Affordable omnidirectional subsurface extended image volumes

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Affordable omnidirectional subsurface extended image volumes

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Why we need image gathers ?

Full subsurface offset volumes allow us to conduct

- AVA analysis
- Geological dip estimation
- Velocity analysis in complex geological enviornment

Extended images

- Given two-way wave equations, source and receiver wavefields are defined as

$$H(\mathbf{m})U = P_s^T Q \quad \text{Forward propagation}$$

$$H(\mathbf{m})^* V = P_r^T D \quad \text{Backward propagation}$$

where

$H(\mathbf{m})$: discretization of the Helmoltz operator

Q : source

D : data matrix

P_s, P_r : samples the wavefield at the source and receiver positions

$\mathbf{m}$: squared-slowness

Extended images

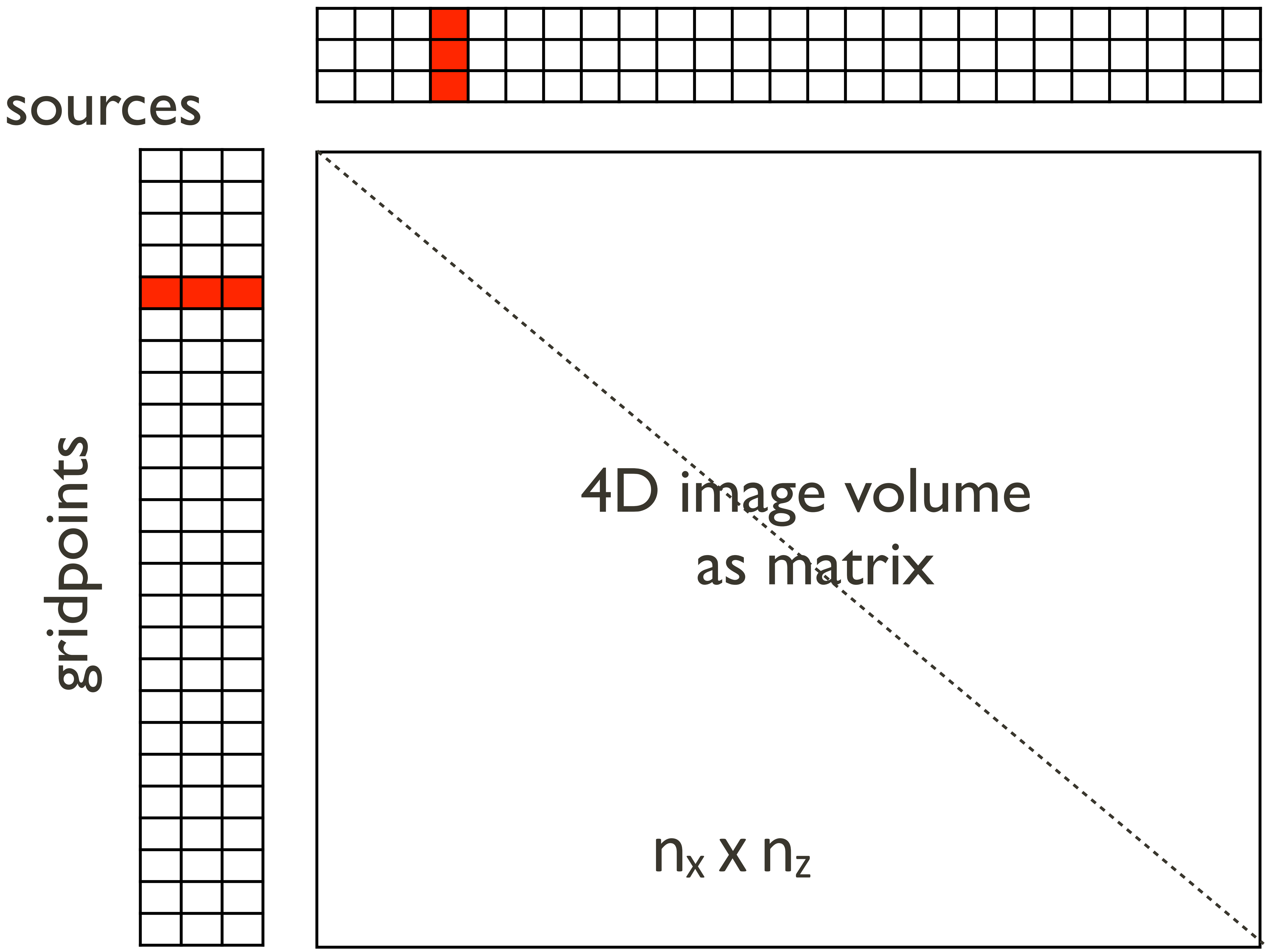
- Organize wavefields in monochromatic data *matrices* where each *column* represents a *common* shot gather

