SVD-free low-rank matrix factorization

-wavefield reconstruction via *jittered* subsampling & reciprocity

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Motivation

- fully sampled data
  - simultaneous shot based FWI & migration
  - estimation of primaries by sparse inversion & SRME

- acquisition challenges
  - missing data

- exploit *low-rank* structure of seismic data
  - randomized sampling **(what is random enough?)** **[Hennenfent et. al. 2008]**
  - *SVD-free* matrix factorization

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[1] Oropeza and M D Sacchi, 2011, Simultaneous seismic data de-noising and reconstruction via Multichannel singular spectrum Analysis (MSSA).

[2] Nadia Kreimer, Aaron Stanton and Mauricio D. Sacchi, Tensor completion based on nuclear norm minimization for 5D seismic data reconstruction, Technical report, 2013

Existing work

- uniform random subsampling
  - no control on the maximum gap size

- SVD computation [1,2]
  - expensive for large scale data

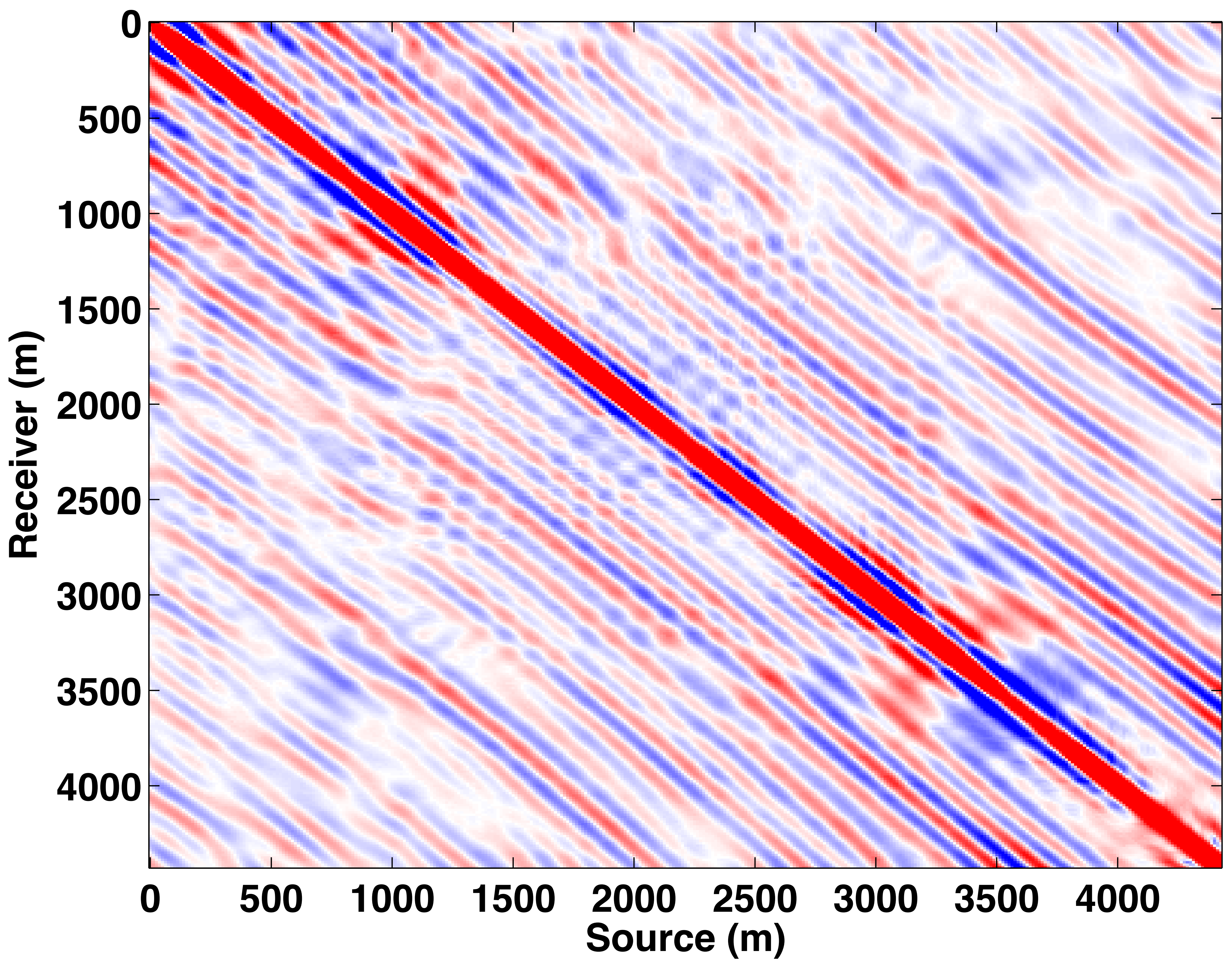
[Candes and Plan 2010, Oropeza and Sacchi 2011]

Matrix completion

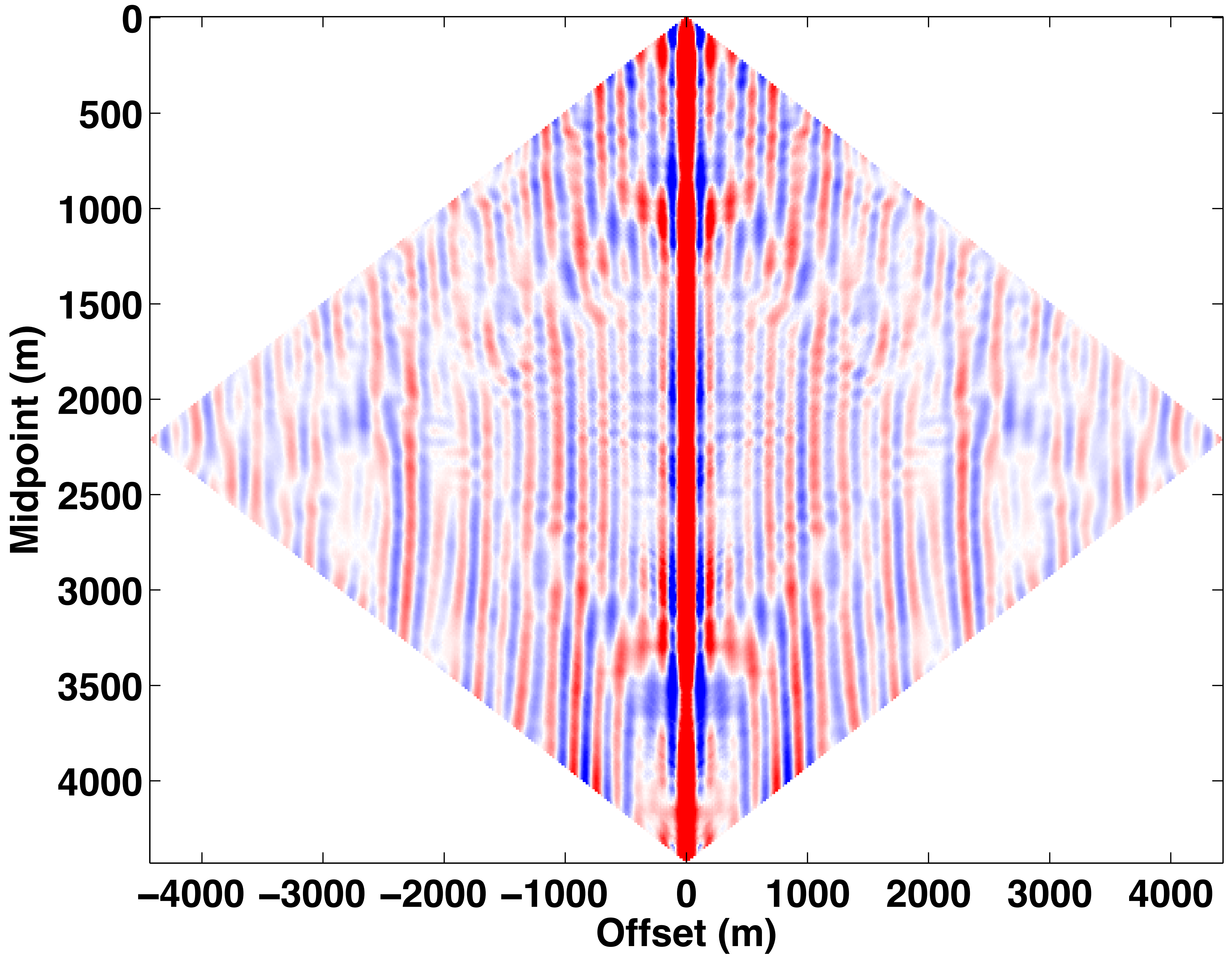
- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Low-rank structure
2-D acquisition

acquisition domain
[source-receiver]

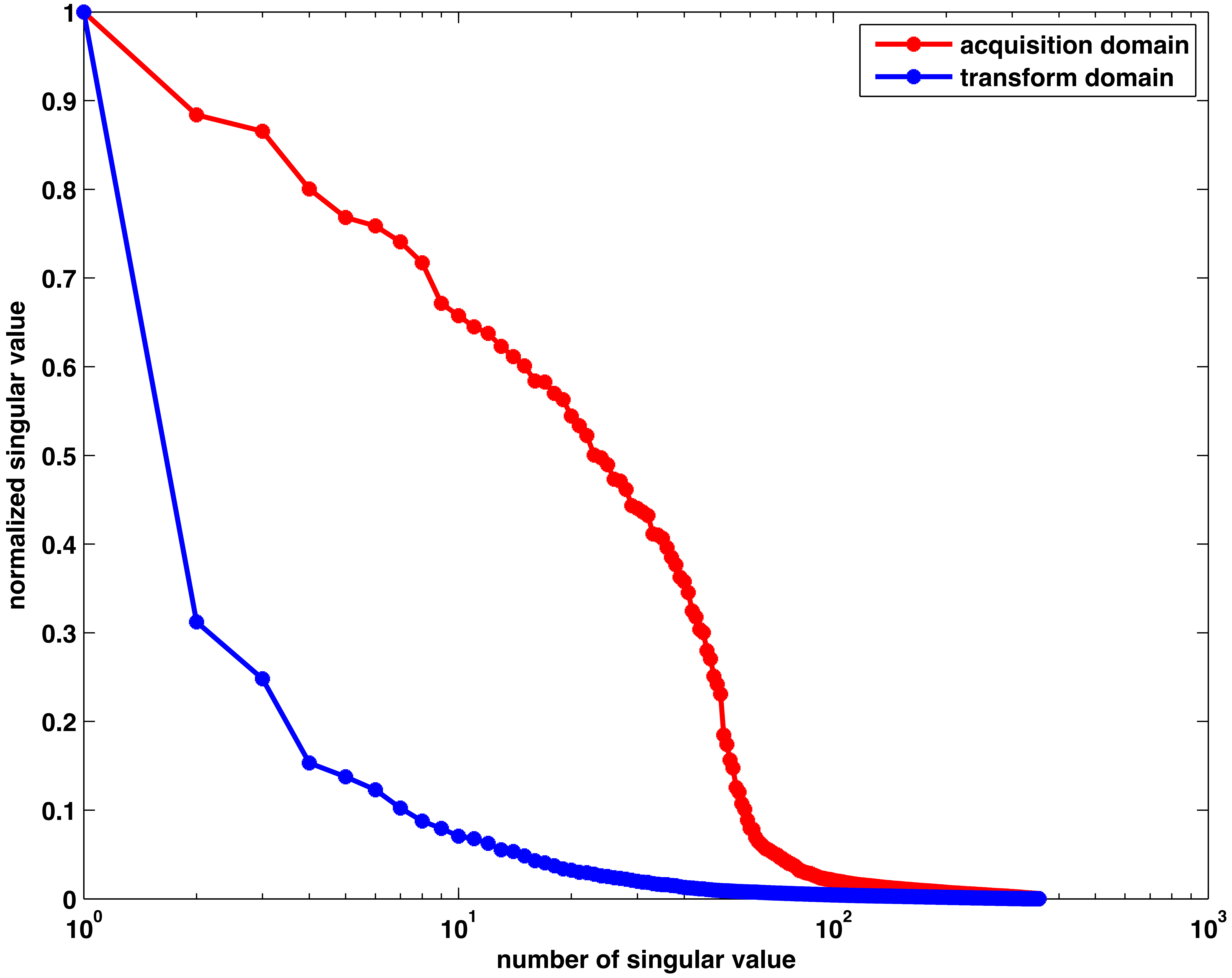


transform domain
[midpoint-offset]



