Rank Minimization via Alternating Optimization

Abstract

We consider an alternating optimization scheme for low-rank matrix completion and discuss the applications to large scale wave field reconstruction.

Low-Rank Matrix Completion

Given a rank r matrix, \( X \in \mathbb{R}^{n \times m} \) we exploit its low dimensional structure to recover \( X \) from limited and noisy samples via

\[
\begin{align*}
\min_Y & \|Y\|_* \quad \text{subject to} \quad \|P_{H}(Y) - b\|_F \leq \epsilon, \\
\text{Since} \quad \|X\|_* &= \min_{L,R} \frac{1}{2} (\|L\|_F^2 + \|R\|_F^2) \quad \text{we equivalently solve} \\
\min_{L,R} & \frac{1}{2} (\|L\|_F^2 + \|R\|_F^2) \quad \text{subject to} \quad \|P_{H}(LR^H) - b\|_F \leq \epsilon.
\end{align*}
\]

We compare our Alternating Optimization approach with LR-BPDN in [1] on a 4001x4001 matrix from a 7 Hz slice of the Gulf Of Mexico dataset. Interpolation was completed in 6 minutes on 1120-core SGI cluster OPTIMUM.

Difference plot of alternating optimization recovery

Recovery via LR-BPDN

SNR = .9 dB

Difference plot of LR-BPDN

Numerical Experiments

Algorithm

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Require: \( P_H, b, r, T \)

1. Initialize: \( L^0 \) to be the top-\( r \) singular vectors of \( b \)

2. for \( t = 0 \) to \( T - 1 \) do

3. \( R^{t+1} = \min_{R \in \mathbb{R}^{m \times m}} \frac{1}{2} ||R||_F^2 \quad \text{s.t.} \quad \|P_H(L^t R^{t+1}) - b\|_F \leq \epsilon 

4. \( L^{t+1} = \min_{L \in \mathbb{R}^{n \times n}} \frac{1}{2} ||L||_F^2 \quad \text{s.t.} \quad \|P_H(L^t R^{t+1}) - b\|_F \leq \epsilon 

5. end for

6. return \( (X^T = L^T R^{t+1})^H \)

Steps 3 and 4 are achieved by solving a series of convex LASSO sub problems [3].

Conclusion

By alternating our optimization between matrix factors and implementing the Pareto curve approach (SPGI1 [3]) we solve a sequence of convex problems that do not require expensive SVD computations and penalize the rank. This results in a tractable procedure, where our experiments highlight the reconstruction SNR improvements in cases where our data matrix is large and highly subsampled (90% missing sources) when compared to LR-BPDN (non-alternating) implementation.

References


5. Raj Kumar et al [2014], SVD-free low-rank matrix factorization : wavefield reconstruction via jittered subsampling and reciprocity. EAGE.


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By alternating our optimization between matrix factors and implementing the Pareto curve approach (SPGI1 [3]) we solve a sequence of convex problems that do not require expensive SVD computations and penalize the rank. This results in a tractable procedure, where our experiments highlight the reconstruction SNR improvements in cases where our data matrix is large and highly subsampled (90% missing sources) when compared to LR-BPDN (non-alternating) implementation.