- Express *image* volume *tensor* for *single* frequency as a *matrix*

$$E = UV^*$$

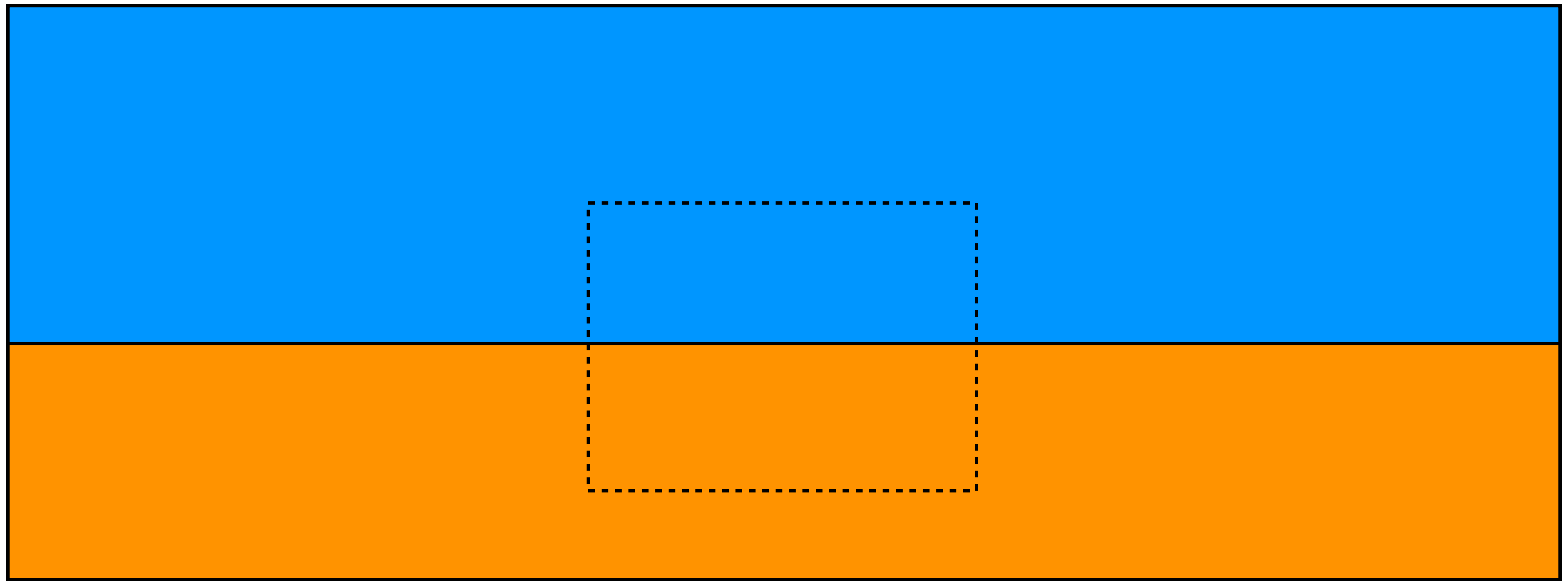
$$E = H^{-1}P_s^T Q D^* P_r H^{-1}$$

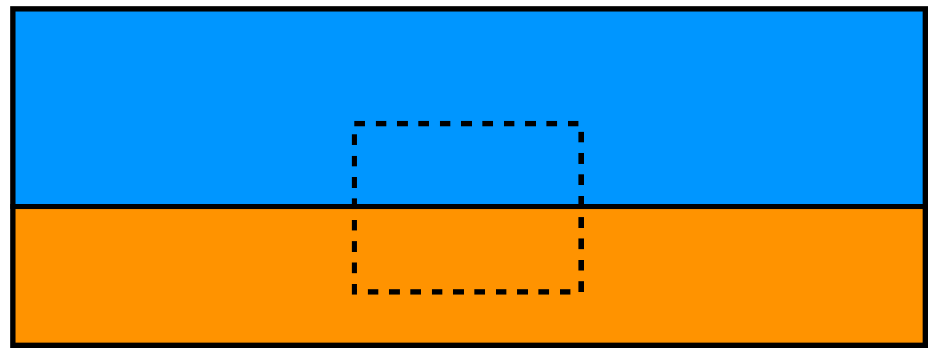
Extended images



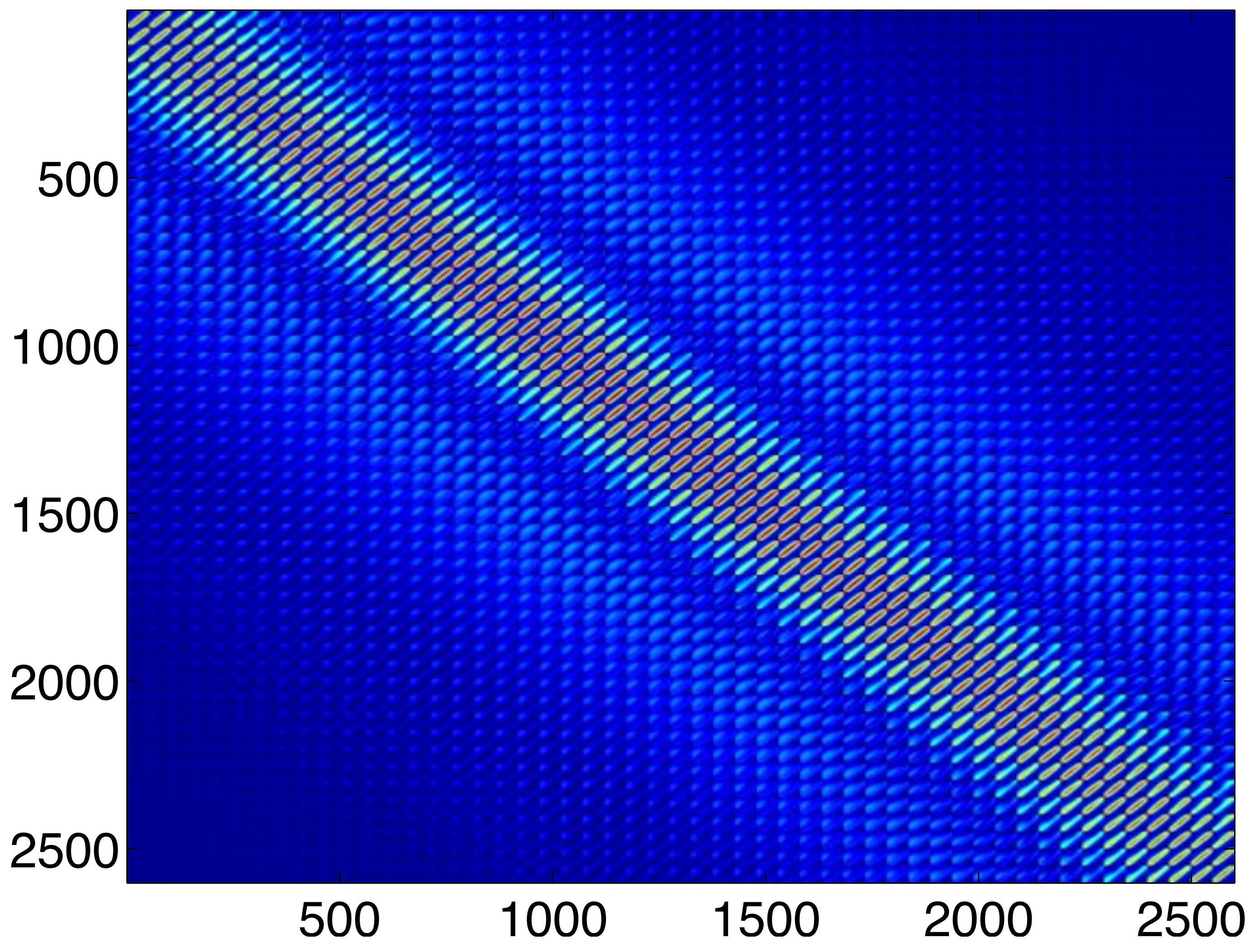
Extended images

example for *one* layer

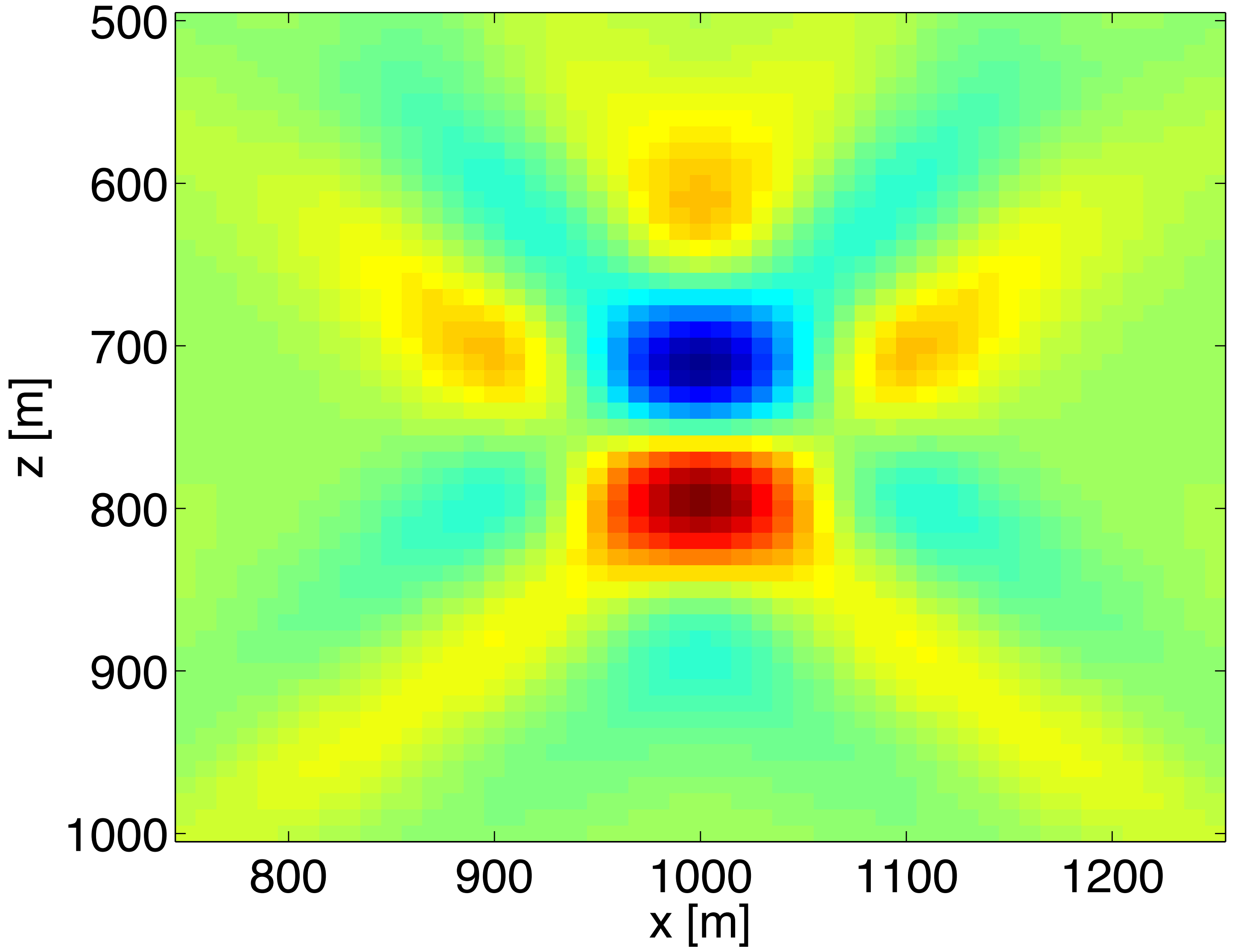




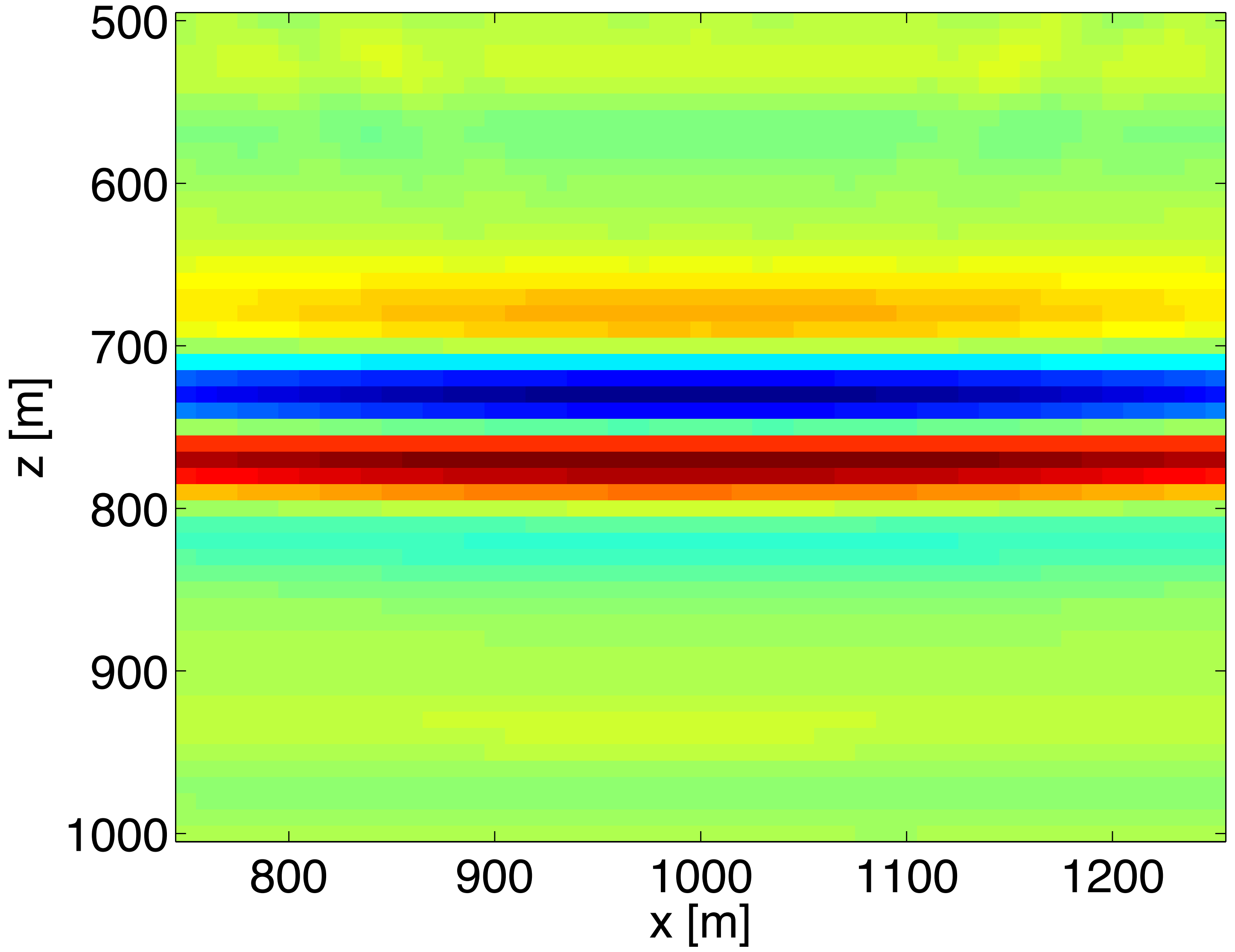
Extended images



full matrix



one column



diagonal

Impediments

Prohibitively expensive to compute and store

Current Workflow

- Compute all the source and receiver wavefields
- Severely subsample the image volume in the subsurface coordinates
- Allowed limited interactions in predefined directions
  - horizontal or vertical based upon geology of interest

Motivation

Can we avoid computing all of the wavefields if not forming the full image volumes?

Motivation

Can we gleans information from the full image volume without requiring a-priori knowledge of the geology?

Extended images

- *Probe* volume with *tall* matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_l]$

$$\widetilde{E} = EW = H^{-1} P_s^T Q D^* P_r H^{-1} W$$

van Leeuwen 2012

where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents *single* scattering points

Computation

- *mat-vec* with extended image :

$$\widetilde{E} = EW = H^{-1} P_s^T Q D^* P_r H^{-1} \mathbf{w}$$

- $\widetilde{\mathbf{d}} = P_r H^{-1} \mathbf{w}$ *(one subsurface source)*
- $\widetilde{\mathbf{y}} = Q D^* \widetilde{\mathbf{d}}$ *(surface source function)*
- $\widetilde{E} = H^{-1} P_s^T \widetilde{\mathbf{y}}$ *(one surface source)*

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Computation

computation of an *image point gather*

	# of PDE solves	"flops for correlations"
conventional	$2N_s$	$N_s \times N_h$
ours	$2N_x$	$N_s \times N_r$

N_s - # of sources
N_r - # of receivers
N_h - # of subsurface offsets
N_x - # of sample points

Computation

computation of an *image point gather*

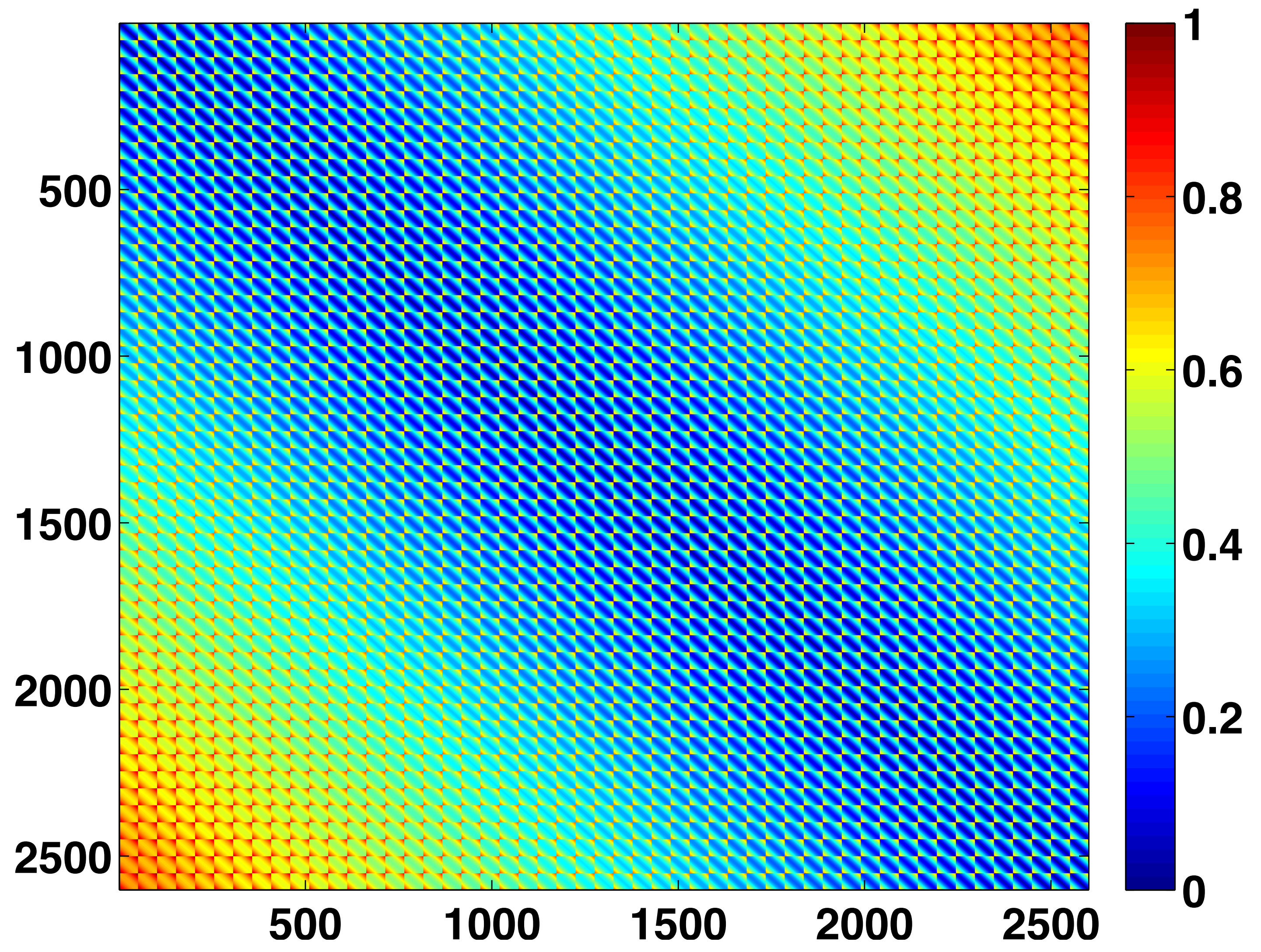
	time (s)	memory (MB)
conventional	23.6	103
ours	2.02	0.03

Application to WEMVA

Biondo & Symes, ’04 , Symes 2008, Sava & Vasconcelos, ’11

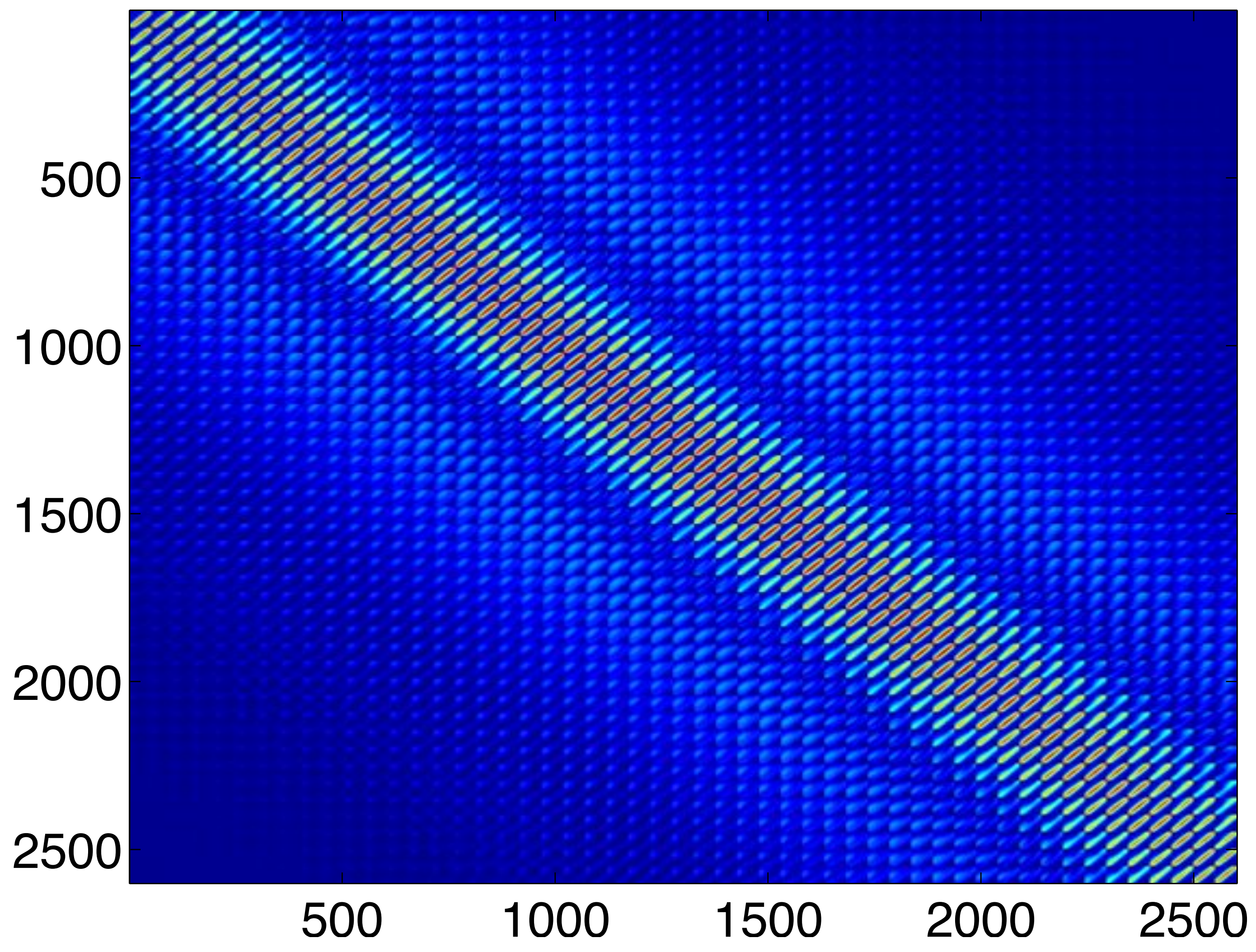
WEMVA

[conventional method]



h

$.*$



E

$.*$ **stand for element-wise multiplication**

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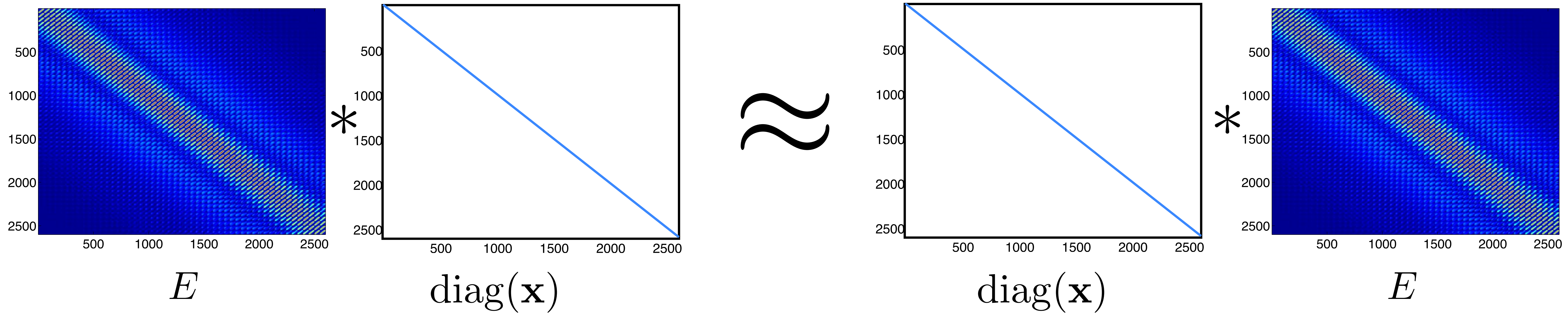
Focusing
[propose method]

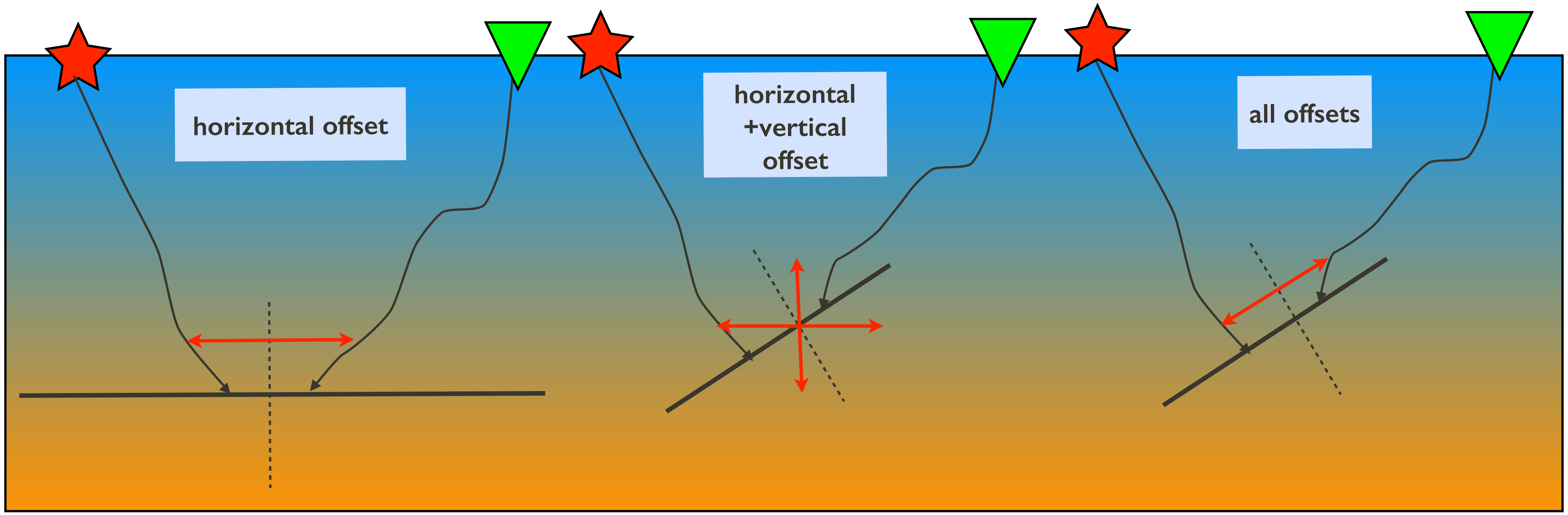
$$E\mathrm{diag}(\mathbf{x}) \approx \mathrm{diag}(\mathbf{x})E$$