Singular value decay

2-D acquisition



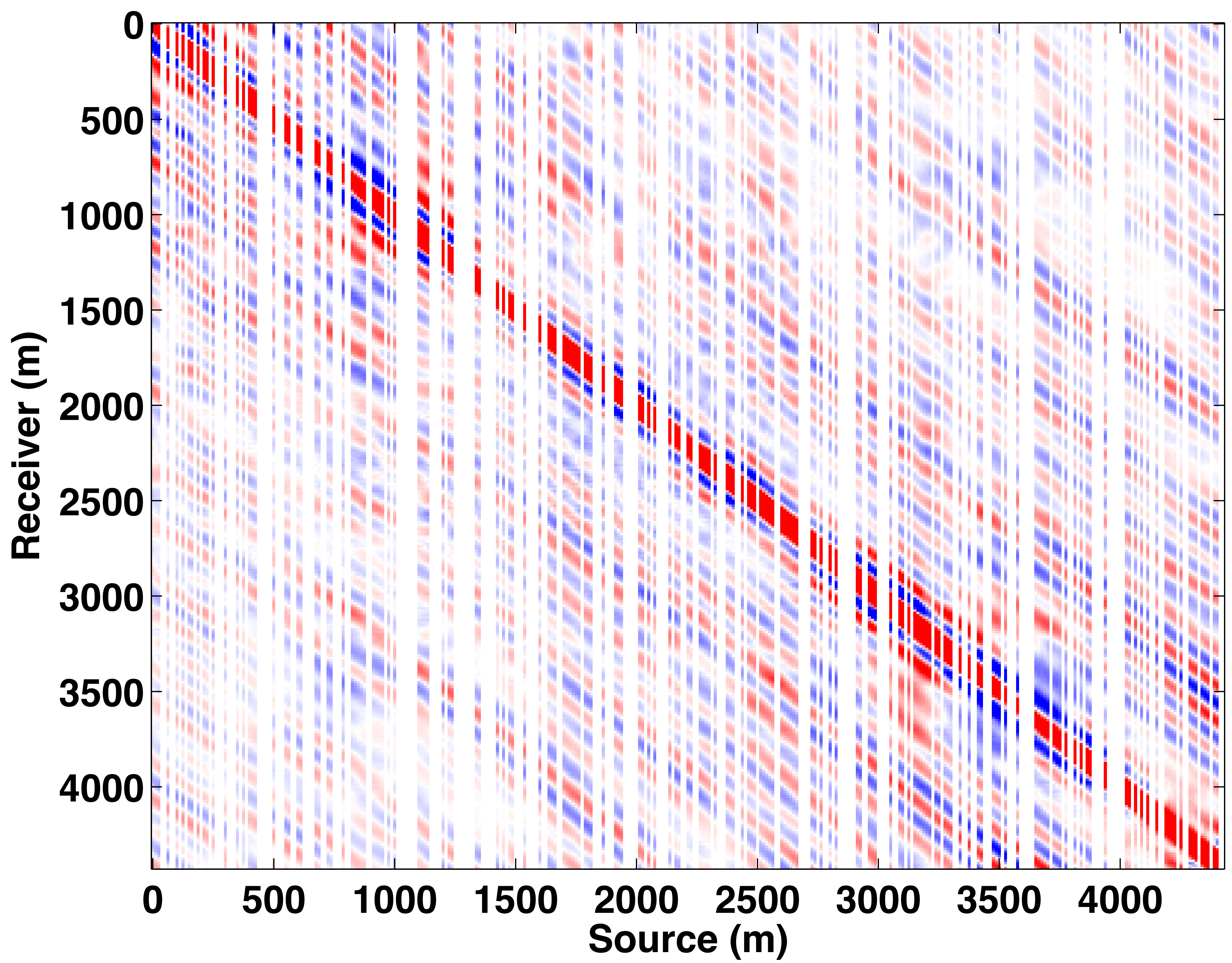
Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

