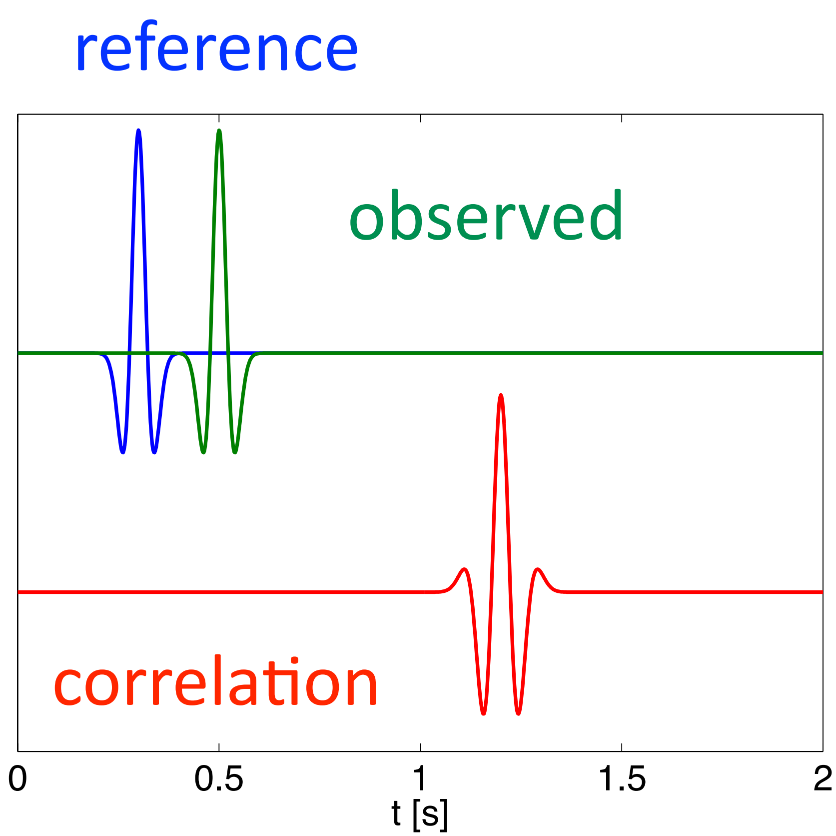
$$G[f](t,\omega,\sigma) = \frac{1}{\sqrt{\sigma}} \int \mathrm{d}t'\, f(t') W[(t-t')/\sigma] \exp[\imath\omega t']$$

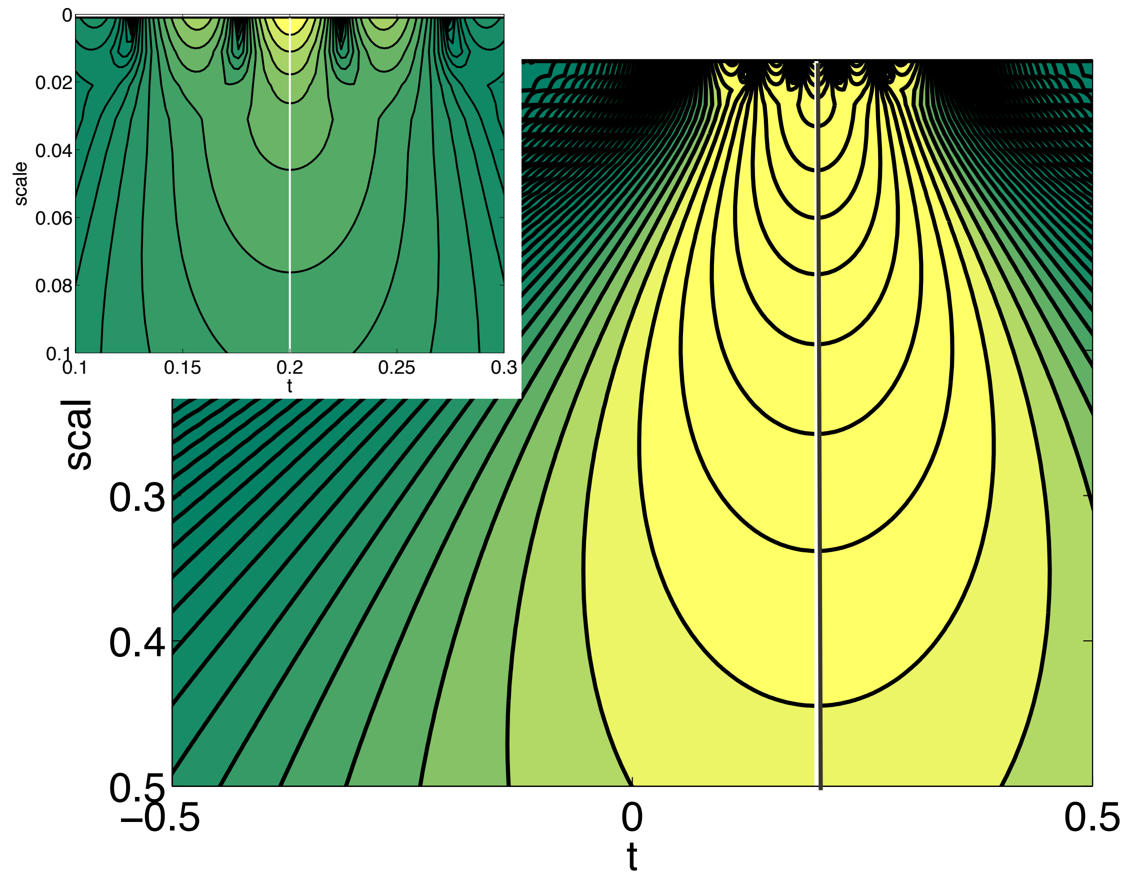
- if $t \not\subset \mathrm{WF}(f)$ then for fixed ω and any $N \in \mathbb{N}$

$$|G[f](t,\omega,\sigma)| \leq \sigma^N \quad \text{as} \quad \sigma \downarrow 0$$

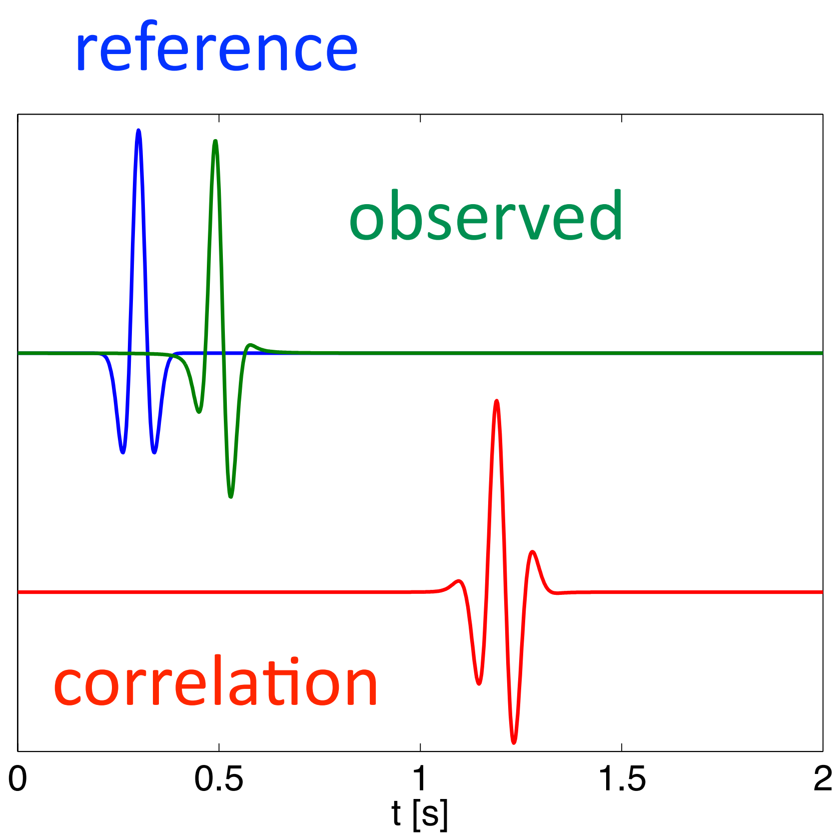
[Hormander 83; Hormann 02; de Hoop 05]