* **matrix-matrix multiplication**

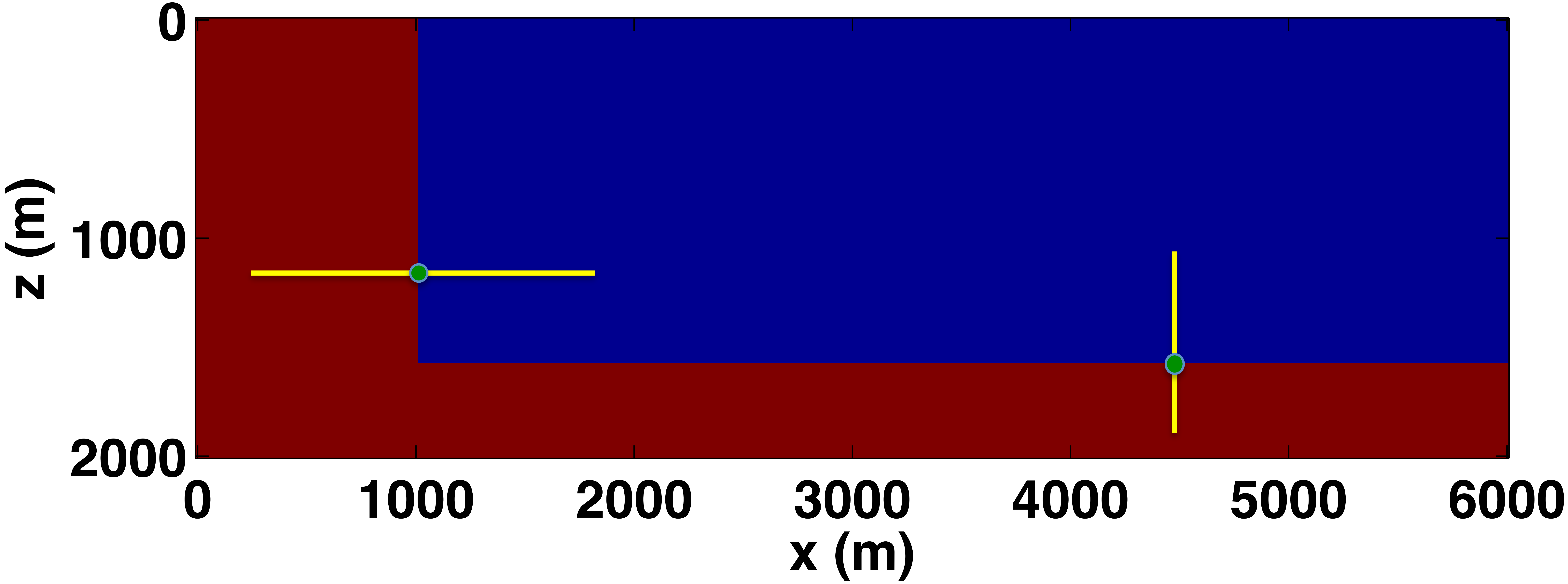
Focusing

where $\mathbf{x}$ represents horizontal, vertical or all offset.

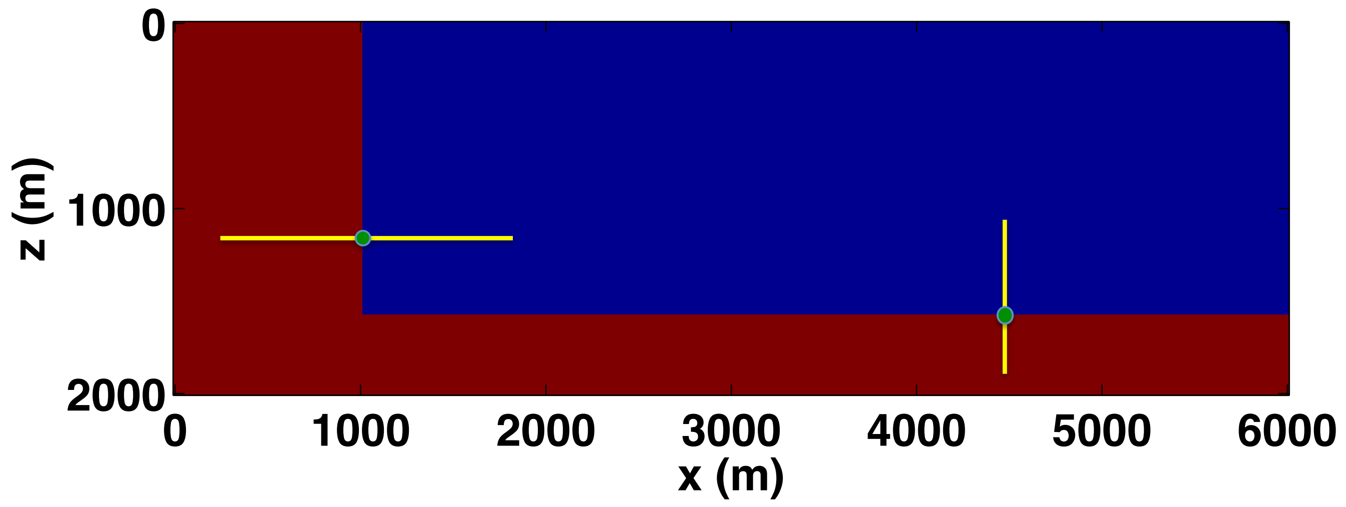


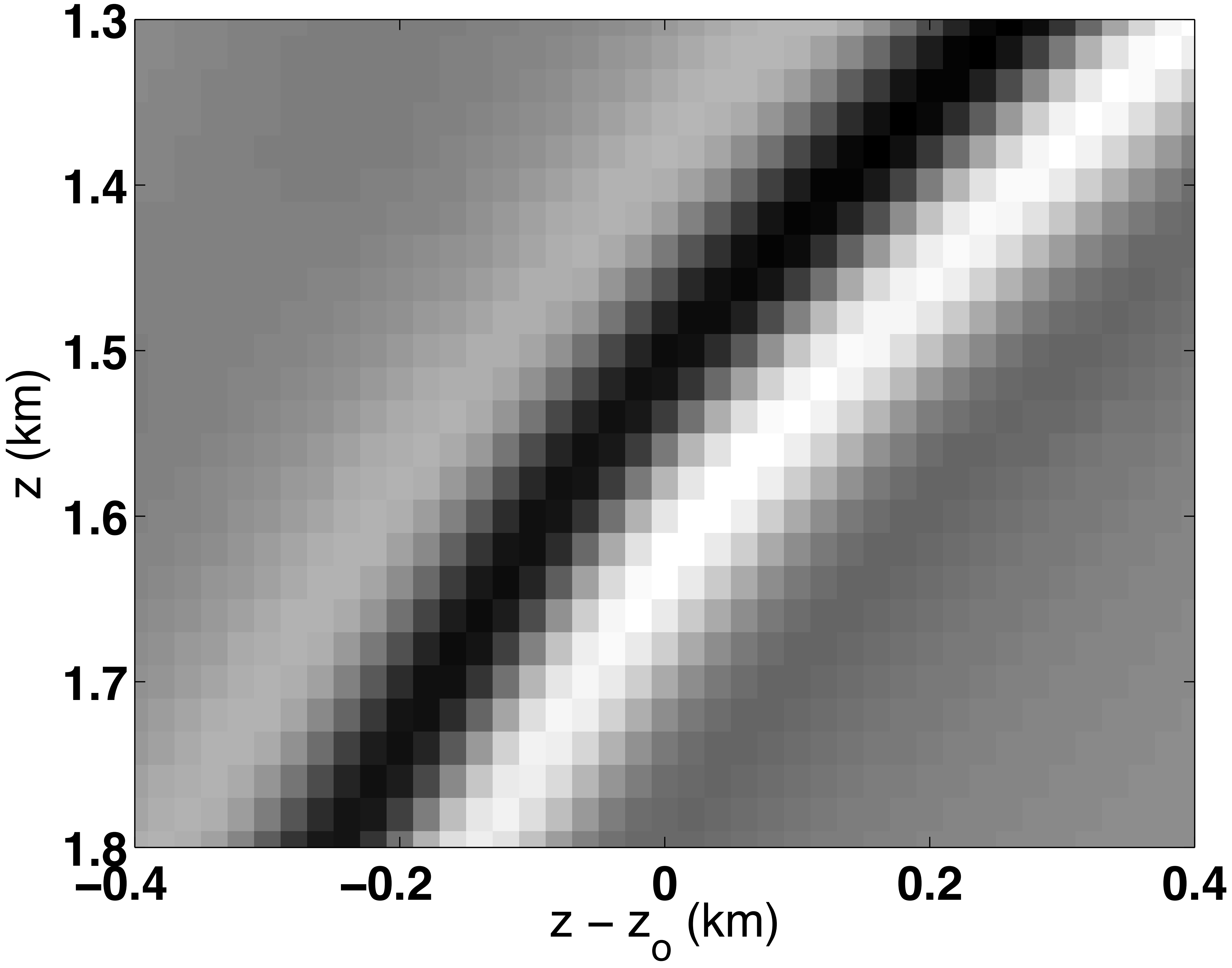
Yang and Sava, 2015, van Leeuwen et. al. 2015

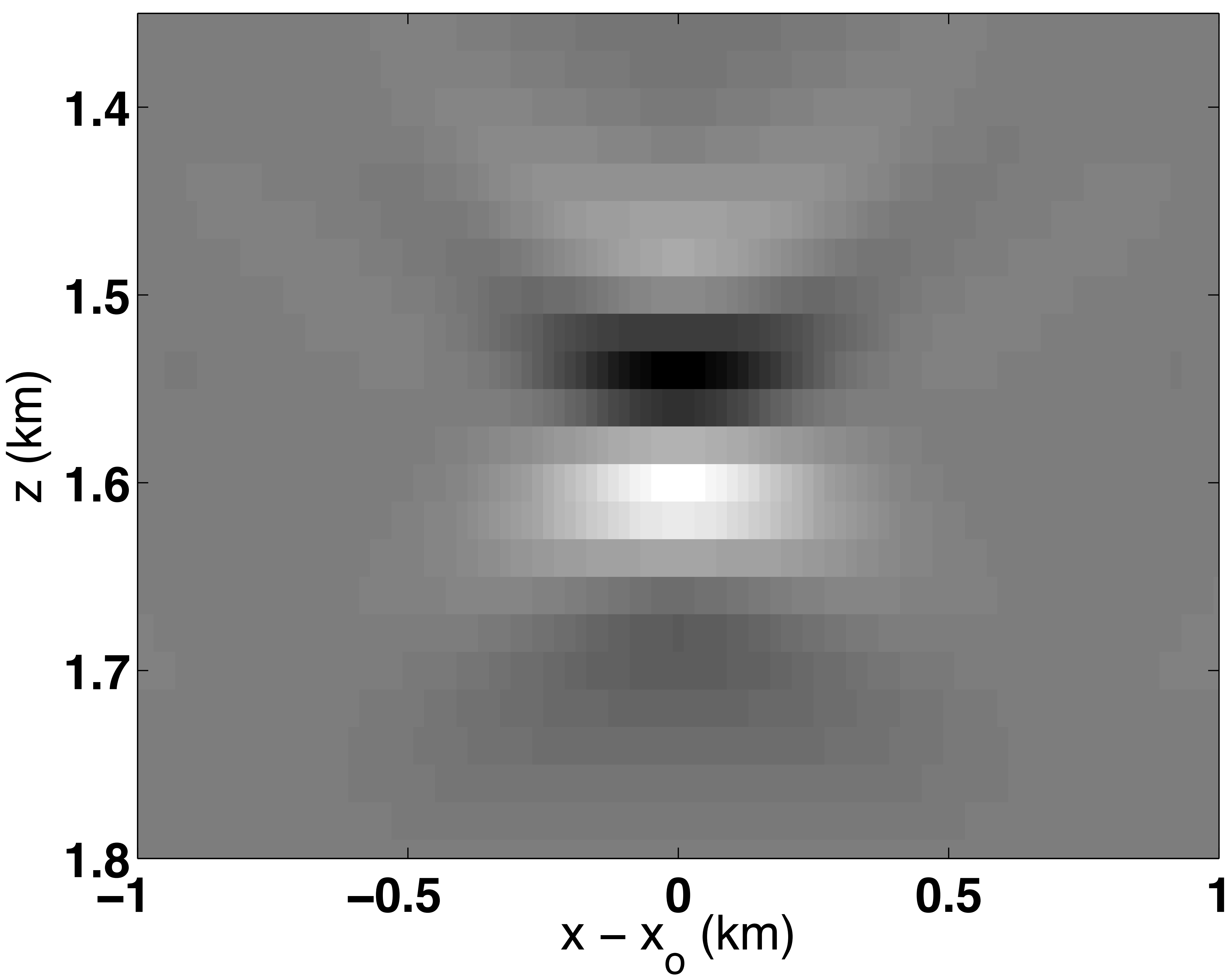
Why do we need all offsets



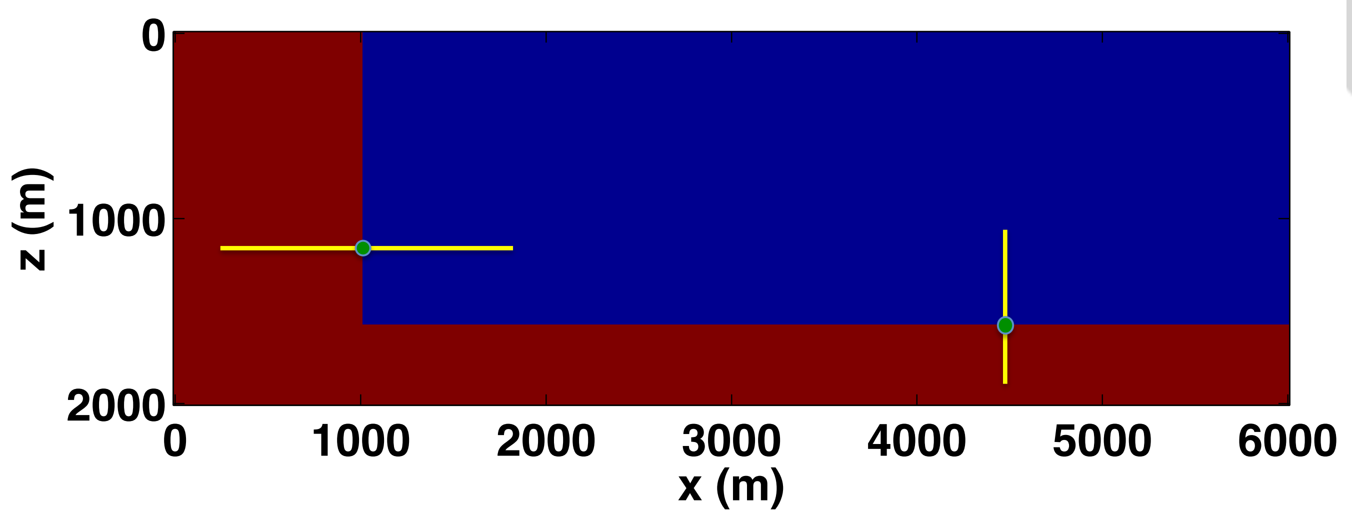
Common image gathers
Horizontal reflector