2-D acquisition
uniform-random sampling

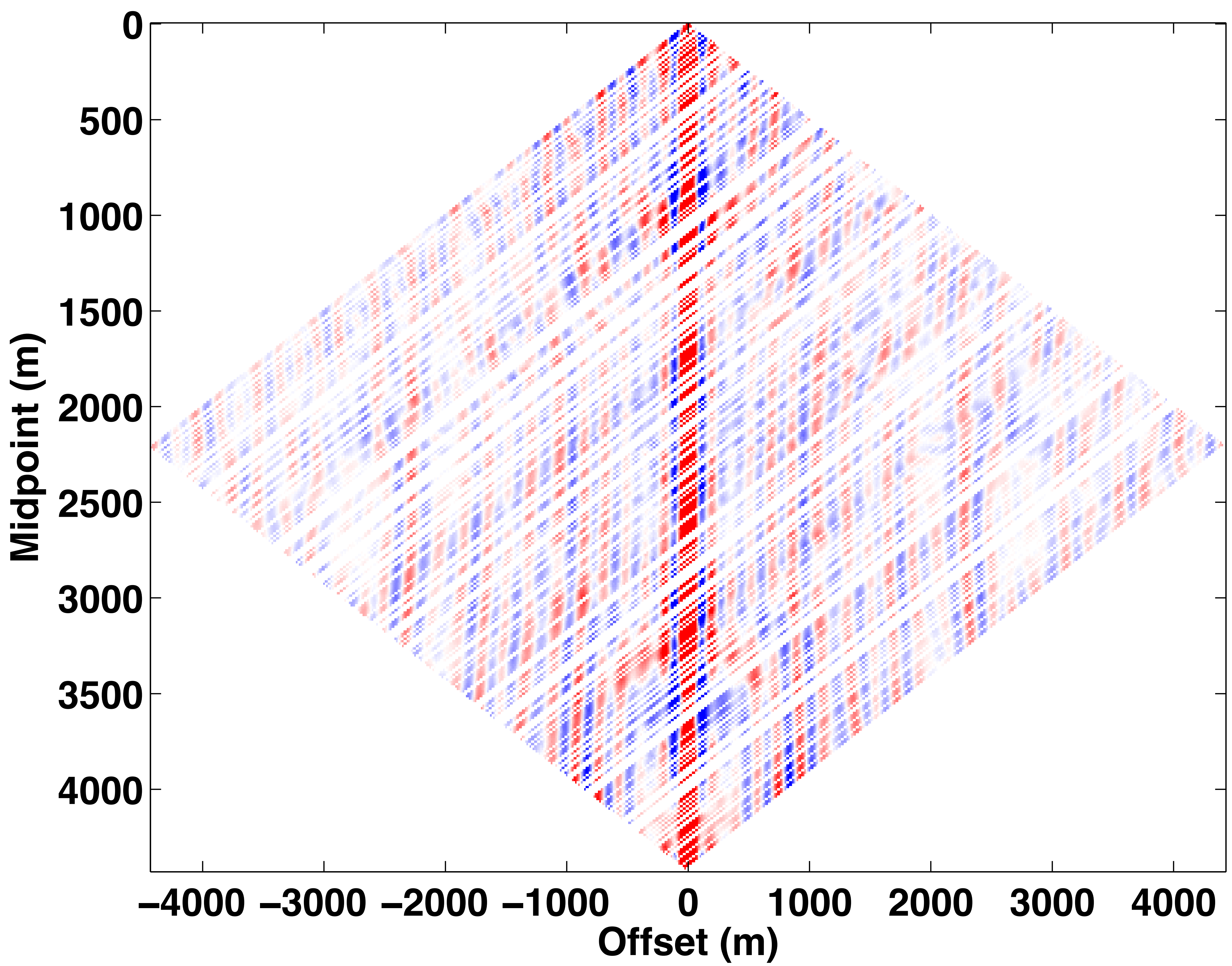
acquisition domain

missing columns *do not* increase rank

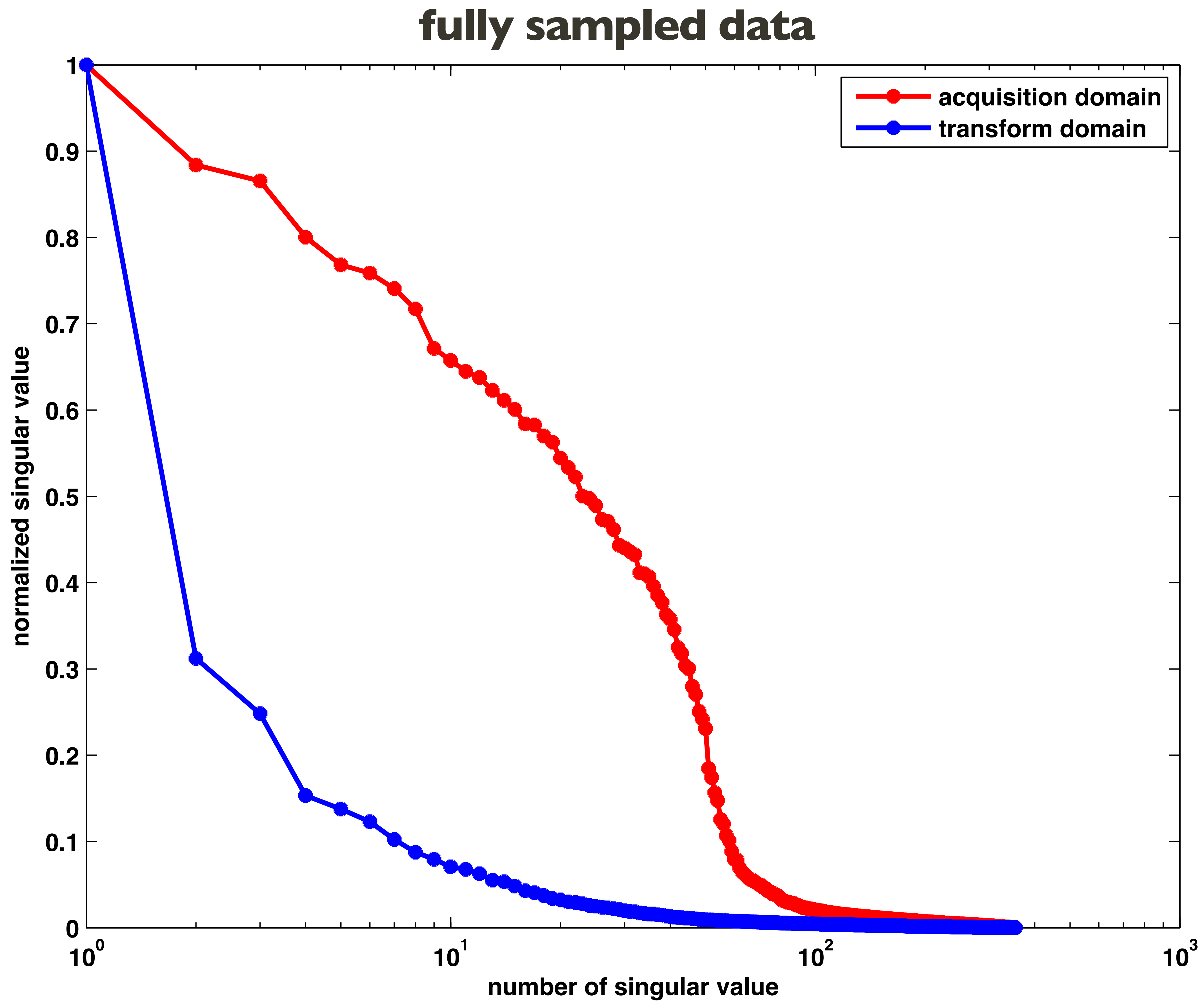


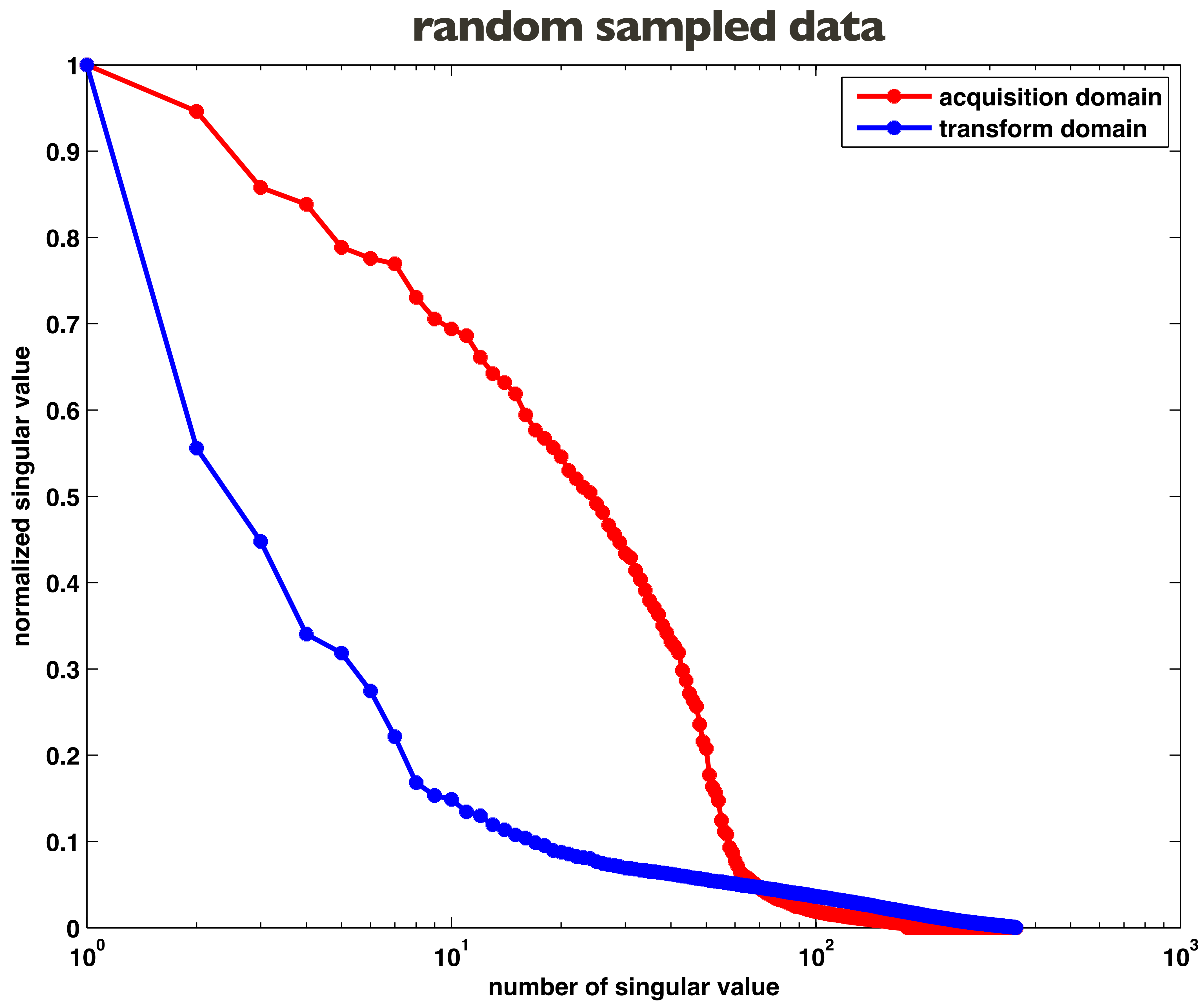
transform domain

missing columns *do* increase rank



Randomized sampling
singular value decay

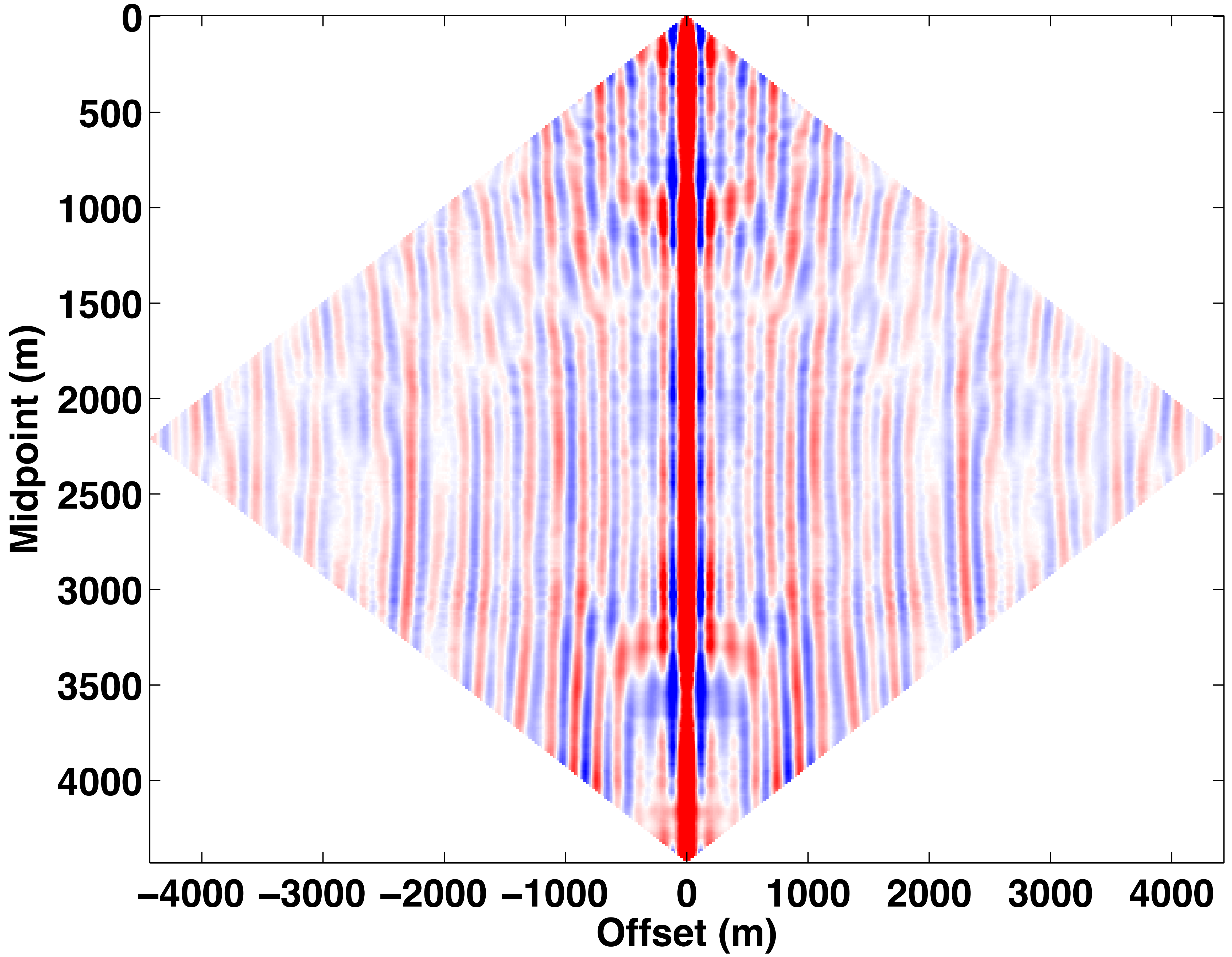




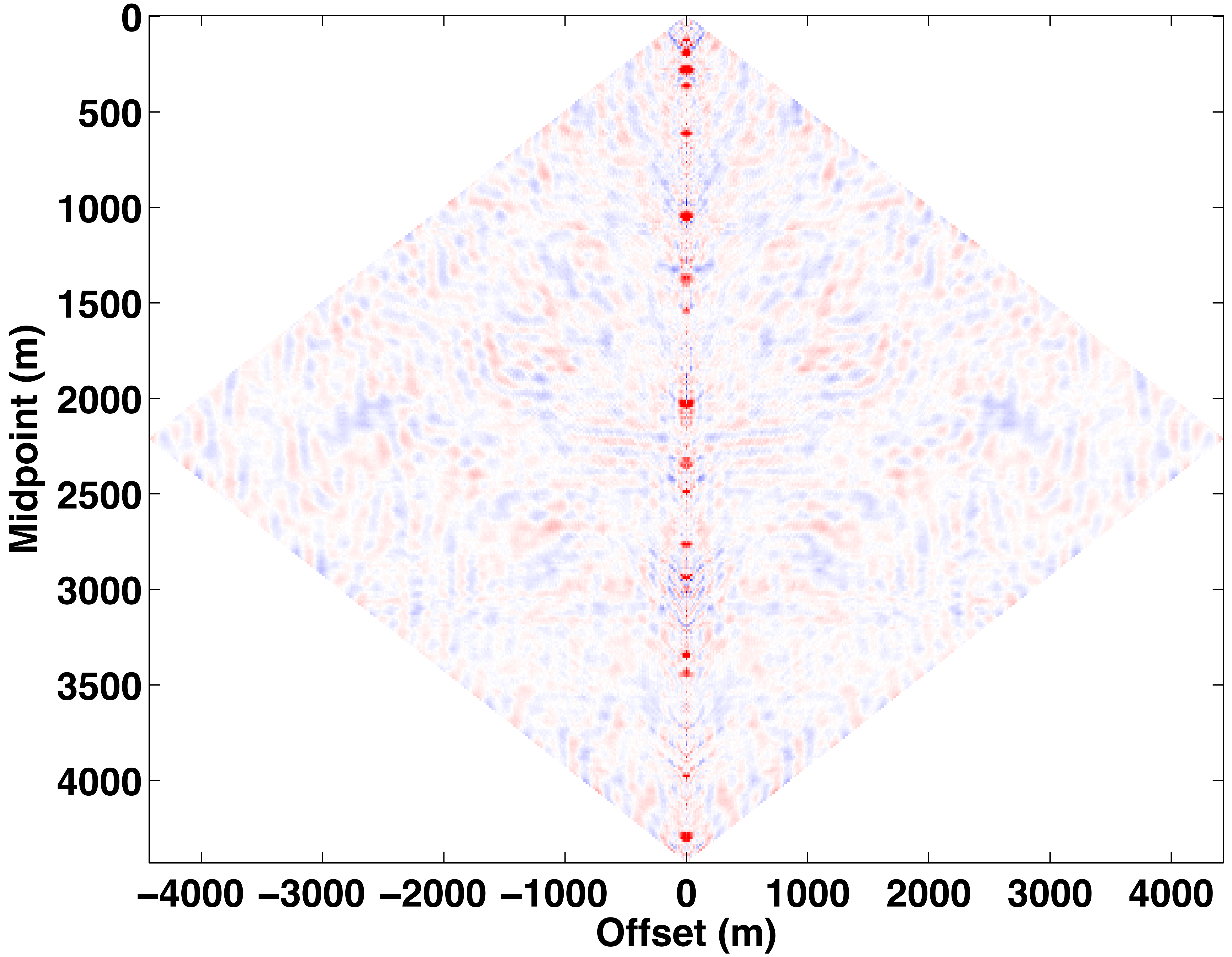
Low-rank interpolation

uniform-random sampling

recovery
[SNR = 14.9 dB]

