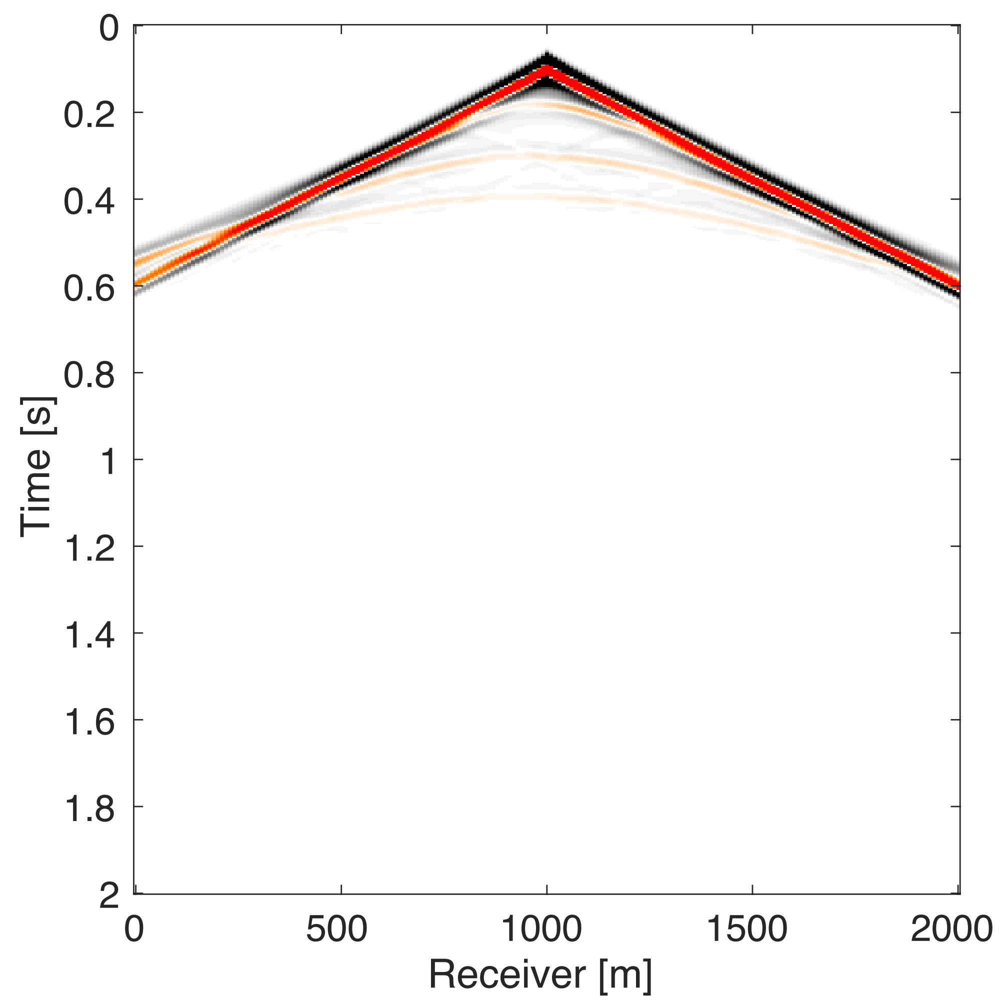
Source estimation for wavefield-reconstruction inversion

