Accelerating an Iterative Helmholtz Solver with FPGAs

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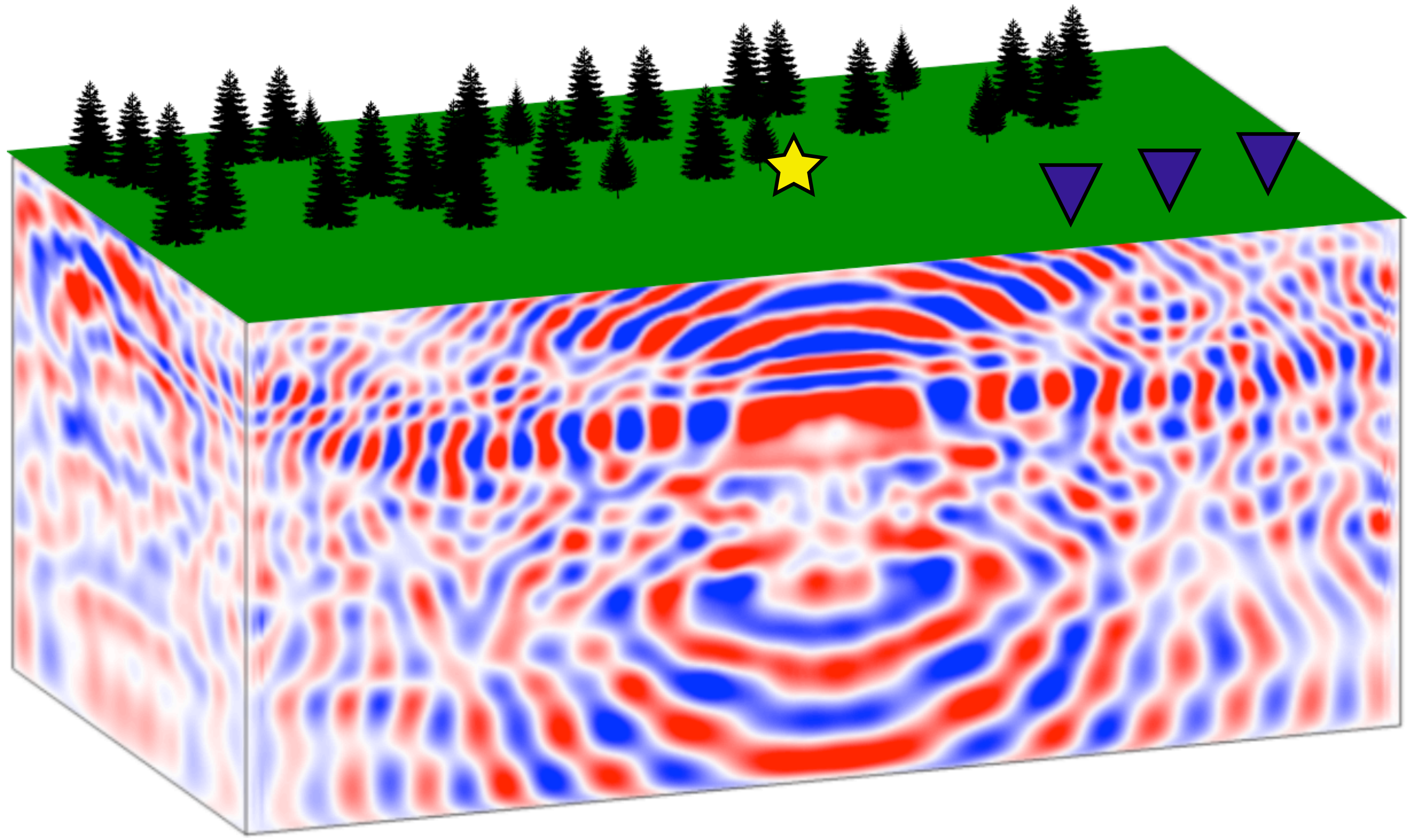
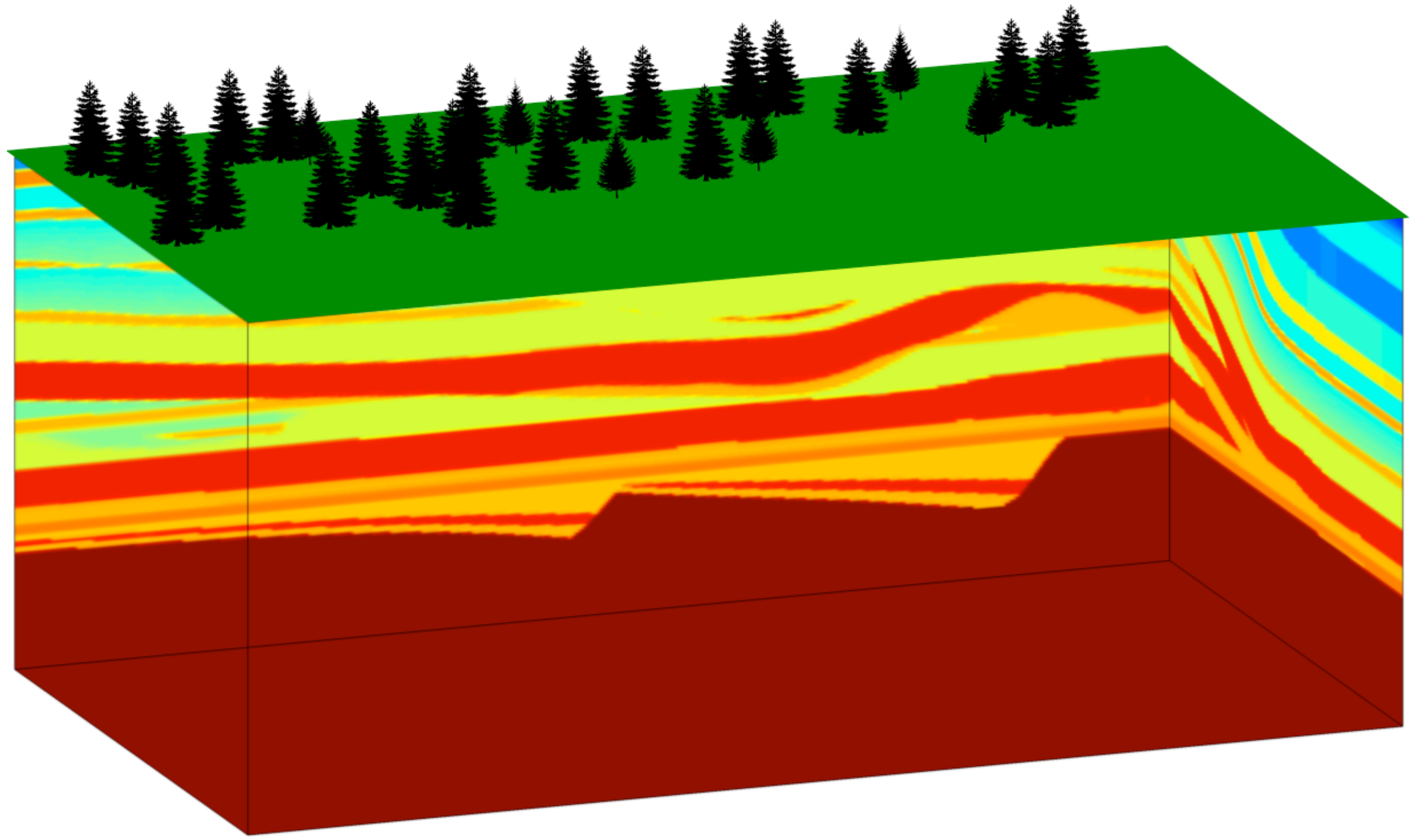
Oh by the way: I have a stutter.

Seismic Wave Simulation

Full-waveform Inversion

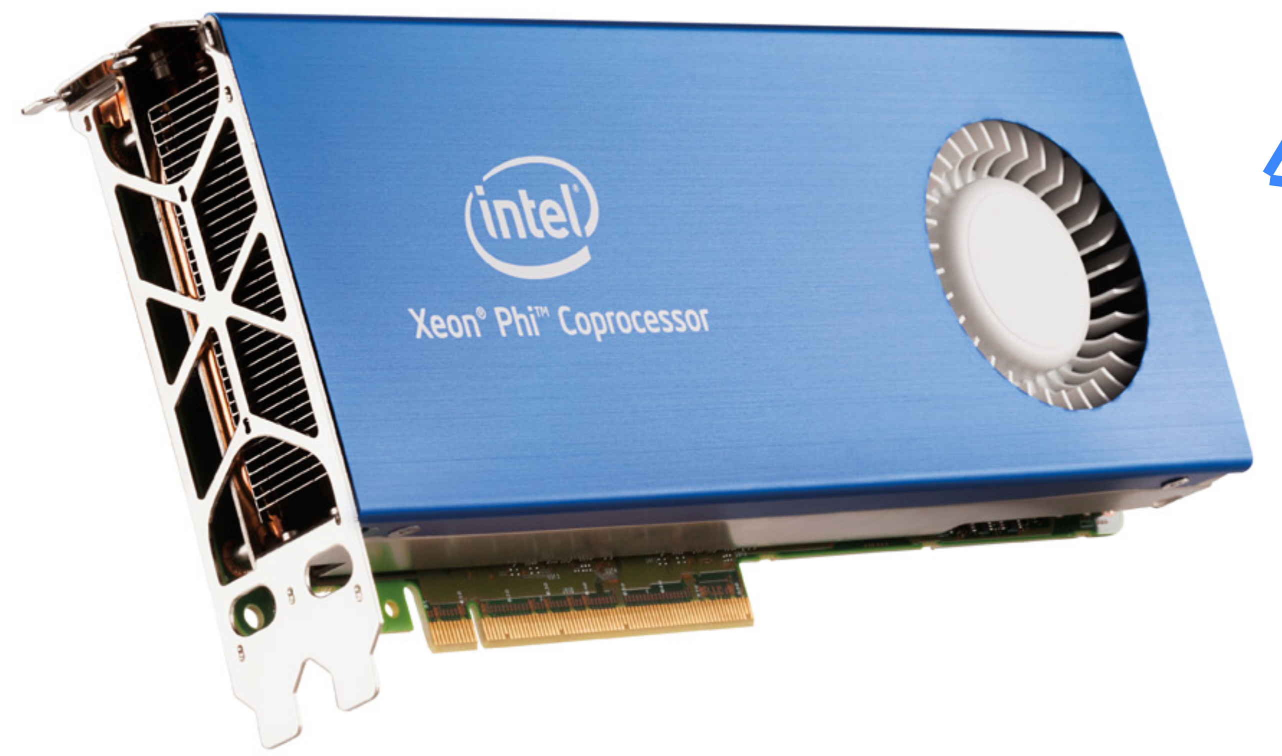


Seismic Wavefield (**u**)