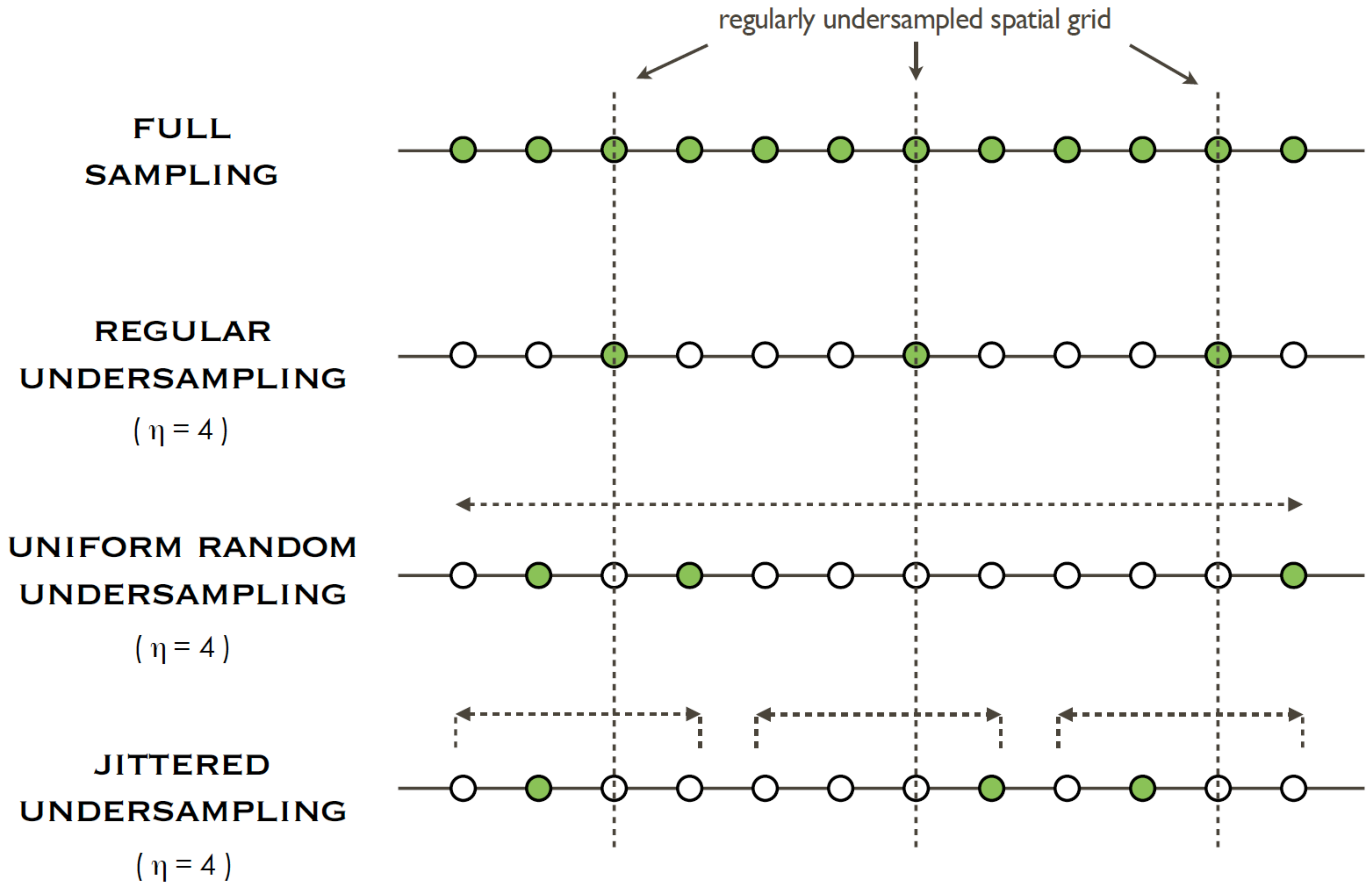


residual



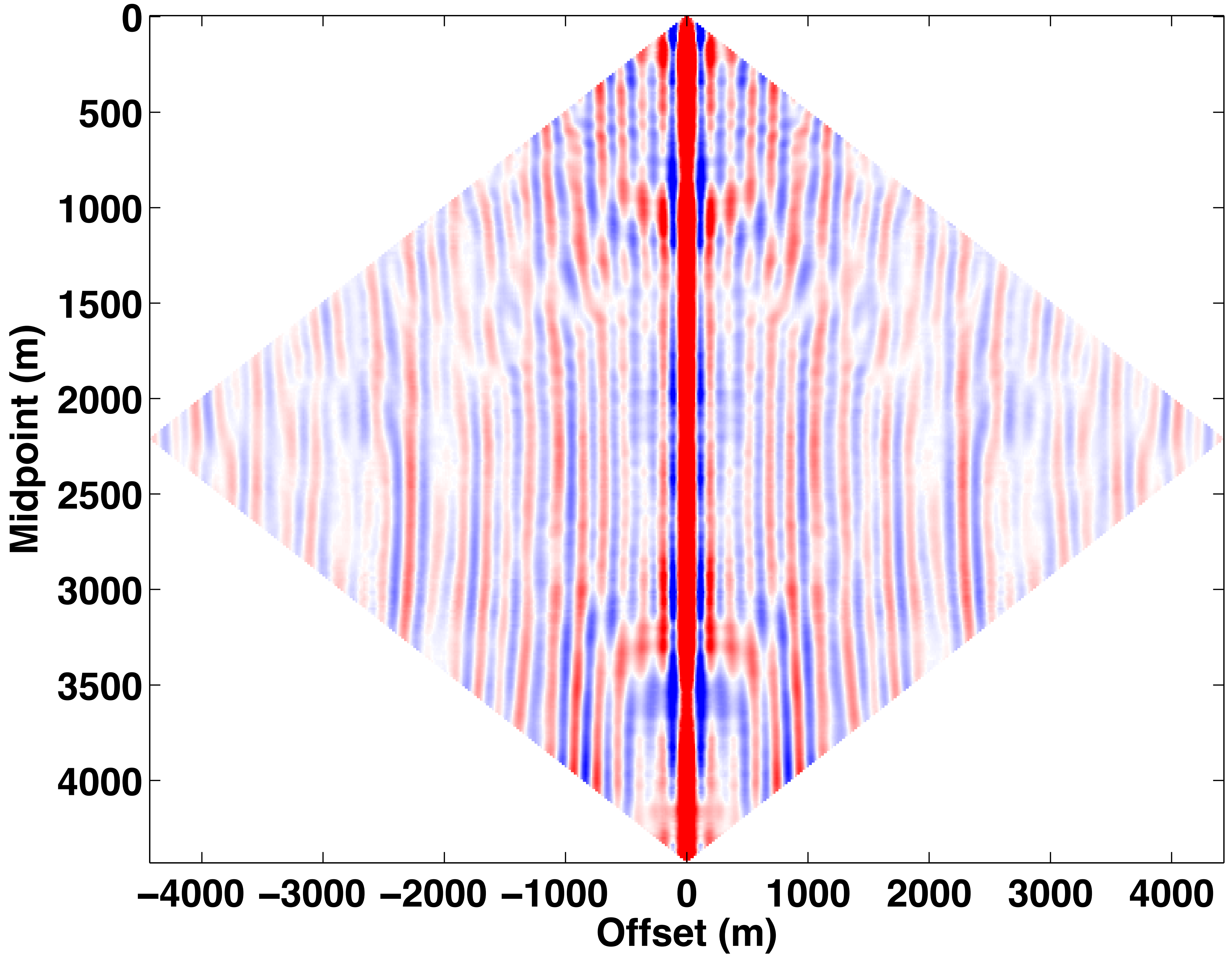
[Hennenfent et. al. 2008]

Sampling schemes

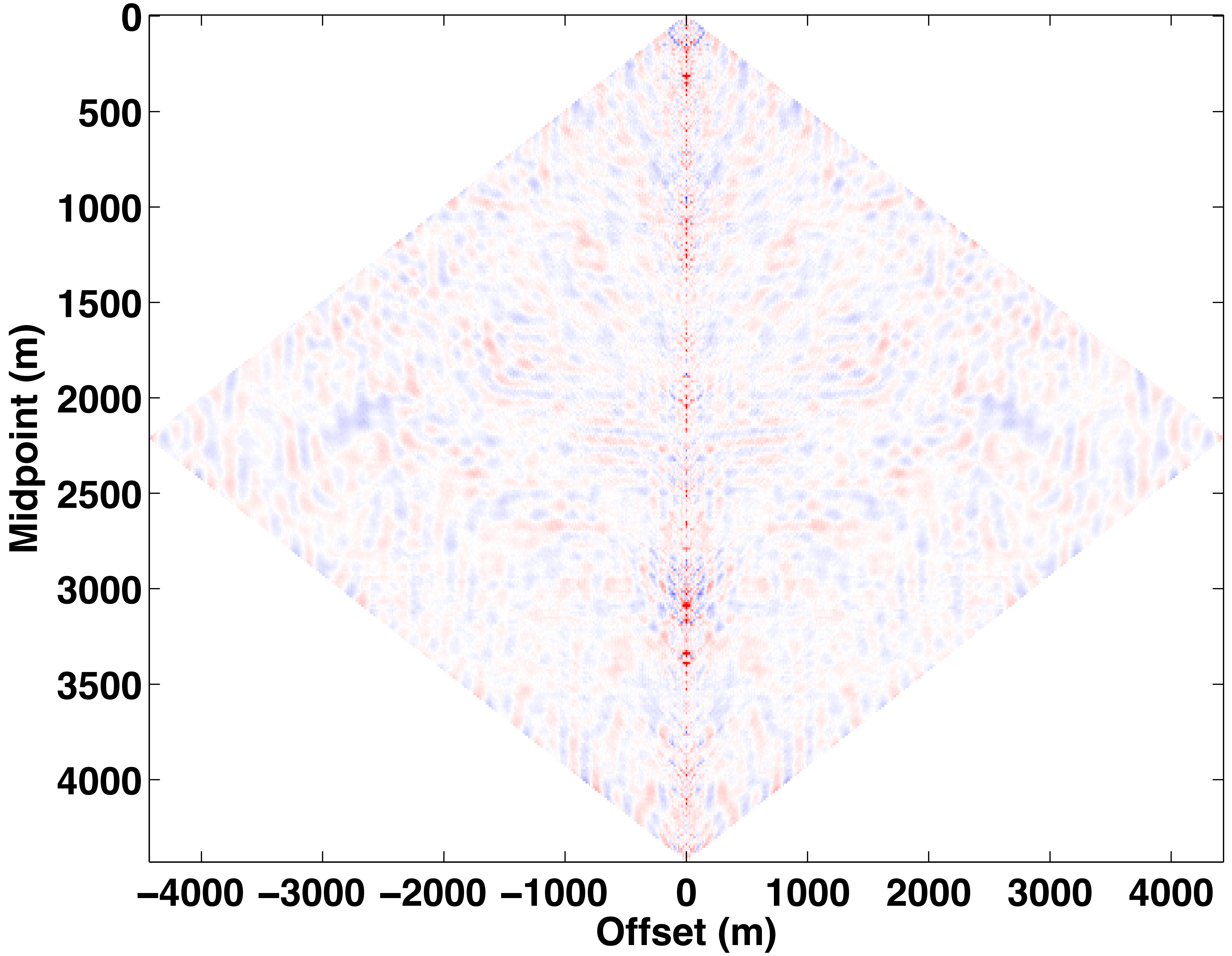


Low-rank interpolation
jittered sampling

recovery
[SNR = 16.9 dB]



