

# Uncertainty-aware time-lapse monitoring of geological carbon storage with learned surrogates

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Georgia Institute of Technology

ML4Seismic

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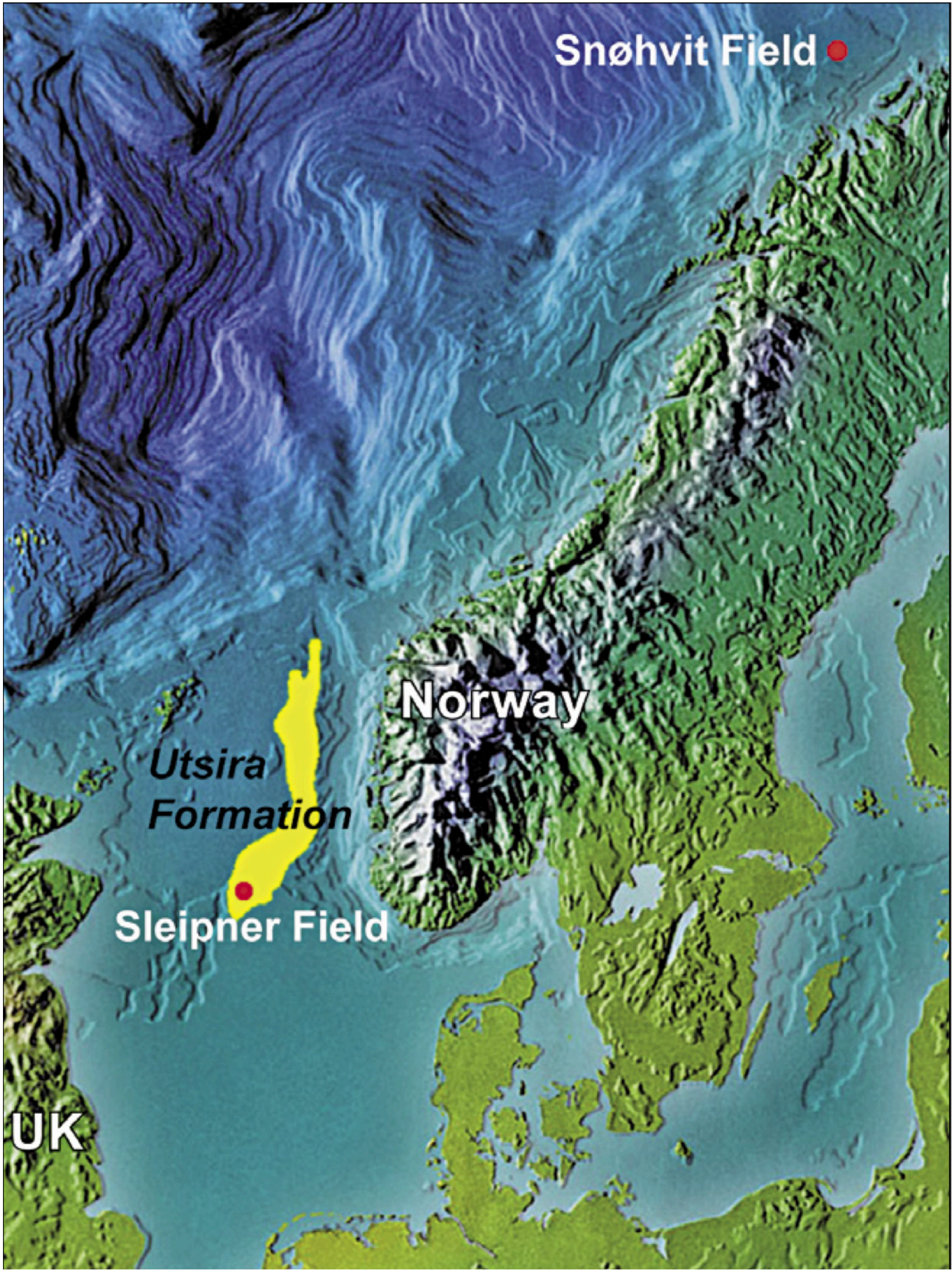
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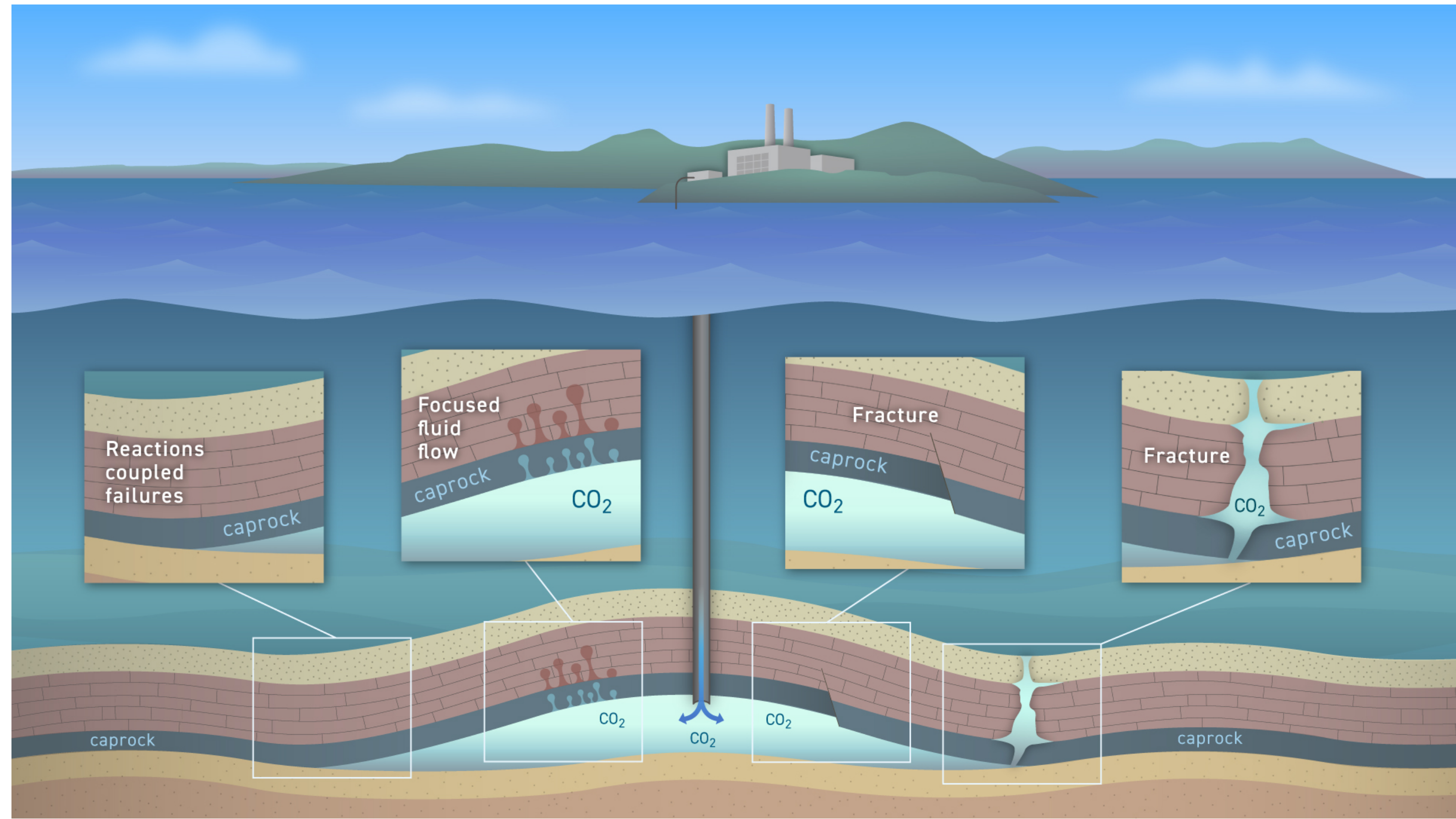
# Seismic monitoring of geological carbon storage

# Geological carbon storage (GCS)

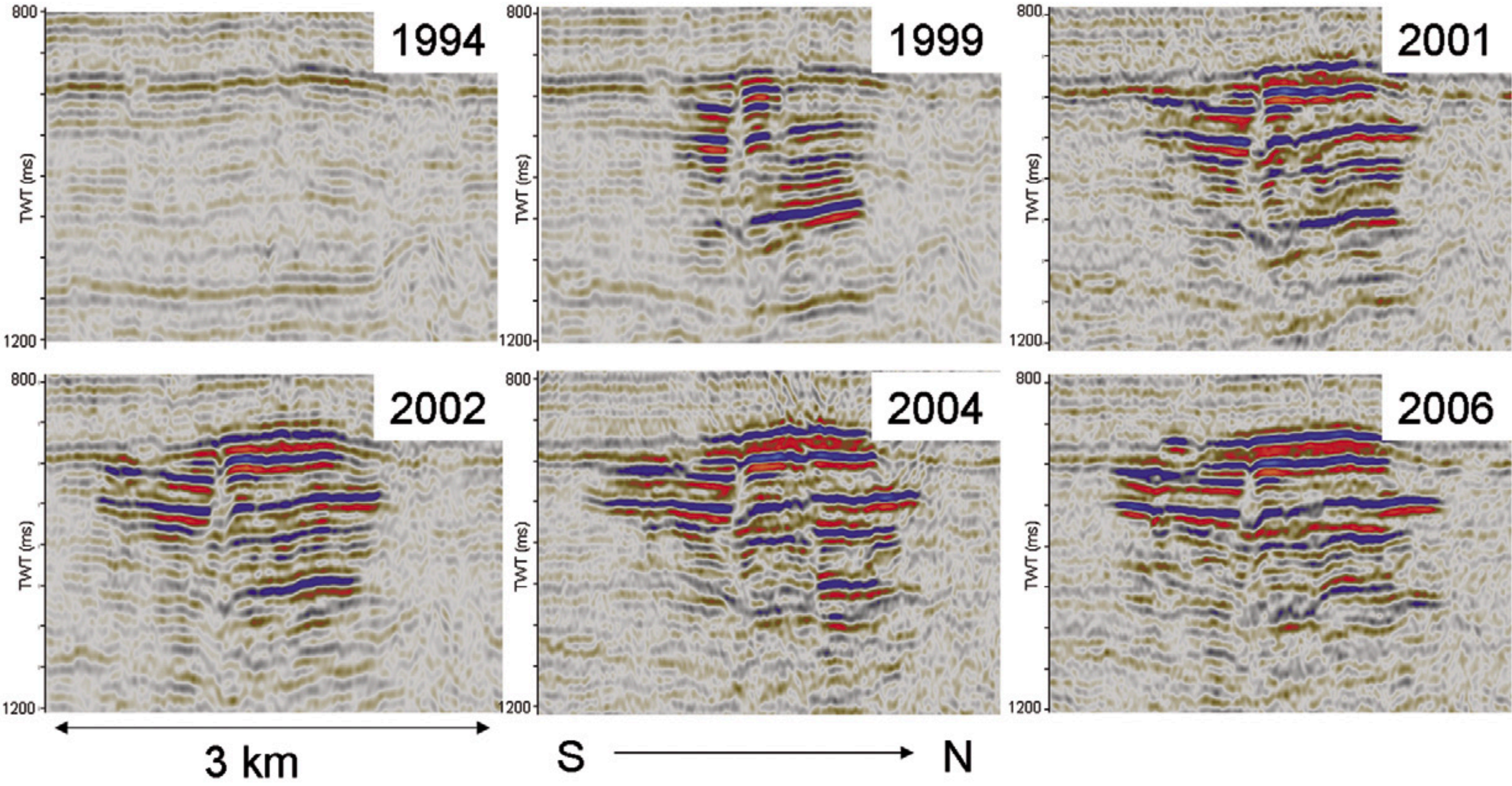
## Sleipner project



Arts, R. J., et al. "Ten years' experience of monitoring CO2 injection in the Utsira Sand at Sleipner, offshore Norway." *First break* 26.1 (2008).



# Seismic response Sleipner project



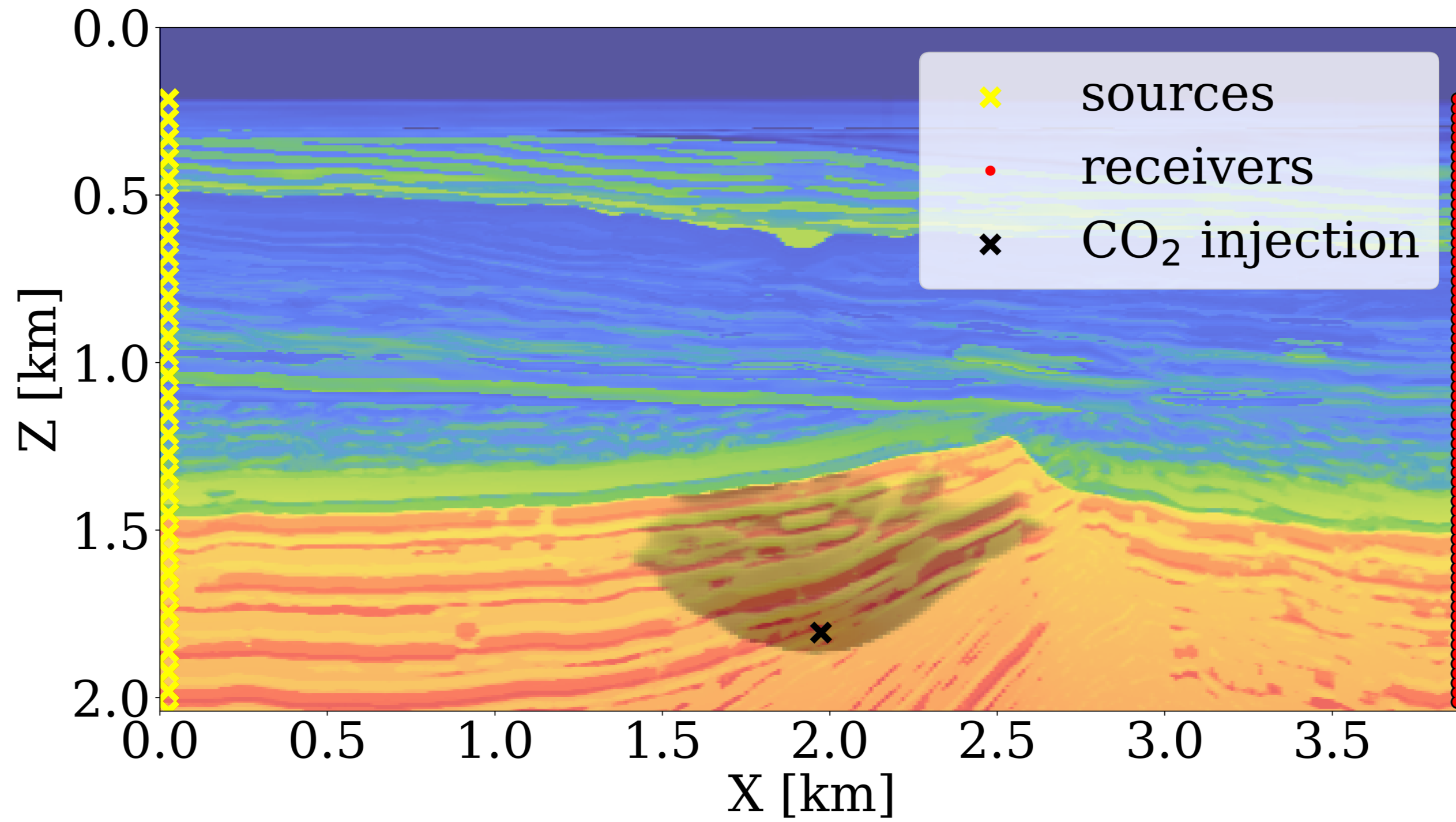
# Motivation

## time-lapse monitoring

Estimate permeability given time-lapse seismic data

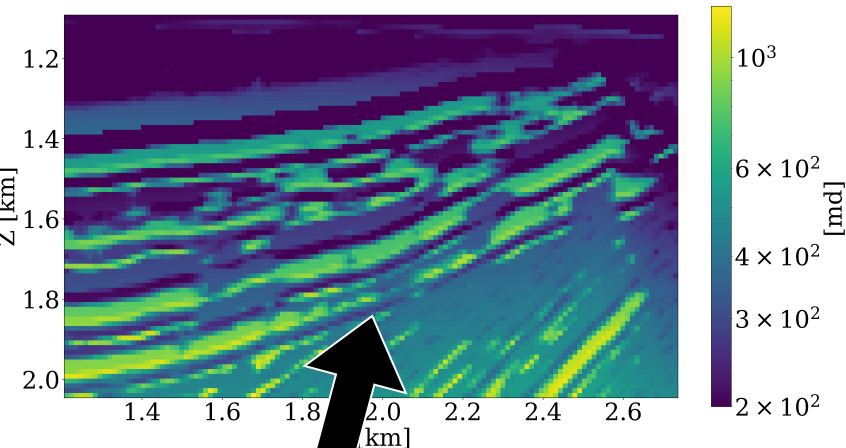
- ▶ end-to-end inversion
- ▶ Fourier neural operator as cheap surrogate for physics
- ▶ **cheap & reliable uncertainty quantification**
- ▶ **uncertainty-aware CO<sub>2</sub> plume forecast**