Zhilong Fang and Felix J. Herrmann

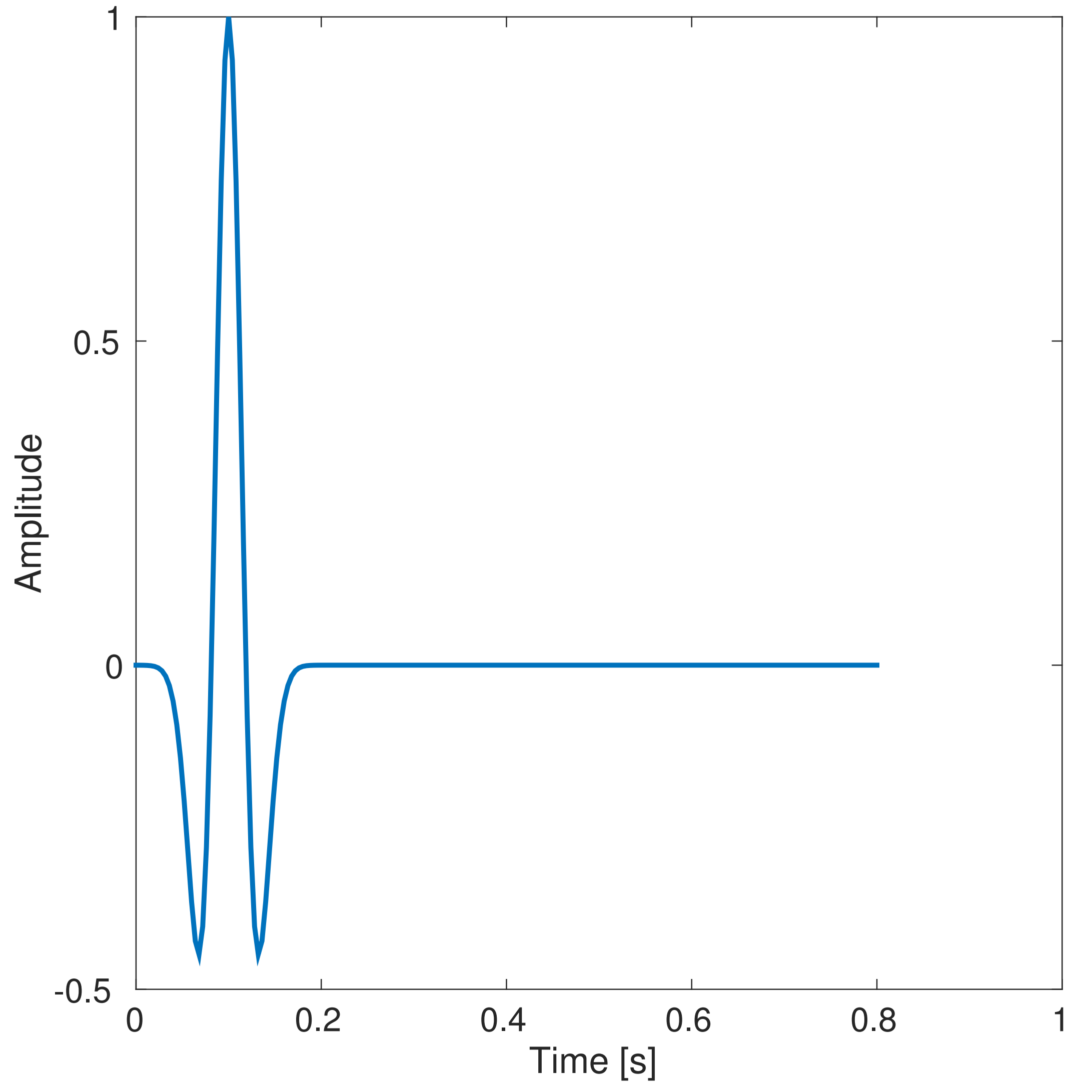
SLIM
University of British Columbia

Motivation



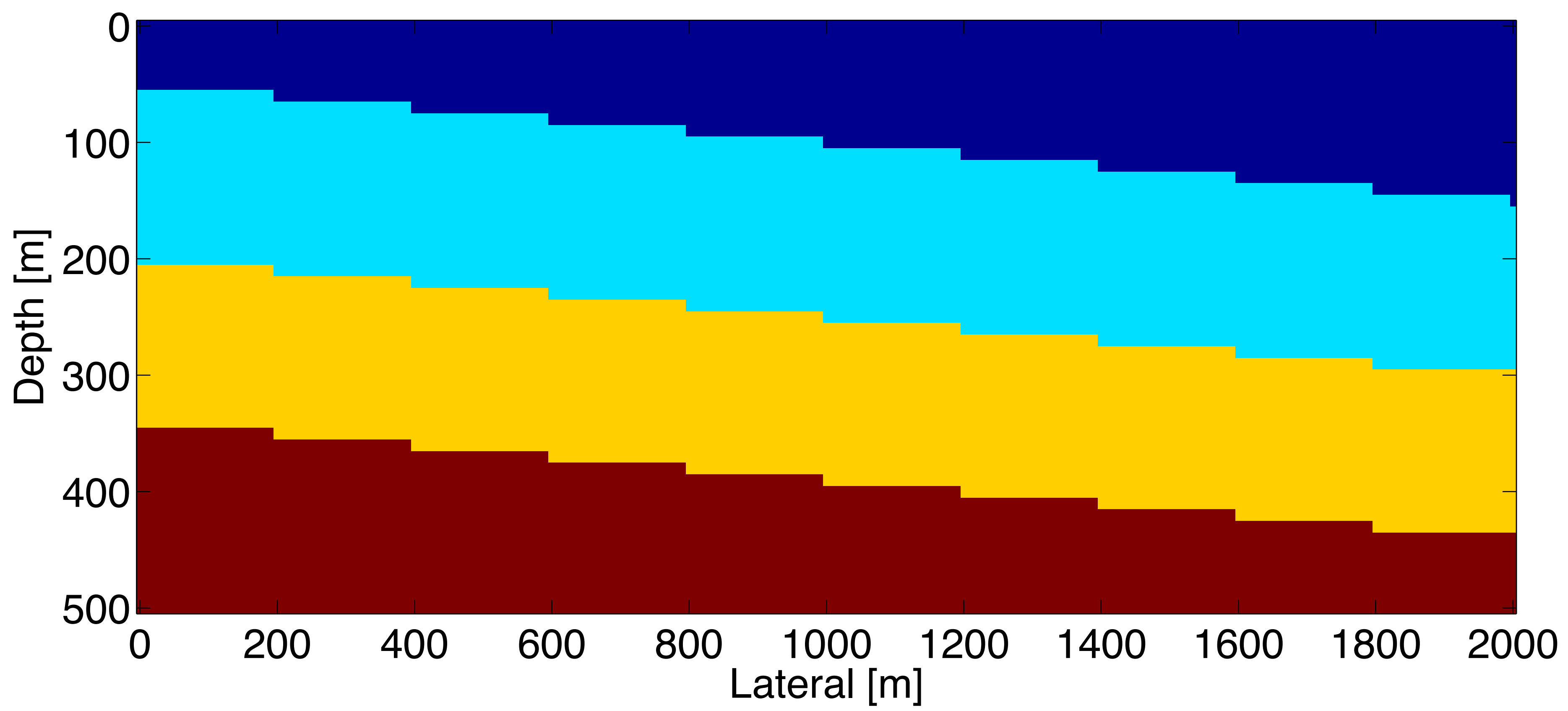
Data

=



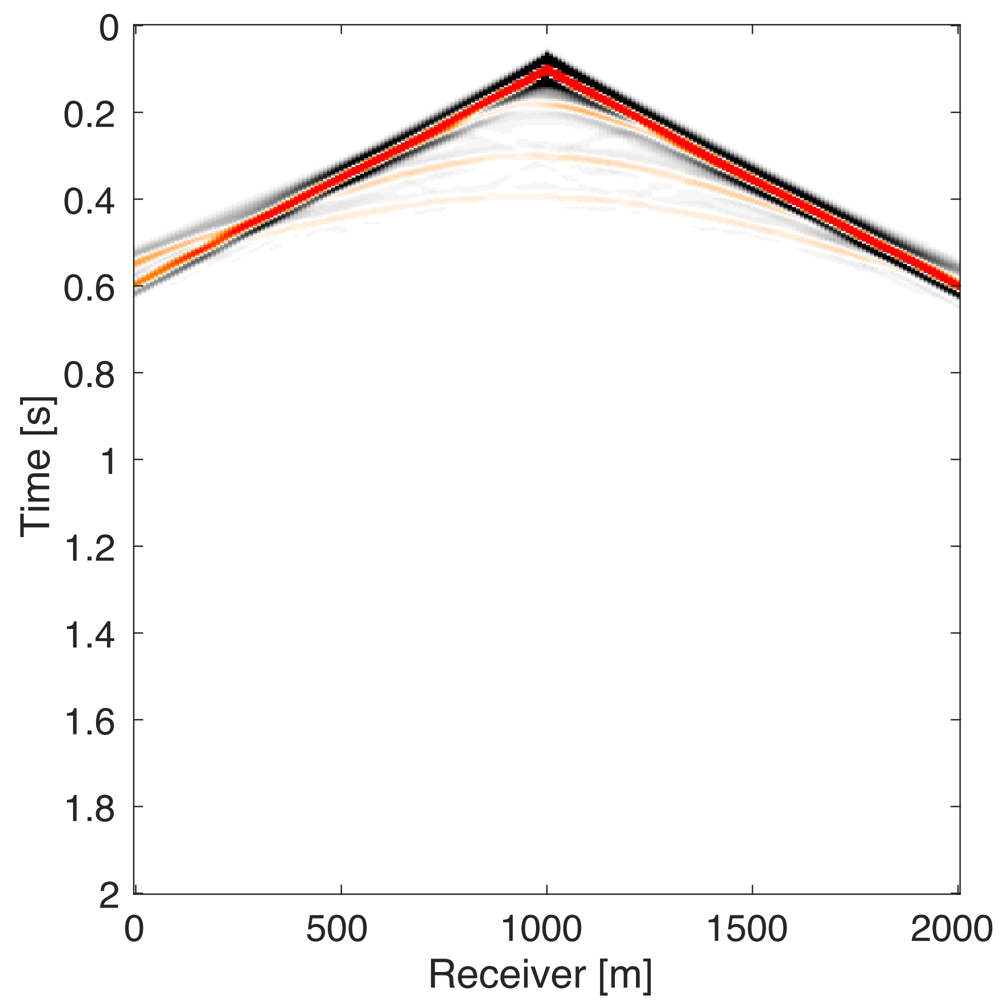
Source wavelet

*



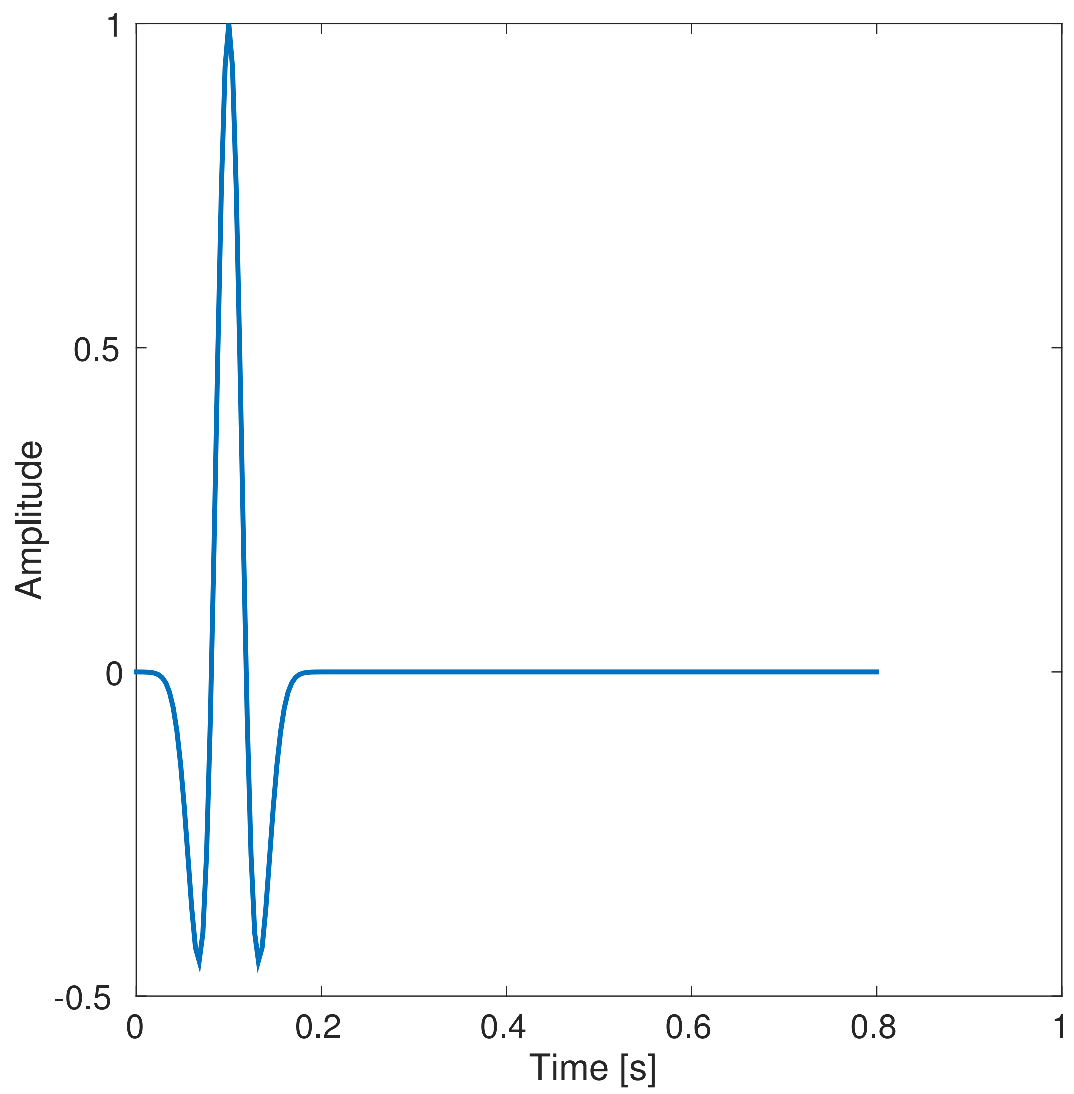
Velocity model

Motivation

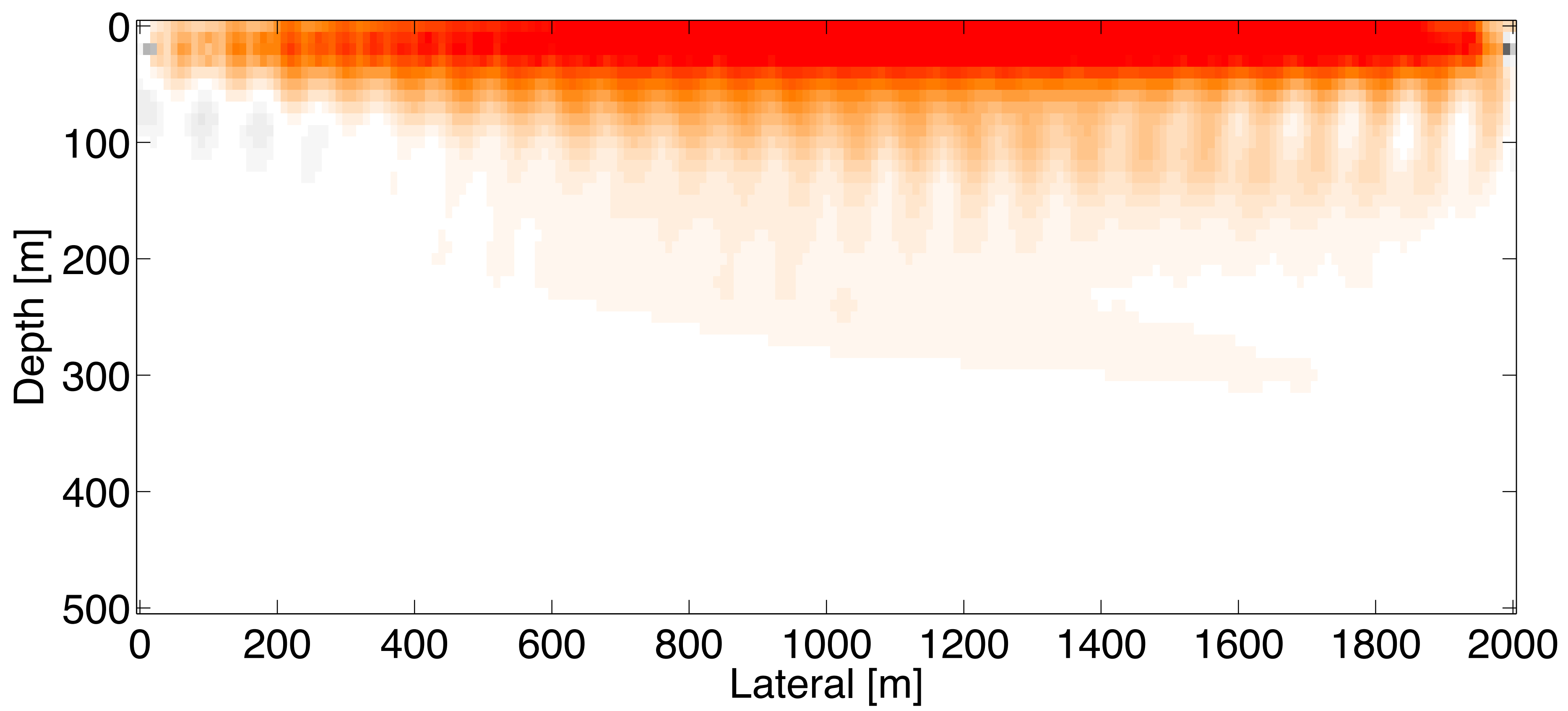


Data

+

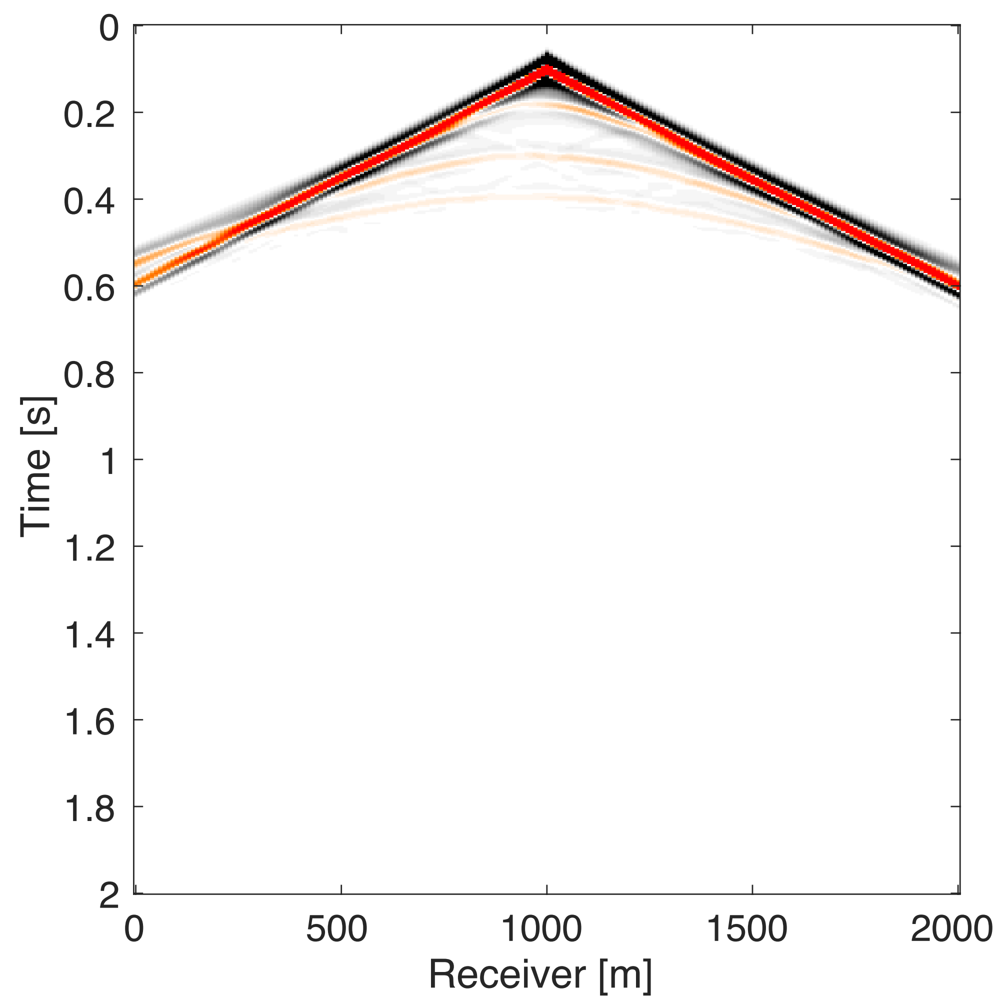


Correct source wavelet

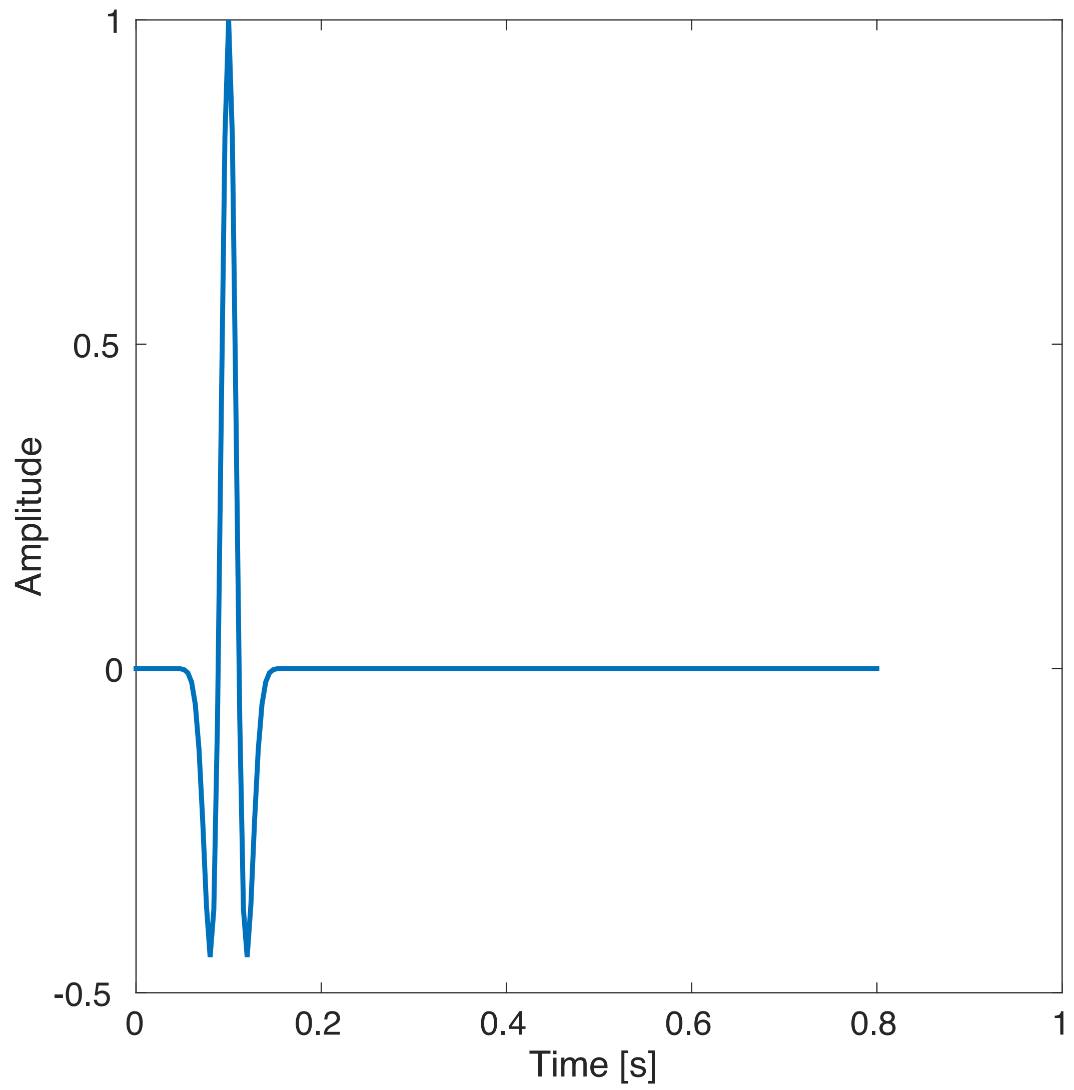


Correct gradient

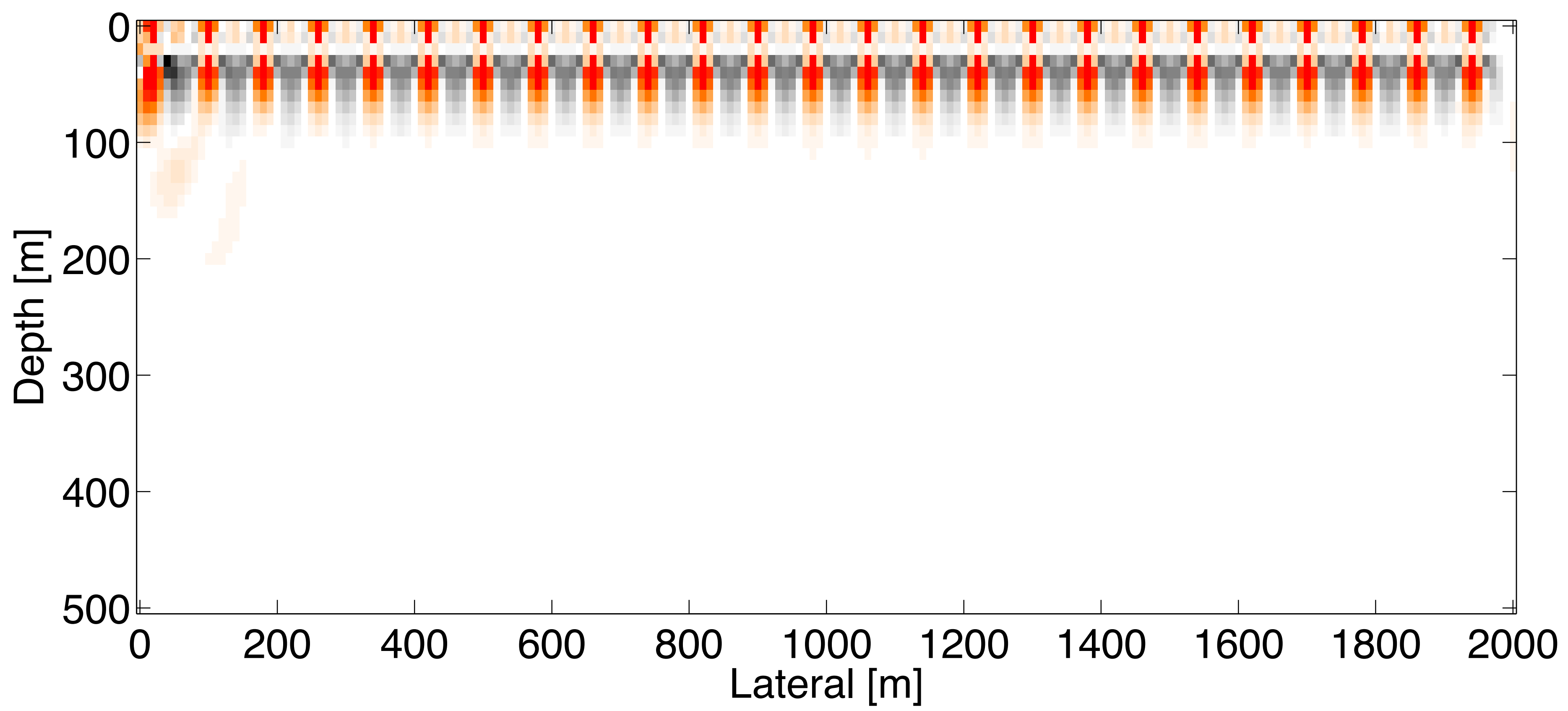
Motivation



Data



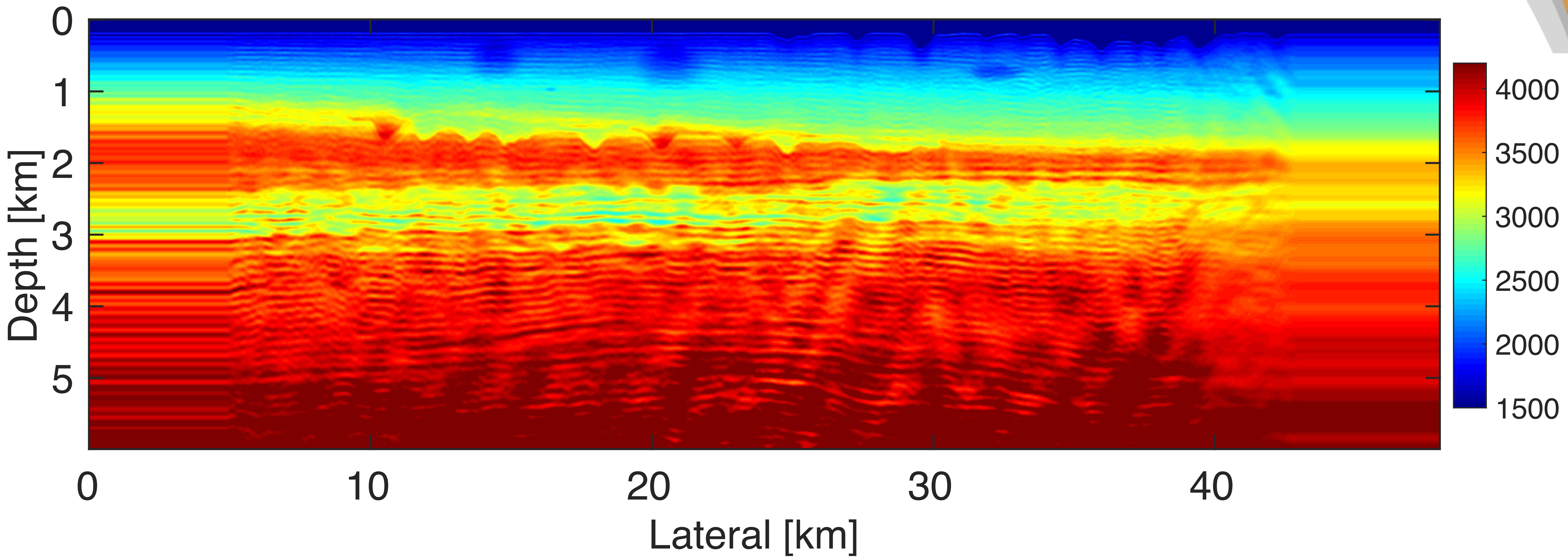
Wrong source wavelet



Wrong gradient

Chevron blind test data

— Wavefield-reconstruction inversion with source estimation



[Tarantola, 1984]

[J. Virieux and S. Operto, 2009]

Full-waveform inversion

Original problem:

$$\underset{\mathbf{u},\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

$$\text{subject to } \mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} = \mathbf{q}_{k,l},$$

where,

$\mathbf{u}_{k,l}$ – Wavefield of the kth shot at lth frequency

$\mathbf{d}_{k,l}$ – Observed data of the kth shot at lth frequency

$\mathbf{q}_{k,l}$ – Source of the kth shot at lth frequency

$\mathbf{A}_{k,l}$ – Helmholtz of the kth shot at lth frequency

$\mathbf{P}_k$ – Receiver projection operator of the kth shot

$\mathbf{m}$ – Squared-slowness

[Tarantola, 1984]

[J. Virieux and S. Operto, 2009]

SLIM

Full-waveform inversion

Reduced/adjoint-state method:

$$\underset{\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1}\mathbf{q}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

with the gradient given by

