POSTER Nº

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A lifted l1/l2 constraint for sparse blind deconvolution

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Blind deconvolution

$$y = w * x = (\alpha w) * (\frac{1}{\alpha} x) = w(t - t_0) * x(t + t_0)$$

The problem is ill-posed—has scaling and shift ambiguities.
Regularization CANNOT avoid these ambiguities.

Usual assumptions and regularizations

- w is short in time
- x is nearly sparse
- ℓ_2 penalty on w
- ℓ_1 penalty on x

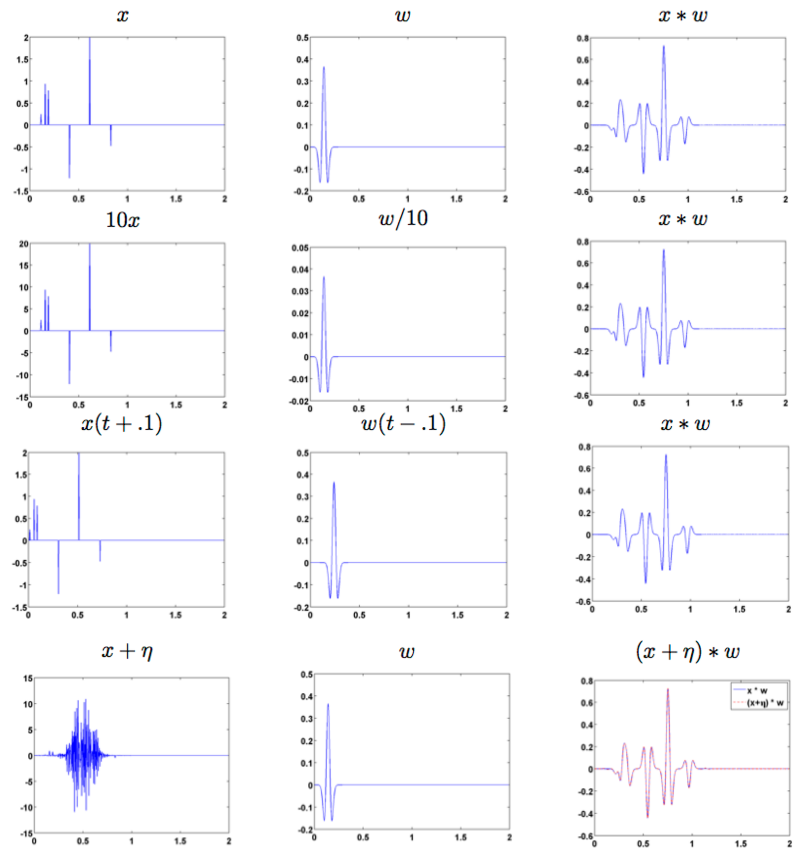


Figure 1. row 1: original signal, kernel and convolution
row 2: scaling ambiguity
row 3: shift ambiguity
row 4: other ambiguity

Blind deconvolution with multiples

$$y = w * x - w * x * x + w * x * x * x \cdots$$
$$= w * x - y * x$$

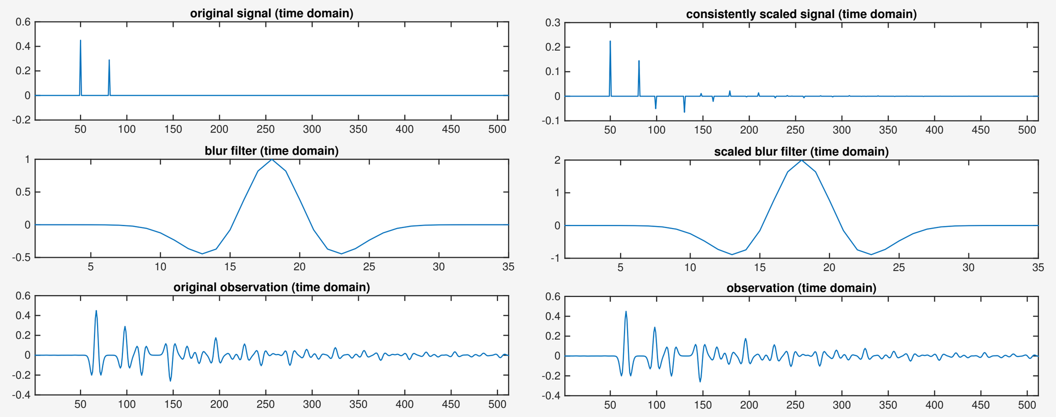


Figure 2. Left: original kernel and signal $\|x\|_1 \approx 0.74$
Right: scaled kernel and signal, $\|\hat{x}_\alpha\|_1 \approx 0.59$ with $\alpha = 2$.