# CO<sub>2</sub> plume prediction



# CO<sub>2</sub> prediction (correct permeability)

permeability  
**K**

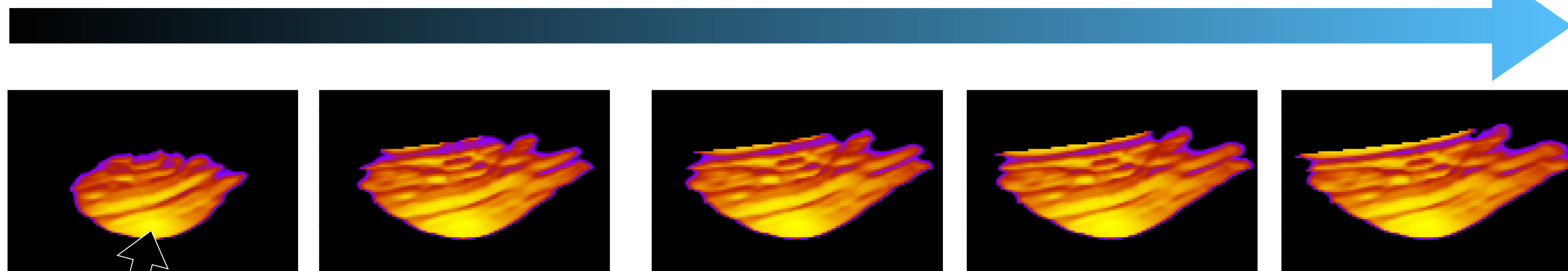


injection  
well

$\mathcal{S}$   
fluid-flow  
physics

CO<sub>2</sub> concentration  
**c**

time

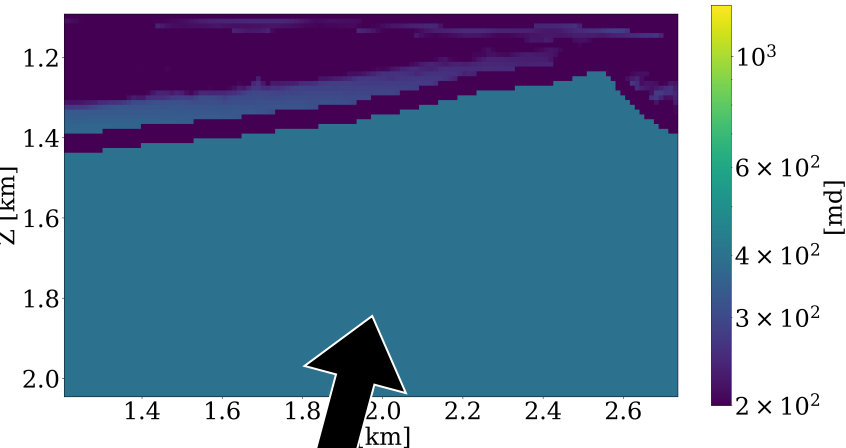


injection  
well

CO<sub>2</sub> flows due to high permeability channels & buoyancy

# CO<sub>2</sub> prediction (wrong permeability)

permeability  
**K**

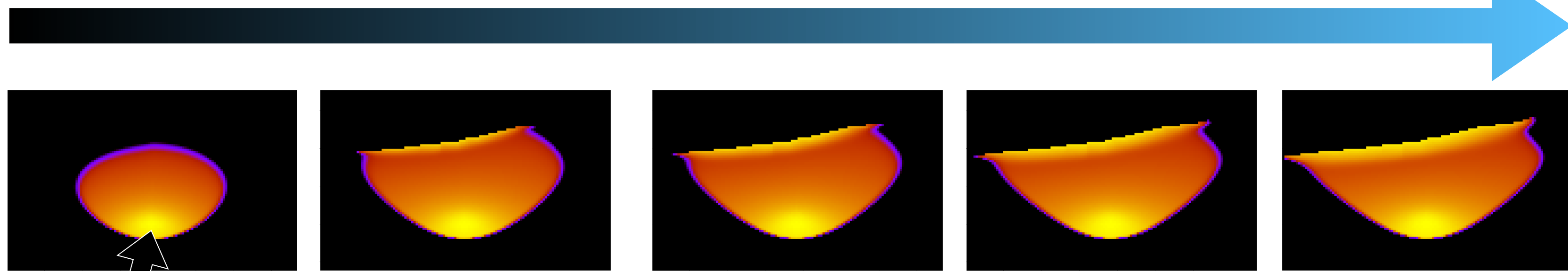


injection well

$\mathcal{S}$   
fluid-flow physics

CO<sub>2</sub> concentration  
**c**

time →



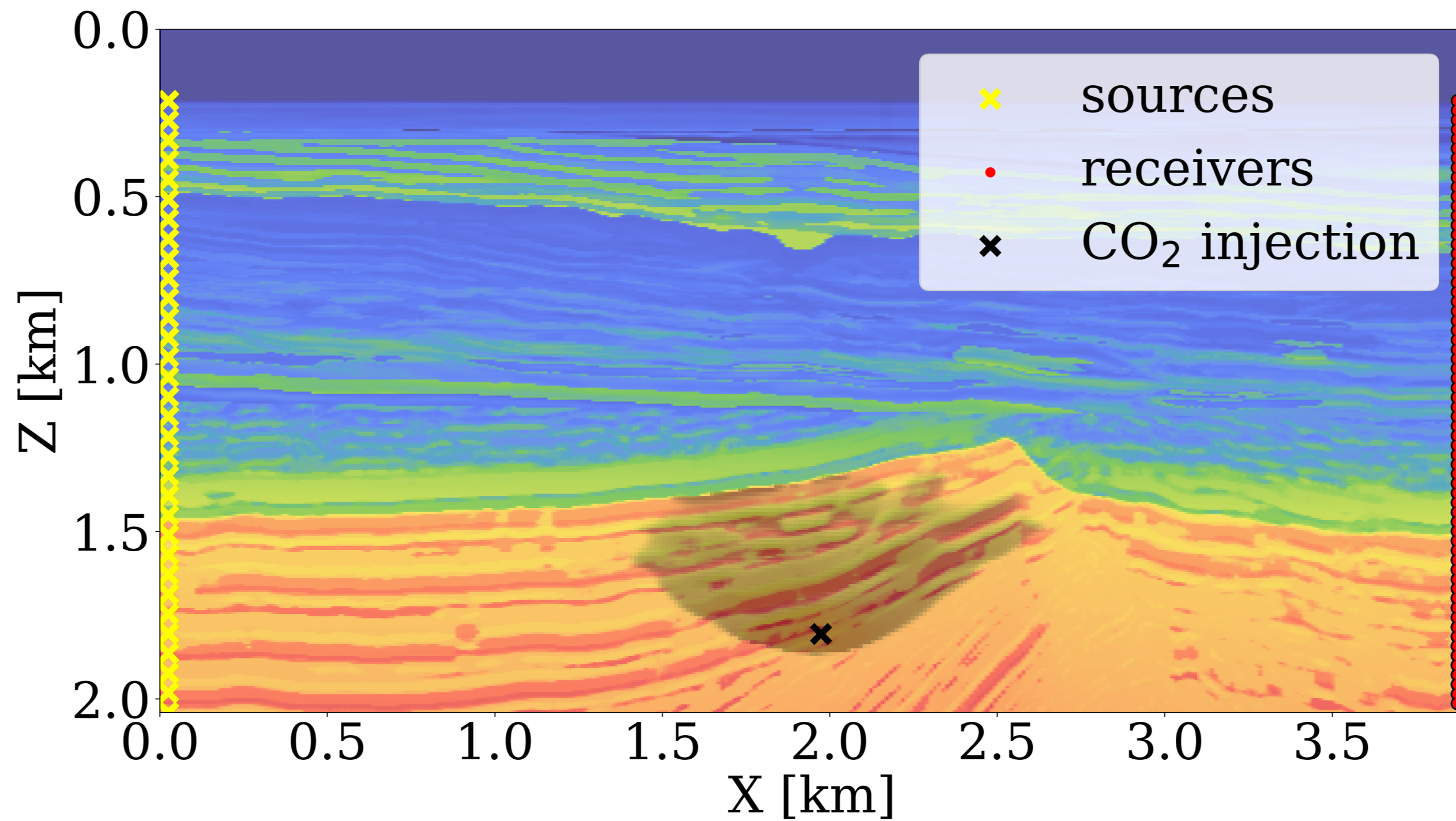
injection well

wrong CO<sub>2</sub> plume – wrong lateral extent

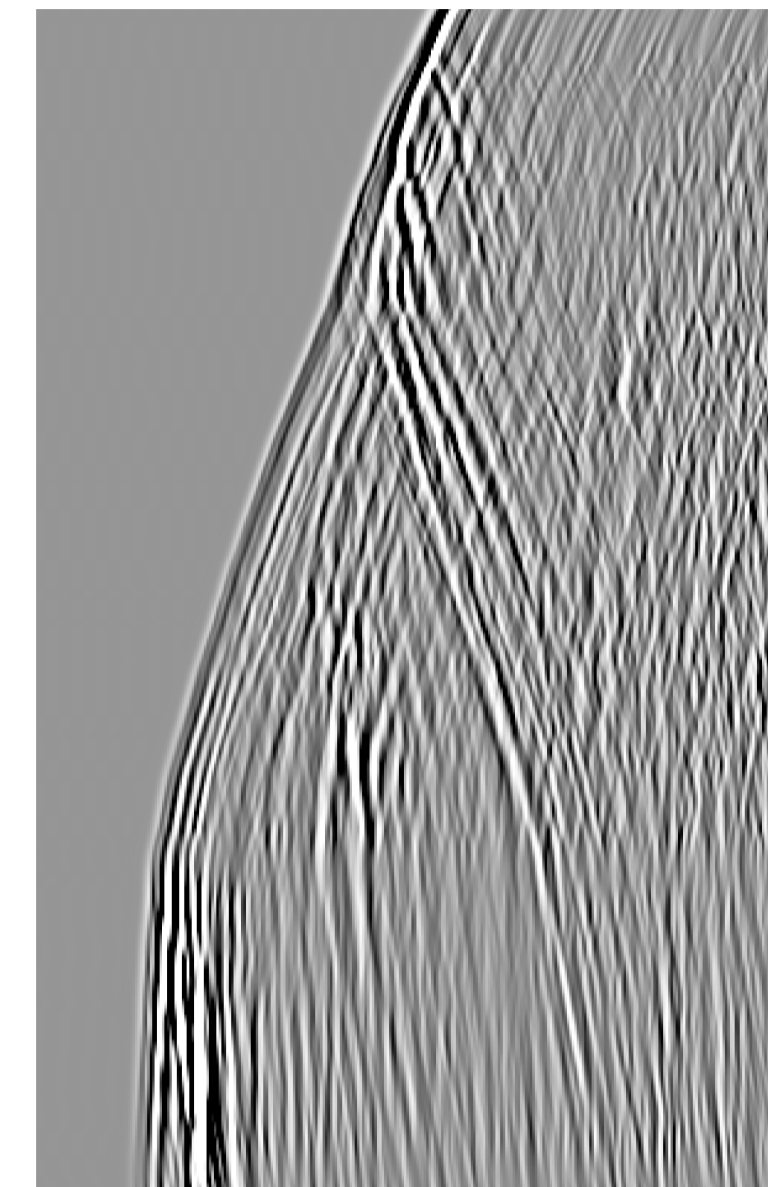
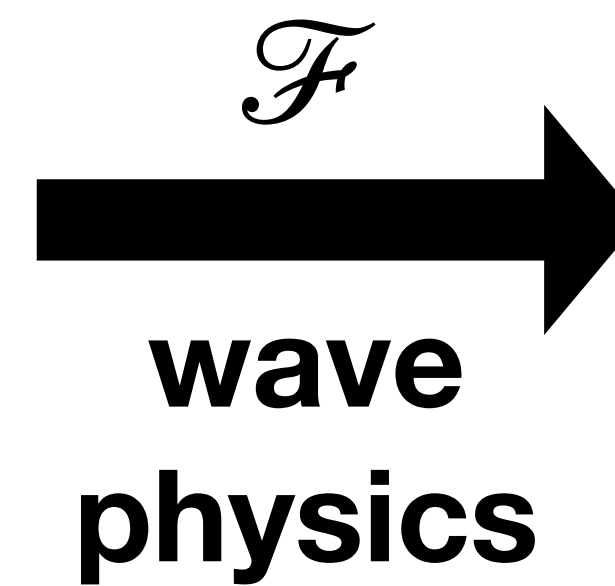


# Time-lapse seismic monitoring

wavespeed  $v$



seismic data  $d$



# Time-lapse monitoring of carbon storage

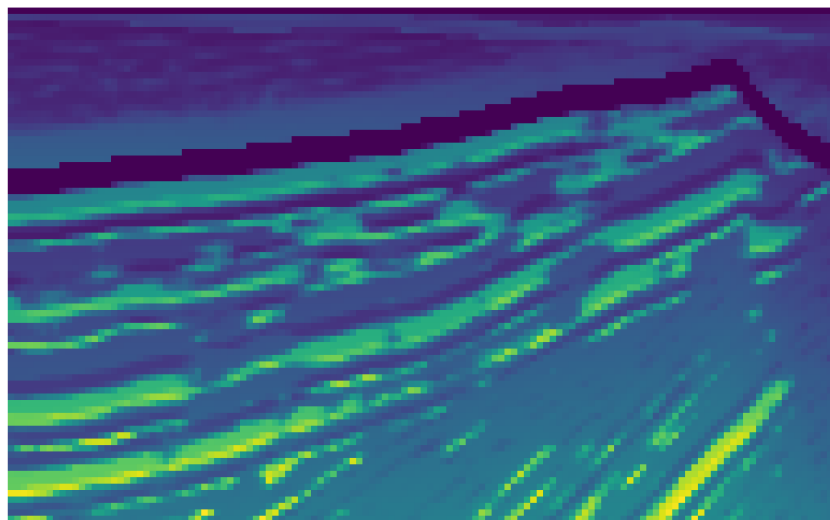
## Multiphysics modeling

permeability  
**K**

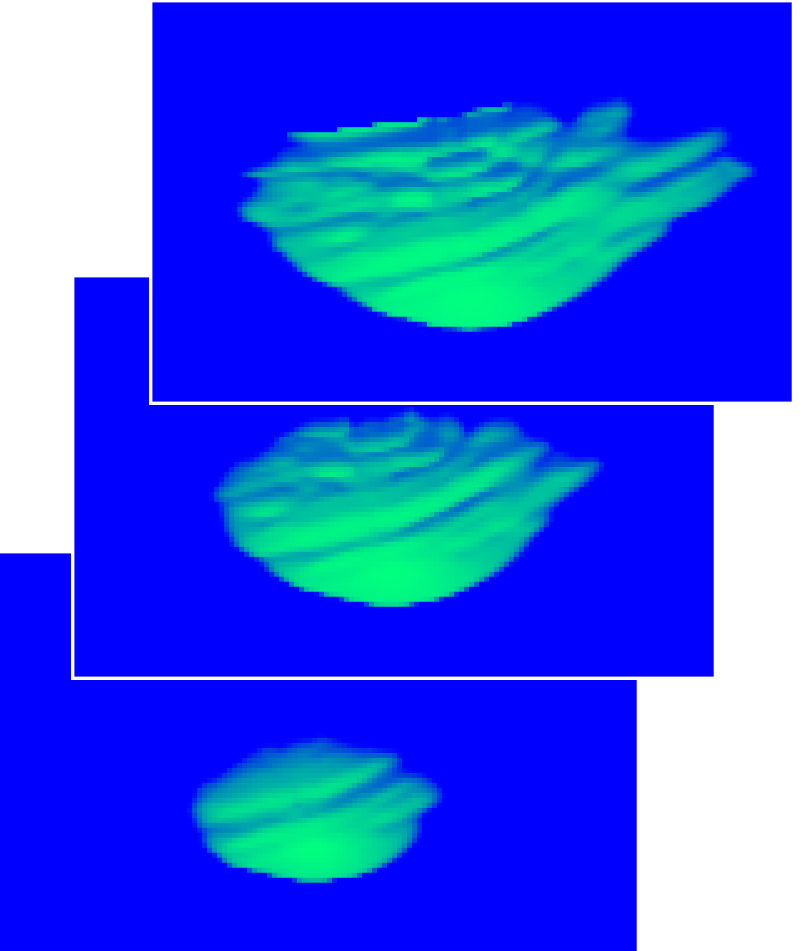
CO<sub>2</sub> concentration  
**c**

wavespeed  
**v**

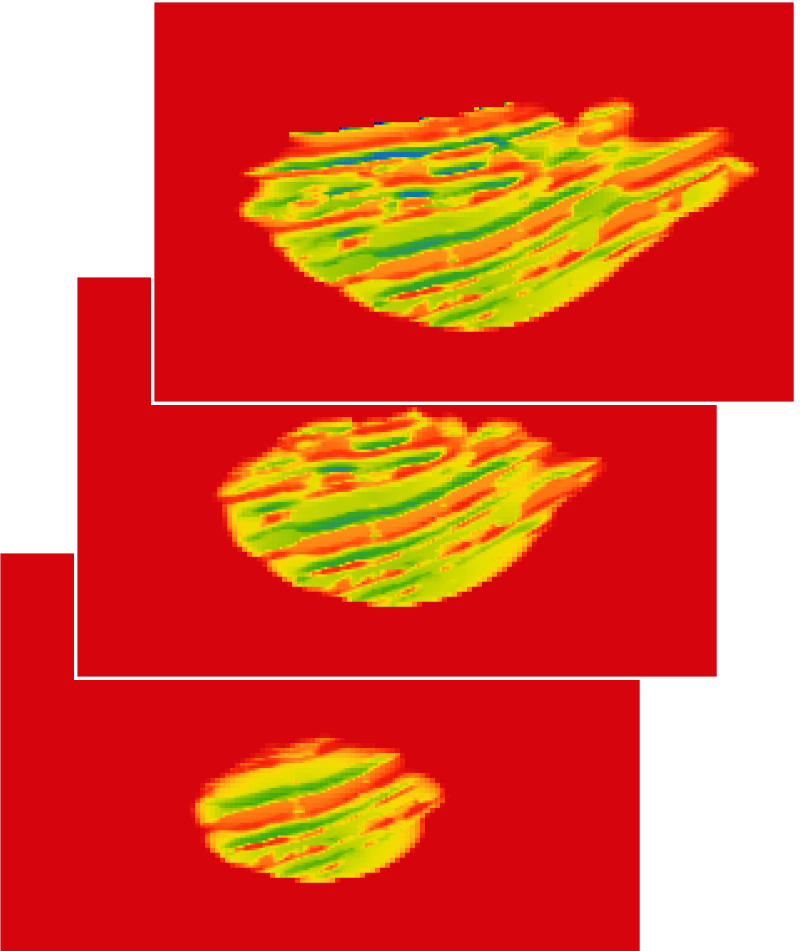
time-lapse data  
**d**



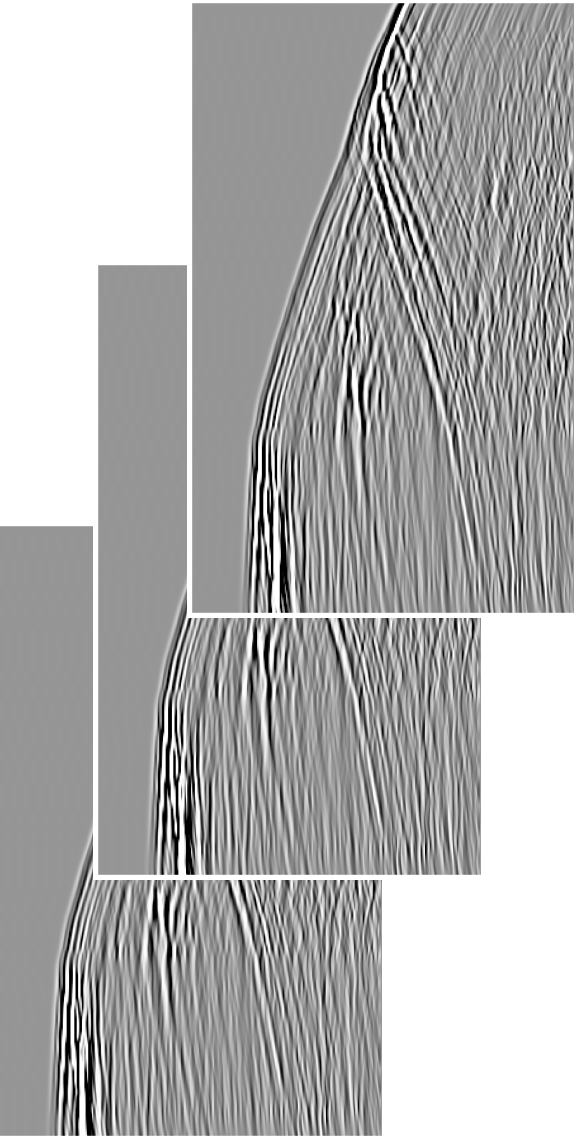
$\mathcal{S}$   
→  
fluid-flow  
physics



$\mathcal{R}$   
→  
rock  
physics



$\mathcal{F}$   
→  
wave  
physics



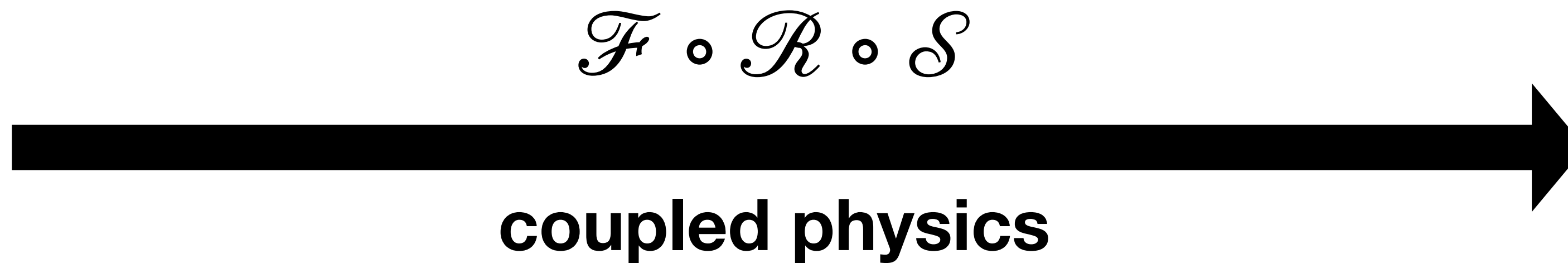
# End-to-end inversion framework

permeability  
 $\mathbf{K}$

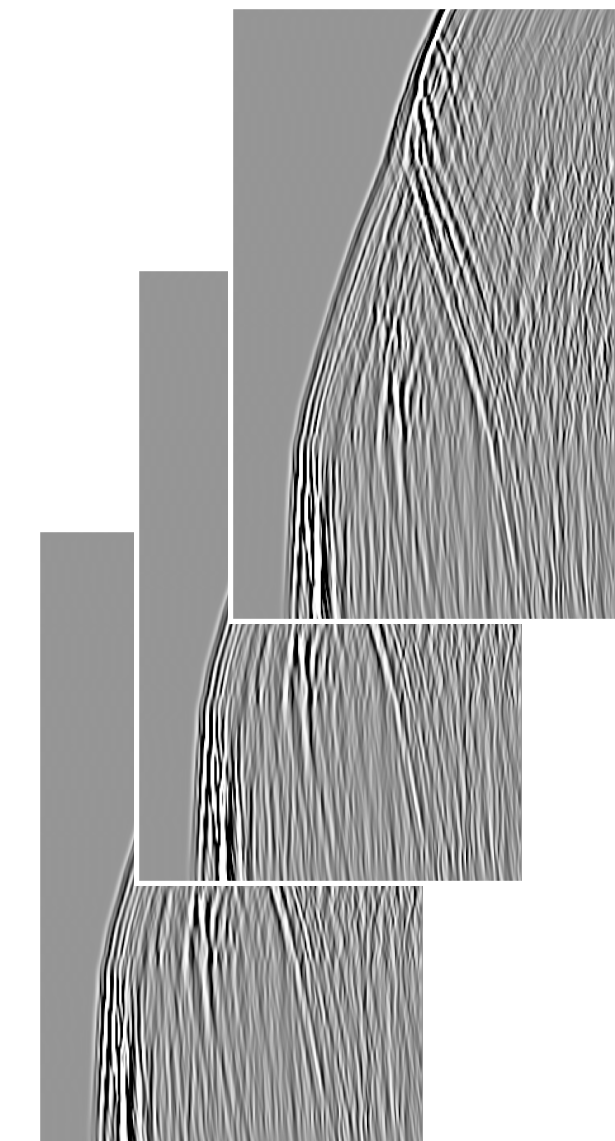
CO<sub>2</sub> concentration  
 $\mathbf{c}$

wavespeed  
 $\mathbf{v}$

time-lapse data  
 $\mathbf{d}$



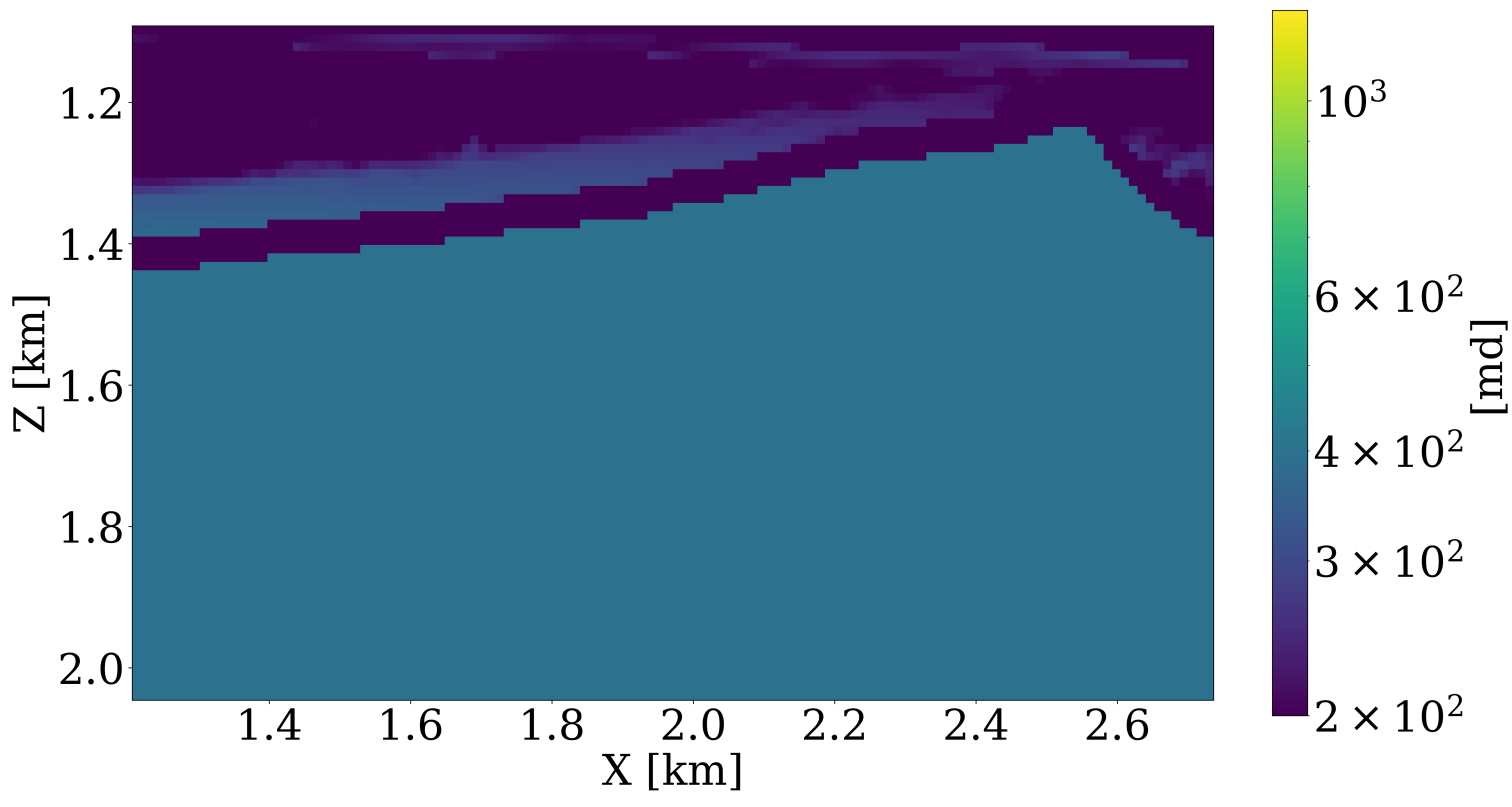
minimize  $\|\mathcal{F} \circ \mathcal{R} \circ \mathcal{S}(\mathbf{K}) - \mathbf{d}\|_2^2$   
 $\mathbf{K}$