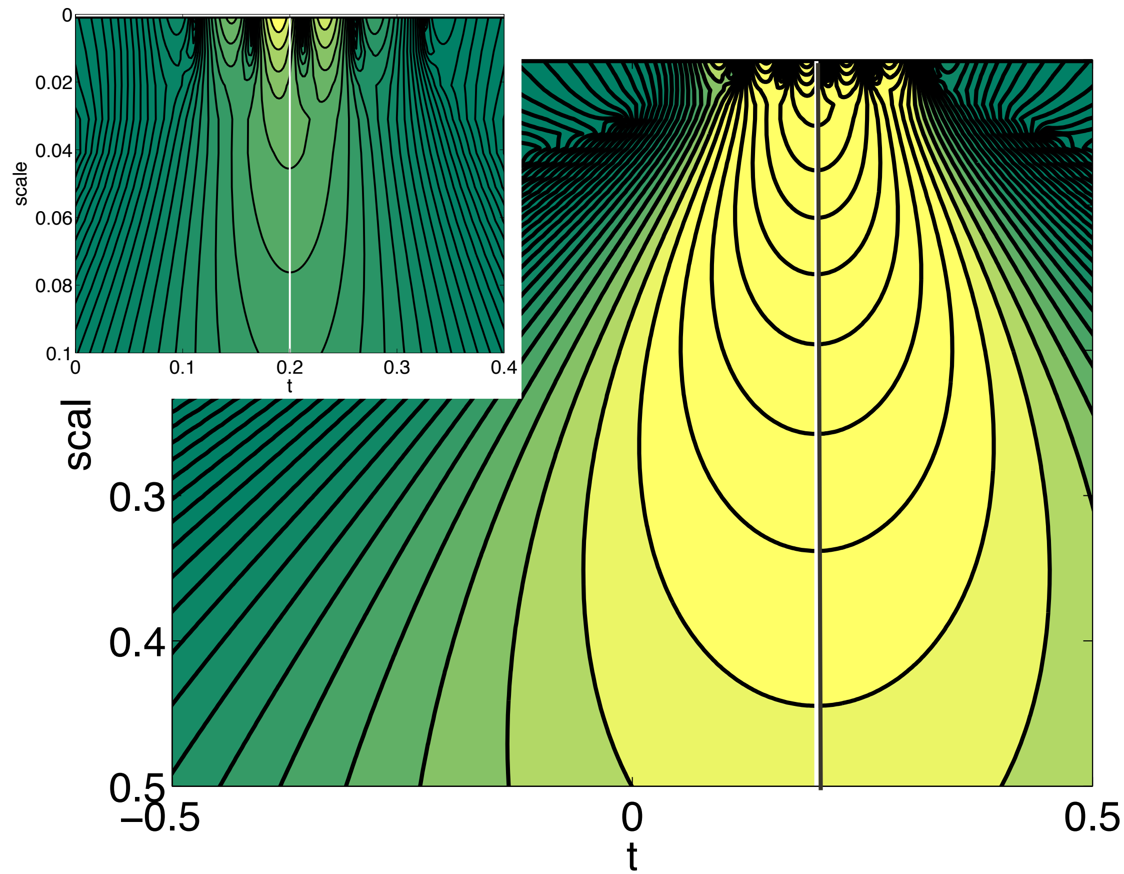
Wavefrontset detection



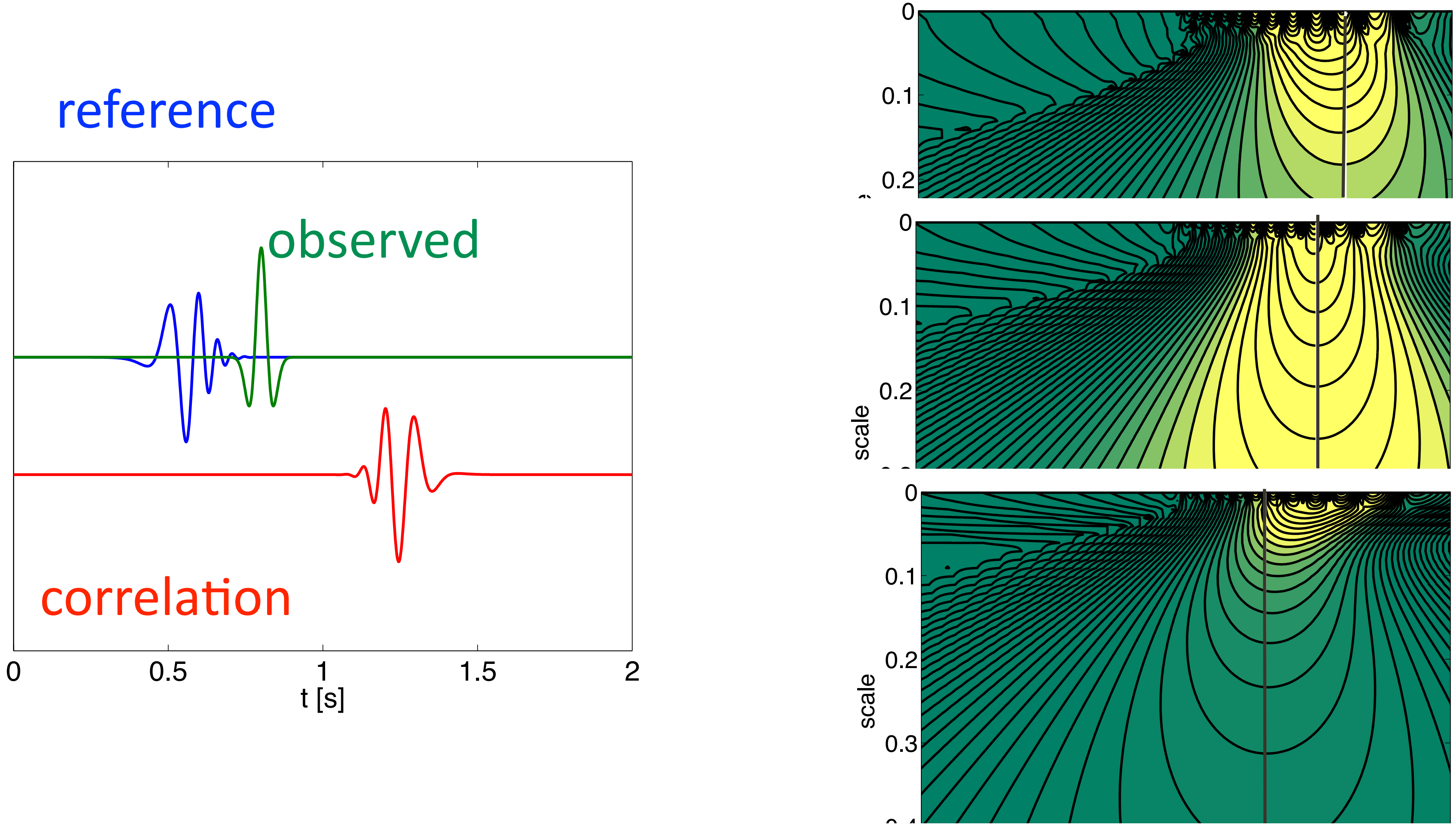


Wavefrontset detection

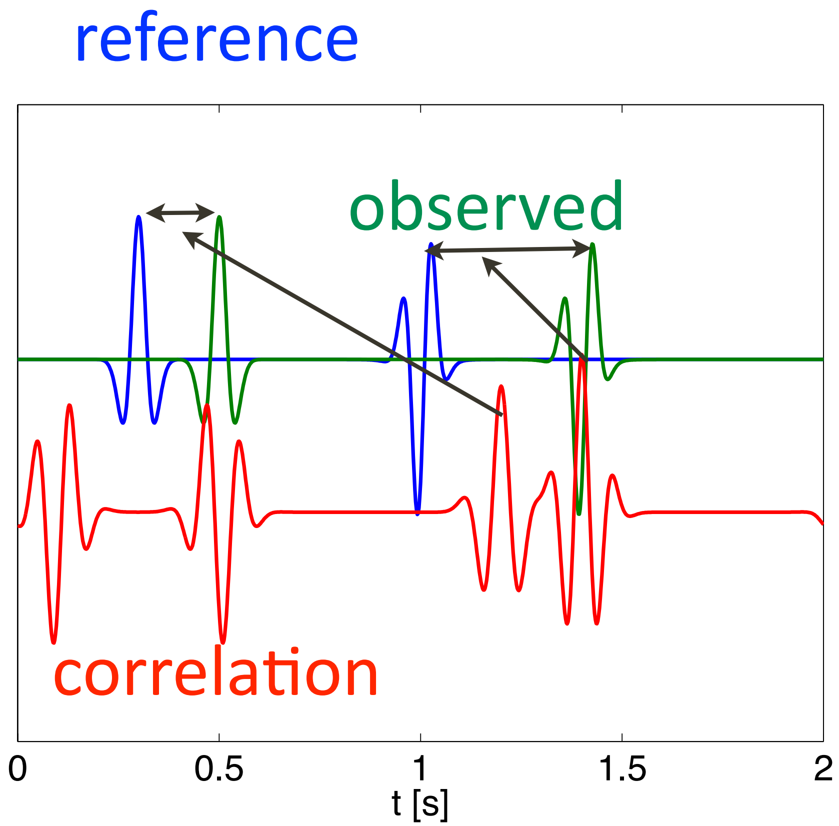


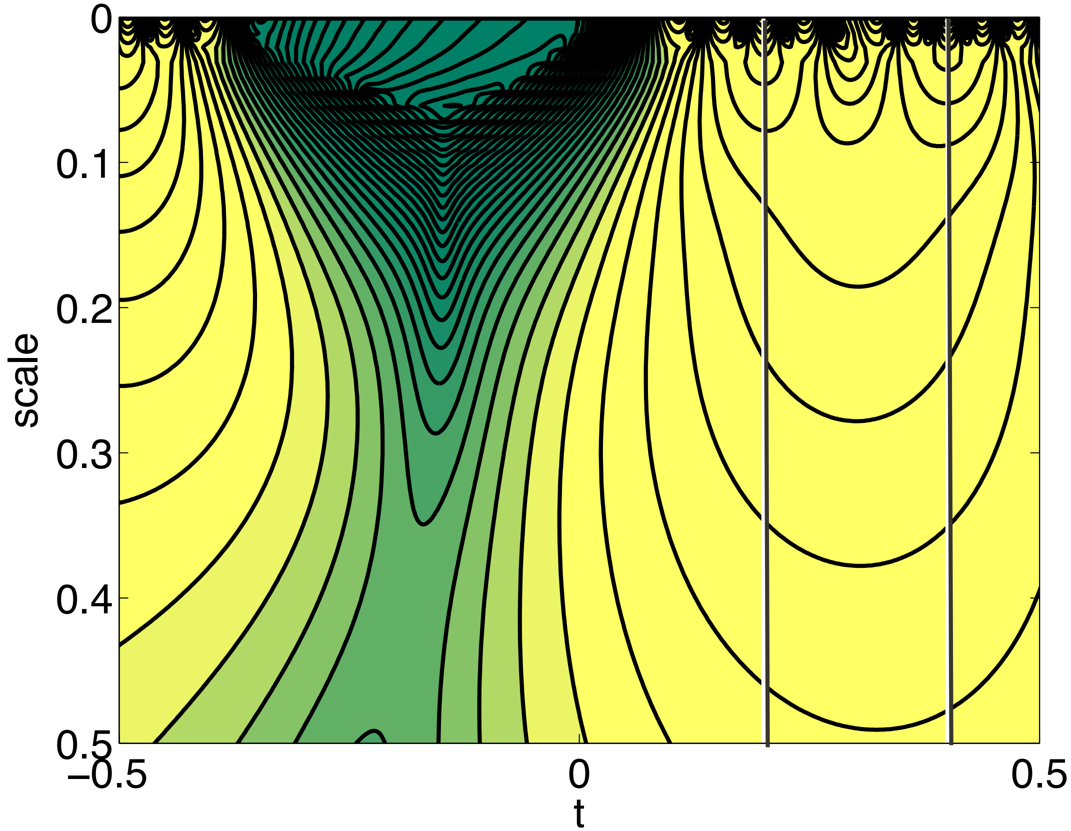


Wavefrontset detection

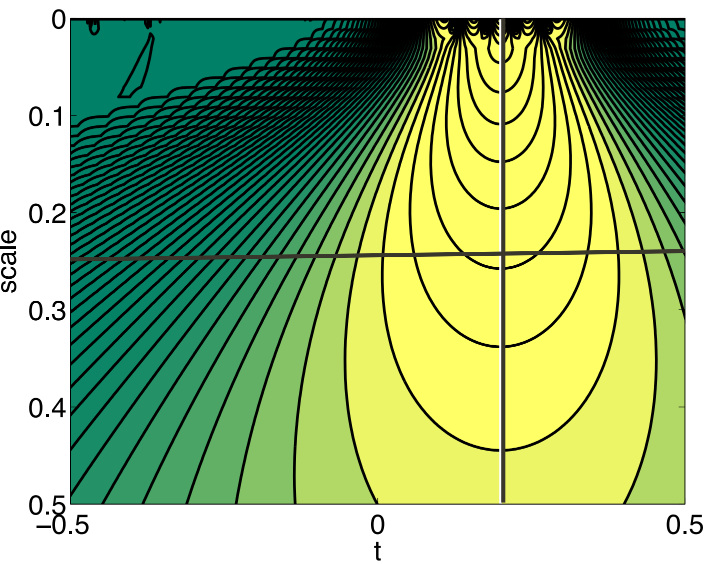


Wavefrontset detection





Misfit criteria



- $\tau[\omega, \sigma] = \text{argmax}_t \, G[\bar{d} * d](t, \omega, \sigma)$ converges to picking approach as $\sigma \downarrow 0$ and $\omega = 0$