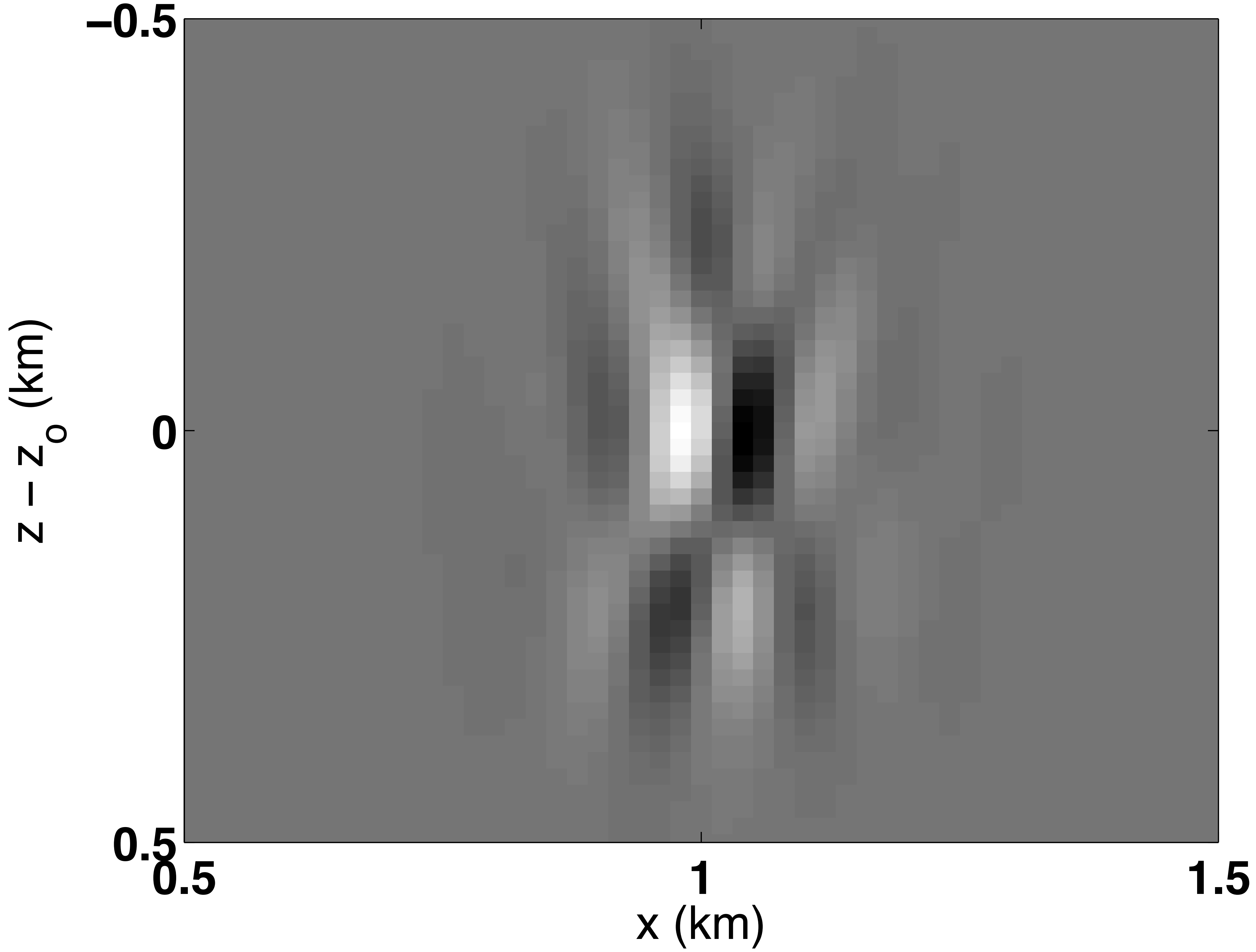


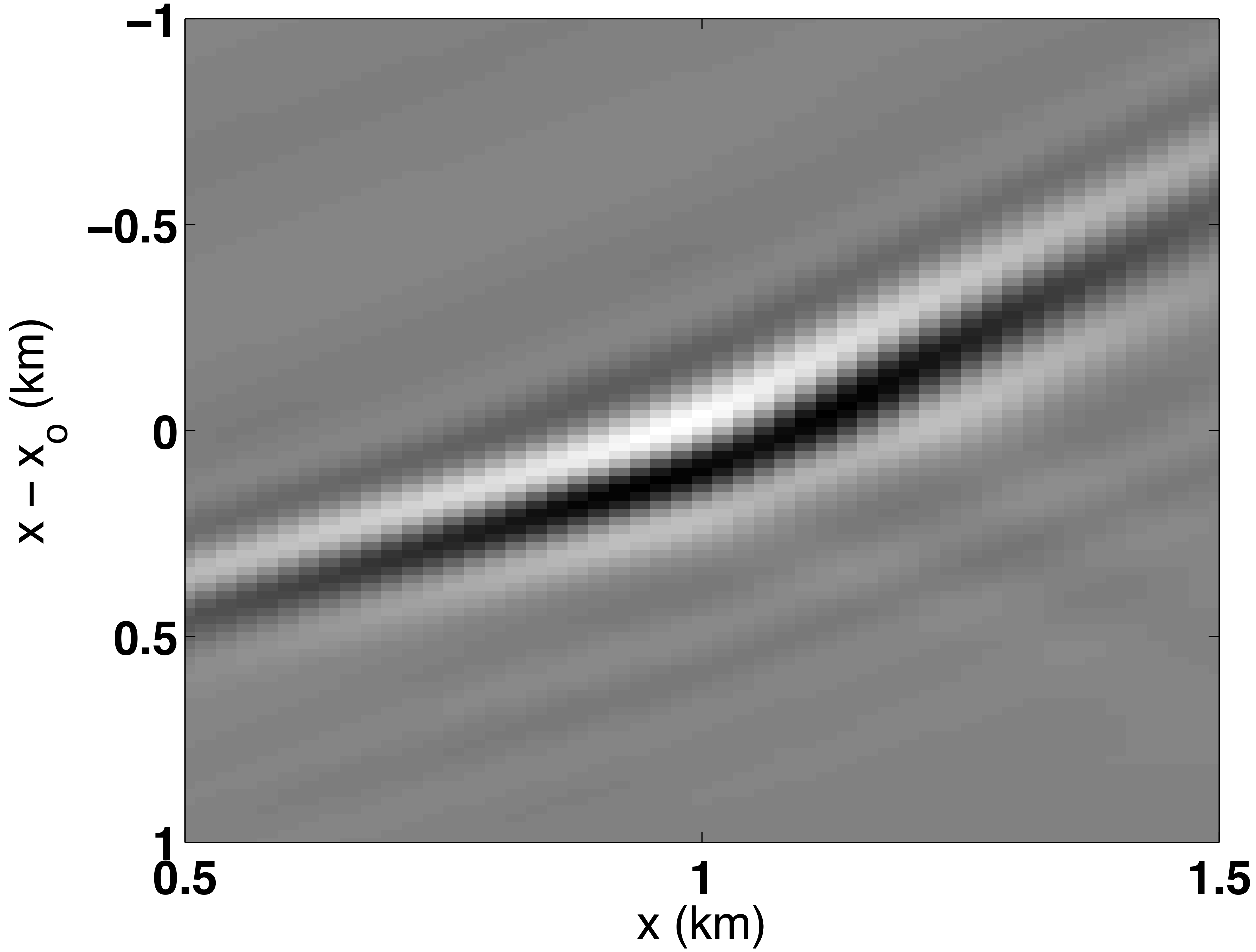




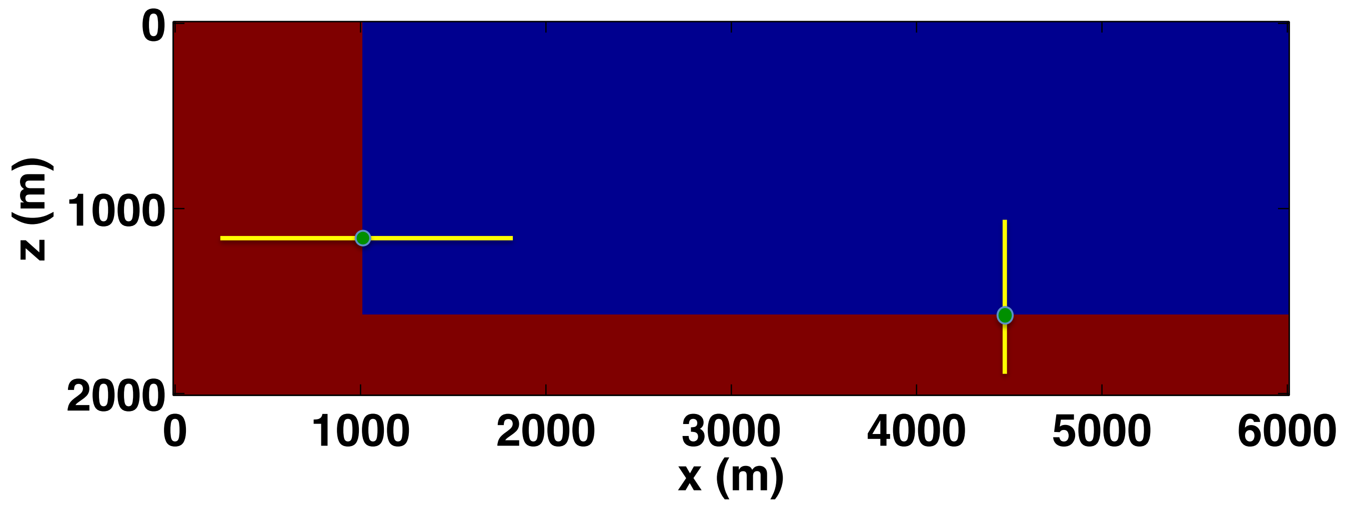
Common image gathers
Vertical reflector

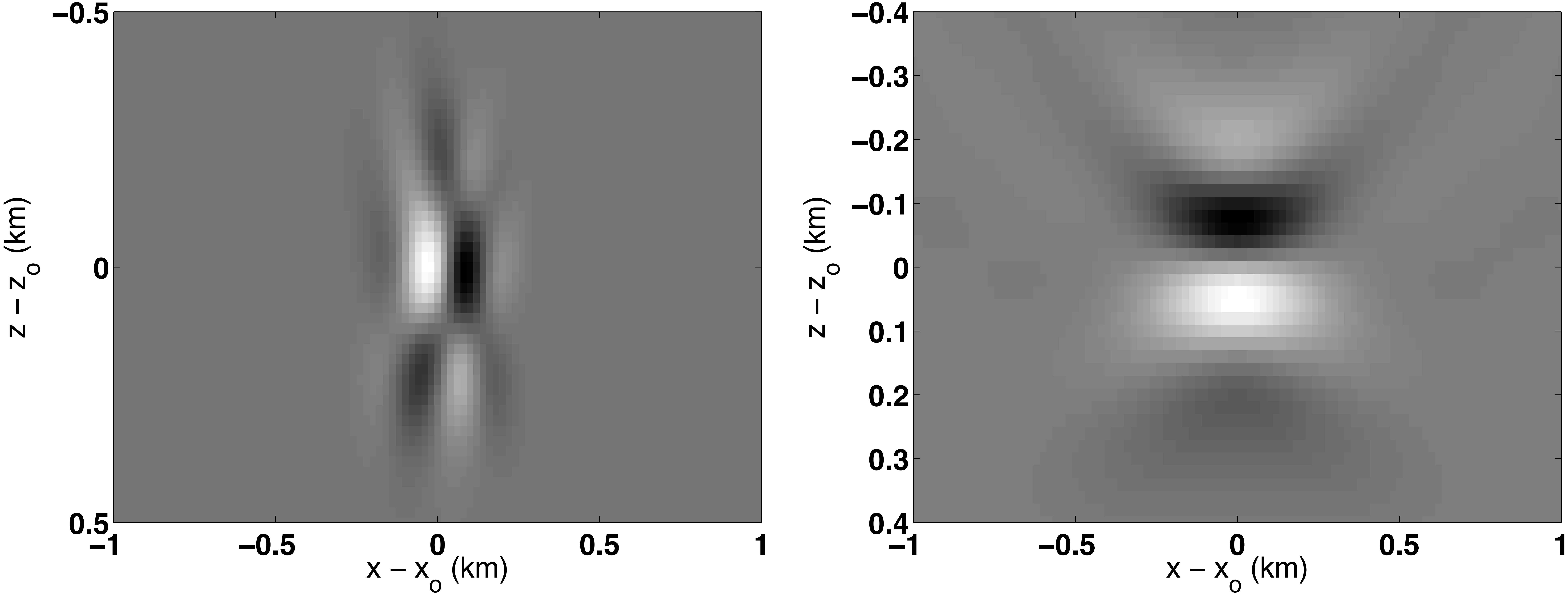






Common image point gathers





Fast WEMVA w/ *randomized* probing

- Measure the error in some norm

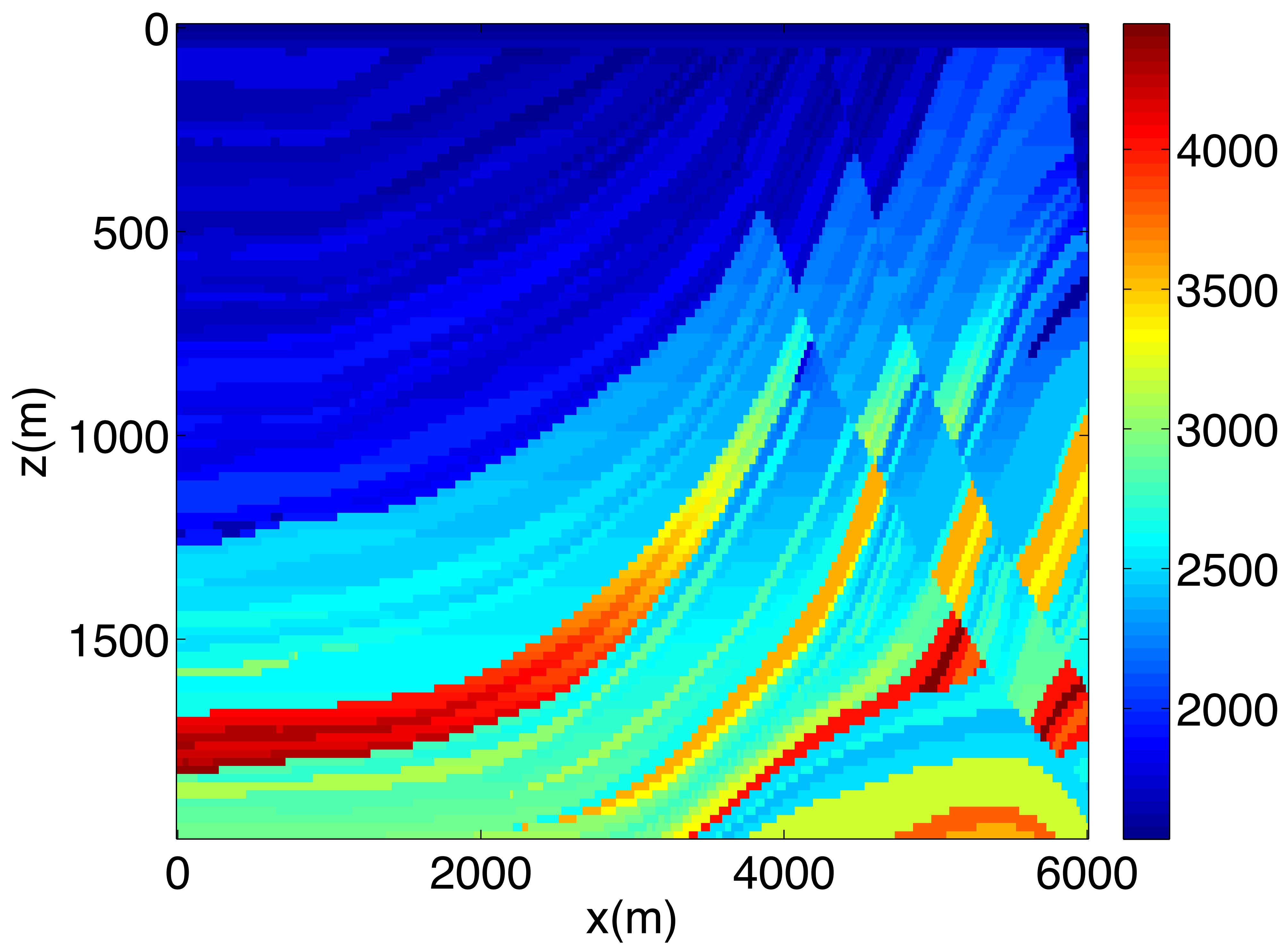
$$\min_{\mathbf{m}} ||E(\mathbf{m})\mathrm{diag}(\mathbf{x}) - \mathrm{diag}(\mathbf{x})E(\mathbf{m})||_?^2$$

- The *Frobenius* norm can be estimated via randomized trace estimation : Avron and Toledo, 2011