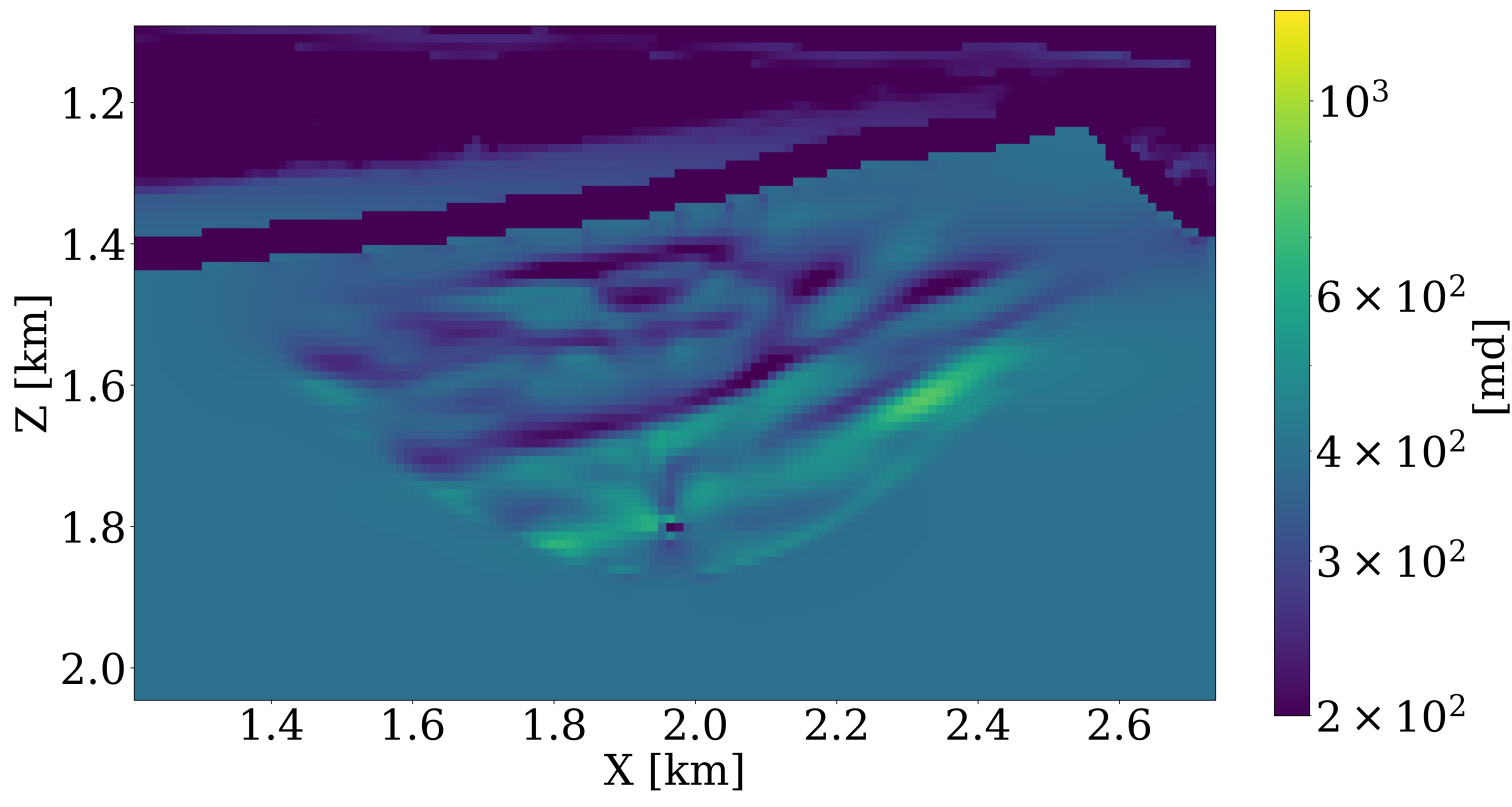
**End-to-end: “find permeability that matches seismic data”**

# Permeability inversion

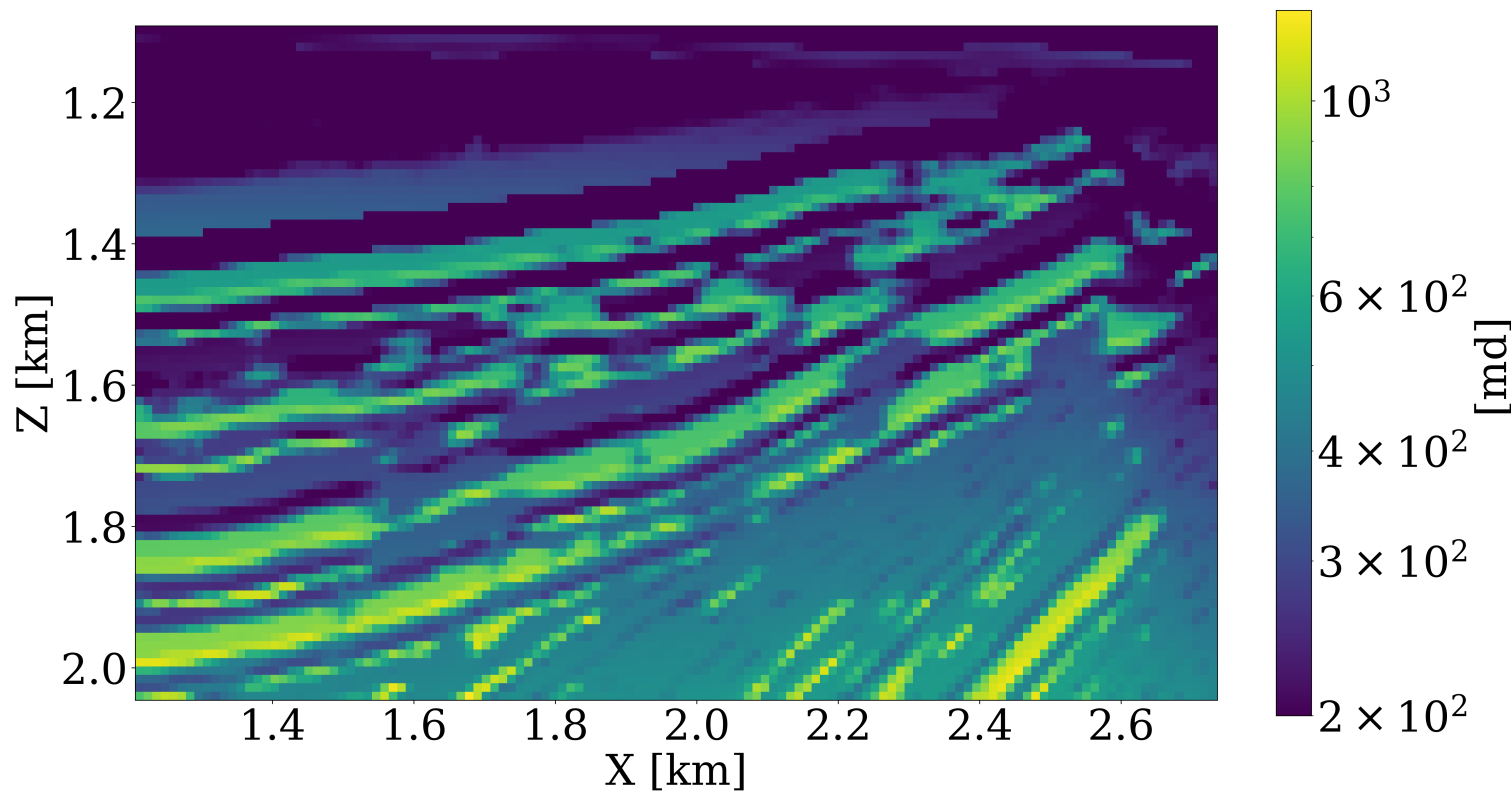
**initial**



**inverted**



**ground truth**



# Fourier neural operator surrogate

## cheap alternative to numerical simulation

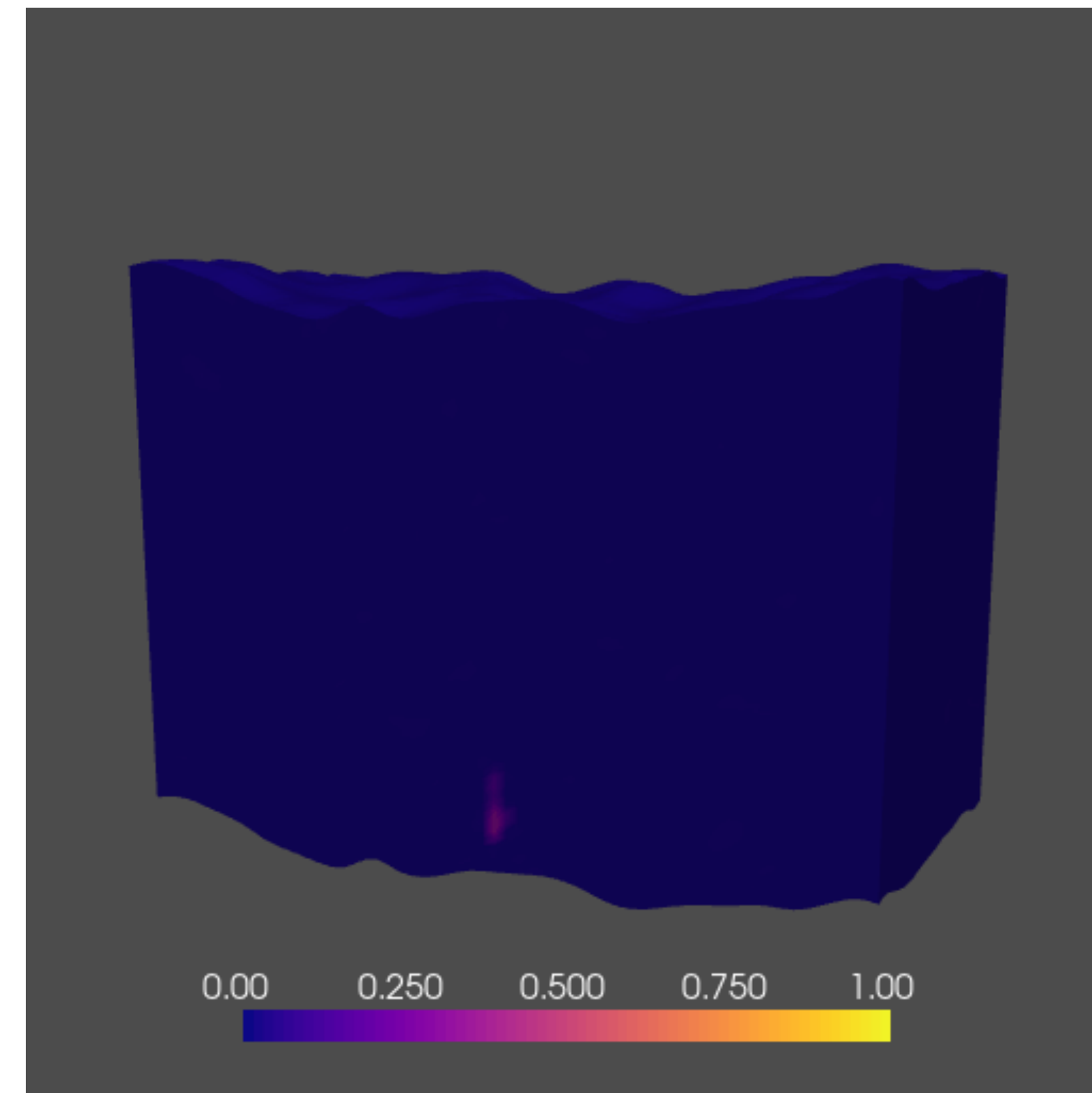
orders of magnitude faster

support surrogate inversion w/ automatic differentiation (AD)

scalable to large-scale 4D via domain decomposition

benefits:

- ▶ reduce computational cost
- ▶ enable uncertainty quantification

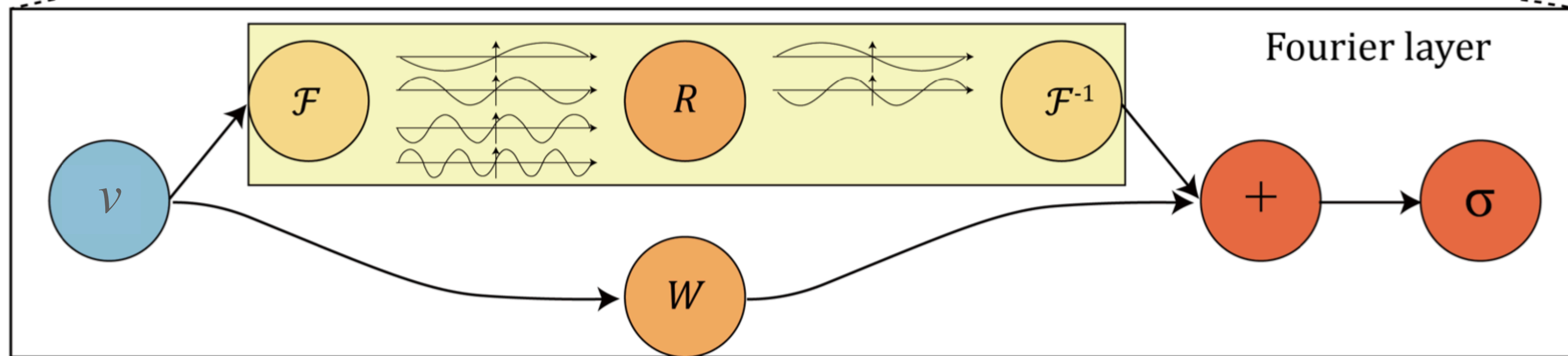
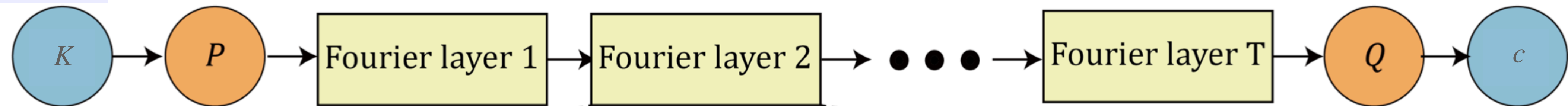


Problem Size	OPM Time (s)	FNO Time (s)	Speedup
$60 \times 60 \times 64 \times 30$	312	1.15	271x
$68 \times 118 \times 263 \times 16$	8291	5.98	1386x

# Surrogate Modeling

## Fourier neural operators – FNOs

FNO learns mappings on low-frequency modes in Fourier space via



Adapted from Li

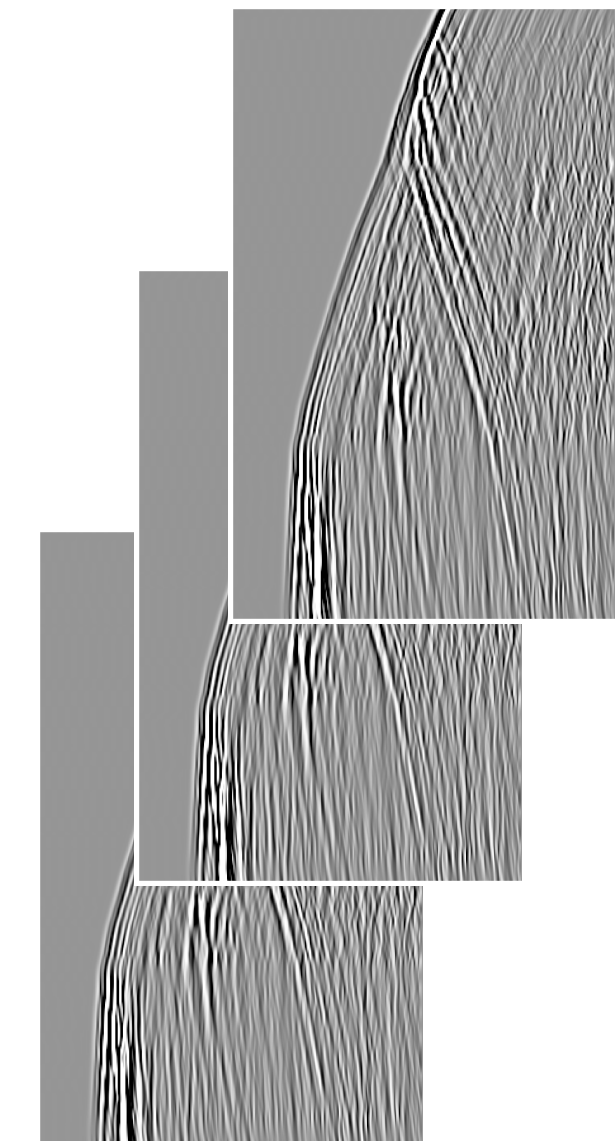
# Learned end-to-end inversion

permeability  
**K**

CO<sub>2</sub> concentration  
**c**

wavespeed  
**v**

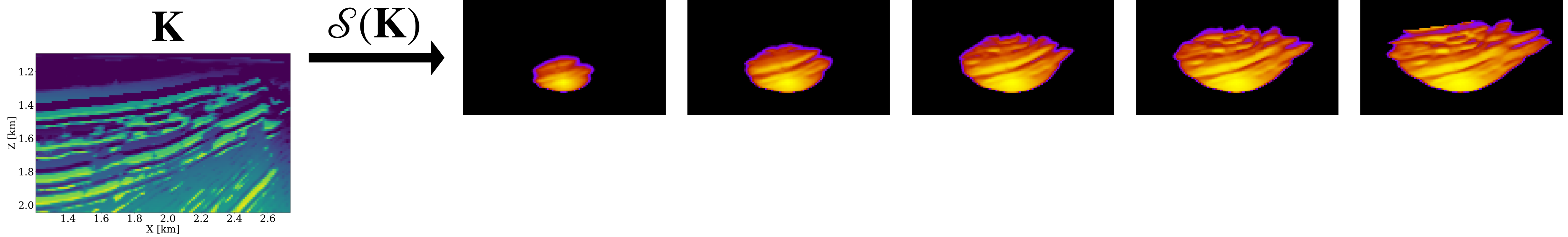
time-lapse data  
**d**



minimize  $\mathbf{K}$   $\|\mathcal{F} \circ \mathcal{R} \circ \mathcal{S}_\theta(\mathbf{K}) - \mathbf{d}\|_2^2$

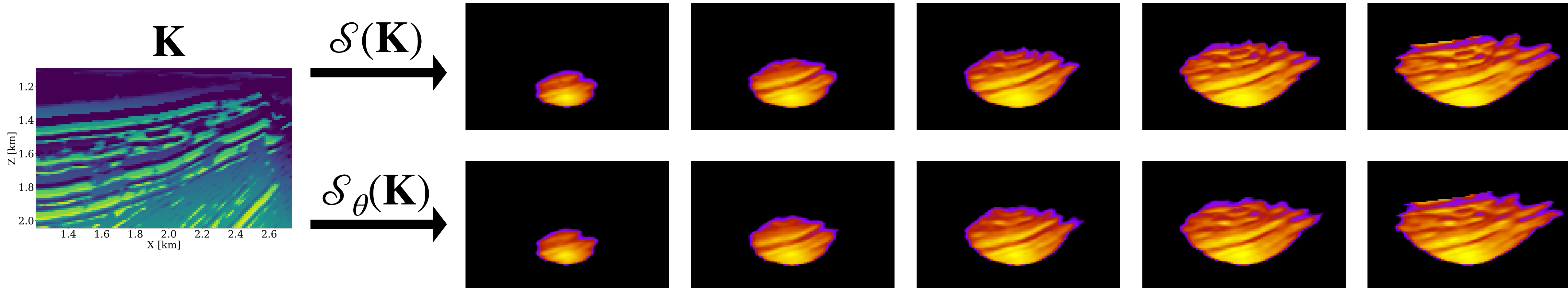
$\mathcal{S}_\theta$  pre-trained FNO: drastically reduce computational cost