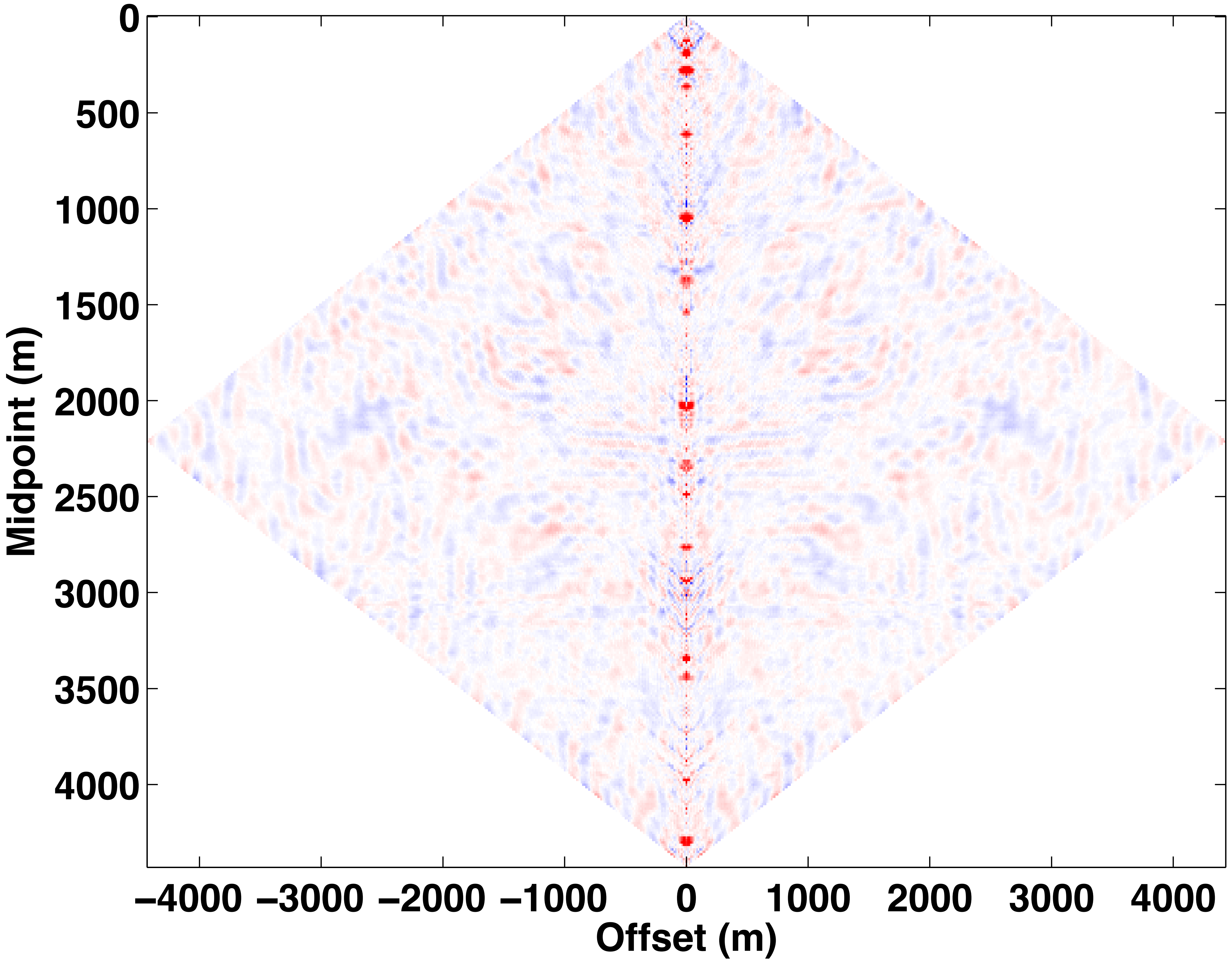
residual



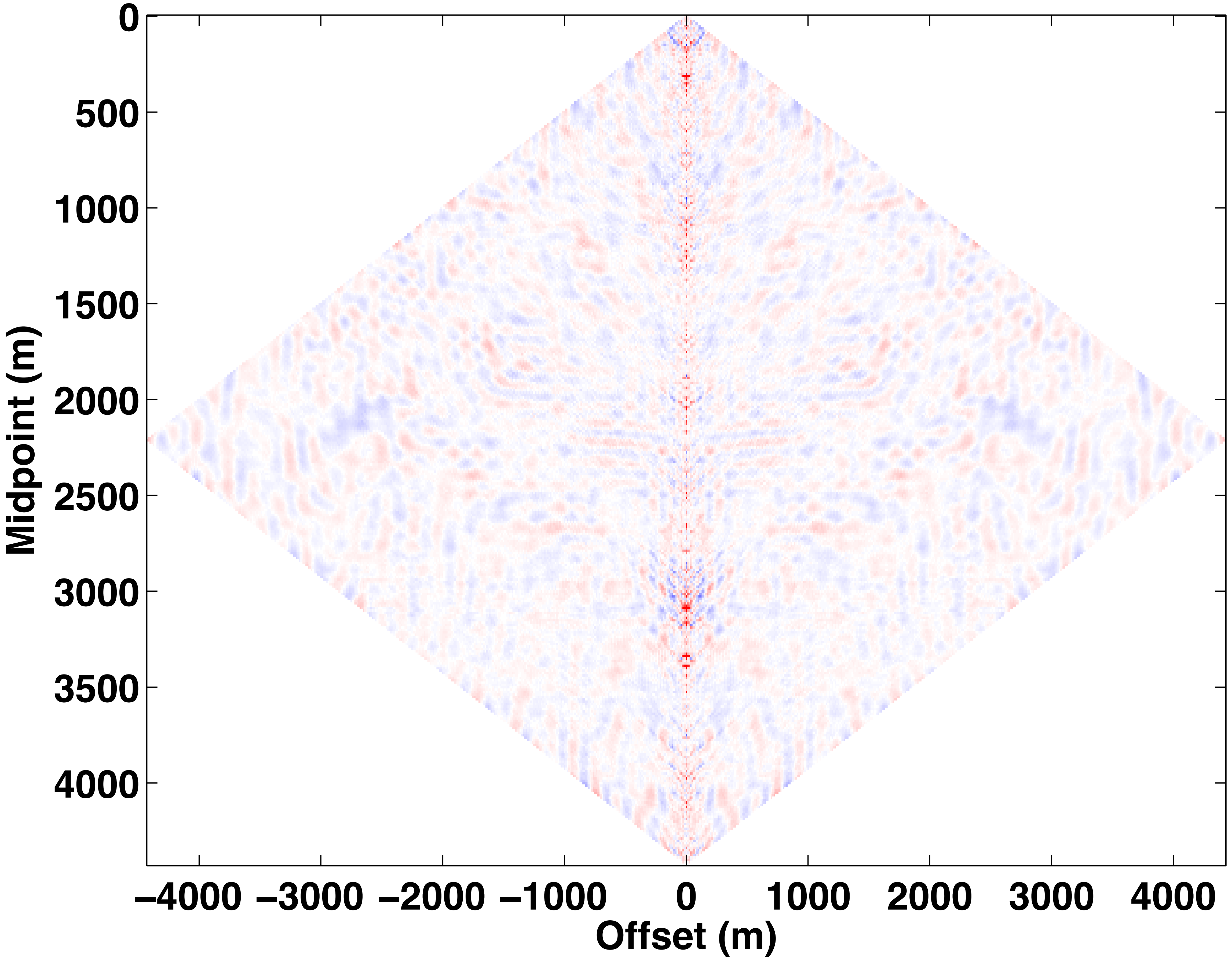
Low-rank interpolation

residual comparison

residual
uniform-random



residual
jittered



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Observations

- sampling become *incoherent* in “transform” domain
- *slow decay* of singular values in “transform” domain
- jittered sampling controlled the gap-size

Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Rank minimization

- given a set of measurements $\mathbf{b}$, aim is to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} \quad \text{rank}(\mathbf{X}) \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where $\text{rank}(\mathbf{X}) =$ number of singular values of $\mathbf{X}$

- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator
$\mathbf{M}$: measurement operator
$\mathcal{S}^H$: transform operator

Rank minimization

- prohibitively *expensive*
  - do not know rank value in advance
  - search over all possible values of rank

- instead solve nuclear-norm minimization
  - convex relaxation of rank-minimization **[Recht et. al. 2010]**

[Recht et. al. 2010]

Nuclear-norm minimization

- we want to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

where λ_i are the *singular* values

Challenges

- requires repeated application of *SVD* for projections

- expensive to compute for large system
  - curse of dimensionality

- can we exploit rank structure "*SVD* free"

[Rennie and Srebro 2005, **Lee et. al. 2010, Recht and Re 2011**]

Factorized formulation

$$\mathbf{X} \in \mathbb{R}^{n \times m} = \mathbf{L} \in \mathbb{R}^{n \times k} \quad \mathbf{R}^H \in \mathbb{R}^{k \times m}$$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$

[Berg and Friedlander 2008, Aravkin et al. 2012b]

Factorized formulation

- reformulate $(BPDN_\sigma)$ formulation

$$\min_{\mathbf{L},\mathbf{R}} ||\mathbf{L}\mathbf{R}^H||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \quad \text{s.t.} \quad \|\mathbf{L}\mathbf{R}^H\|_* \leq \tau$$

where τ is a rank regularization parameter

[Rennie and Srebro 2005]

Factorized formulation

- Upper-bound on nuclear norm is defined as

$$\|\mathbf{L}\mathbf{R}^H\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2$$

where $\|\cdot\|_F^2$ is sum of squares of all entries

- choose k *explicitly* & avoid costly *SVD's*

Computational cost

with and without SVD

	σ	0.2	0.1	0.09	0.08
Matrix completion w/ SVD	SNR (dB)	12.8	17	17.4	17.9
	time (sec)	30.4	42.8	32.9	58.8
Matrix completion w/o SVD	SNR (dB)	13.1	17.1	17.4	18
	time (sec)	1.6	2.9	3.2	4

Experiments and Results

- Gulf of Suez
  - 2-D seismic line
  - 50 % missing traces
  - rank adjusted from low to high frequency
  - 150 iterations