- Maximize $||G[\bar{d} * d](0, ., \sigma)||_2^2$
- Minimize $||\partial_t G[\bar{d} * d](0, ., \sigma)||_2^2$

Misfit criteria

Rewrite:

$$G[f](0,\omega,\sigma) = (\widehat{W_\sigma \cdot f})(\omega)$$

$$\partial_t G[f](0,\omega,\sigma) = (\widehat{W'_\sigma \cdot f})(\omega)$$

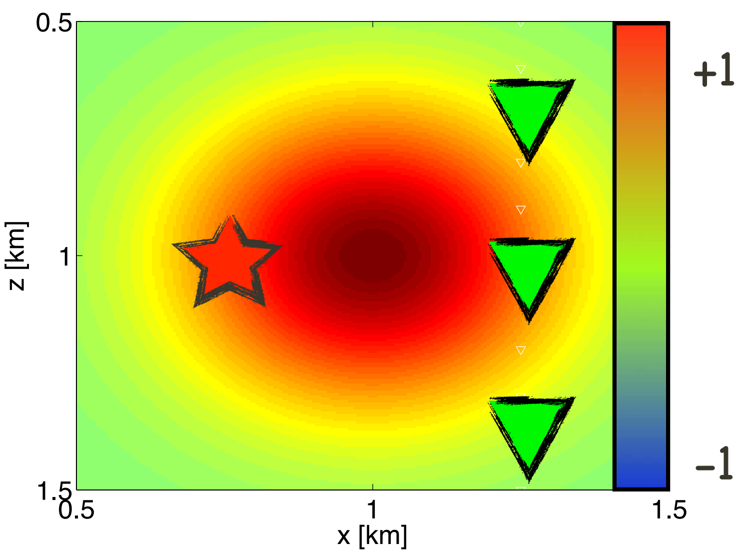
where $W_\sigma(t) = \frac{1}{\sqrt{\sigma}} \exp[-(t/\sigma)^2]$

Misfit:

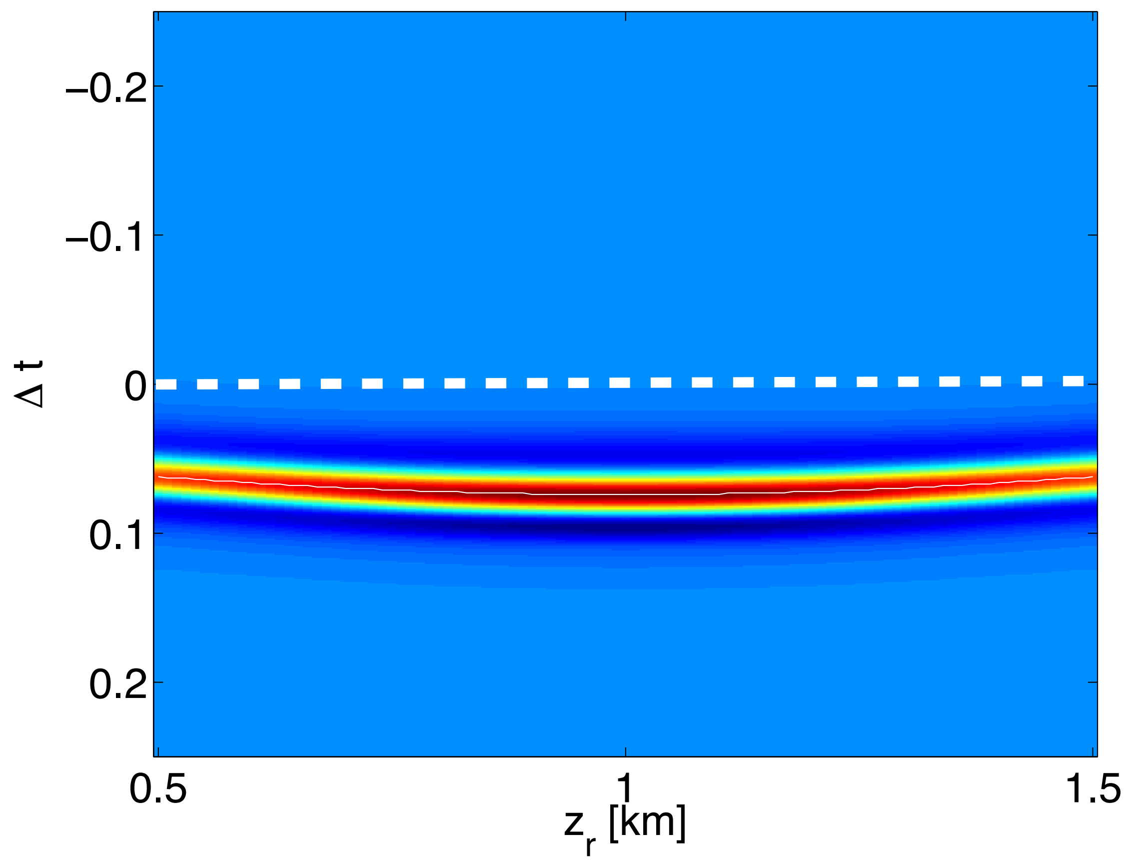
$$\phi = \frac{||W_\sigma \cdot (\bar{d} * d)||_2^2}{||d||_2^2}$$

