

# Capturing velocity-model uncertainty and two-phase flow with Fourier Neural Operators

Ali Siahkoohi, Thomas Grady, Abhinav P Gahlot, Hüseyin Tuna Erdinc, and Felix J. Herrmann

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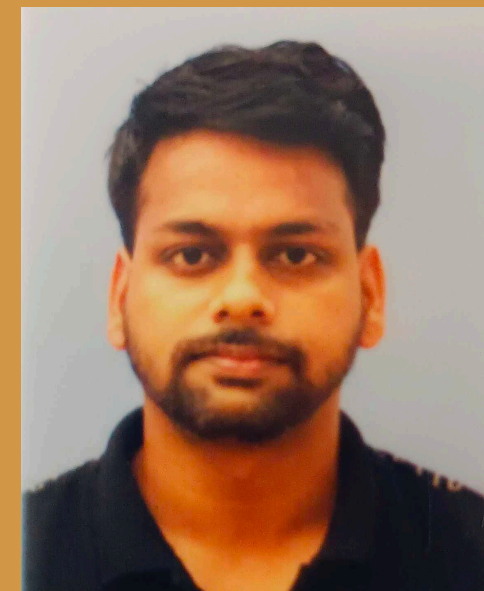
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# Motivation

## increasing complexity

Geophysical challenges such as *geological carbon storage* call for

- high-*resolution* & highly *sensitive* imaging of *weak* time-lapse signals
- *complex* multi-phase *flow* simulations
- *coupling* of wave & fluid-flow physics
- approaches that are *uncertainty* aware

**Results in a need for *surrogate* models to make simulations computationally *feasible***

Today's focus is on recent developments enabled by neural operators



# What are neural operators?

# Neural operators

## Conventional Neural Networks:

- learn *discretized* image-to-image mappings
- generalize poorly to different discretization & sources (e.g. well locations)

## Neural Operators:

- learn mappings between function spaces (e.g. PDE solution operators)
- finite-dimensional (fixed grid)  $\implies$  infinite-dimensional (gridless)

# Learning neural operators

Given  $\{a^{(i)}(\mathbf{x}), u^{(i)}(\mathbf{x})\}_{i=1}^N$  from nonlinear map between function spaces  $\mathcal{A}, \mathcal{U}$ :

$$\mathcal{G}: \mathcal{A} \mapsto \mathcal{U}$$

taking values on  $\mathbb{R}^{d_a}$  and  $\mathbb{R}^{d_u}$ , operator learning entails minimizing

$$\mathbf{w}^* = \operatorname{argmin}_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^N \| \mathcal{G}_{\mathbf{w}}(a^{(i)}) - u^{(i)} \|_2^2$$

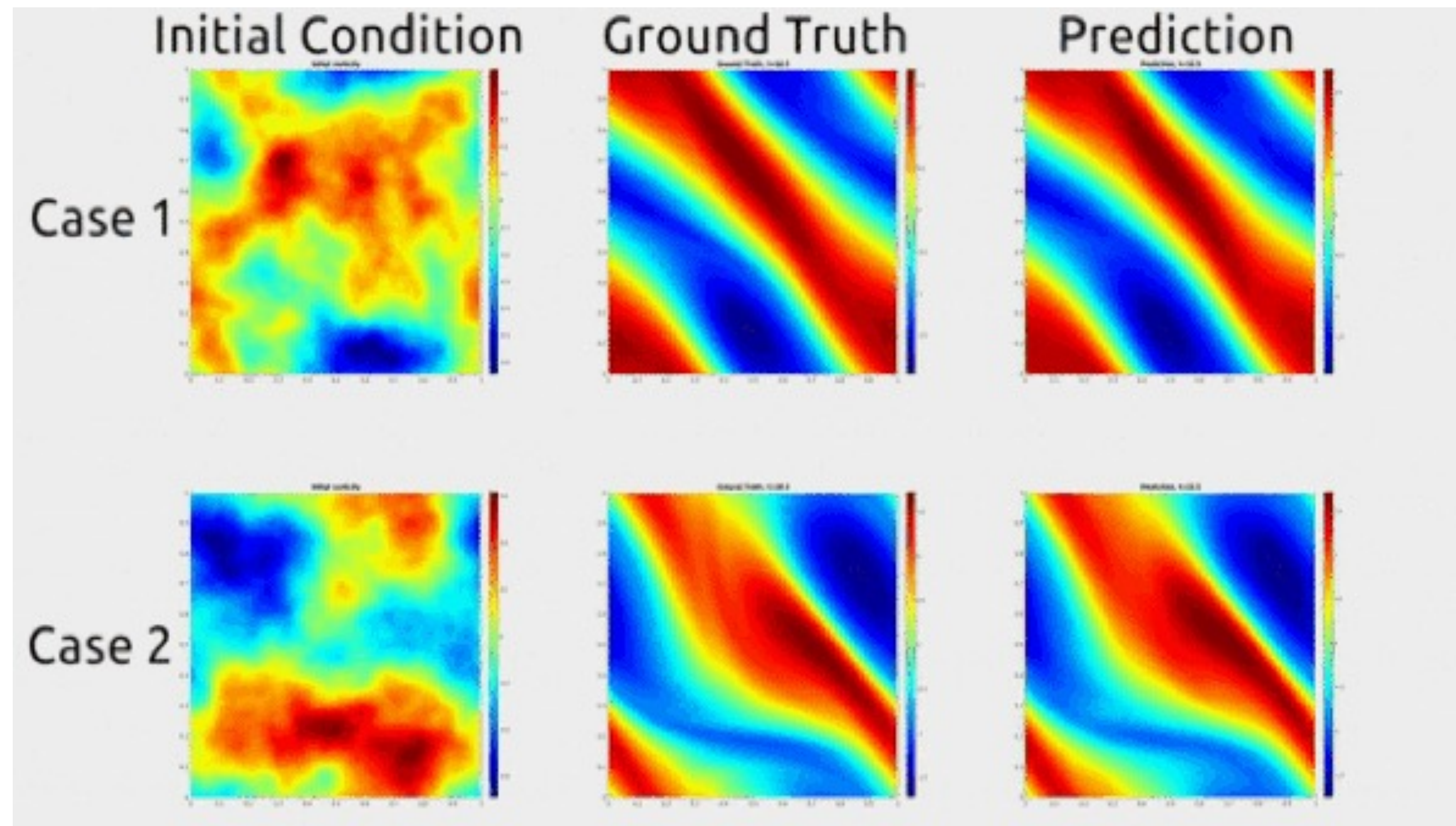
yielding the approximate mapping

$$\mathcal{G}_{\mathbf{w}^*}: \mathcal{A} \mapsto \mathcal{U}$$



# Applications

## Fourier Neural Operators





# So what?

# Surrogate model

## two-phase flow equations

mass balance equation  $\frac{\partial}{\partial t}(\phi S_i \rho_i) + \nabla \cdot (\rho_i \mathbf{v}_i) = \rho_i q_i, i = 1, 2$

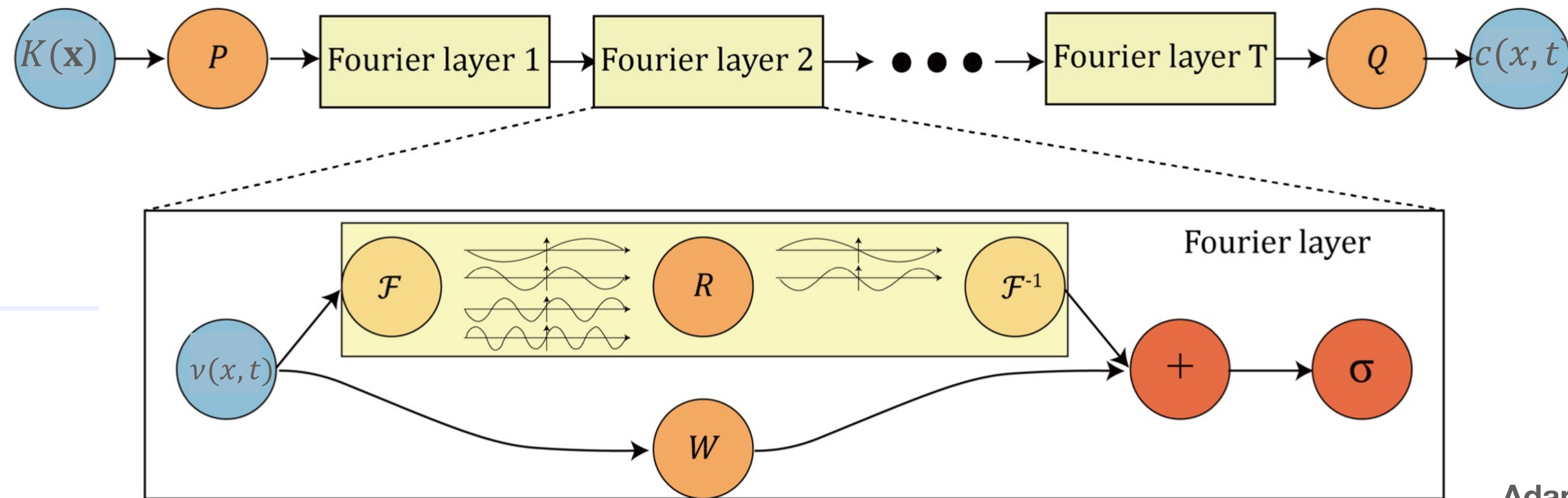
inject CO<sub>2</sub> to replace water  $S_1 + S_2 = 1$

Darcy's law  $\mathbf{v}_i = -\frac{K k_{ri}}{\mu_i} (\nabla P_i - g \rho_i \nabla Z), i = 1, 2$

Corey model  $k_{ri}(S_i) = S_i^2$

fluid pressure  $P_2 = P_1 - P_c(S_2)$

# Fourier Neural Operators



Adapted from Li

Maps permeability  $K$  in  $(\mathbf{x}, K(\mathbf{x}))$  and outputs  $c(\mathbf{x}, t)$

$P$  lifts to higher latent dimension and  $Q$  projects back to target dimension

Fourier layer:  $v_{j+1} = \sigma \left( W v_j + \mathcal{F}^{-1} \left( R_{\phi} \cdot (\mathcal{F} v_j) \right) \right)$

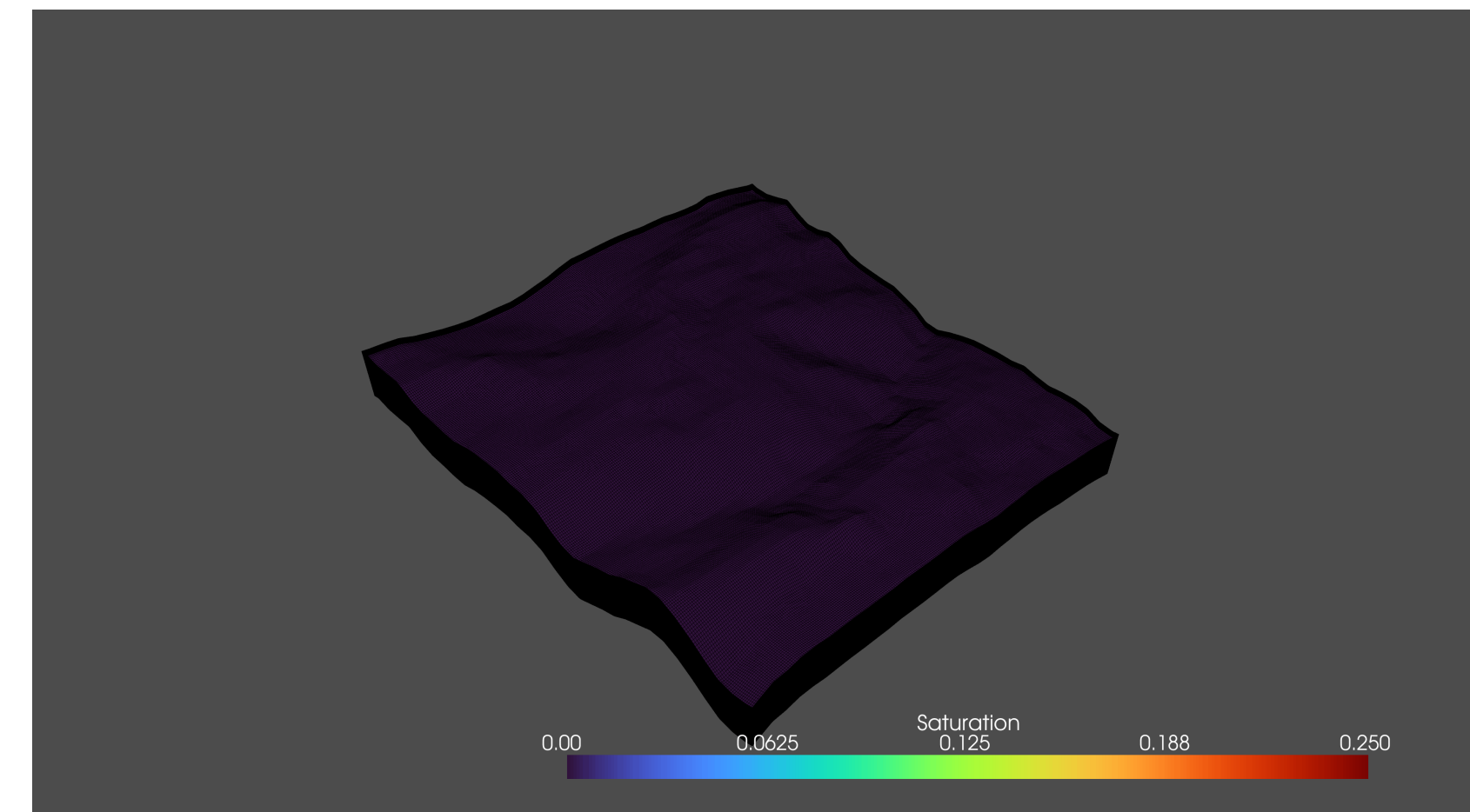
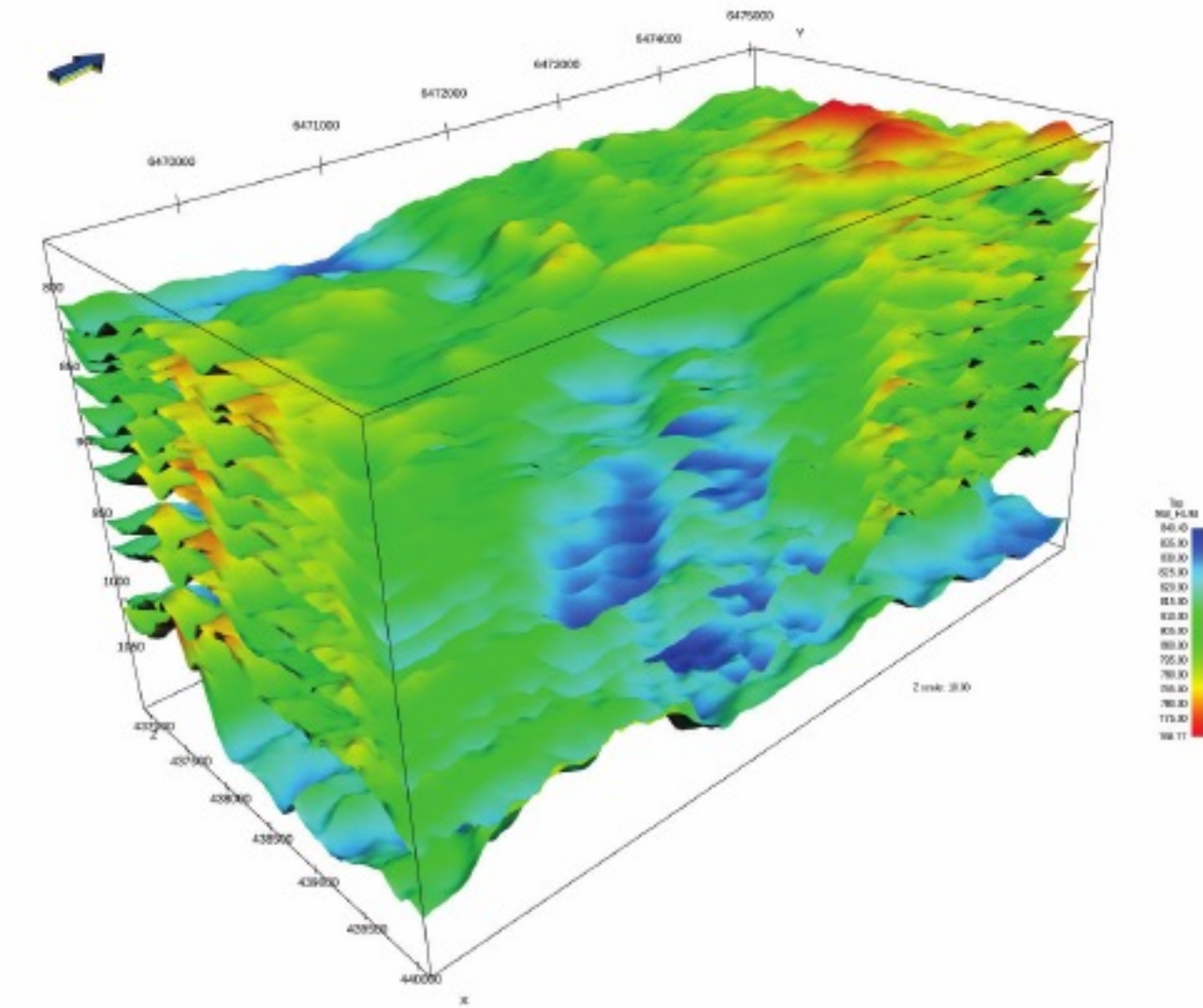
$R_{\phi}$  selects a subset of the modes, usually a low-pass filter