Earth model (**m**)

The Accelerators Have Arrived



Top 10 of "Top 500" Supercomputers

Rank	Site	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	National Super Computer Center in Guangzhou China	**Tianhe-2 (MilkyWay-2)** - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P NUDT	3,120,000	33,862.7	54,902.4	17,808
2	DOE/SC/Oak Ridge National Laboratory United States	**Titan** - Cray XK7 , Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x Cray Inc.	560,640	17,590.0	27,112.5	8,209
3	DOE/NNSA/LLNL United States	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom IBM	1,572,864	17,173.2	20,132.7	7,890
4	RIKEN Advanced Institute for Computational Science (AICS) Japan	K computer, SPARC64 VIIIfx 2.0GHz, Tofu interconnect Fujitsu	705,024	10,510.0	11,280.4	12,660
5	DOE/SC/Argonne National Laboratory United States	**Mira** - BlueGene/Q, Power BQC 16C 1.60GHz, Custom IBM	786,432	8,586.6	10,066.3	3,945
6	Swiss National Supercomputing Centre (CSCS) Switzerland	**Piz Daint** - Cray XC30, Xeon E5-2670 8C 2.600GHz, Aries interconnect , NVIDIA K20x Cray Inc.	115,984	6,271.0	7,788.9	2,325
7	Texas Advanced Computing Center/Univ. of Texas United States	**Stampede** - PowerEdge C8220, Xeon E5-2680 8C 2.700GHz, Infiniband FDR, Intel Xeon Phi SE10P Dell	462,462	5,168.1	8,520.1	4,510
8	Forschungszentrum Juelich (FZJ) Germany	**JUQUEEN** - BlueGene/Q, Power BQC 16C 1.600GHz, Custom Interconnect IBM	458,752	5,008.9	5,872.0	2,301
9	DOE/NNSA/LLNL United States	**Vulcan** - BlueGene/Q, Power BQC 16C 1.600GHz, Custom Interconnect IBM	393,216	4,293.3	5,033.2	1,972
10	Leibniz Rechenzentrum Germany	**SuperMUC** - iDataPlex DX360M4, Xeon E5-2680 8C 2.70GHz, Infiniband FDR IBM	147,456	2,897.0	3,185.1	3,423

FPGAs: Reconfigurable Hardware Accelerators

The Punchline

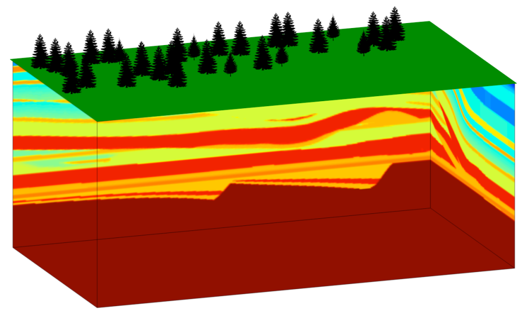
Modelling Seismic Waves
Mathematical Formulation

Modelling Seismic Waves: The Wave Equation

$$\left(\omega^2 \mathbf{m} + \Delta\right)\mathbf{u} = \mathbf{q}$$

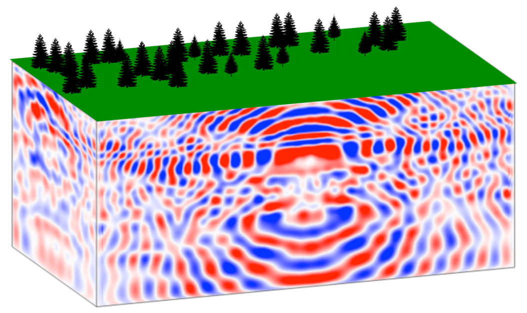
Frequency

Earth Model



Laplacian

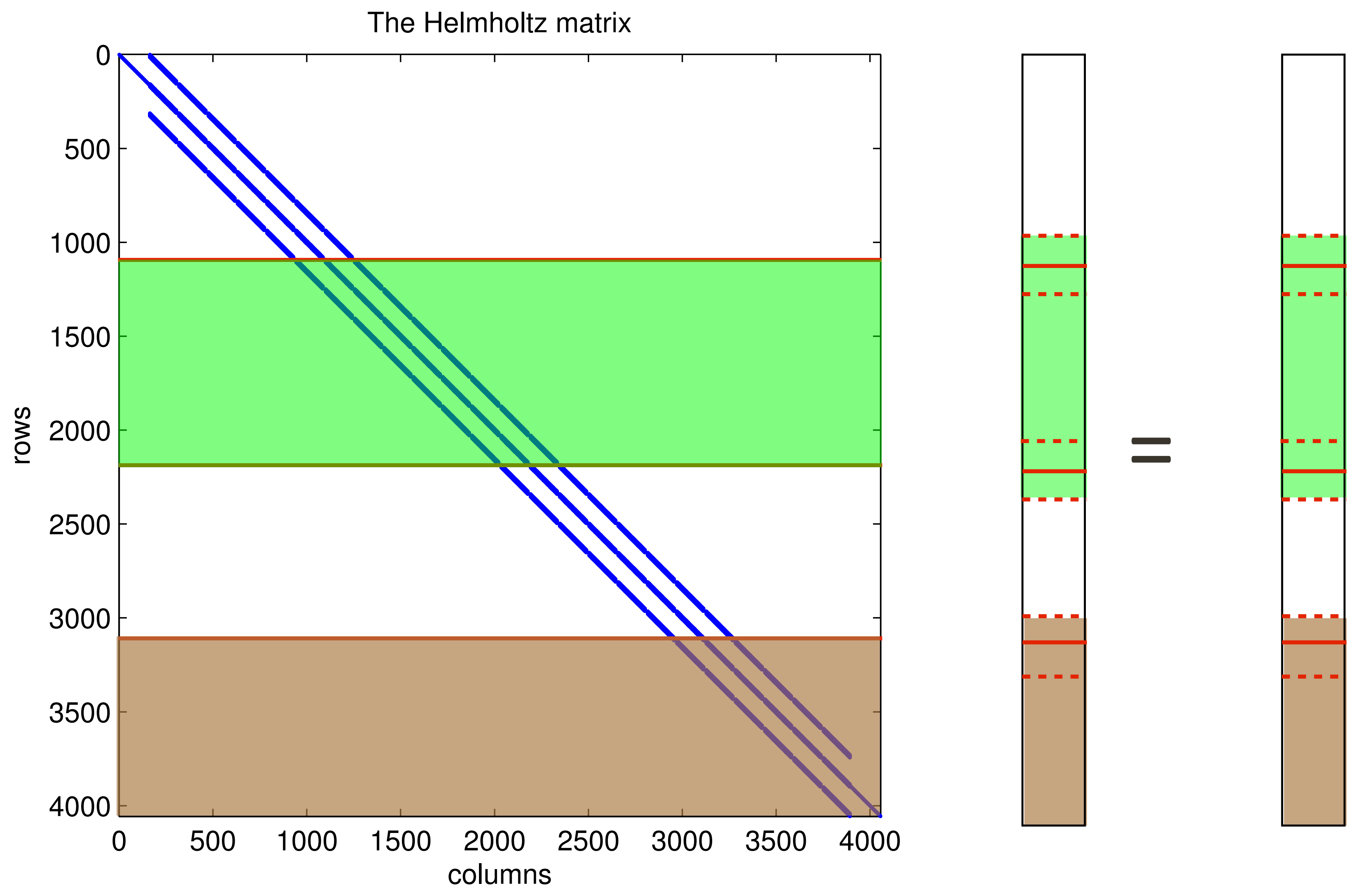
Wavefield (iterate)



Seismic Source (right-hand side)

Modelling Seismic Waves: Discretization
[Operto, 2007]