[TvL 08; TvL 10]

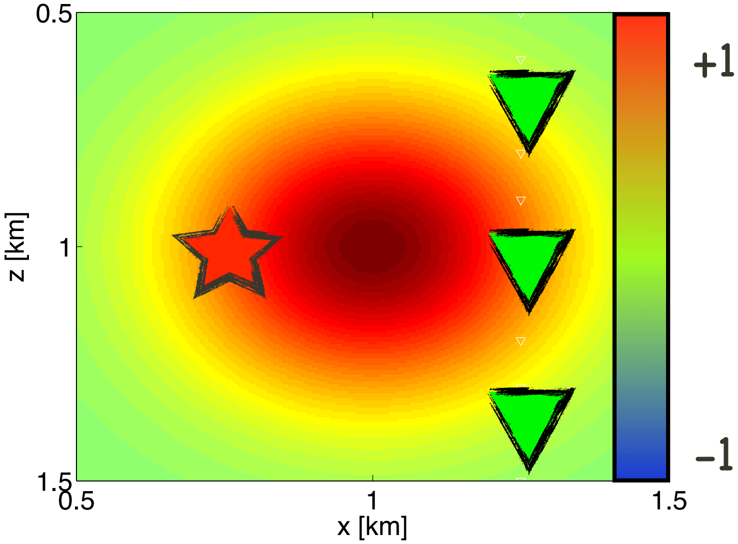
Misfit criteria



velocity perturbation

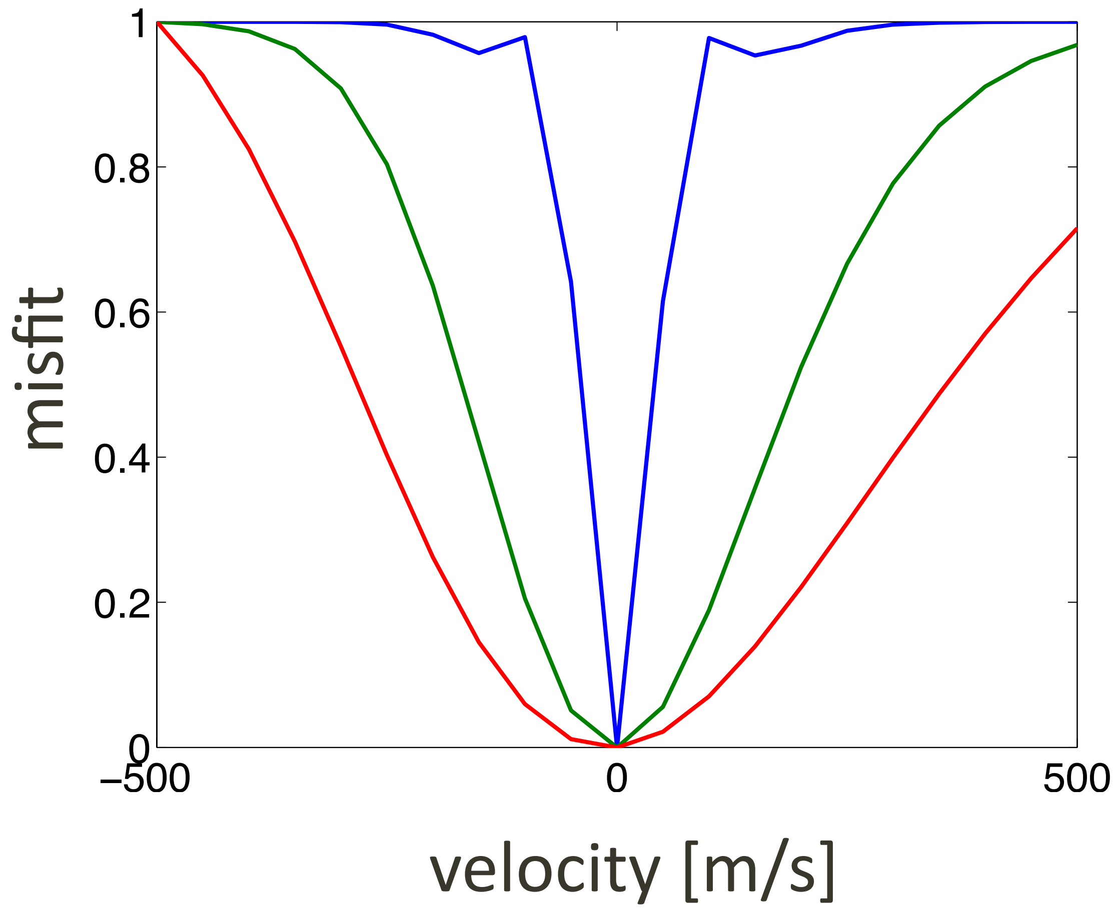


Misfit criteria

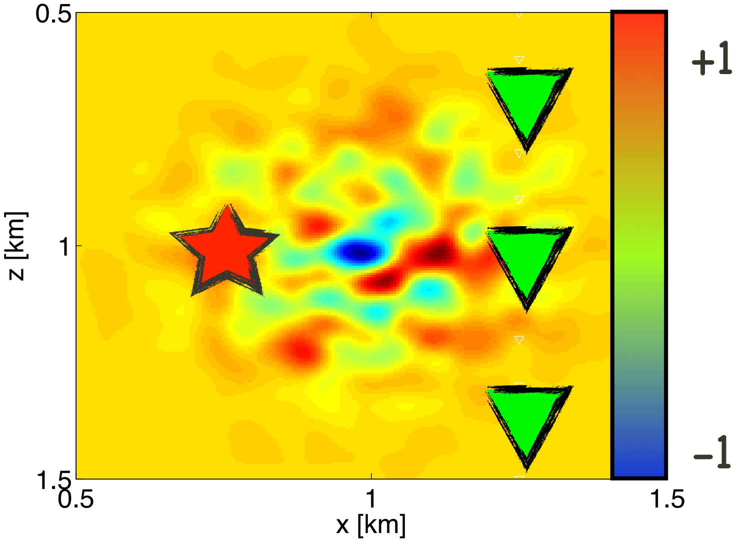


velocity perturbation

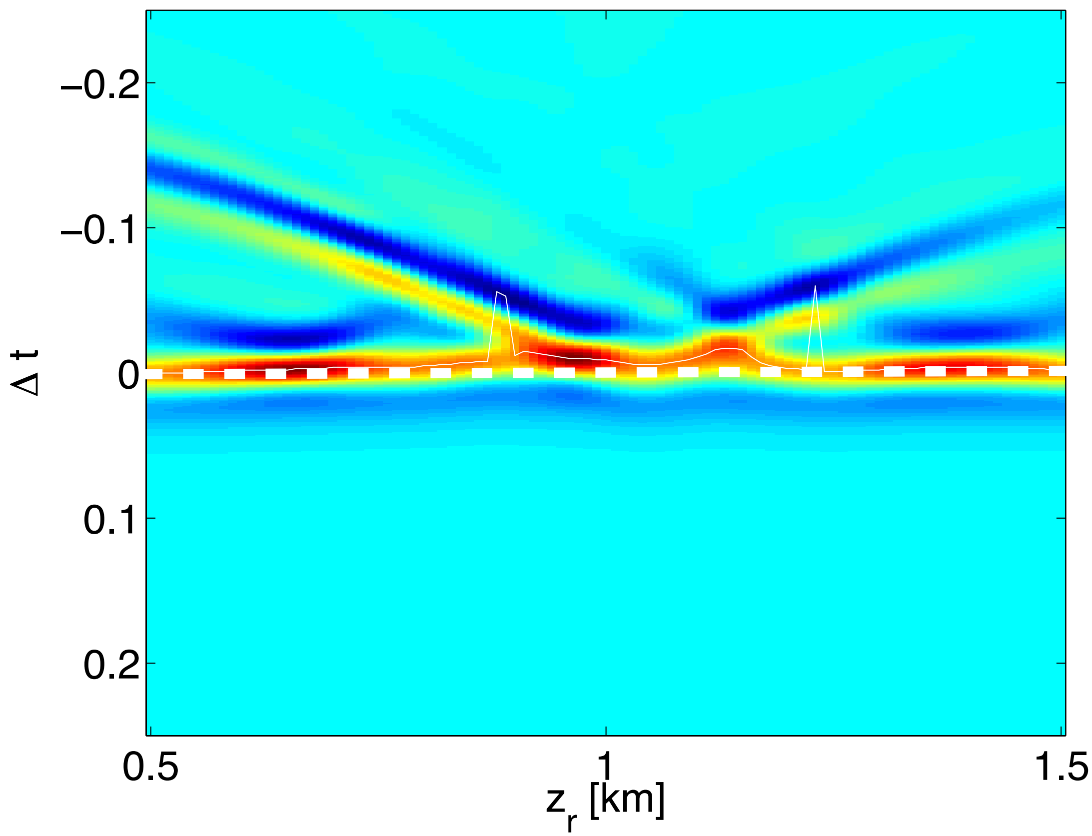
small, medium, large



