Introduction

Ocean bottom multicomponent seismic data contains rich shear (S-wave) information that can be used for various applications such as imaging through a gas chimney and reservoir monitoring (Stewart et al., 2003). Up to now We are unable to directly record compressional P-waves and S-waves. What is measured in the field is the acoustic pressure and the three particle velocity components. This multi-component data can be decomposed to obtain the P- and S-waves. There exist different P- and S-wave decomposition methods. Data can be decomposed based on polarity as P- and S-waves propagate with different polarities. Another decomposition method is the elastic wavefield decomposition derived from the elastic wave-equation (Wapenaar and Berkhout, 2014). This method which we implement in this work has the advantage of providing up- and down-going P- and S-waves. However, recording the S-waves with the multicomponent measurements is associated with prohibitively high acquisition costs. This is due to the fact that the spatial sampling of conventional acquisition designs is determined by the Nyquist sampling criterion, which is directly proportional to the subsurface apparent velocity. To properly sample the S-waves, source and/or receiver intervals need to be much finer compared with the requirements of the P-waves due to the slower velocity of the earlier. Therefore we seek a solution by which we reconstruct the S-waves at a cost much lower than required by the Nyquist sampling criterion. To achieve this, we design a compressed sensing based acquisition (Candès and Wakin, 2008), where sources and receivers are placed on a subsampled grid at random. This results in a coarser acquisition grid with missing sources and/or receivers, thus we call for interpolation to obtain densely sampled data at a finer grid. It has been shown by Hennenfent and Herrmann (2006, 2008); Herrmann et al. (2012) that borrowing insights from compressive sensing and using randomized acquisition provides better interpolated results. Different interpolation methods have been proposed including but not limited to transformation-based methods such as the Fourier transform (Zwartjes and Sacchi, 2007), curvelet transform (Hennenfent and Herrmann, 2006, 2008), Radon transform (Trad et al., 2002; Kabir and Verschuur, 1995), and frequency-wavenumber transform (Stanton and Sacchi, 2013). In this work, we use an SVD-free matrix-factorization technique for low rank interpolation to interpolate multi-component seismic data volumes. This method is computationally and memory efficient compared to the other sparsity-based interpolation approaches for large scale seismic data volumes (Kumar et al., 2015).

SVD-free low-rank matrix factorization

As outlined in Kumar et al. (2015), the success of rank-minimization based seismic data interpolation hinges on the fact that fully sampled data should exhibit a low-rank structure in some transform domain while subsampling increases the rank in that transform domain. Under these assumptions, a rank-minimization problem can be solved to reconstruct the missing-traces during seismic data acquisition. Kumar et al. (2015) showed that for 2D seismic data acquisition, midpoint-offset domain is one such domain to exploit the low-rank structure of seismic data. Here, low-rank means that the seismic data can be well approximated by doing a decomposition based on few singular vectors associated with few largest singular values. Following the same ideas, for multi-component data, we observe that monochromatic frequency slices have low-rank structure in the m-h domain, whereas, subsampling increases the rank. Hence, rank-minimization based techniques can be used to interpolated the missing-traces for multi-component seismic data acquisition. Given a matrix $\mathbf{X}$ in $\mathbf{C}^{n\times m}$ and a linear measurement operator $\mathscr{A}$ that maps from $\mathbf{C}^{m\times n} \rightarrow \mathbf{C}^{p}$ with $p \ll n\times m$, the solution to the rank-minimization problem can be found by solving the following convex optimization problem (Recht et al., 2010)

$$\min_{\mathbf{X}} \|\mathbf{X}\|_* \quad \text{subject to} \quad \|\mathscr{A}(\mathbf{X}) - b\|_2 \leq \varepsilon, \qquad (\text{BPDN}_\sigma)$$

