Sparsity-promoting least-squares migration with the linearized inverse scattering imaging condition

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Introduction

Reverse-time migration (RTM) and least-squares RTM suffer from low-frequency artifacts when the migration velocity model contains strong velocity contrasts on the scale of the dominant wave length (e.g. salt bodies). Strategies for addressing this problem can be categorized as follows:

- Wavefield filtering (e.g. using Poynting vectors)
- Image filtering (e.g. Laplacian preconditioning)
- Alternatives to cross-correlation imaging condition

Op't Root et al. (2012) introduced the linearized inverse scattering imaging condition (ISIC), which is an approximate inverse of the linearized Born migration operator and naturally suppresses these artifacts. We extend this work and reformulate RTM with ISIC as the action of an (adjoint) linear operator and then derive the corresponding forward operator, which allows to use this forward/adjoint pair in a LS-RTM workflow.

Linearized inverse scattering imaging condition

RTM with the conventional cross-correlation imaging condition (CCIC) can be formulated as the action of the adjoint Born modeling operator on the observed reflection data: $\delta\mathbf{m} = \mathbf{J}^\top\delta\mathbf{d}$, where

$$\mathbf{J}^\top\delta\mathbf{d} = -\sum_t \mathrm{diag}\Big(\ddot{\mathbf{u}}[t]\Big)\underbrace{\Big(\mathbf{A}(\mathbf{m})^{-\top}\mathcal{P}_r^\top\delta\mathbf{d}\Big)[t]}_{\mathbf{v}[\mathbf{t}]}. \qquad (1)$$

$\mathbf{A}(\mathbf{m})$: discretized wave equation with velocity model
$\delta\mathbf{m}$: image, $\delta\mathbf{d}$: observed (reflection) data
$\mathcal{P}_r$: receiver projection operator
$\ddot{\mathbf{u}}[t]$: 2nd time derivative of source wavefield
$\mathbf{v}[t]$: receiver wavefield

RTM with ISIC (Op't Root et al., 2012) can be formulated accordingly as the action of a modified adjoint Born modeling operator:

$$\widehat{\mathbf{J}}^\top\delta\mathbf{d} = \sum_t \Bigg\{\mathrm{diag}\Big(\ddot{\mathbf{u}}[t]\odot\mathbf{m}\Big)\Big(\mathbf{A}(\mathbf{m})^{-\top}\mathcal{P}_r^\top\delta\mathbf{d}\Big)[t] + \sum_{i=1}^{3}\mathrm{diag}\left(\frac{\partial\mathbf{u}[t]}{\partial\mathbf{x}_i}\right)\frac{\partial}{\partial\mathbf{x}_i}\left(\mathbf{A}(\mathbf{m})^{-\top}\mathcal{P}_r^\top\delta\mathbf{d}\right)[t]\Bigg\}. \qquad (2)$$

$\widehat{\mathbf{J}}^\top$: modified Born migration operator w/ ISIC
$\frac{\partial}{\partial\mathbf{x}_i}$: spatial derivative in i^{th} direction

The ISIC imaging operator $\widehat{\mathbf{J}}^\top$ is similar to the conventional migration operator, but contains and additional term with spatial partial derivatives. As noted in Whitmore and Crawley (2012), the individual contributions contain low-frequency artifacts with reverse signs, that cancel upon summing (figure 1).