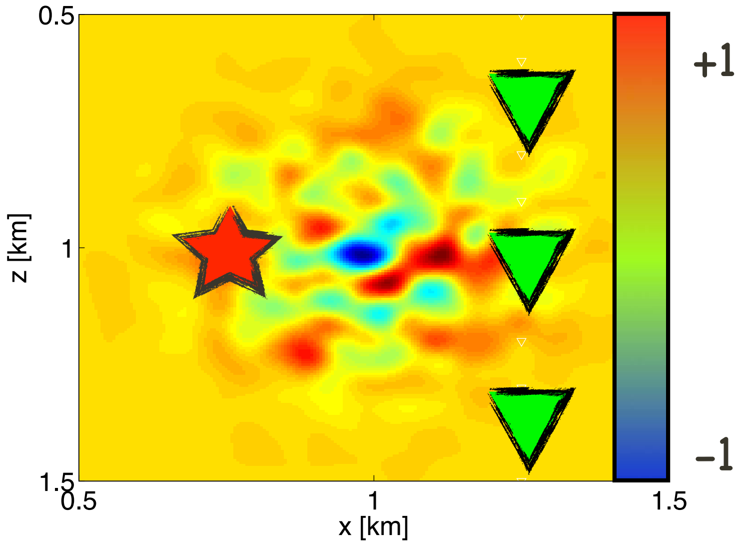
Misfit criteria



velocity perturbation

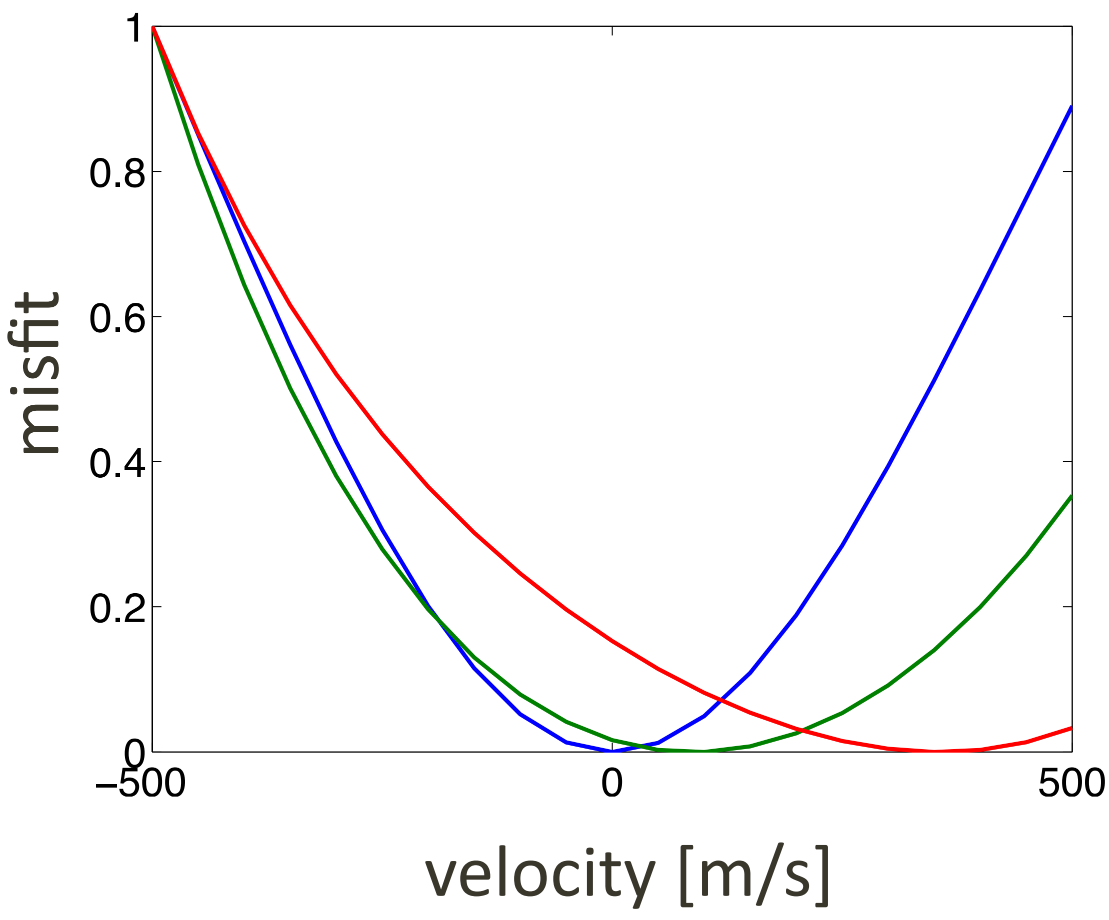


Misfit criteria



velocity perturbation

small, medium, large



Misfit criteria

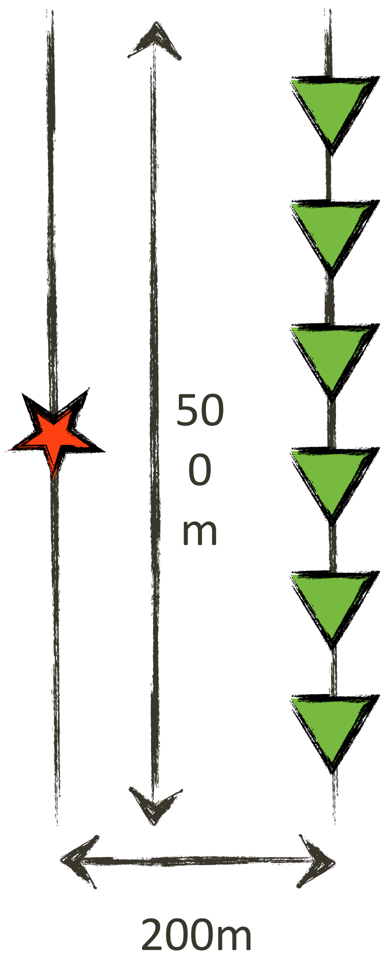
Multiscale WF detection allows us to move from

