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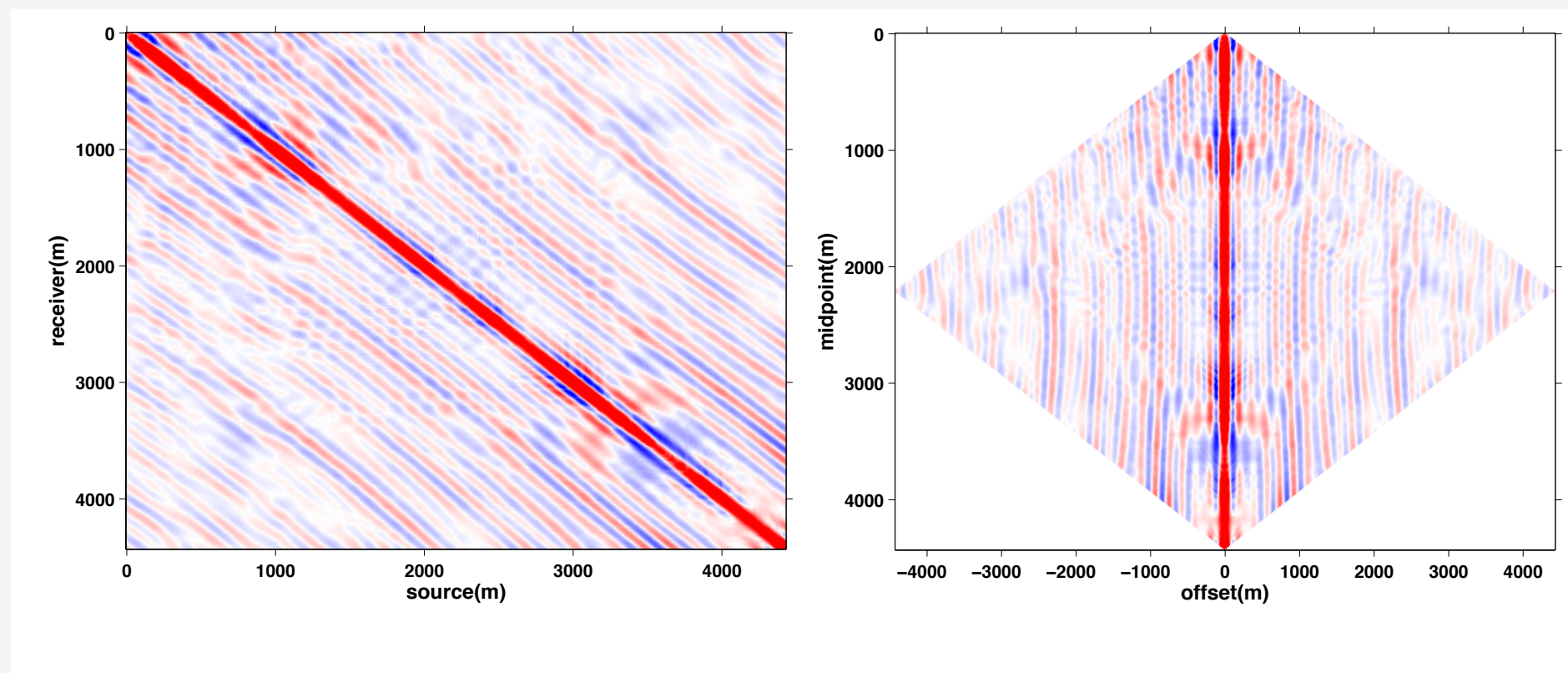
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Matrix Completion on Unstructured Grids: 2-D Seismic Data Regularization and Interpolation



Introduction

- acquisition challenges
- need fully sampled data
- regularization & interpolation
- exploit low-rank structure of seismic data



Quantifying regularization error

Let $\mathbf{X}_r \in \mathbb{C}^{n \times m}$, $\mathbf{b} = \mathbf{RM}(\mathbf{X}_{ir}) + e$ with $\|e\| \leq \eta$, and $\|\mathbf{N}(\mathbf{X}_r) - \mathbf{X}_{ir}\|_F \leq \epsilon$.
Let $\tilde{\mathbf{X}}$ be the solution of $\text{BPDN}_{\epsilon+\sigma}$, then

$$\|\mathbf{X}_r - \tilde{\mathbf{X}}\|_F \leq \frac{C_1}{\sqrt{k}} \underbrace{\sum_{i=k+1}^l \sigma_i(\mathbf{X}_r)}_{\text{Rank approximation}} + \underbrace{C_2 \epsilon}_{\text{Regularization}} + \underbrace{C_2 \eta}_{\text{noise}}$$

where

$$l = \min\{n, m\}$$

$$C_1 \text{ and } C_2 > 0$$



[†] John "Ernie" Esser (May 19th 1980 - March 8th 2015)