# Challenges

## Scaling FNOs to realistic problems is challenging

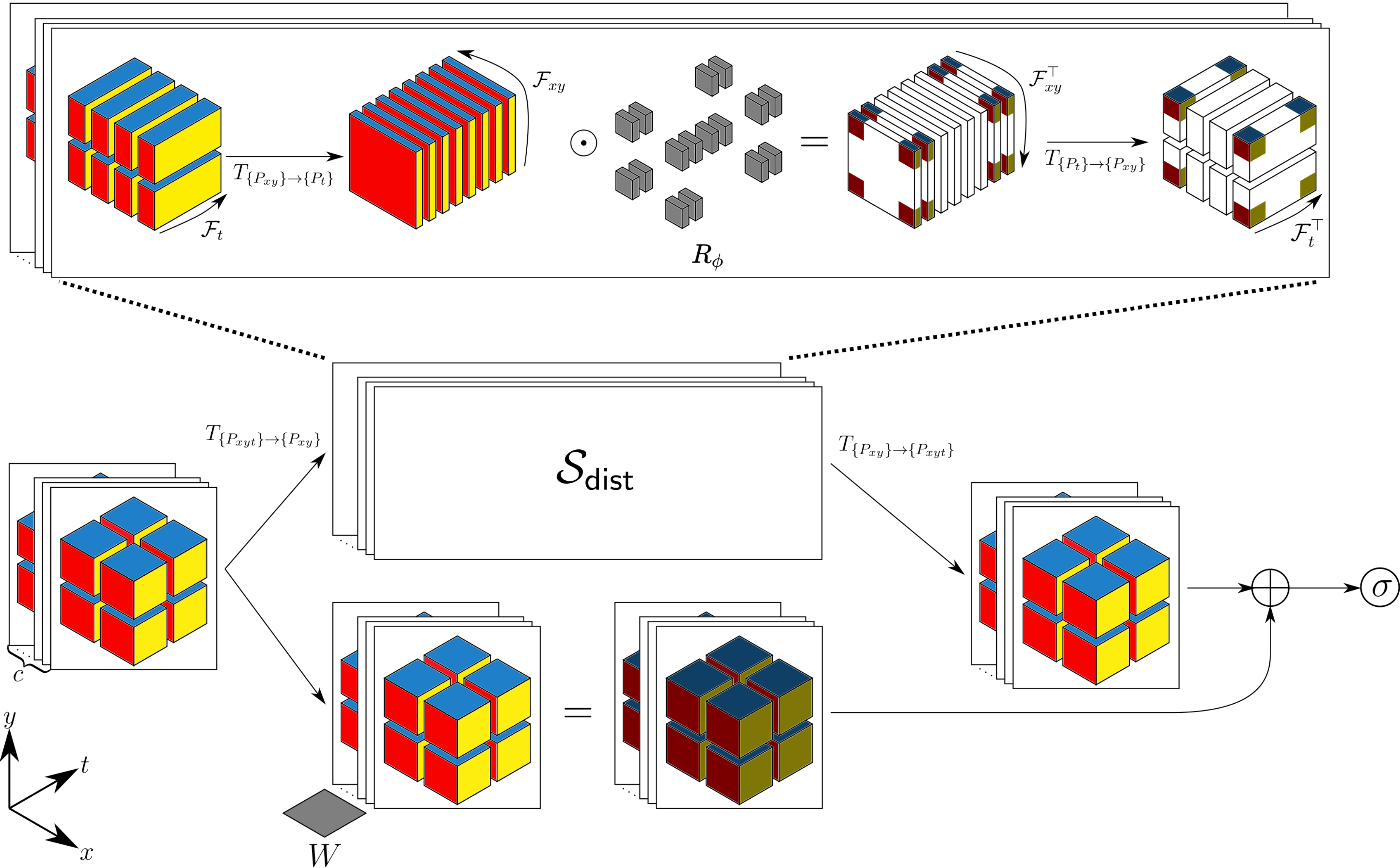
- 3D permeability  $(x, y, z) \rightarrow$  4D CO<sub>2</sub> evolution  $(x, y, z, t)$
- problems beyond  $64^3(x, y, z)$  do not fit w/i GPUs
- real problems are often much larger, e.g. Sleipner (low-resolution) is  $64 \times 118 \times 263$
- need high-dimensional model-parallelism on distributed-memory systems





# FNO Block w/ domain decomposition

Implemented w/ DistDL



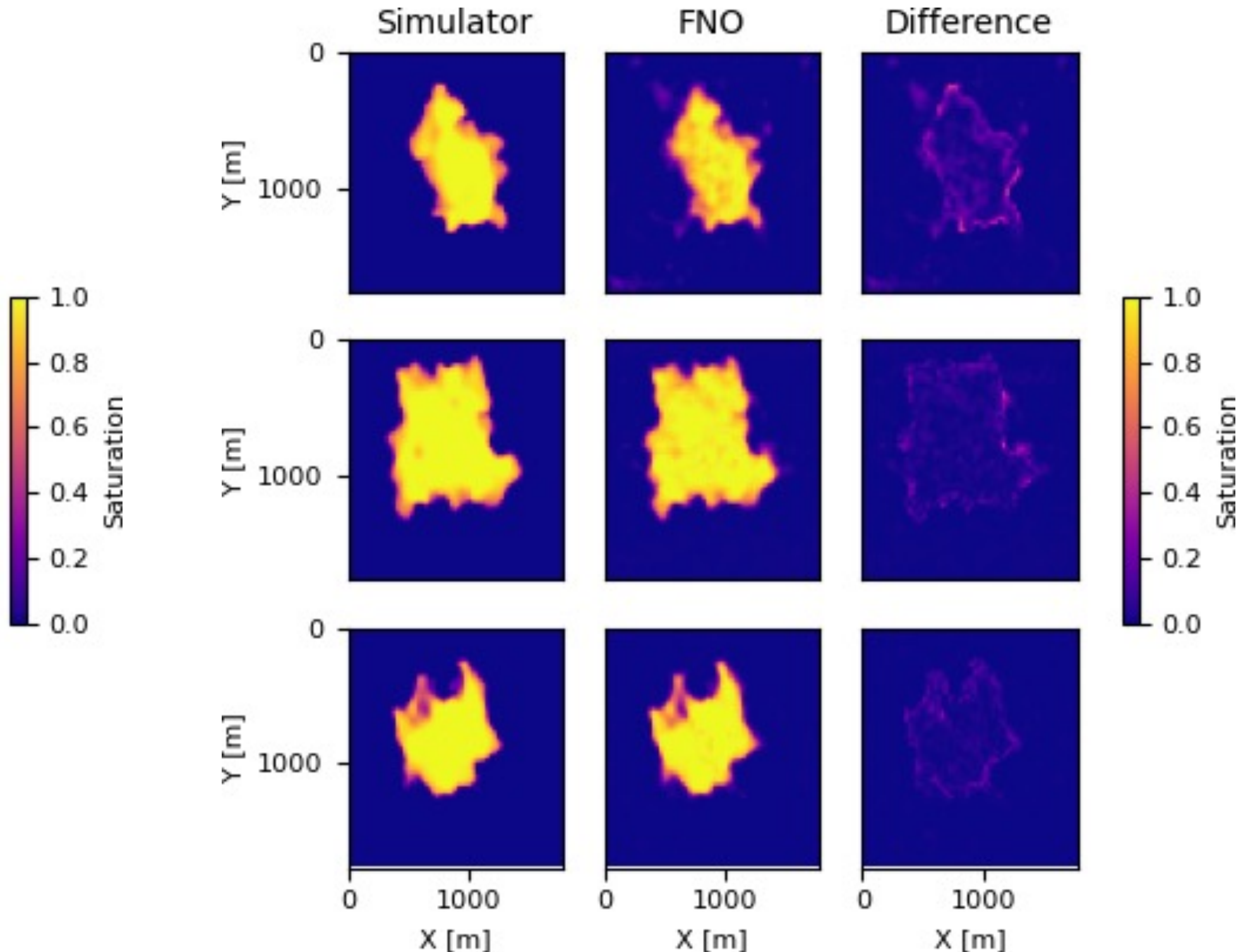
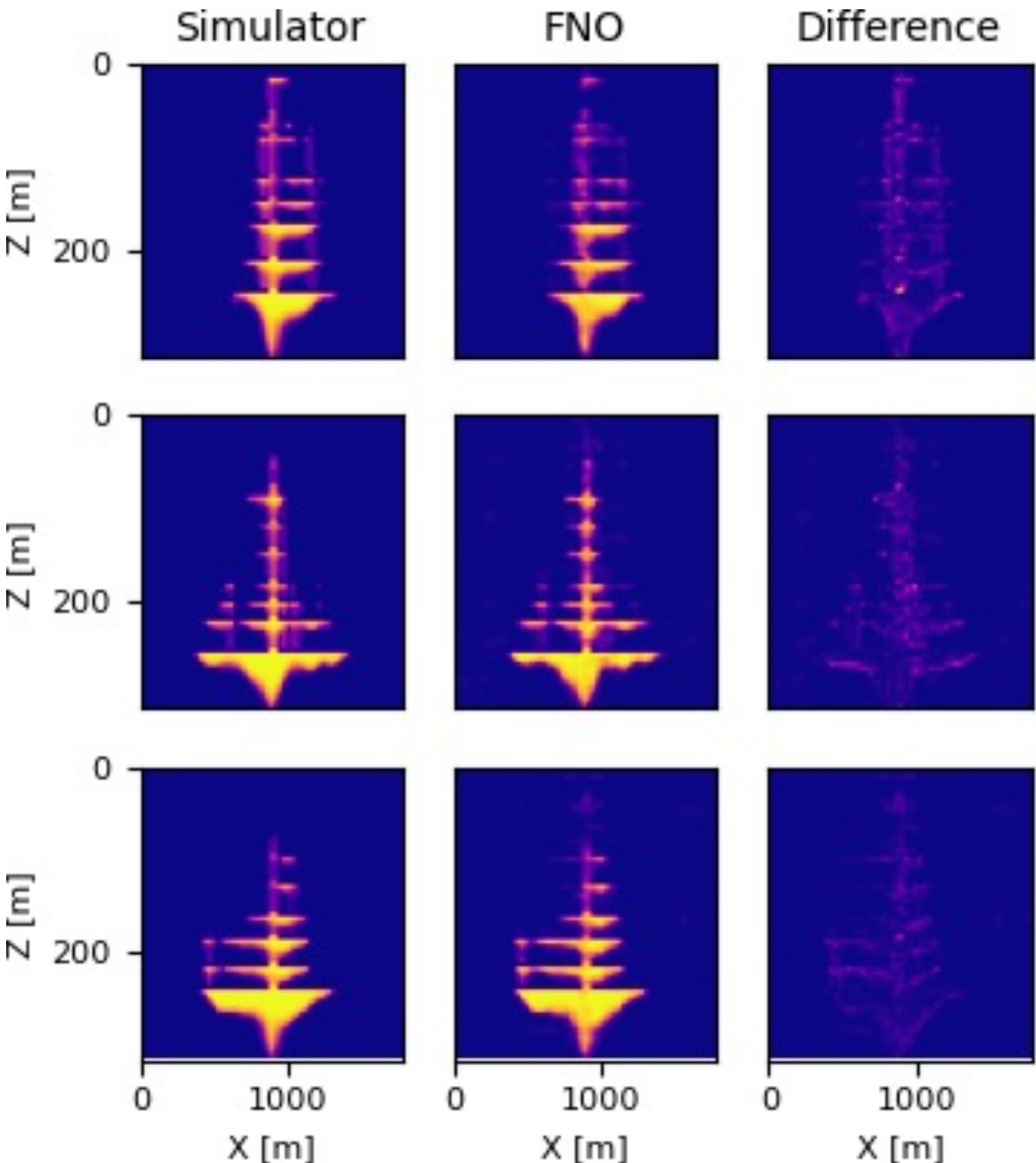
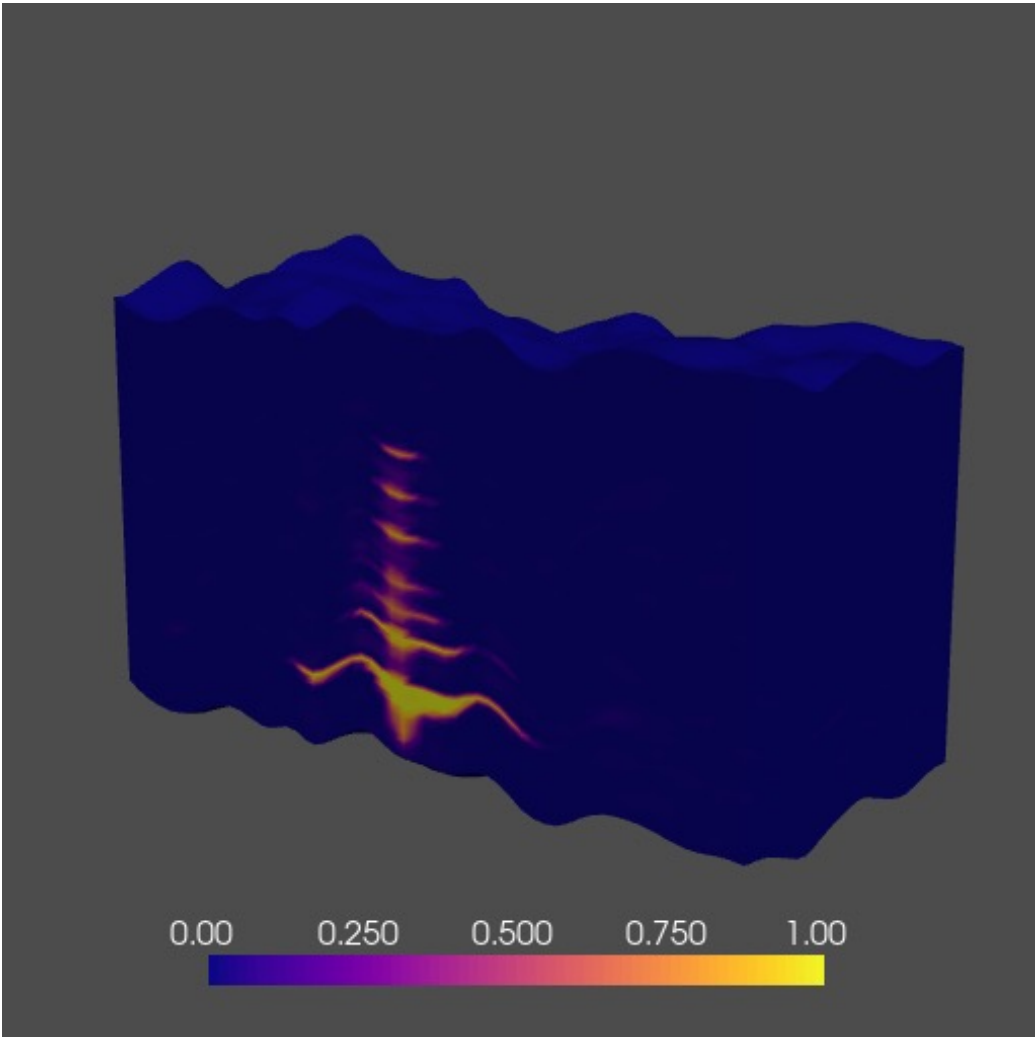
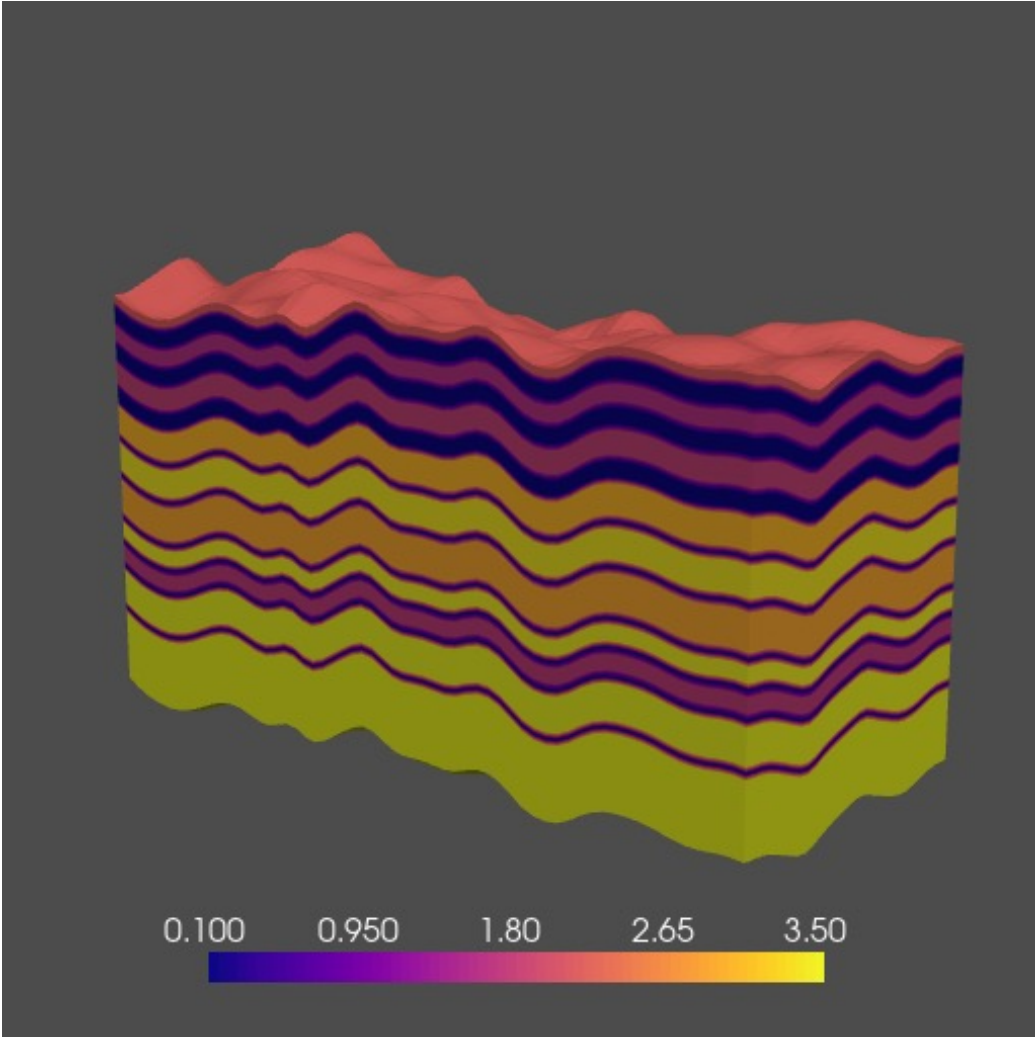