Traditional solver (EPSI)

- Assume w is short in time: $w = Ch$, $C = [I, 0]$.
- Put ℓ_1 penalty on x, $(\hat{w}, \hat{x}) = \arg\min_{w,x} \|y - w * x + y * x\|_2 + \lambda \|x\|_1$
- Alternatively update x and w.

Observation: the scaling(shift) ambiguity is still there

$$\hat{y} = \hat{x}(\hat{w} - \hat{y}) = \hat{x}_\alpha(\hat{w}_\alpha - \hat{y}) \text{ where } w_\alpha = \alpha w, \quad \hat{x}_\alpha = \frac{\hat{x}}{(\alpha - 1)\hat{x} + \alpha}$$

Resolving the scaling issue with an ℓ_1/ℓ_2 penalty

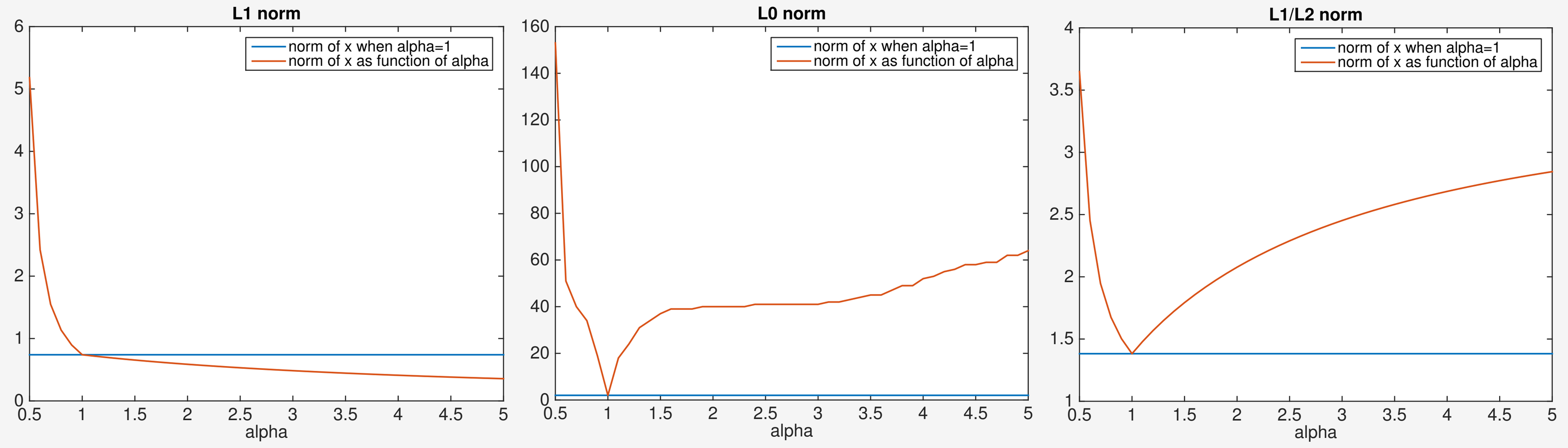


Figure 3.Which norm to choose? How do $\|x_\alpha\|_0$, $\|x_\alpha\|_1$ and $\|x_\alpha\|_1/\|x_\alpha\|_2$ change with α?