# Fourier neural operators learns physics

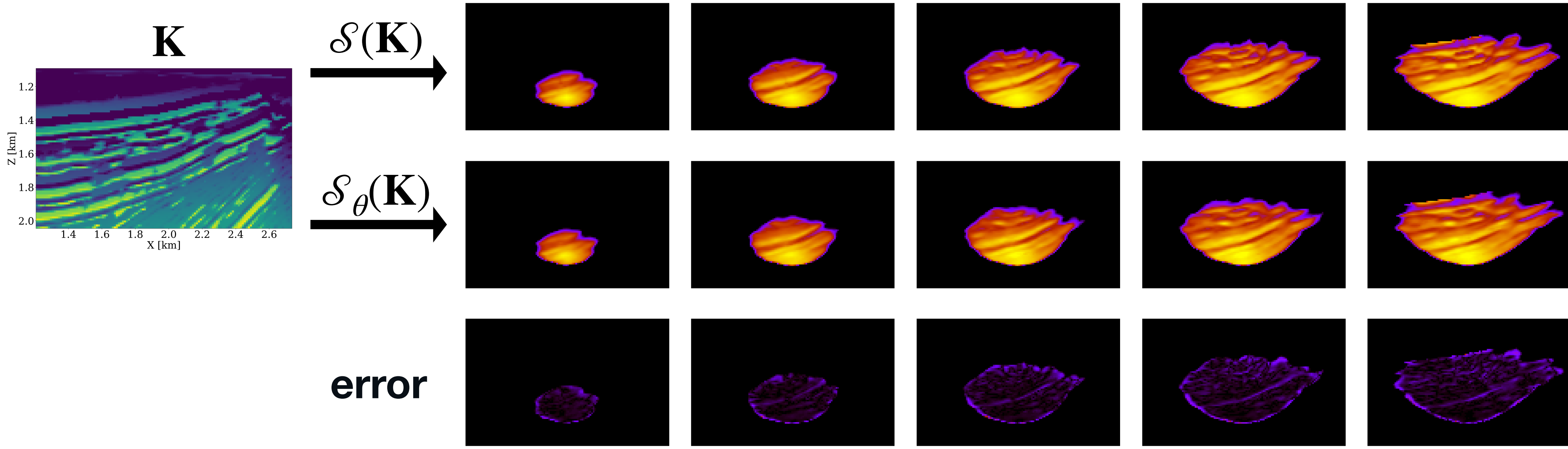




# Fourier neural operators learns physics



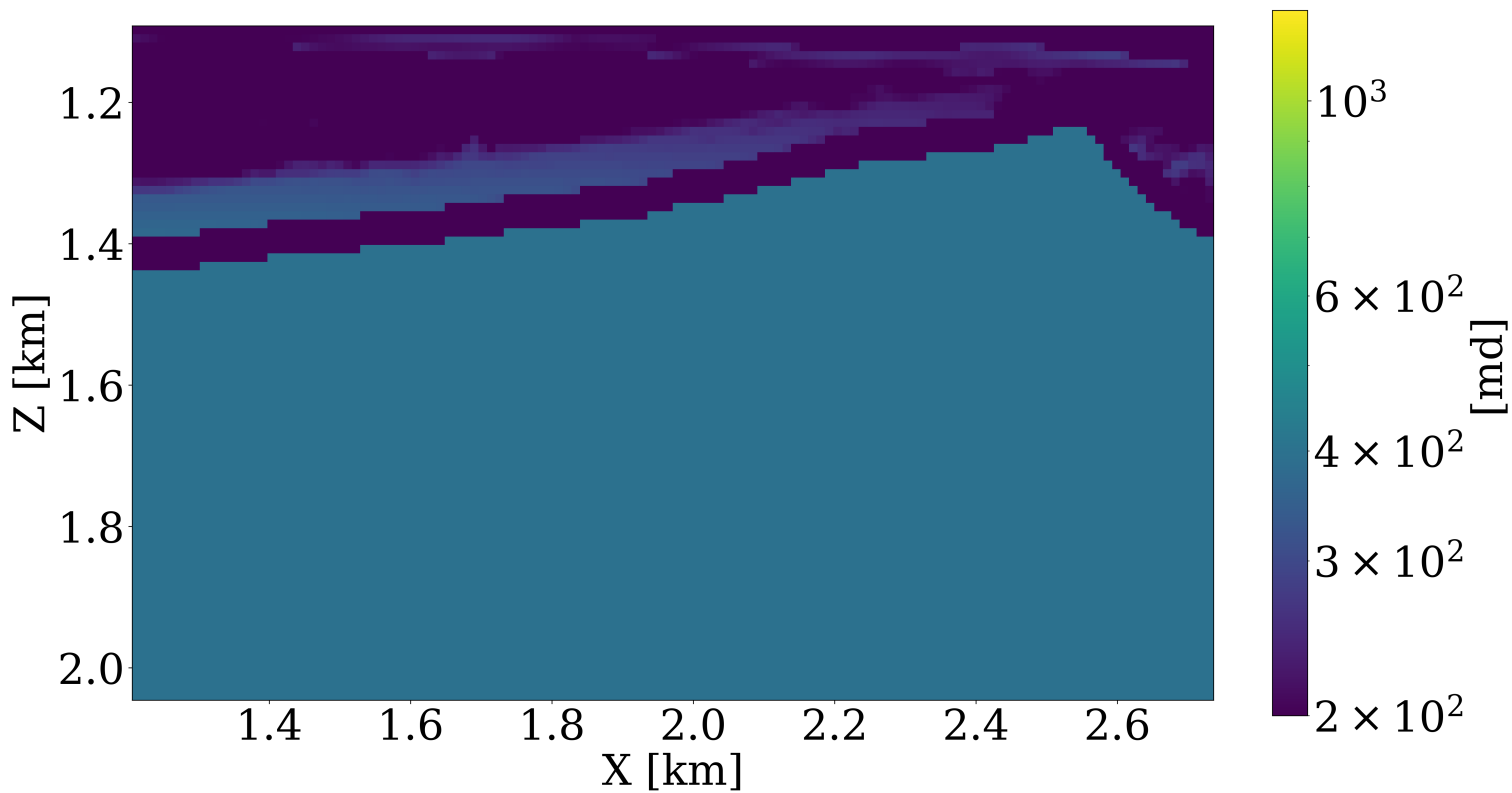
# Fourier neural operators learns physics