# Results

## 3D two-phase flow for CCS

1386X speedup on Sleipner-size model

Problem Size	OPM Time (s)	FNO Time (s)	Speedup
$60 \times 60 \times 64 \times 30$	312	1.15	271x
$68 \times 118 \times 263 \times 16$	8291	5.98	1386x





# Leakage detection

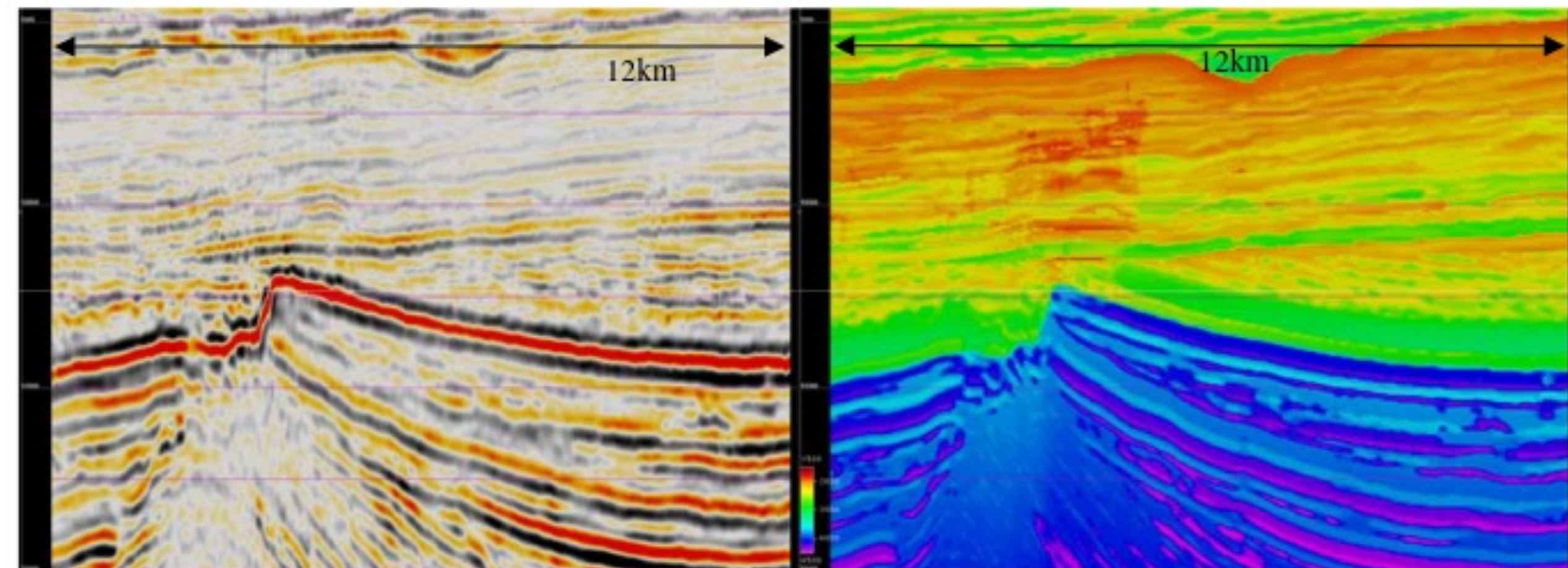
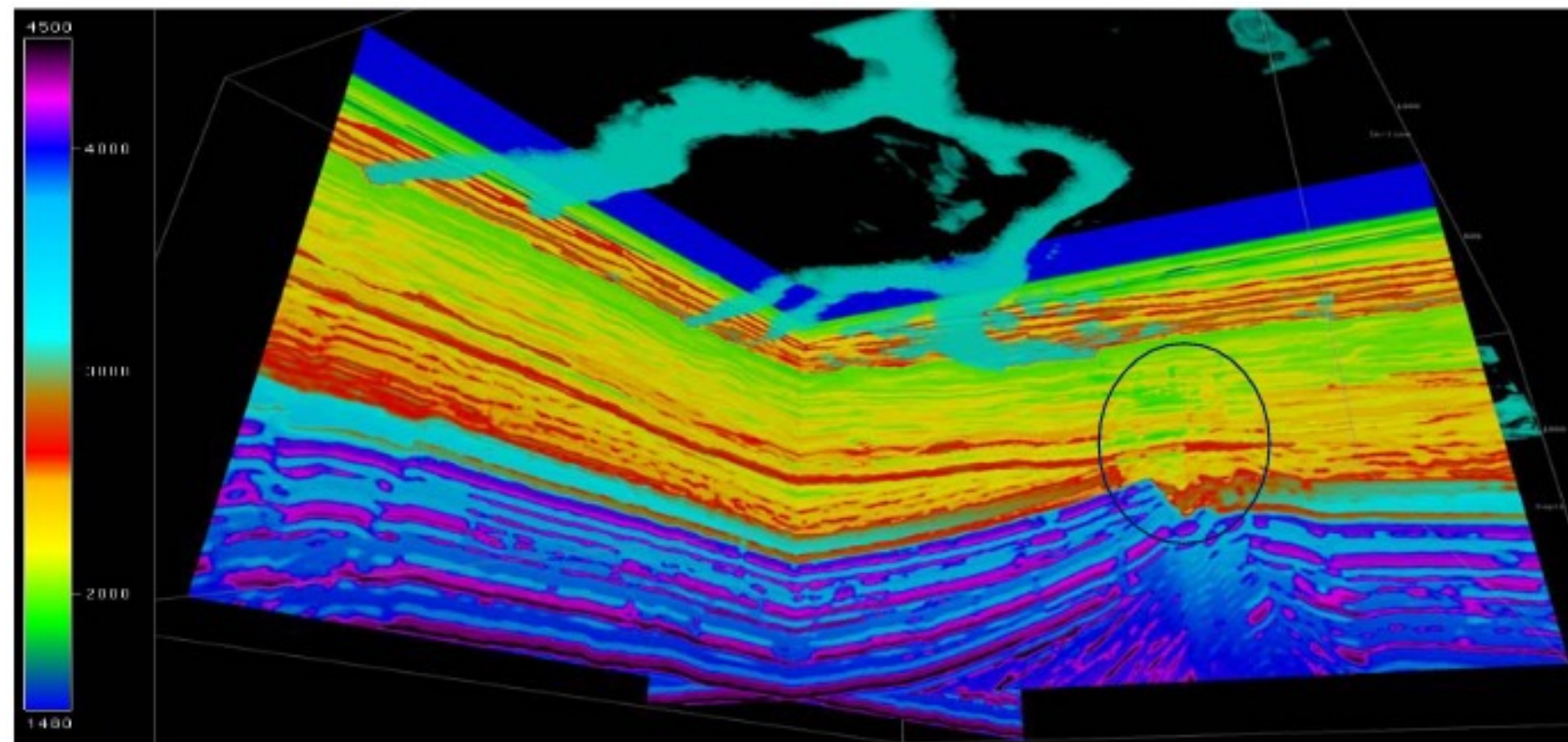