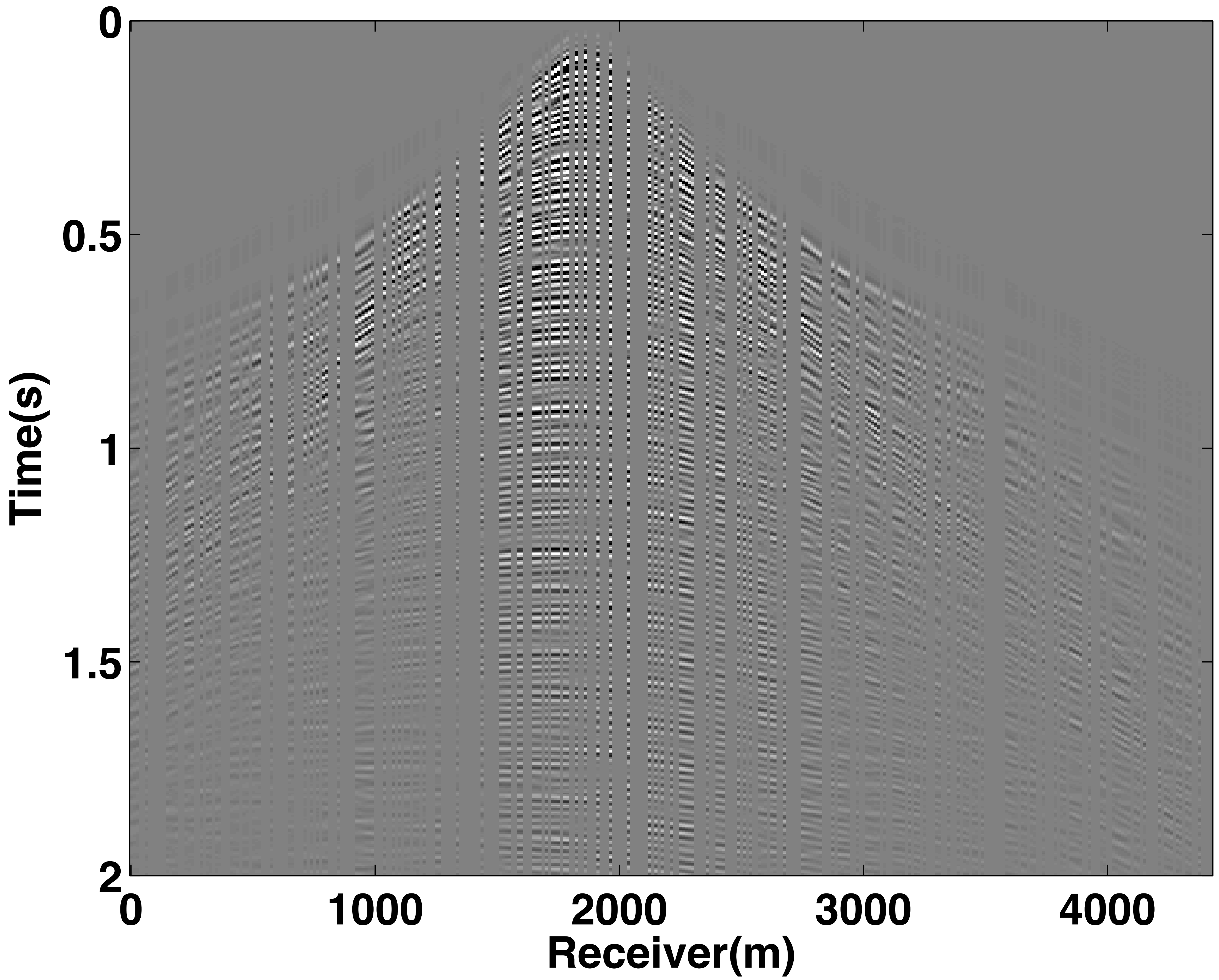
Experiments and Results

Case 1 : Uniform random subsampling

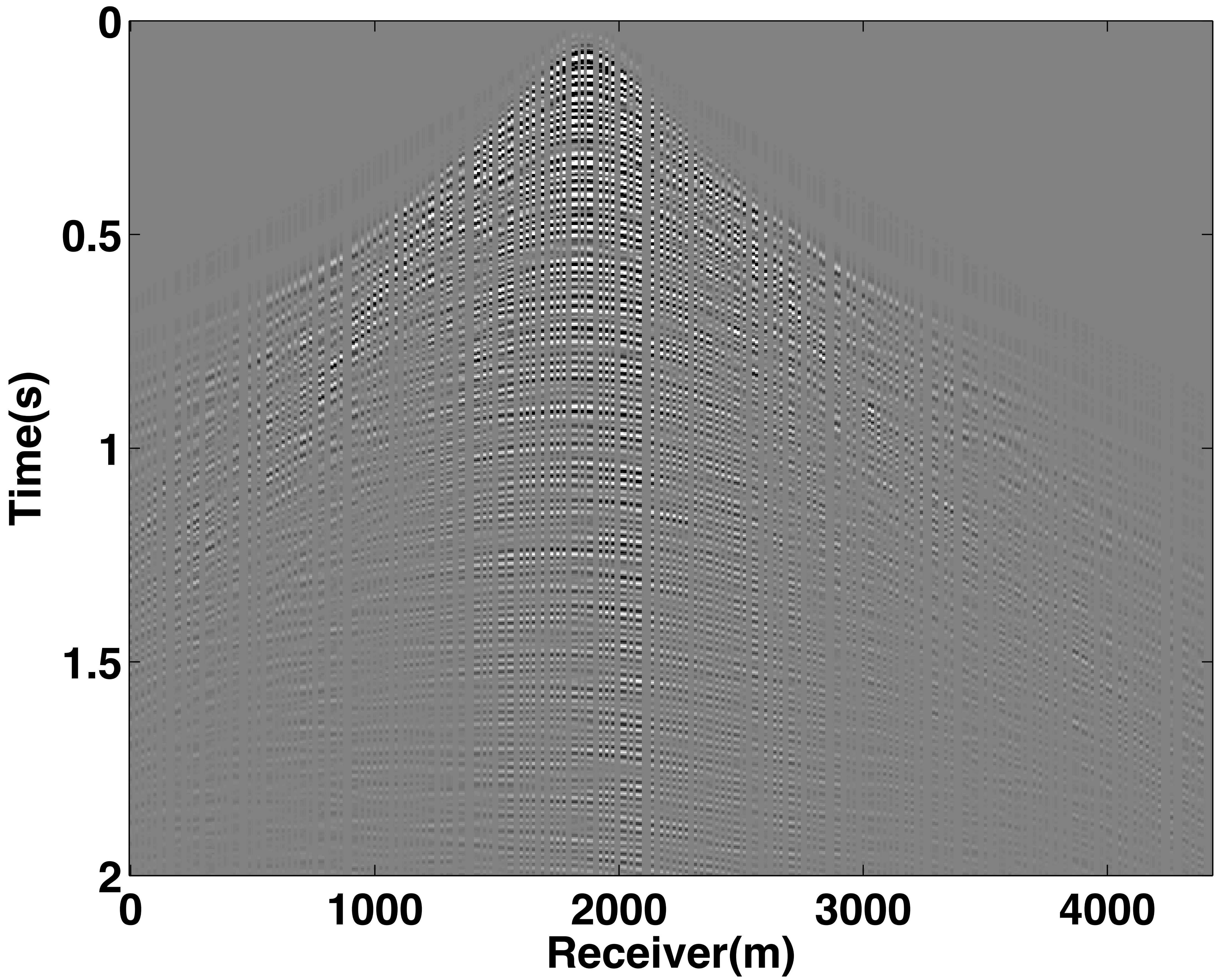
Case 2 : Jittered subsampling

Case 3 : Jittered + Reciprocity

Uniform random v/s Jittered



Discrete random



Jittered

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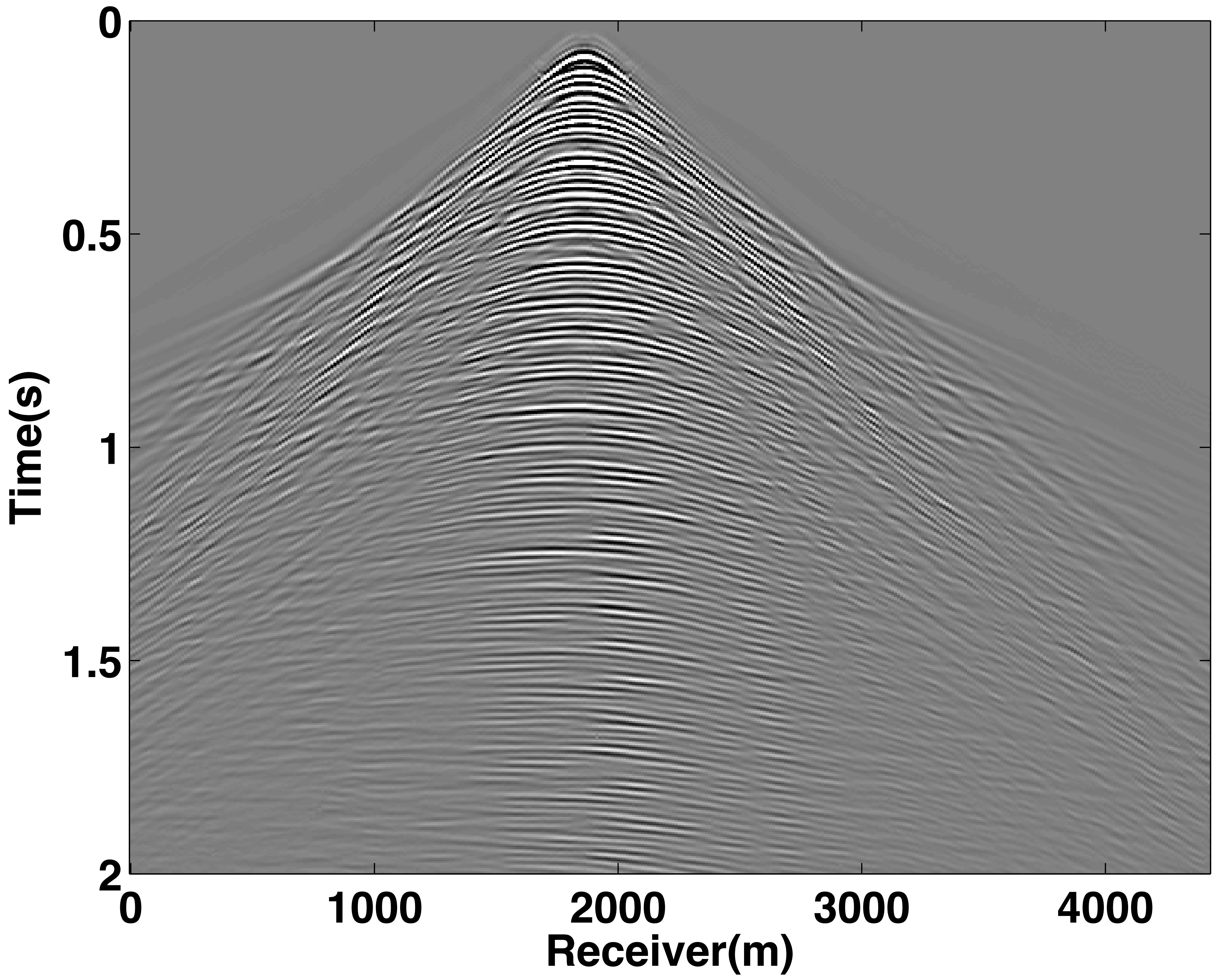
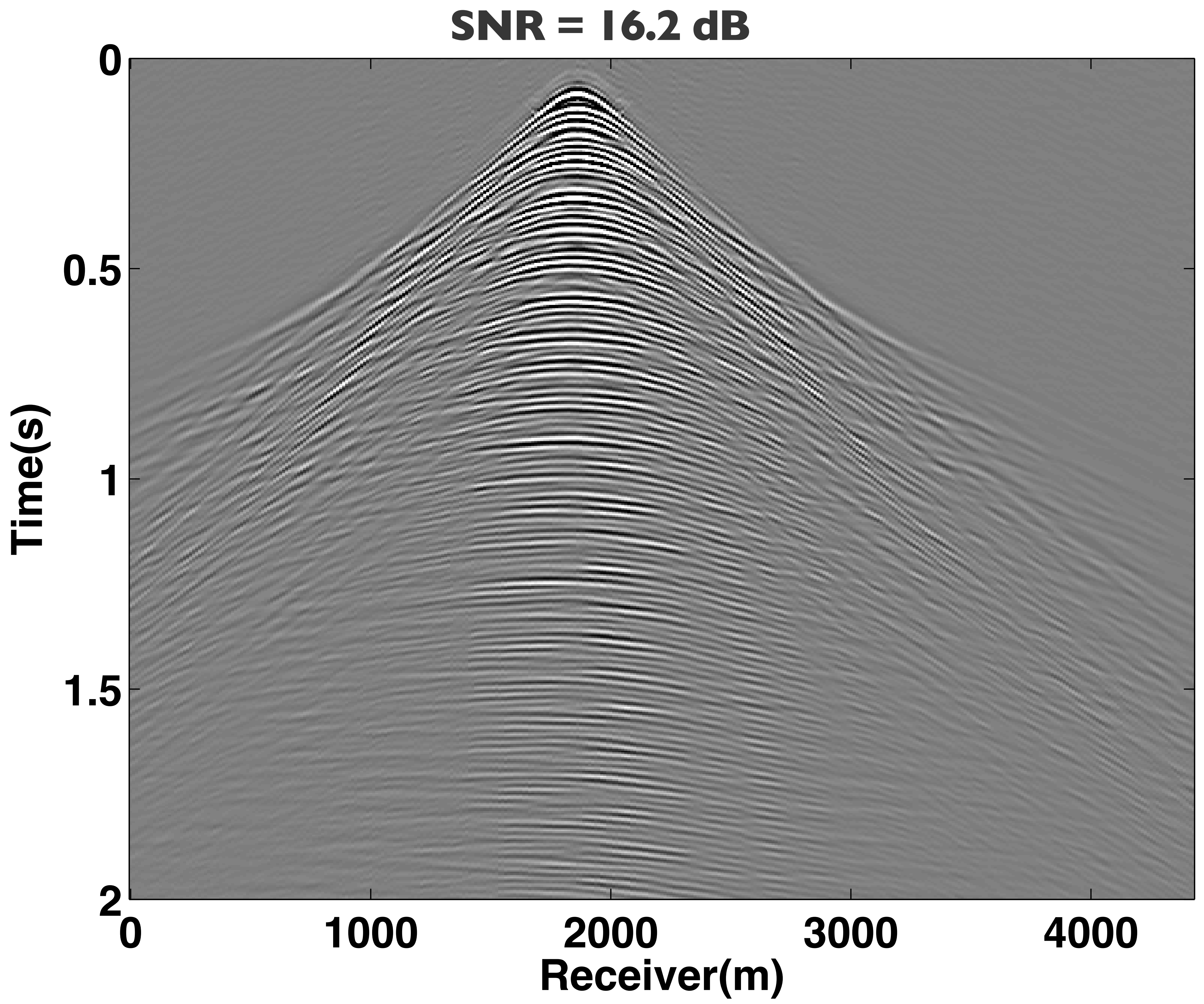
Case I