So we will use the ℓ_1/ℓ_2 constraint and solve

$$\min \log \frac{\|x\|_1}{\|x\|_2} \quad \text{subject to } \|y - w * x + y * x\|_2 \le \epsilon$$

Solving the optimization problem (method of multipliers)

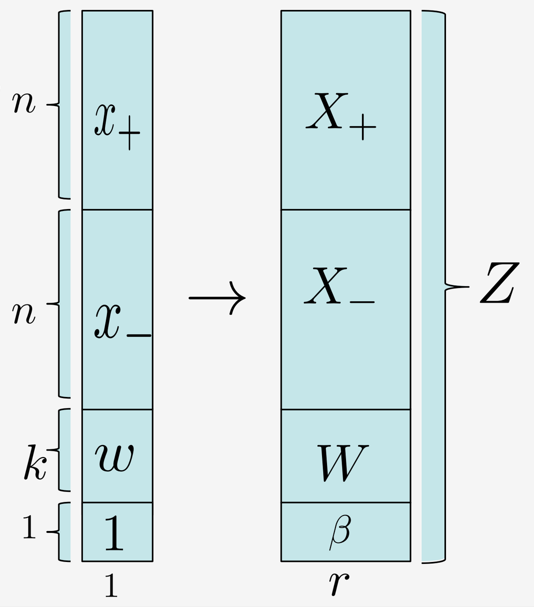
Original problem (non-convex, non differentiable)

$$\min_{x,w} \log(\|x\|_1/\|x\|_2)$$

subject to $\|\hat{y} - \mathrm{diag}(\hat{w}\hat{x}^T) - \mathrm{diag}(\hat{y}\hat{x}^T)\|_2 \le \epsilon$ data constraint
$\|x\|_\infty \le 1$ box constraint

Lifting: Mitigate local minima

n : length of x
k : length of w
r : lifted rank



Split x into positive and negative parts (non-convex, differentiable at $x \neq 0$) $x = x_+ - x_-$

$$\min_{x,w} \log(\sum x_+ + \sum x_-)/(\|x_+ + x_-\|_2)$$

subject to $\|\hat{y} - \mathrm{diag}(\hat{w}\hat{x}^T) - \mathrm{diag}(\hat{y}\hat{x}^T)\|_2 \le \epsilon$ data constraint
$0 \le x_+, x_-, \le 1$ box constraint
$\langle x_+, x_- \rangle = 0$ non-overlapping constraint

Final optimization problem

Low rank penalty — L1 norm — L2 norm

$$\min_{X_+, X_-, W, \beta} \mathrm{Trace}(Z^T Z) - \|Z^T Z\|_F + \log \frac{\mathbf{1}^T(X_+ + X_-)(X_+ + X_-)^T \mathbf{1}}{\mathrm{Trace}(X_+ - X_-)(X_+ - X_-)^T}$$

subject to $\|\hat{y} - \mathrm{diag}(\hat{W}\hat{X}^T) - \mathrm{diag}(\hat{y}(\hat{X}\beta^T)^T)\|_2 \le \epsilon$ data constraint
$0 \le X_+, X_- \le 1$ box constraint
$\langle X_+, X_- \rangle = 0$ non-overlapping constraint
$\|\beta\|_2 = 1$ weights sum up to 1

Reconstruct (x, w) from Z

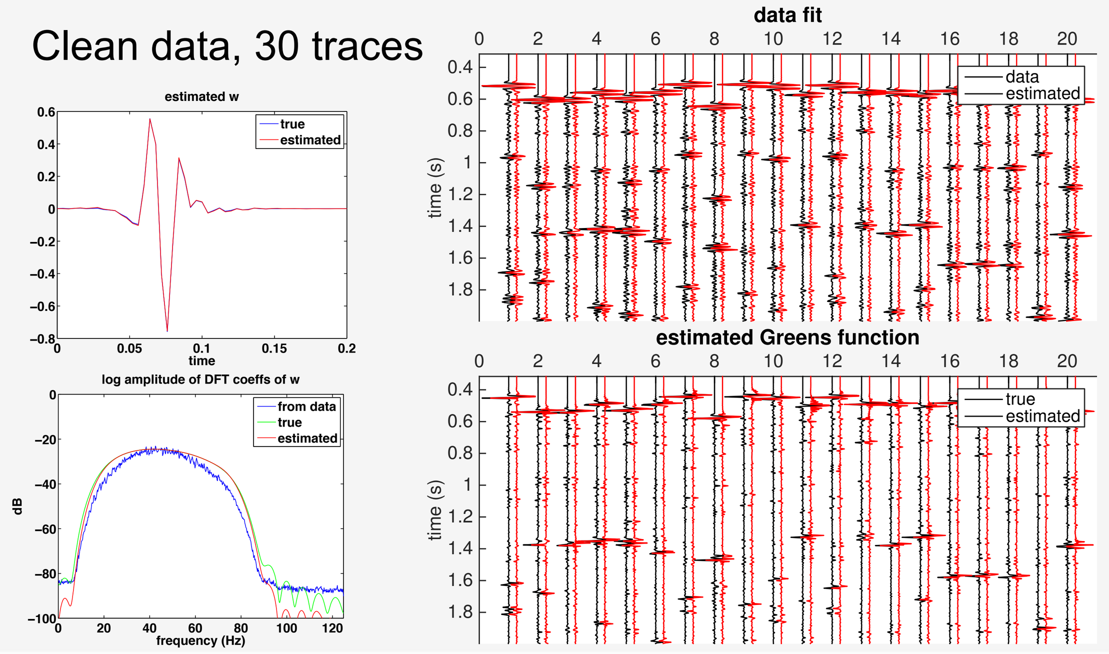
extract the first left singular vector v, and singular value σ of Z