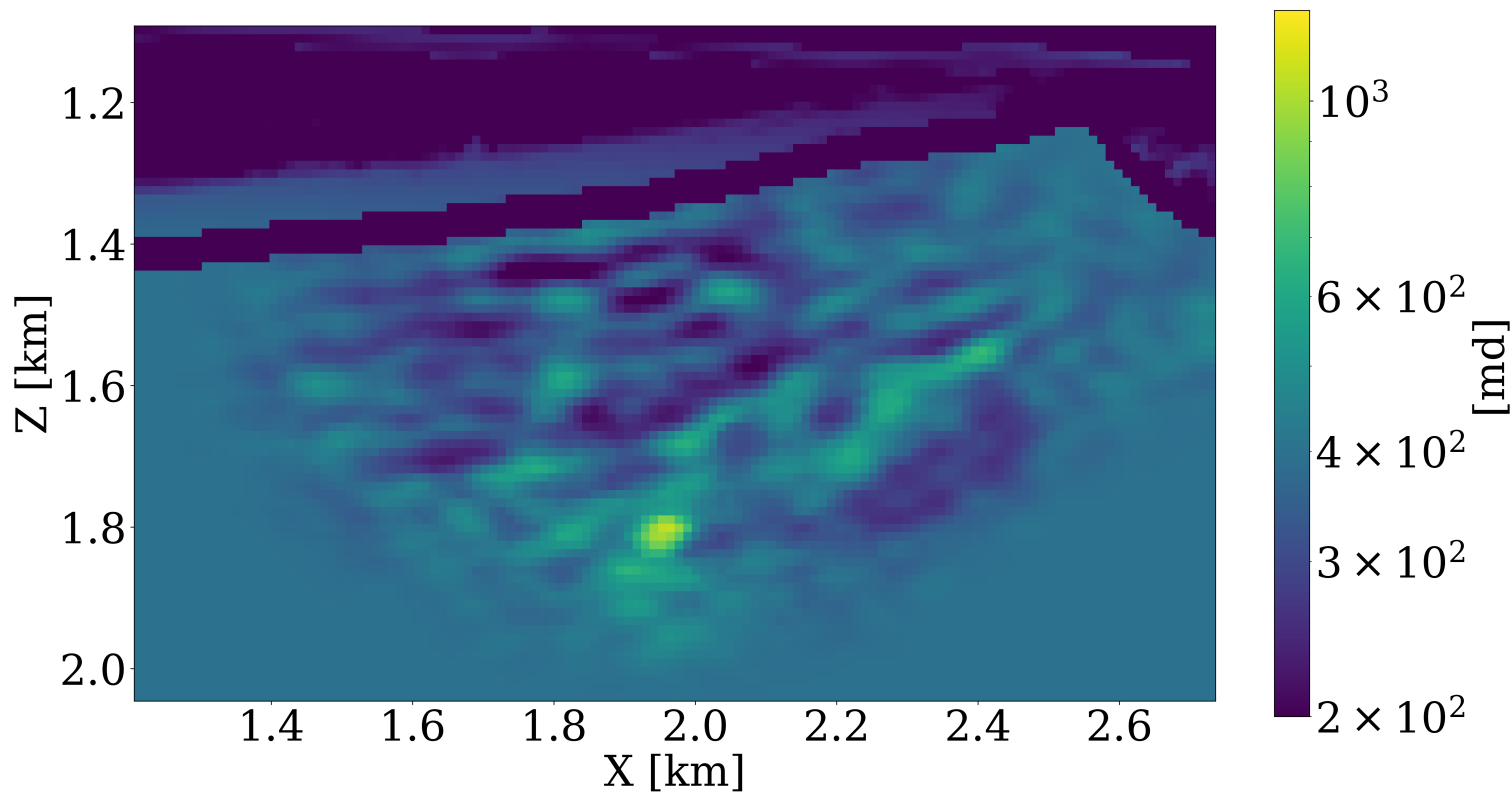
# Permeability inversion

## FNO surrogate

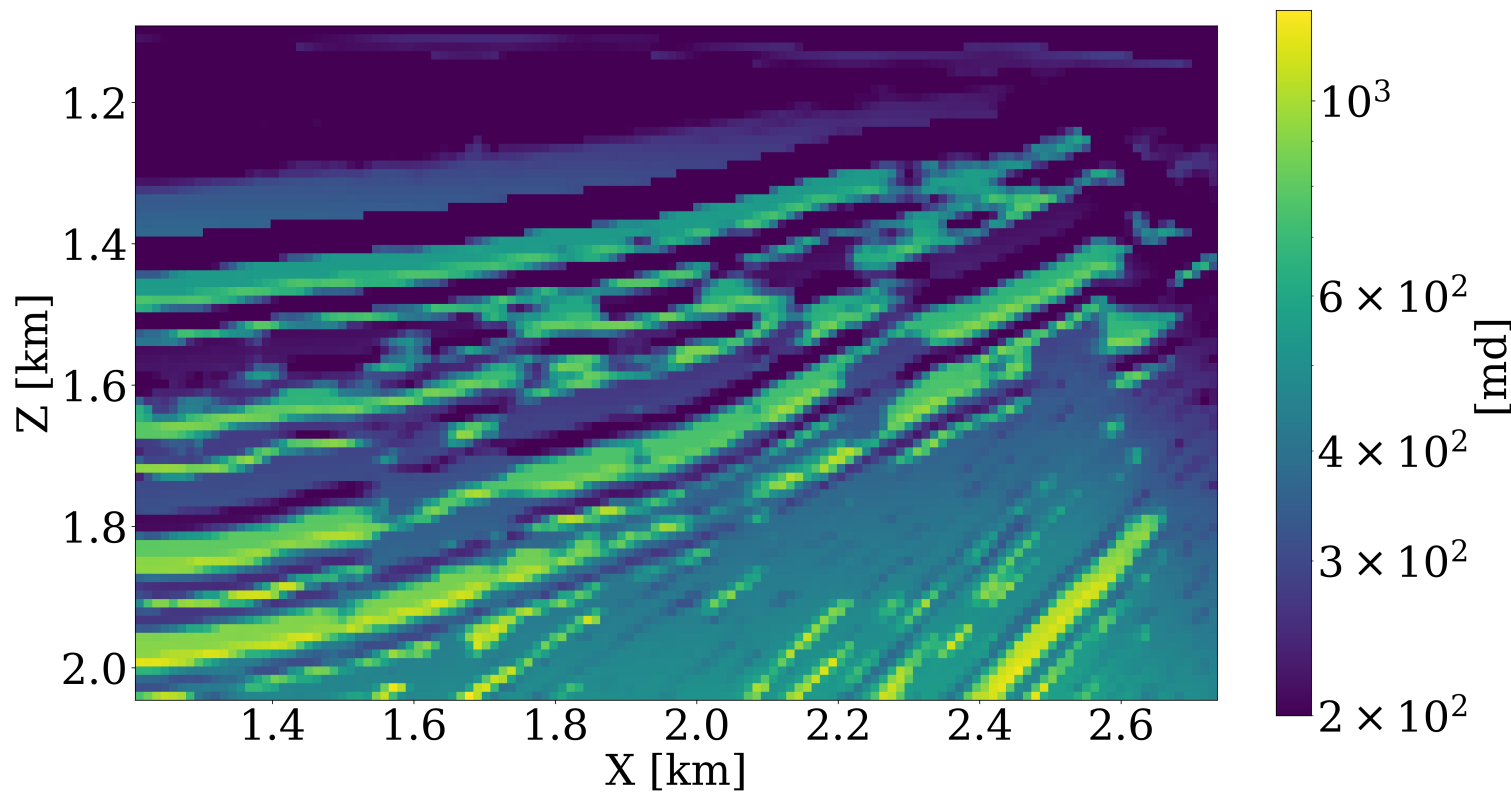
**initial**



**inverted**



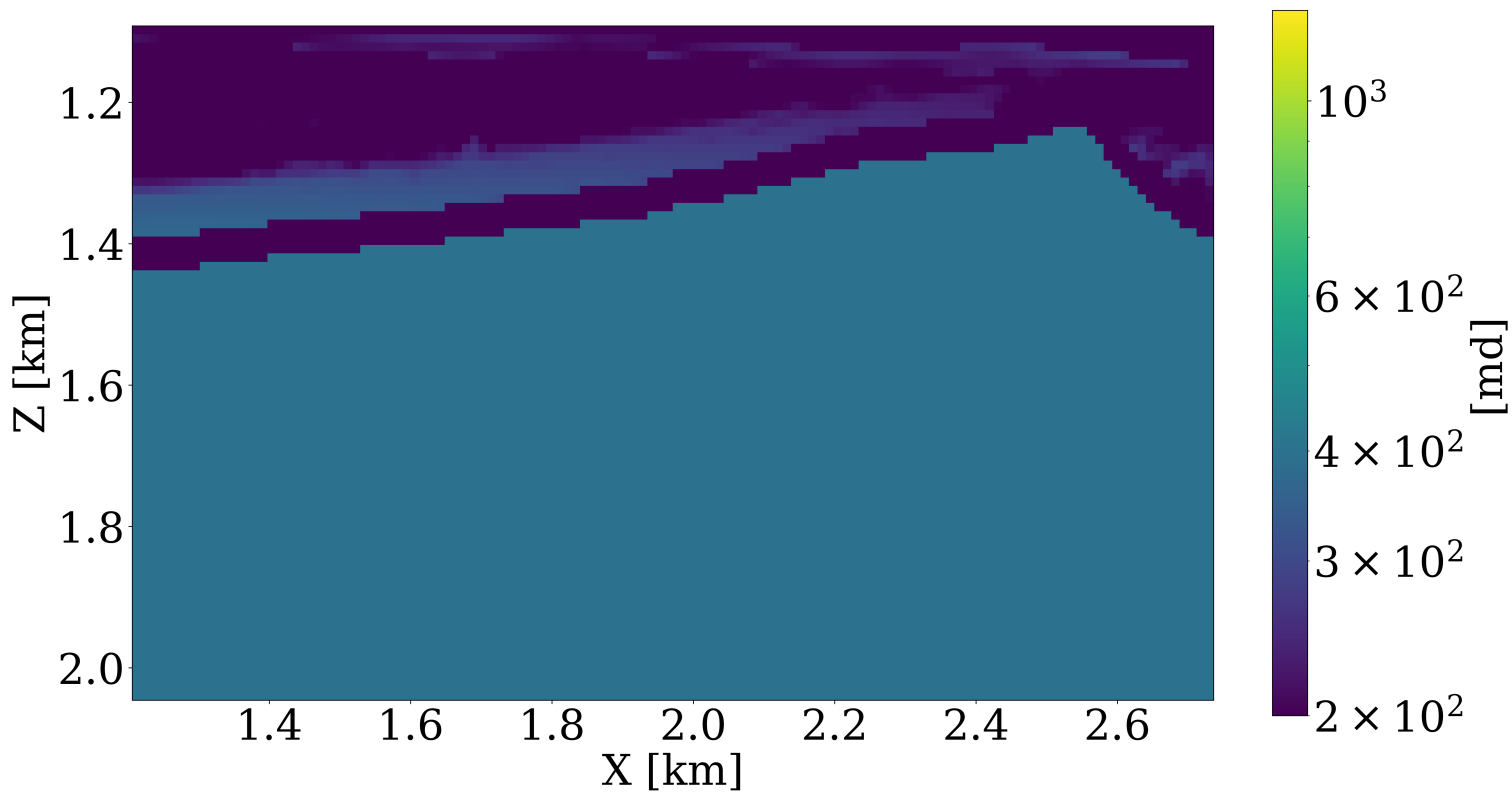
**ground truth**



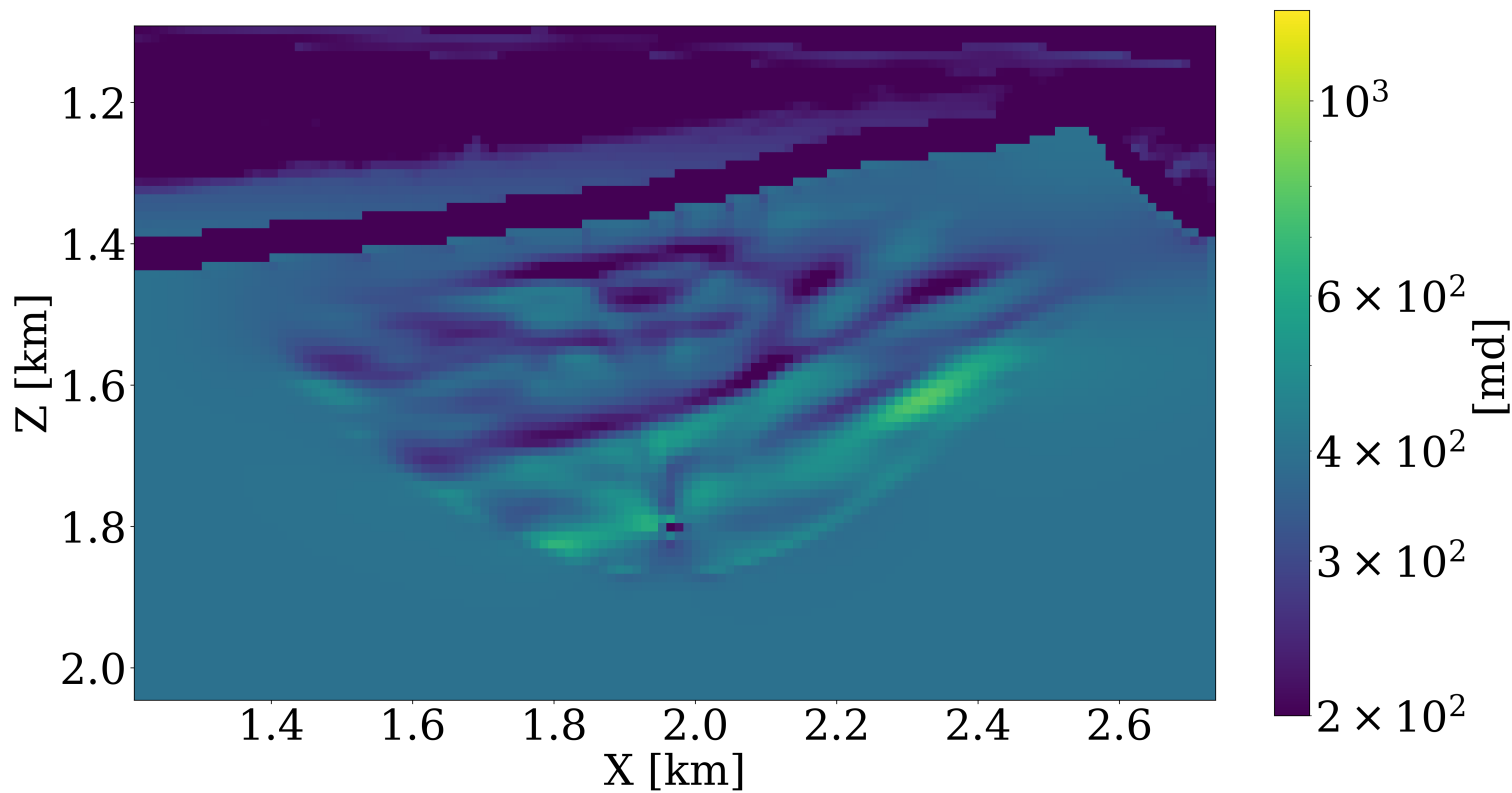
# Permeability inversion

## physics-based inversion

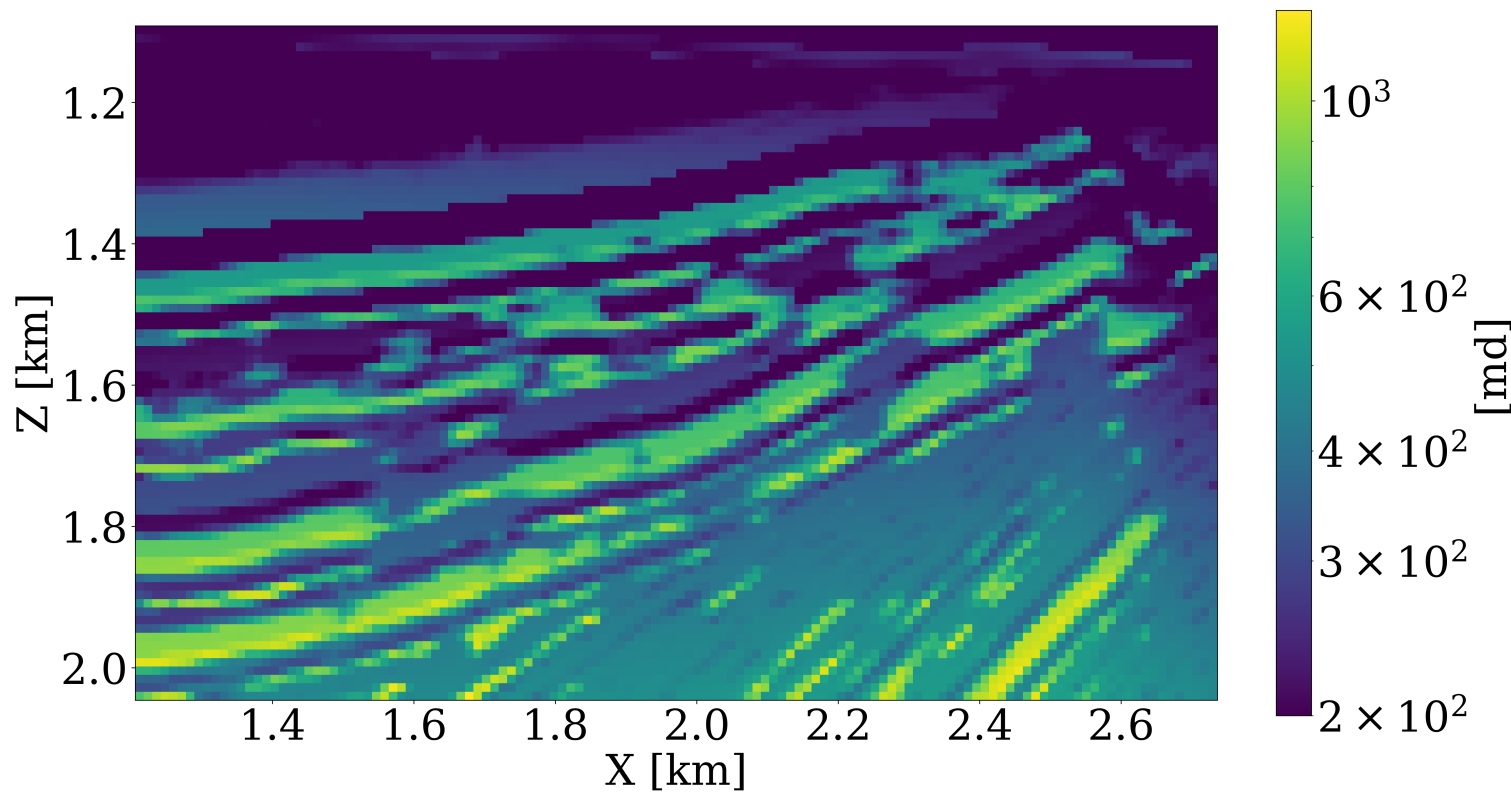
initial



inverted

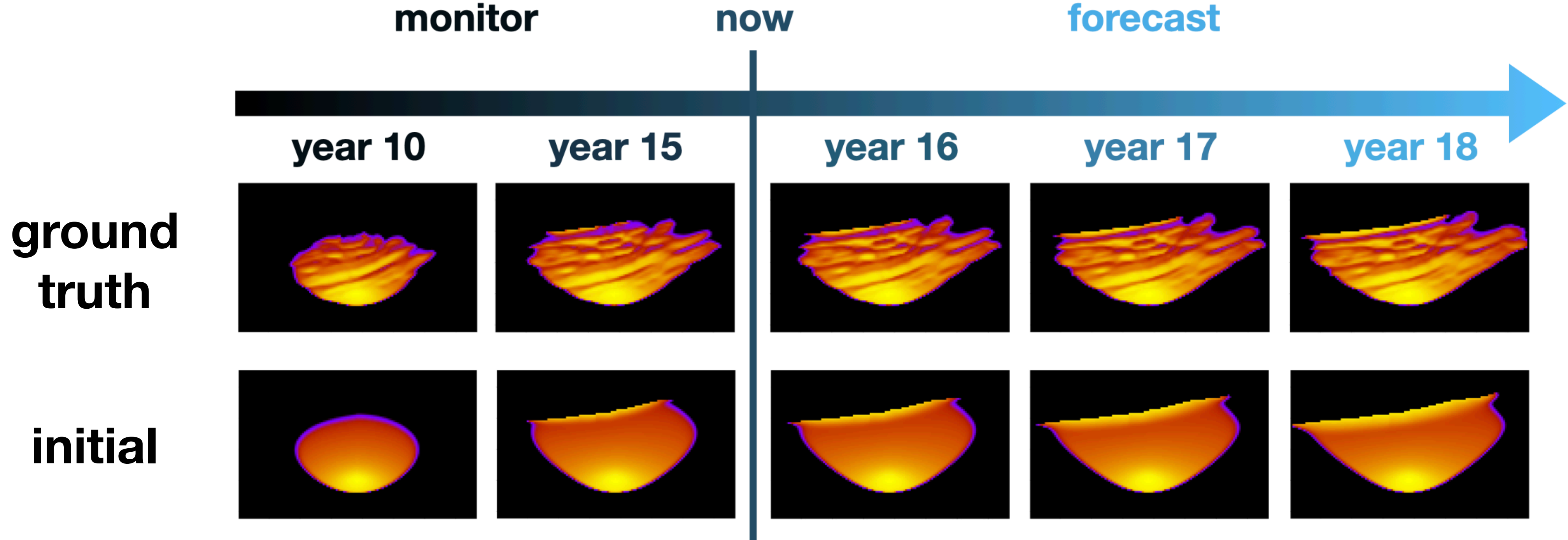


ground truth



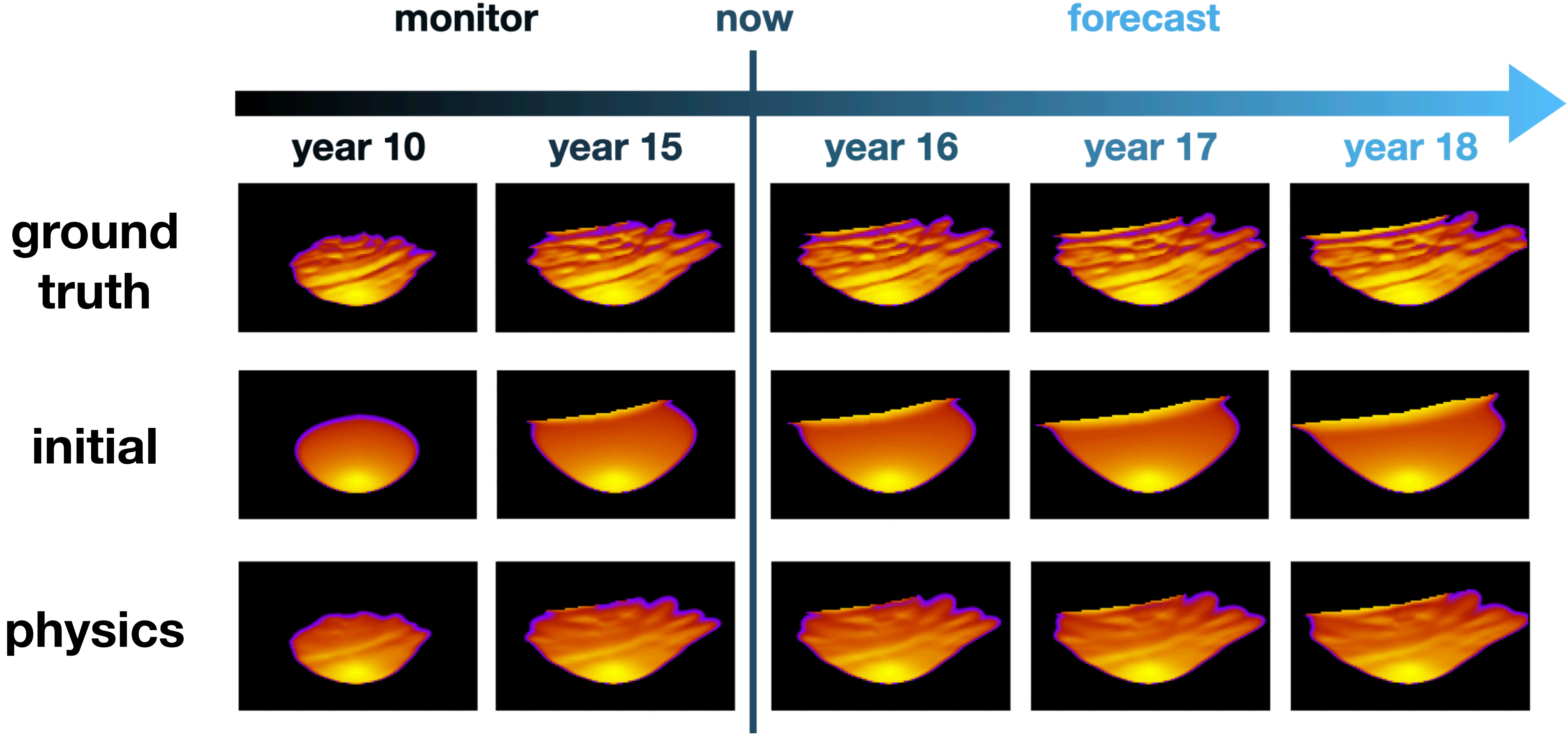
# End-to-end inversion

## CO<sub>2</sub> plume



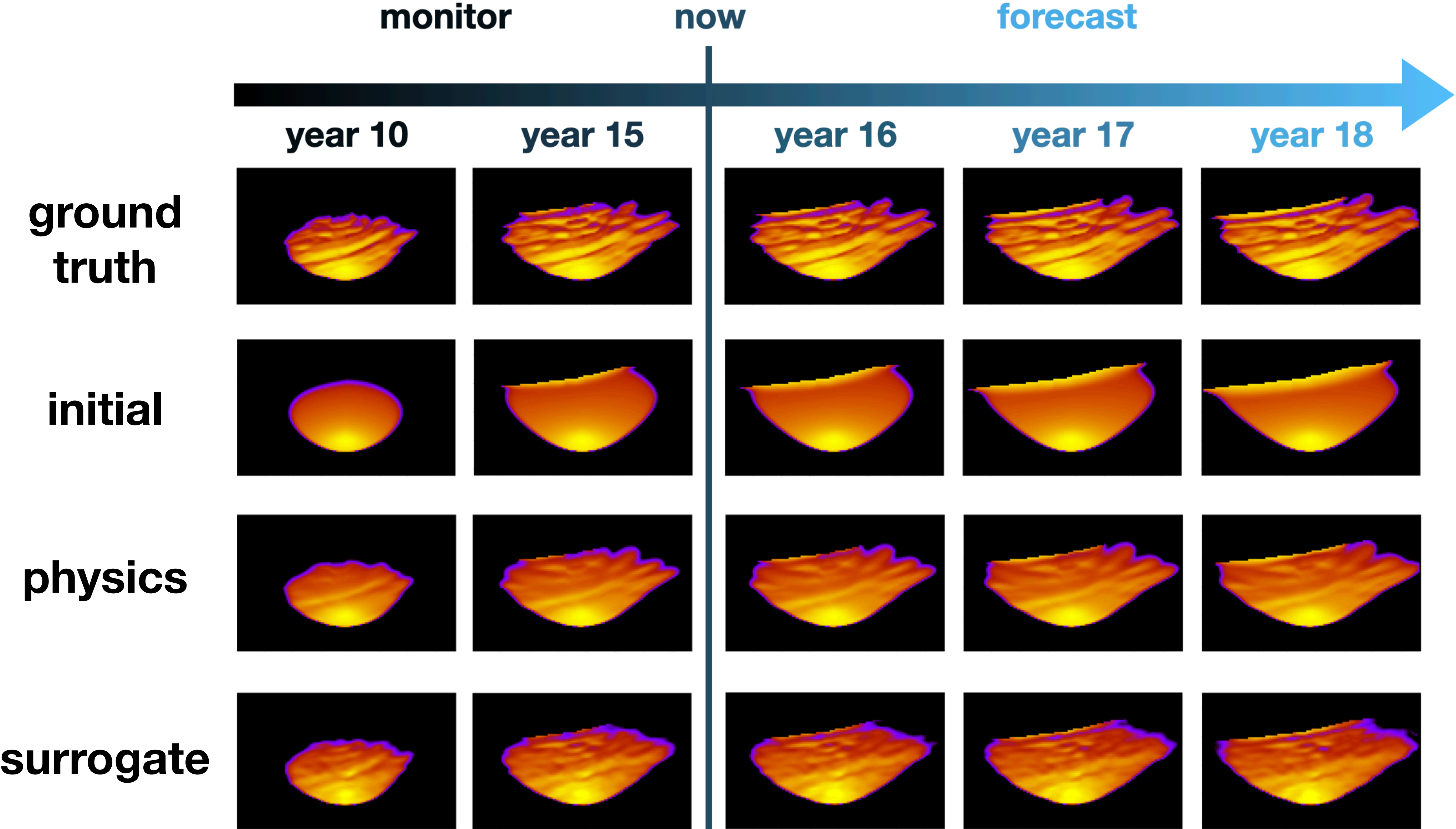
# End-to-end inversion

## CO<sub>2</sub> plume



# End-to-end inversion

## CO<sub>2</sub> plume



# Uncertainty quantification



# Bayesian Uncertainty Quantification

Bayesian posterior  $p_{\text{post}}(\mathbf{K} | \mathbf{d}) \propto p_{\text{like}}(\mathbf{d} | \mathbf{K})p_{\text{prior}}(\mathbf{K})$

**K** unknown model parameters (permeability)

**d** observed data

$p(\mathbf{d} | \mathbf{K})$  data likelihood

$p(\mathbf{K})$  prior

# Bayesian Uncertainty Quantification

## stochastic gradient Langevin dynamics (SGLD)

$$\mathbf{K}_{k+1} = \mathbf{K}_k - \frac{\alpha_k}{2} \nabla_{\mathbf{K}} \log p_{\text{post}}(\mathbf{K} | \mathbf{d}) + \boldsymbol{\eta}_k$$

- ▶ gradient-based MCMC
- ▶  $\alpha_k$  step size,  $\boldsymbol{\eta}_k$  noise

# Fast Uncertainty Quantification

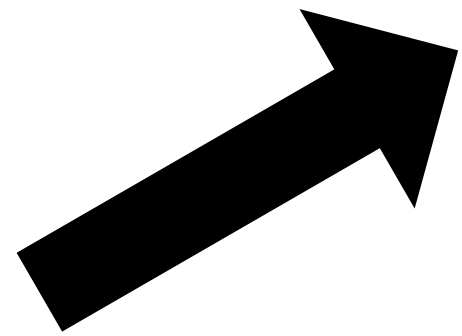
learned surrogate & prior

$$p_{\text{post}}(\mathbf{K} | \mathbf{d}) \propto p_{\text{like}}(\mathbf{d} | \mathbf{K})p_{\text{prior}}(\mathbf{K})$$

# Fast Uncertainty Quantification

learned surrogate & prior

$$p_{\text{post}}(\mathbf{K} | \mathbf{d}) \propto p_{\text{like}}(\mathbf{d} | \mathbf{K})p_{\text{prior}}(\mathbf{K})$$



**Fourier  
neural  
operators**

# Fast Uncertainty Quantification

learned surrogate & prior

$$p_{\text{post}}(\mathbf{K} | \mathbf{d}) \propto p_{\text{like}}(\mathbf{d} | \mathbf{K})p_{\text{prior}}(\mathbf{K})$$

Fourier  
neural  
operators

The diagram features a central mathematical equation. Below the equation, two arrows point towards the terms  $p_{\text{like}}(\mathbf{d} | \mathbf{K})$  and  $p_{\text{prior}}(\mathbf{K})$ . The arrow from the left points to  $p_{\text{like}}(\mathbf{d} | \mathbf{K})$  and is labeled 'Fourier neural operators'. The arrow from the right points to  $p_{\text{prior}}(\mathbf{K})$  and is labeled 'normalizing flows'.


normalizing  
flows

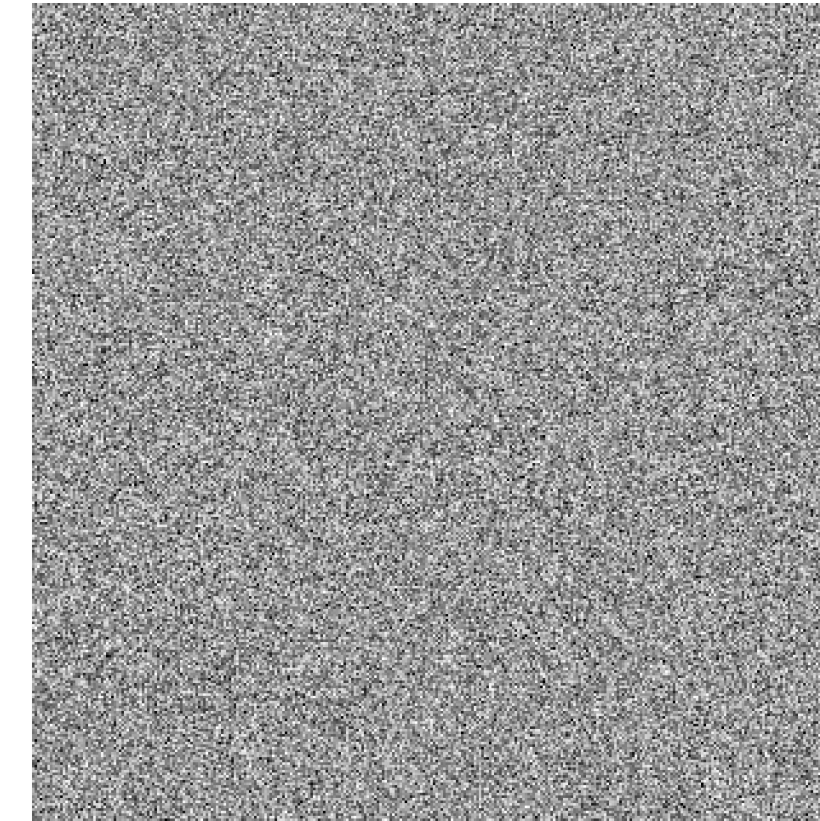
# Normalizing flows (NFs)