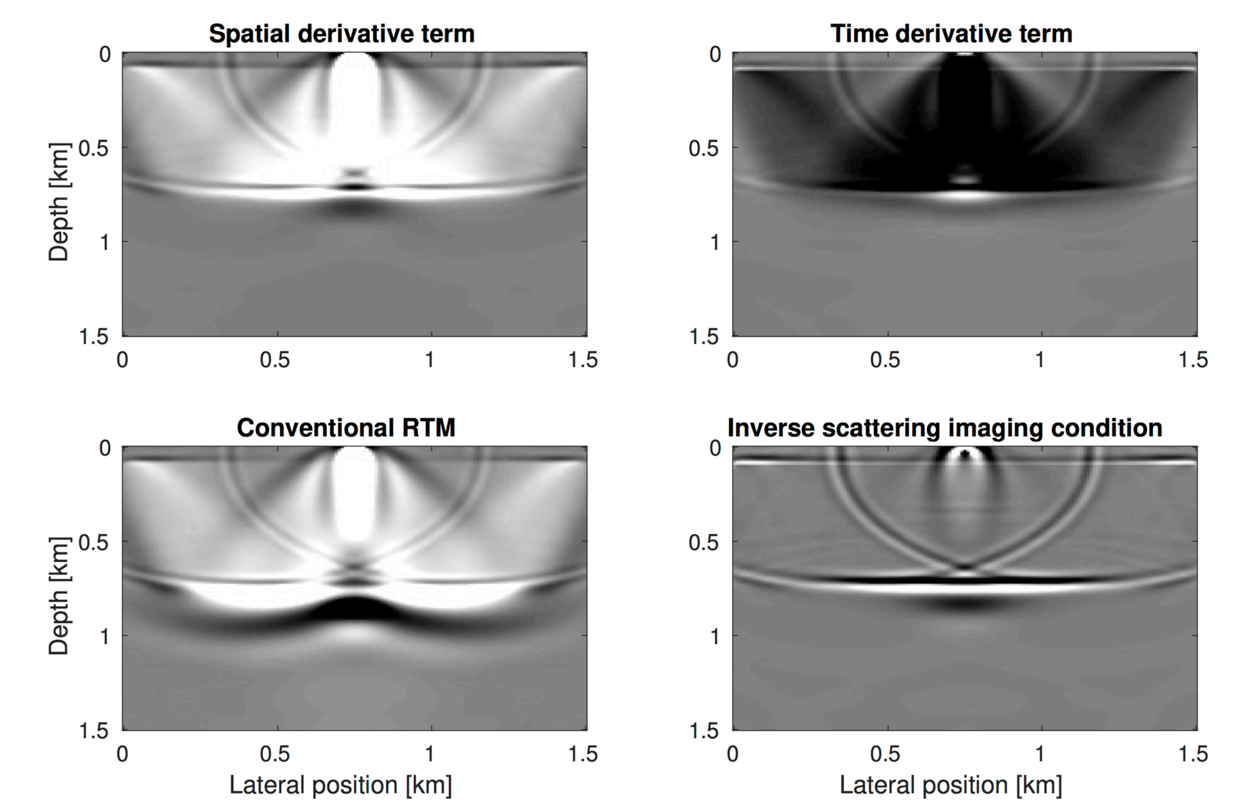

Figure 1: Individual term of the ISIC migration operator (top row) for imaging a two layer velocity model. Summing the terms leads to the figure on the bottom right, whereas conventional RTM with CCIC leads to the figure on the bottom left.

To use ISIC for least-squares migration, the corresponding forward operator of $\widehat{\mathbf{J}}^\top$ is required, which is obtained by transposing equation 2:

$$\delta\mathbf{d} = \widehat{\mathbf{J}}\delta\mathbf{m} = \Bigg\{\mathcal{P}_r\mathbf{A}(\mathbf{m})^{-1}\mathrm{diag}\Big(\ddot{\mathbf{u}}[t]\odot\mathbf{m}\Big)\delta\mathbf{m} + \mathcal{P}_r\sum_{i=1}^{3}\mathbf{A}(\mathbf{m})^{-1}\frac{\partial}{\partial\mathbf{x}_i}\mathrm{diag}\left(\frac{\partial\mathbf{u}[t]}{\partial\mathbf{x}_i}\right)\delta\mathbf{m}\Bigg\}.$$

Application to SPLS-RTM

With the forward-adjoint pair of the modified Born modeling operator, the linearized inverse scattering imaging condition can be incorporated into LS-RTM, using any solver that relies matvec products of $\widehat{\mathbf{J}}^\top$ and $\widehat{\mathbf{J}}$, e.g. gradient descent, LSQR or the linearized Bregman method.

For our example, we incorporate the modified operator into a sparsity-promoting LS-RTM workflow, in which the following objective function is minimized:

$$\underset{\delta\mathbf{m}}{\mathrm{minimize}} \quad \lambda||\mathbf{C}\delta\mathbf{m}||_1 + \frac{1}{2}||\mathbf{C}\delta\mathbf{m}||_2^2$$
$$\text{subject to: } ||\widehat{\mathbf{J}}\delta\mathbf{m} - \delta\mathbf{d}||_2 \leq \sigma.$$

$\mathbf{C}$: forward Curvelet transform
σ : data noise level
λ : trade-off parameter to control sparsity

The combination of ℓ_1- and ℓ_2- norms in the objective is called an elastic net and has the effect of making the objective strongly convex. This problem can be solved using the linearized Bregman method:

1. **for** $k = 1, ..., n$
2. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k\widehat{\mathbf{J}}_{r(k)}^\top(\widehat{\mathbf{J}}_{r(k)}\mathbf{x}_k - \delta\mathbf{d}_{r(k)})\cdot\max(0, 1 - \frac{\sigma}{||\widehat{\mathbf{J}}_{r(k)}\mathbf{x}_k - \delta\mathbf{d}_{r(k)}||_2})$
3. $\mathbf{x}_{k+1} = \mathbf{C}^\top S_\lambda(\mathbf{C}\mathbf{z}_{k+1})$
4. **end for**

where $\mathbf{x}_k, \mathbf{z}_k$ are the primal and dual variables, t_k is the step length, and S_λ is the soft-thresholding function. The algorithm allows to work with a subset of randomly selected shots in each iteration as defined by the sequence $r(k)$ (Yin, 2010; Herrmann et al., 2015). Through source subsampling, the overall number of PDE solves can be reduced significantly, while sparsity promotion on the image controls the subsampling artifacts.

We demonstrate the algorithm with the modified ISIC operator on the Sigsbee 2A model, using the following setup:

- 960 observed shot records (inverse crime), 10 s recording time
- 25 iterations of SPLS-RTM w/ 100 shots per iteration
- compare SPLS-RTM w/ cross-correlation vs. inverse scattering imaging condition (no Laplace filtering etc.)