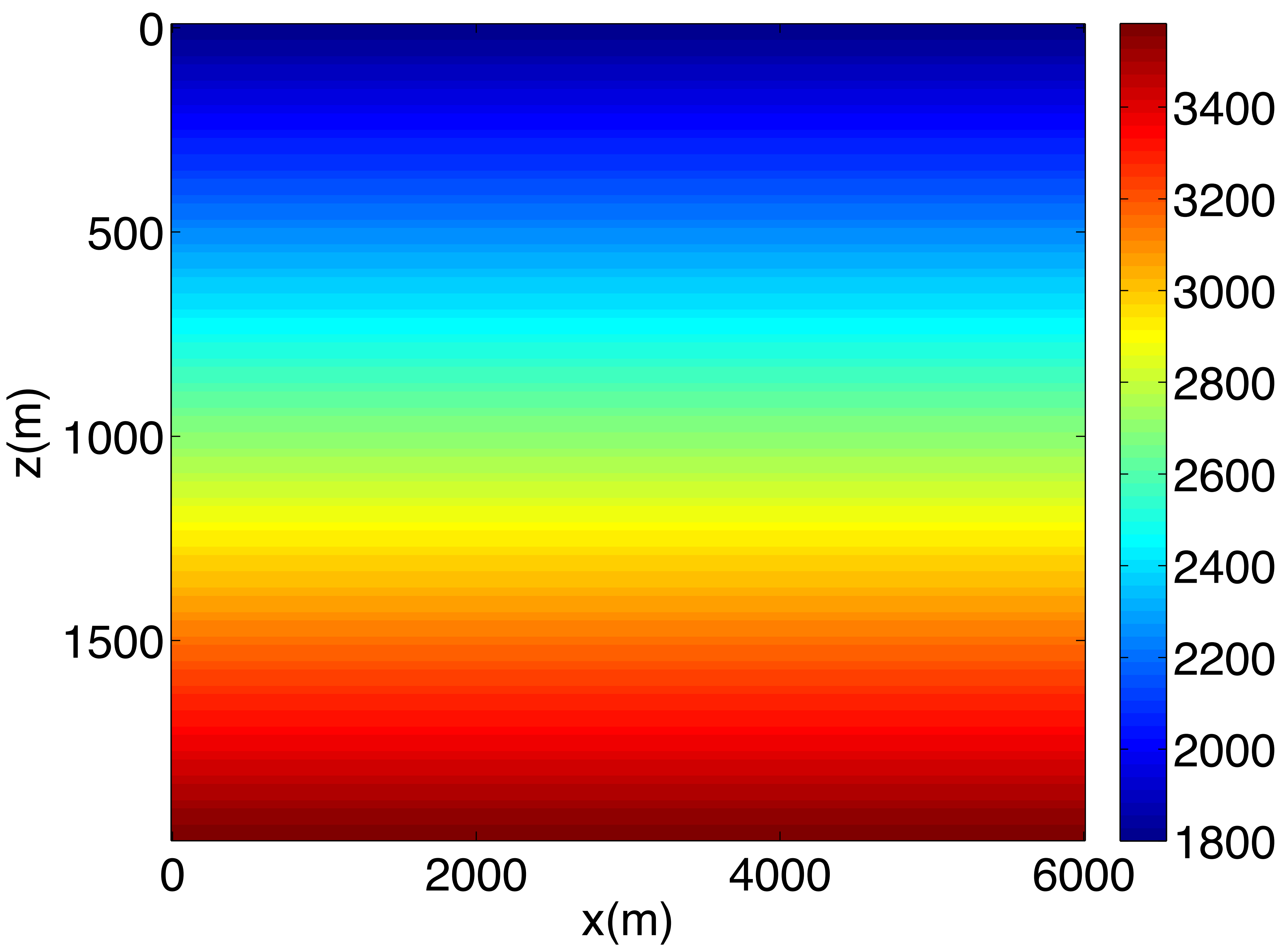
$$||A||_F^2 = \mathrm{trace}(A^T A)$$
$$\approx \sum_{i=1}^{K} \mathbf{w}_i^T A^T A \mathbf{w}_i = \sum_{i=1}^{K} ||A\mathbf{w}_i||_2^2$$

where $\sum_{i=1}^{K} \mathbf{w}_i \mathbf{w}_i^T \approx I$

Randomized probing
[reflection]



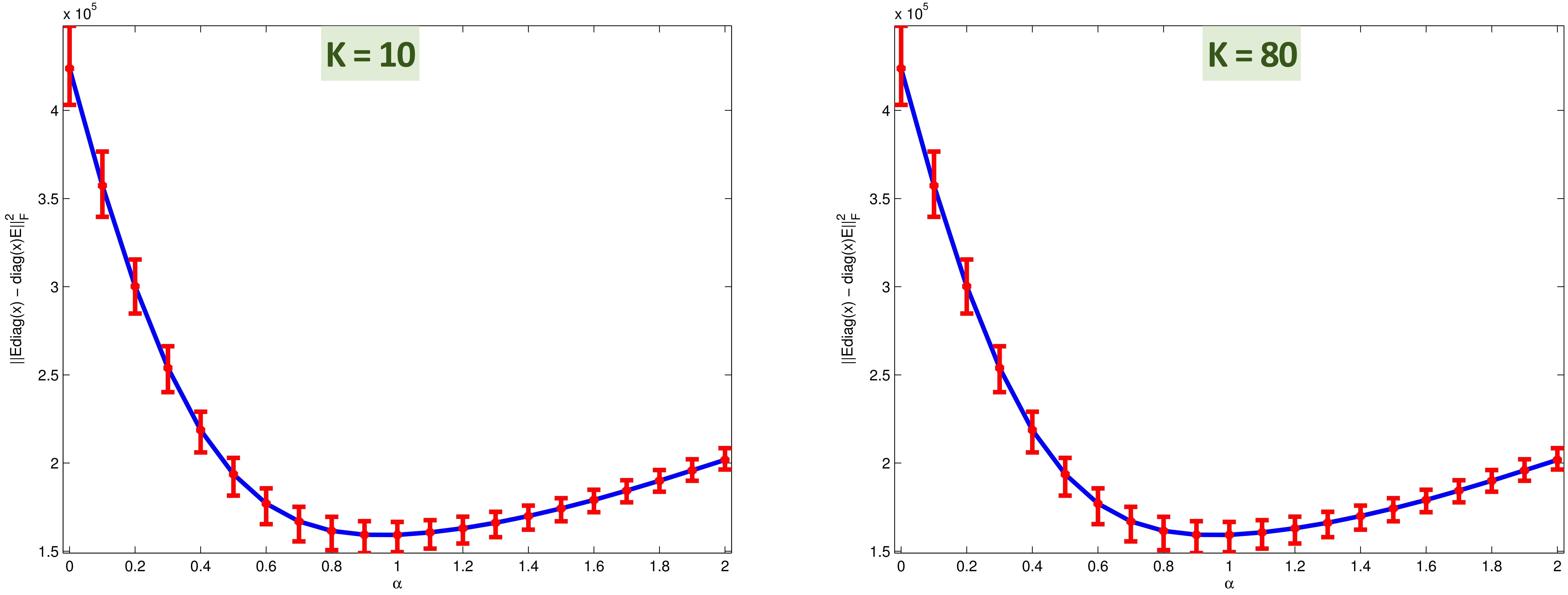
true model



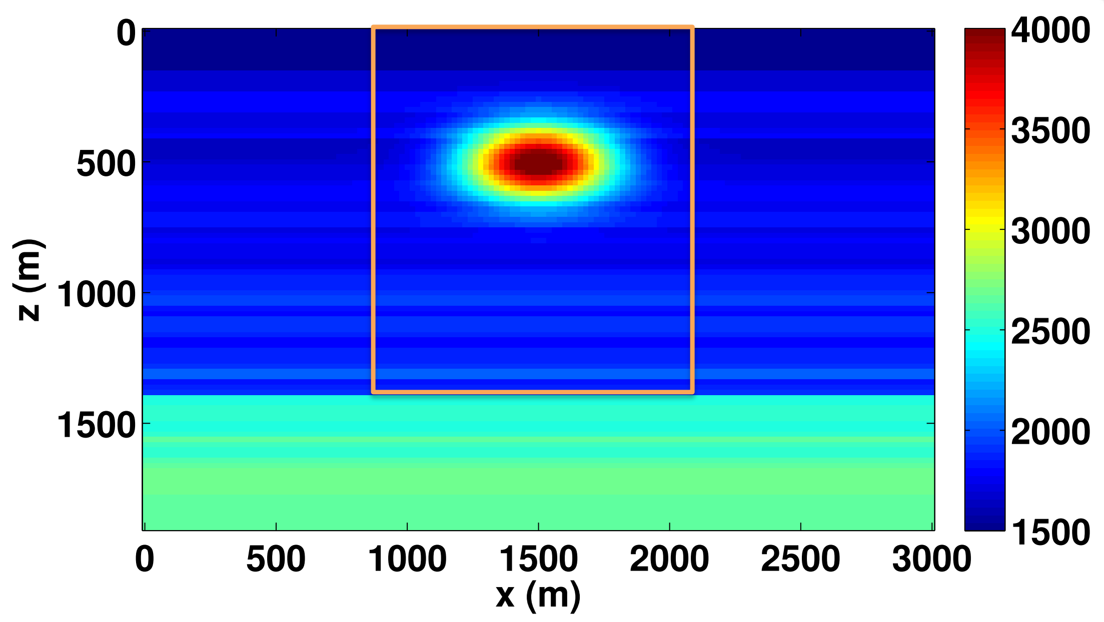
initial model

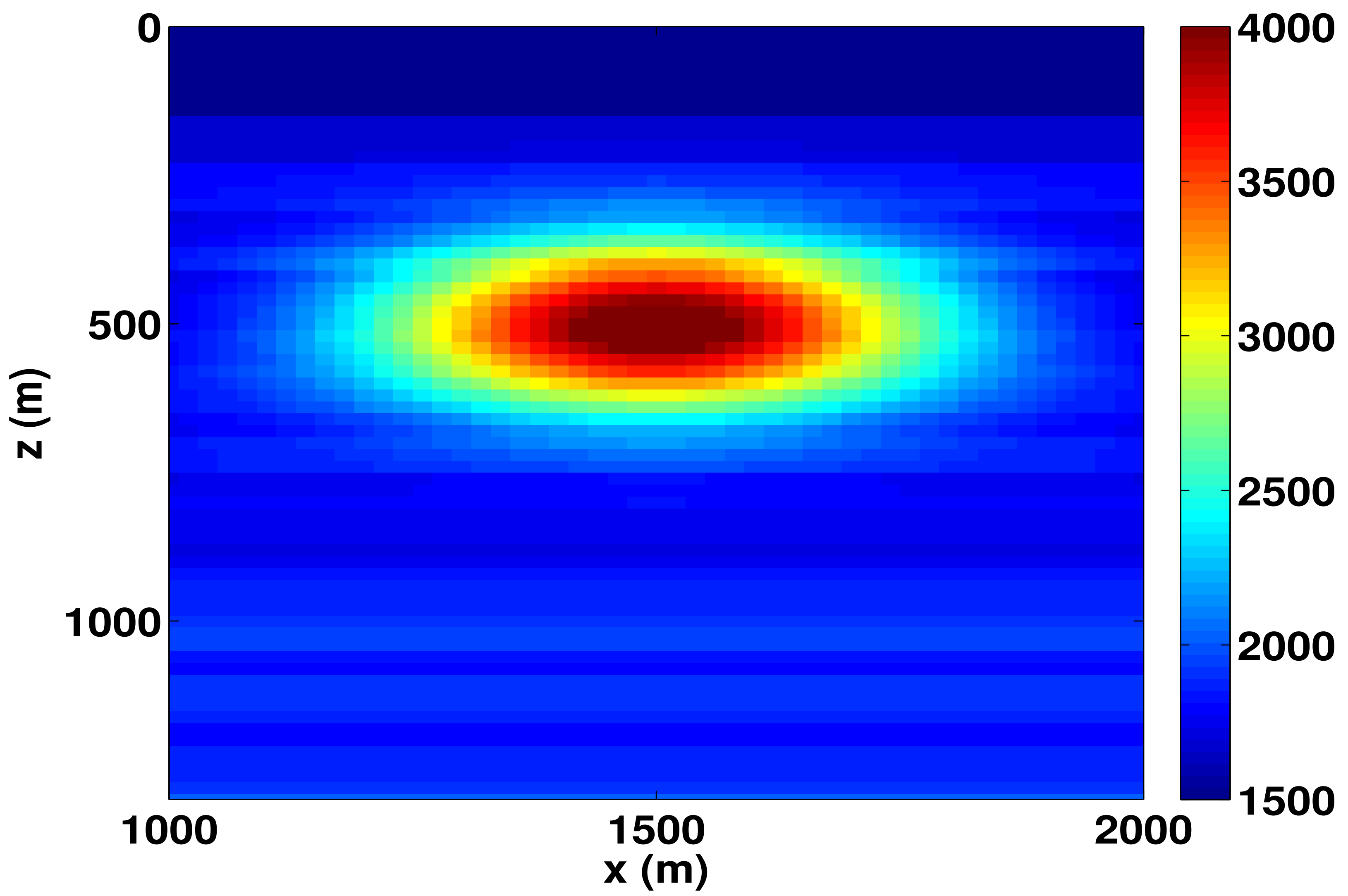
Randomized probing
[reflection]

- Exact
- Error bar of approximated objective function
  (different color represents different random realization)