**Training:**



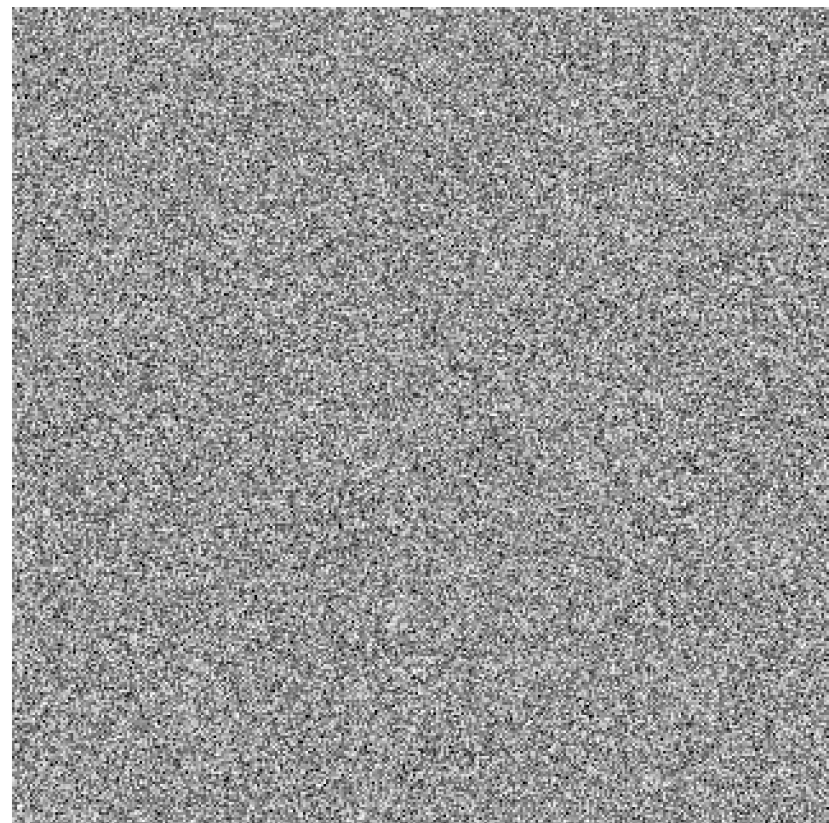
$$\mathbf{x} \sim p_X(\mathbf{x})$$

$$\mathcal{G}_w^{-1}(\mathbf{x})$$





$$\mathbf{z} \sim p_Z(\mathbf{z})$$

**Sampling:**



$$\mathbf{z} \sim p_Z(\mathbf{z})$$

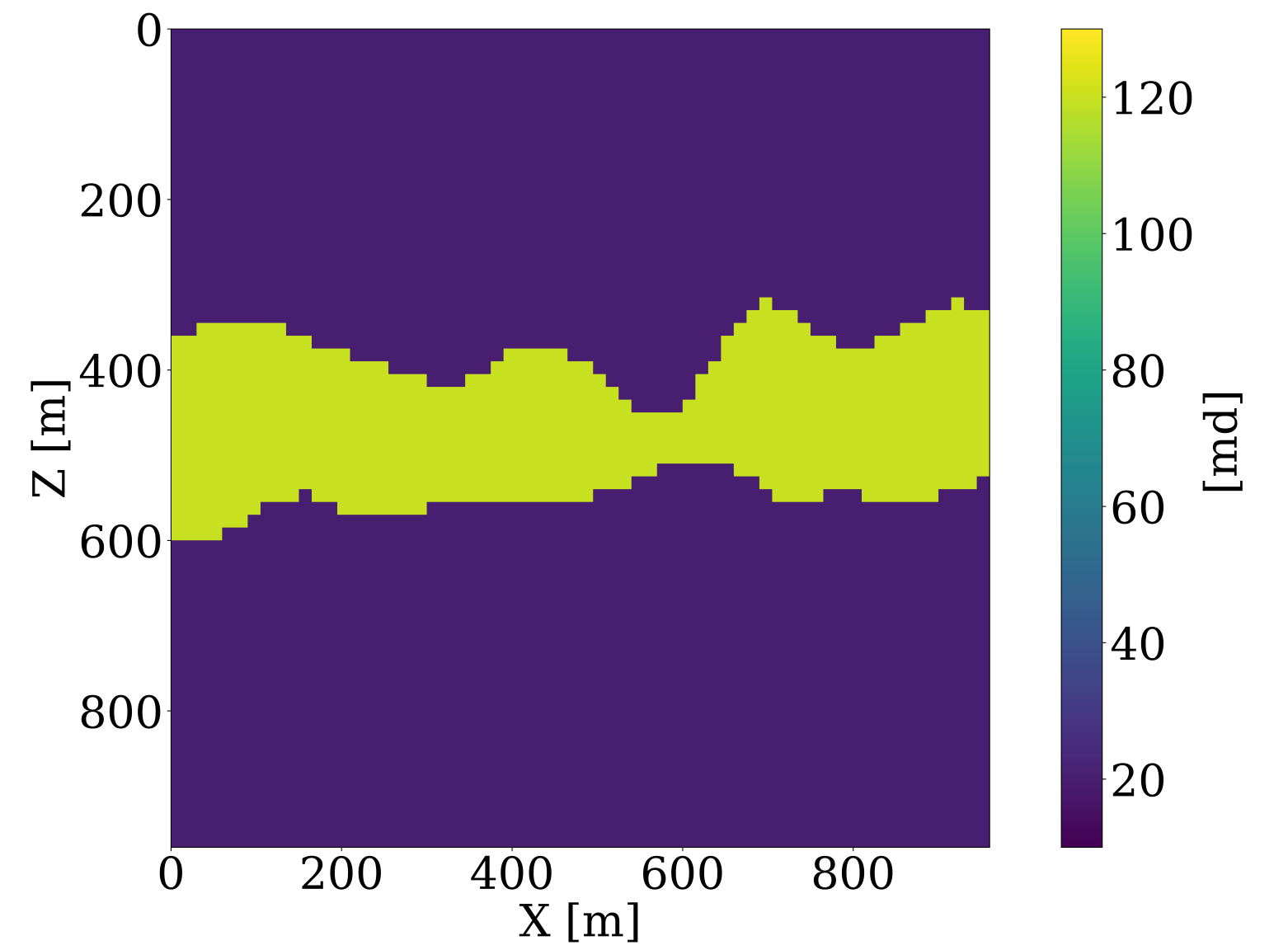
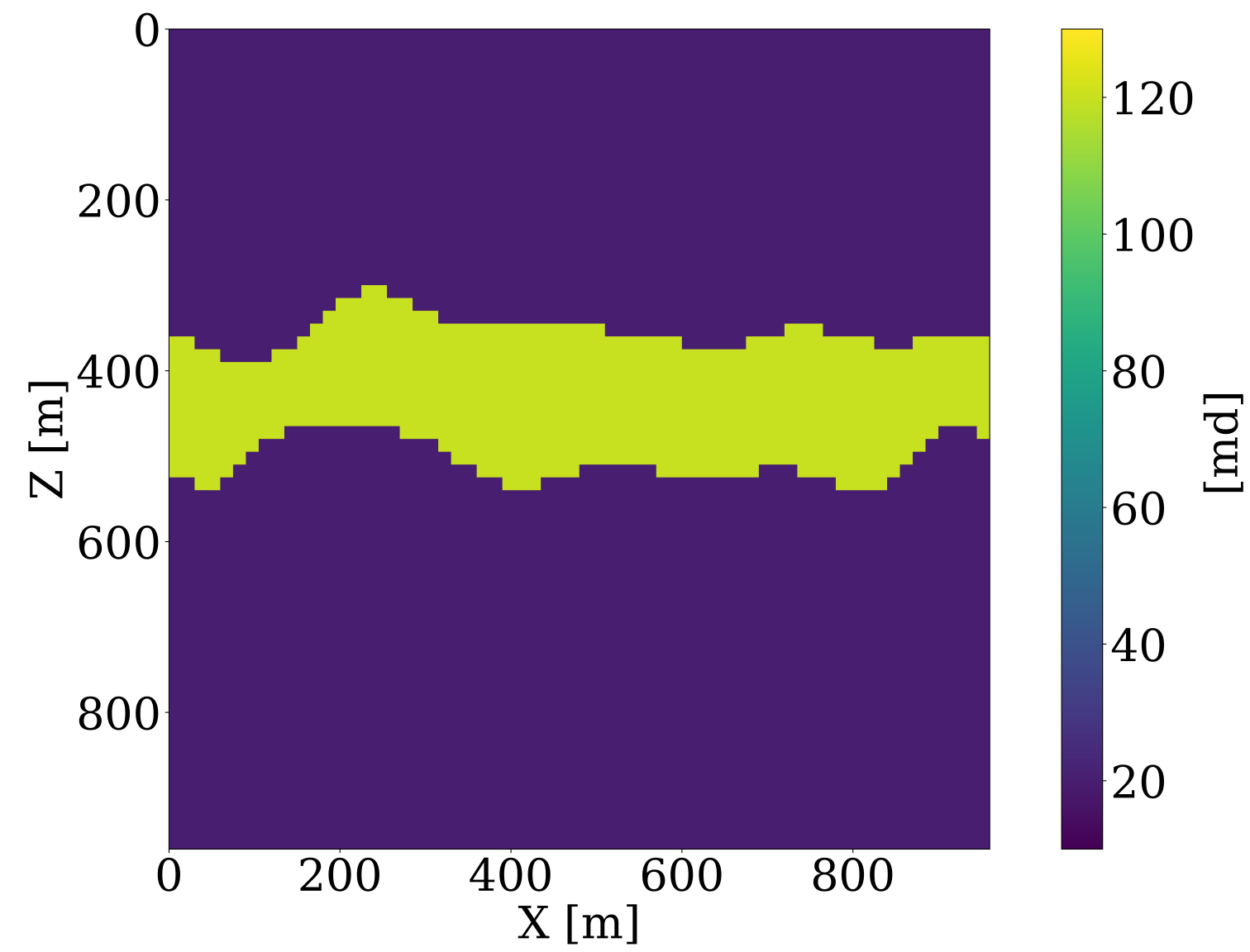
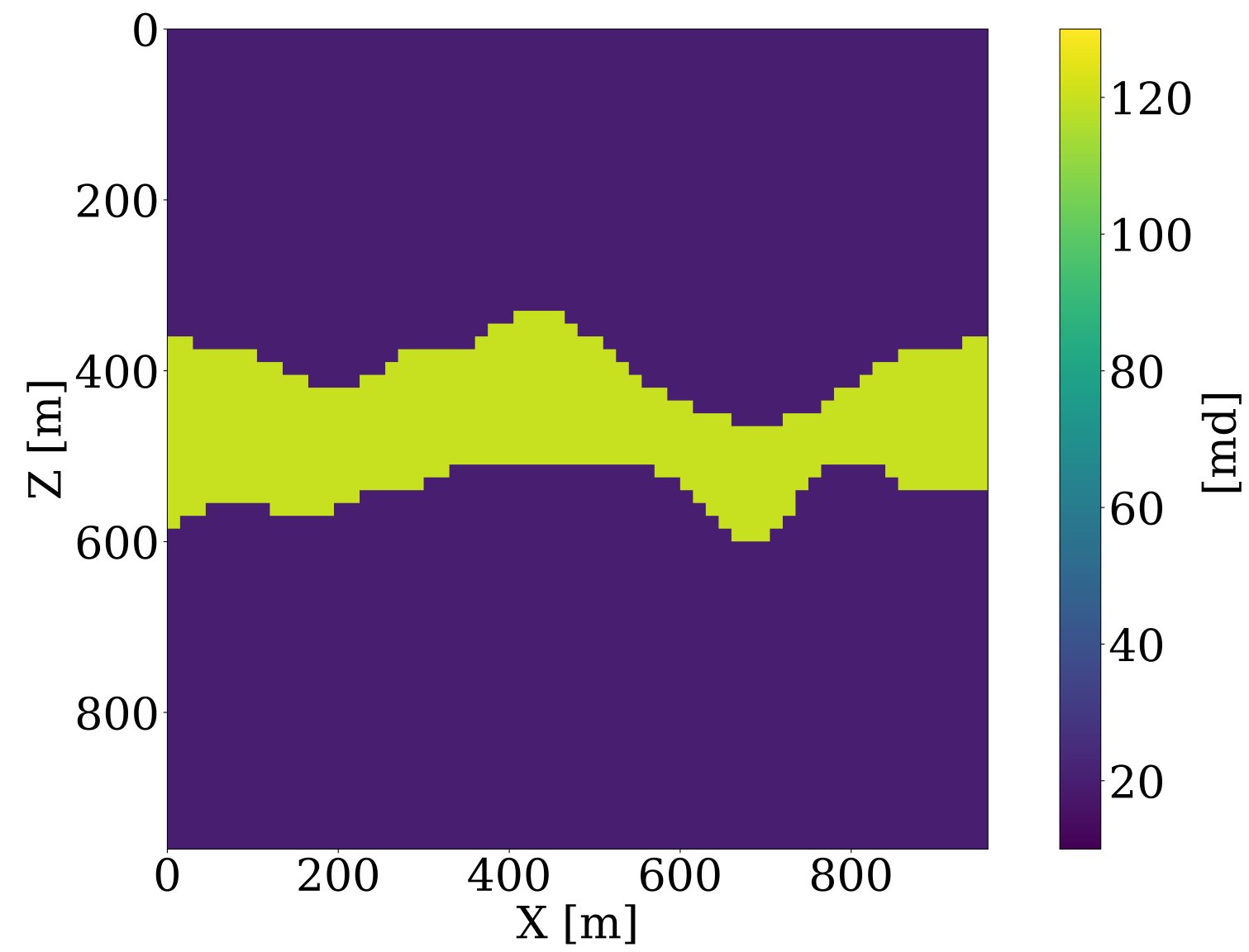
$$\mathcal{G}_w(\mathbf{z})$$




$$\mathbf{x} \sim p_X(\mathbf{x})$$

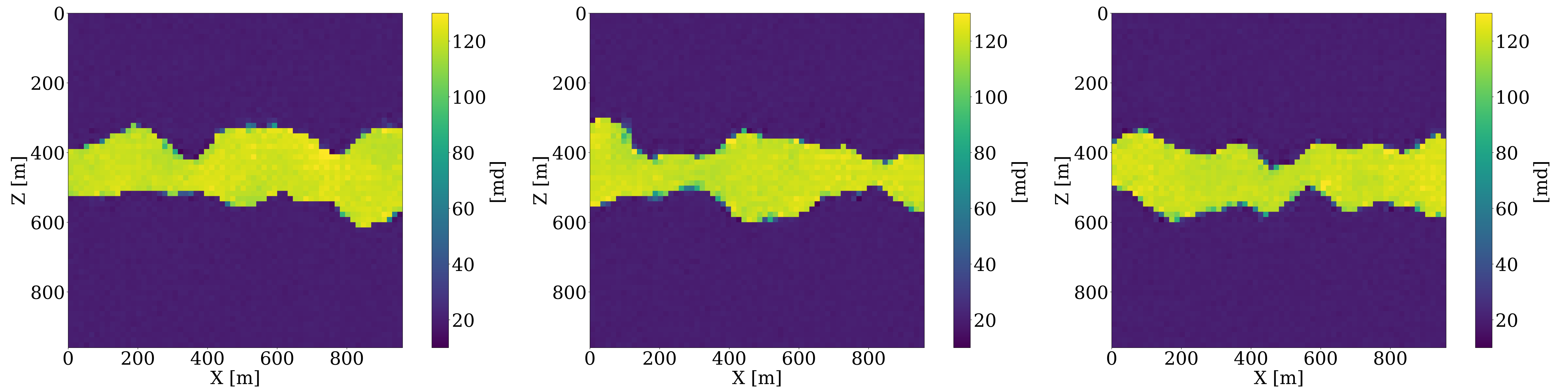
# Training samples

## permeability models



# Generative samples

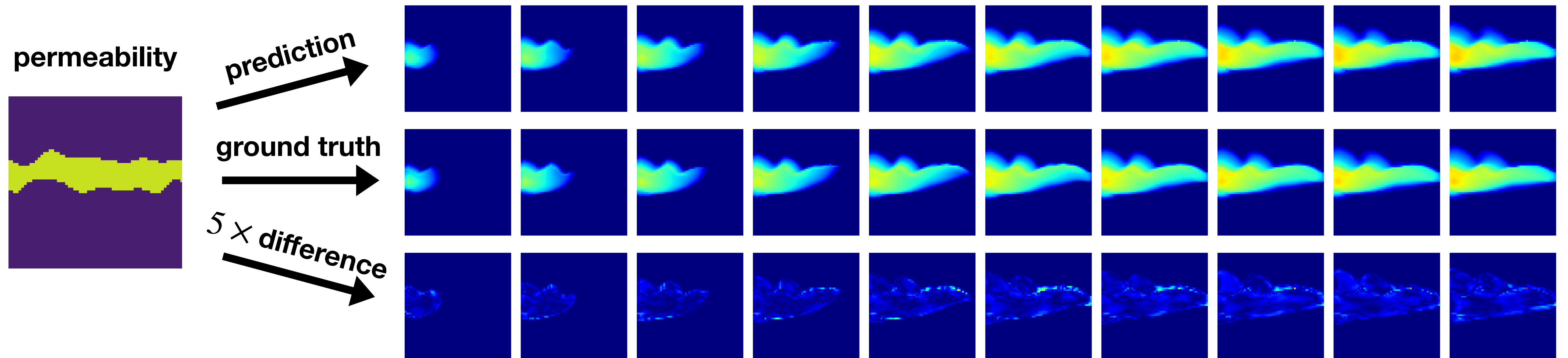
## “fake” permeability models





# Fourier neural operators

likelihood



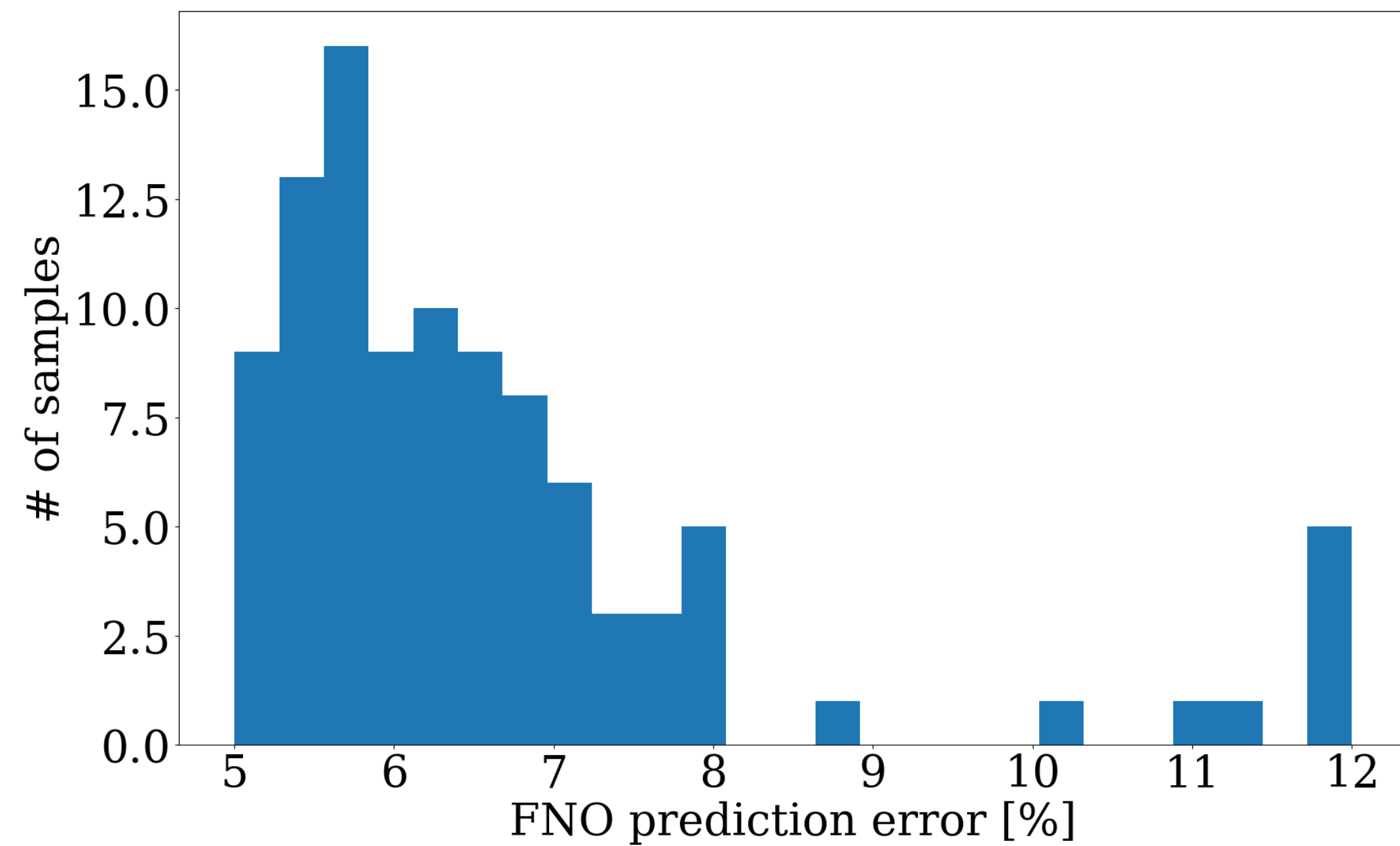
2000 random permeability channels

FNO & NF share the same training samples

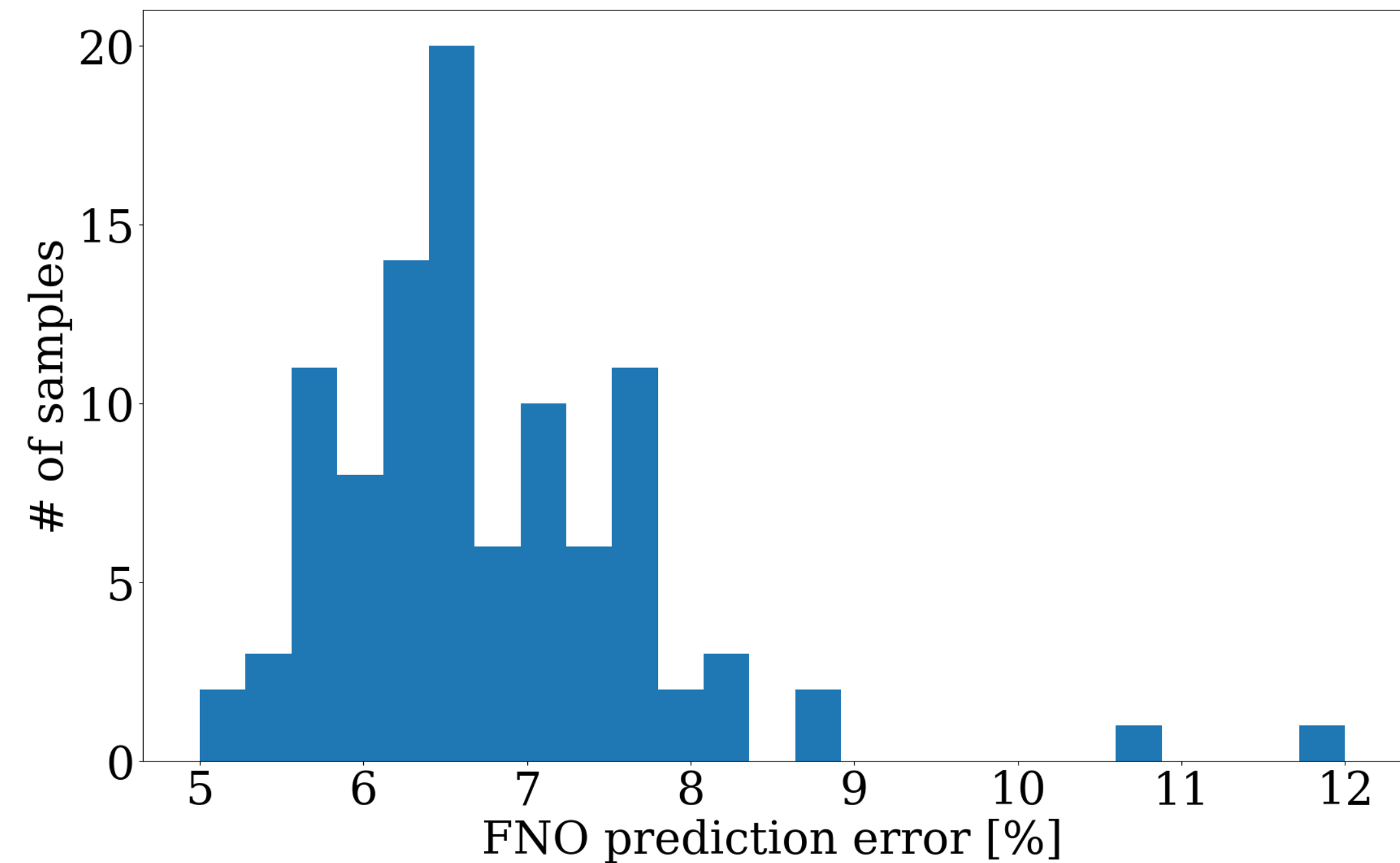
# FNO prediction error

**FNO gives low prediction error on generative samples from NF**

**real samples**



**generative samples**



# Bayesian Uncertainty Quantification

## stochastic gradient Langevin dynamics (SGLD)

$$\mathbf{K}_{k+1} = \mathbf{K}_k - \frac{\alpha_k}{2} \nabla_{\mathbf{K}} \log p_{\text{post}}(\mathbf{K} | \mathbf{d}) + \boldsymbol{\eta}_k$$