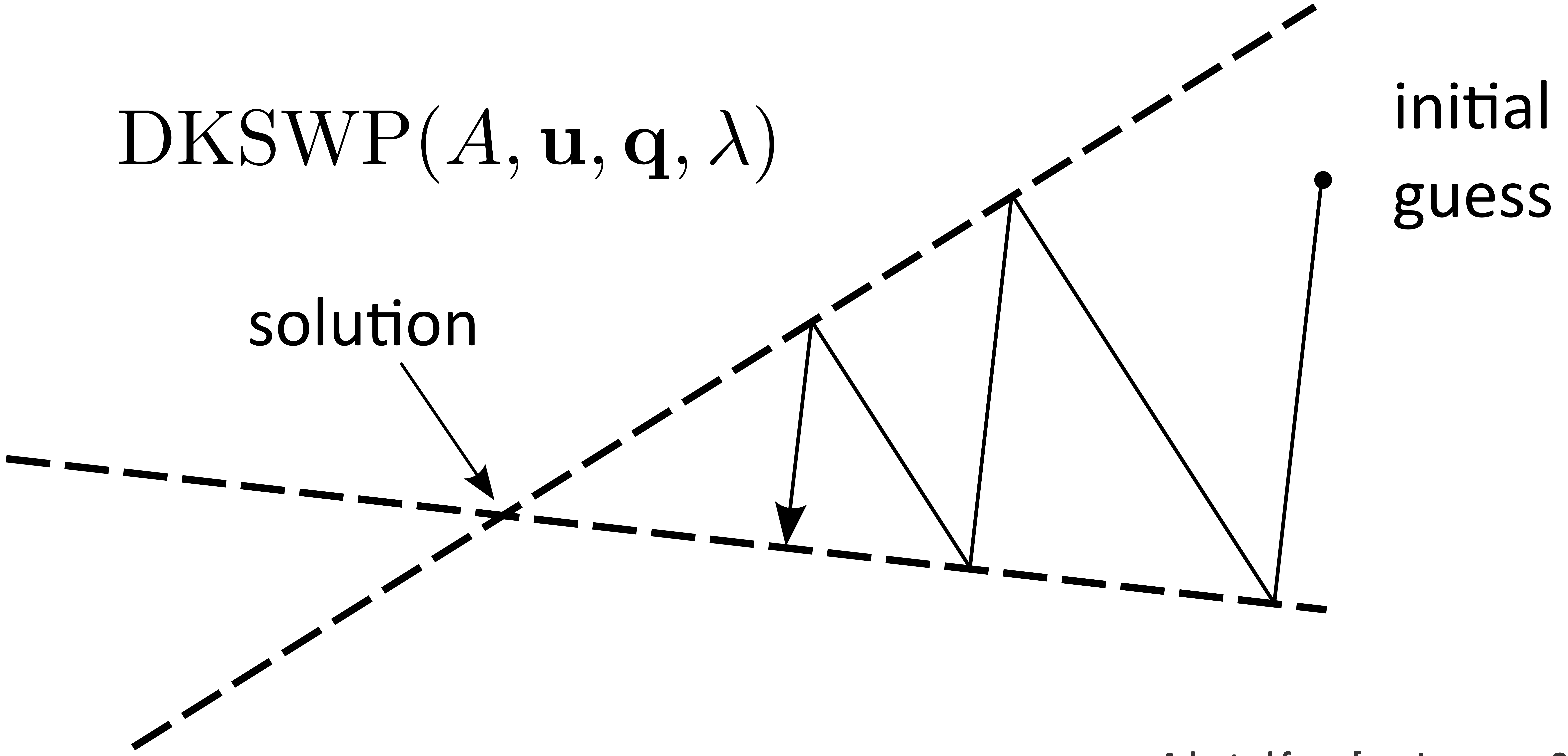
$$A(\mathbf{m}, \omega)\mathbf{u} = \mathbf{q}$$



Solving the Helmholtz System

The Kaczmarz Algorithm
[Kaczmarz, 1937]

$$\mathrm{DKSWP}(A, \mathbf{u}, \mathbf{q}, \lambda)$$



Adapted from [van Leeuwen, 2012]

The Kaczmarz Algorithm: Equivalent to SSOR-NE
[Björck and Elfving, 1979]

Double Kaczmarz sweep on the original system:

$$A\mathbf{u} = \mathbf{q}$$

↔

One iteration of SSOR on the normal equations:

$$AA^*\mathbf{y} = \mathbf{q}$$
$$A^*\mathbf{y} = \mathbf{u}$$

Both are computed as:

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \lambda(b_i - \langle \mathbf{a}_i, \mathbf{u}_k \rangle) \frac{\mathbf{a}_i^*}{\|\mathbf{a}_i\|^2}$$

$$k : 1 \rightarrow 2N$$
$$i : 1 \rightarrow N, N \rightarrow 1$$

Kaczmarz + CG = CGMN

[Björck & Elfving 1979]

CGMN: Solves for Fixed Point of Kaczmarz Row Projections

$$\begin{aligned} \mathrm{DKSWP}(A, \mathbf{u}, \mathbf{q}, \lambda) &= Q_1 \cdots Q_N Q_N \cdots Q_1 \mathbf{u} + R\mathbf{q} \\ &= Q\mathbf{u} + R\mathbf{q}. \end{aligned}$$

Assume **u** is a solution and re-arrange:

$$(I - Q)\mathbf{u} = R\mathbf{q}$$

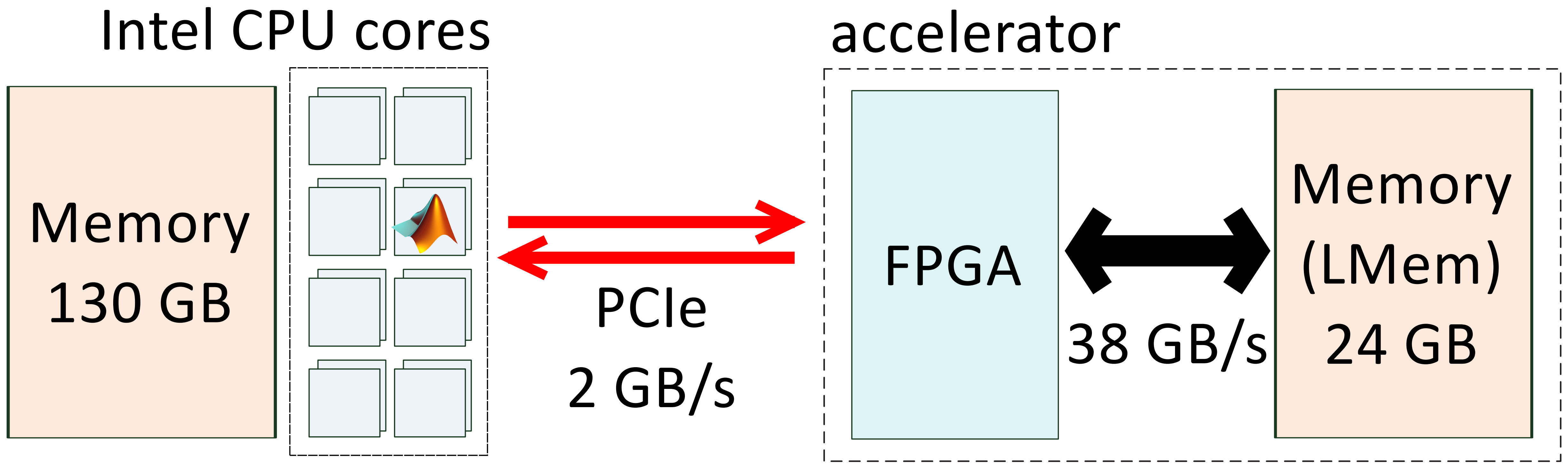
Block parallelization of Kaczmarz + CG = CARP-CG

[Gordon & Gordon, 2010]

Contribution of This Work

Compute Node Overview

[Maxeler Technologies, 2011]



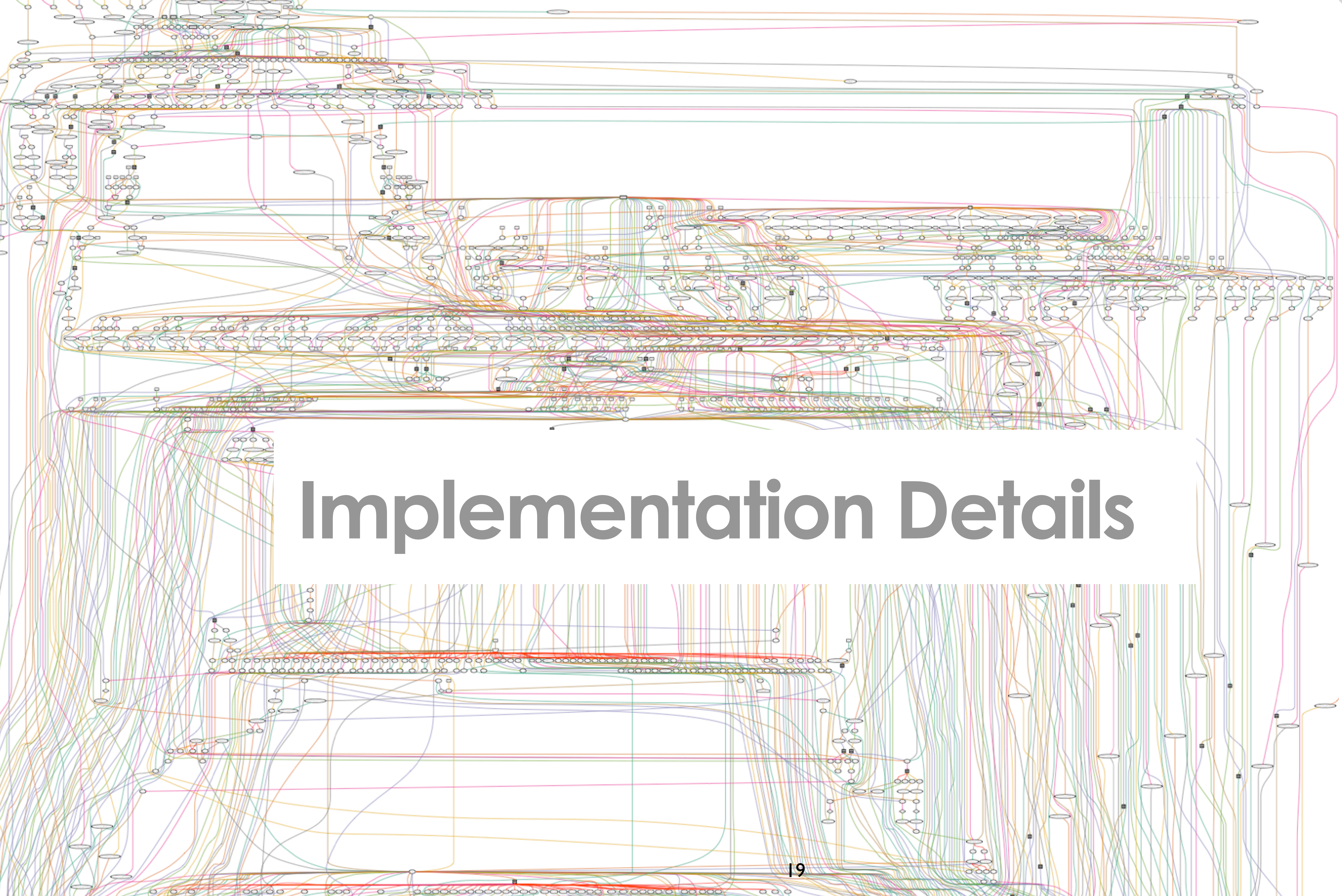
Algorithm 1 CGMN (Björck and Elfving [4])