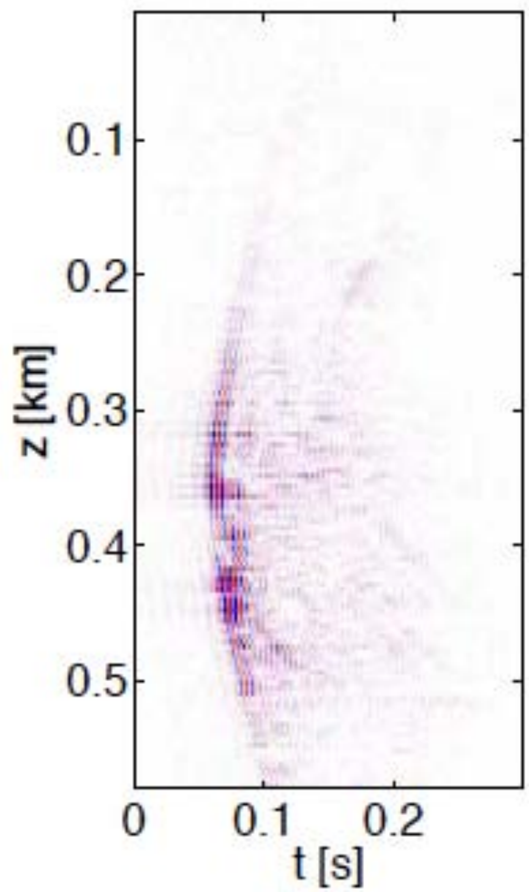
- Traveltime fitting at large scale

to

- `Stack power' at small scale

Numerical example II



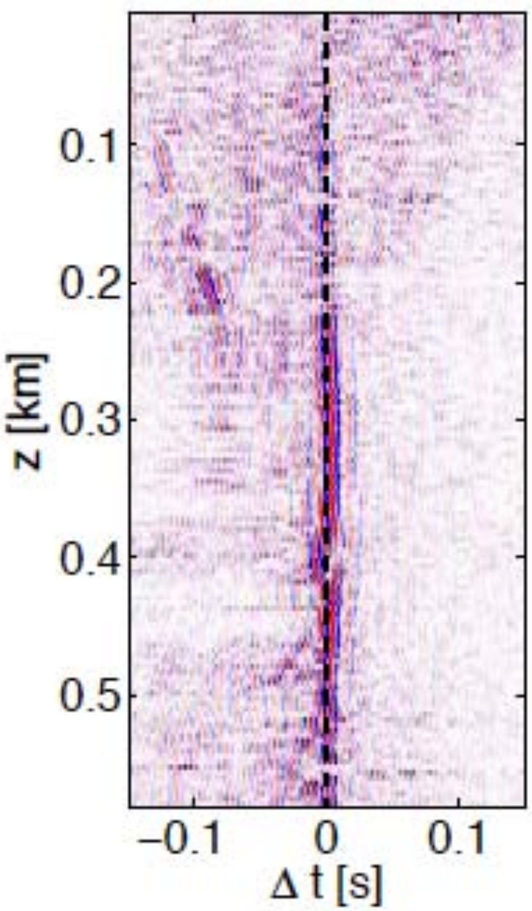


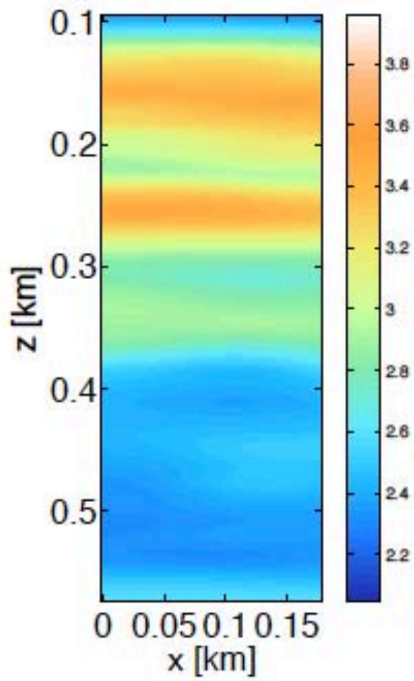
[TvL 10;]

Real cross-well data set

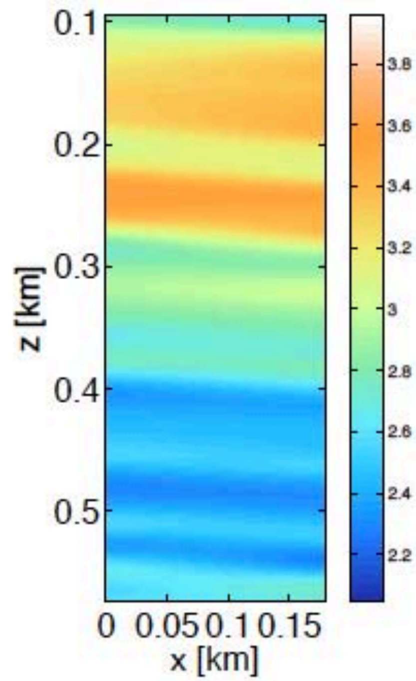
- Frequency domain FD
- Adjoint-state for gradient
- L-BFGS for optimization
- different stages using different basis functions

