Uncertainty-aware time-lapse monitoring of geological carbon storage with learned surrogates

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Seismic monitoring of geological carbon storage
Geological carbon storage (GCS)

Sleipner project

Seismic response
Sleipner project

Motivation

time-lapse monitoring

Estimate permeability given time-lapse seismic data

- end-to-end inversion
- Fourier neural operator as cheap surrogate for physics
- cheap & reliable uncertainty quantification
- uncertainty-aware CO$_2$ plume forecast
CO₂ plume prediction
CO₂ prediction (correct permeability)

Permeability $K$

CO₂ concentration $c$

Time

Fluid-flow physics

Injection well

CO₂ flows due to high permeability channels & buoyancy
CO\textsubscript{2} prediction (wrong permeability)

permeability $K$

$\mathcal{S}$ fluid-flow physics

CO\textsubscript{2} concentration $c$

injection well

injection well

wrong CO\textsubscript{2} plume — wrong lateral extent

time
Time-lapse seismic monitoring

wavespeed $v$

seismic data $d$

wave physics
Time-lapse monitoring of carbon storage
Multiphysics modeling

permeability $K$

CO$_2$ concentration $c$

wavespeed $v$

time-lapse data $d$

fluid-flow physics $\mathcal{S}$

rock physics $\mathcal{R}$

wave physics $\mathcal{F}$
End-to-end inversion framework

**End-to-end:** “find permeability that matches seismic data”

\[
\text{minimize } K \| \mathcal{F} \circ \mathcal{R} \circ \mathcal{S}(K) - d \|^2_2
\]

- **Permeability**: $K$
- **CO$_2$ concentration**: $c$
- **Wavespeed**: $v$
- **Time-lapse data**: $d$

Permeability inversion

initial

inverted

ground truth
Fourier neural operator surrogate
cheap alternative to numerical simulation

orders of magnitude faster

support surrogate inversion w/ automatic differentiation (AD)

scalable to large-scale 4D via domain decomposition

benefits:

- reduce computational cost
- enable uncertainty quantification

Surrogate Modeling

Fourier neural operators – FNOs

FNO learns mappings on low-frequency modes in Fourier space via

Learned end-to-end inversion

\[ \mathcal{F} \circ \mathcal{R} \circ \mathcal{S}_\theta \] coupled physics

\[ \min_{\mathbf{K}} \| \mathcal{F} \circ \mathcal{R} \circ \mathcal{S}_\theta(\mathbf{K}) - \mathbf{d} \|_2^2 \]

\[ \mathcal{S}_\theta \text{ pre-trained FNO: drastically reduce computational cost} \]
Fourier neural operators learns physics

\( \mathcal{S}(K) \)
Fourier neural operators learns physics

$\mathcal{S}(K)$

$\mathcal{S}_\theta(K)$
Fourier neural operators learns physics
Permeability inversion
FNO surrogate

**initial**

**inverted**

**ground truth**
Permeability inversion
physics-based inversion

initial

inverted

ground truth
End-to-end inversion
CO$_2$ plume

<table>
<thead>
<tr>
<th>monitor</th>
<th>now</th>
<th>forecast</th>
</tr>
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<tbody>
<tr>
<td>year 10</td>
<td>year 15</td>
<td>year 16</td>
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<td>year 17</td>
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<td>year 18</td>
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</tbody>
</table>

ground truth

initial
End-to-end inversion

CO₂ plume

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- **ground truth**
- **initial**
- **physics**
End-to-end inversion
CO$_2$ plume

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<tbody>
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<tr>
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<td><img src="image5.png" alt="Image" /></td>
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</table>

- **ground truth**
- **initial**
- **physics**
- **surrogate**
Uncertainty quantification
Bayesian Uncertainty Quantification

Bayesian posterior \( p_{\text{post}}(K \mid d) \propto p_{\text{like}}(d \mid K)p_{\text{prior}}(K) \)

\( K \) unknown model parameters (permeability)
\( d \) observed data
\( p(d \mid K) \) data likelihood
\( p(K) \) prior
Bayesian Uncertainty Quantification
stochastic gradient Langevin dynamics (SGLD)

\[ K_{k+1} = K_k - \frac{\alpha_k}{2} \nabla_K \log p_{\text{post}}(K | d) + \eta_k \]

- gradient-based MCMC
- \(\alpha_k\) step size, \(\eta_k\) noise

Fast Uncertainty Quantification
learned surrogate & prior

\[ p_{\text{post}}(K \mid d) \propto p_{\text{like}}(d \mid K)p_{\text{prior}}(K) \]
Fast Uncertainty Quantification
learned surrogate & prior

\[ p_{post}(K \mid d) \propto p_{like}(d \mid K)p_{prior}(K) \]

Fourier neural operators
Fast Uncertainty Quantification
learned surrogate & prior

\[ p_{\text{post}}(K \mid d) \propto p_{\text{like}}(d \mid K)p_{\text{prior}}(K) \]

Fourier neural operators

normalizing flows
Normalizing flows (NFs)

Training:

\[ x \sim p_X(x) \quad \xrightarrow{G_w^{-1}(x)} \quad z \sim p_Z(z) \]

Sampling:

\[ z \sim p_Z(z) \quad \xrightarrow{G_w(z)} \quad x \sim p_X(x) \]

Training samples
permeability models
Generative samples
“fake” permeability models
Fourier neural operators
likelihood

2000 random permeability channels
FNO & NF share the same training samples
FNO gives low prediction error on generative samples from NF

- **real samples**
- **generative samples**
Bayesian Uncertainty Quantification

stochastic gradient Langevin dynamics (SGLD)

\[ K_{k+1} = K_k - \frac{\alpha_k}{2} \nabla K \log p_{\text{post}}(K | d) + \eta_k \]

- gradient-based MCMC
- \( \alpha_k \) step size, \( \eta_k \) noise

Bayesian Uncertainty Quantification
stochastic gradient Langevin dynamics (SGLD)

\[ z_{k+1} = z_k - \frac{\alpha_k}{2} \nabla_{z_k} \log p_{\text{post}}(z | d) + \eta_k \]

- gradient-based MCMC
- \( \alpha_k \) step size, \( \eta_k \) noise
- \( \mathcal{G}(z) = K \), \( \mathcal{G} \) trained NF, \( z \) latent variable
FNO + Normalizing Flow prior
Numerical experiment
posterior samples
Numerical experiment
UQ - permeability

well recovered channel
error & variance aligned
higher uncertainty on far end
Numerical experiment
UQ - plume

ground truth

posterior sample 1

posterior sample 2

posterior sample 3
Numerical experiment
UQ - plume

ground truth

posterior mean

5X error

posterior variance

higher uncertainty
► edge
► far end of the plume
► later time steps
► aligned w/ error
Conclusions

End-to-end inversion

- estimate permeability
- predict & forecast CO₂ plume

Fast & reliable UQ

- Fourier neural operators — cheap likelihood
- normalizing flows — important prior
- FNO + normalizing flows — low FNO error enables fast & reliable UQ
Acknowledgement

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