# Proxy model

derived from imaged 3D seismic & well data

convert velocity model into

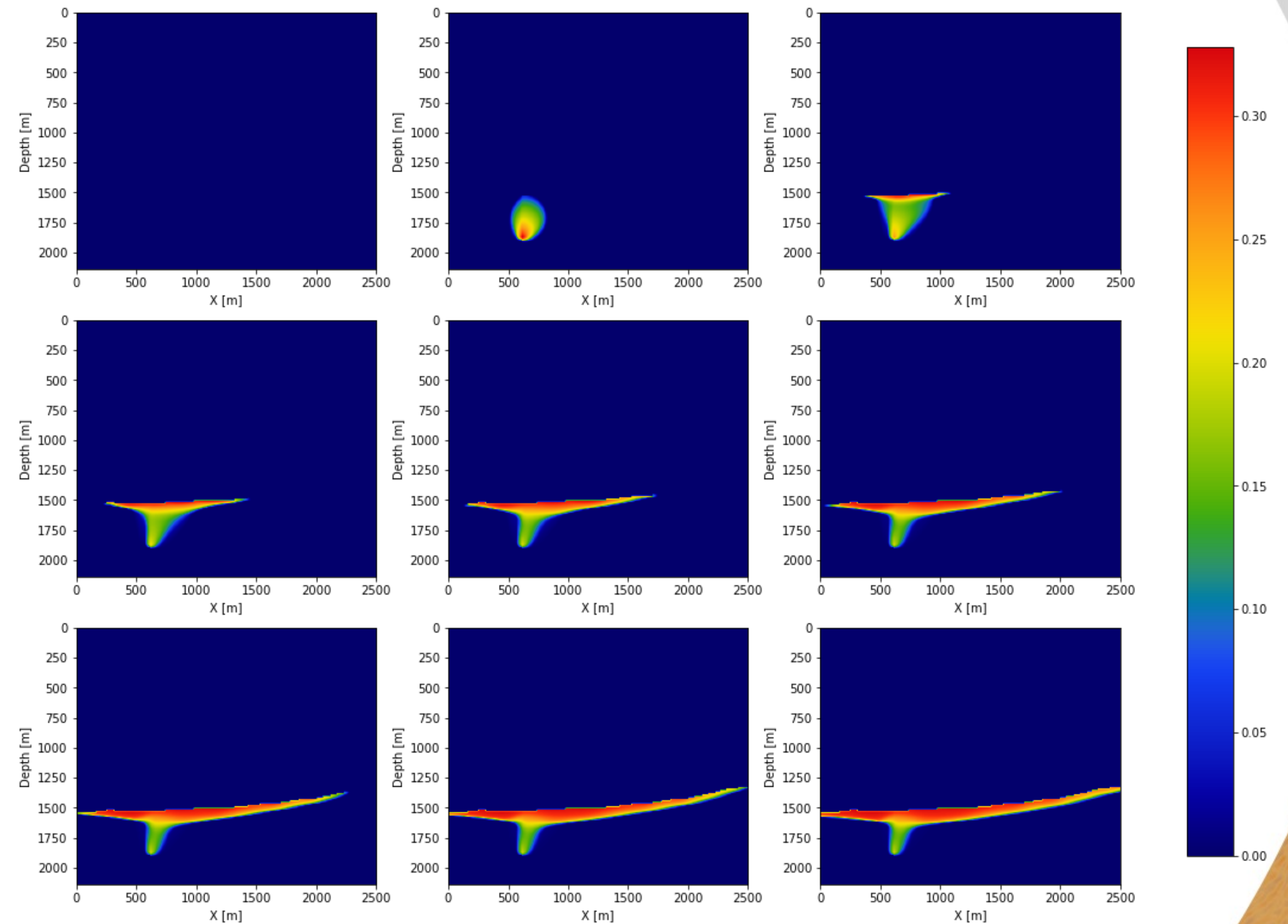
- permeability
- porosity





# Simulations

## regular CO2 plume

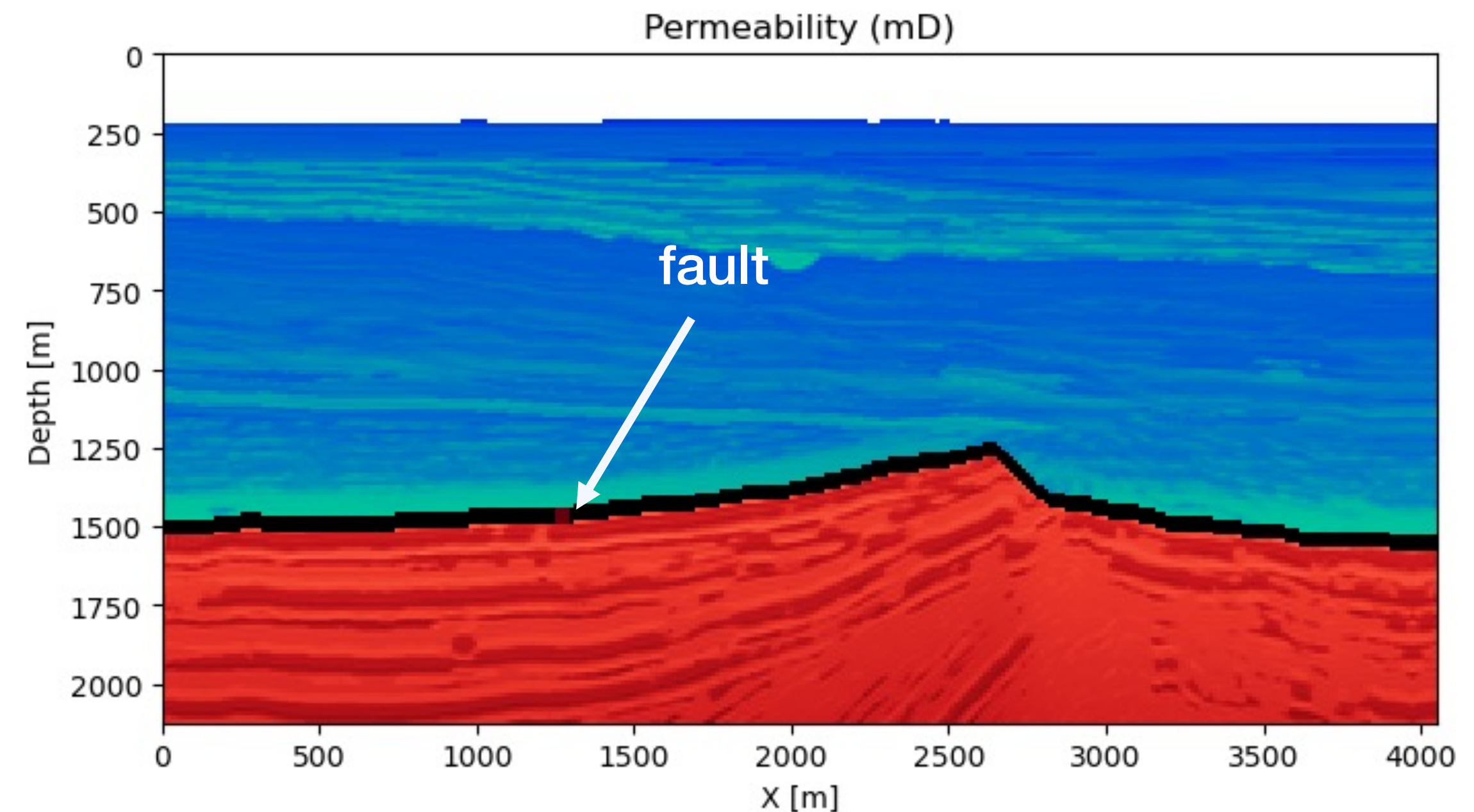
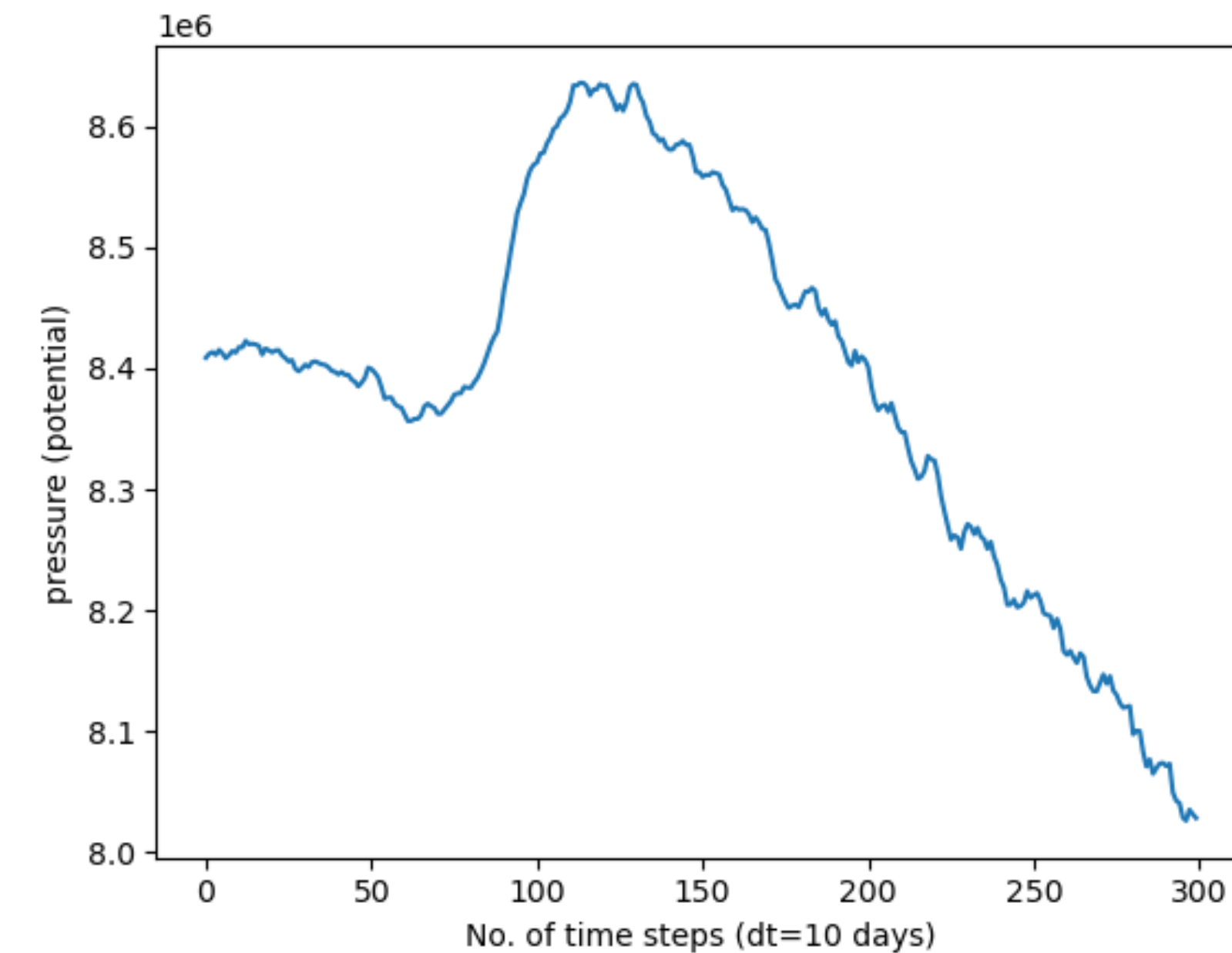




# Leakage scenario

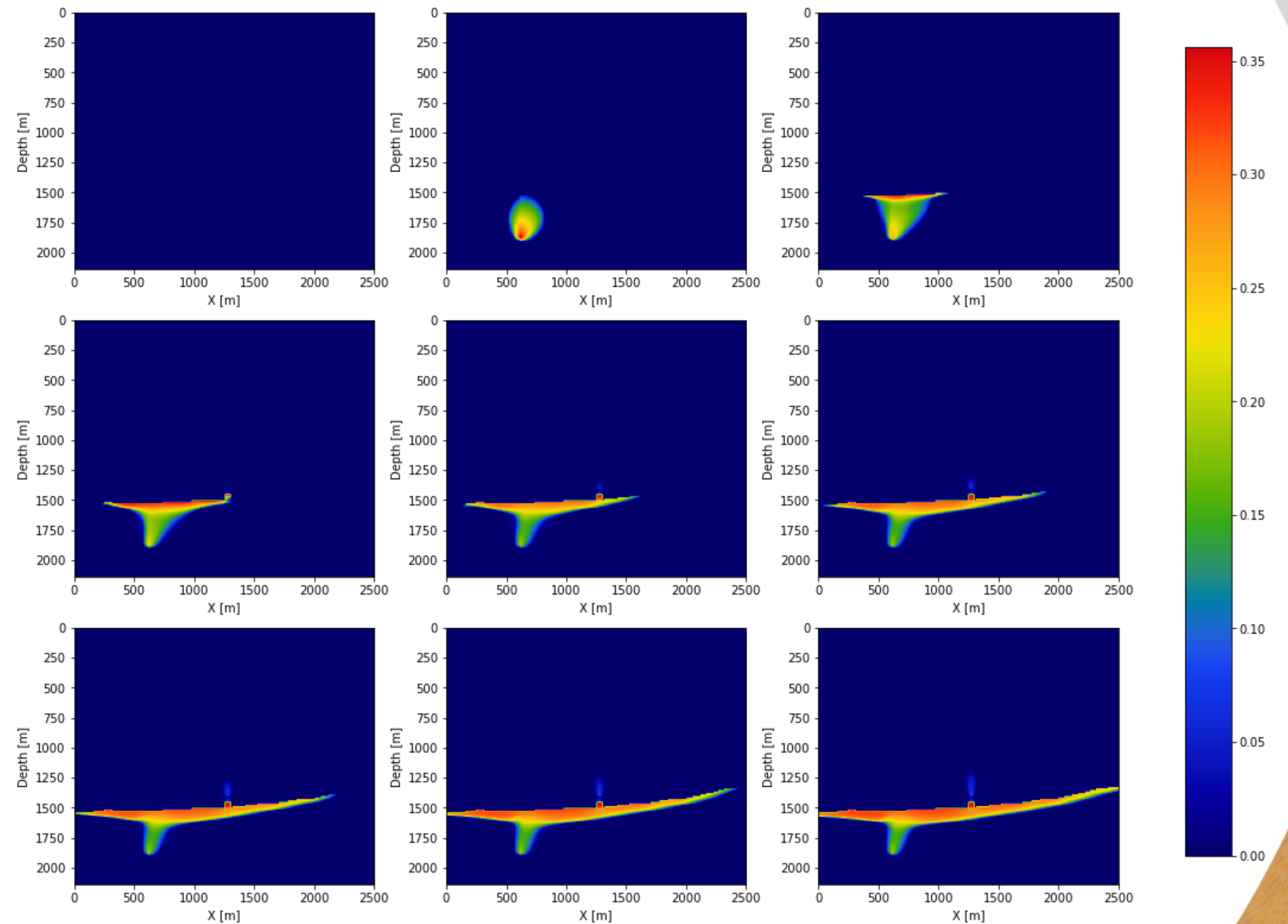
## pressure-induced fault activation

- two-phase-flow simulations
- pressure  $\geq 15\text{MPa}$  induces a fault in the seal
- increase in permeability leads to leakage through seal
- leak location in seal selected at random w/ random widths