Numerical example

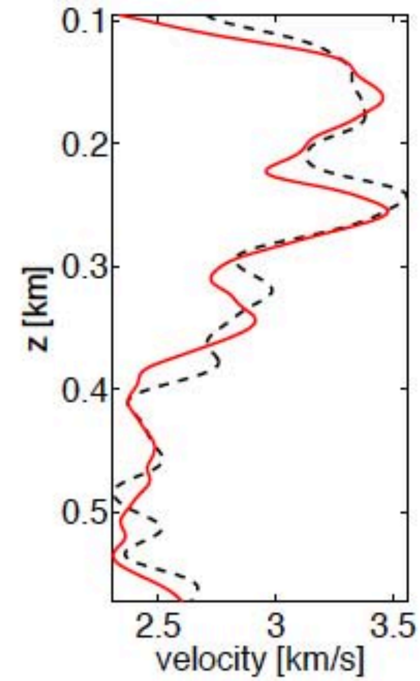




result

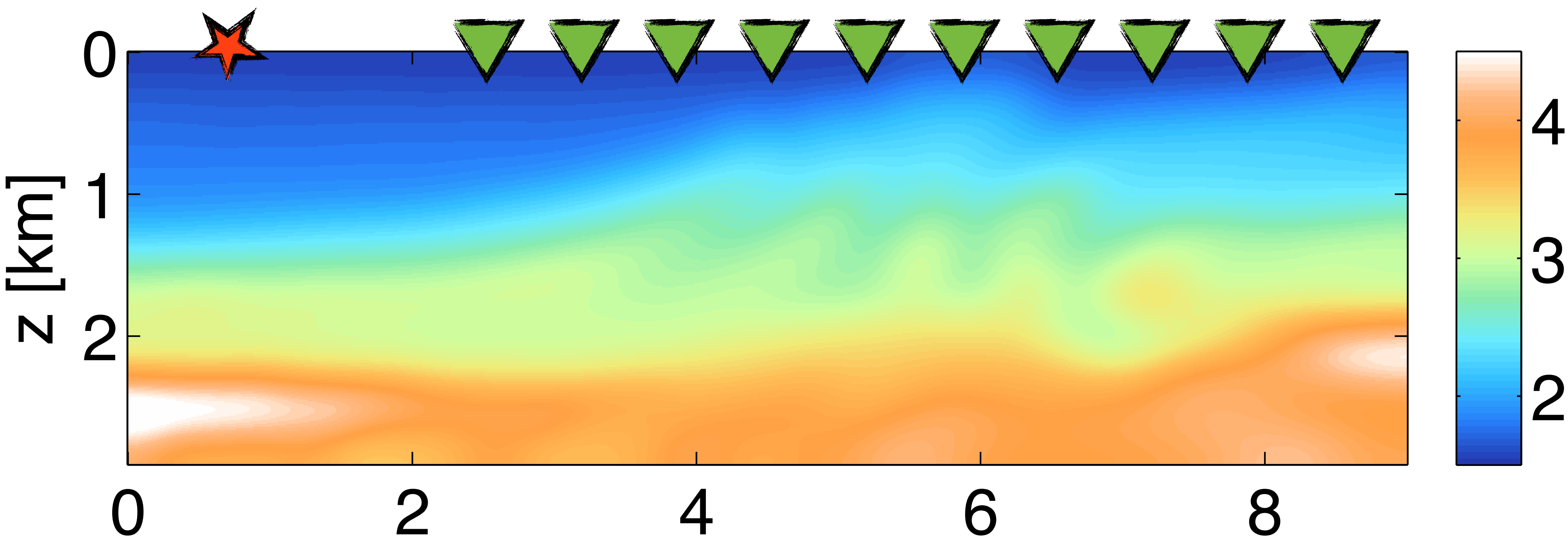


reference

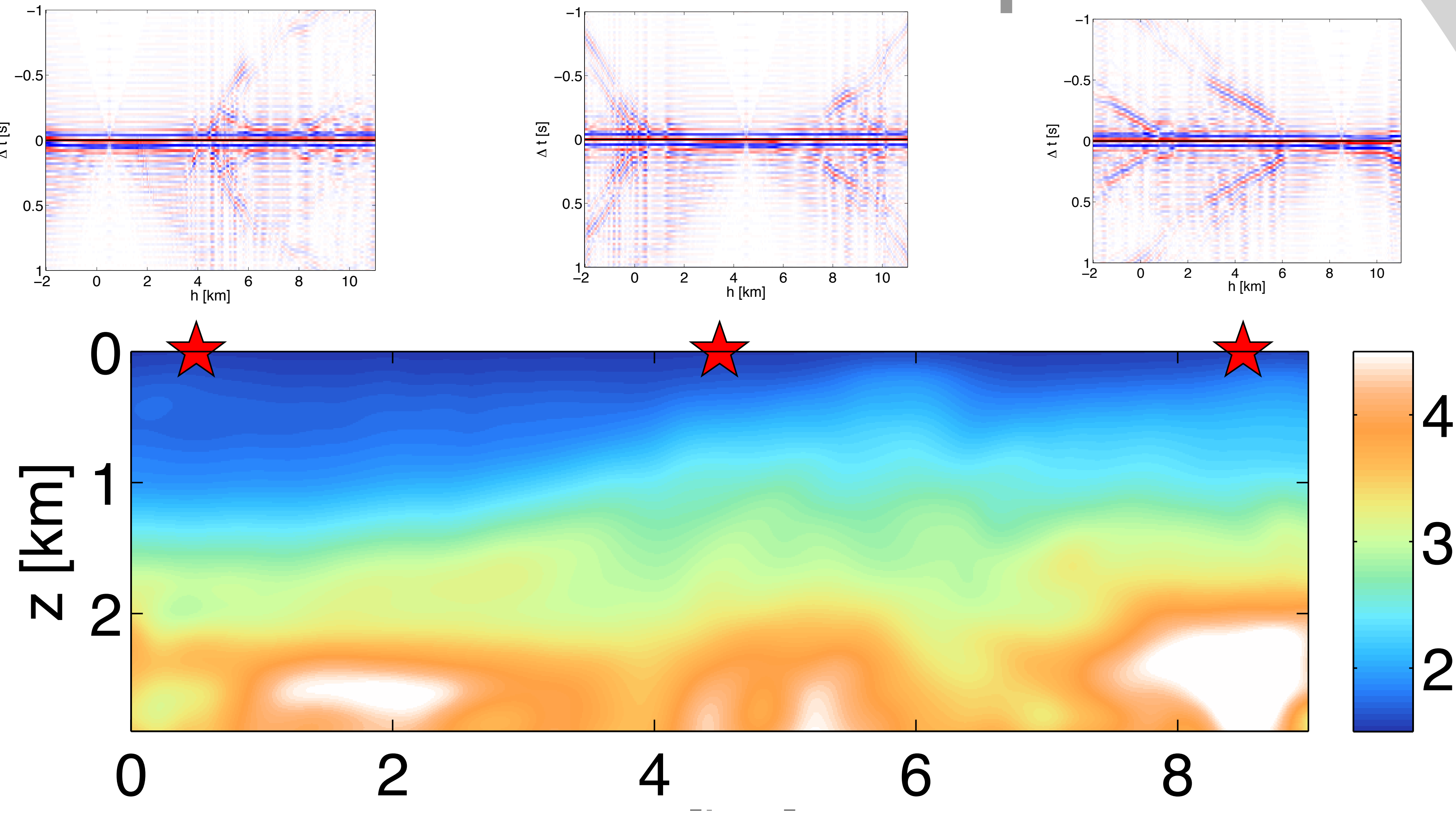


Numerical example II

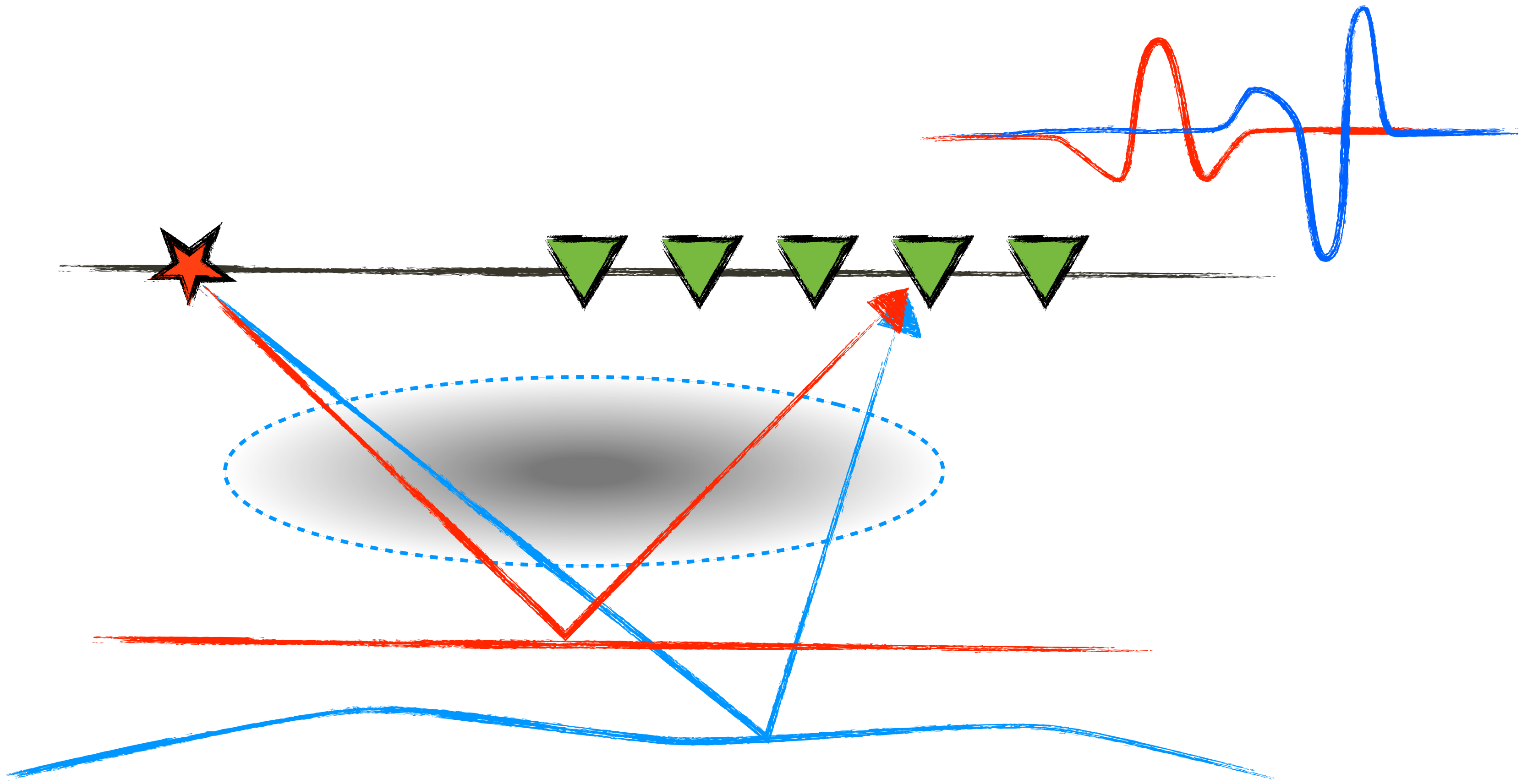
`Diving-wave tomography'



Numerical example II

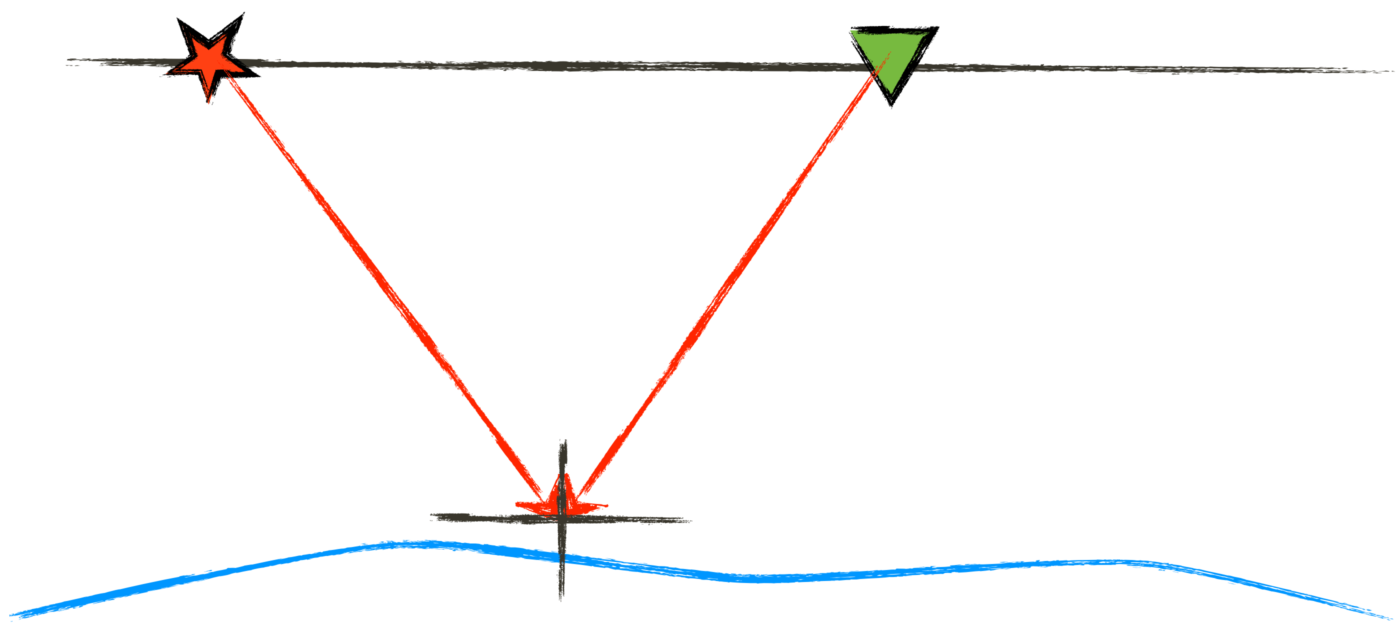


Reflection tomography



Reflection tomography

- Correlate wavefields in space $(\Delta x, \Delta z)$
- Produces *image volume*
- Measure focusing with Gaussian weight