Input: A, $\mathbf{u}$, $\mathbf{q}$, λ

$R\mathbf{q} \leftarrow \mathrm{DKSWP}(A, \mathbf{0}, \mathbf{q}, \lambda)$
$\mathbf{r} \leftarrow R\mathbf{q} - \mathbf{u} + \mathrm{DKSWP}(A, \mathbf{u}, \mathbf{0}, \lambda)$
$\mathbf{p} \leftarrow \mathbf{r}$
while $\|\mathbf{r}\|^2 > tol$ **do**
  $\mathbf{s} \leftarrow (I - Q)\mathbf{p} = \mathbf{p} - \mathrm{DKSWP}(A, \mathbf{p}, \mathbf{0}, \lambda)$
  $\alpha \leftarrow \|\mathbf{r}\|^2 \,/\, \langle \mathbf{p}, \mathbf{s} \rangle$
  $\mathbf{u} \leftarrow \mathbf{u} + \alpha \mathbf{p}$
  $\mathbf{r} \leftarrow \mathbf{r} - \alpha \mathbf{s}$
  $\beta \leftarrow \|\mathbf{r}\|^2_{\mathrm{curr}} \,/\, \|\mathbf{r}\|^2_{\mathrm{prev}}$
  $\|\mathbf{r}\|^2_{\mathrm{prev}} \leftarrow \|\mathbf{r}\|^2_{\mathrm{curr}}$
  $\mathbf{p} \leftarrow \mathbf{r} + \beta \mathbf{p}$
end while