# Simulations

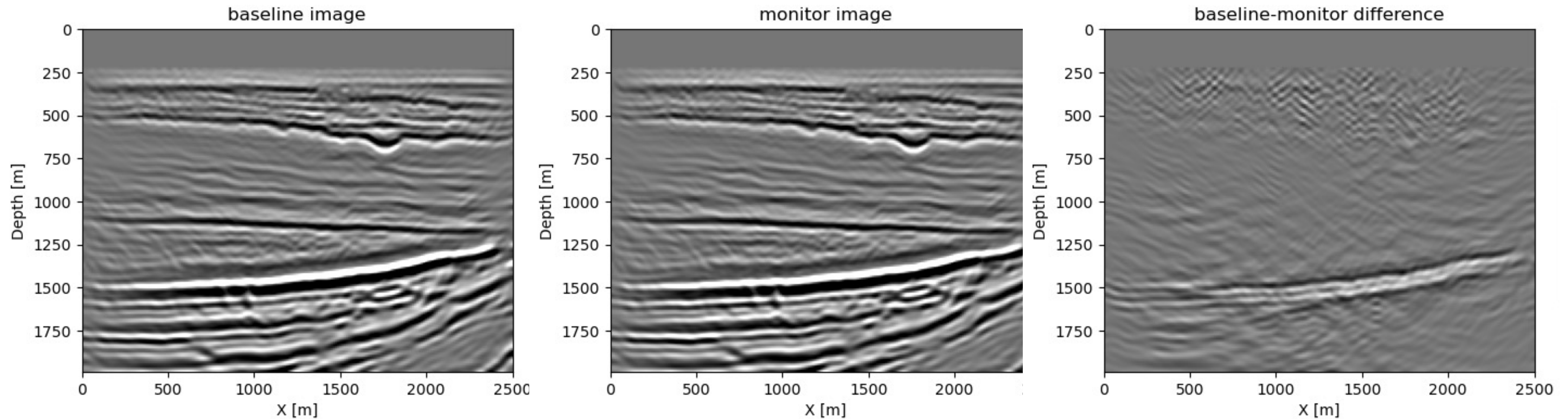
## irregular CO2 plume





# Time-lapse images

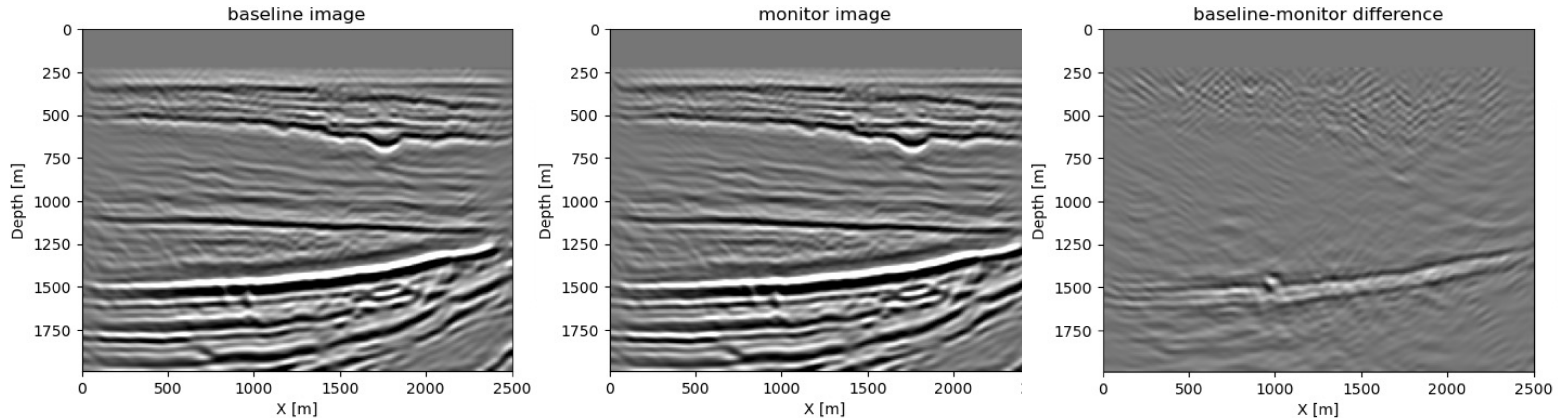
regular flow – no replication acquisition





# Time-lapse images

irregular flow – no replication acquisition





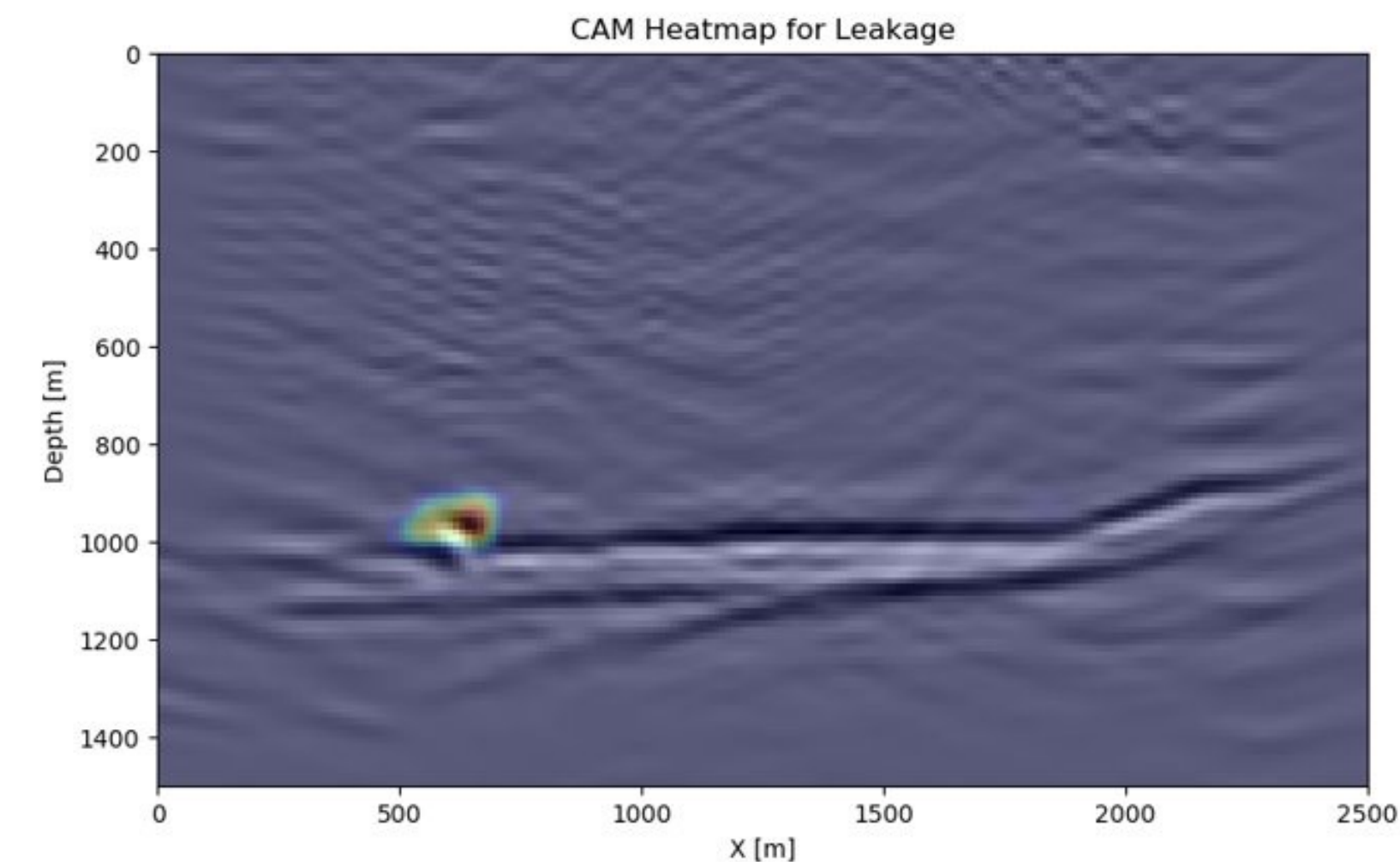
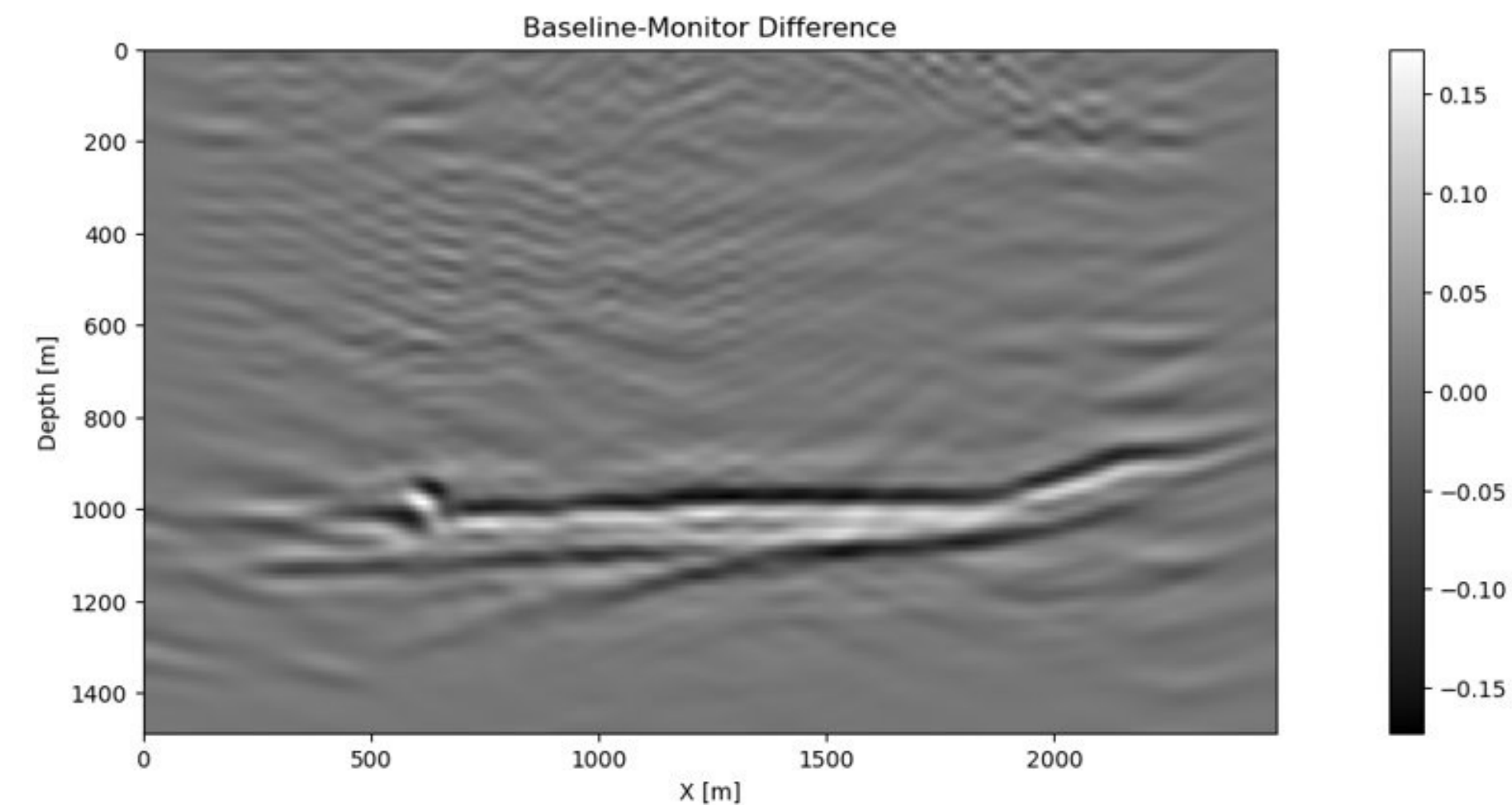
# Leak / no leak classification

TABLE I  
Performance Metric Comparison

Metrics	Mean Deviation
Accuracy	$0.797 \pm 0.066$
Precision	$0.711 \pm 0.105$
Recall	$0.954 \pm 0.023$
F1 Score	$0.818 \pm 0.068$

Precision: measures how many leakages model predicted correctly, out of all predicted leakages

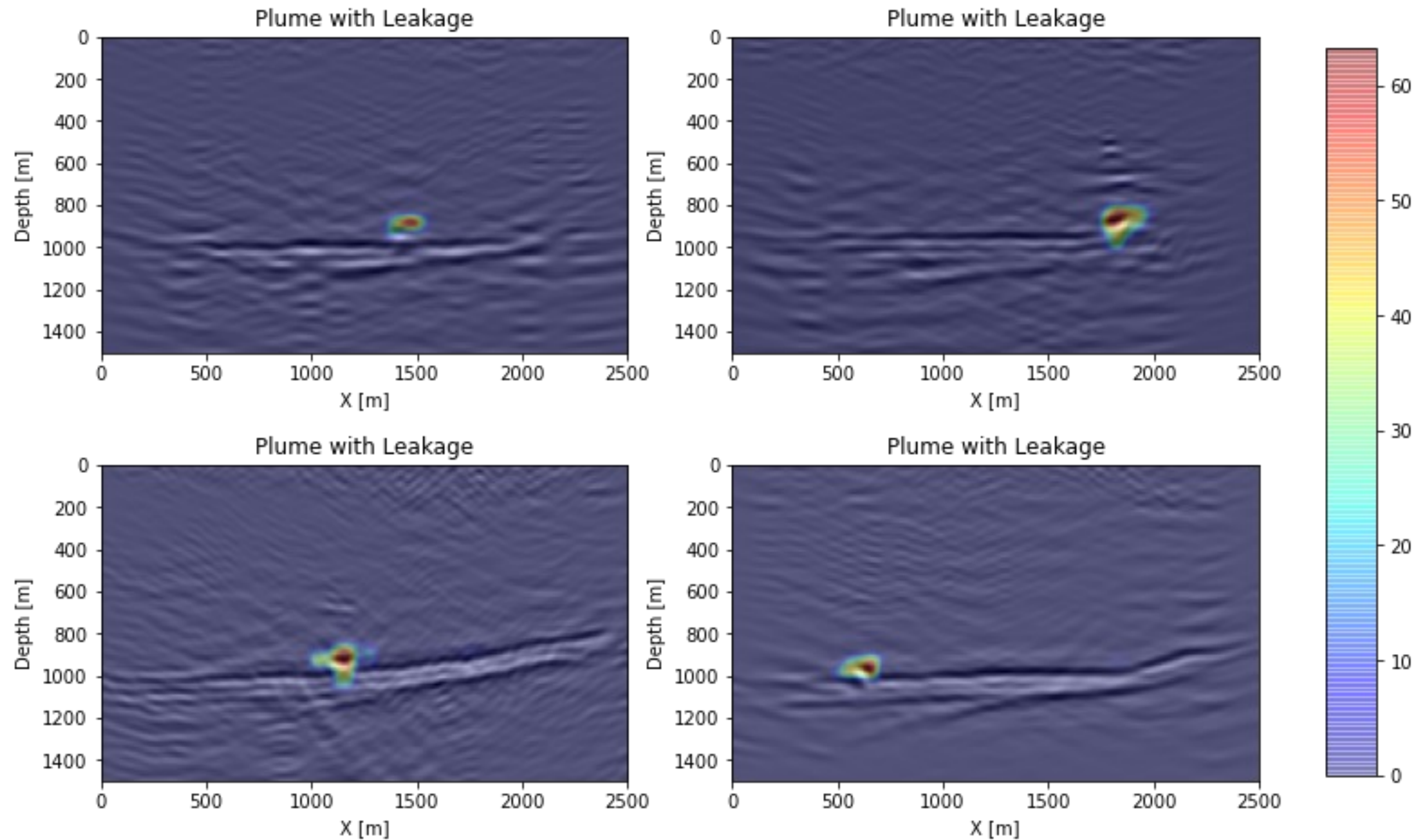
Recall: measures how many leakages model predicted correctly, out of all actual leakages