$$\mathbf{g} = \sum_{k,l} \mathbf{u}^*_{k,l} \frac{\partial \mathbf{A}^*_{k,l}}{\partial \mathbf{m}} \mathbf{v}_{k,l}$$

$$\mathbf{u}_{k,l} = \mathbf{A}_{k,l}(\mathbf{m})^{-1}\mathbf{q}_{k,l}$$

$$\mathbf{v}_{k,l} = \mathbf{A}^{-*}_{k,l}(\mathbf{m})\mathbf{P}^*_k \mathbf{r}_{k,l}$$

$$\mathbf{r}_{k,l} = \mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1}\mathbf{q}_{k,l} - \mathbf{d}_{k,l}$$

2 PDE solves are required !

[van Leeuwen, T and Herrmann, F J , 2013]
[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]
[Golub, G H and Pereyra, V , 1973]

Wavefield-reconstruction inversion

Joint optimization problem:

$$\underset{\mathbf{u},\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \mathbf{q}_{k,l}\|_2^2$$

Eliminating $\mathbf{u}$ w/ variable projection:

$$\overline{\mathbf{u}} = \arg\min_{\mathbf{u}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \mathbf{q}_{k,l}\|_2^2$$

[van Leeuwen, T and Herrmann, F J , 2013]

[Golub, G and Pereyra, V, 1973]

Wavefield-reconstruction inversion

Corresponds to solving the following augmented system:

$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} \\ \mathbf{P}_k \end{pmatrix} \overline{\mathbf{u}}_{k,l} = \begin{pmatrix} \lambda \mathbf{q}_{k,l} \\ \mathbf{d}_{k,l} \end{pmatrix}$$

with the gradient

1 augmented system solves is required !

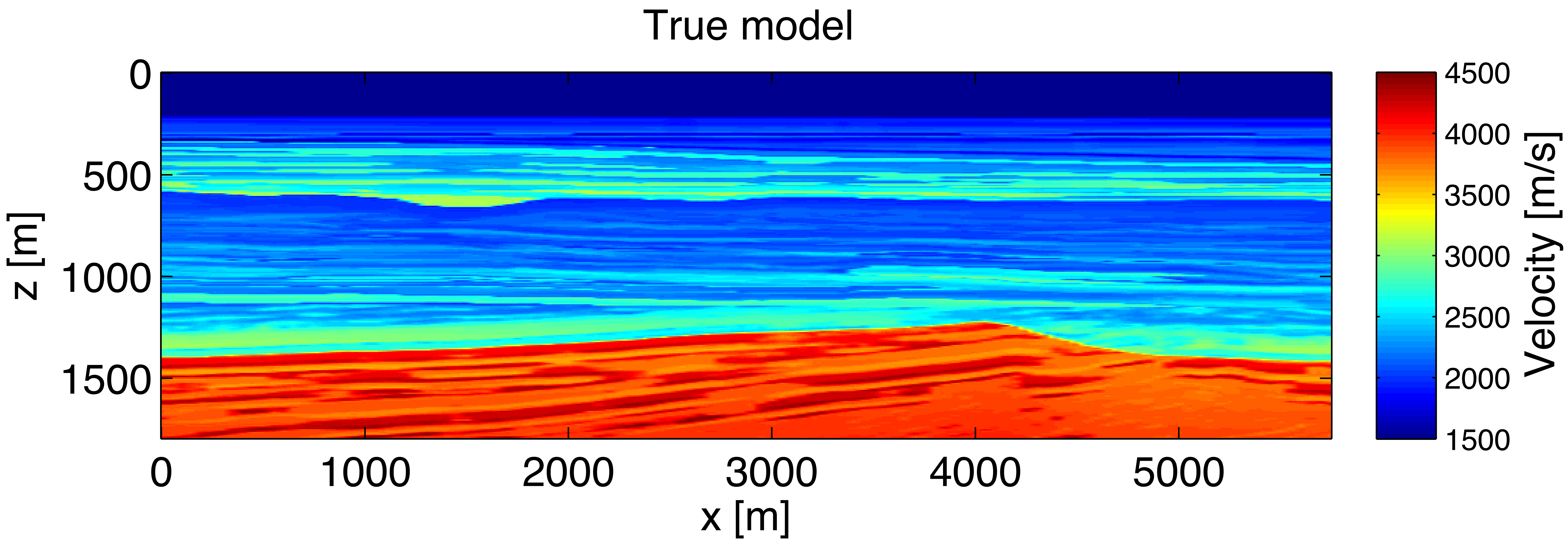
$$\mathbf{g} = \sum_{k,l} \overline{\mathbf{u}}^*_{k,l} \frac{\partial \mathbf{A}^*_{k,l}}{\partial \mathbf{m}} \overline{\mathbf{v}}_{k,l}$$

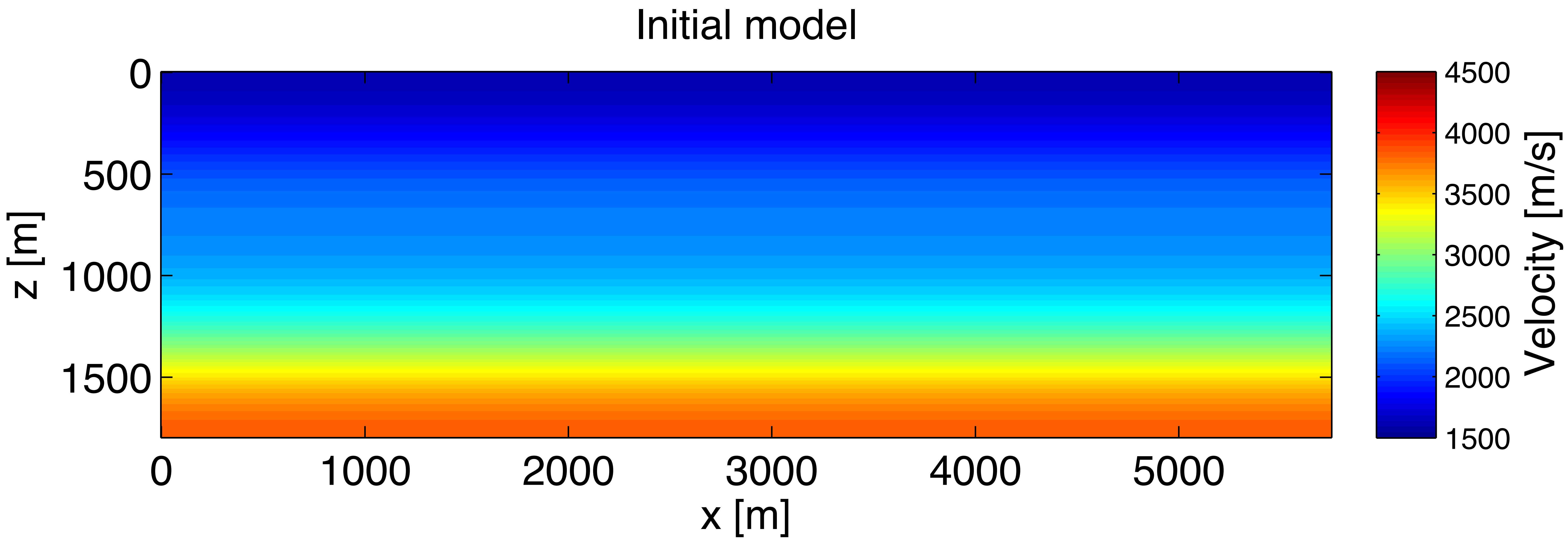
$$\overline{\mathbf{v}}_{k,l} = \mathbf{A}_{k,l}(\mathbf{m})\overline{\mathbf{u}}_{k,l} - \mathbf{q}_{k,l}$$

True & initial model

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]