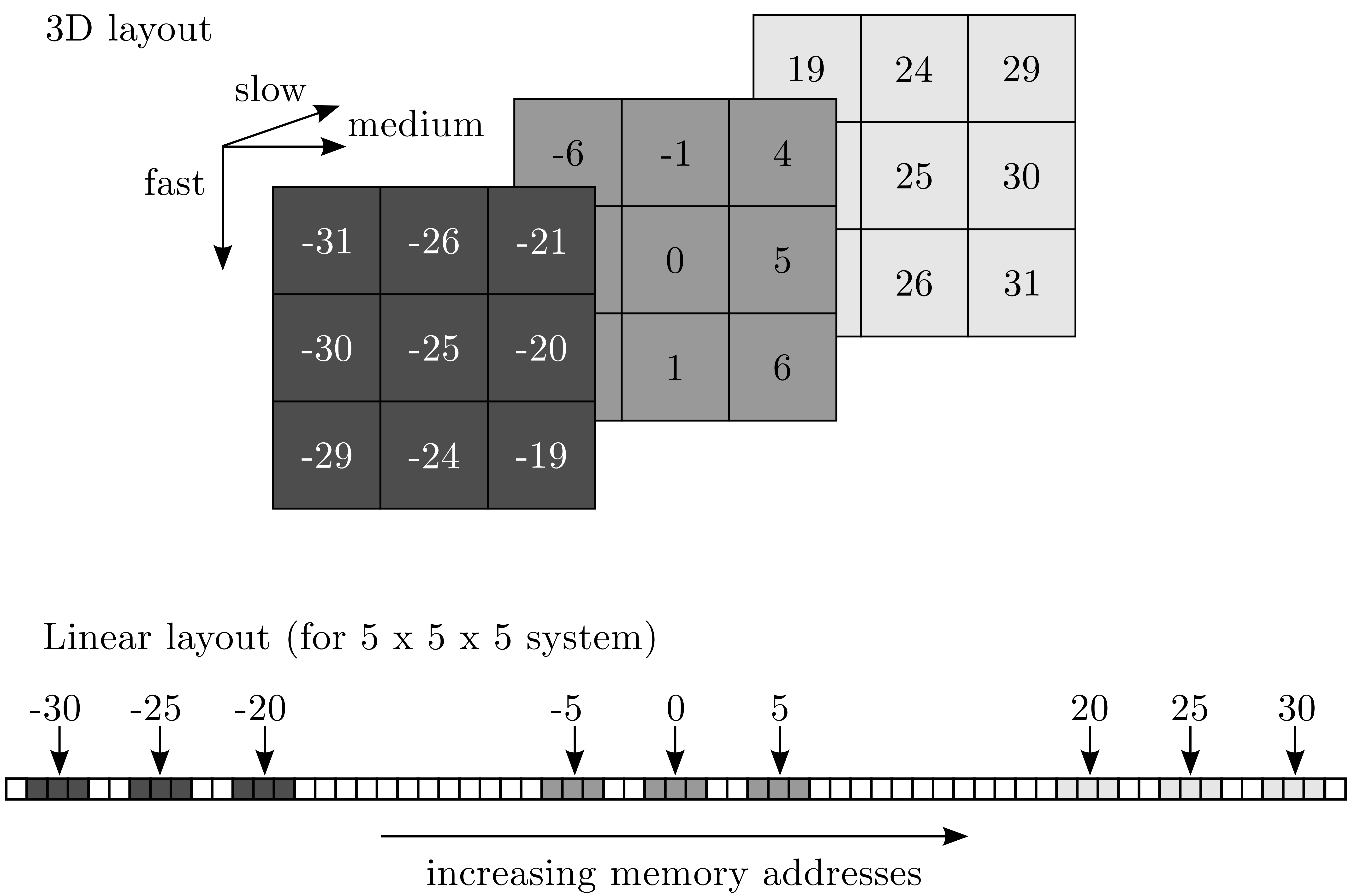
Output: $\mathbf{u}$

Kernel: running on accelerator

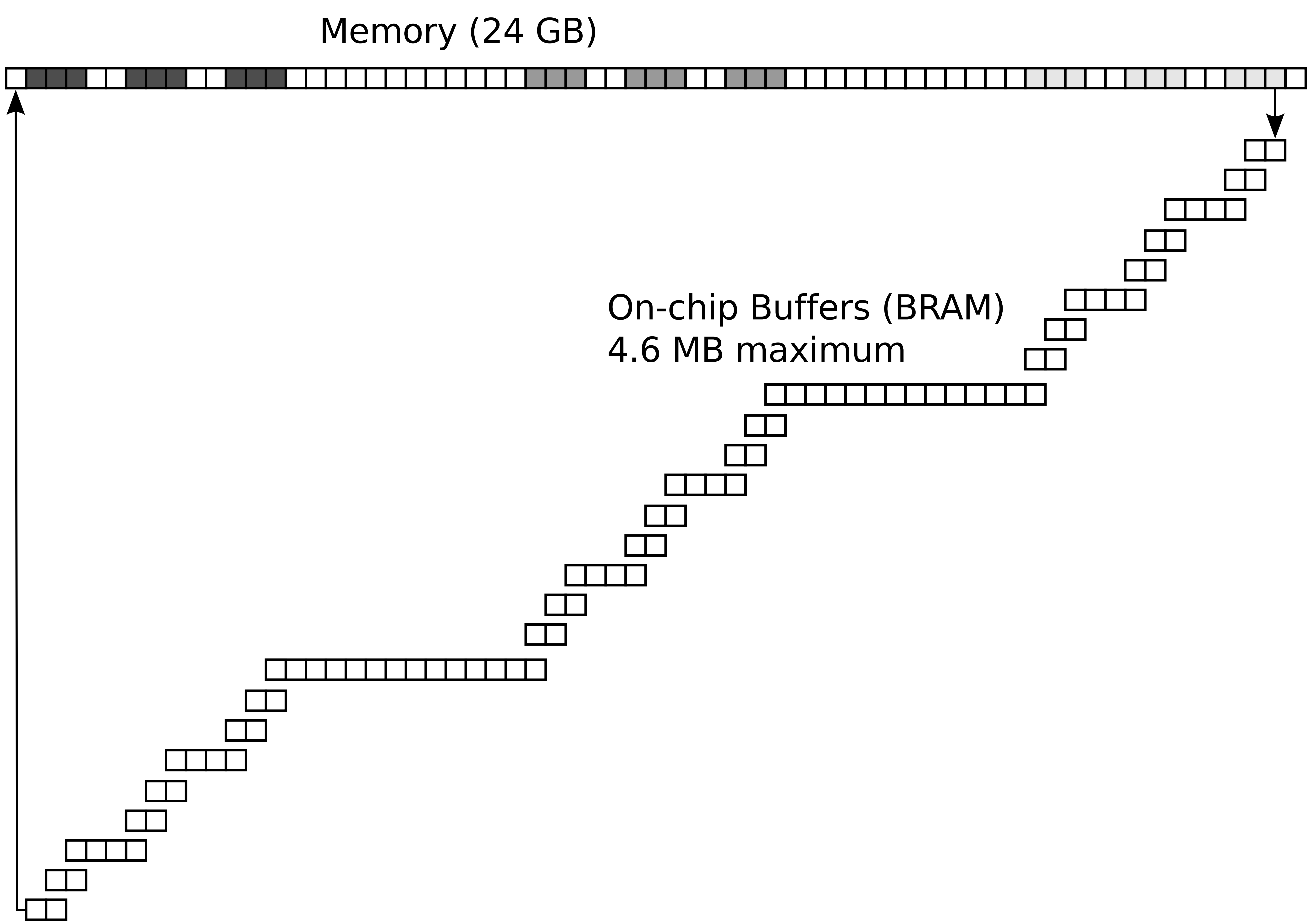


Implementation Details

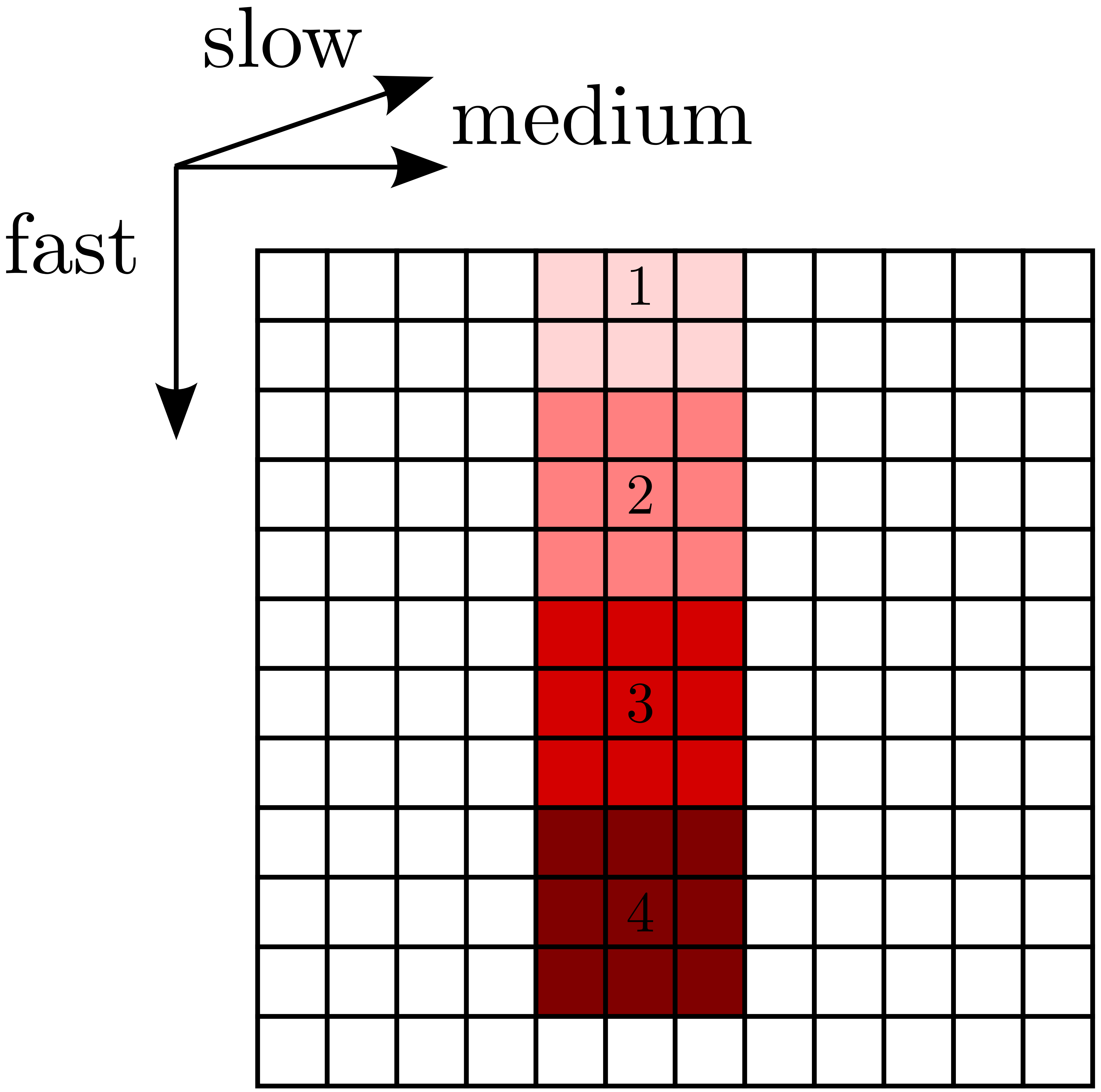
Layout of 3D Wavefields in 1D Memory



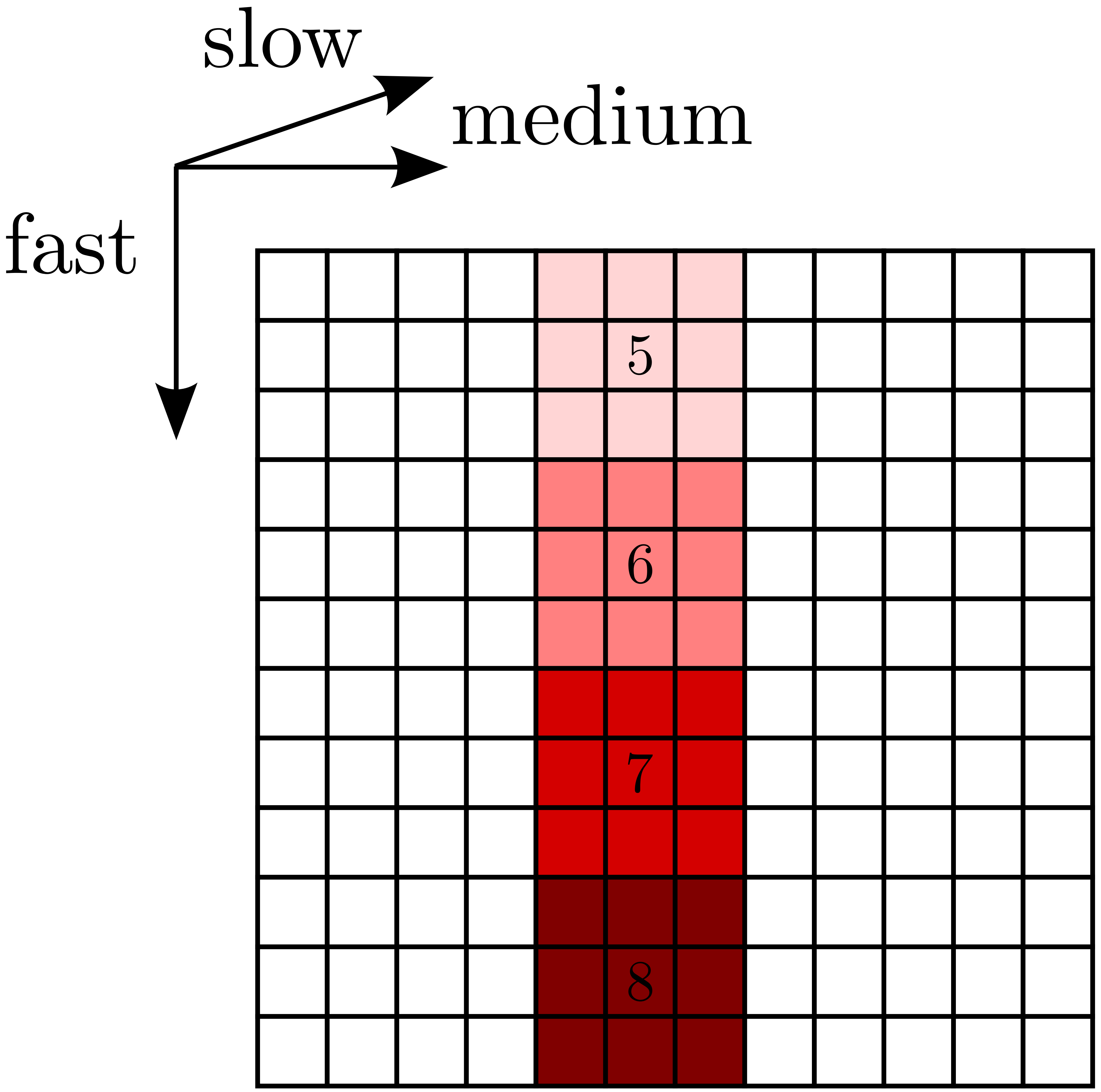
Buffering: Overcoming Latency of Memory Access