# Leakage detection

## examples

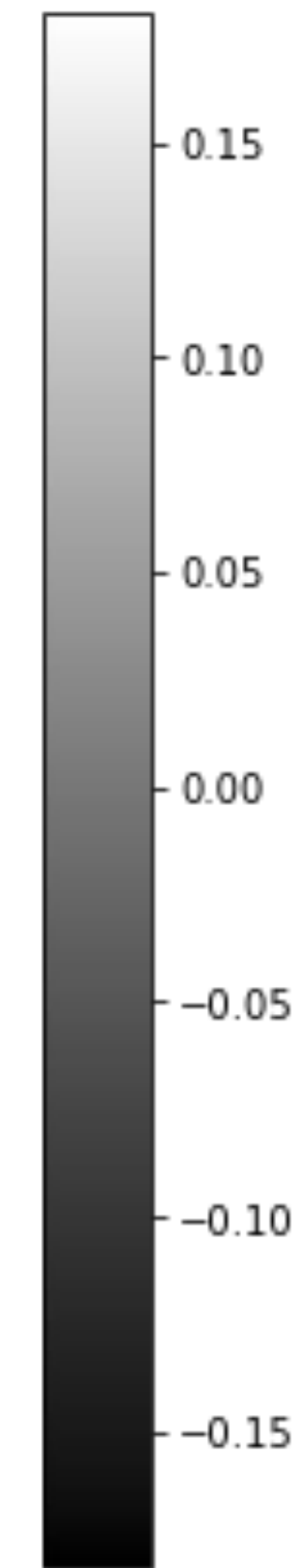
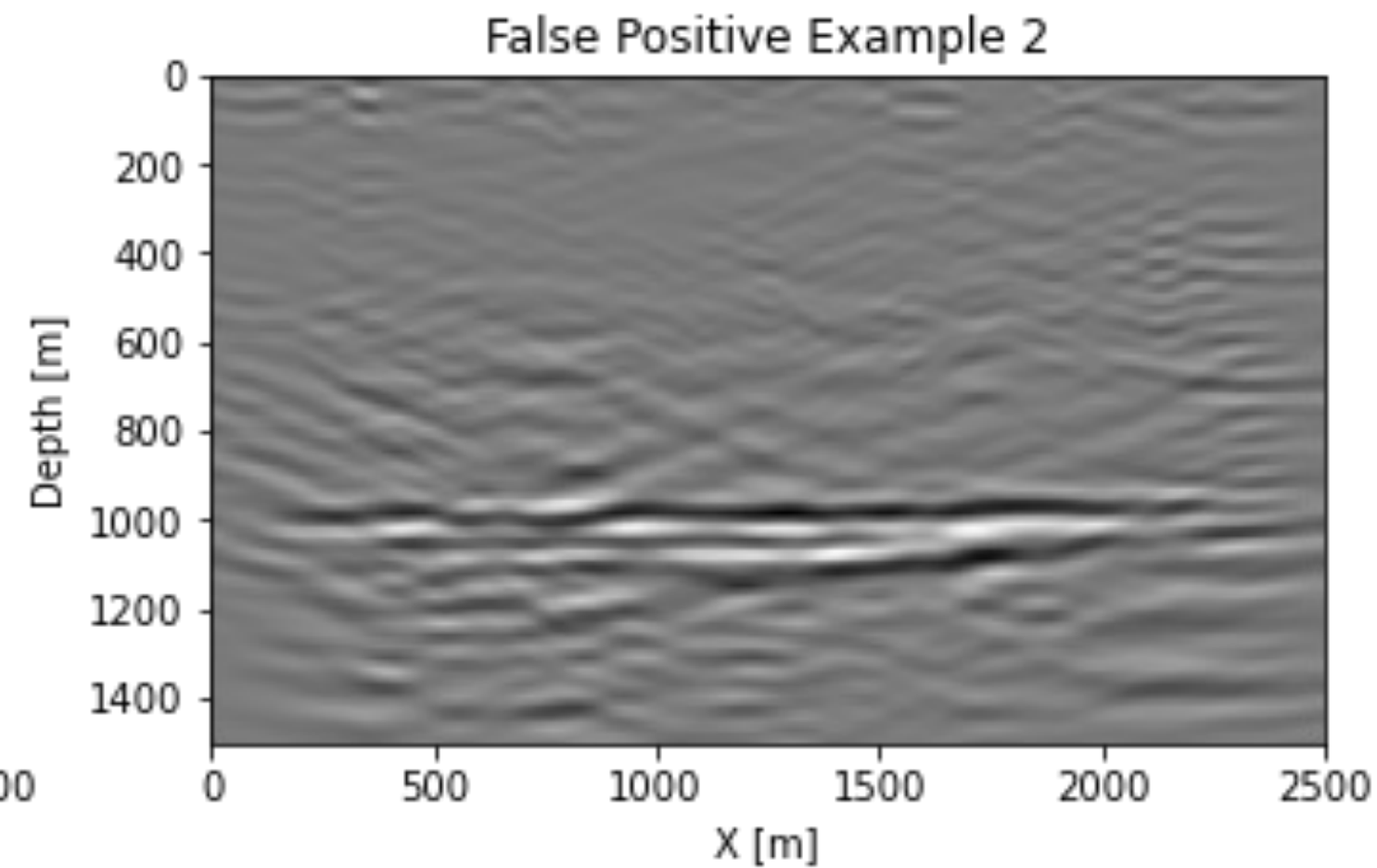
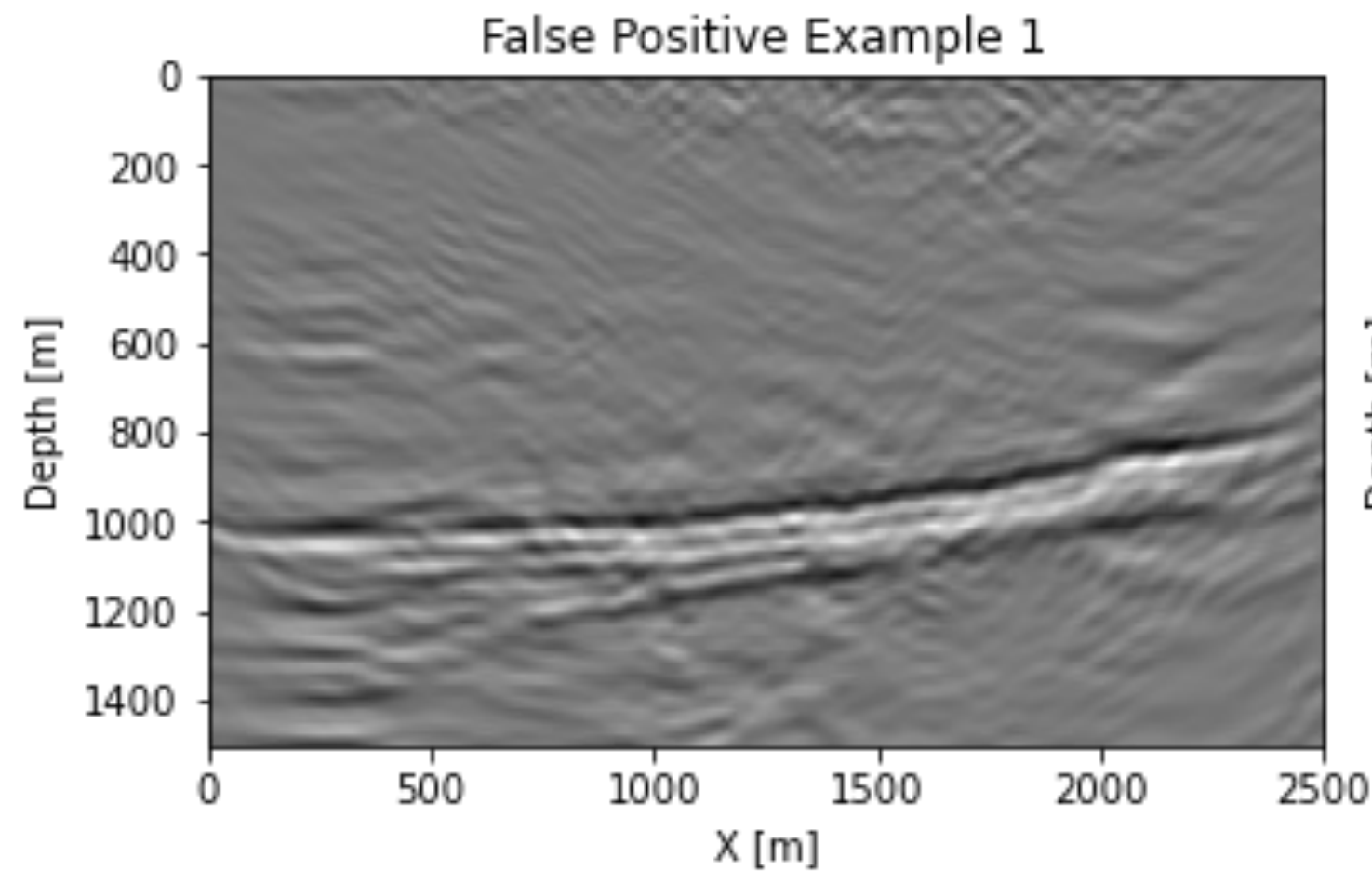




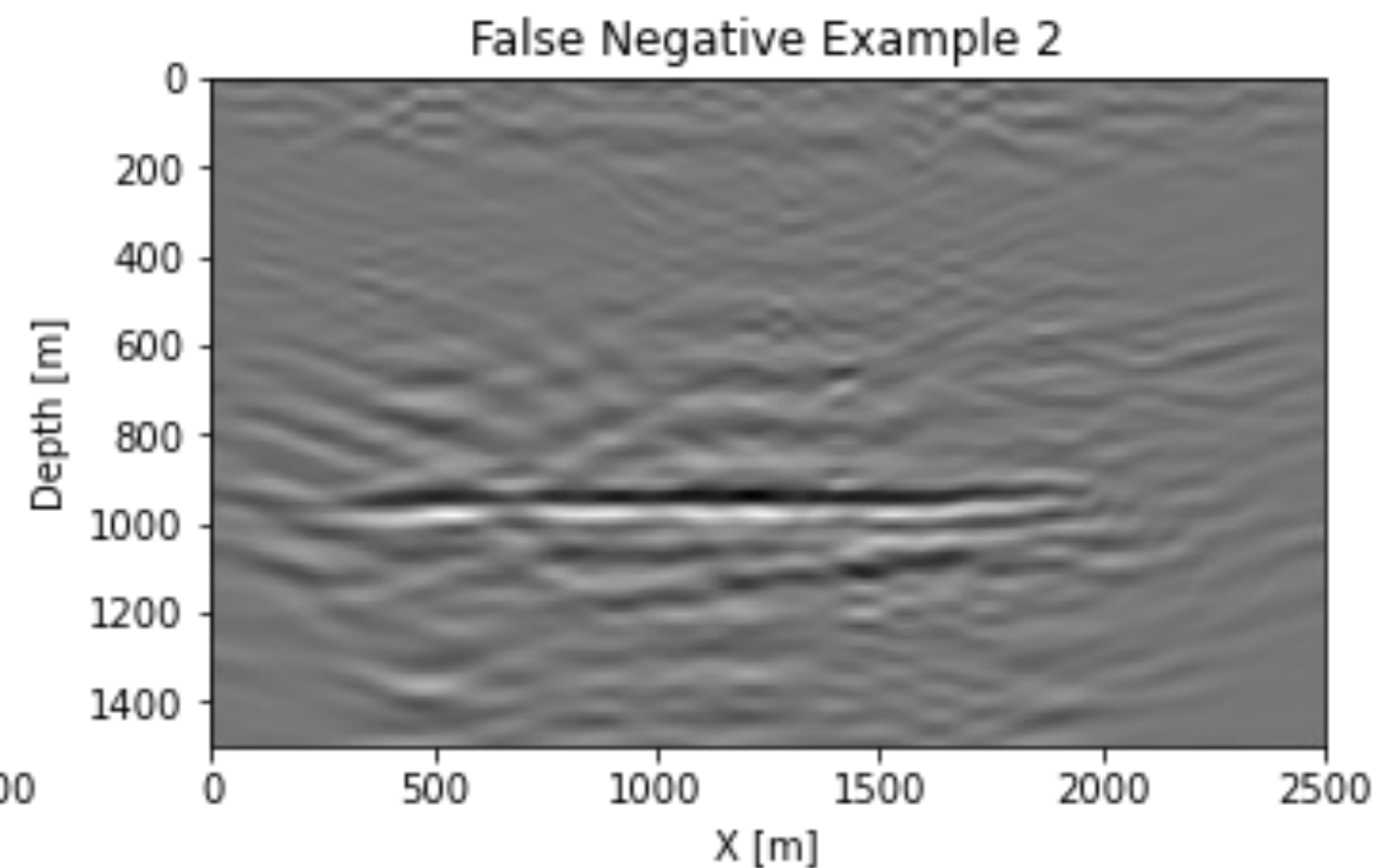
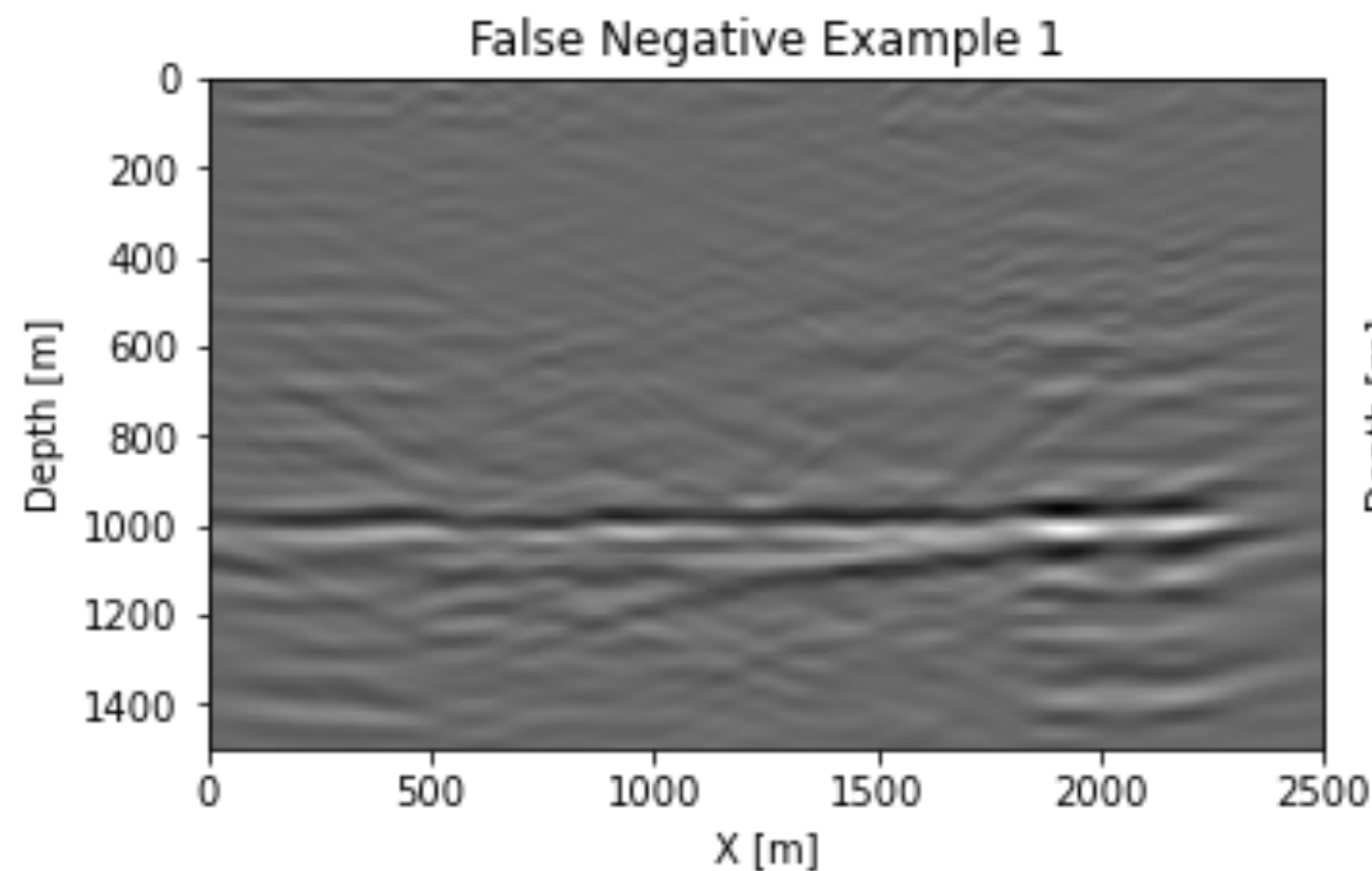
# False positives/negatives

## examples

**false  
positives**



**false  
negatives**



# What about uncertainty?



e.g. w.r.t. background velocity  
models...