- ▶ gradient-based MCMC
- ▶  $\alpha_k$  step size,  $\boldsymbol{\eta}_k$  noise

# Bayesian Uncertainty Quantification

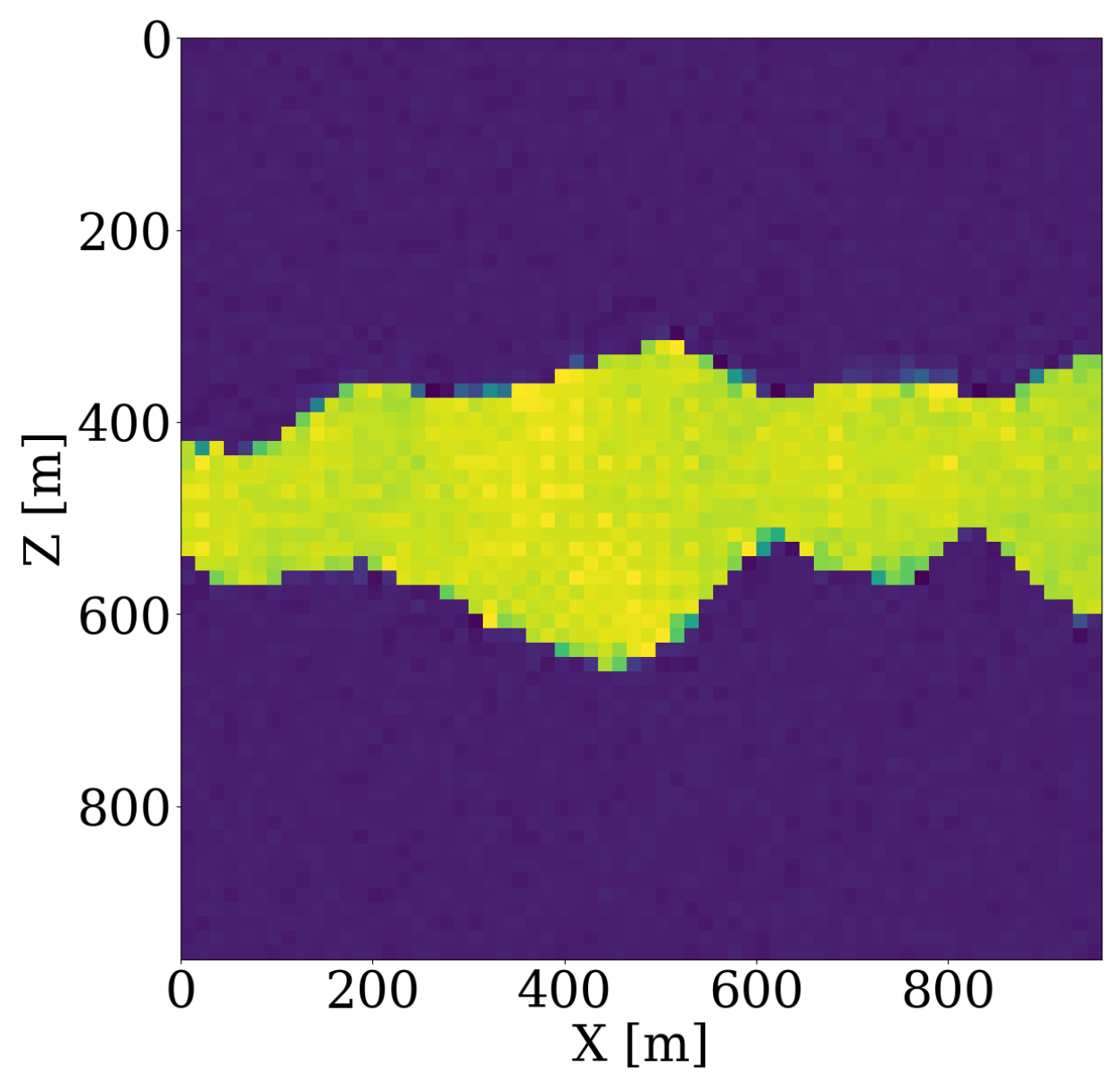
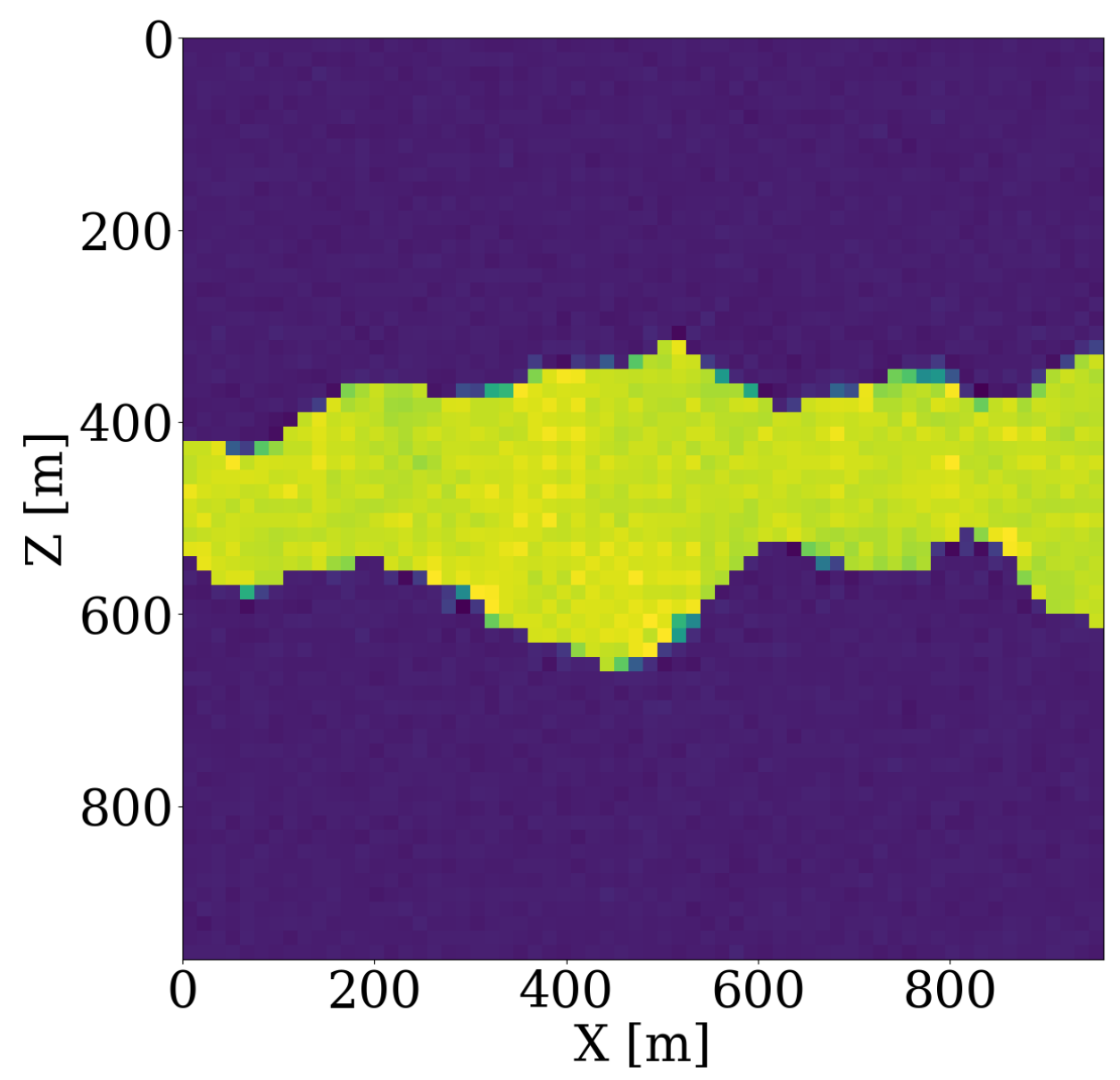
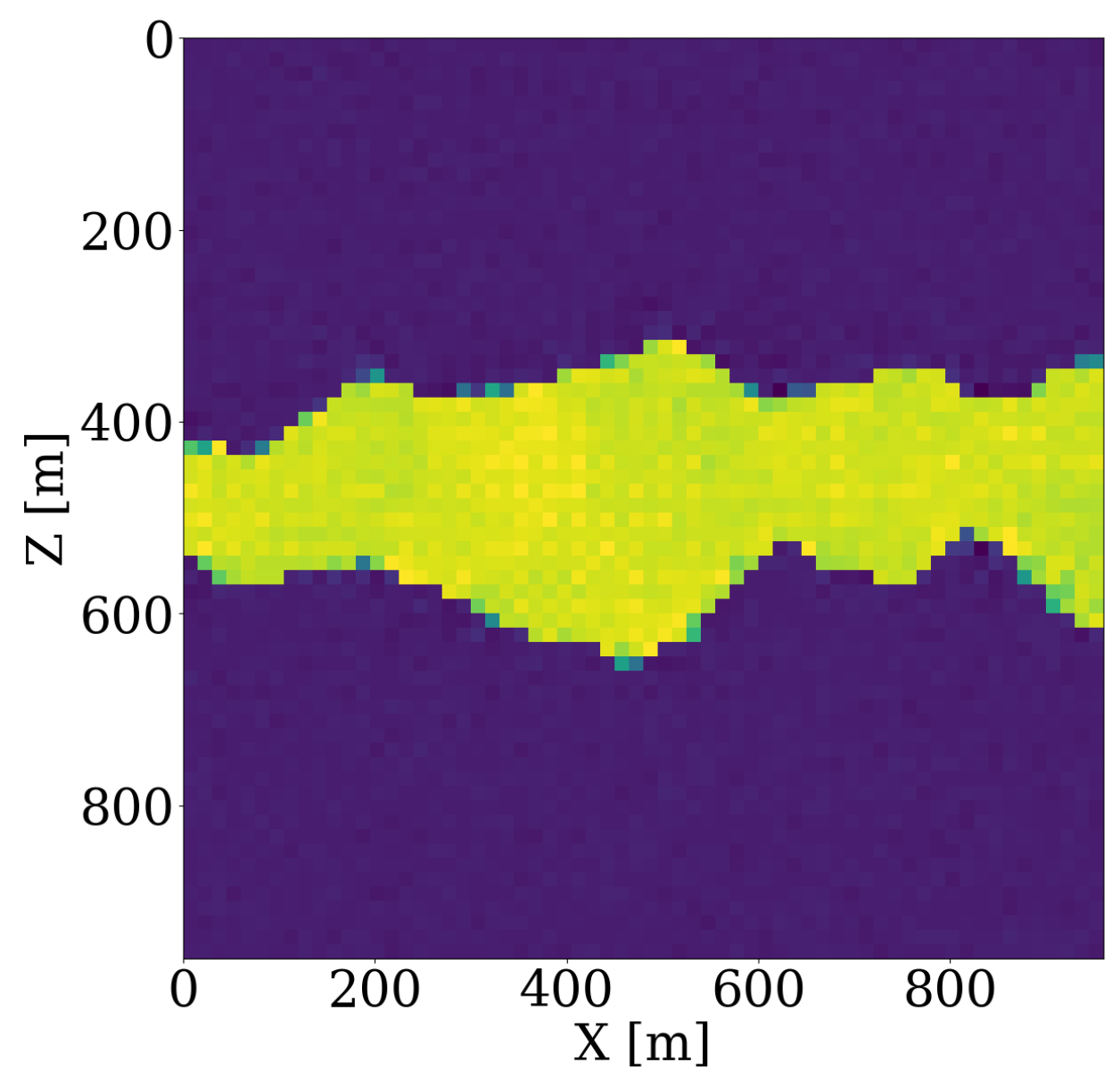
## stochastic gradient Langevin dynamics (SGLD)

$$\mathbf{z}_{k+1} = \mathbf{z}_k - \frac{\alpha_k}{2} \nabla_{\mathbf{z}_k} \log p_{\text{post}}(\mathbf{z} | \mathbf{d}) + \boldsymbol{\eta}_k$$

- ▶ gradient-based MCMC
- ▶  $\alpha_k$  step size,  $\boldsymbol{\eta}_k$  noise
- ▶  $\mathcal{G}(\mathbf{z}) = \mathbf{K}$ ,  $\mathcal{G}$  trained NF,  $\mathbf{z}$  latent variable

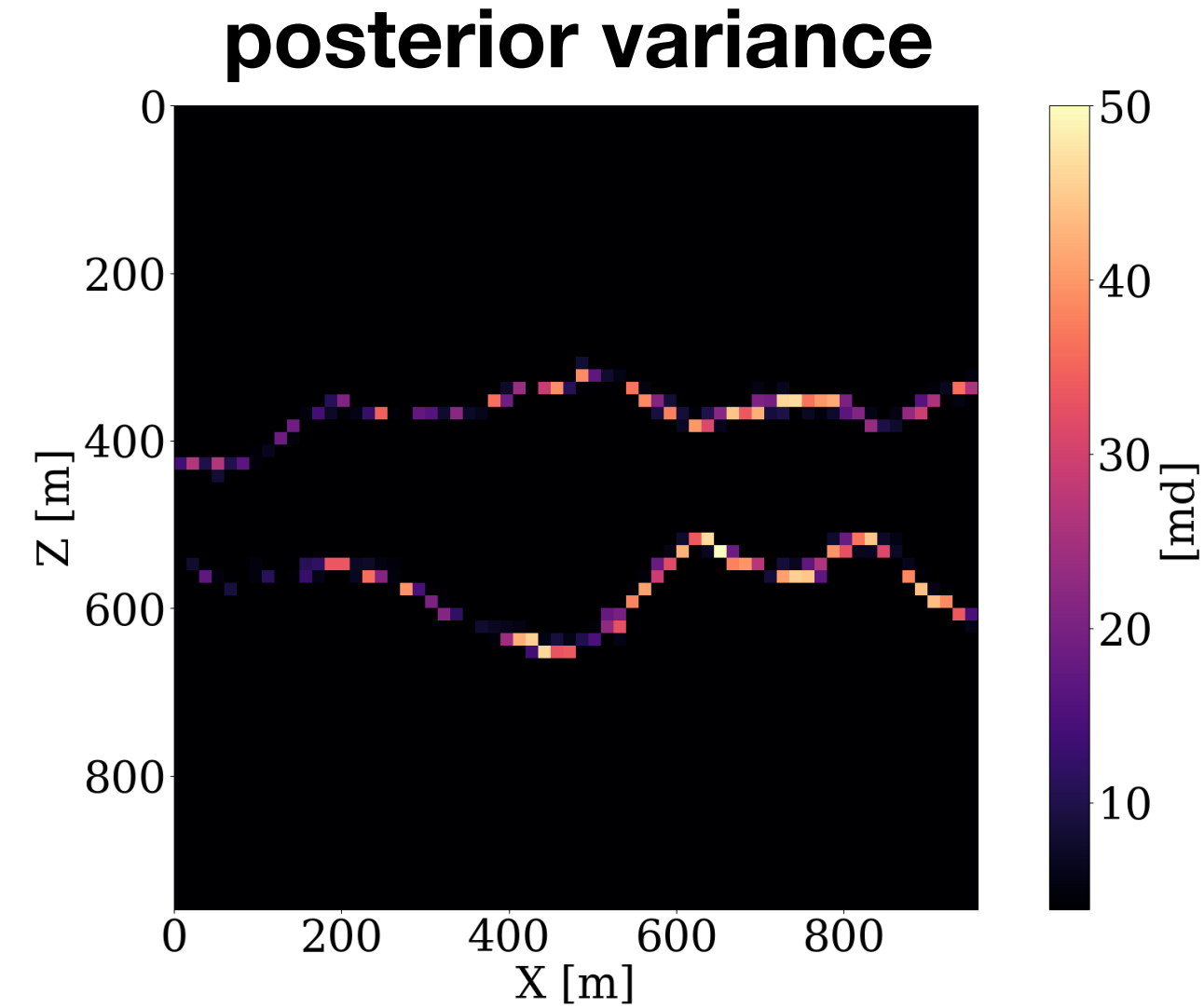
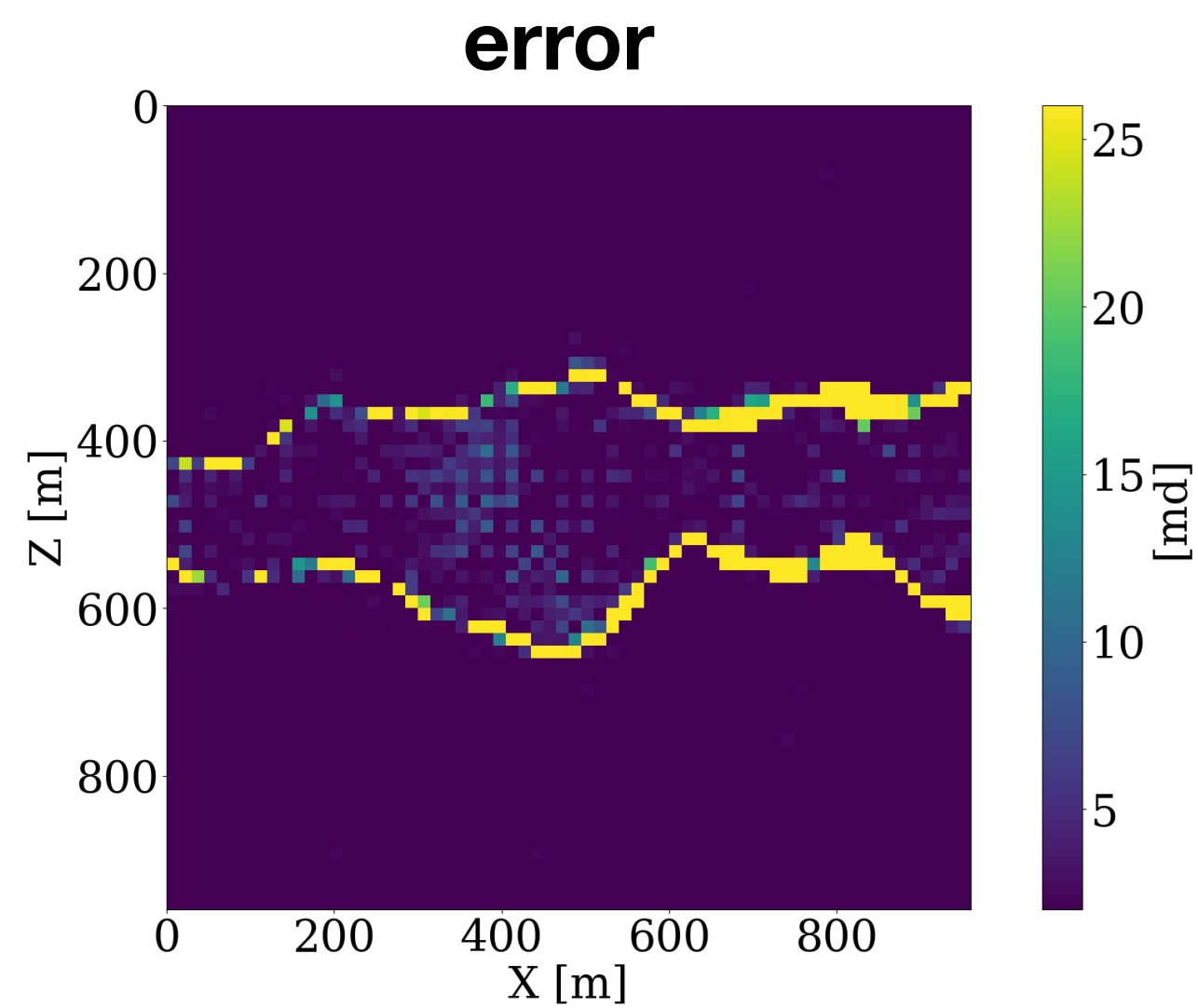
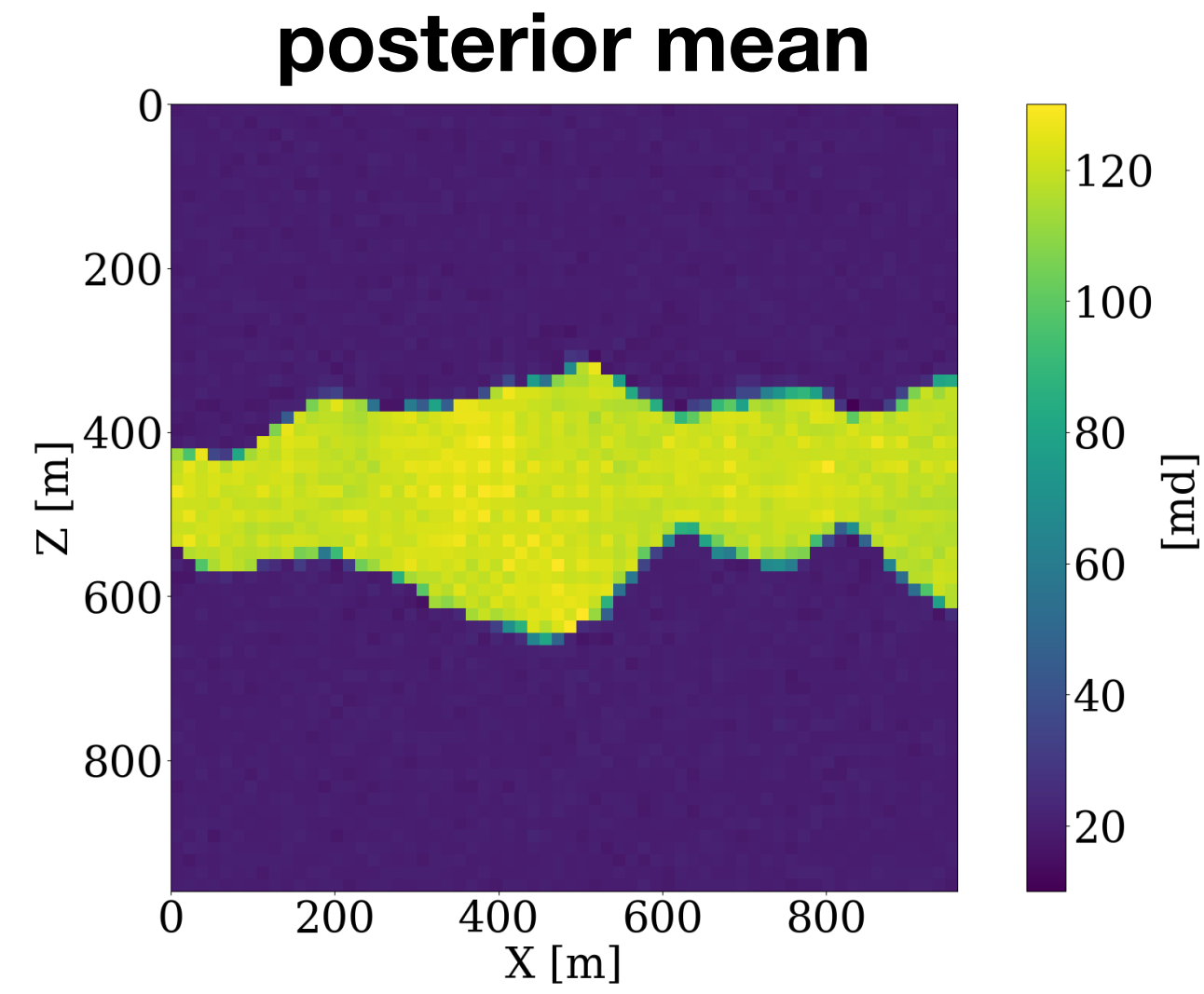
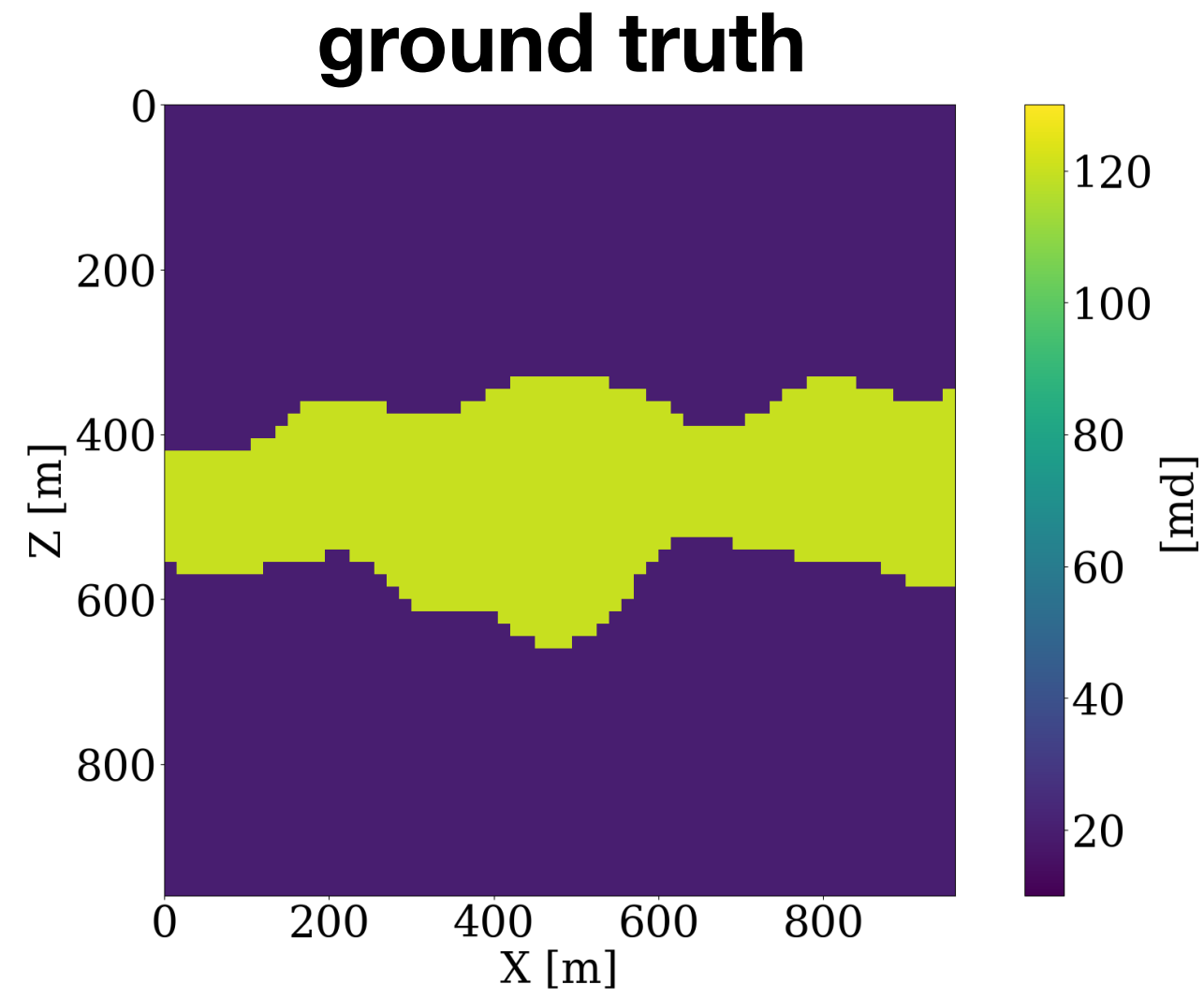
# FNO + Normalizing Flow prior