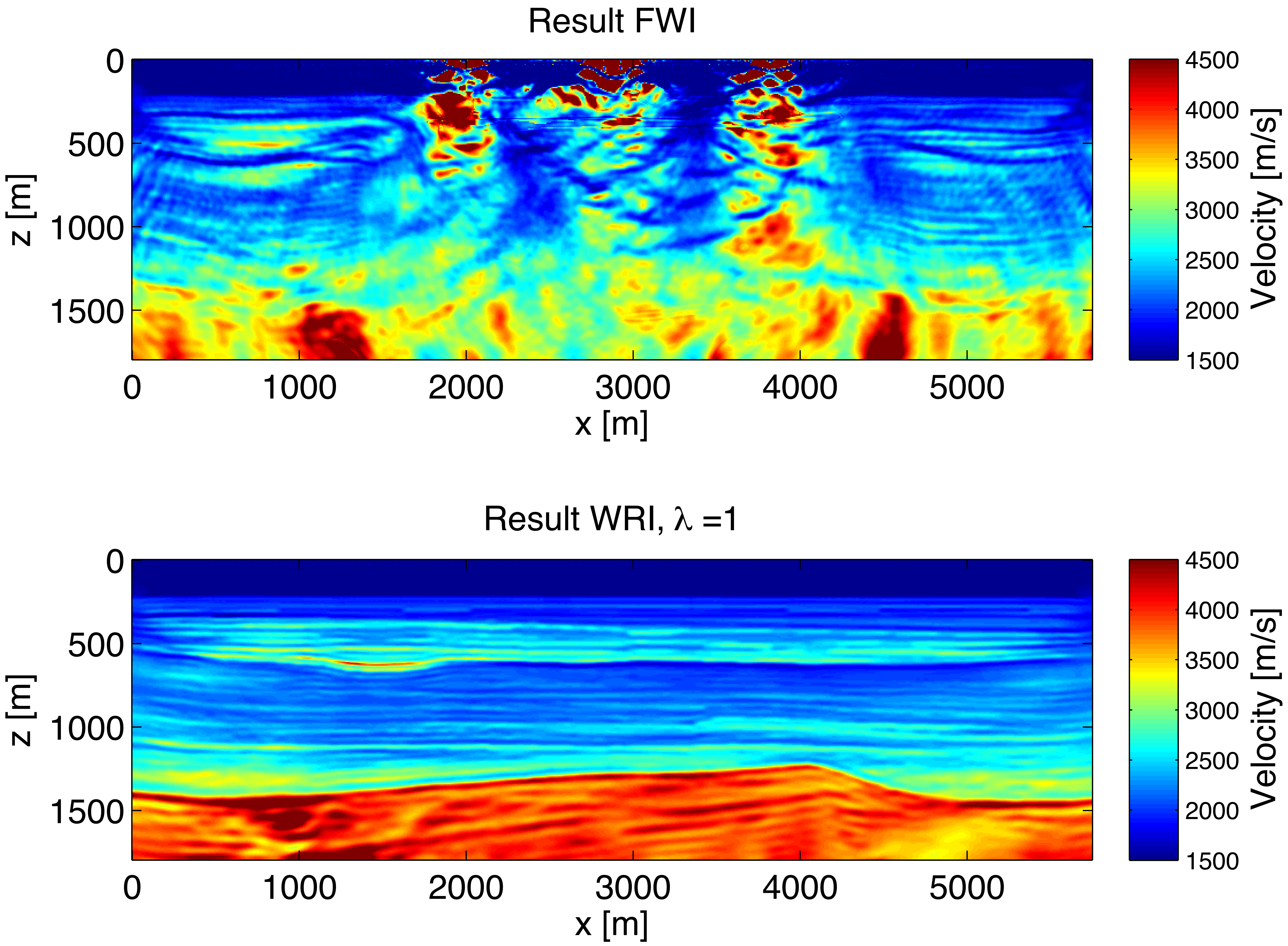




FWI vs WRI

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]



WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u},\mathbf{m},\alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \alpha_{k,l}\mathbf{e}_{k,l}\|_2^2$$

WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u},\mathbf{m},\alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \alpha_{k,l}\mathbf{e}_{k,l}\|_2^2$$

Eliminate $\mathbf{u}$ and α jointly w/ variable projection:

$$[\overline{\mathbf{u}}, \overline{\alpha}] = \arg\min_{\mathbf{u},\alpha} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \alpha_{k,l}\mathbf{e}_{k,l}\|_2^2$$

WRI with source estimation

Corresponds to solving the following augmented system:

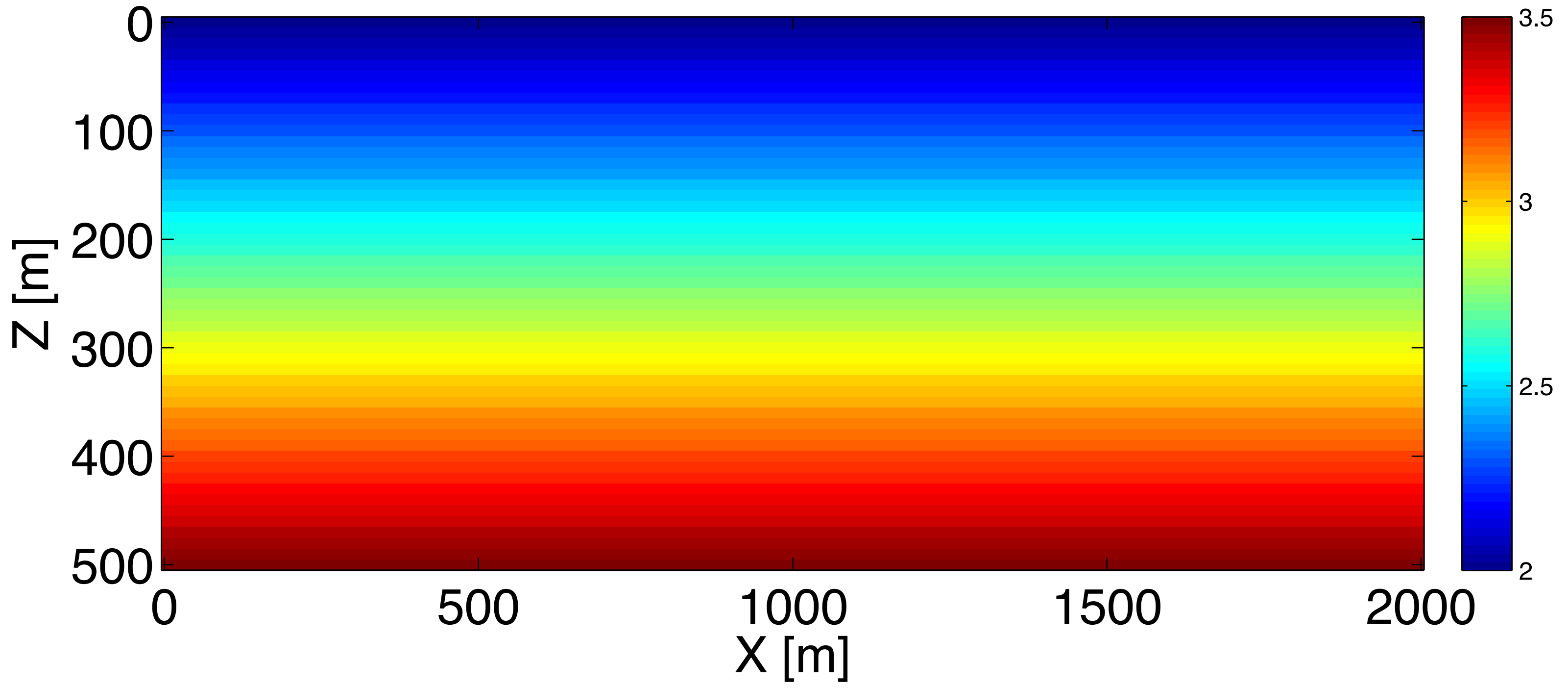
$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} & -\lambda \mathbf{e}_{k,l} \\ \mathbf{P}_k & 0 \end{pmatrix} \begin{pmatrix} \overline{\mathbf{u}}_{k,l} \\ \overline{\alpha}_{k,l} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d}_{k,l} \end{pmatrix}$$

Cf. original augmented system:

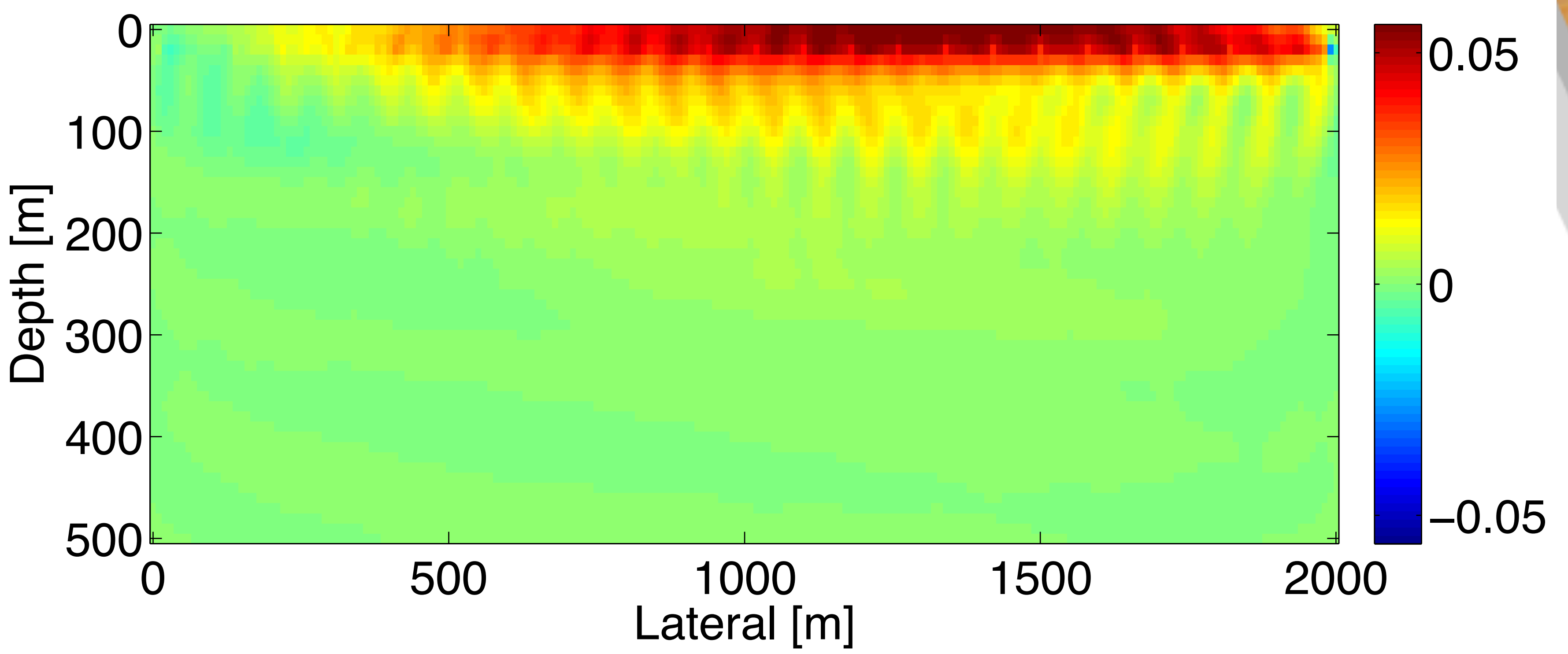
Full column rank!
No additional computational cost!

$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} \\ \mathbf{P}_k \end{pmatrix} \overline{\mathbf{u}}_{k,l} = \begin{pmatrix} \lambda \mathbf{q}_{k,l} \\ \mathbf{d}_{k,l} \end{pmatrix}$$

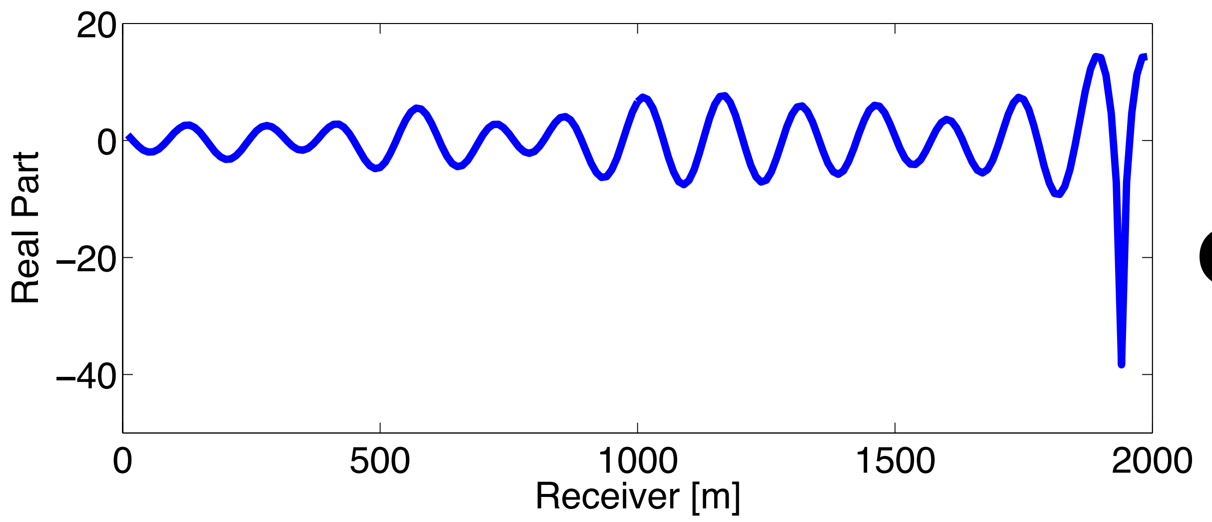
WRI with source estimation



$\mathbf{m}$



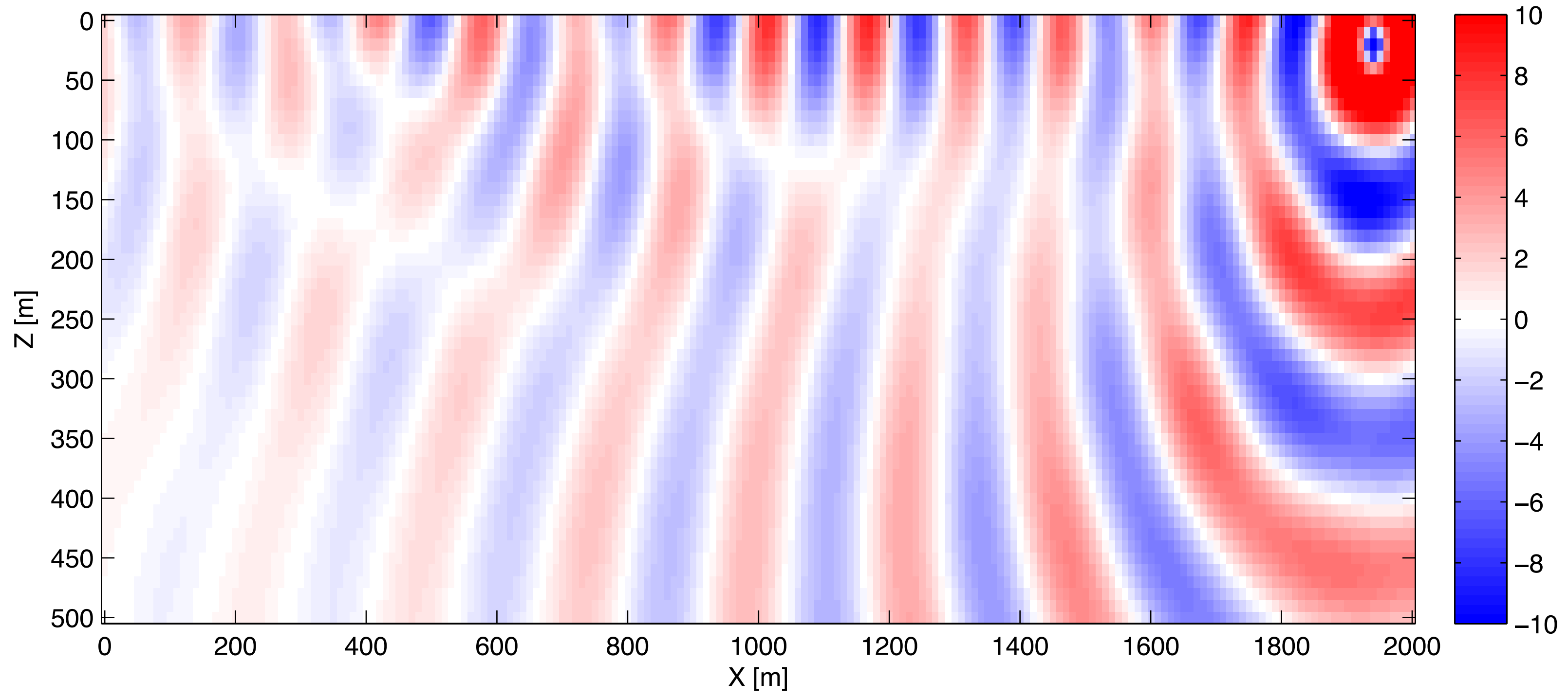
$\mathbf{g}$



$\mathbf{d_{obs}}$

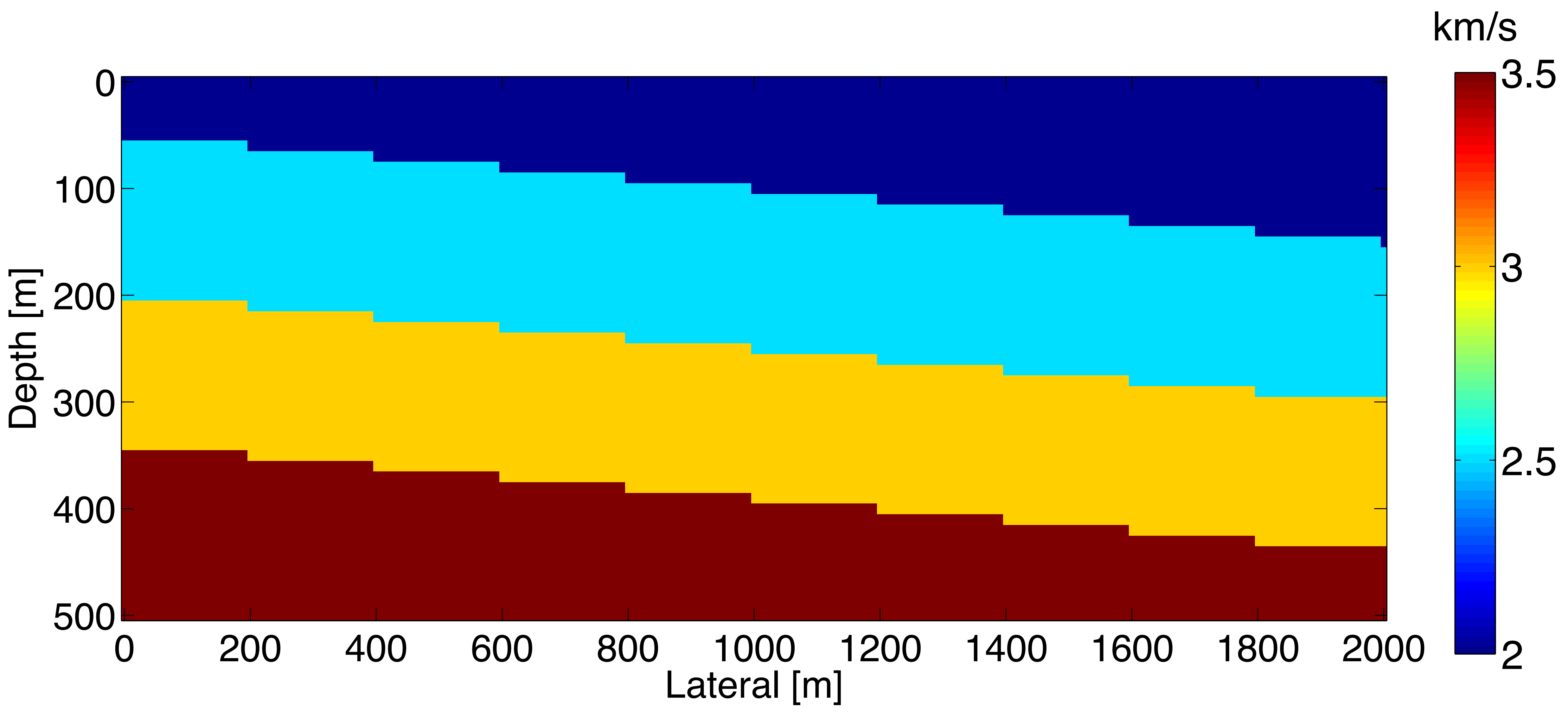
$$\begin{pmatrix} \lambda \mathbf{A} & -\lambda \mathbf{e} \\ \mathbf{P} & 0 \end{pmatrix} \begin{pmatrix} \overline{\mathbf{u}} \\ \overline{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d_{obs}} \end{pmatrix}$$