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Matrix Completion

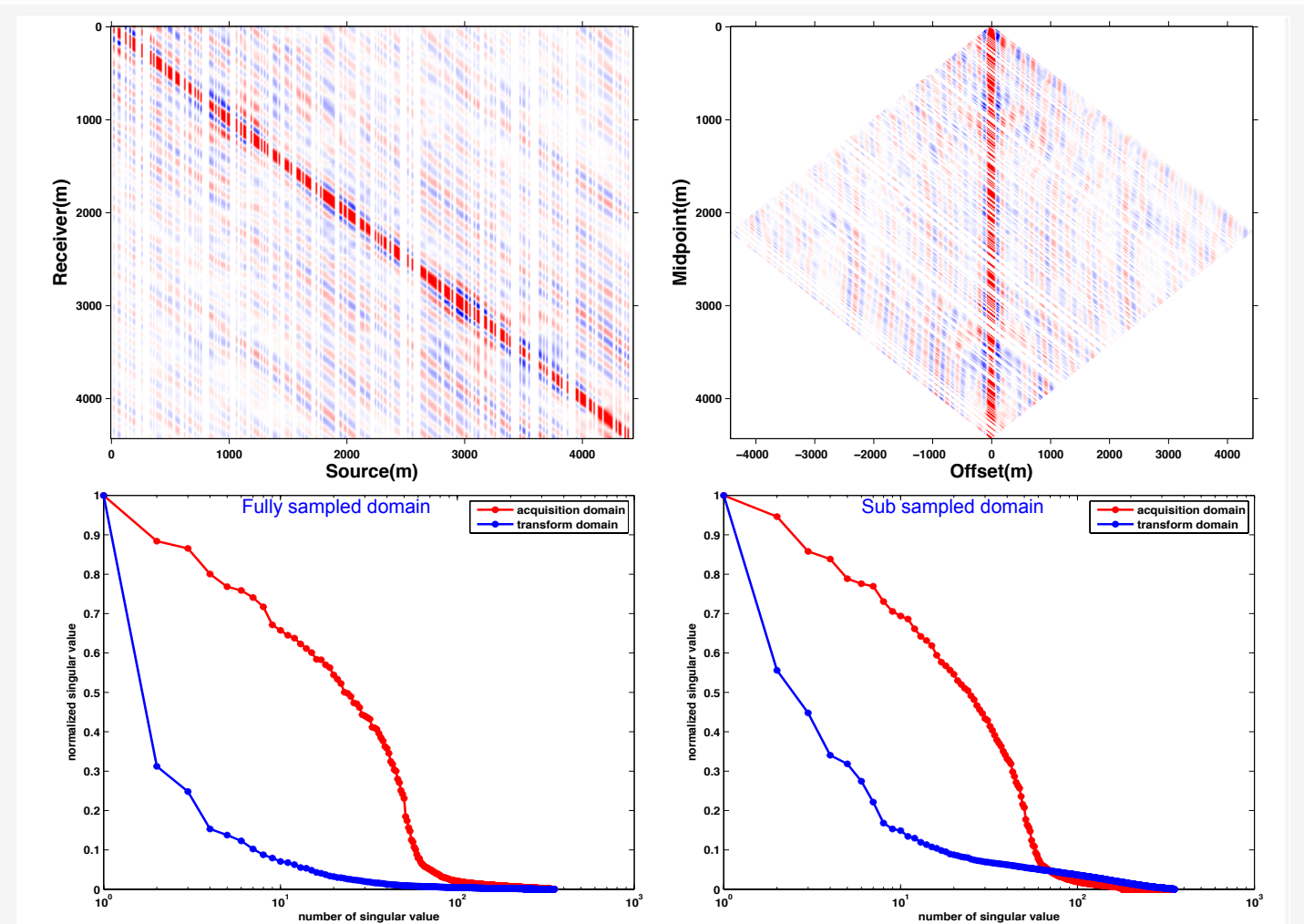
signal structure

- Low rank/fast decay of singular values

sampling scheme

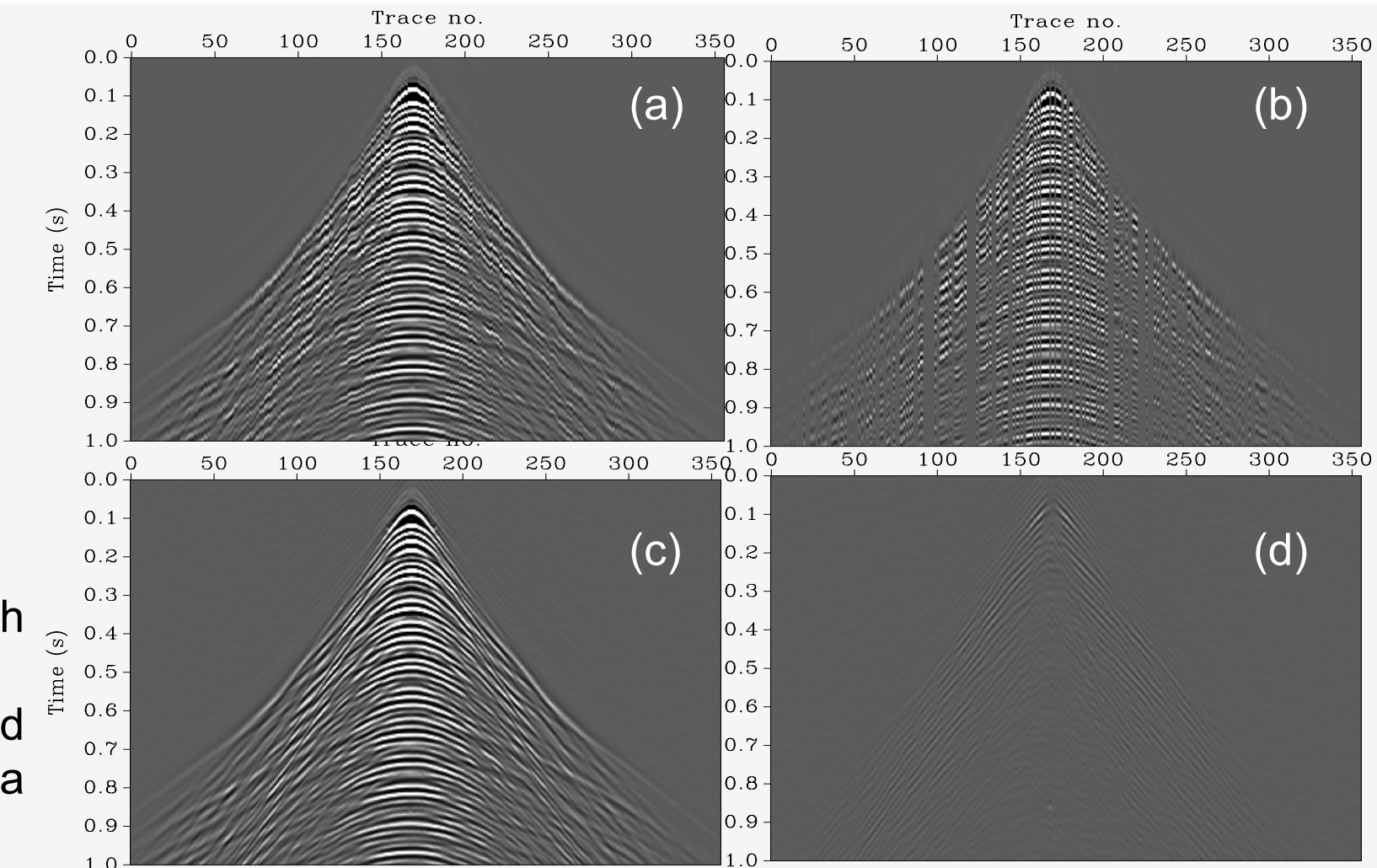
- missing data increase rank in "transform domain"

recovery using rank penalization scheme



Regularization & Interpolation

Figure 2: Regularization and Interpolation. Ground truth at (a) irregular grid, (b) and with missing traces. (c, d) Recovery and difference using proposed rank-minimization on unstructured grids with a SNR of 19.3 dB.