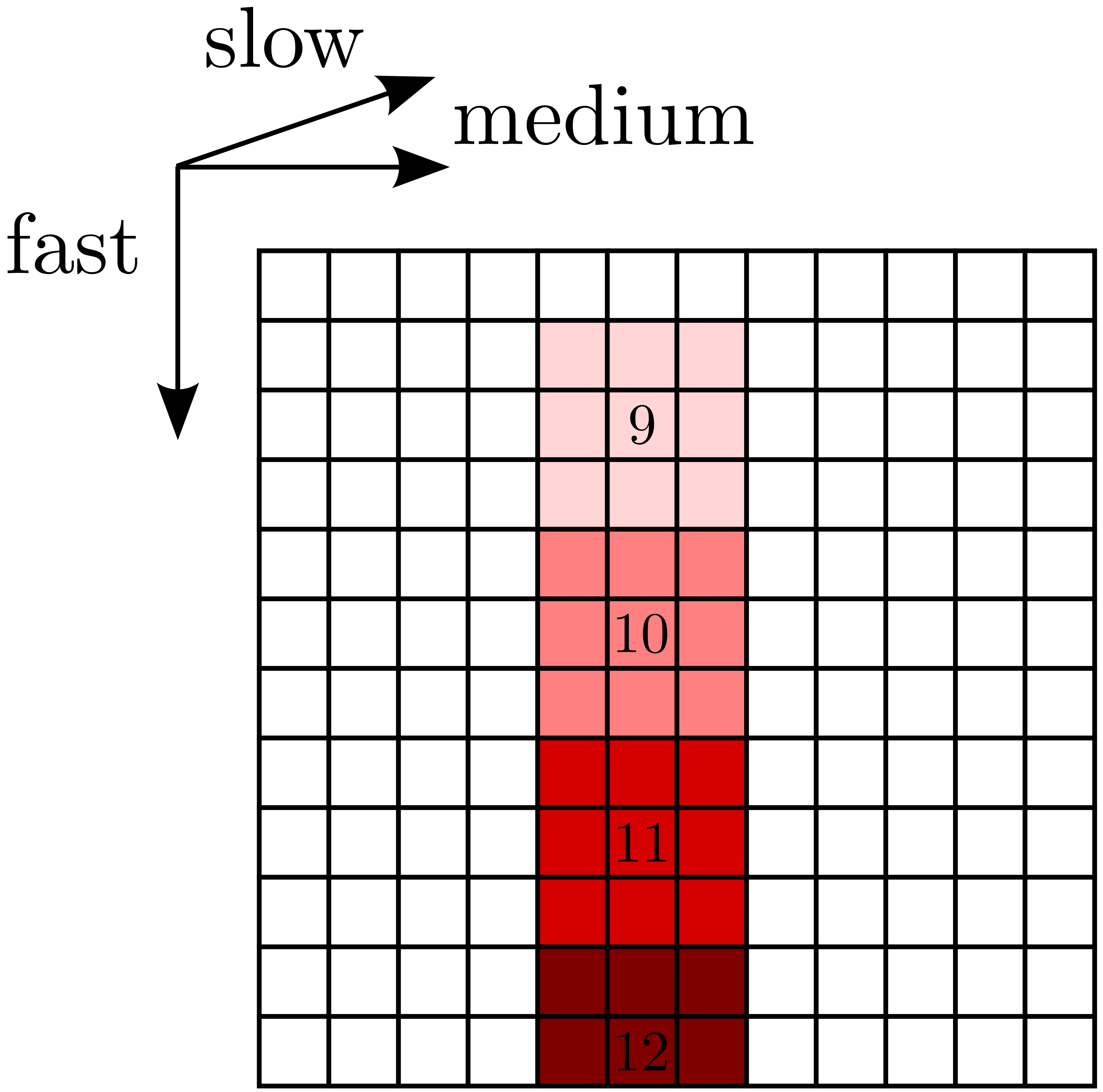
Pipelining: Overcoming Latency of Computation



Pipelining: Overcoming Latency of Computation



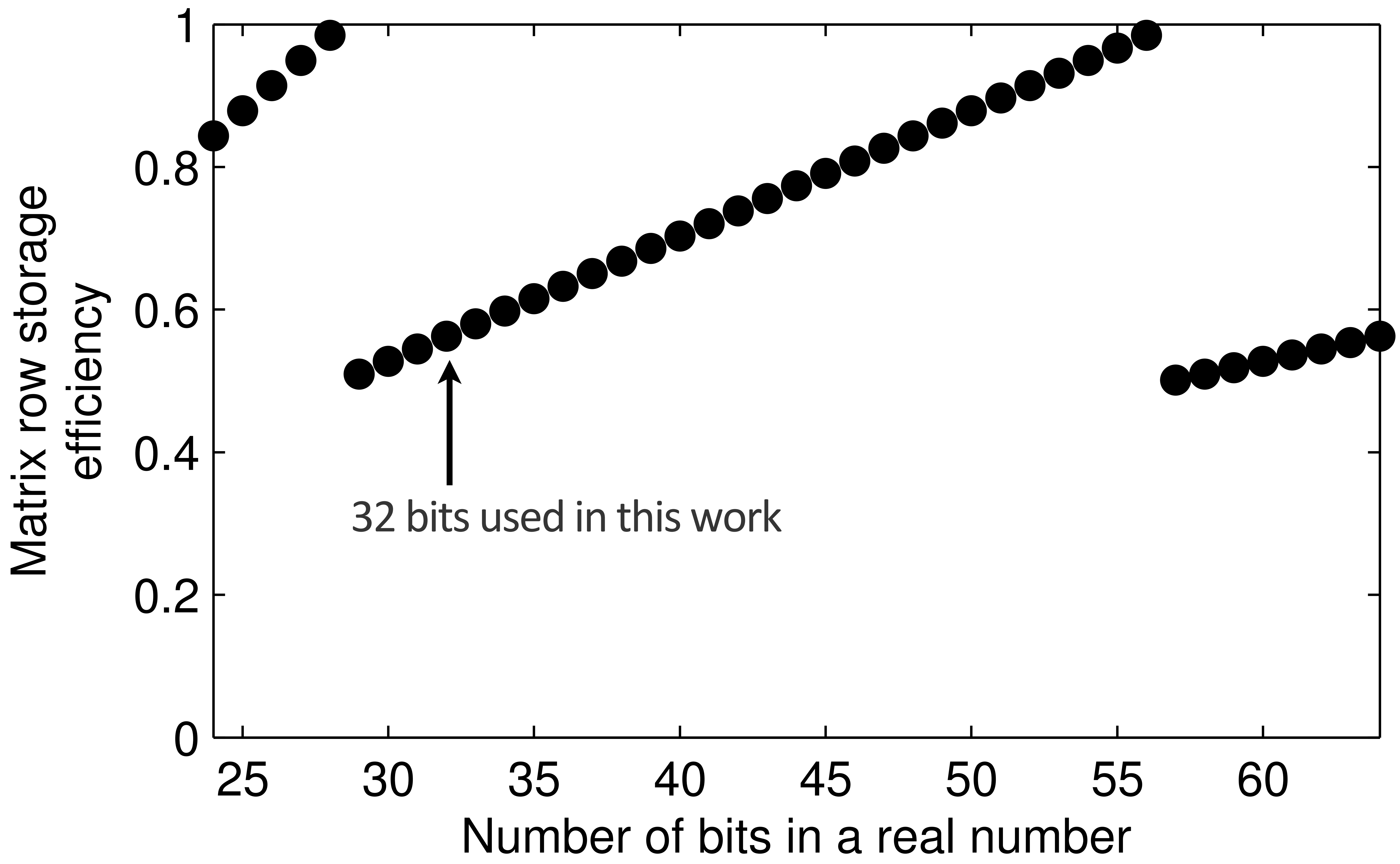
Pipelining: Overcoming Latency of Computation



Granular Memory Access: 384 bytes / burst

Number of bits in a real number	Number of bits in a complex number	Complex numbers per burst
24	48	64
32 (single precision)	64	48
48	96	32
64 (double precision)	128	24

Matrix Coefficient Representation



Results & Discussion

What is being compared?