where $\|\mathbf{X}\|_* = \|\sigma\|_1$, and σ is the vector of singular values. In our case, $\mathbf{X}$ represents densely sampled monochromatic frequency slices from the multi-component data in the midpoint-offset domain, b is randomly subsampled data volume, $\mathscr{A}$ is the sampling-transformation operator that is composed of a restriction operator R and transformation operator $\mathscr{S}$ that maps the data from the source-receiver domain to the midpoint-offset domain. In order to solve (BPDN_σ) for large-scale seismic data, we use an extension of the SPGℓ_1 solver (Berg and Friedlander, 2008) developed for the (BPDN_σ) problem in Aravkin et al. (2012). The SPGℓ_1 algorithm finds the solution to the (BPDN_σ) by solving a sequence of LASSO subproblems:

$$\min_{\mathbf{X}} \|\mathscr{A}(\mathbf{X}) - b\|_2 \quad \text{s.t.} \ \|\mathbf{X}\|_* \leq \tau, \qquad (\text{LASSO}_\tau)$$

where τ is relaxed by traversing the Pareto curve. Unfortunately, solving each LASSO subproblem is expensive for large-scale seismic data, where the unknowns are of the order 10^{10} to 10^{12}. This is due to the required projection onto the nuclear norm ball $\|\mathbf{X}\|_* \leq \tau$ during each iteration by performing a singular value decomposition followed by thresholding the singular values. To overcome the need of computing SVD, we adopt a recent factorization-based approach (Rennie and Srebro, 2005; Lee et al., 2010; Recht and Ré, 2013), which parametrizes the matrix $\mathbf{X} \in \mathbf{C}^{n\times m}$ as the product of two low rank factors $\mathbf{L} \in \mathbf{C}^{n\times k}$ and $\mathbf{R} \in \mathbf{C}^{m\times k}$, such that, $\mathbf{X} = \mathbf{L}\mathbf{R}^H$, where $(^H)$ represents the hermitian transpose. This factorization approach significantly reduces the size of the decision variable from $2nm$ to $2k(n+m)$ when $k \ll m,n$. Rennie and Srebro (2005) showed that the nuclear norm obeys the relationship

$$\|\mathbf{X}\|_* \leq \frac{1}{2}\|\mathbf{L}\|_F^2 + \|\mathbf{R}\|_F^2 \,, \tag{1}$$

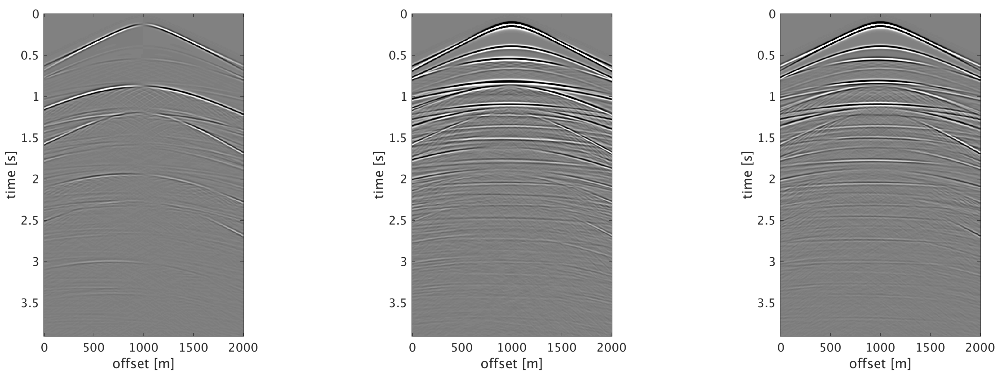
where $\|\cdot\|_F^2$ is the Frobenius norm of the matrix (sum of the squared entires). In the next sectionss, we demonstrate the efficacy of interpolation followed by elastic wavefield decomposition to obtain up- and down-going S-waves.