$\overline{\alpha}$ and

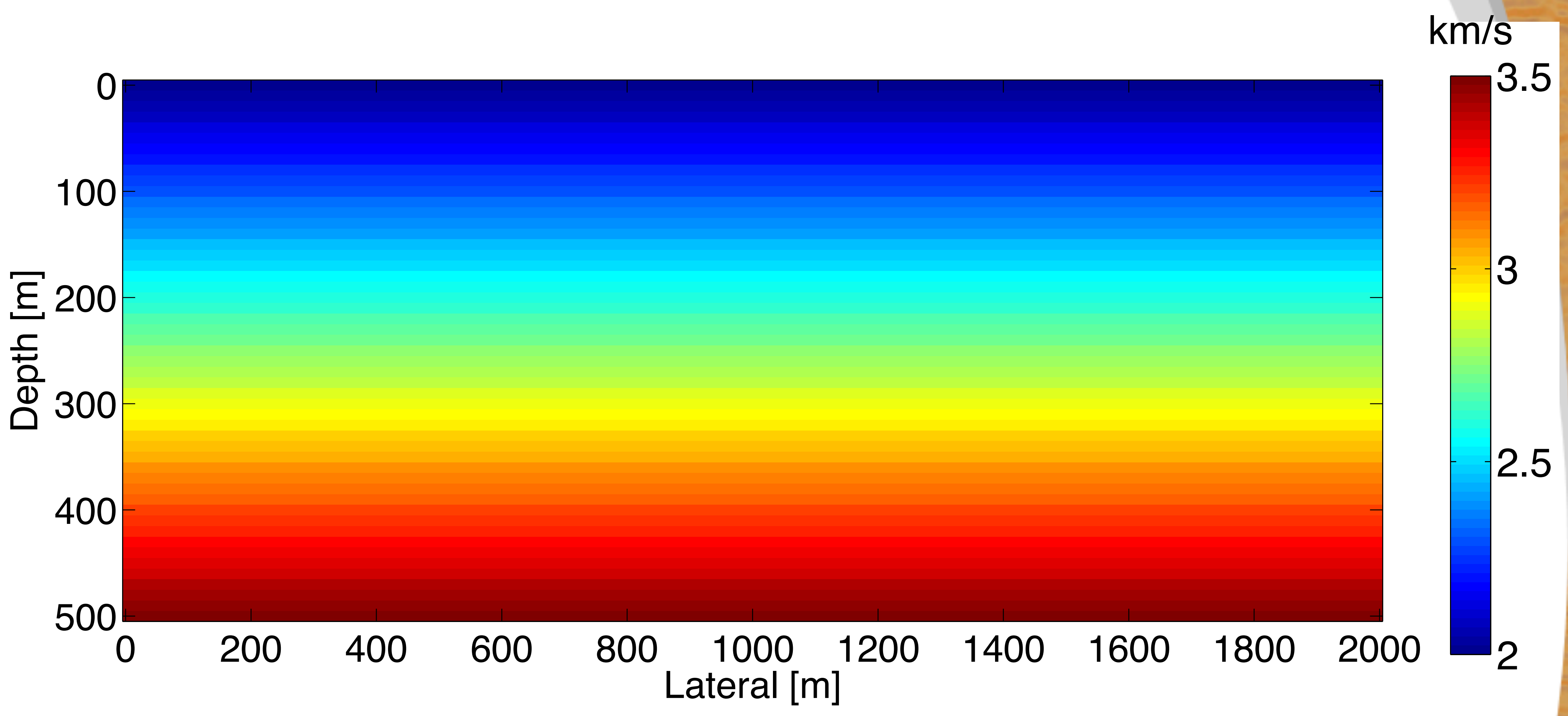


$\overline{\mathbf{u}}$

Synthetic example

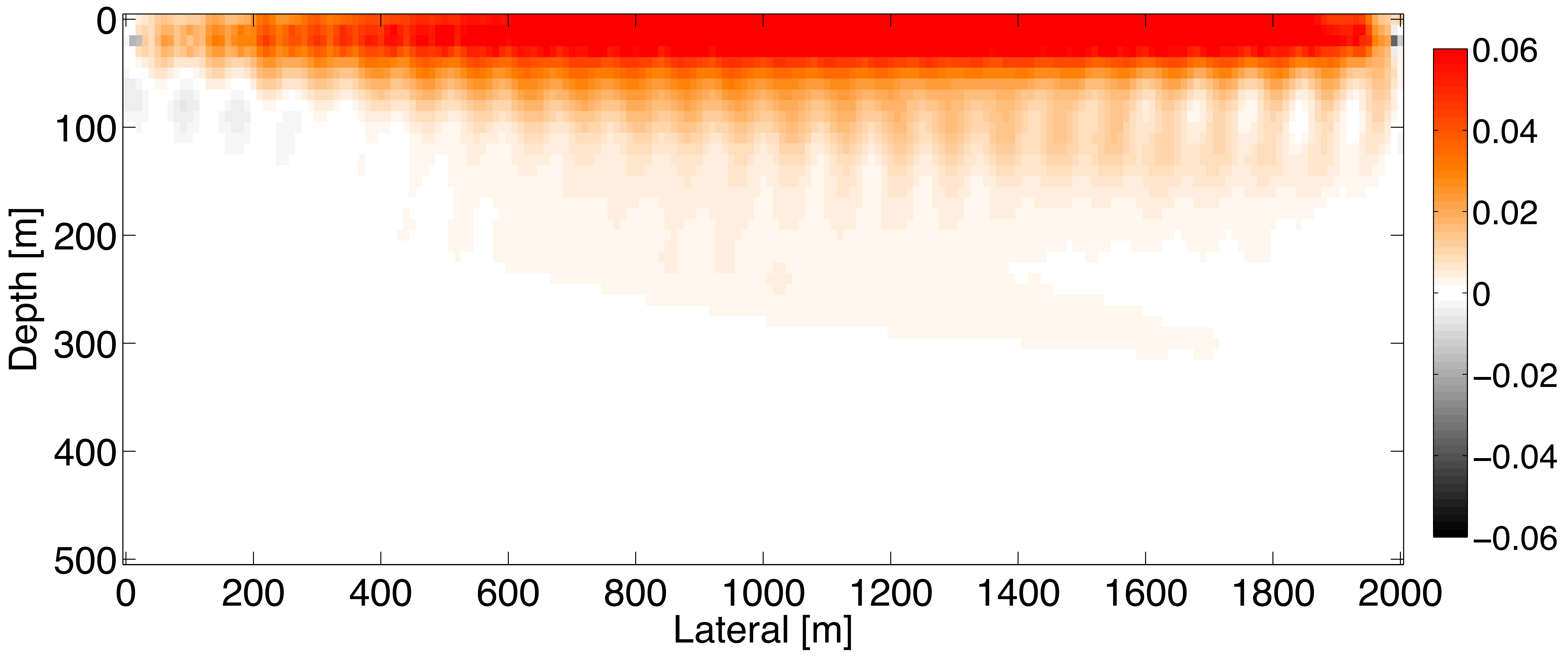


True Model

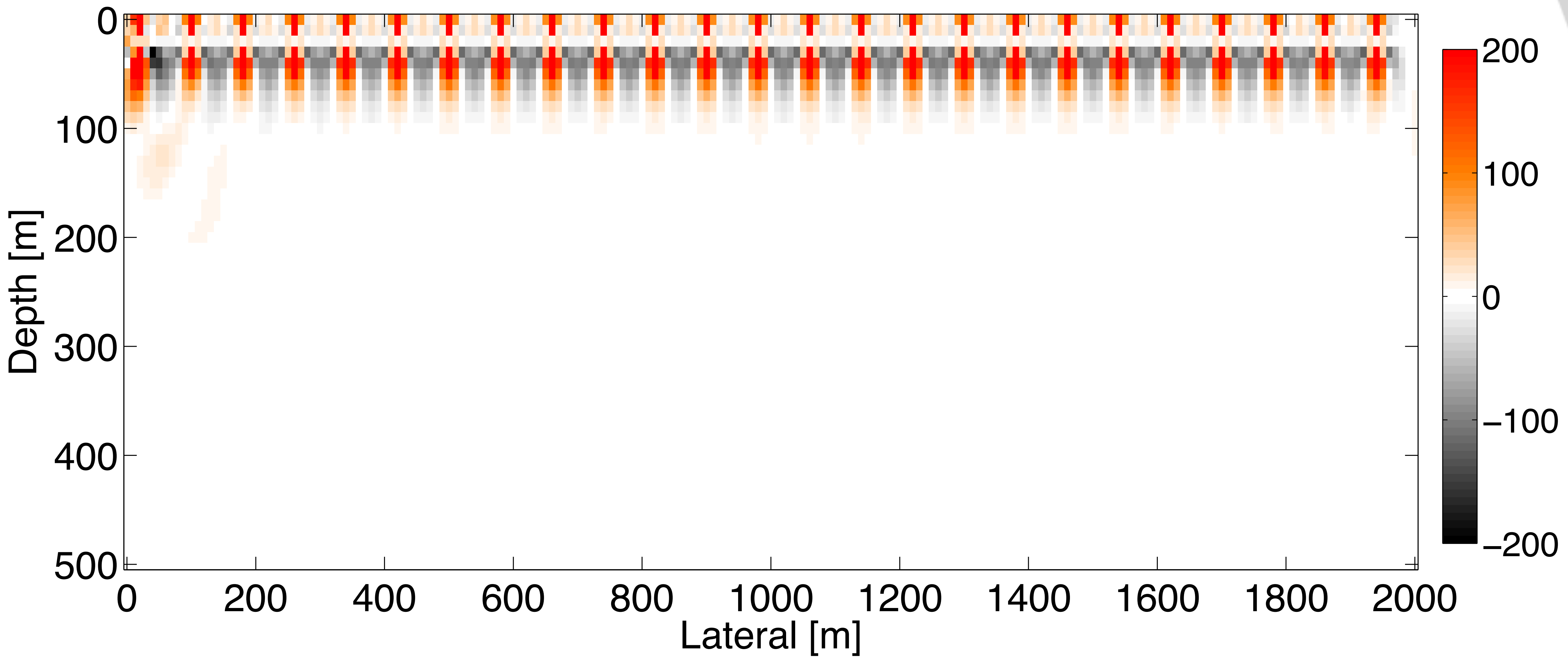


Initial Model

Gradient comparison

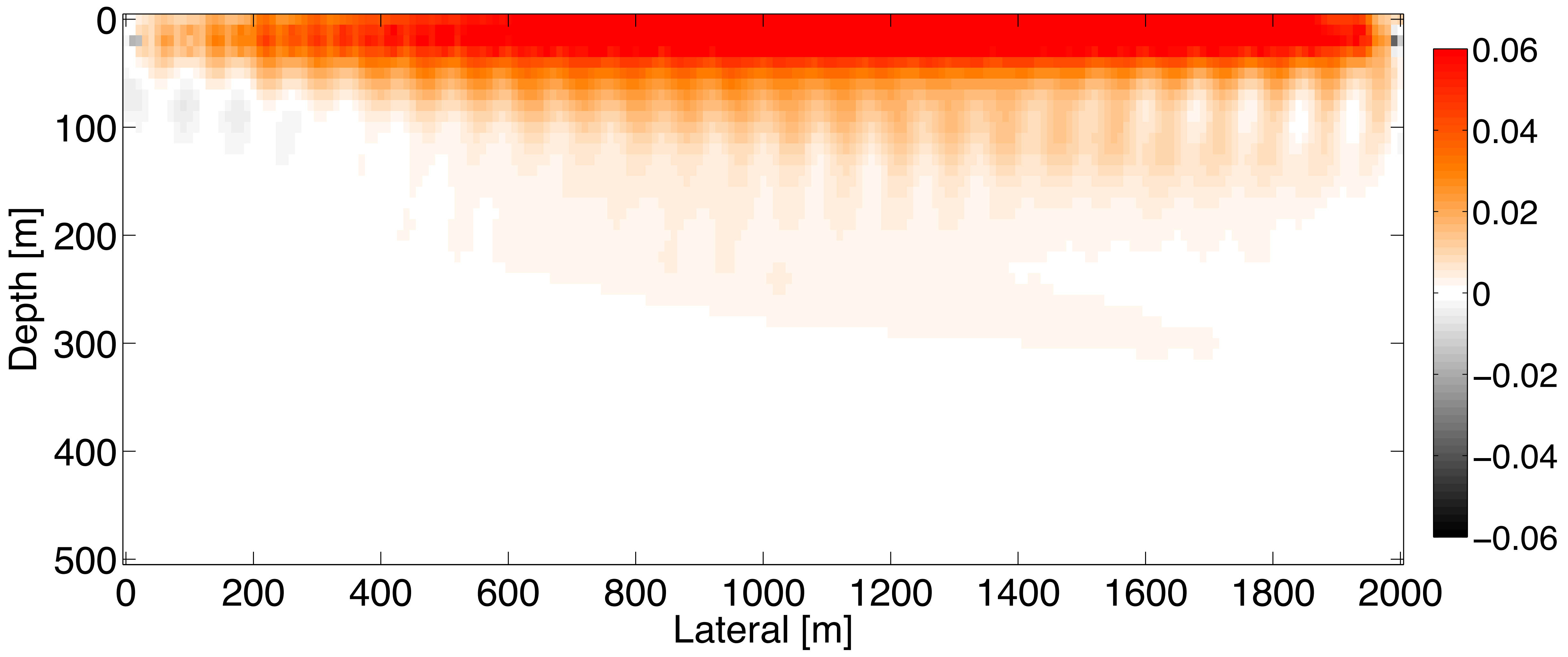


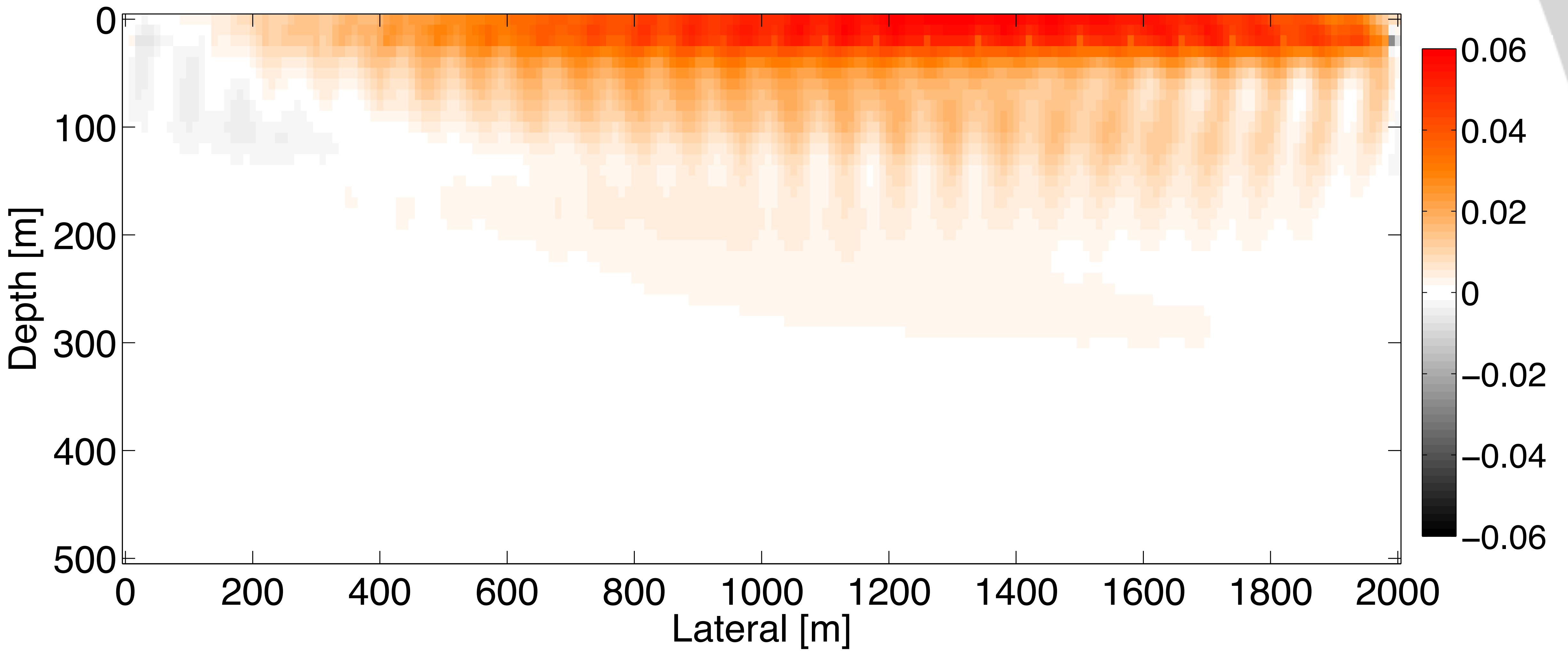
Gradient with true source wavelet



Gradient with wrong source wavelet

Gradient comparison

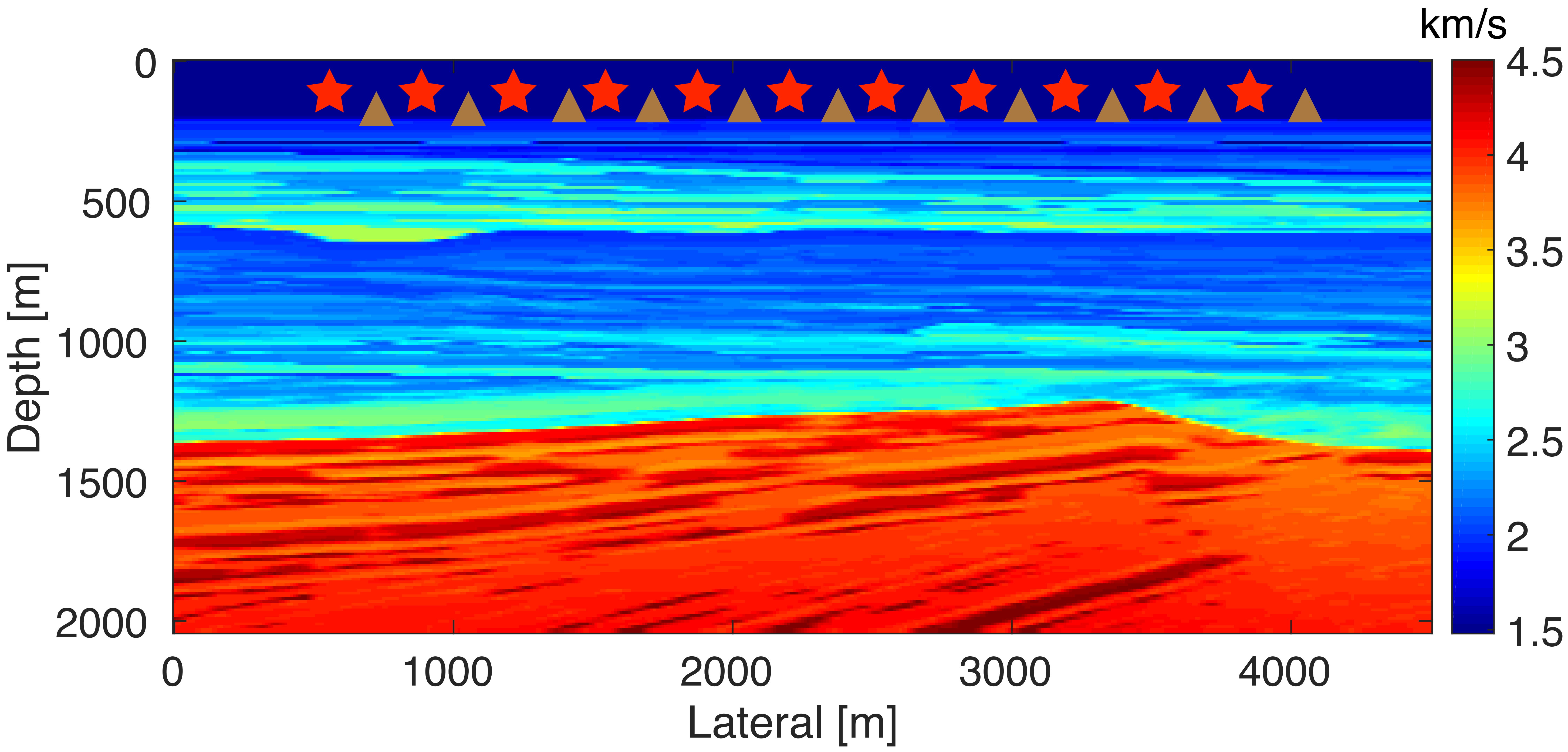




Gradient with true source wavelet

Gradient with estimated source wavelet

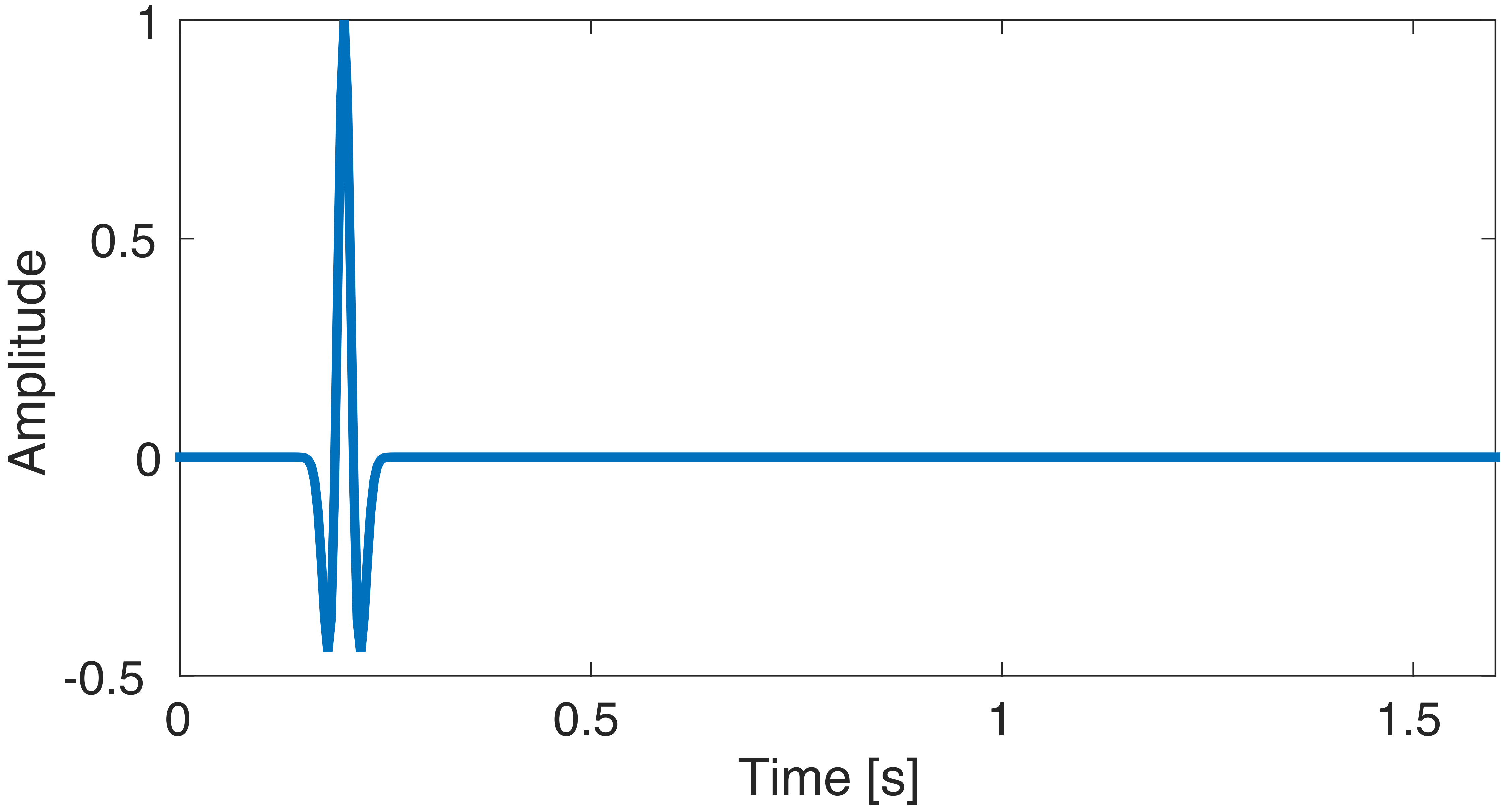
BG model



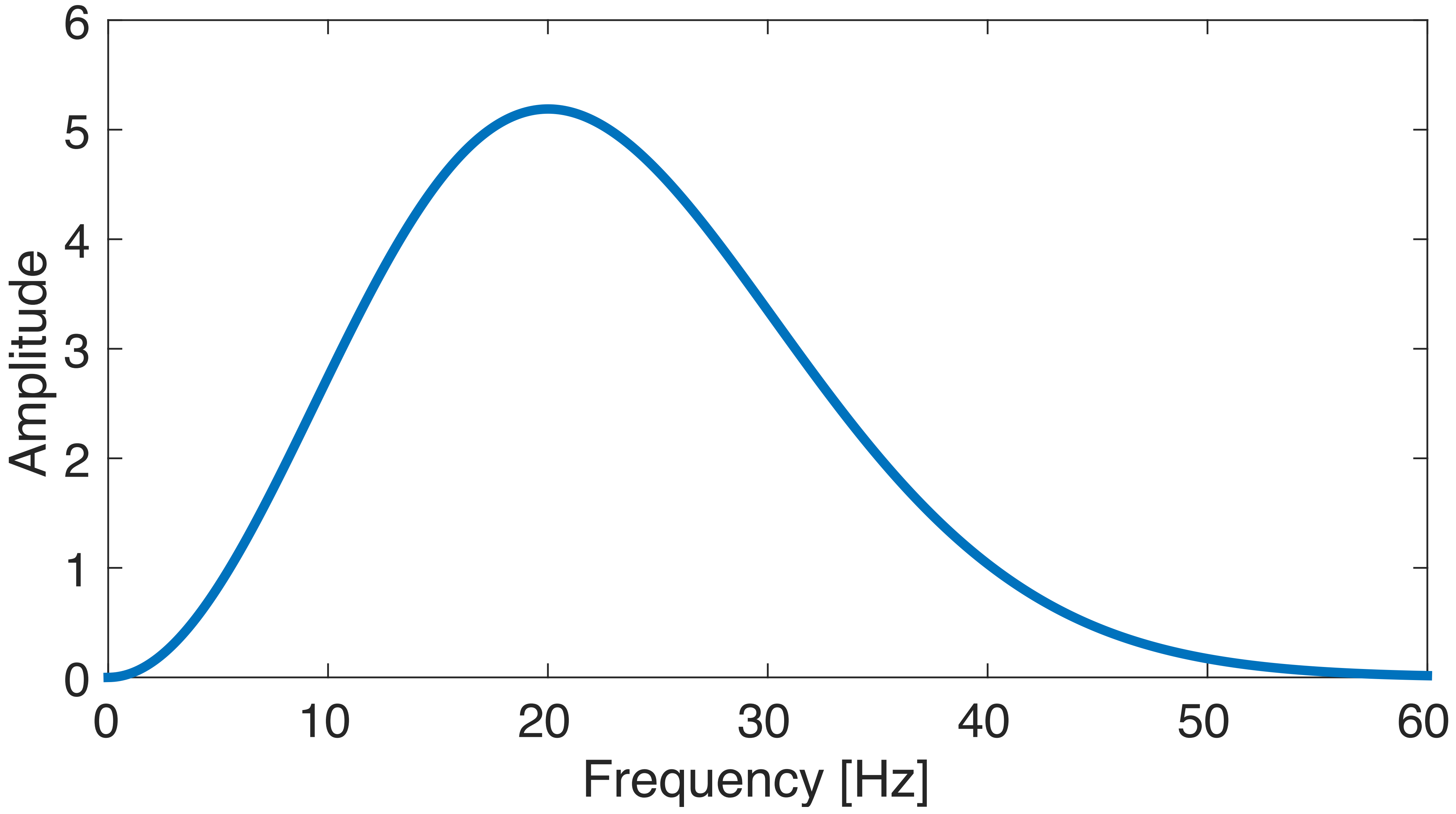
Modeling information:
Model size: 2000m x 4500m
Source spacing: 50m
Receiver spacing: 10m
Fixed spread 4.5km
Frequency : 2~31 Hz

Inversion information:
Optimization Solver: Gauss-Newton
Iterations per frequency band: 21
Batch size: 15