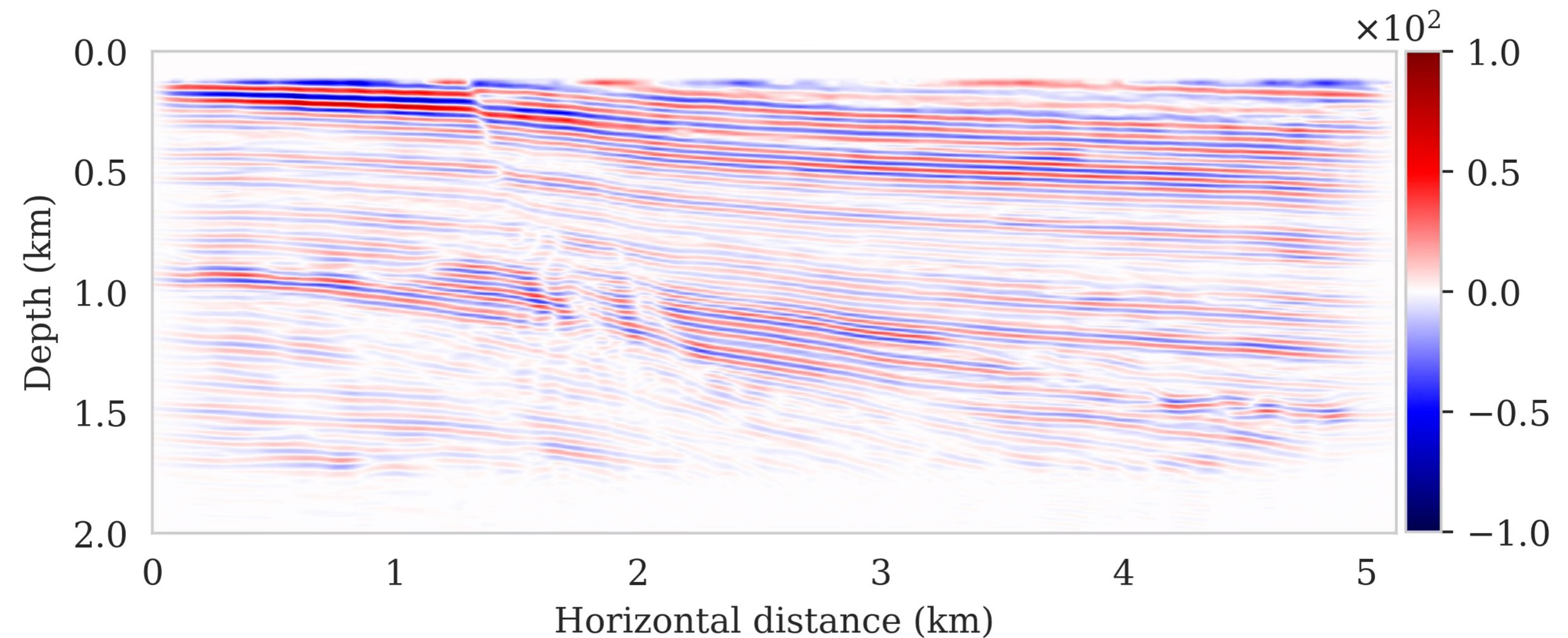
# Training set

## RTMs for different background velocity models

Samples from  $p(\mathbf{m}_0 \mid \mathbf{d})$   
for the background  
velocity model



Corresponding RTMs  
$$\delta \mathbf{m}_{RTM} = \sum_{i=1}^{n_s} \mathbf{J}(\mathbf{m}_0, \mathbf{q}_i)^\top \delta \mathbf{d}_i$$





# Velocity continuation

w/ FNOs

Learn mapping  $\mathcal{T}_{(\mathbf{m}_{\text{init}}, \mathbf{m}_{\text{target}})}: \delta\mathcal{M} \rightarrow \delta\mathcal{M}$  from training pairs

$$\{((\mathbf{m}_0^{(i)}, \delta\mathbf{m}_{\text{init}}), \delta\mathbf{m}_{\text{RTM}}^{(i)}) | i = 1, \dots, N\}$$

- $(\mathbf{m}_0^{(i)}, \delta\mathbf{m}_{\text{init}})$  input target background and initial seismic image training pair
- $\delta\mathbf{m}_{\text{RTM}}^{(i)}$  target seismic image

by minimizing

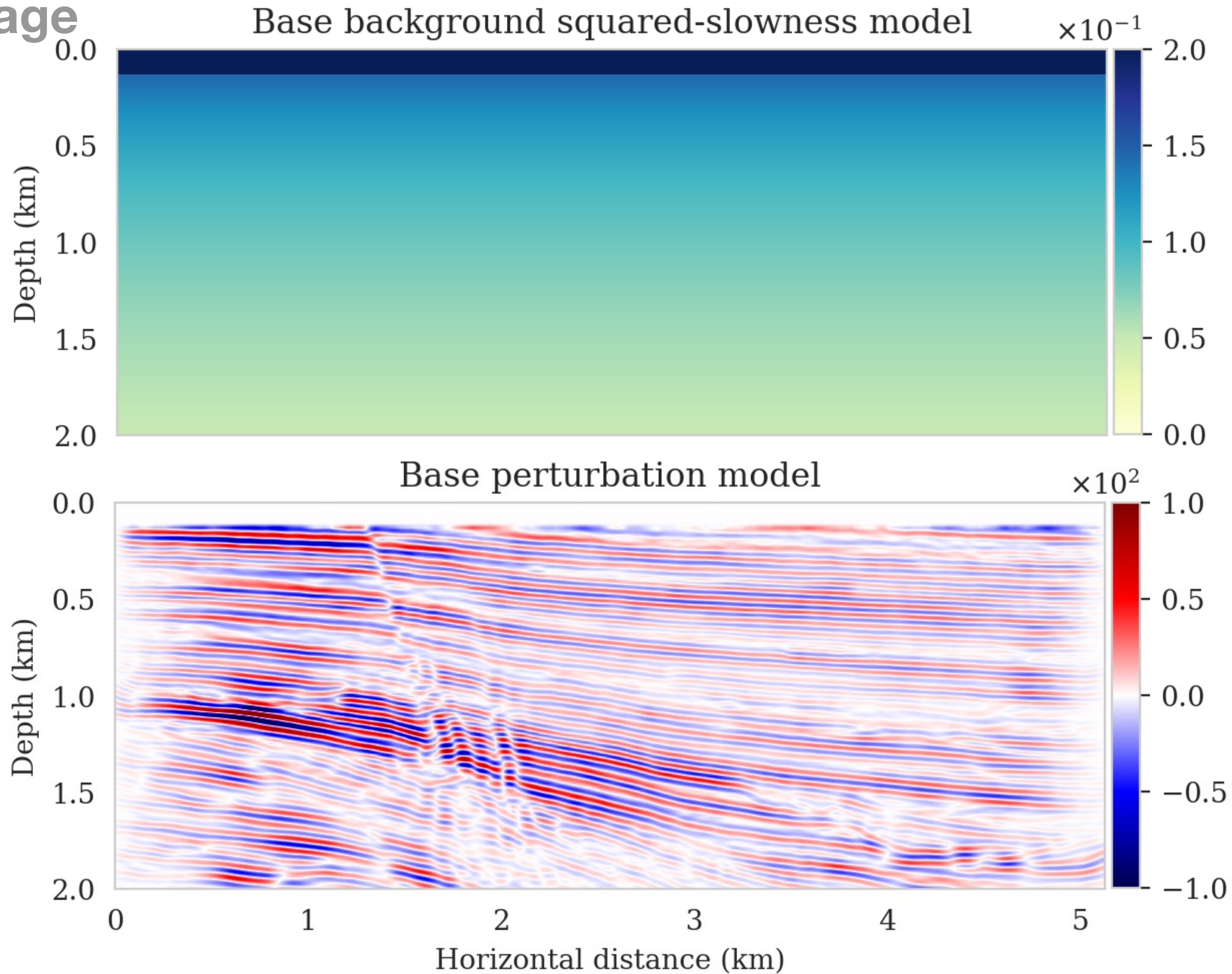
$$\mathbf{w}^* = \underset{\mathbf{w}}{\operatorname{argmin}} \frac{1}{N} \sum_{i=1} \|\mathcal{G}_{\mathbf{w}}(\mathbf{m}_0^{(i)}, \delta\mathbf{m}_{\text{init}}) - \delta\mathbf{m}_{\text{RTM}}^{(i)}\|_2^2$$

yielding the learned FNO  $\mathcal{G}_{\mathbf{w}^*}: \mathcal{M} \times \delta\mathcal{M} \rightarrow \delta\mathcal{M}$ .