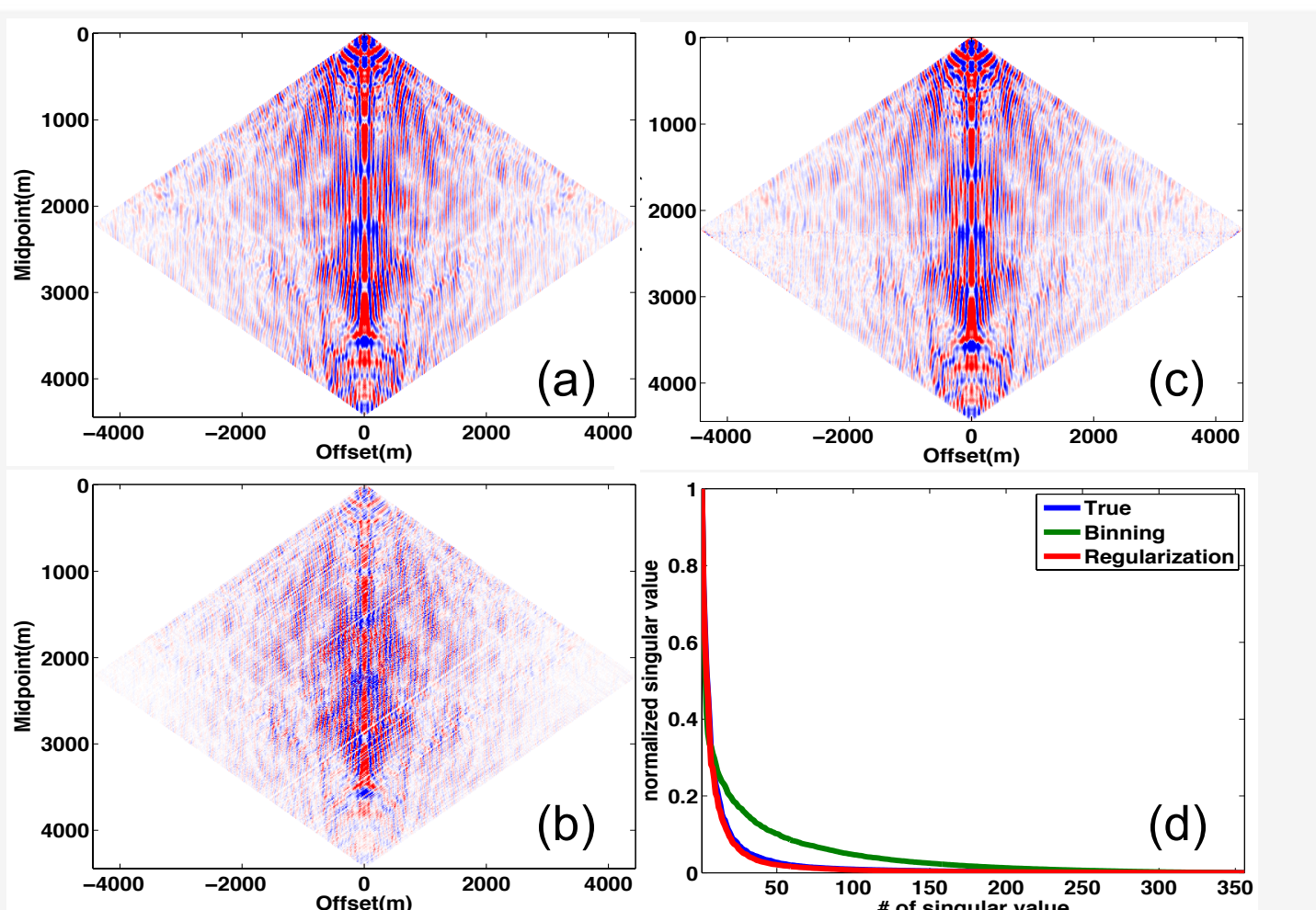


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Regularization

Figure 1: Impact of regularization on singular values decay in the transform-domain. Monochromatic frequency slice at 35 Hz in the midpoint-offset domain, (a) ground truth, (b) binning using nearest-neighbor interpolation, (c) unstructured sampling operator based rank-minimization. (d) Singular values decay in the midpoint-offset domain.



Conclusion

- Binning does not preserve the underlying low-rank structure of seismic data
- Incorporation of grid-irregularity in matrix-completion benefits the regularization & interpolation process and preserve the low-rank structure of seismic data
- Propose matrix-completion framework is computationally affordable for large-scale problems since its SVD-free