Source wavelet

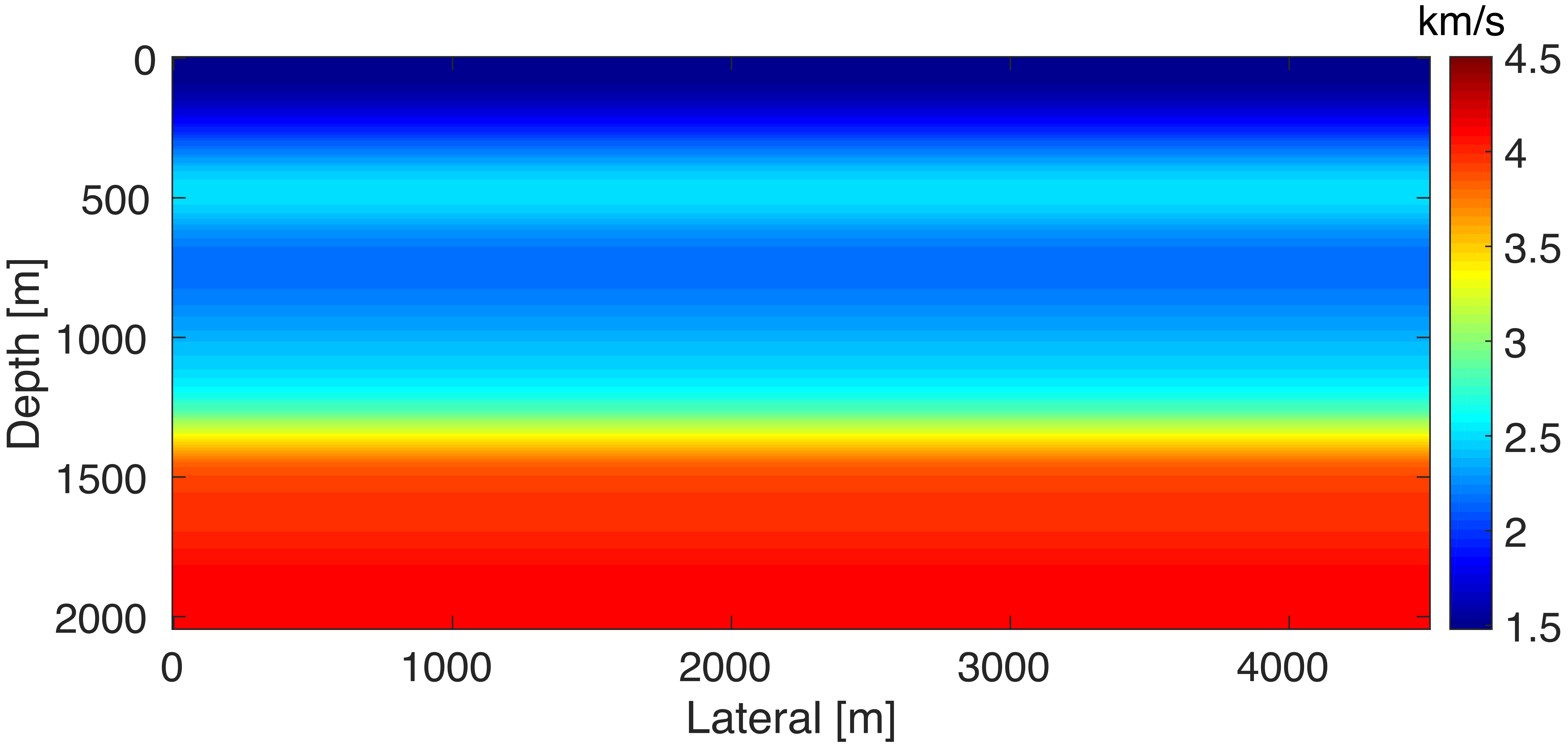


Source Wavelet

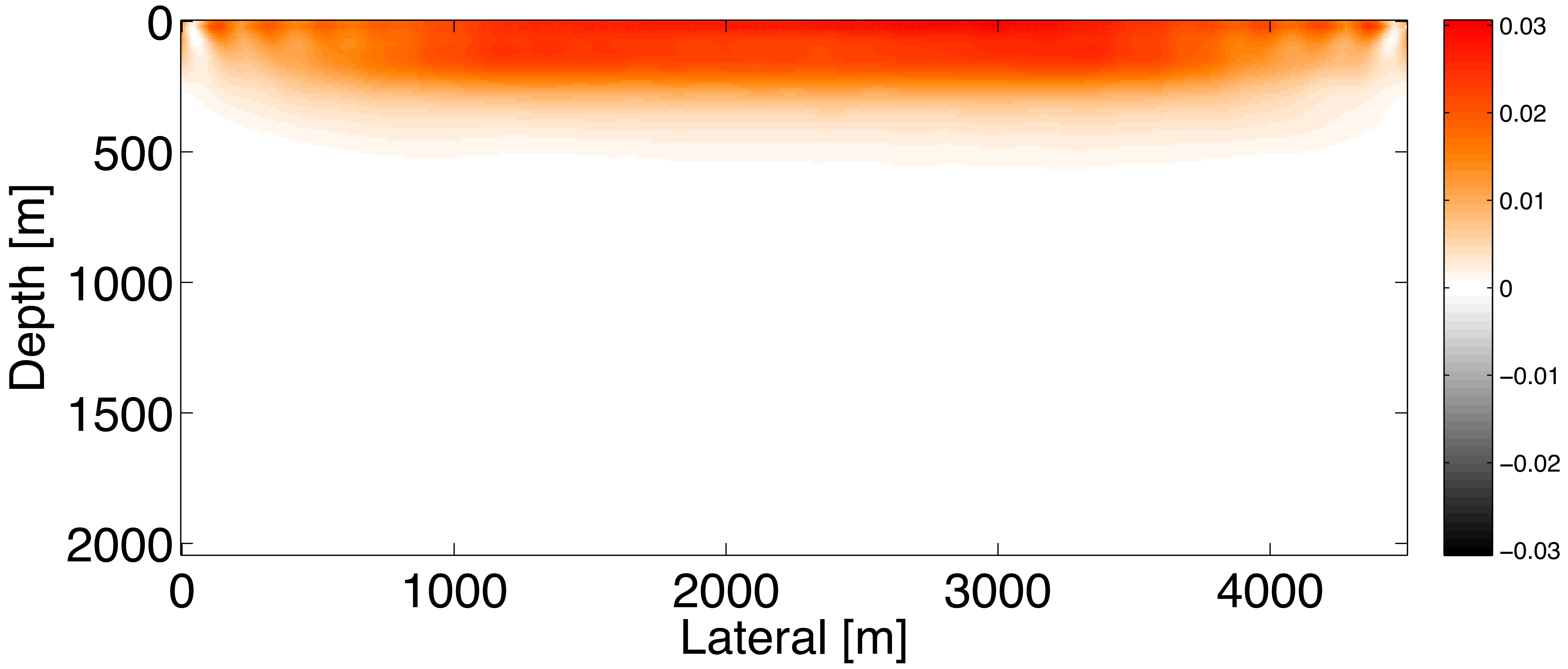


Spectrum

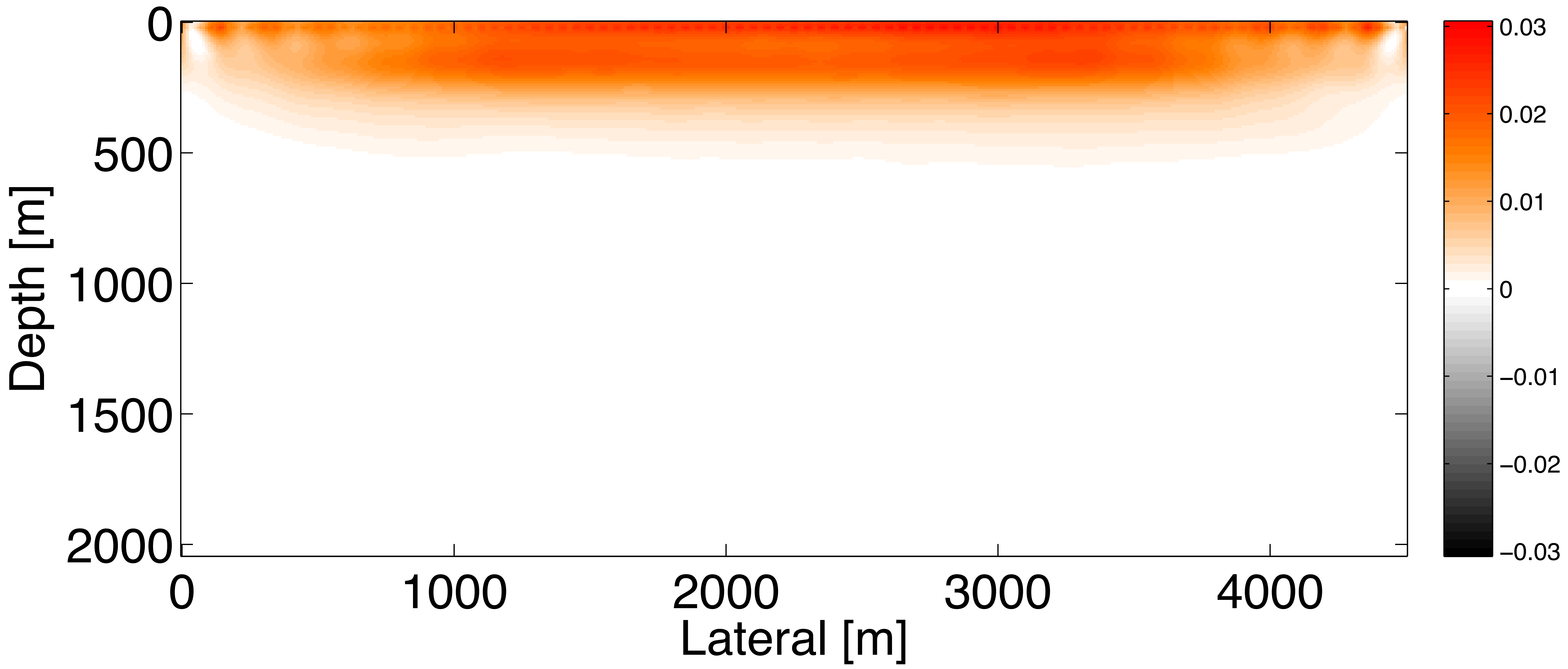
Initial model



First gradient comparison

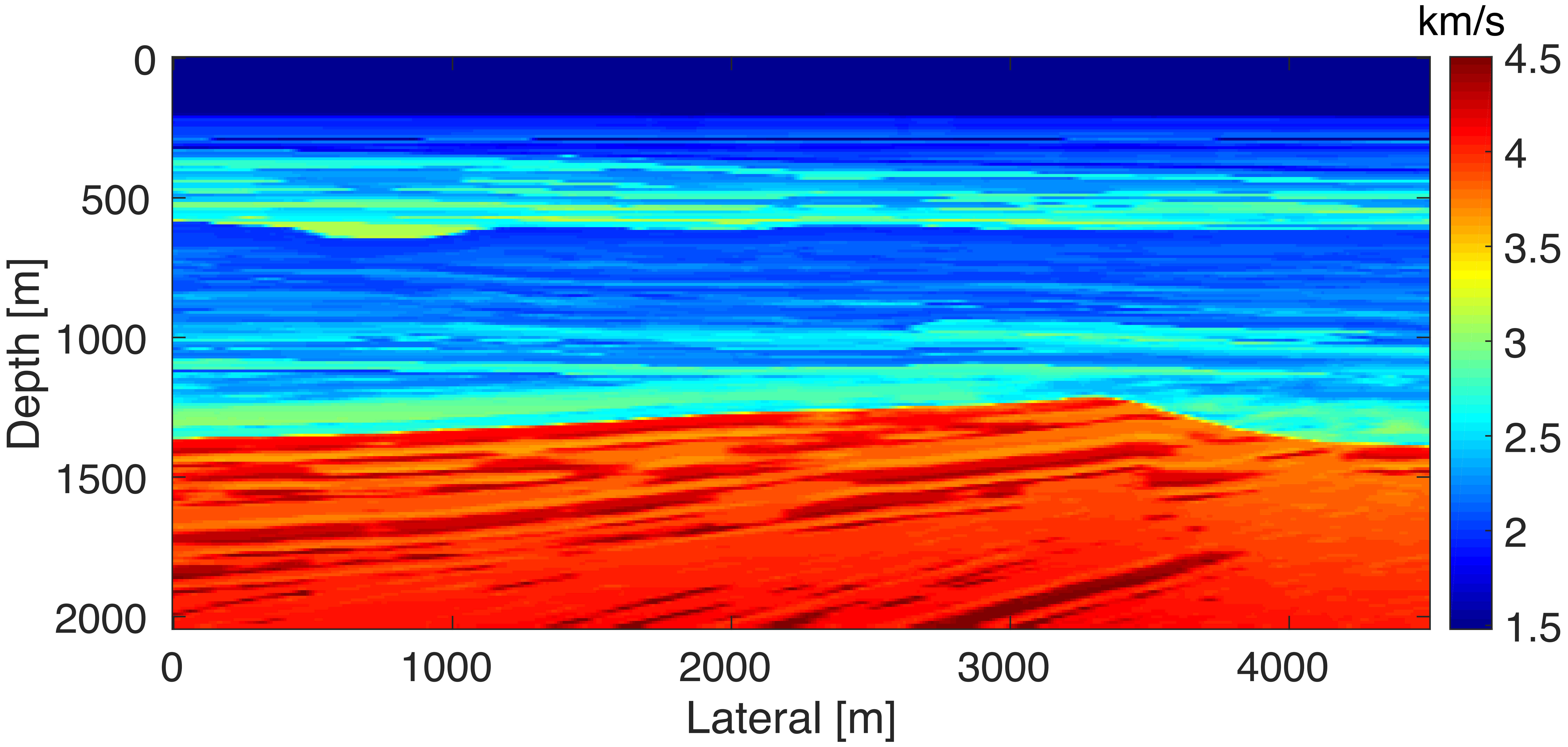


Gradient with true source wavelet



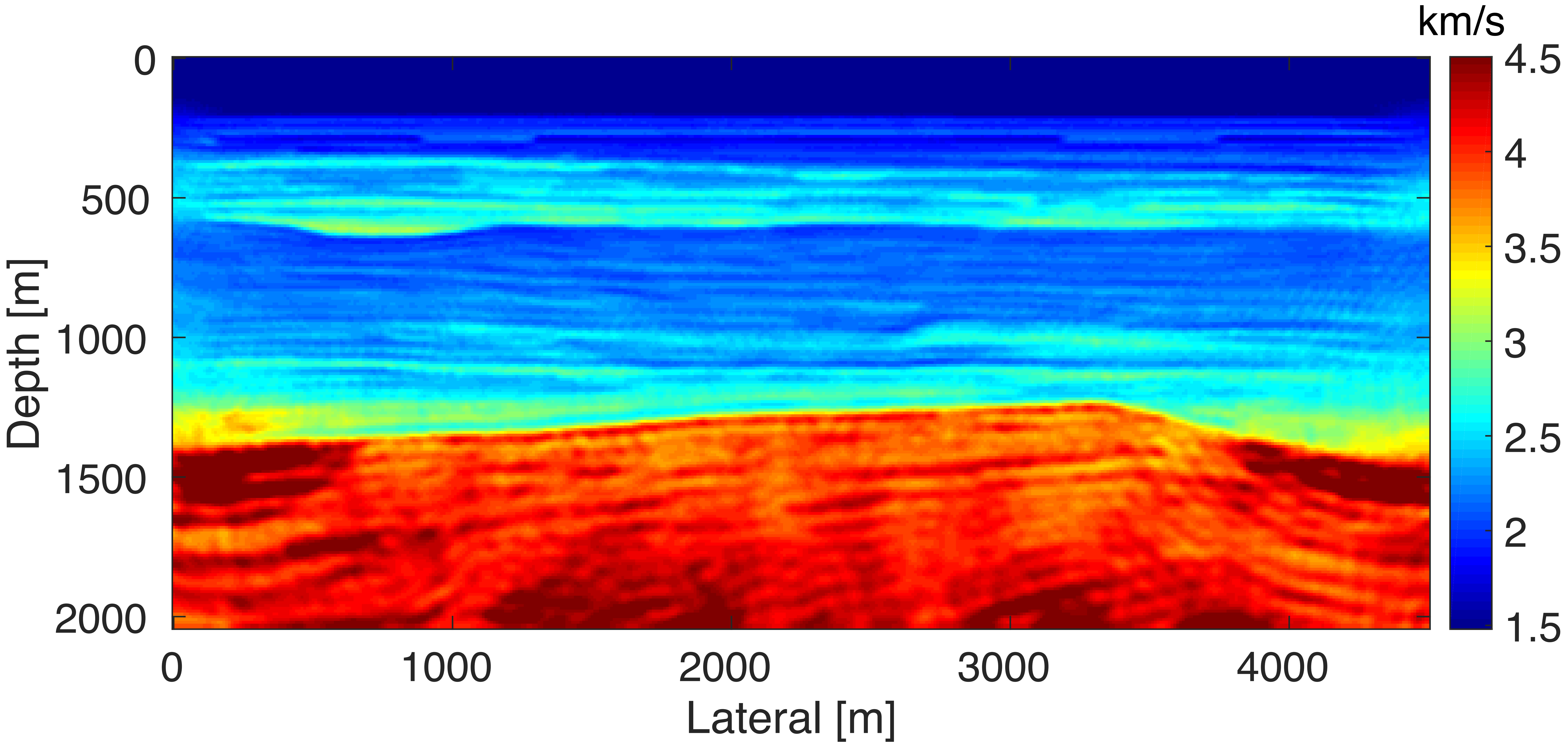
Gradient with estimated source wavelet

True Model

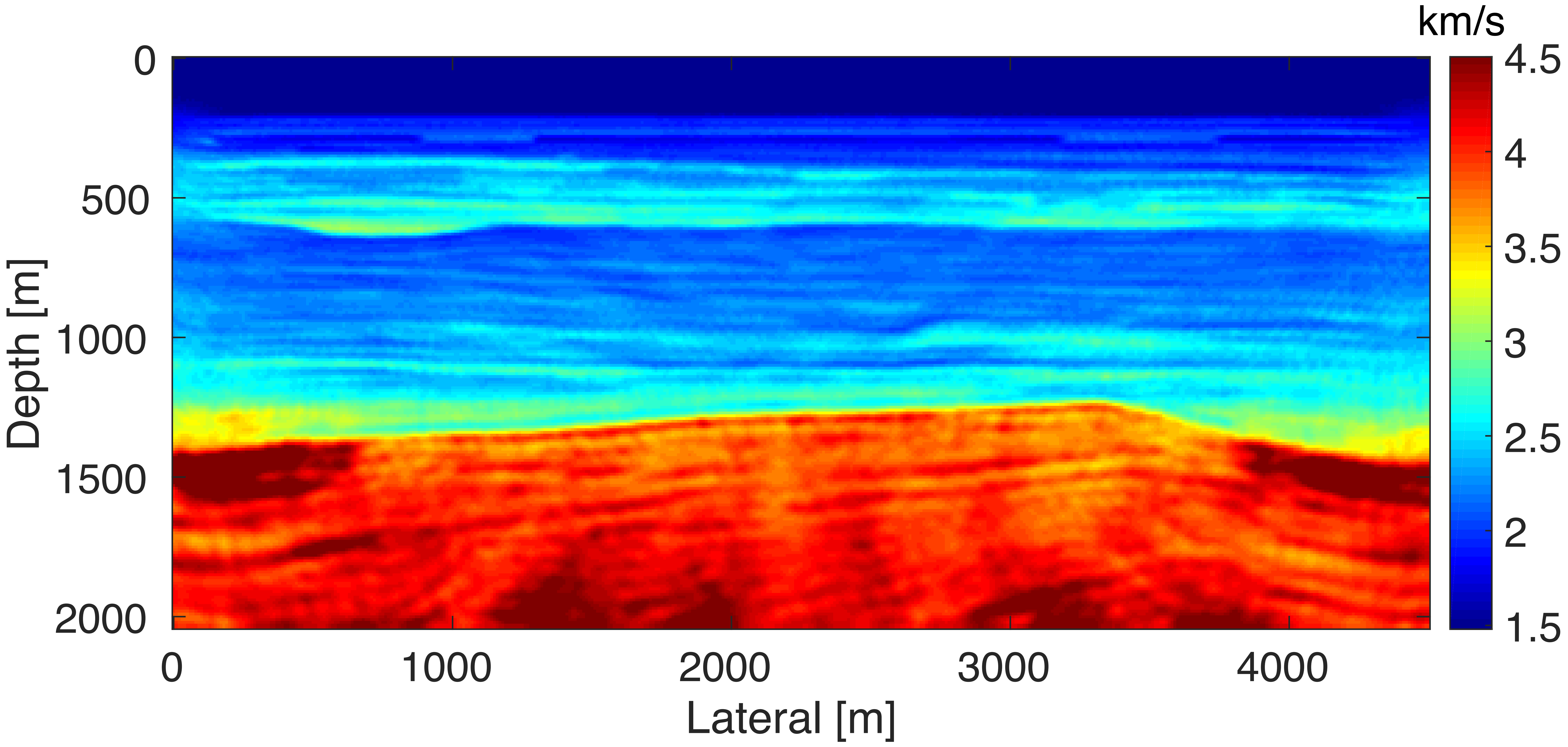


km/s
4.5
4
3.5
3
2.5
2
1.5
0
500
1000
1500
2000
Depth [m]
0
1000
2000
3000
4000
Lateral [m]