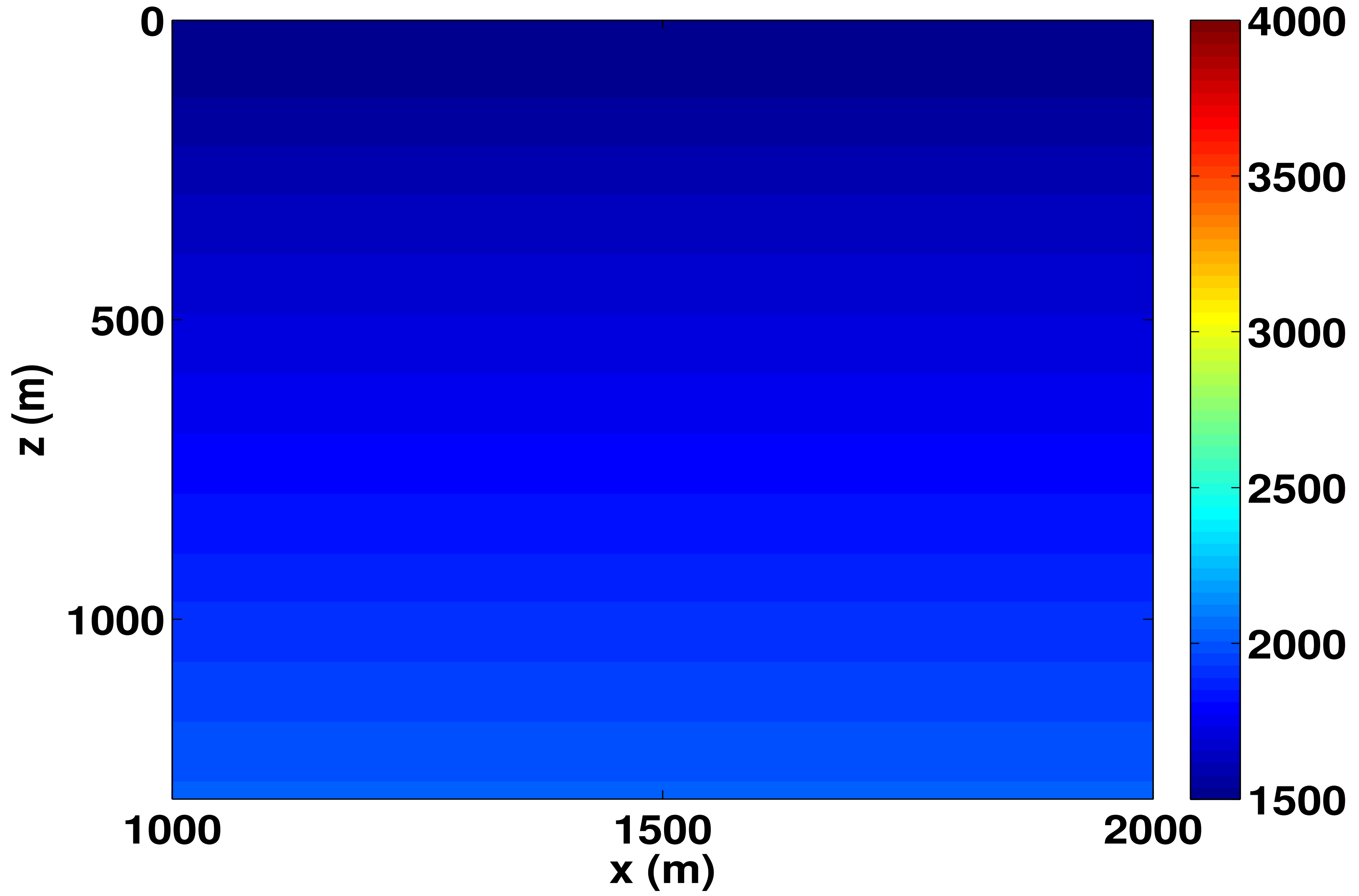


Lens Model

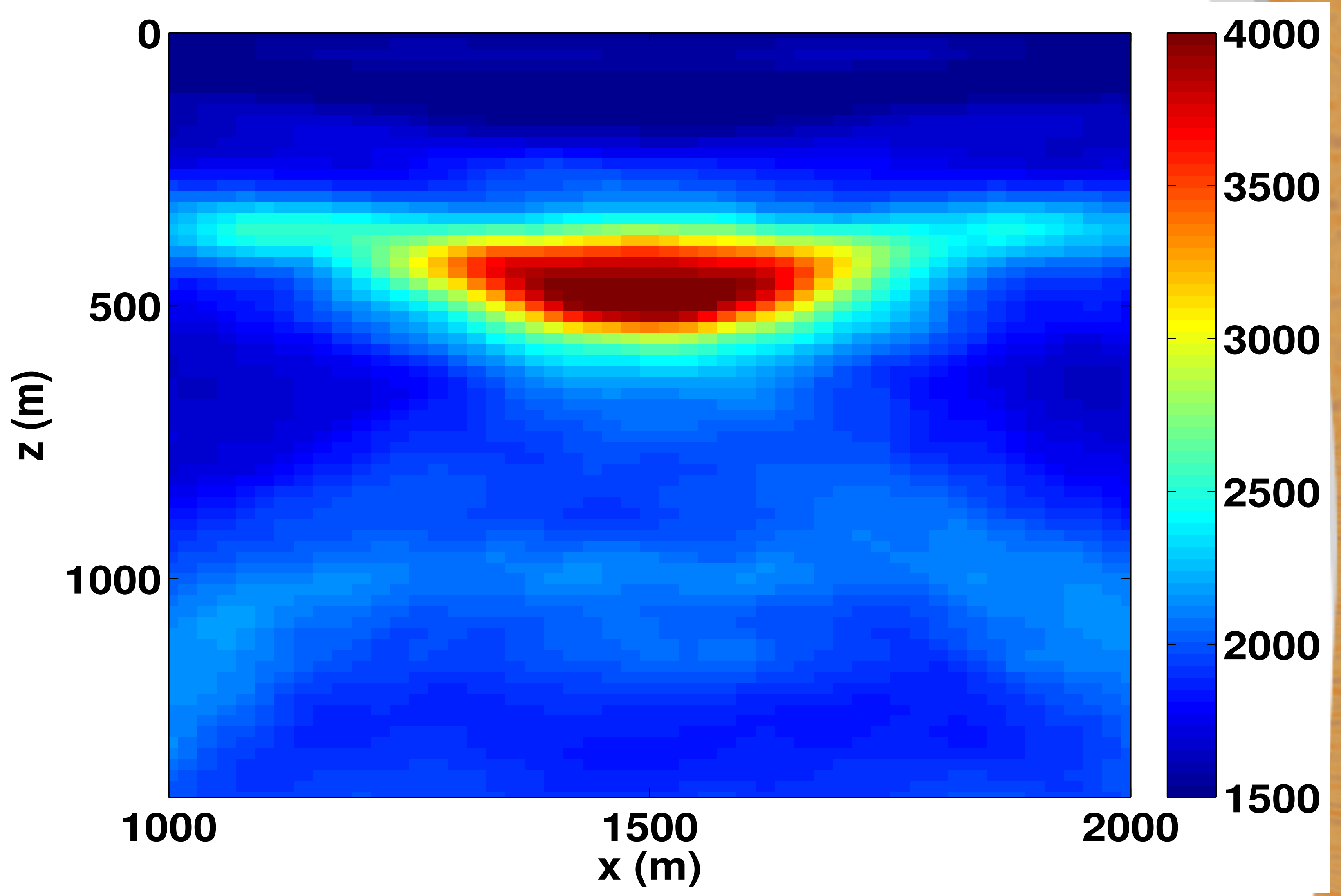




True model

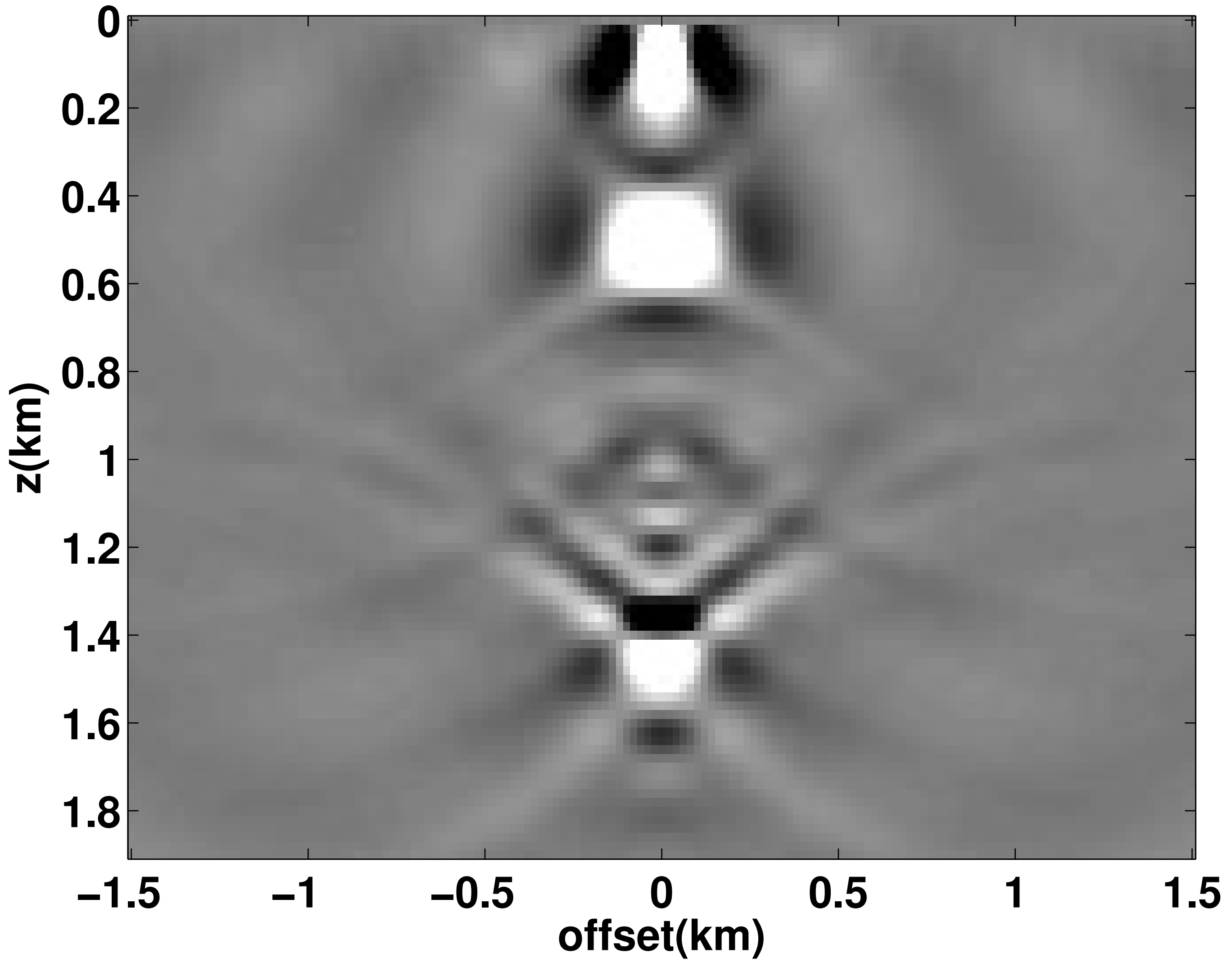


Initial model

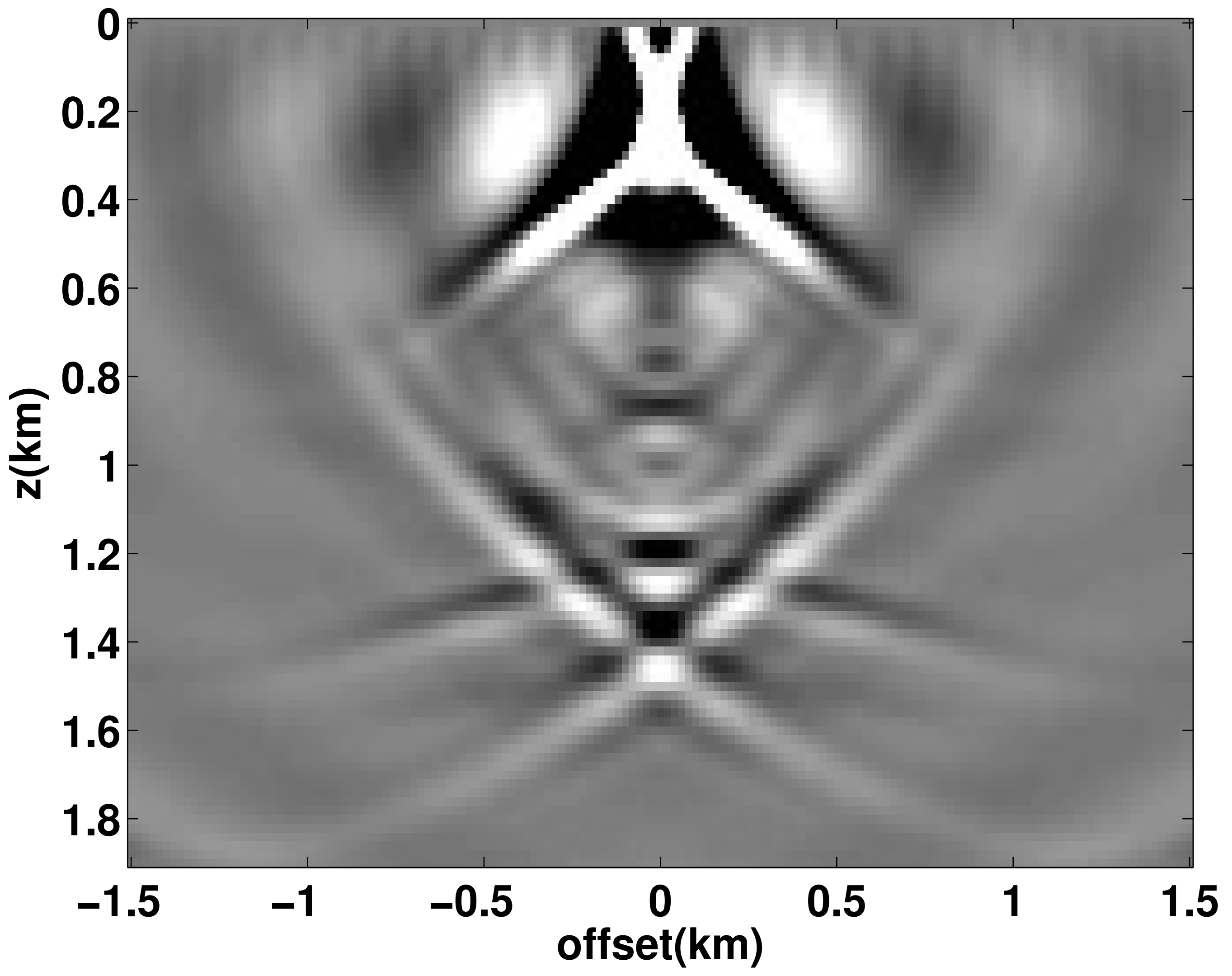


WEMVA

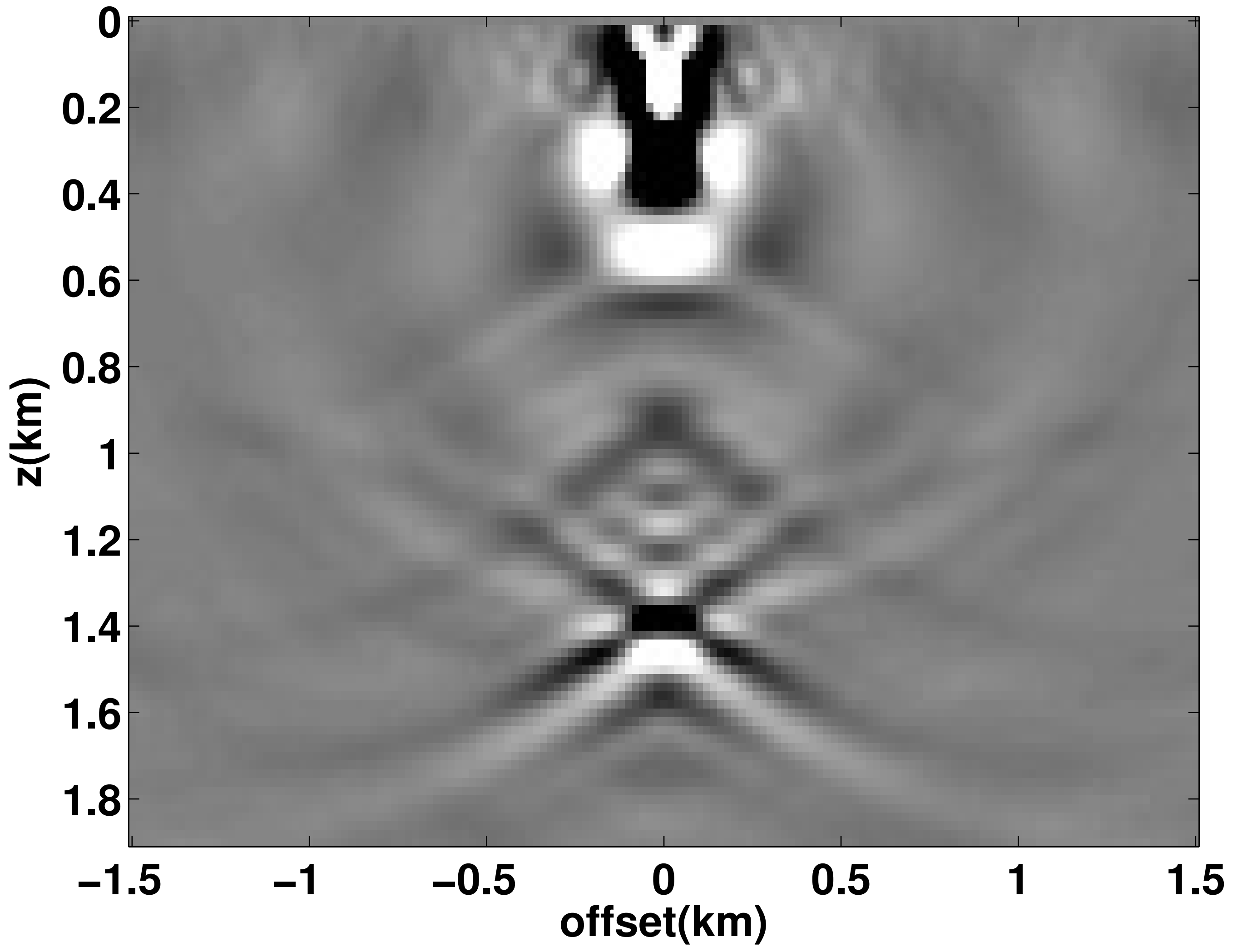
Lens Model
[image gathers]