[Claerbout ; Rickett; Sava ;Symes]

Reflection tomography

Spatial correlation:

$$E = VU^*$$

where $HU = Q$ and $H^*V = R$

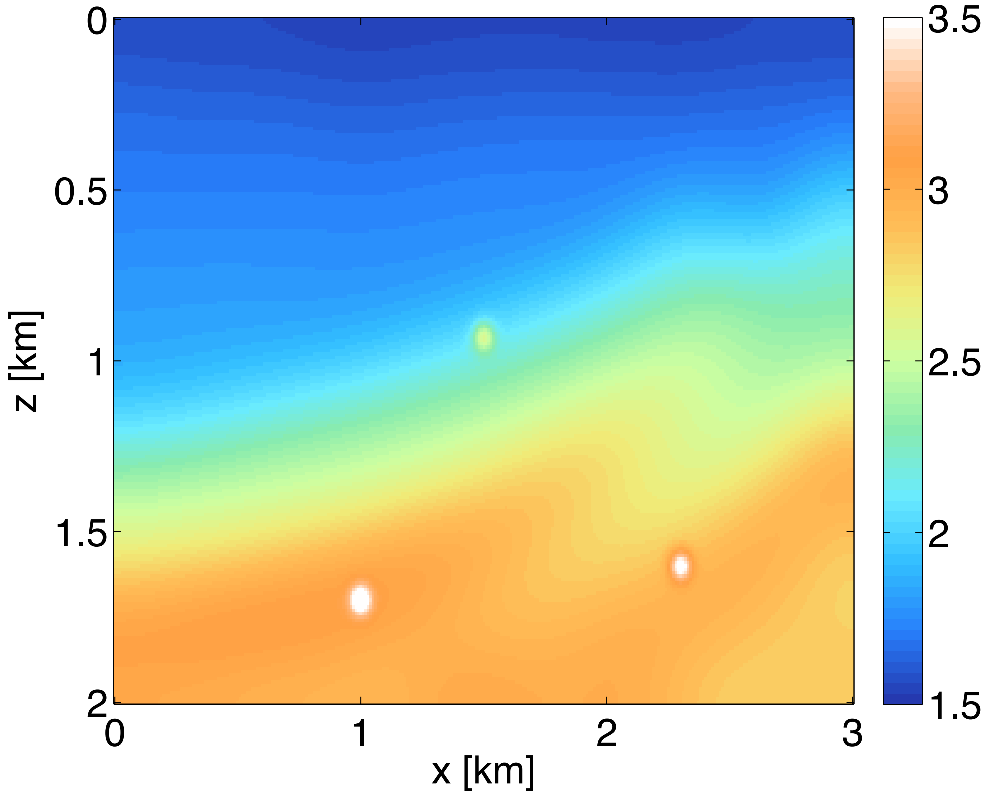
many r.h.s. !!

Action on a vector:

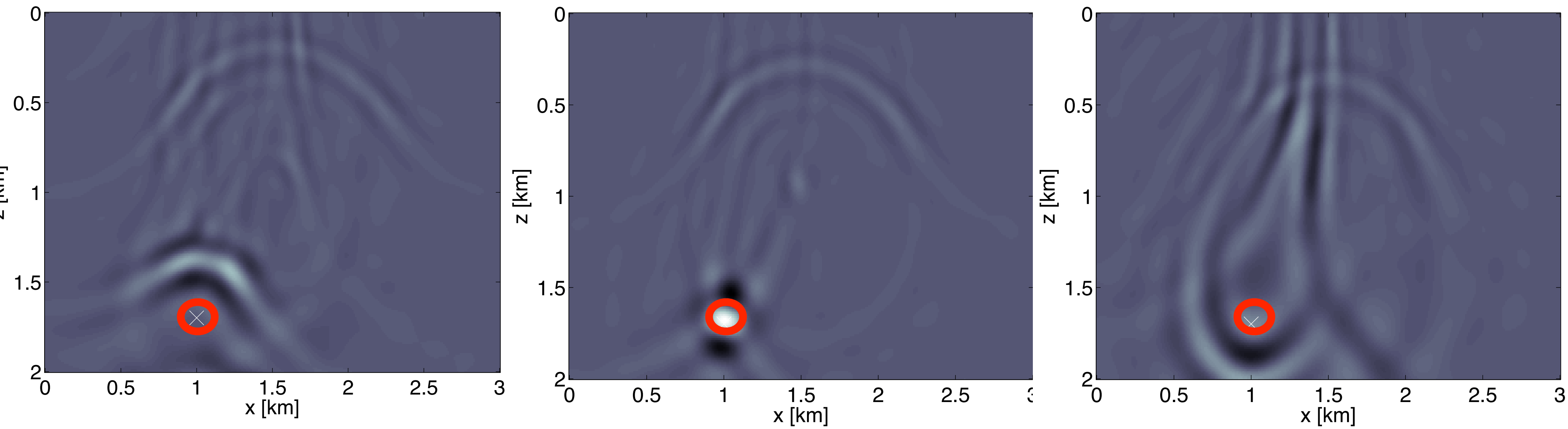
$$E\mathbf{x} = V\underbrace{(U^*\mathbf{x})}_{\mathbf{y}} = H^{-*}(R\mathbf{y})$$

one r.h.s. !!

Reflection tomography



Reflection tomography



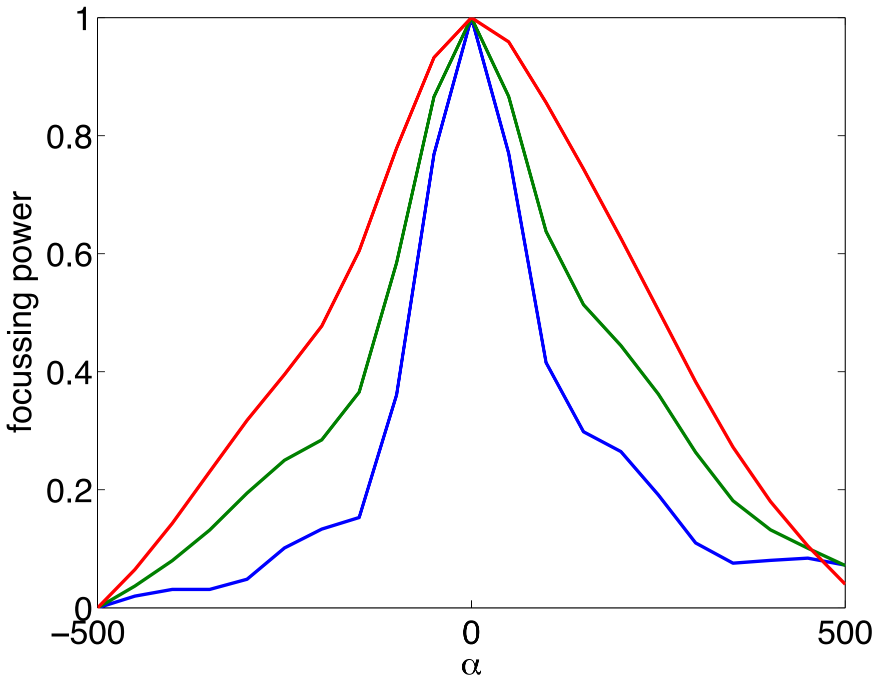
low velocity

high velocity

[TvL 11]

Reflection tomography

focussing power for small, medium and large scale



Conclusions & Future work

- Natural way to move from traveltime to amplitude fitting, and overcome loopskipping
- Multiscale WF detection might be extended to dispersion and stereo tomography
- Similar ideas might be applied in reflection case

Acknowledgements

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