Elastic wavefield decomposition

The two-way wavefields can be decomposed into one-way wavefields using a decomposition matrix $\mathbf{N}$ in the frequency-wavenumber (f-k) domain as follows: (Wapenaar and Berkhout, 2014):

$$\begin{pmatrix} \phi^+(z) \\ \psi_y^+(z) \\ \phi^-(z) \\ \psi_y^-(z) \end{pmatrix} = \begin{pmatrix} \mathbf{N}_1^+(z) & \mathbf{N}_2^+(z) \\ \mathbf{N}_1^-(z) & \mathbf{N}_2^-(z) \end{pmatrix} \begin{pmatrix} -t_{xz}(z) \\ -t_{zz}(z) \\ v_x(z) \\ v_z(z) \end{pmatrix}, \tag{2}$$

where ϕ and ψ are the P- and S-wave potentials, v_x and v_z are the horizontal and vertical particle velocity components, t_{xz} and t_{zz} are the shear and normal tractions, respectively. z is the depth level at which decomposition is performed, and the superscripts $^-$ and $^+$ indicates up- and down-going wavefields respectively. For each shot record in the f-k domain, we perform the decomposition over frequencies and wavenumbers. The decomposition sub-matrices require the ocean floor properties to be known, which we assume as it is beyond the scope of this paper. However, they can be estimated using an adaptive wavefield decomposition scheme (Schalkwijk et al., 2003), or using wavefield tomography (Alfaraj et al., 2015). At the ocean bottom ($z = z_1$), the shear traction vanishes ($t_{xz}(z_1) = 0$), while the normal traction equals the negative of the acoustic pressure ($t_{zz}(z_1) = -p$), (Aki and Richards, 2002). We can now obtain the up- and down-going wavefields from alias-free measurements at the ocean bottom using a hydrophone and the two particle velocity components. However, the quality of wavefield decomposition suffers due to the coarser sampling of conventional multi-component data. Therefore, interpolating the data at a finer grid is an important processing step before wavefield decomposition to get the desired P- and S-waves.