Reference implementation:

- Solution Algorithm
  - CARP-CG
  - written in MATLAB
  - single-threaded
  - double precision
  - running on Intel Xeon E5-2670
- Computational Kernel
  - CARP sweeps
  - written in C
  - 32 threads
  - double precision
  - running on Intel Xeon E5-2670

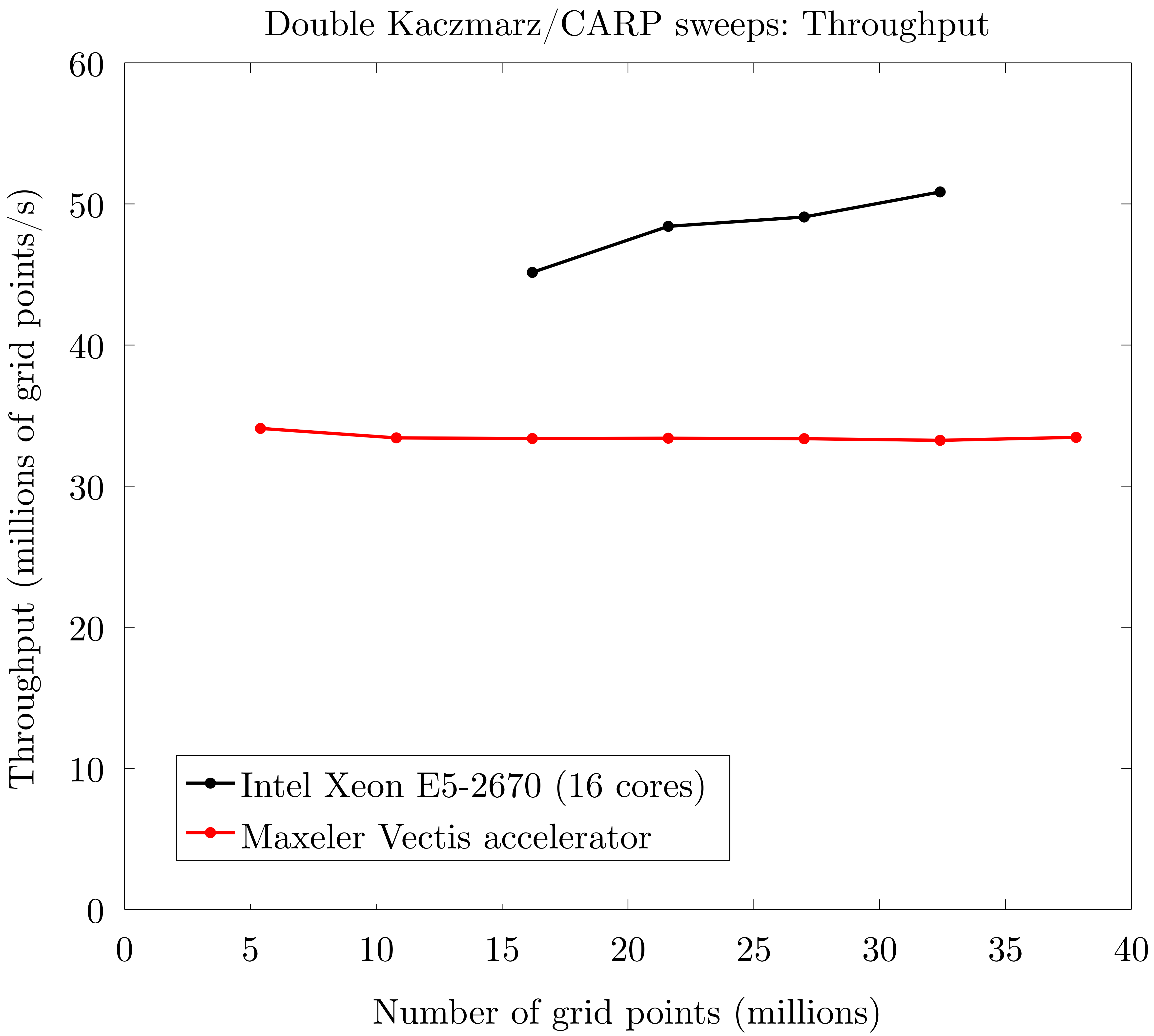
Accelerator implementation:

- Solution Algorithm
  - CGMN
  - written in MATLAB
  - single-threaded
  - single precision
  - running on Intel Xeon E5-2670
- Computational Kernel
  - Kaczmarz sweeps
  - single precision
  - running on Maxeler Vectis accelerator at 100 MHz
  - Memory clock at 303 MHz

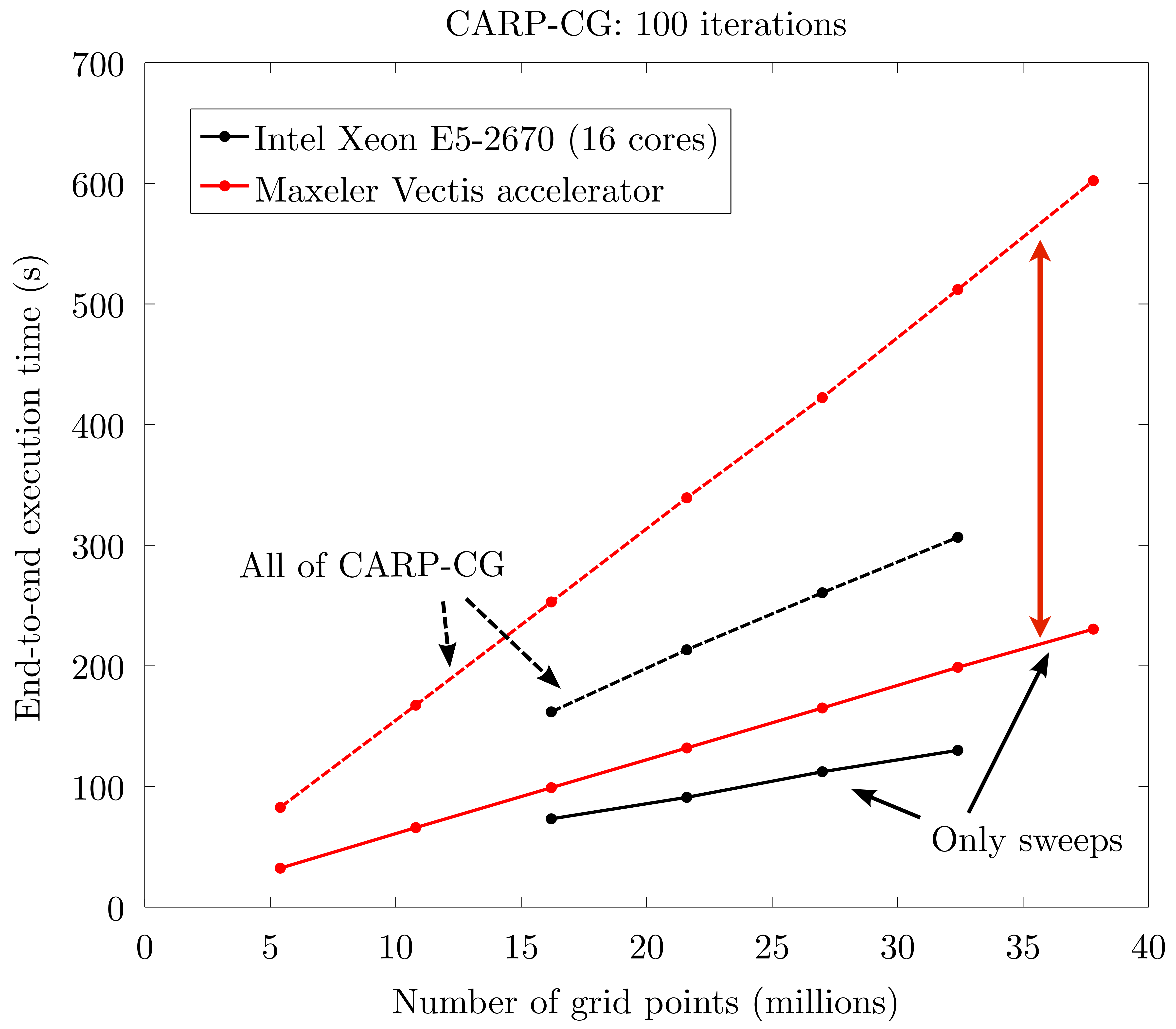
Experimental Set-up

- Solve part of the SEG/EAGE Overthrust System: 432 x 500 x Z.
- Point source.
- Zero initial guess.
- Run 100 CARP-CG/CGMN iterations.

Throughput of Kaczmarz/CARP sweeps



CARP-CG: End-to-end execution time

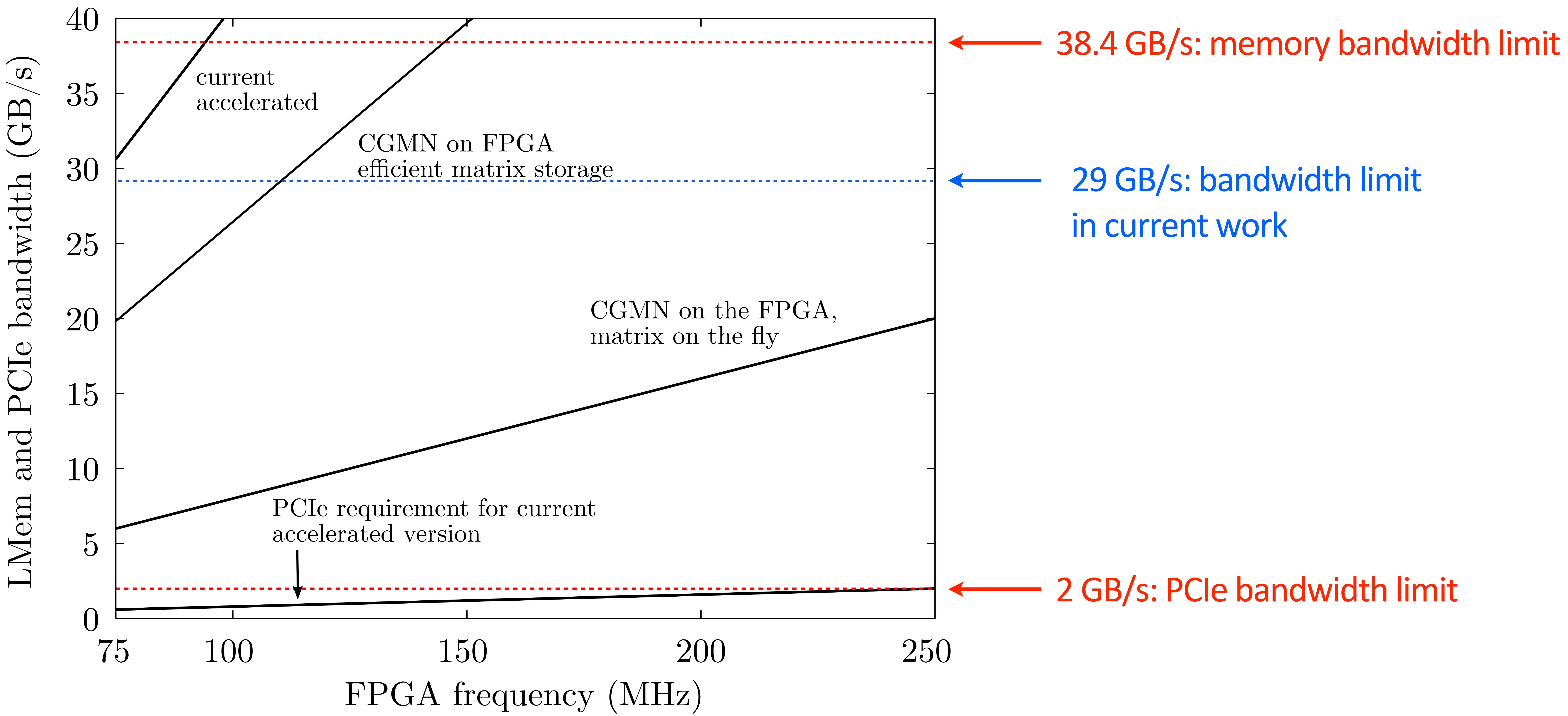


Lots of overhead!