True model

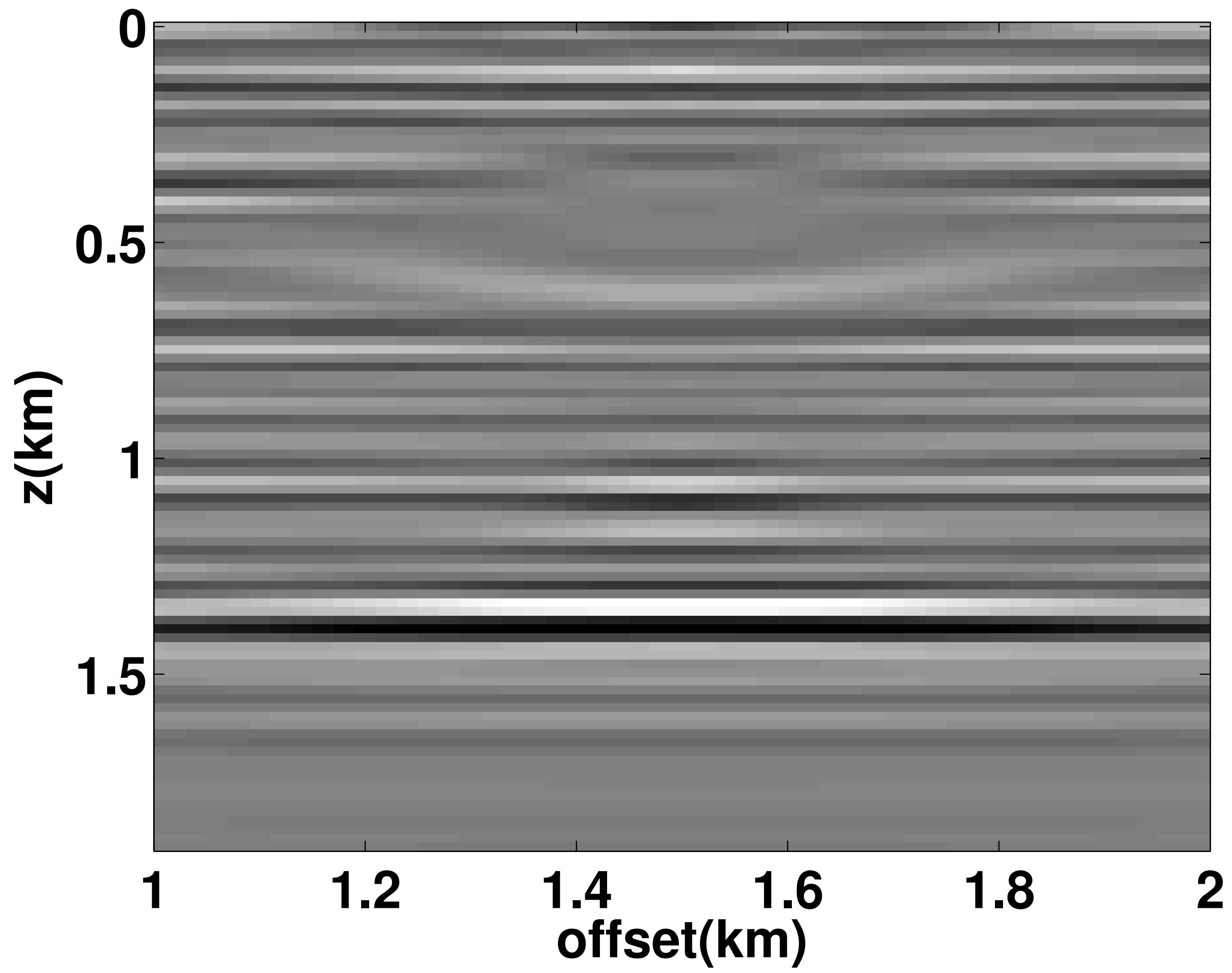


Initial model

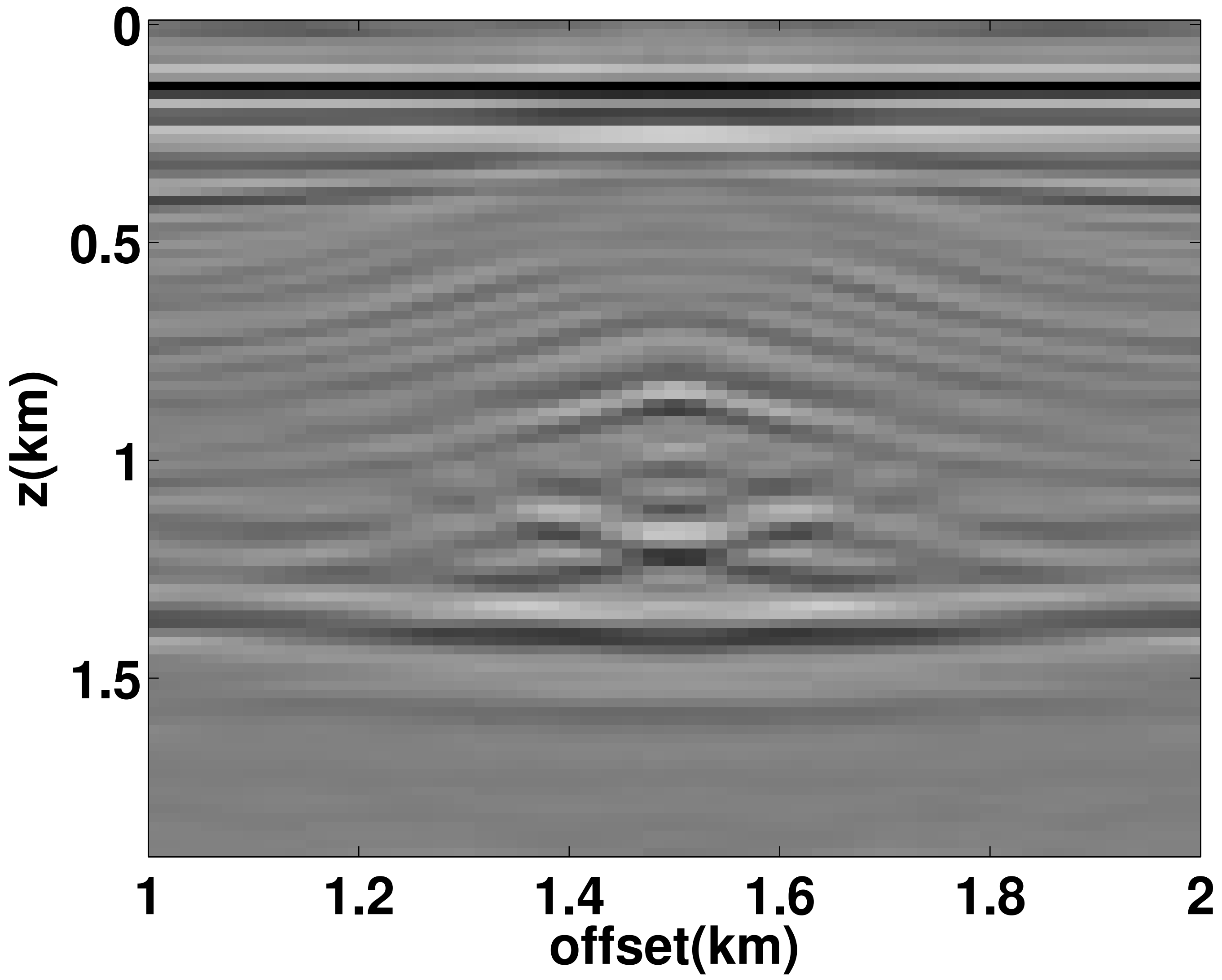


WEMVA

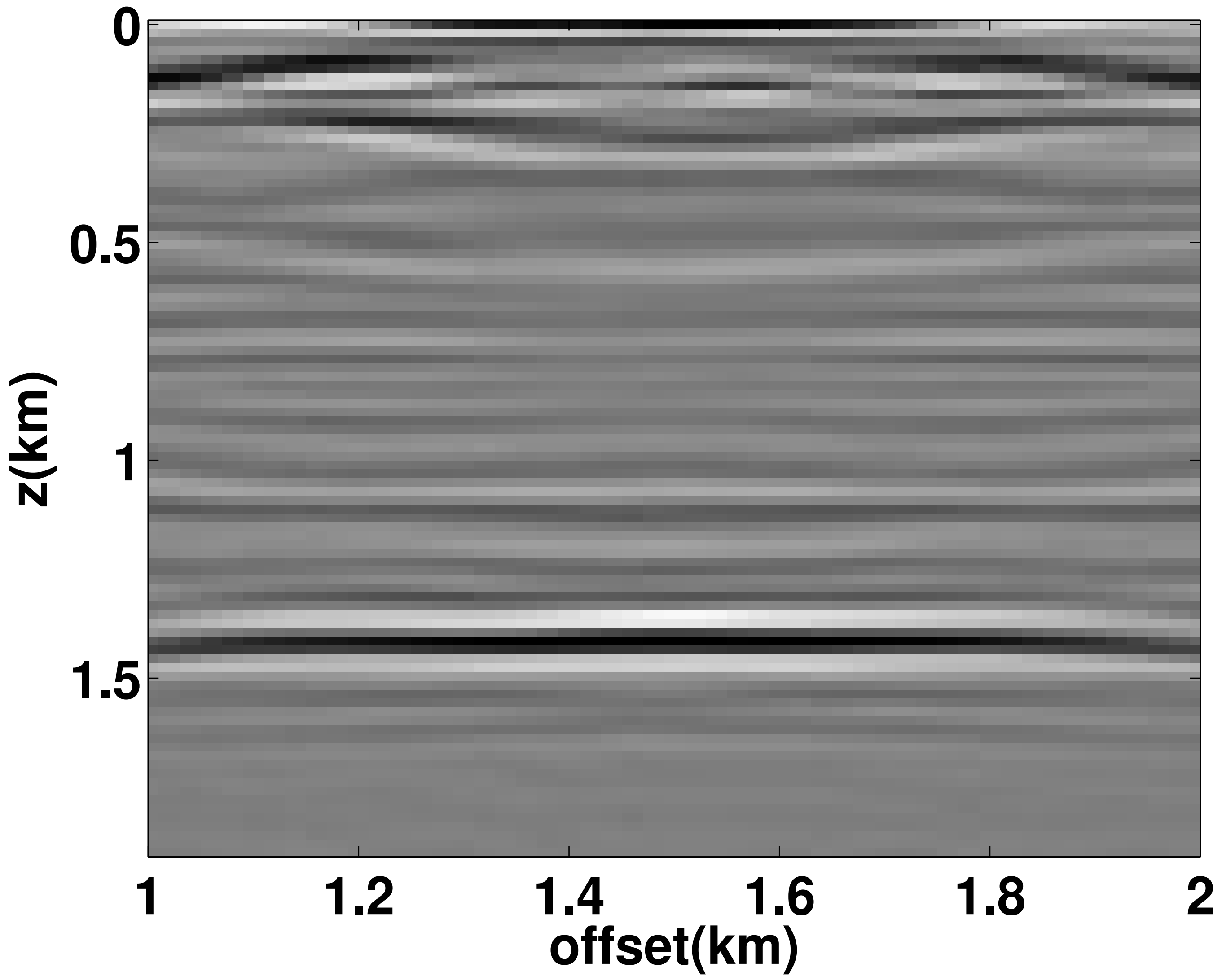
Least-squares RTM Images



True model



Initial model



WEMVA

Conclusions

- *probings* allows us to get *offset* information for *all sub-surface direction*

- *Prior knowledge of geology is not required*

- *randomized* trace estimation allows us to *compute* WEMVA objective cheaply

Image gathers w/ surface-related multiples

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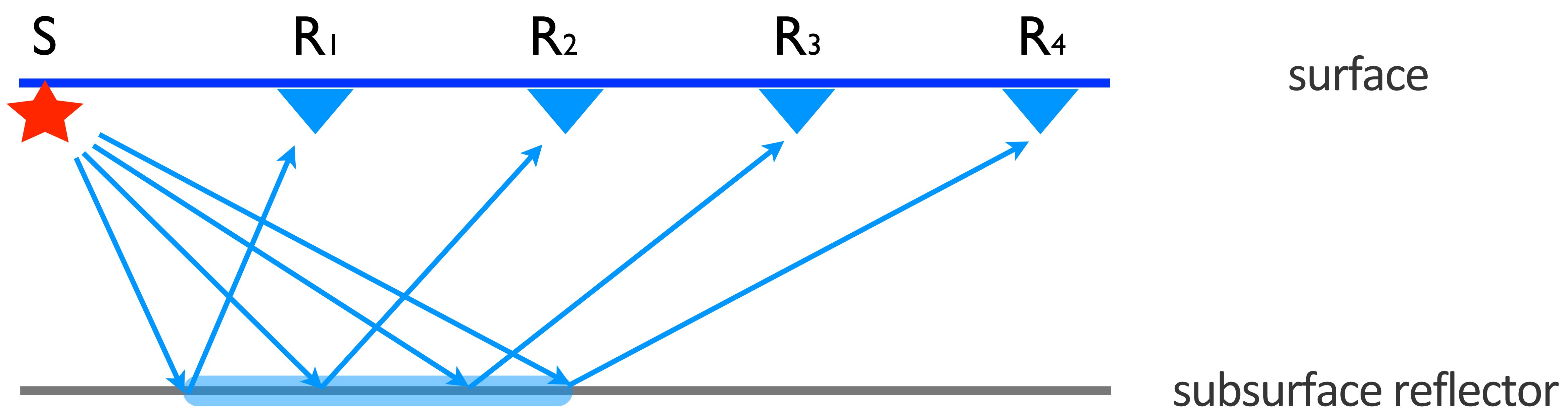
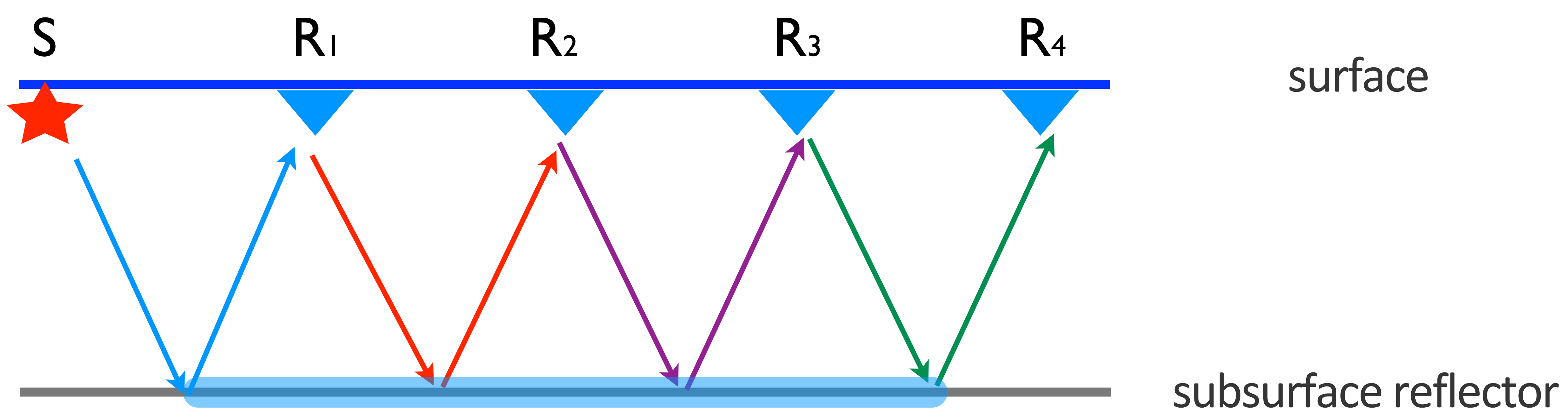
Why need multiples ?