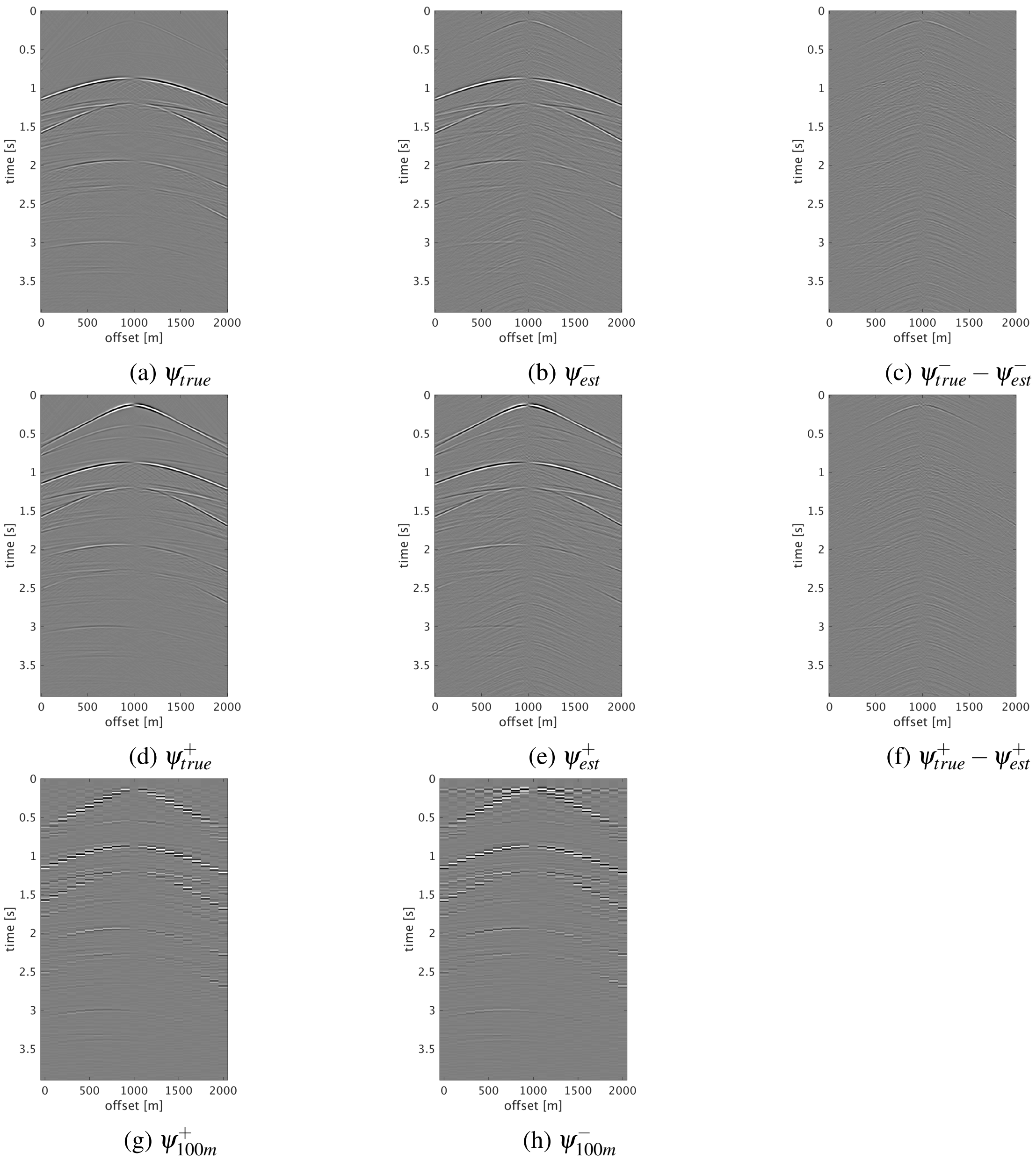
(a) v_x (b) v_z (c) p

Figure 1: Synthetic multicomponent ocean bottom shot gathers.

Interpolation and decomposition of multicomponent data

We test the feasibility of using randomized subsampling followed by low rank interpolation to obtain finely sampled S-wave data. We model multicomponent ocean bottom data using a 2D elastic finite difference modelling scheme (Thorbecke and Draganov, 2011), figure (1). We model 200 shots and

200 receivers spaced at 10 m with a maximum frequency of 60 Hz. We subsample the data by only using 25% of the original data sampled at random. The largest and smallest gaps between consecutive samples are 130 m and 10 m, respectively. We use the SVD-free low rank interpolation to interpolate the subsampled data and use it as an input for the elastic decomposition scheme to obtain the estimated up- and down-going S-waves, which we compare with the true S-wave sections obtained by directly decomposing the densely sampled data, figure 2. There is a high accuracy match between the true and estimated results. However, we can obviously see the effect of aliasing when comparing the results obtained by conventional 100m source and receiver spacing with the densely sampled S-waves. This proves that acquiring data in an under-sampled random fashion outperforms coarse regular sampling to get reasonably accurate S-waves.



(a) ψ^{-}_{true} (b) ψ^{-}_{est} (c) $\psi^{-}_{true} - \psi^{-}_{est}$

(d) ψ^{+}_{true} (e) ψ^{+}_{est} (f) $\psi^{+}_{true} - \psi^{+}_{est}$

(g) ψ^{+}_{100m} (h) ψ^{-}_{100m}

Figure 2: Decomposed up- and down-going S-waves from: data sampled regularly at 10m (a,d), 75% randomly subsampled then interpolated data (b,e), and 100m sampled data (g,h). c and f are the residuals of the difference between (a,b) and (d,e), respectively.

Conclusions

It is prohibitively expensive to record shear waves with conventional acquisition designs. By borrowing ideas from compressive sensing, we were able to show that randomized under-sampled acquisition is of favour for reconstructing the S-waves. Doing so, aliasing is turned into random noise. Using SVD-free matrix factorization for low rank interpolation, we interpolate the subsampled data to reconstruct the alias-free S-waves. Current on-going work include joint elastic decomposition interpolation where we solve for the interpolated decomposed wavefields in one optimization scheme.

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