# Numerical experiment posterior samples



# Numerical experiment

## UQ - permeability



**well recovered channel**

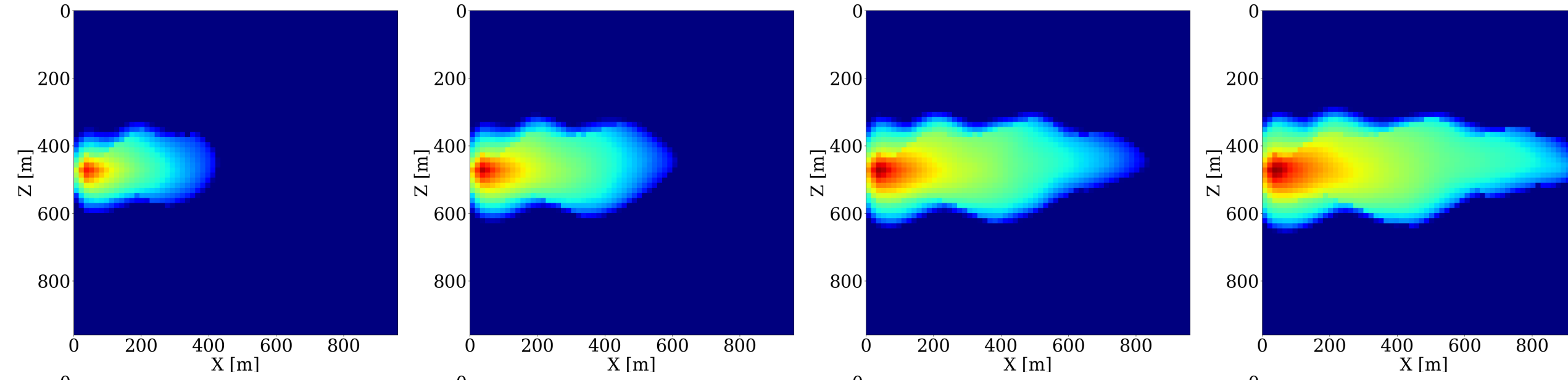
**error & variance aligned**

**higher uncertainty on far end**

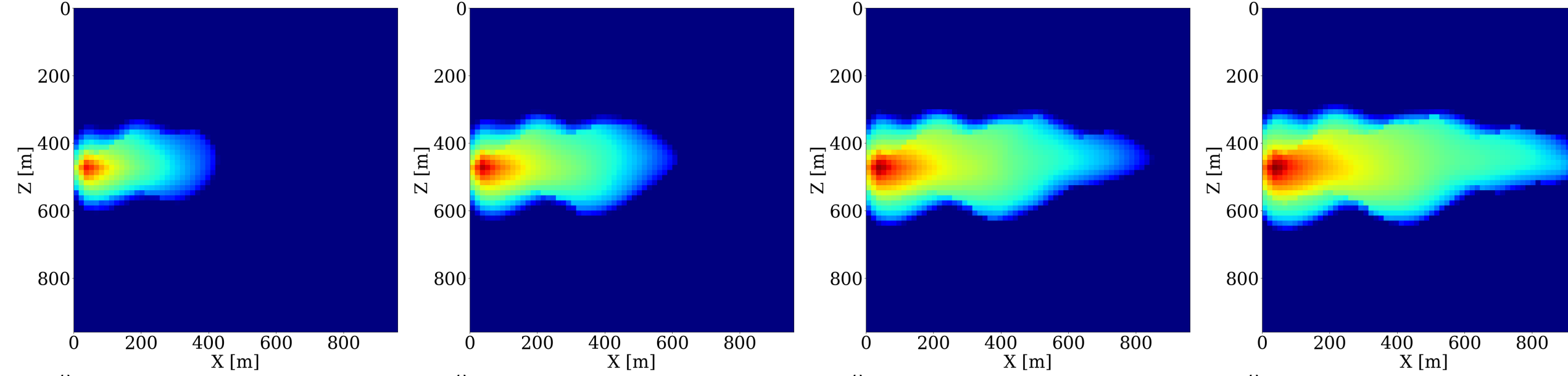
# Numerical experiment

## UQ - plume

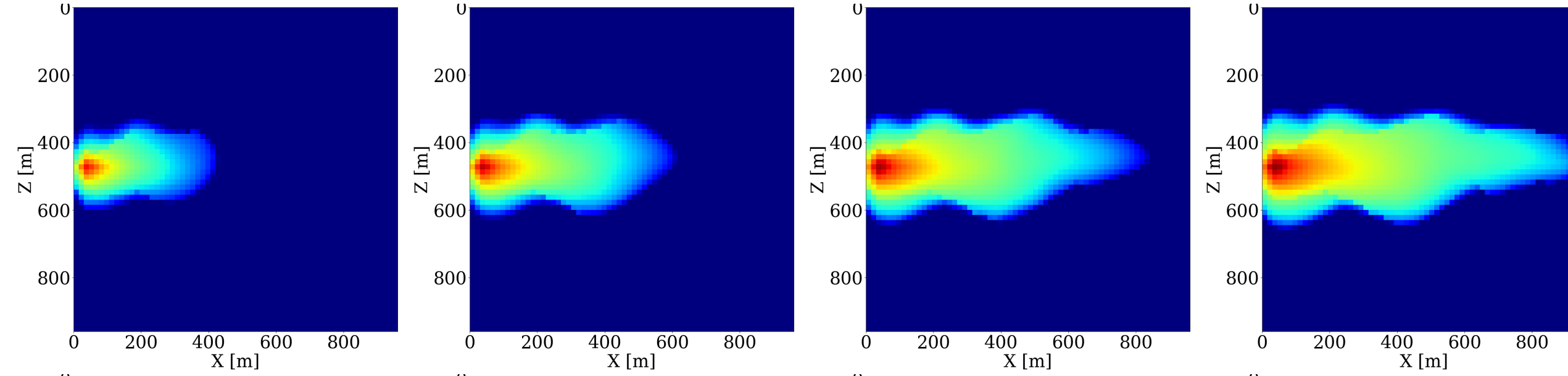
**ground truth**



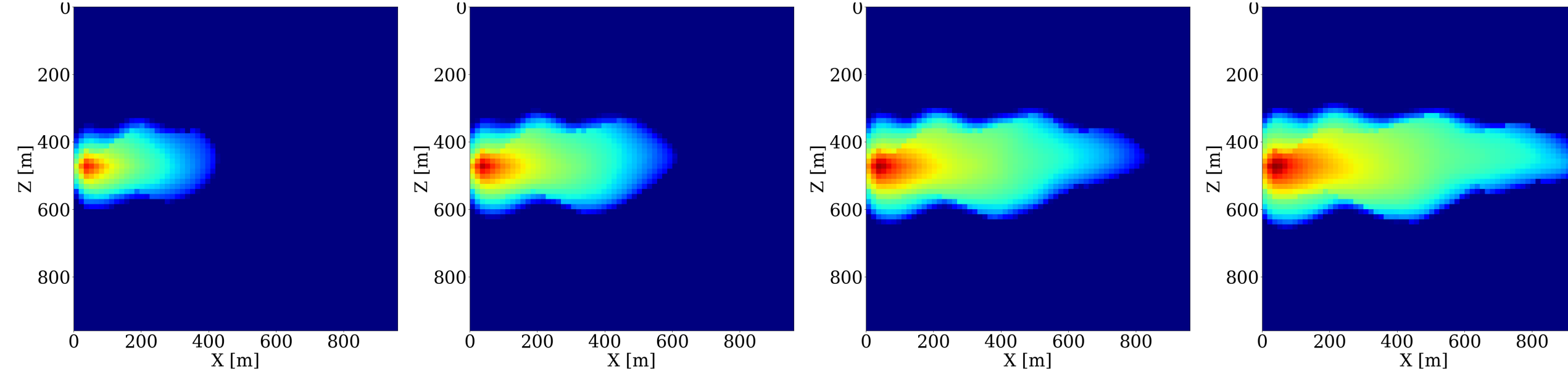
**posterior sample 1**



**posterior sample 2**



**posterior sample 3**

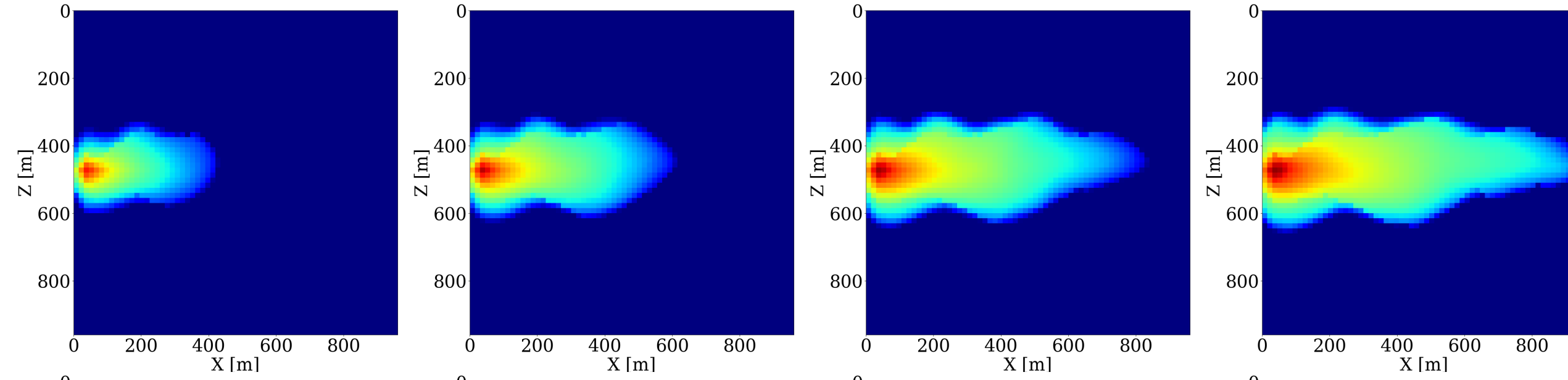




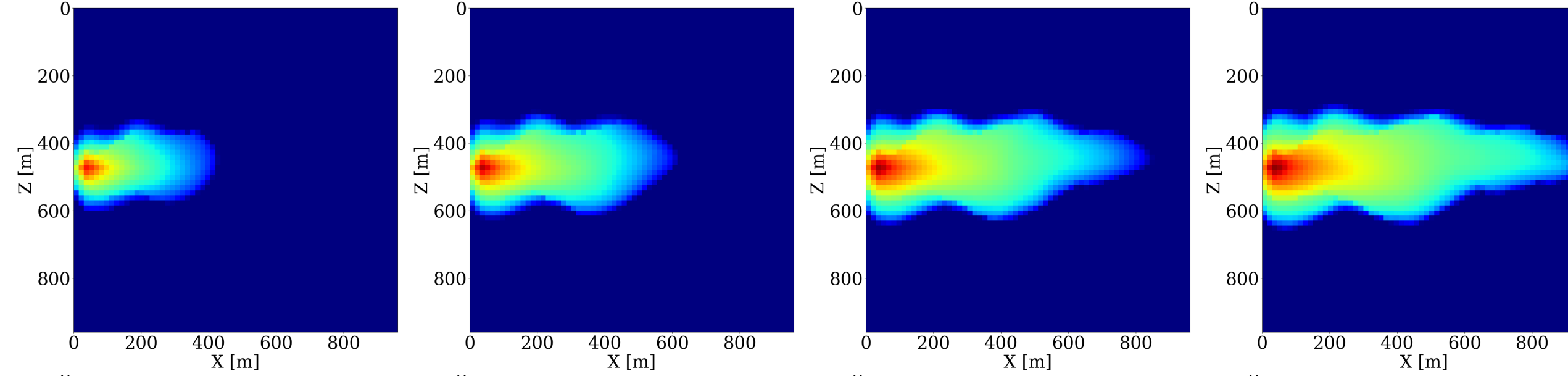
# Numerical experiment

## UQ - plume

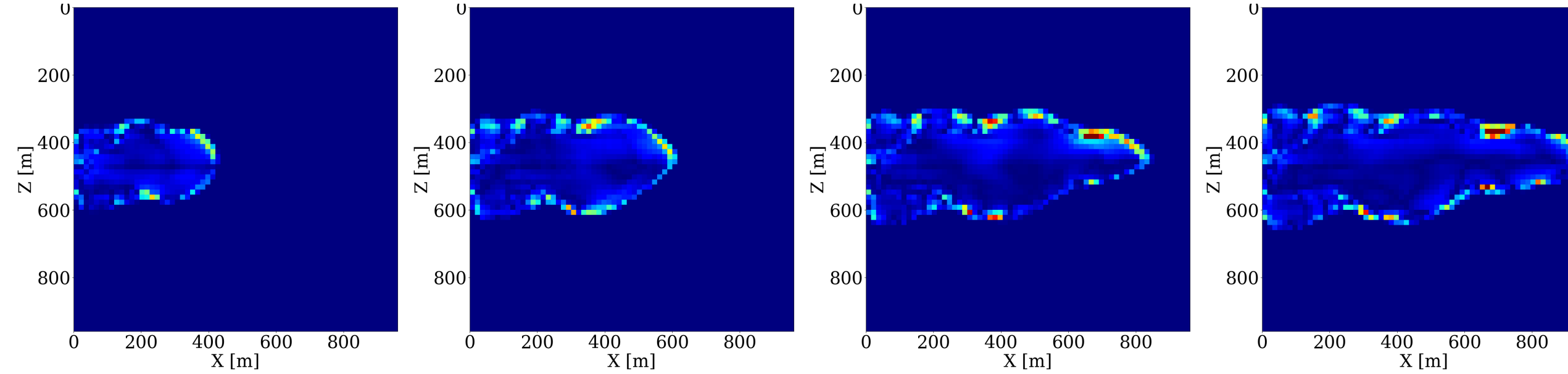
ground truth



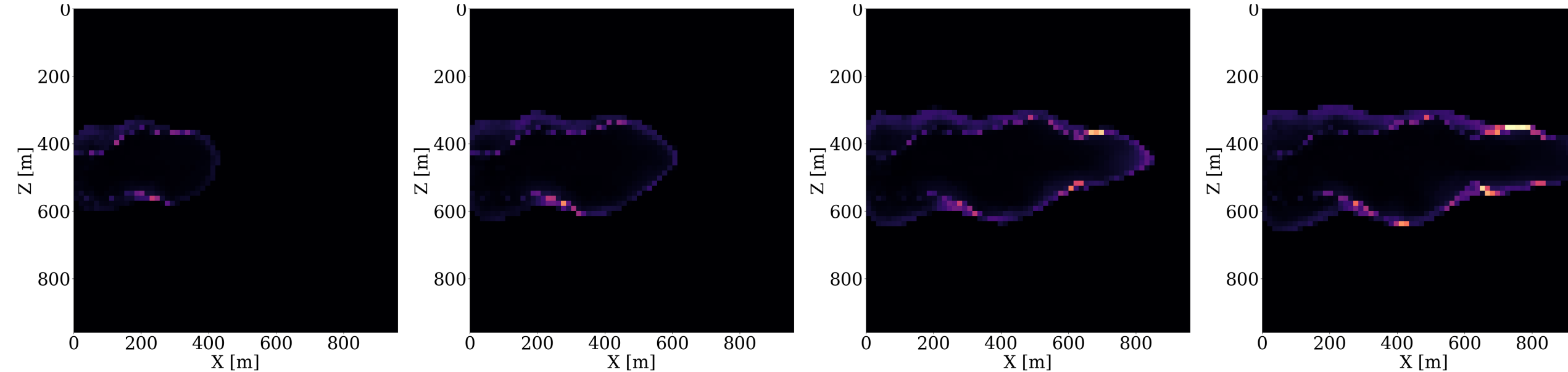
posterior mean



5X error



posterior variance



higher  
uncertainty

- ▶ edge
- ▶ far end of the plume
- ▶ later time steps
- ▶ aligned w/ error

# Conclusions

## End-to-end inversion

- ▶ estimate permeability
- ▶ predict & forecast CO<sub>2</sub> plume

## Fast & reliable UQ

- ▶ Fourier neural operators — cheap likelihood
- ▶ normalizing flows — important prior
- ▶ FNO + normalizing flows — low FNO error enables fast & reliable UQ

# Acknowledgement

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