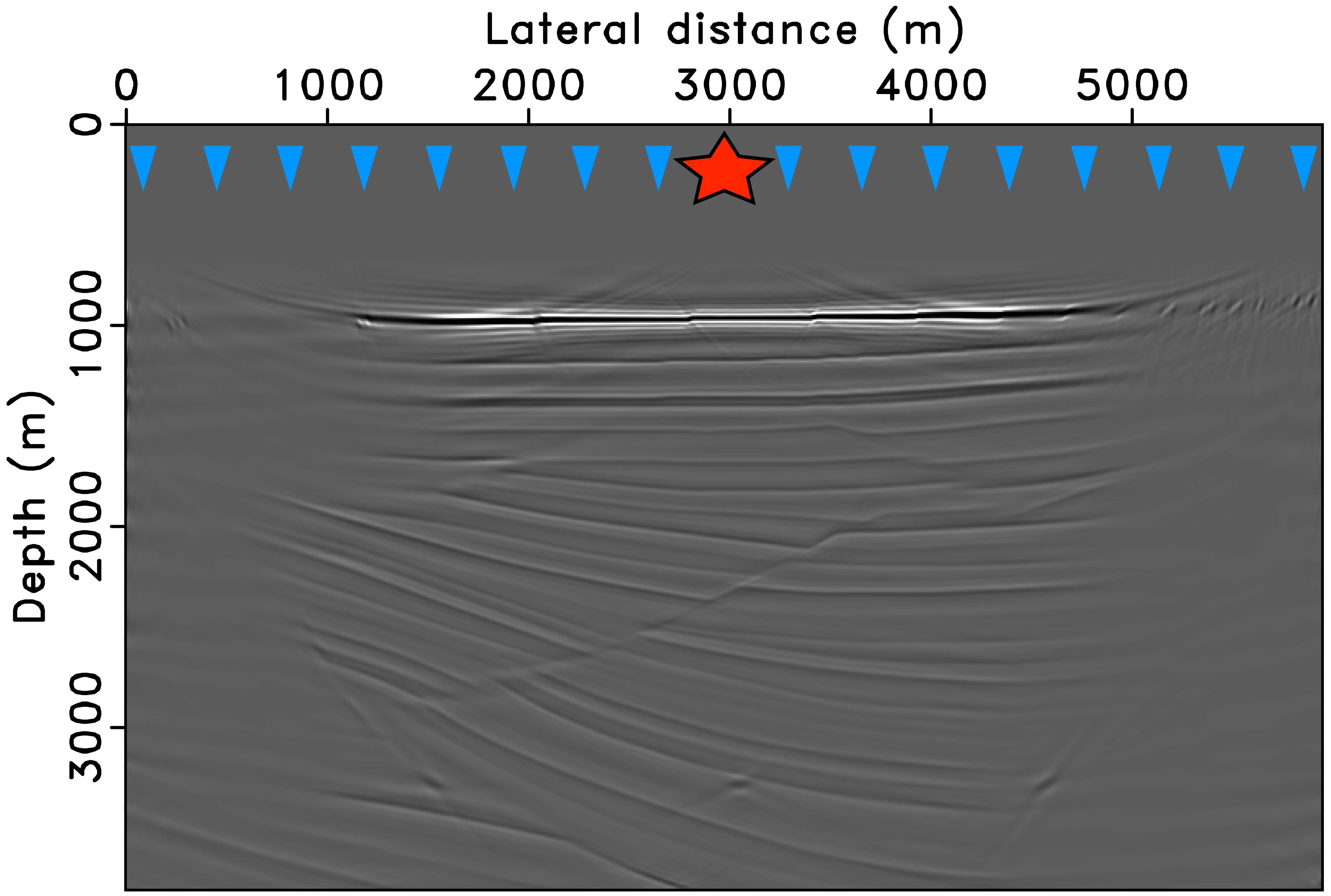


illumination by primaries

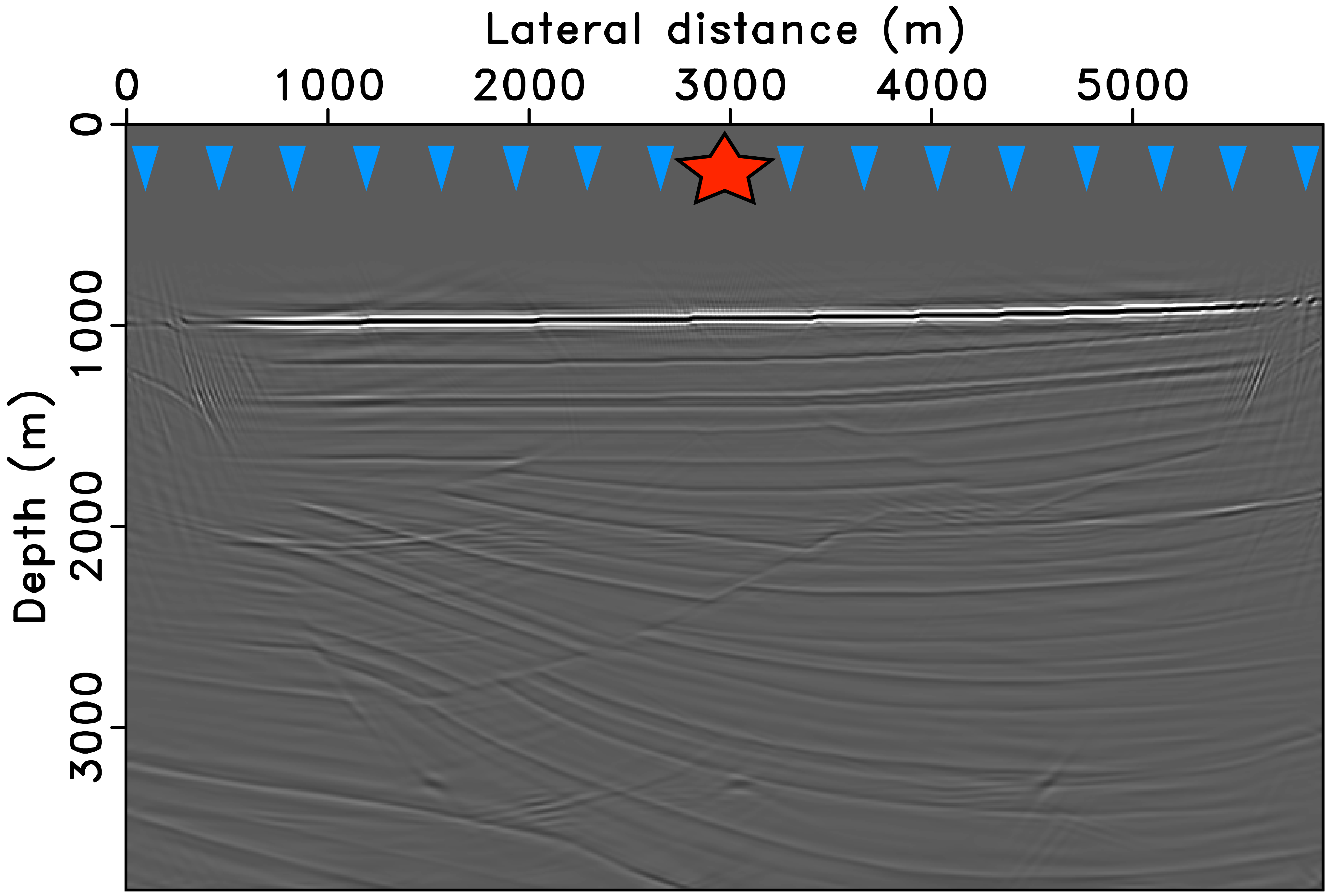


illumination by multiples

Tu and Herrmann, 2015



Least-squares imaging: *primary-only* shot gather



Least-squares imaging: *multiple-only* shot gather

Motivation

- *Leverage benefits of SRME*
  - highly accurate data-driven multiple prediction
- *All in one go* method
  - we combine SRME within the extended imaging condition

Extended imaging with multiples

$$\widetilde{E} = EW = H^{-1} P_s^T (Q - P) P^* P_r H^{-1} W$$

where

$(Q - P)$: areal source

P : total upgoing wavefield

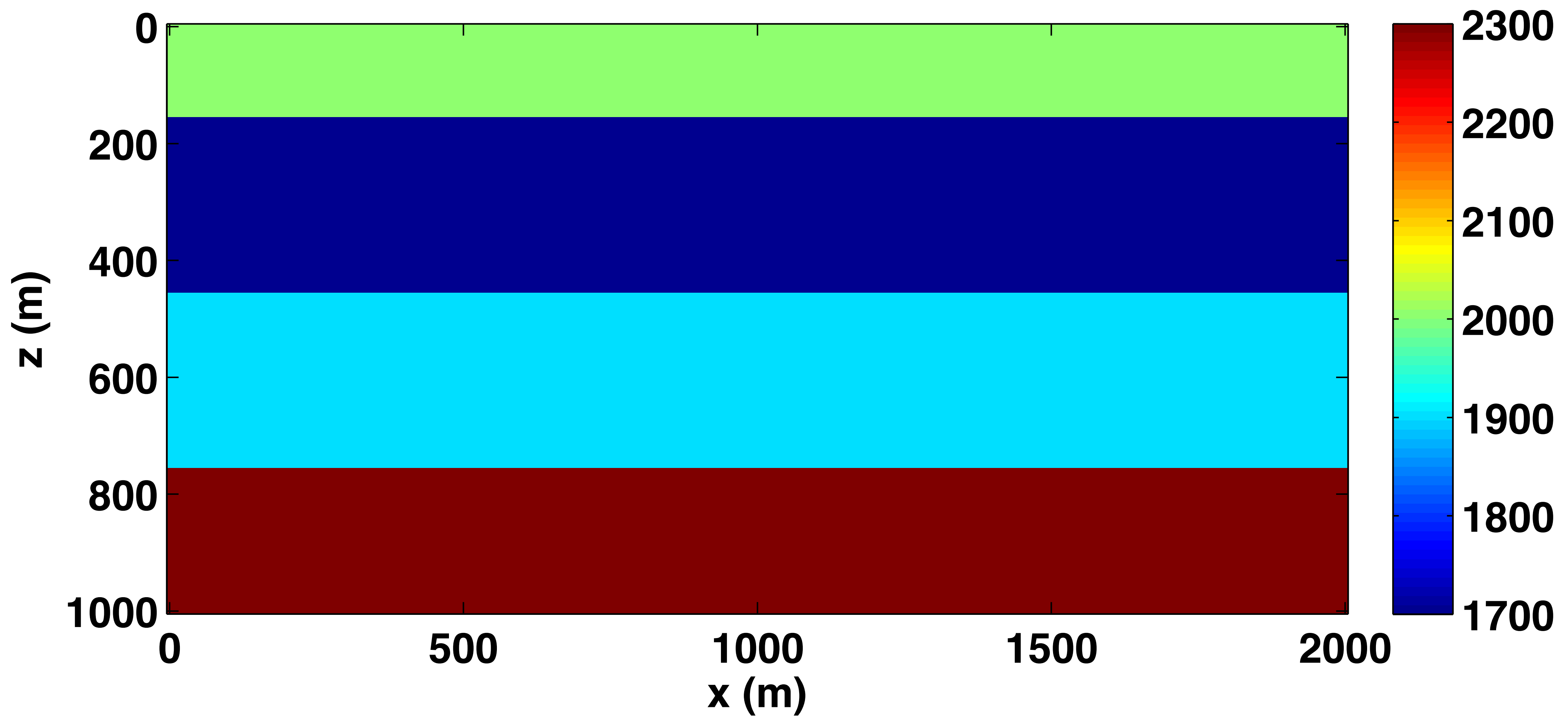
Least-square extended imaging

$$\underset{\widetilde{E}}{\text{minimize}} \quad \frac{1}{2}\|\mathcal{F}(\widetilde{E}) - P\|_F^2,$$

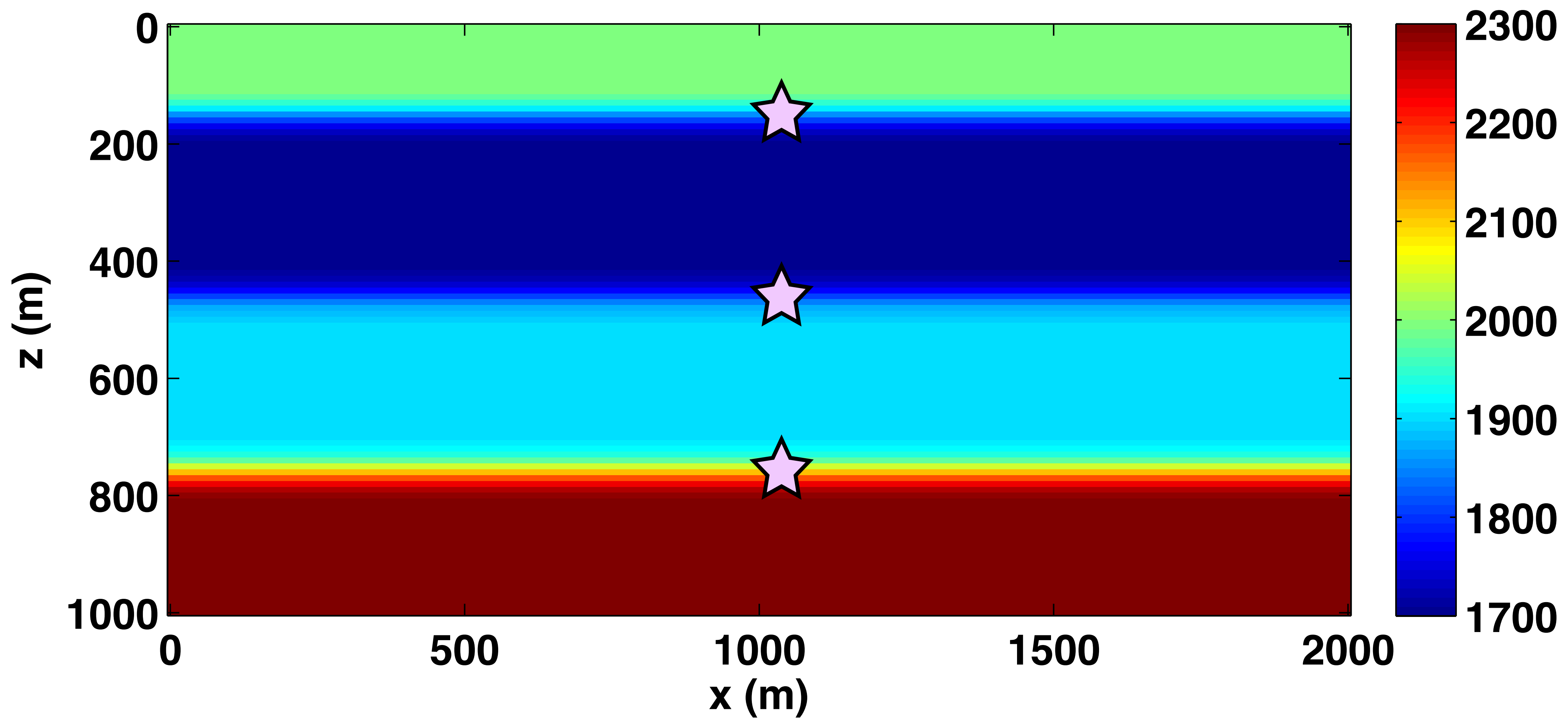
where

$$\mathcal{F}(\widetilde{E}) = P_r H^{-1} E W^T H^{-1} P_s^T (Q - P),$$

Velocity model

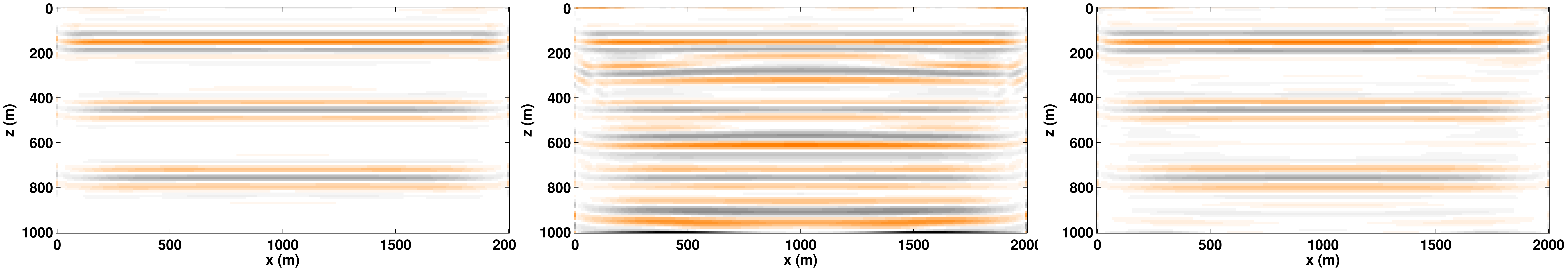


True model



Initial model

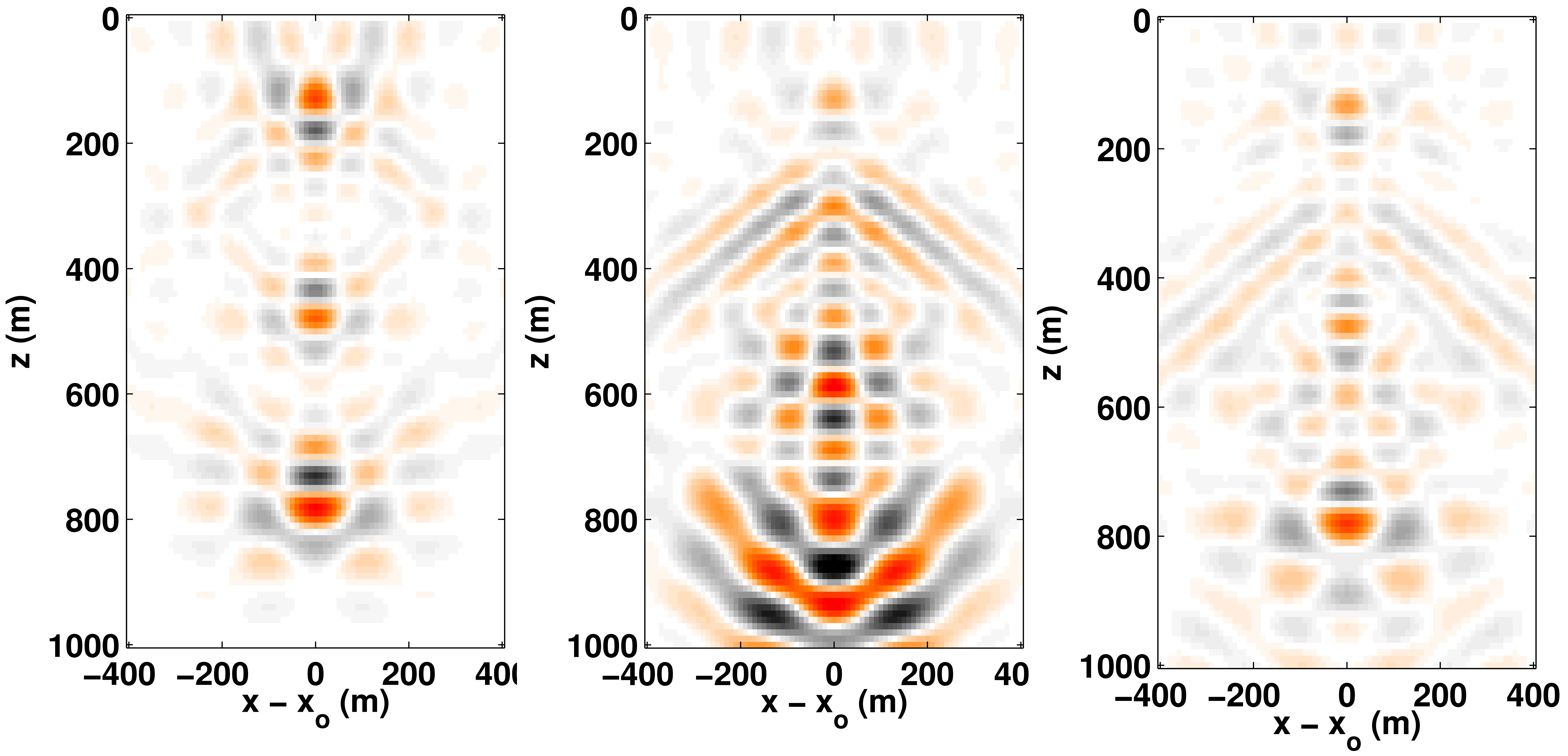
Least-squares RTM images



Primary only

Primary + multiples **w/o** areal sources

Primary + multiples **with** areal sources



Primary only

Primary + multiples
w/o areal sources

Primary + multiples
with areal sources

Conclusions

Multiples provide extra illumination that can complement primaries.

Multiples can be used with primaries to form subsurface image gathers via least-squares inversion.

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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