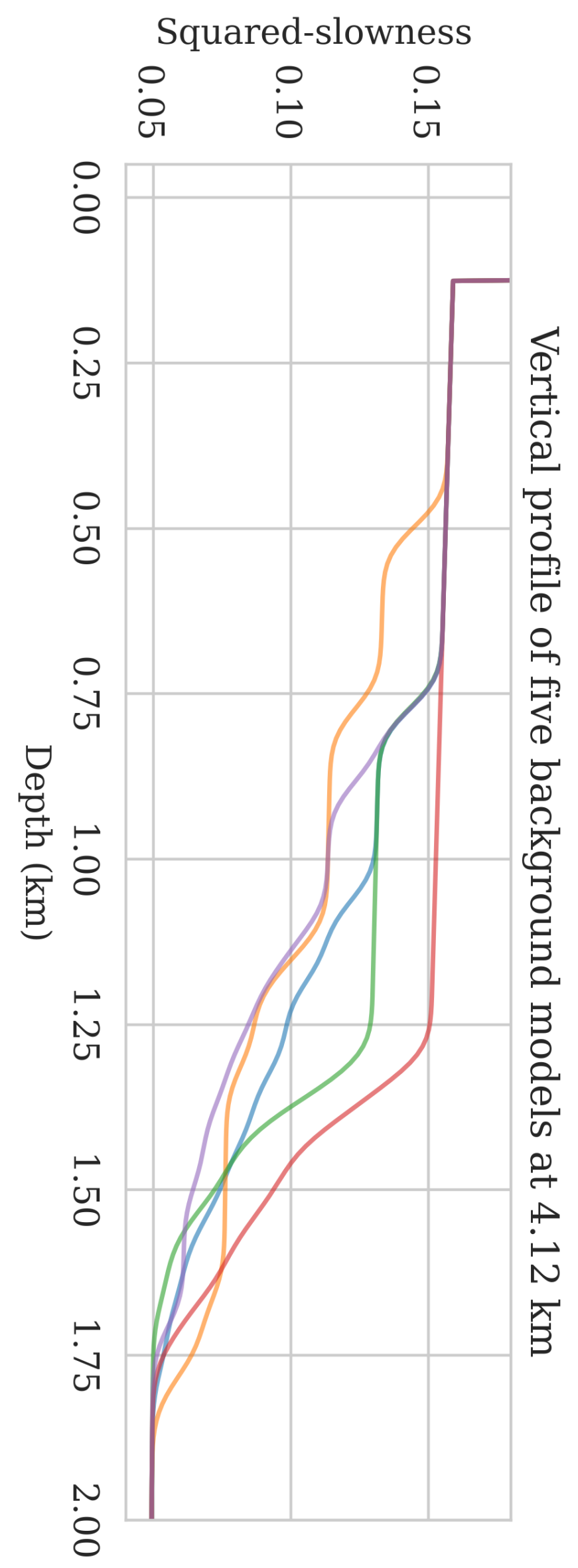
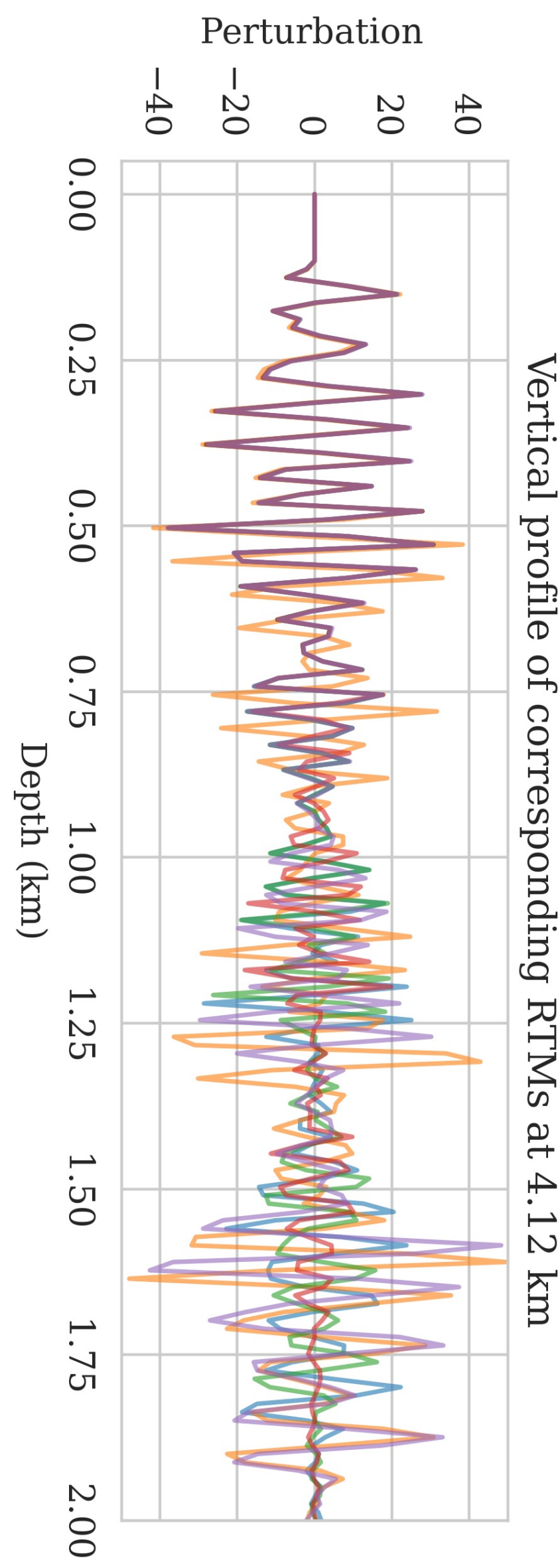
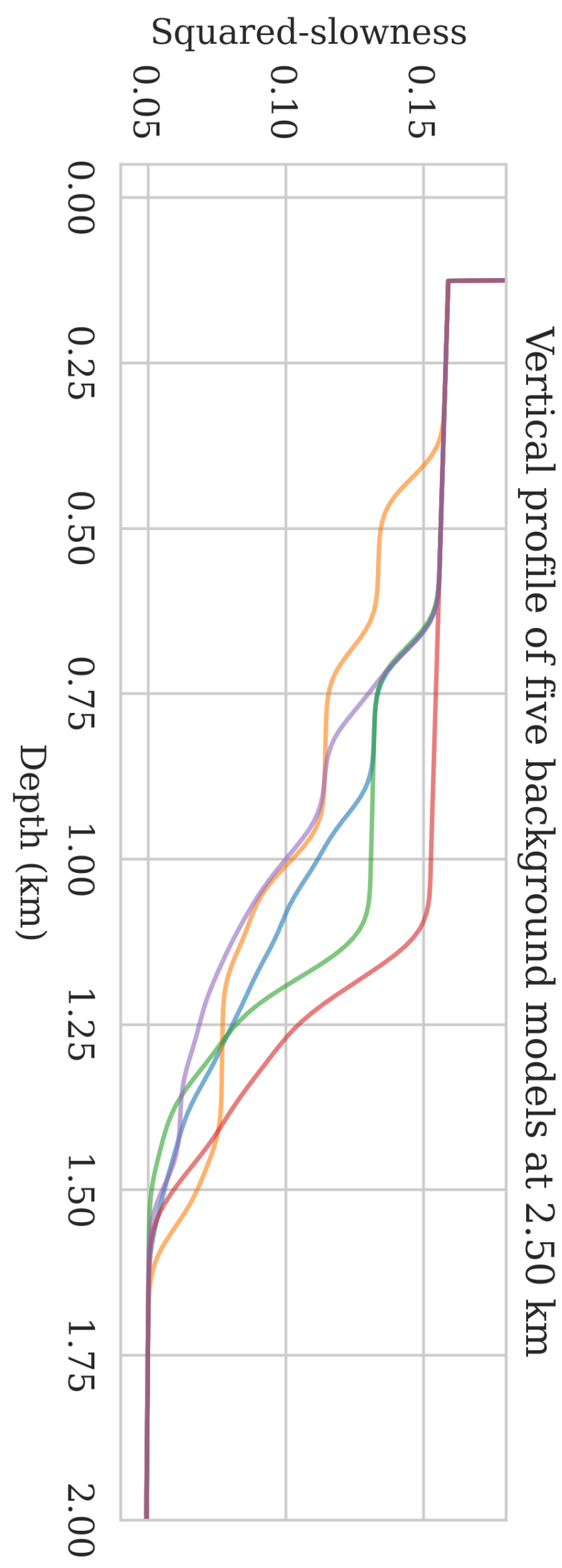
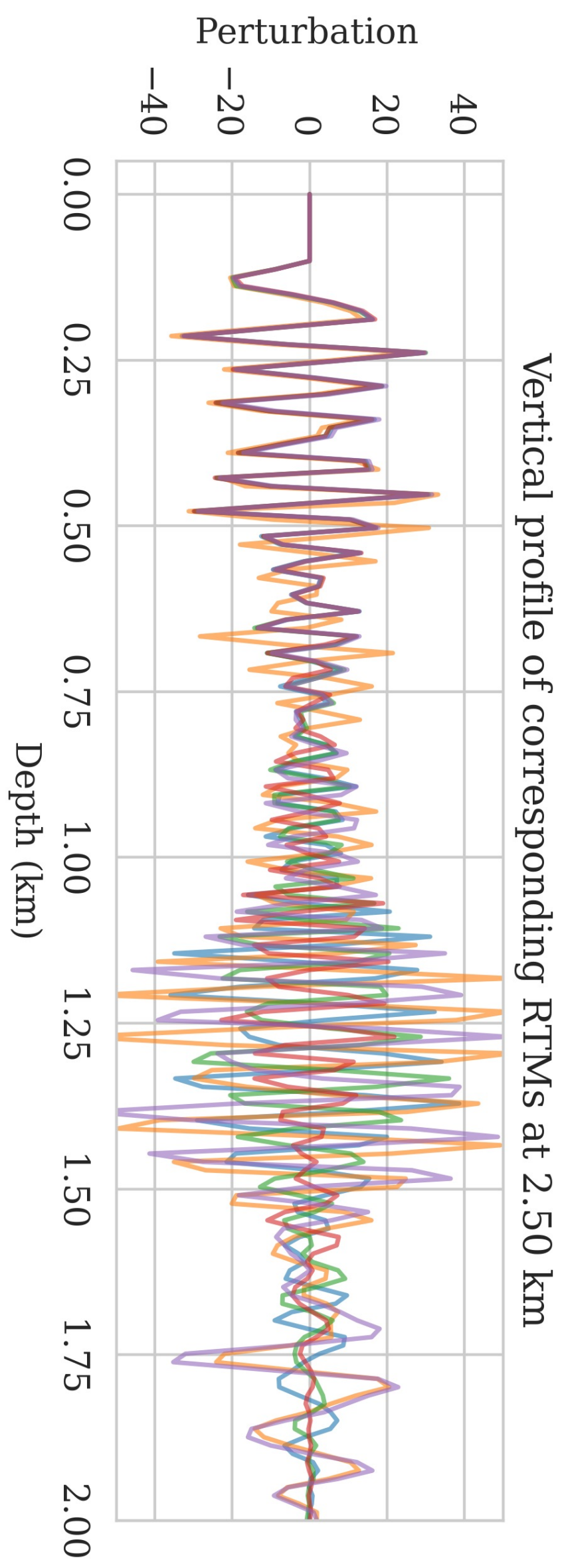


# Reference

## velocity & image





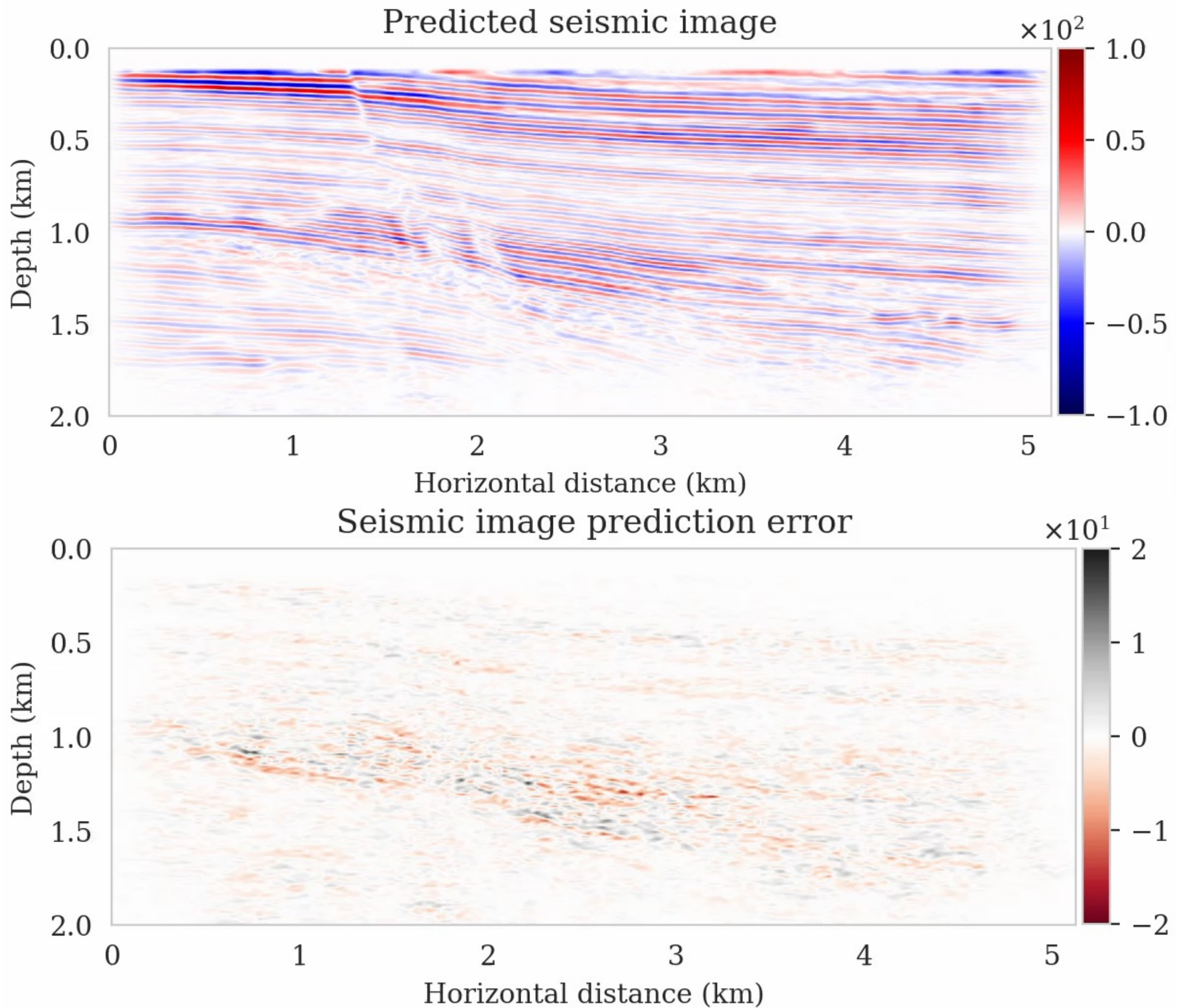
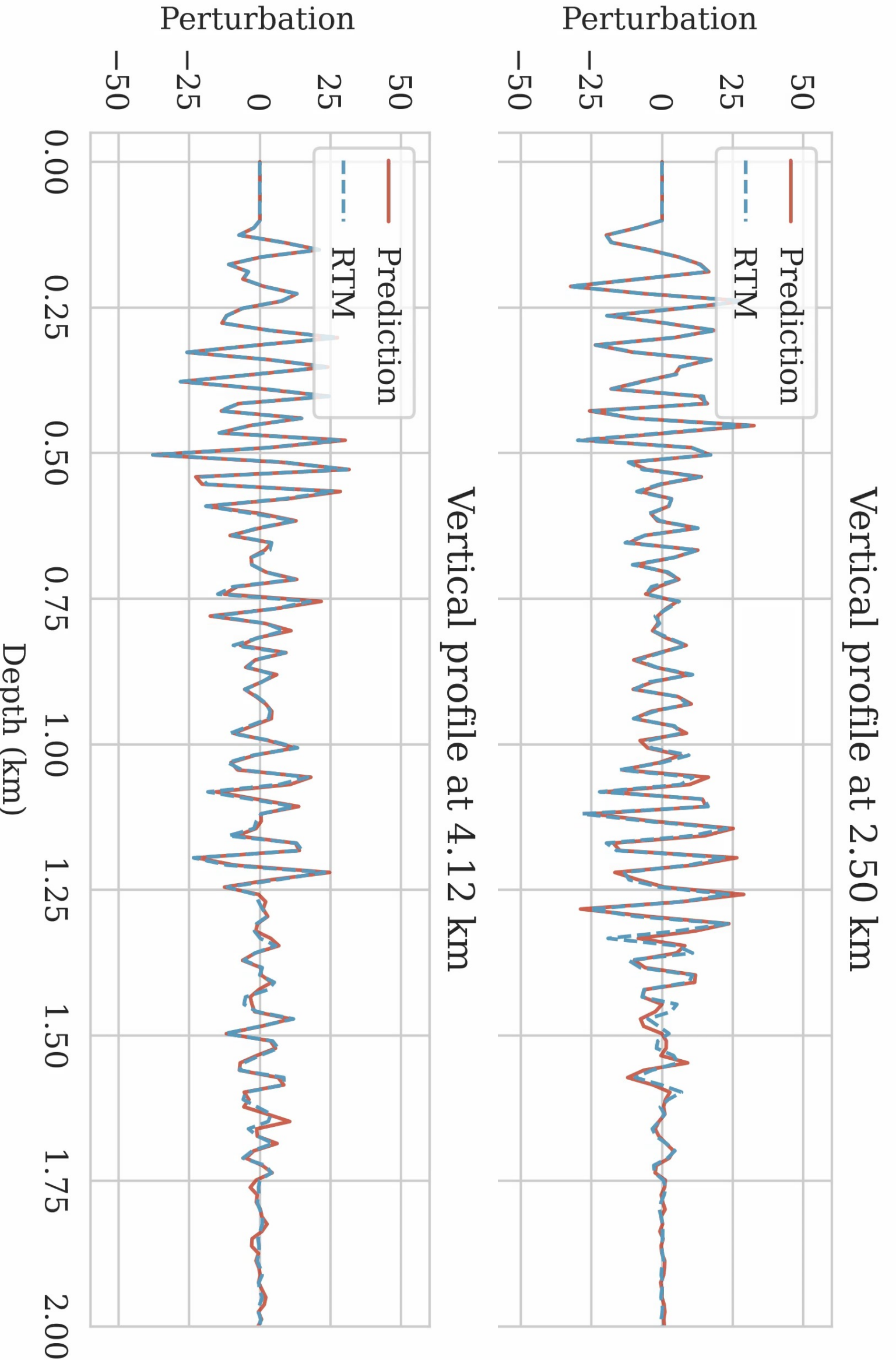


# Differences

## RTM images



# Predicted reflectivity models





# Conclusions

Neural Networks and Neural Operators can act as surrogates to capture complicated physics:

- two-phase flow
- velocity continuations

Bayes Inference allows for risk analysis for geological carbon storage

FNO's opens to possibility to

- carry out inversions using AD
- conduct UQ

# Acknowledgment

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