[uniform random subsampling]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth

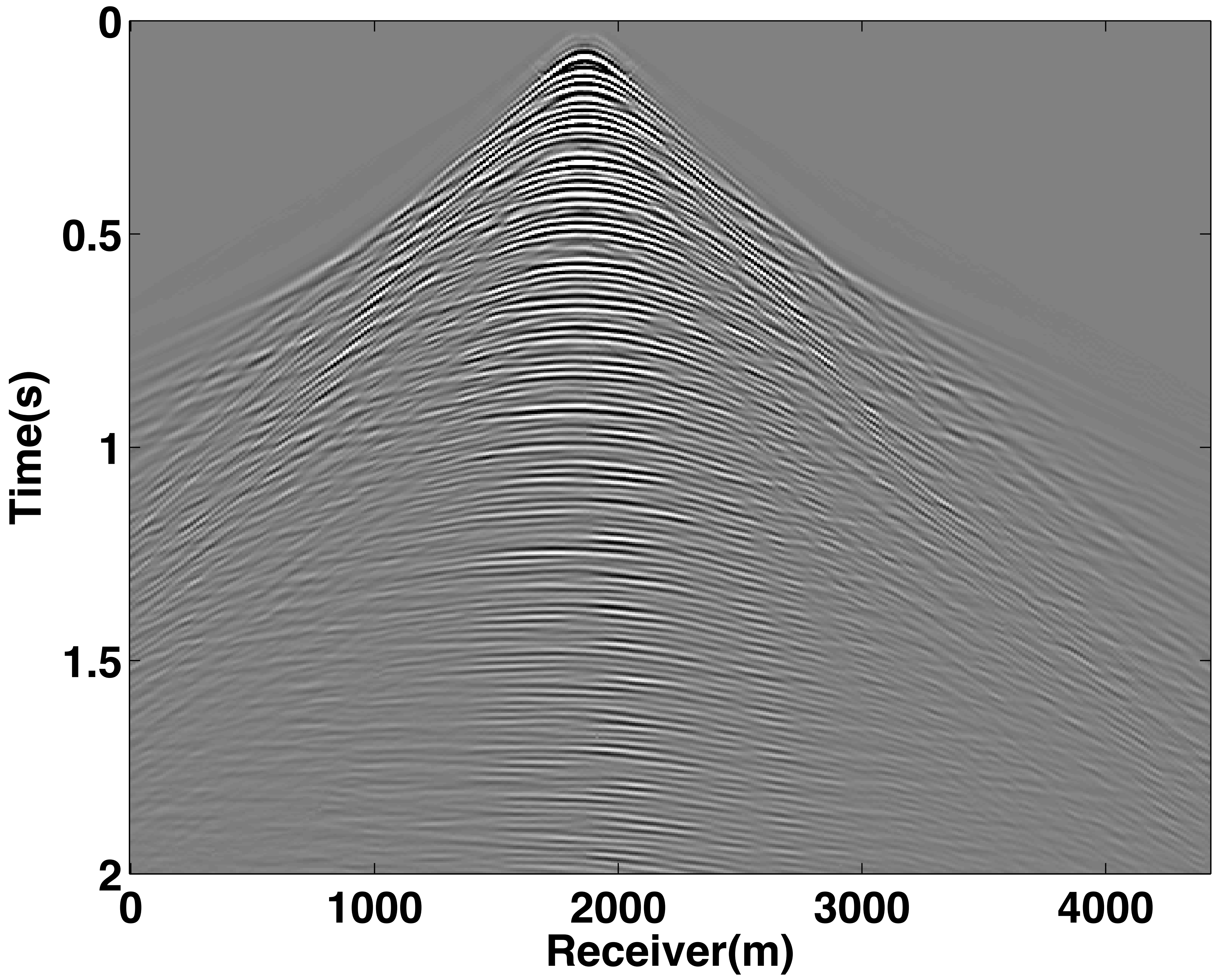


Recovery

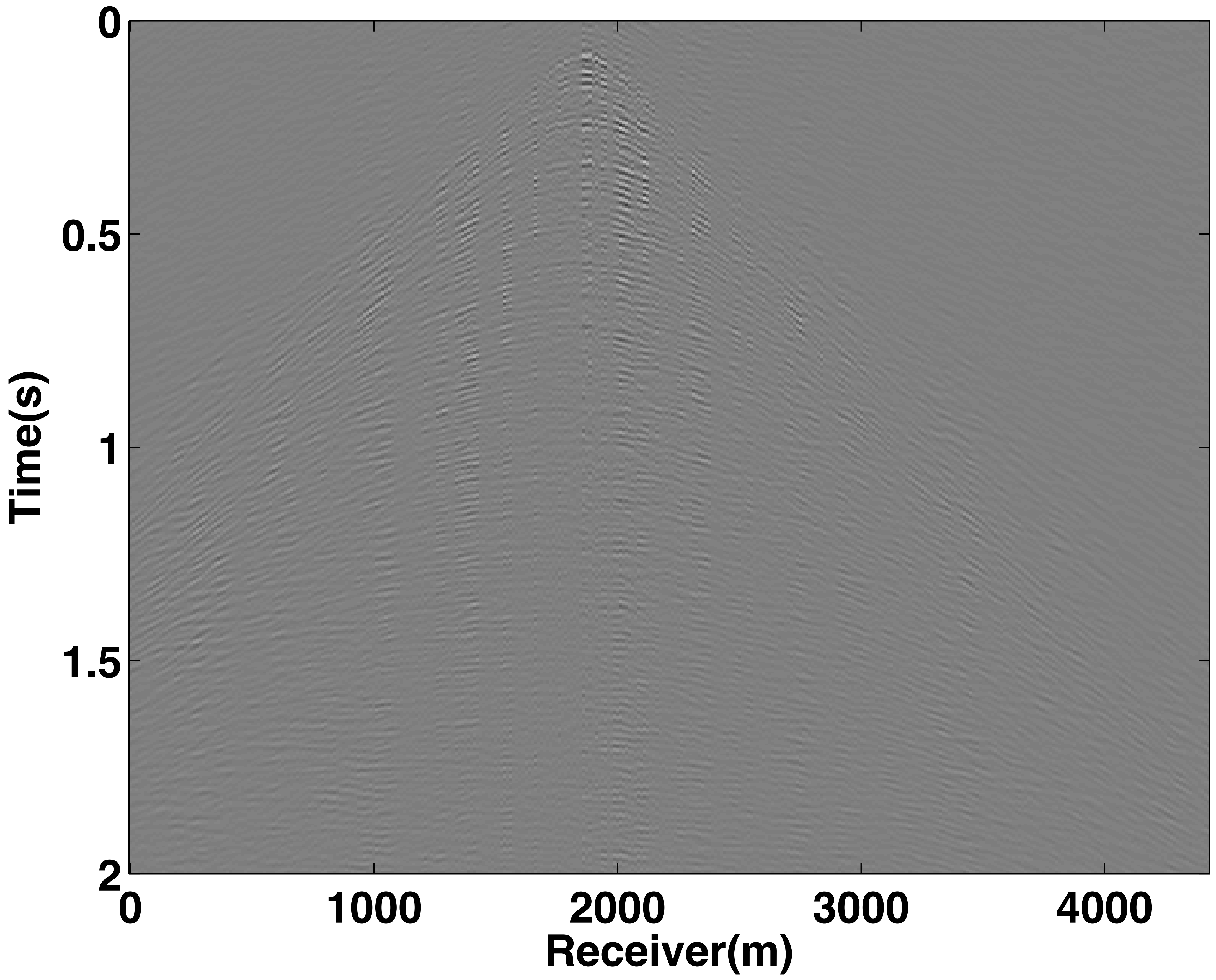
$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$

Case I

[uniform random subsampling]