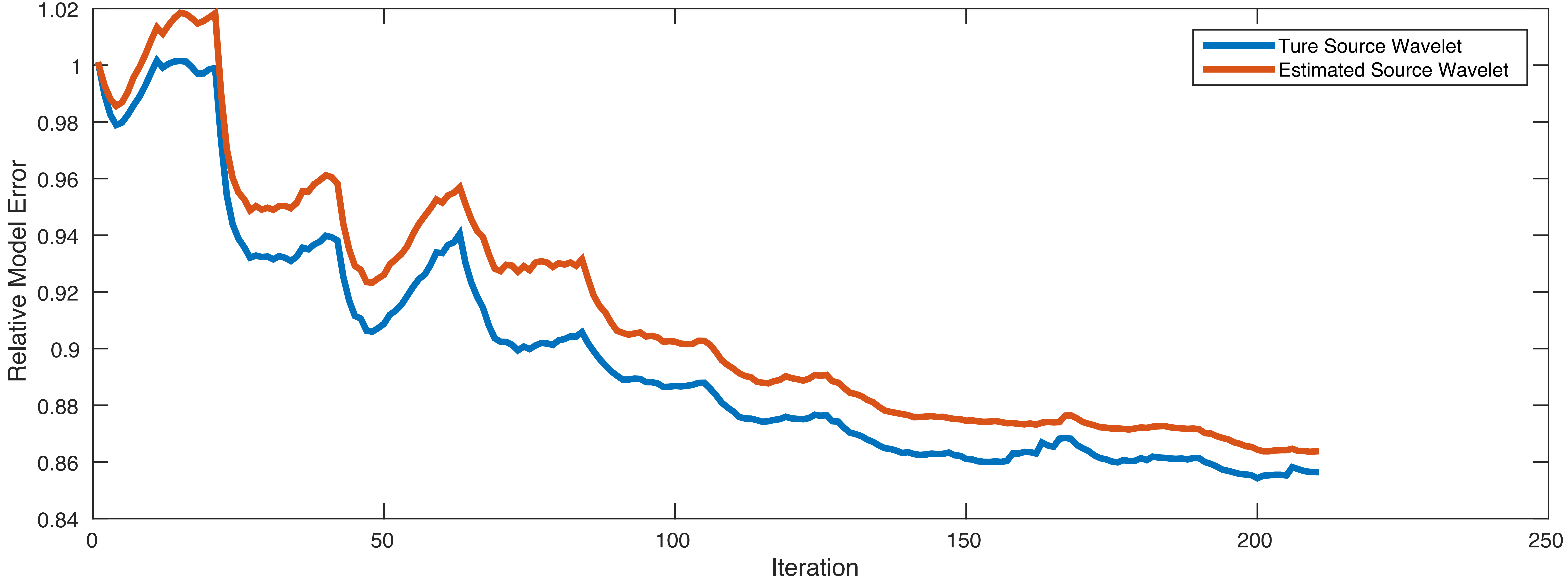
Result with true source wavelet



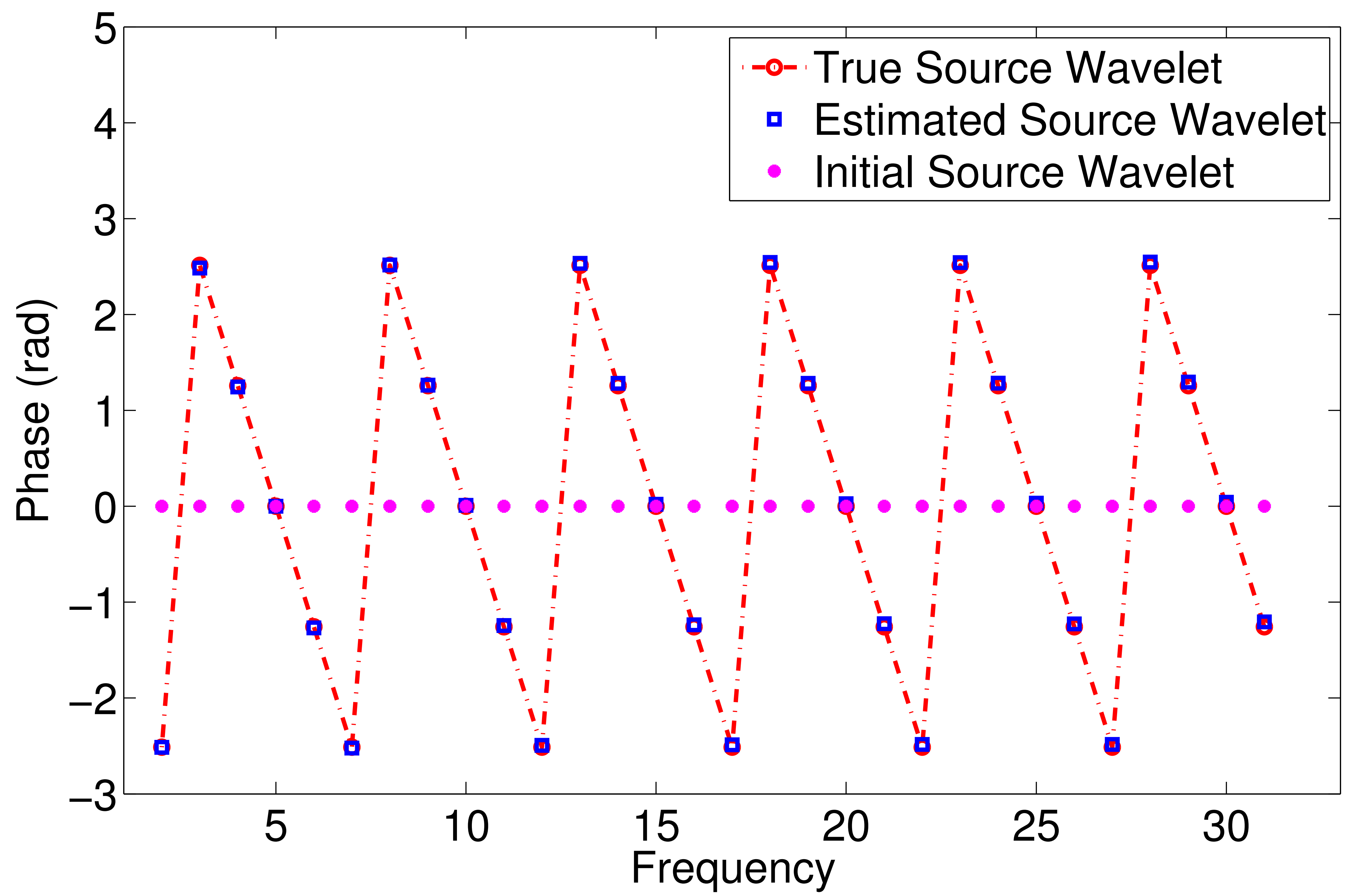
Result with estimated source wavelet



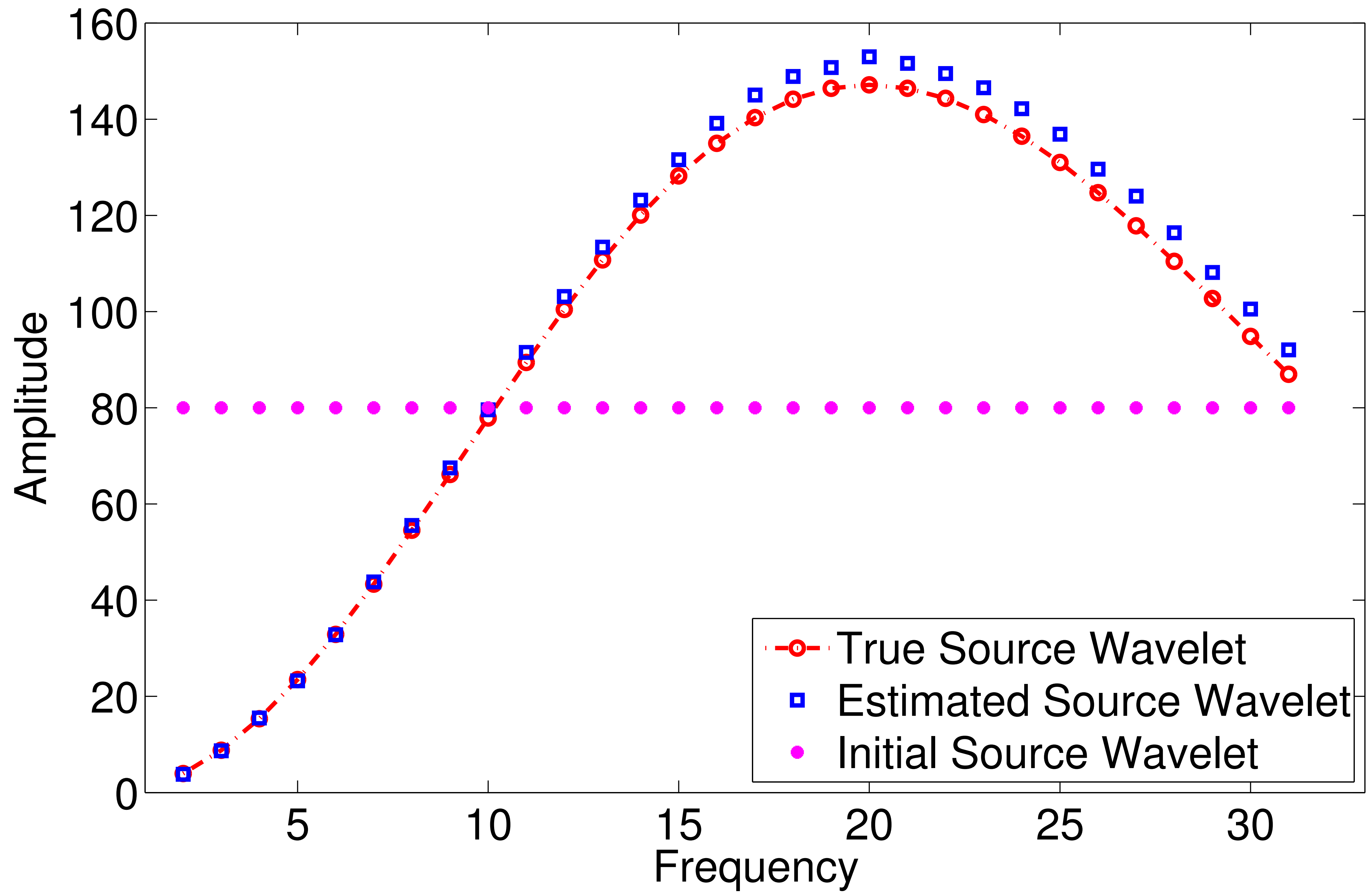
Relative model-error comparison



Source wavelet comparison

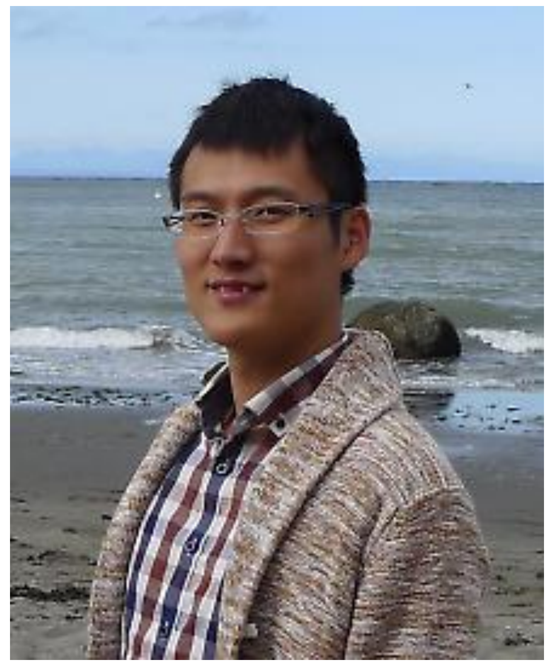


Phase

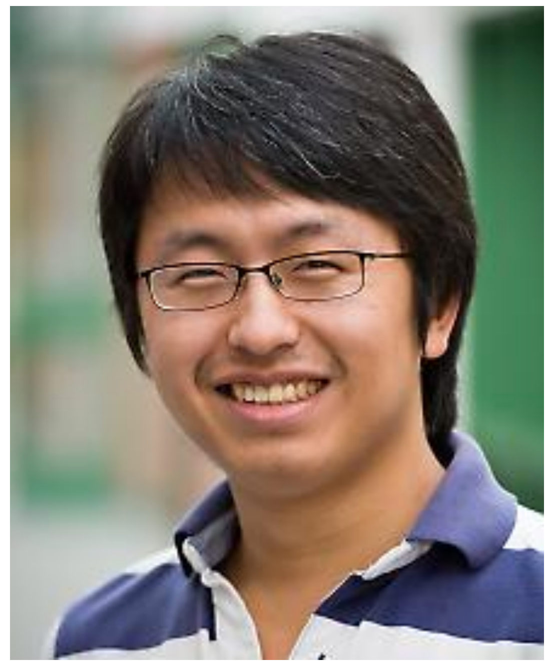


Amplitude

Chevron blind test data



Zhilong Fang



Xiang Li

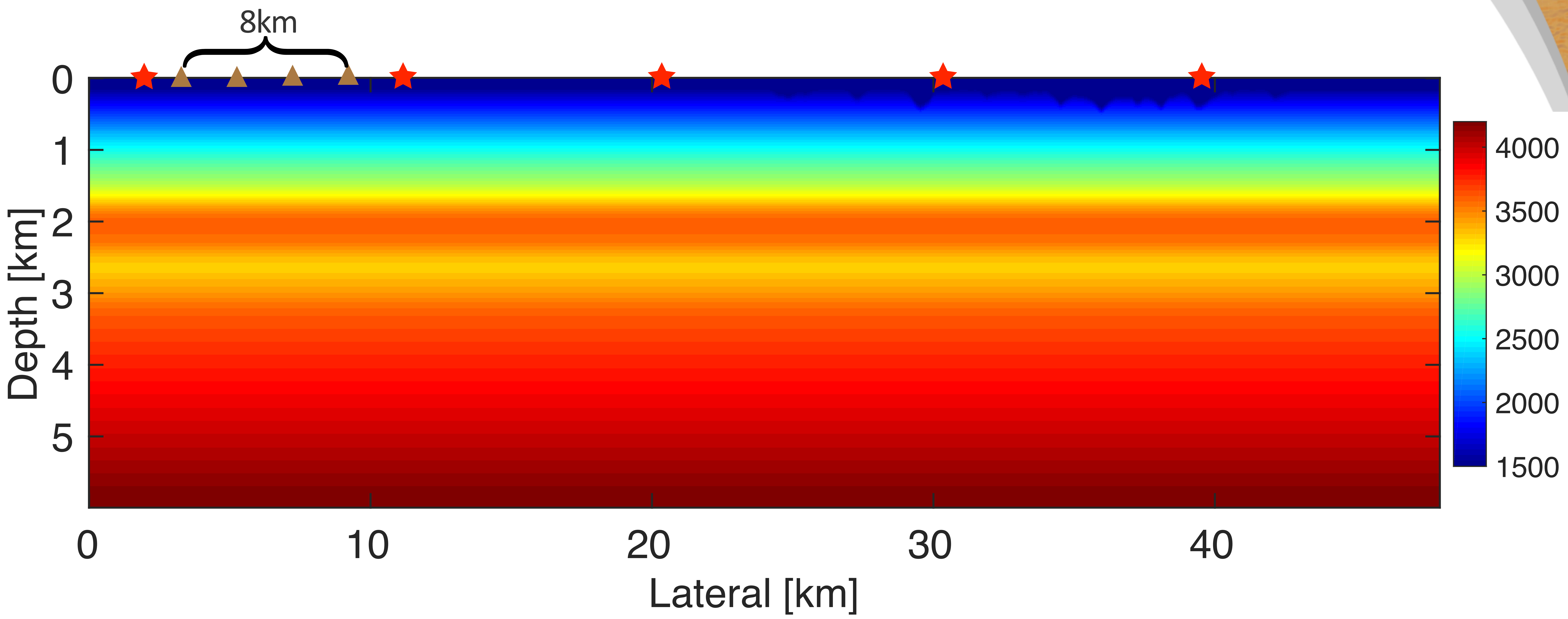
Bas Peters

Brendan Smithyman

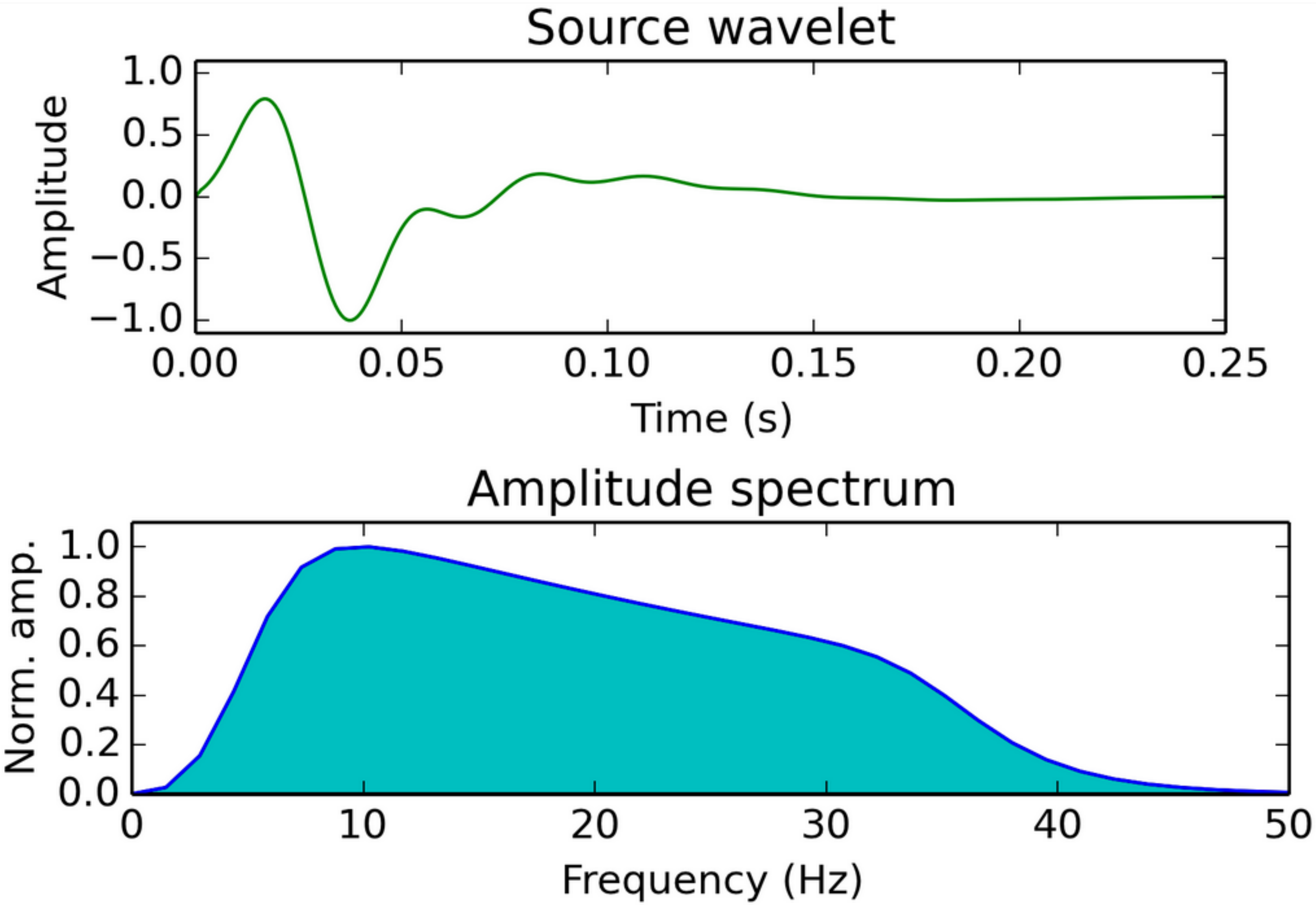
Mengmeng Yang

Felix J. Herrmann

Chevron blind test data



Chevron blind test data



Chevron blind test data

Data-set information:

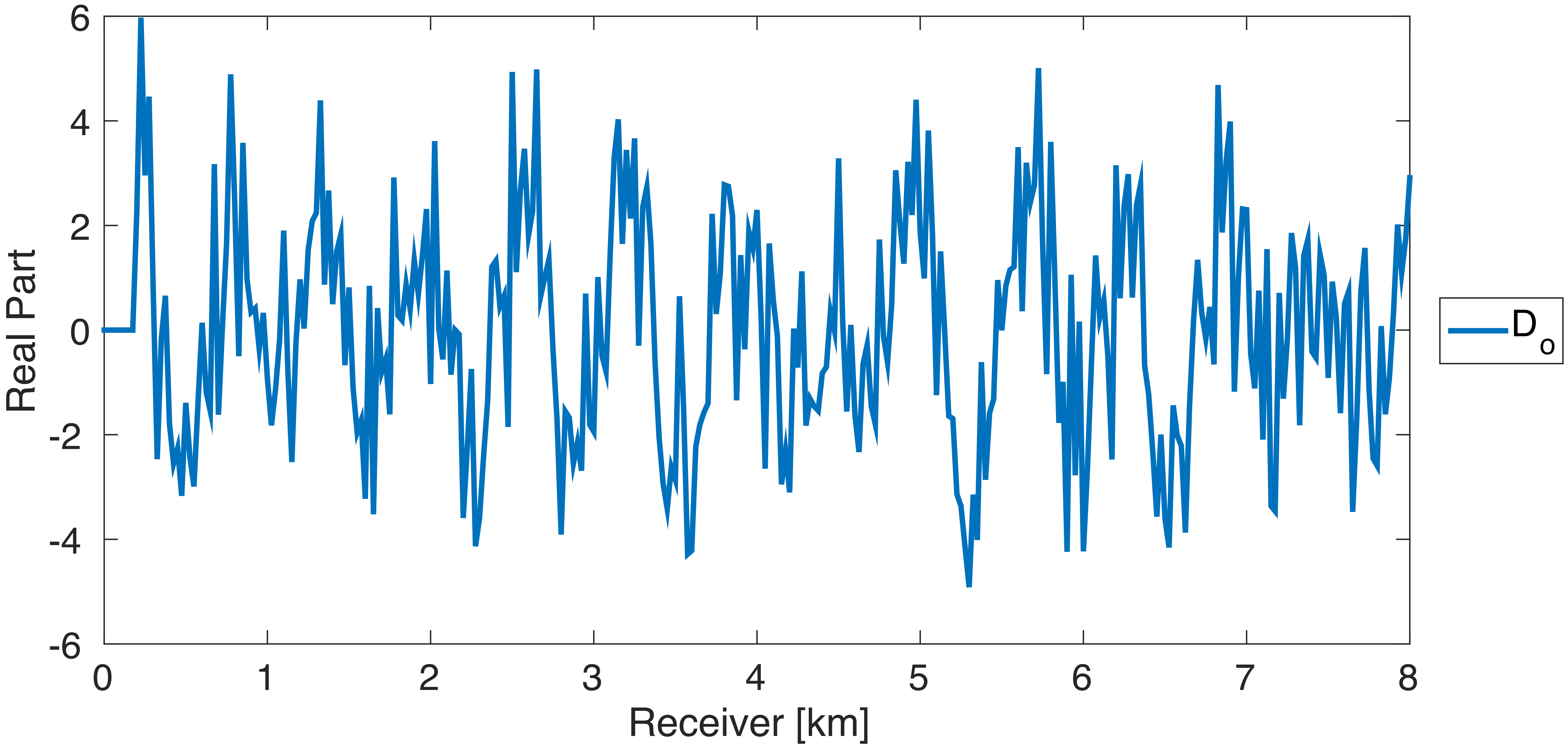
1. 1600 shots: ds = 25 m, Source depth = 15 m;
2. 321 hydrophone recs/shot: dr = 25 m, Receiver depth = 15 m;
3. Maximum offset = 8000 m;
4. Record time = 8.0 s, sample rate 4 ms;
5. Vp water = constant = 1510 m/s;
6. With free surface multiples present in the data;
7. Isotropic Elastic.

Chevron blind test data

Inversion strategy:

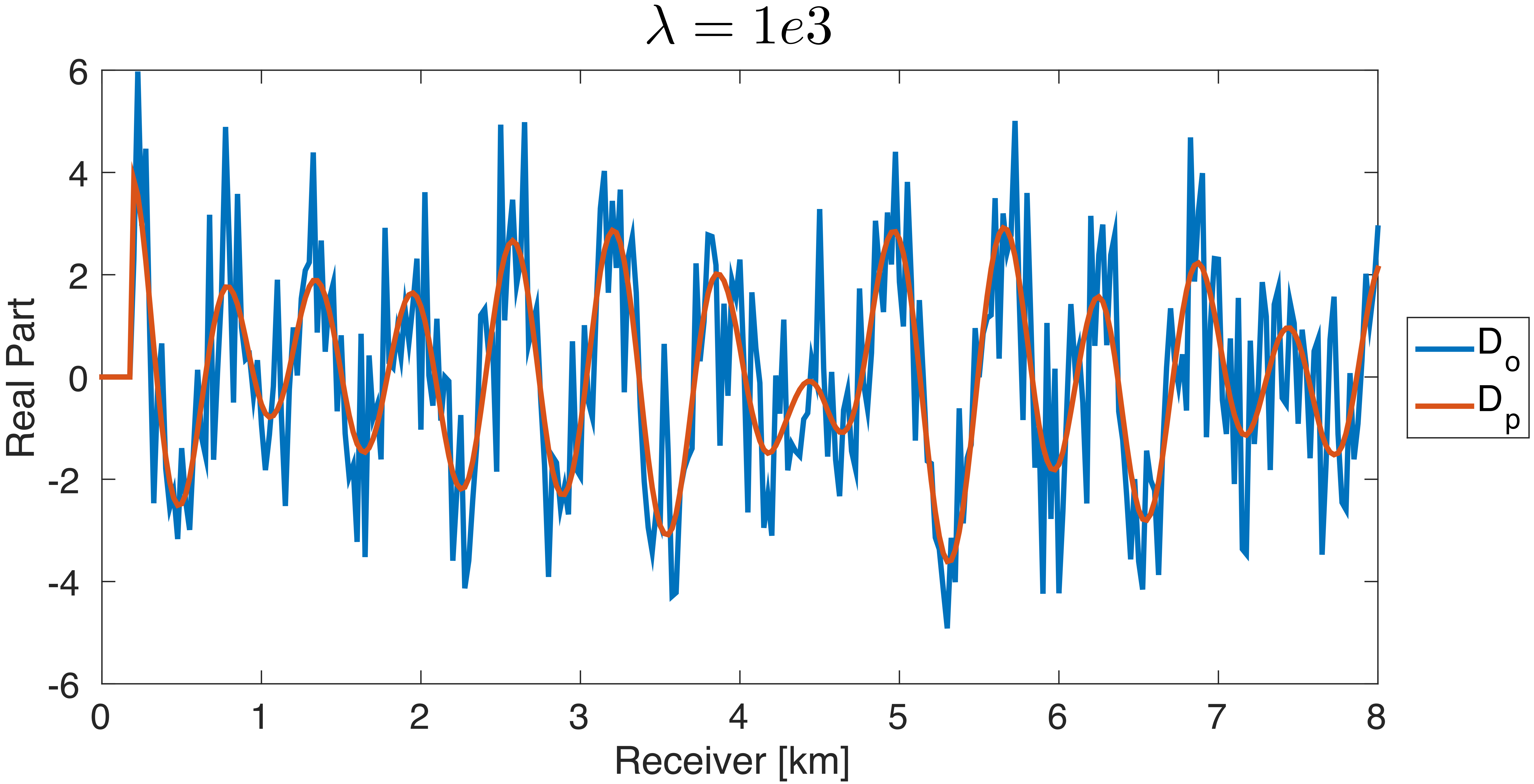
1. Frequency domain WRI with Source estimation;
2. Frequency bands: [3:0.2:5]Hz, [3:0.2:7]Hz, [3:0.2:9]Hz, [3:0.2:11]Hz, [3:0.2:19]Hz;
3. Batch sizes of random frequency subsets: 3, 6, 10, 10, 15;
4. Batch size of random source subsets: 300;
5. Optimization solver: l-BFGS with 20 iterations per frequency band;
6. 4 passes of WRI at frequency 3-11 Hz and 1 pass to 19 Hz;
7. Grid size: 20m for 3-11Hz and 12m for 3-19Hz;
8. No pre-processing !!!