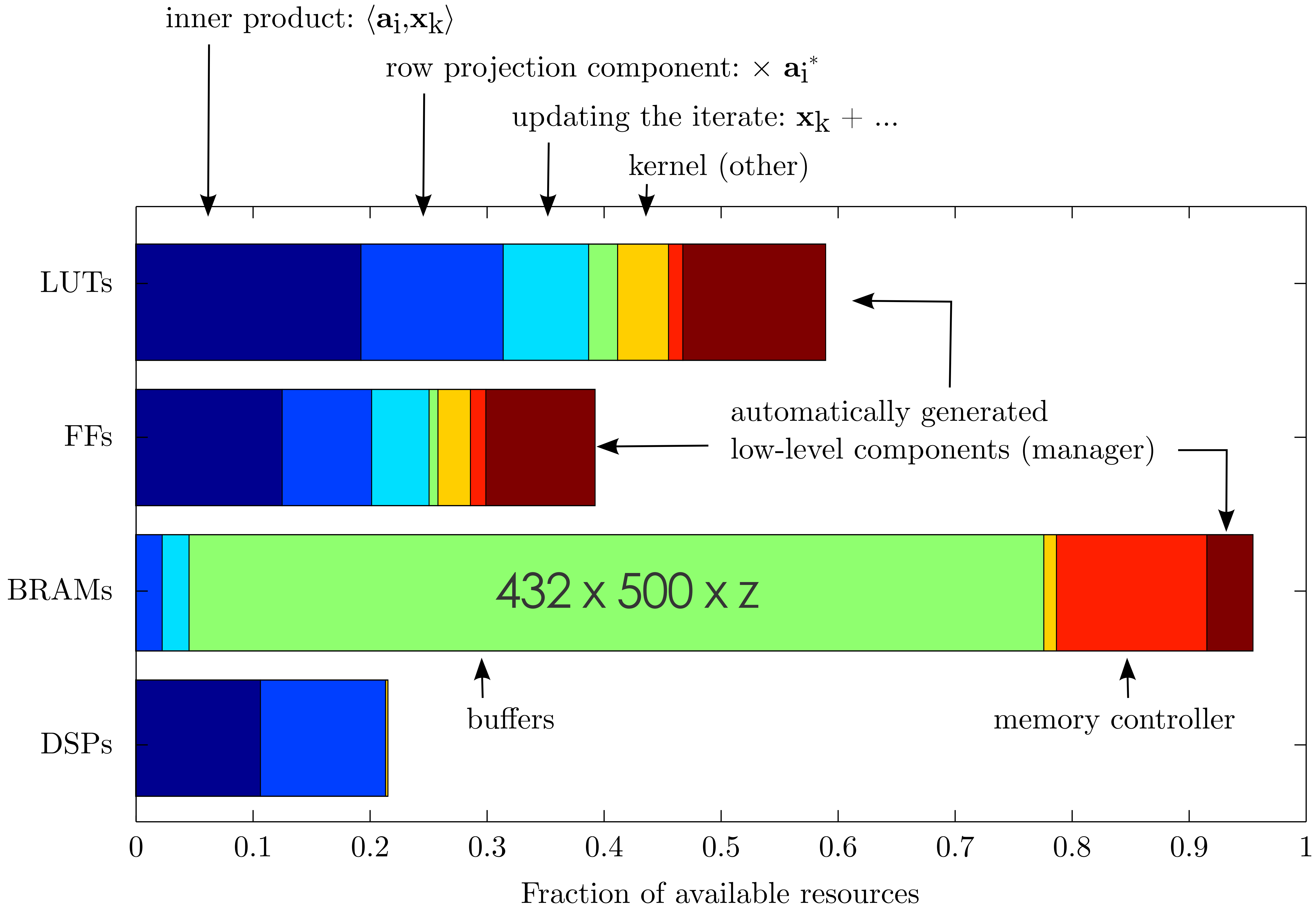
Kaczmarz sweeps are 39% of run time.

Future Solution: Port all of CGMN to accelerator.

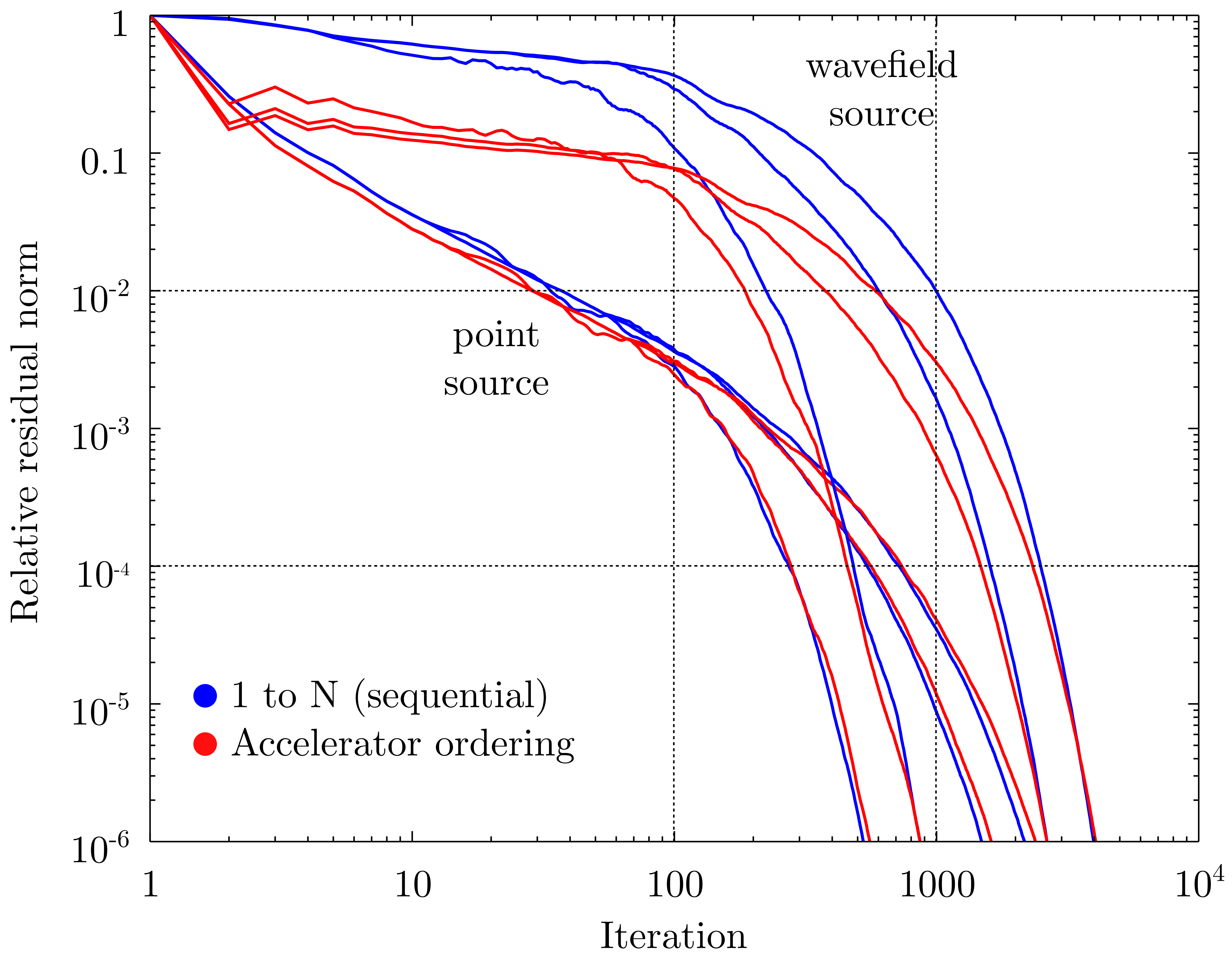
Avoiding Future Communication Bottlenecks



FPGA Resource Usage: Room for parallelism?



Effect of matrix row ordering on CGMN convergence



Conclusion

Have **implemented** frequency-domain wave simulation using reconfigurable hardware.

More work needed to realize full potential of accelerator system.

SLIM

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SINBAD

NSERC CRSNG

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