$$x_+ = \sigma v(1, .., n)$$
$$x_- = \sigma v(n + 1, ..., 2n)$$
$$w = \sigma v(2n + 1, ..., 2n + k)$$

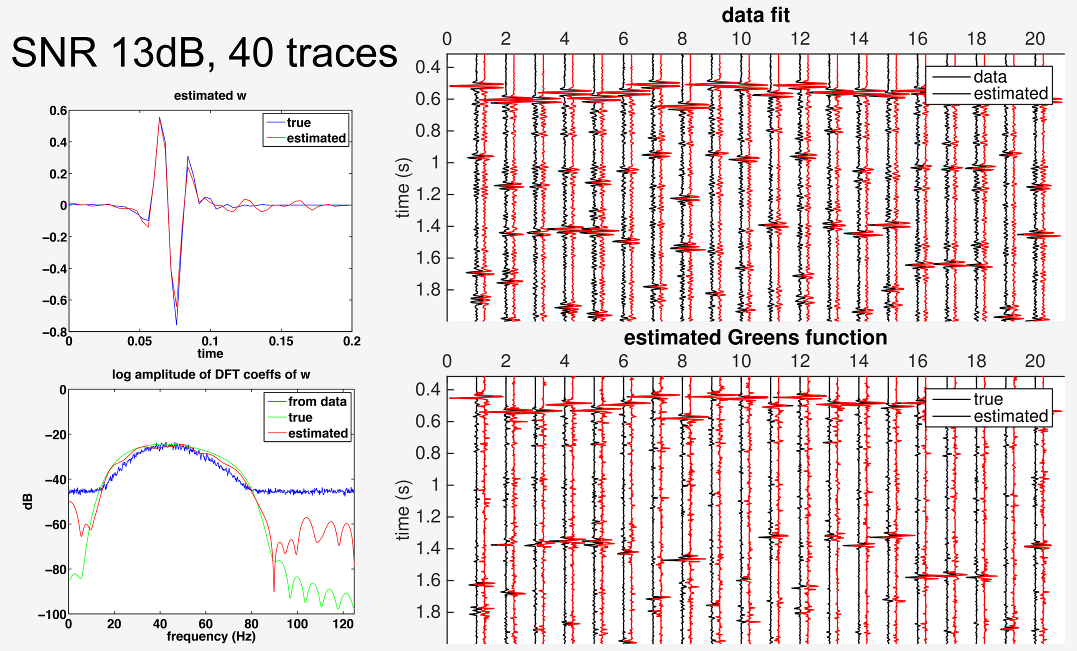
Pluto1.5 data

Initial guess: $w = 0$, $x =$ normalized random Gaussian vector

Clean data, 30 traces



SNR 13dB, 40 traces



†John "Ernie" Esser (May 19, 1980 – March 8, 2015)

This work is a reflection of Ernie's extraordinary contributions to this challenging problem. Unfortunately, Ernie was not able to see the final results of his original work. We miss him dearly, and will continue to work on this exciting approach.

Acknowledgements

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