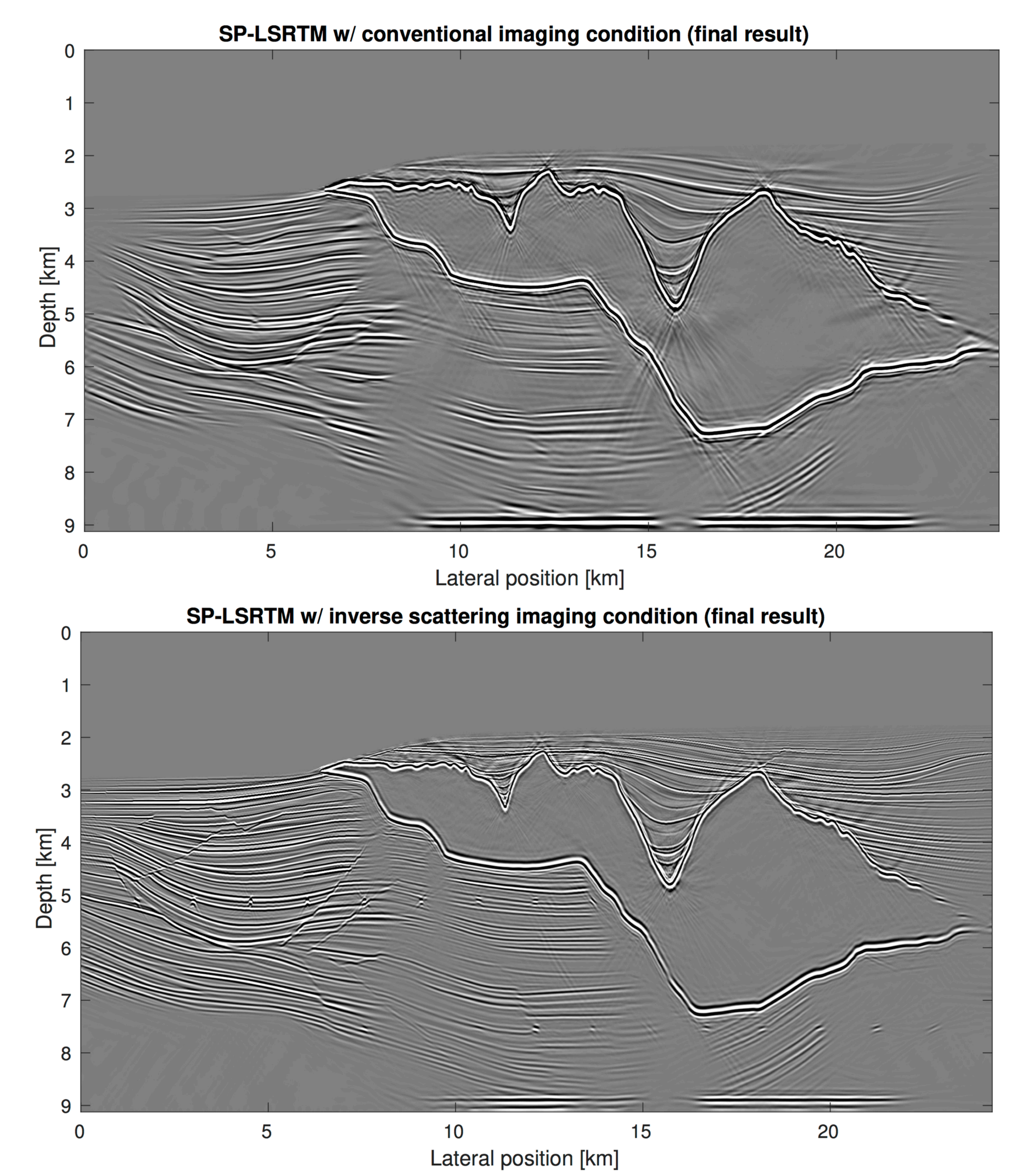

Figure 2: Results after 25 iterations of SPLS-RTM using the conventional cross-correlation imaging condition (top) versus the inverse scattering imaging condition (bottom). A post-migration Laplace filter was applied to the final results to remove residual artifacts.

Conclusions

By reformulating RTM with the linearized inverse scattering imaging condition as the action of an (adjoint) linear operator, a forward-adjoint pair of the operator can be derived and incorporated into a least-squares migration workflow.

In combination with our sparsity-promoting LS-RTM workflow using the linearized Bregman method, this leads to:

- increased convergence of the imaging algorithm through supression of low-frequency artifacts
- least-squares imaging at a fraction of conventional LS-RTM through source subsampling (only 2 passes through data)
- ISIC operator introduces no significant additional computational cost (only need to apply additional spatial derivatives)
- no additional wavefields need to be stored in memory
- no image or wavefield filtering required during inversion

Combining the benefits of SPLS-RTM using the linearized Bregman method, which allows to work with random subsets of sources in each iteration, with the linearized inverse scattering imaging condition, leads to a cost efficient imaging workflow that can be applied to large-scale 3D data sets in challenging geological settings.

Acknowledgements

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