Data comparison
— 3 Hz Data of 800th shot

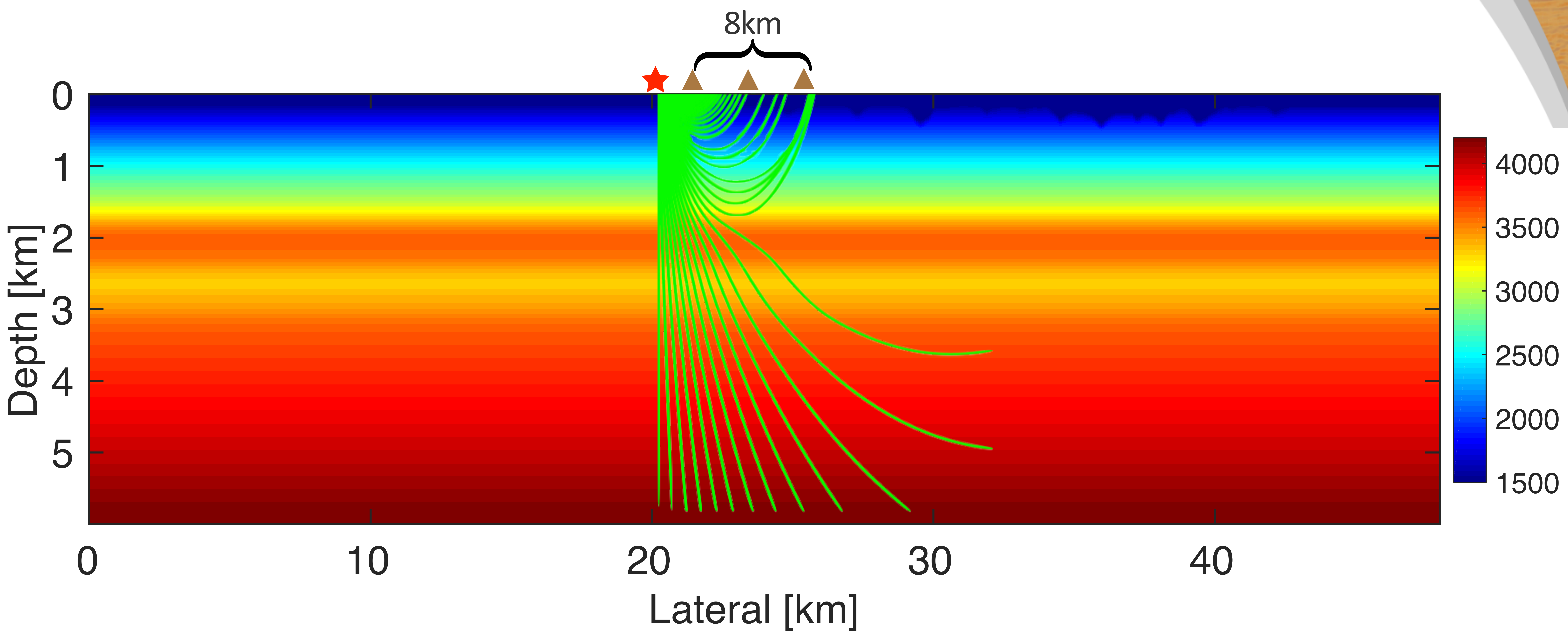


Data comparison

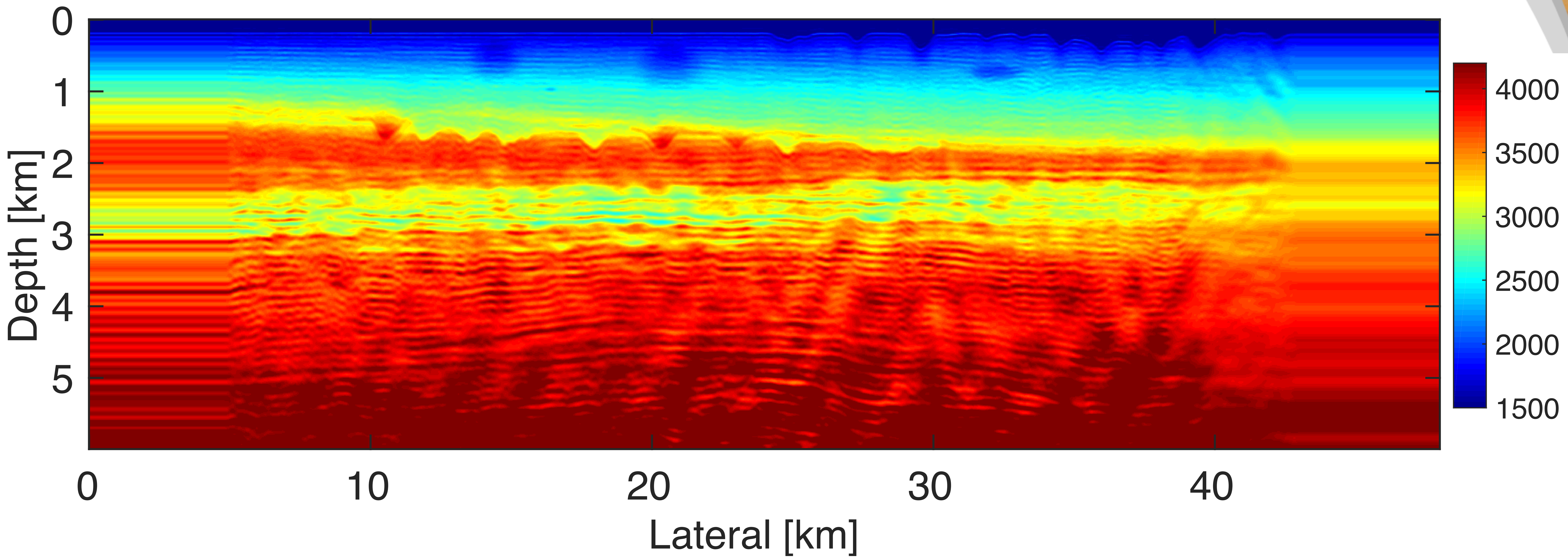
— 3 Hz Data of 800th shot



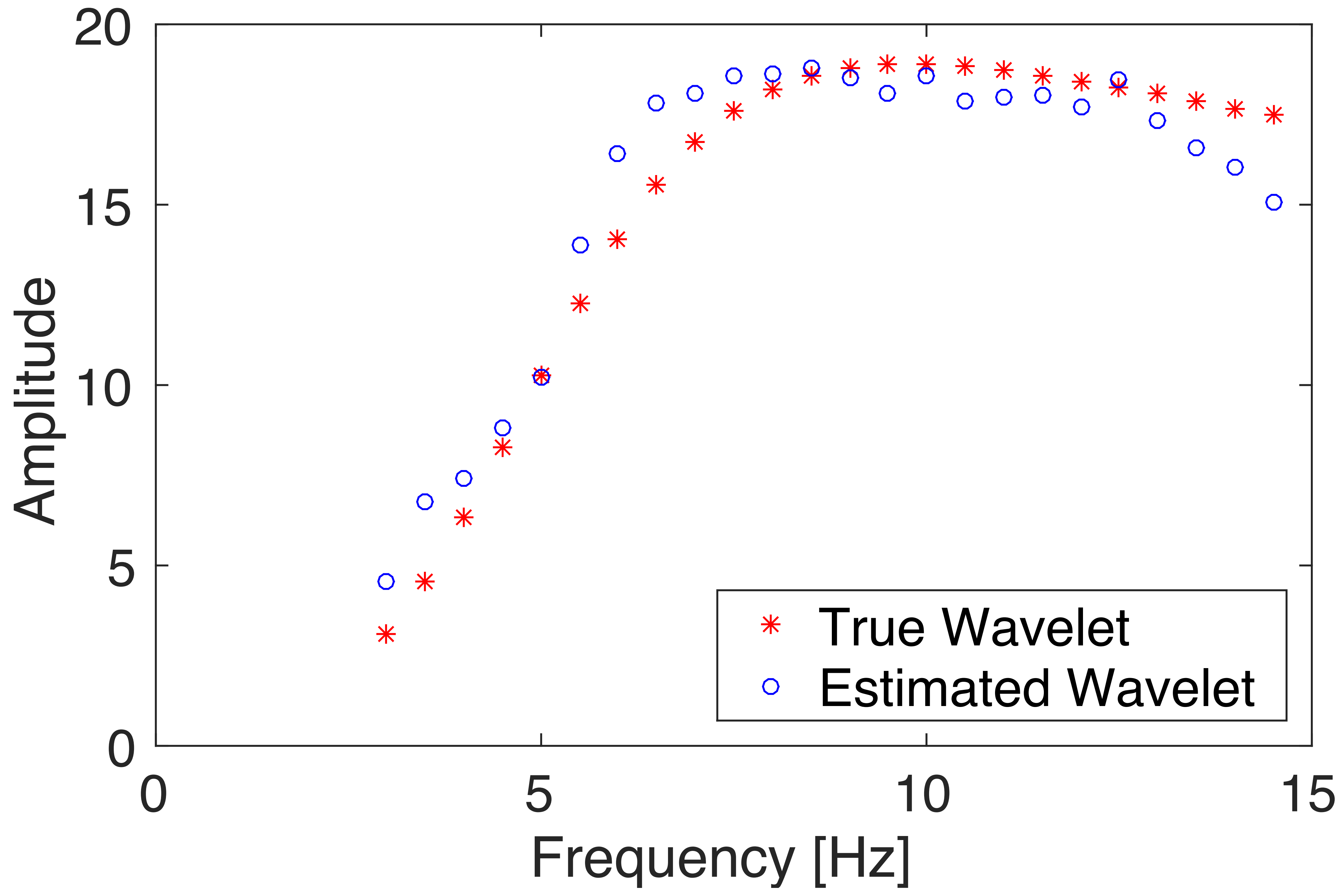
Initial model



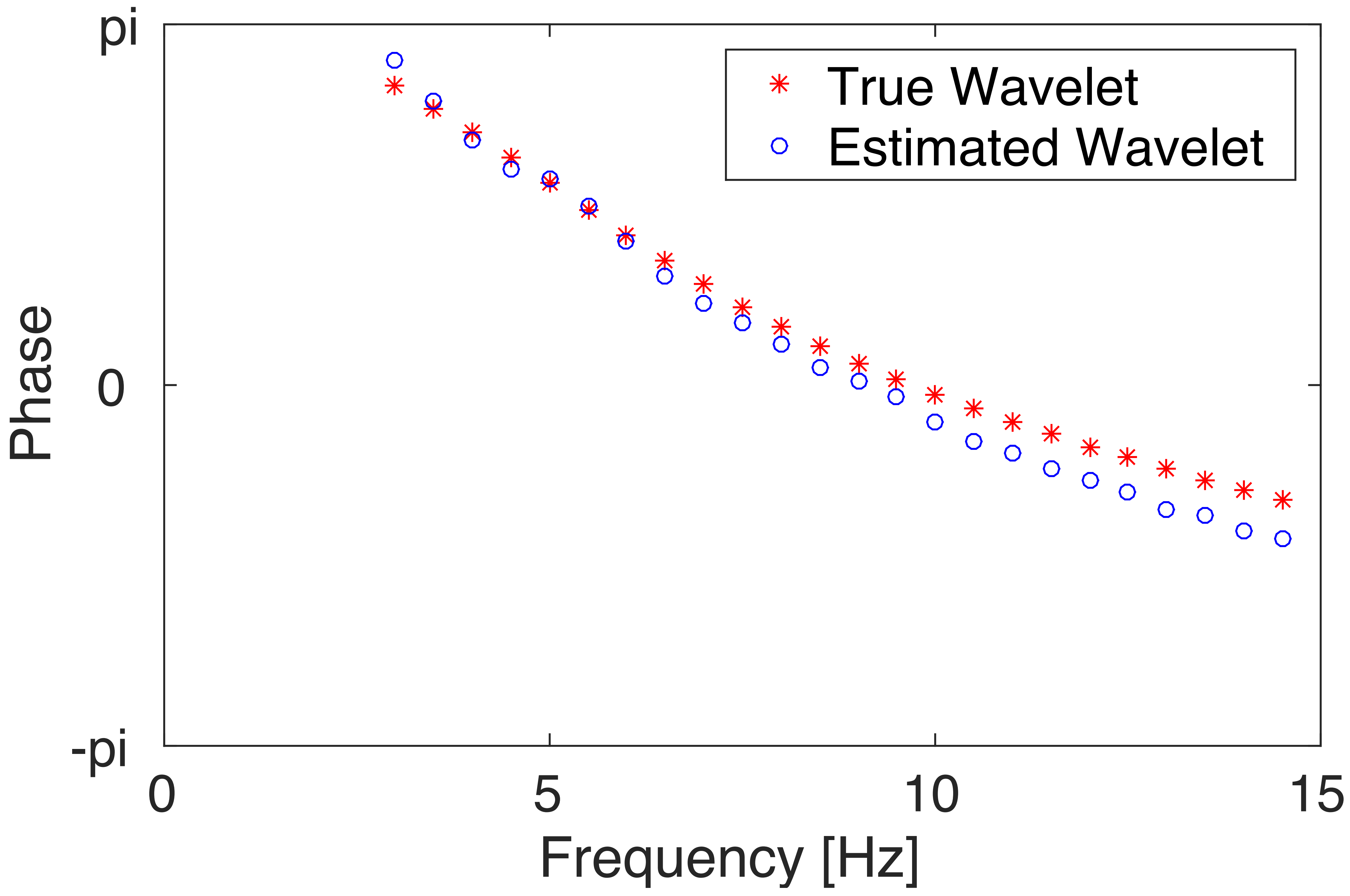
Inversion result



Source wavelet comparison

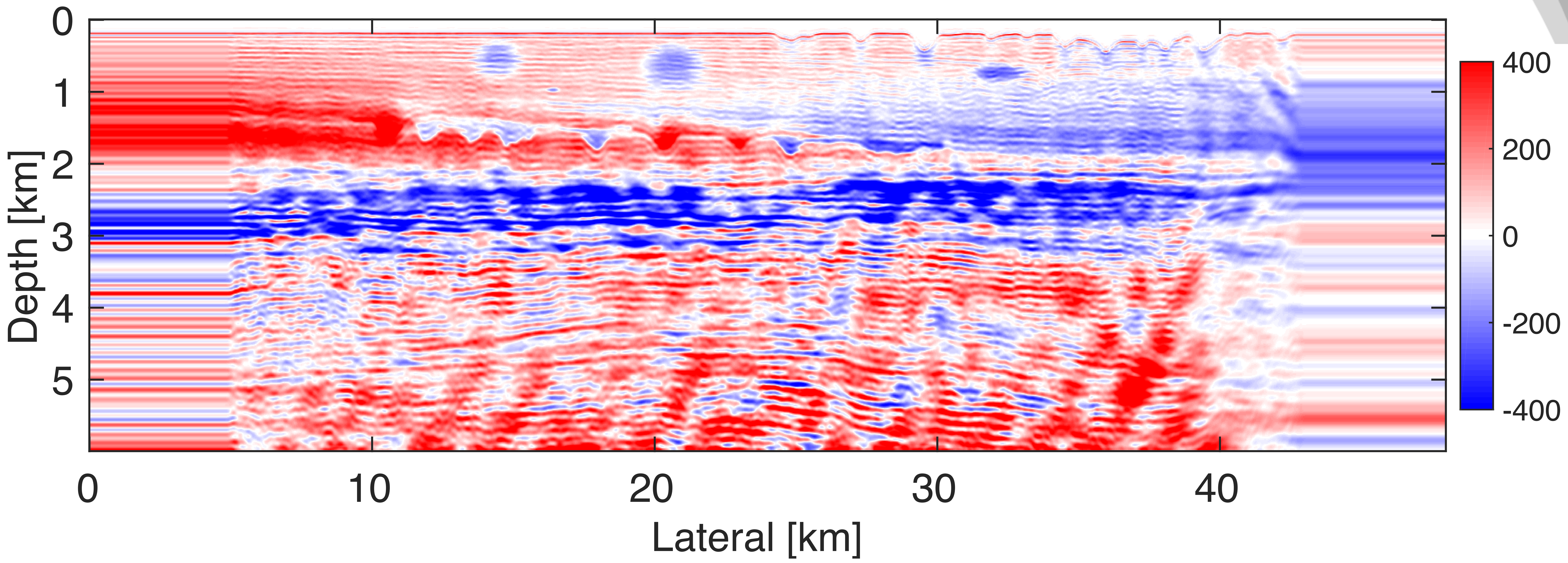


Amplitude

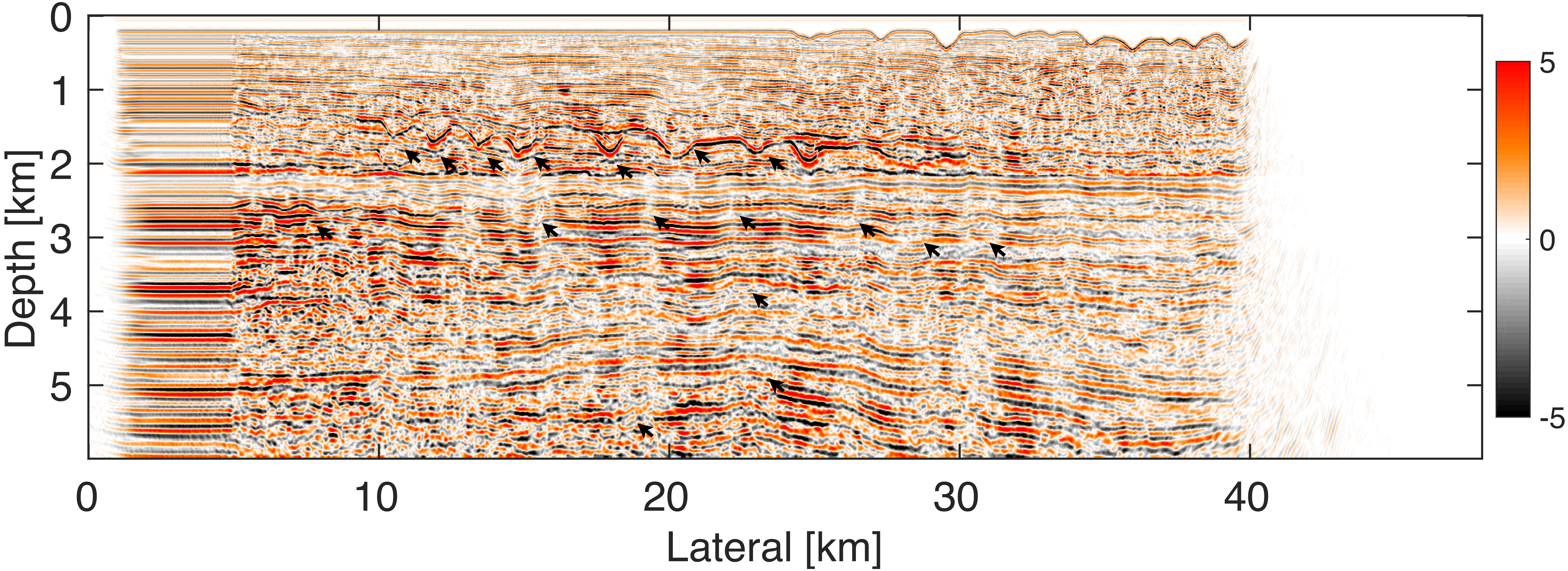


Phase

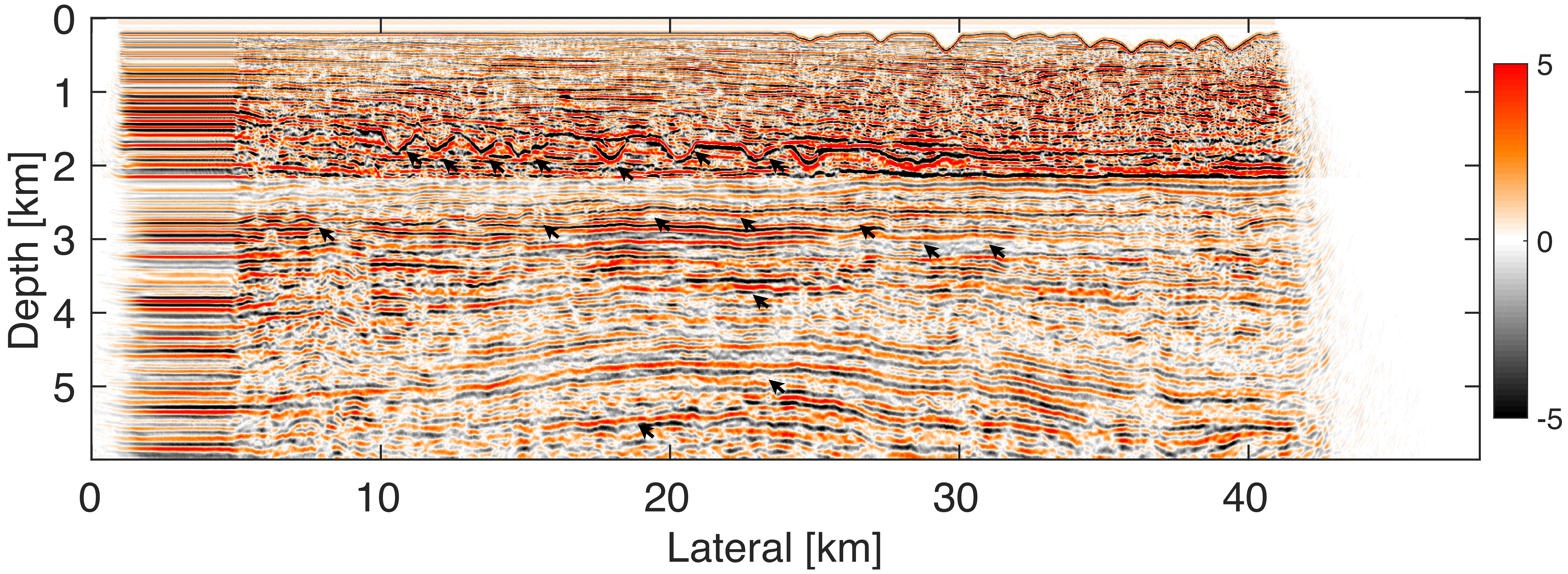
Model update



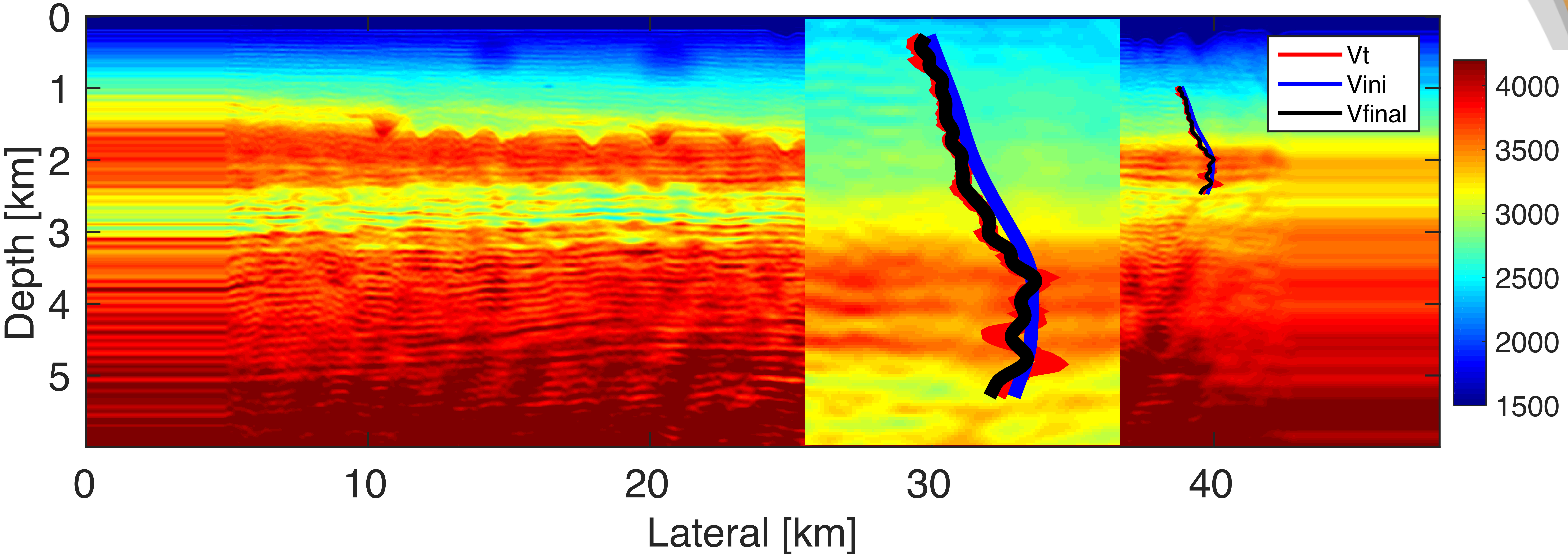
Kirchhoff migration
—Initial model



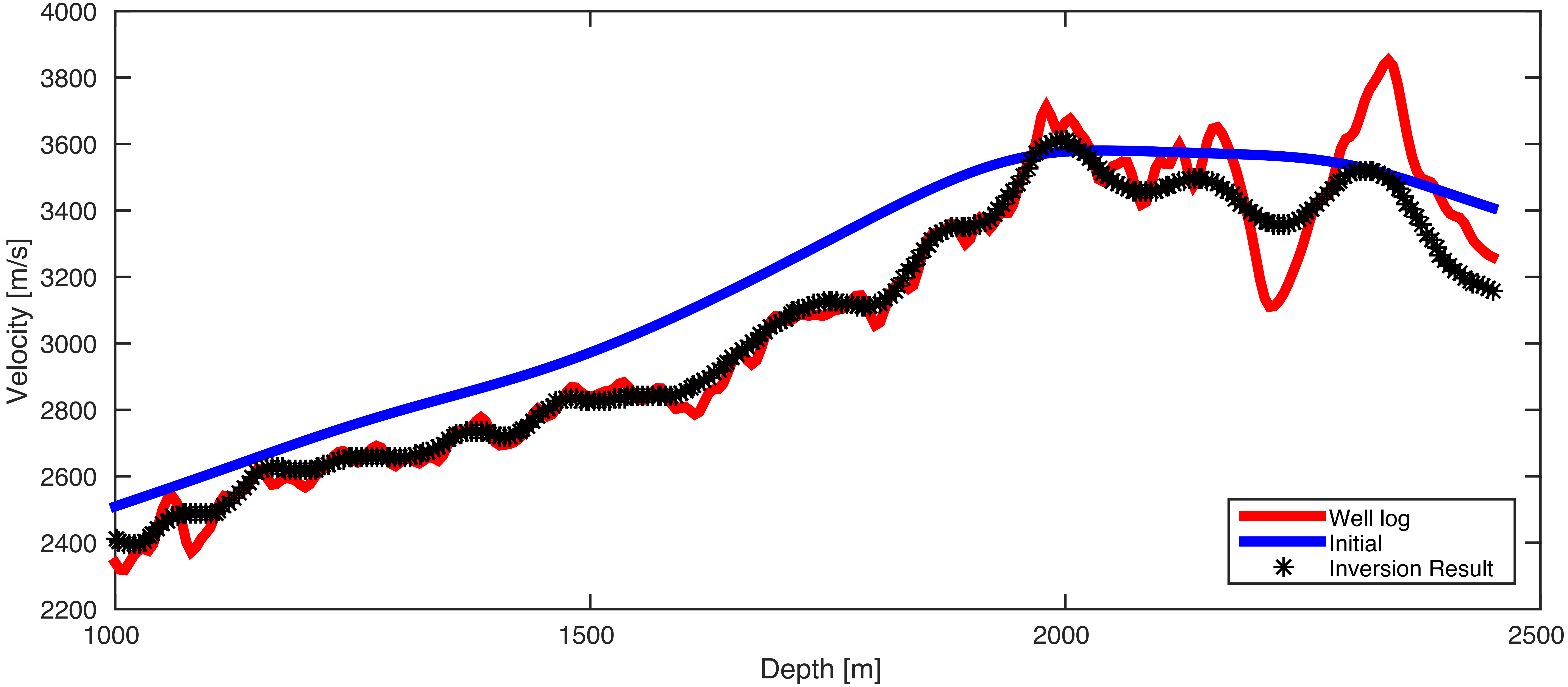
Kirchhoff migration
—Inversion result



Well-log comparison



Well-log comparison



Conclusions

1. Using the variable projection method, we can estimate the source wavelet for the WRI.
    - Synthetic BG model

2. Source estimation enhances the robustness of WRI for field seismic data.
    - Chevron blind test data

Acknowledgements

Thanks for help of our passed Post-Doctoral Fellow Ernie Esser.

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada via the Collaborative Research and Development Grant DNOISEII (375142--08). This research was carried out as part of the SINBAD II project which is supported by the following organizations: BG Group, BGP, CGG, Chevron, ConocoPhillips, DownUnder GeoSolutions, Hess, Petrobras, PGS, Schlumberger, Sub Salt Solutions and Woodside