Ground Truth

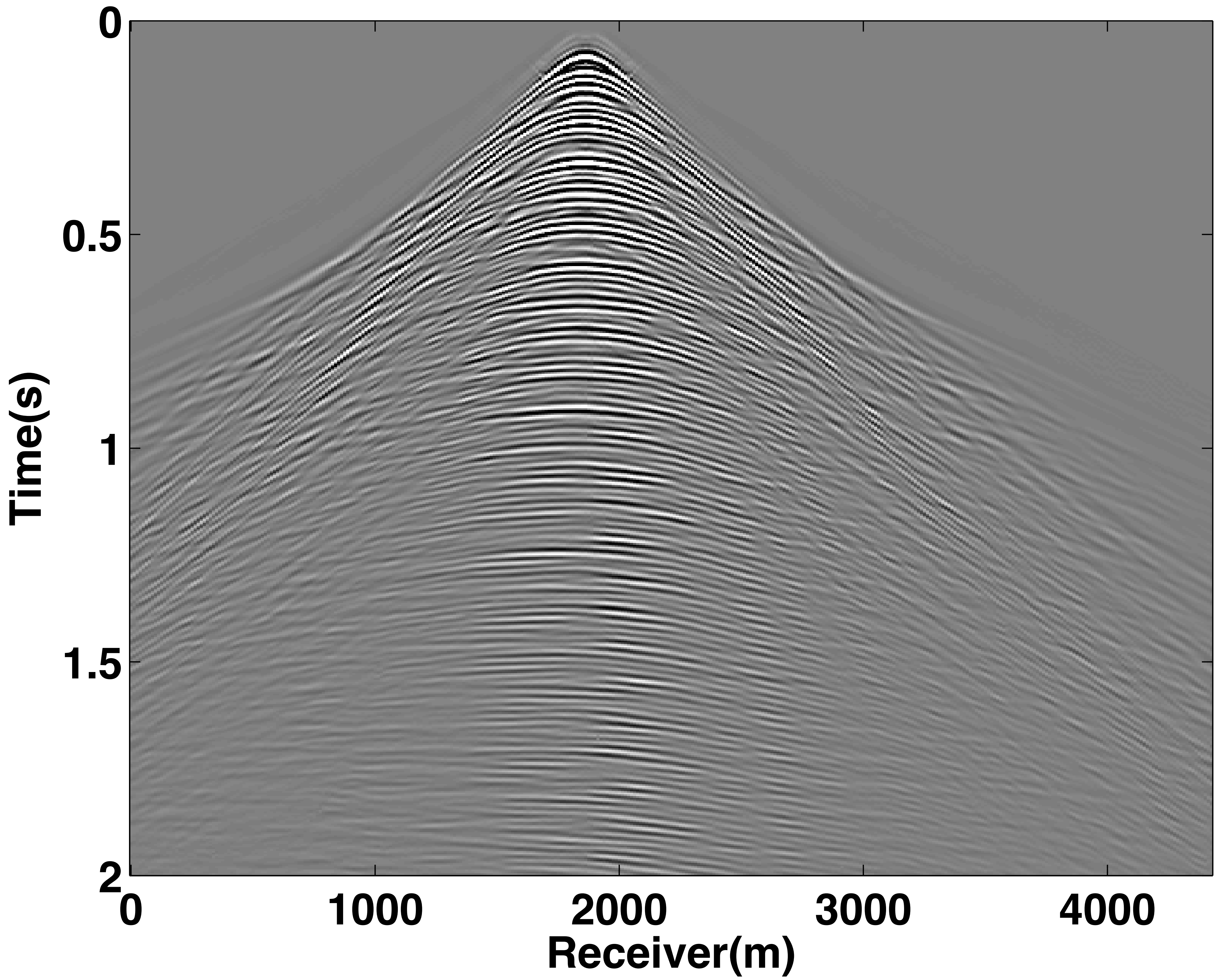


Diference

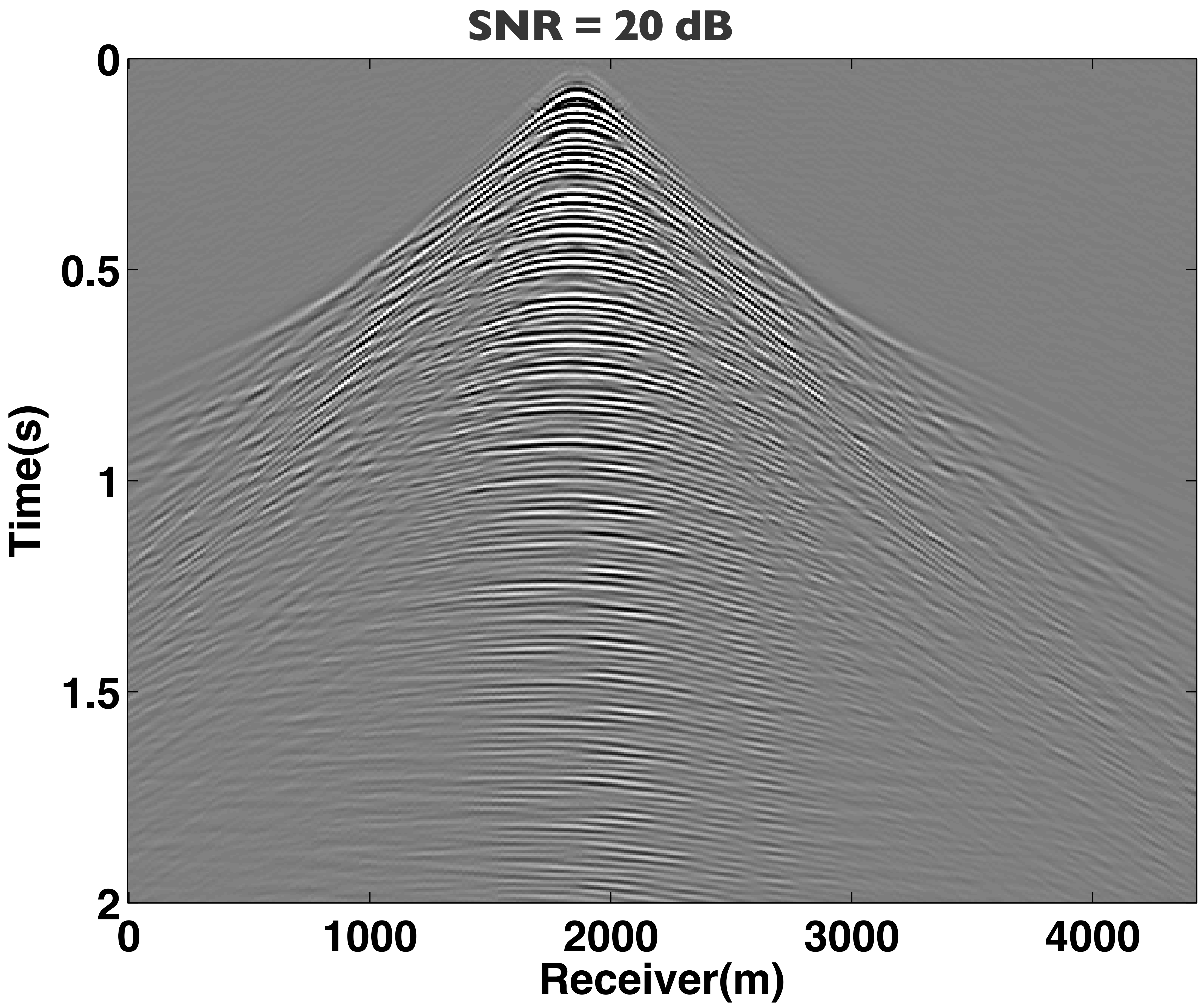
Case 2

[jittered subsampling]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth

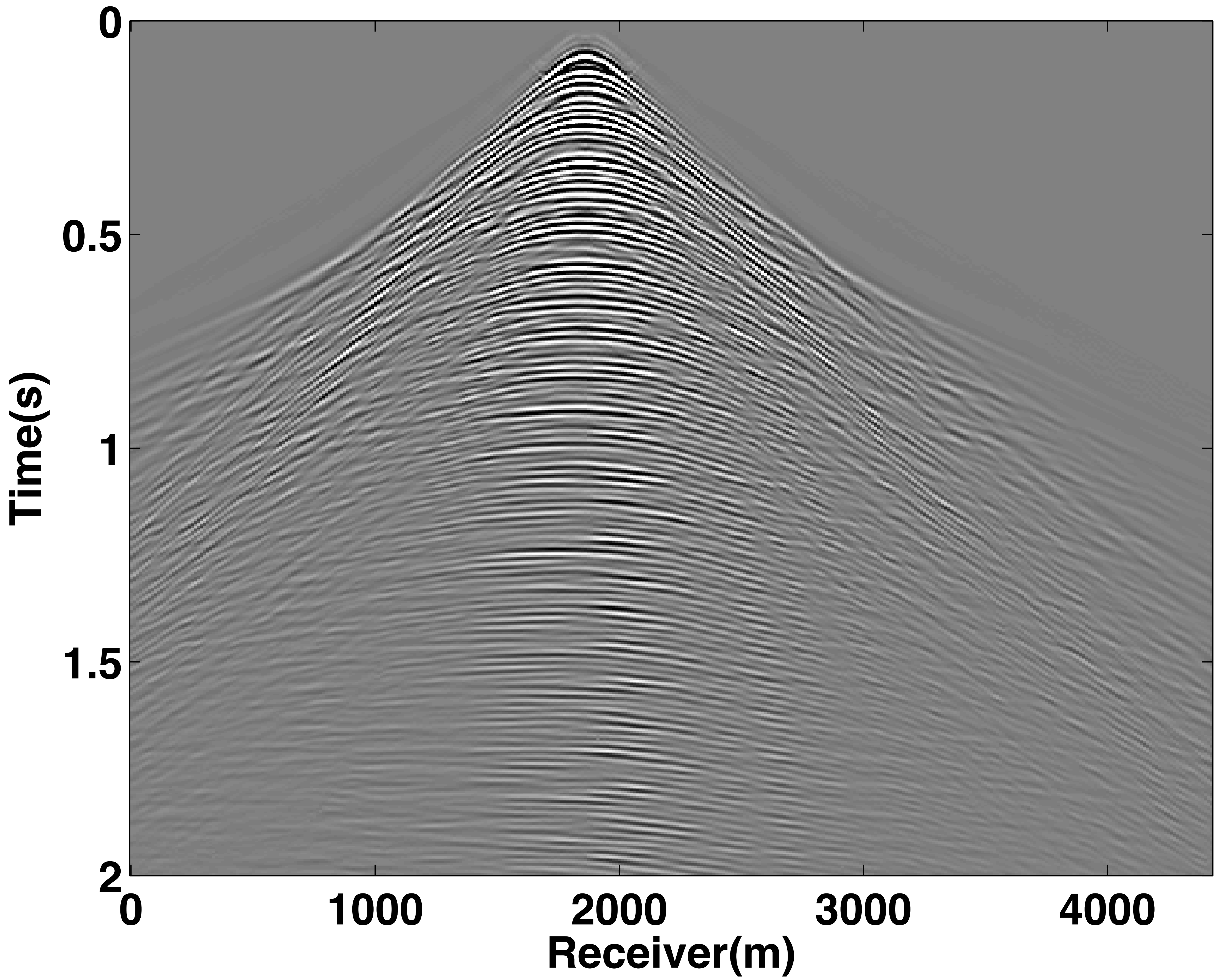


Recovery

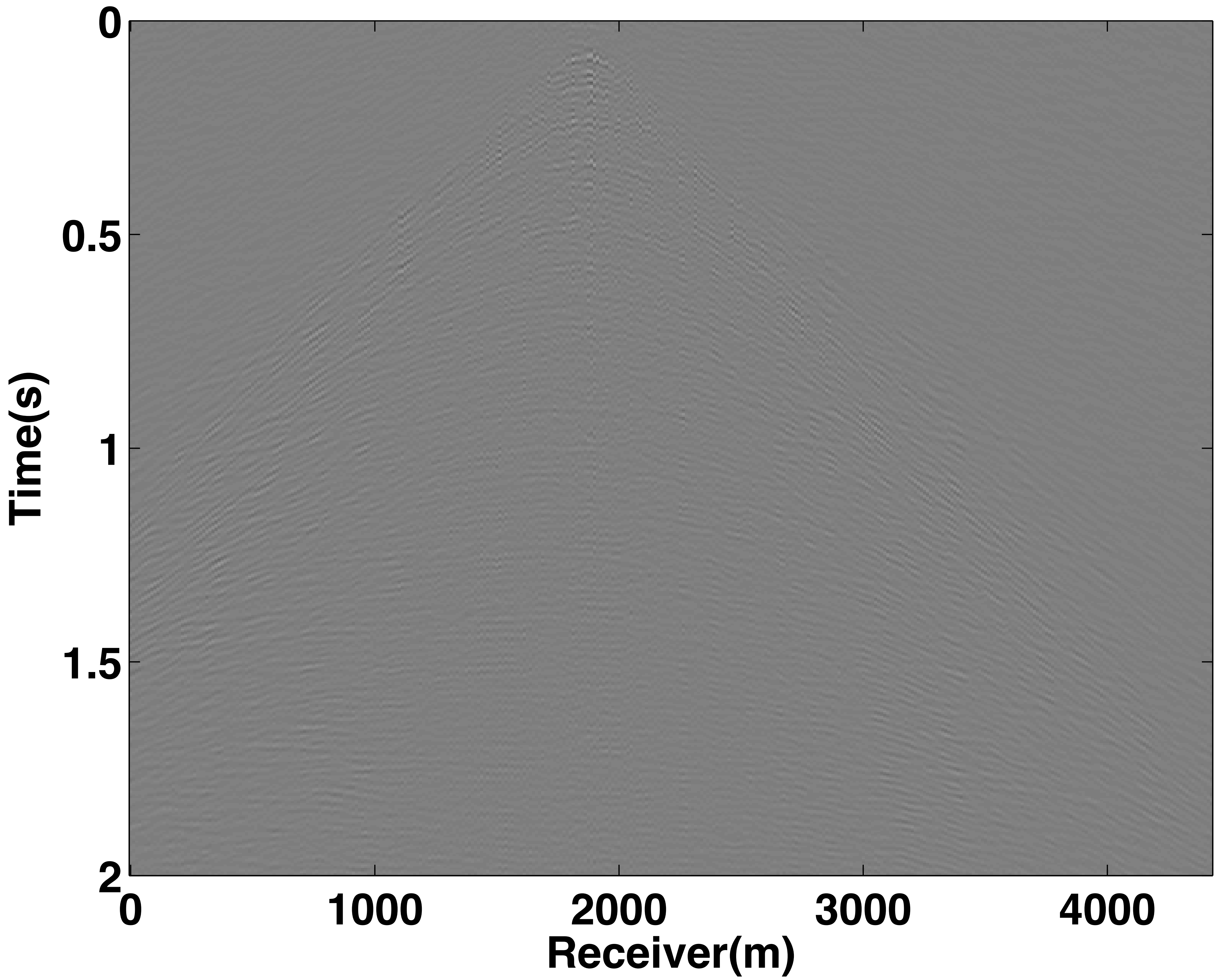
Case 2

[jittered subsampling]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth



Diference

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[Fenati et. al. 1984, Johnson et. al. 2013]

Source-Receiver reciprocity

- Seismic data obeys reciprocity
  - within the noise level
- deviations from this property caused by acquisition geometry and source directivity
- projection of the data into a symmetry subspace

Source-Receiver reciprocity

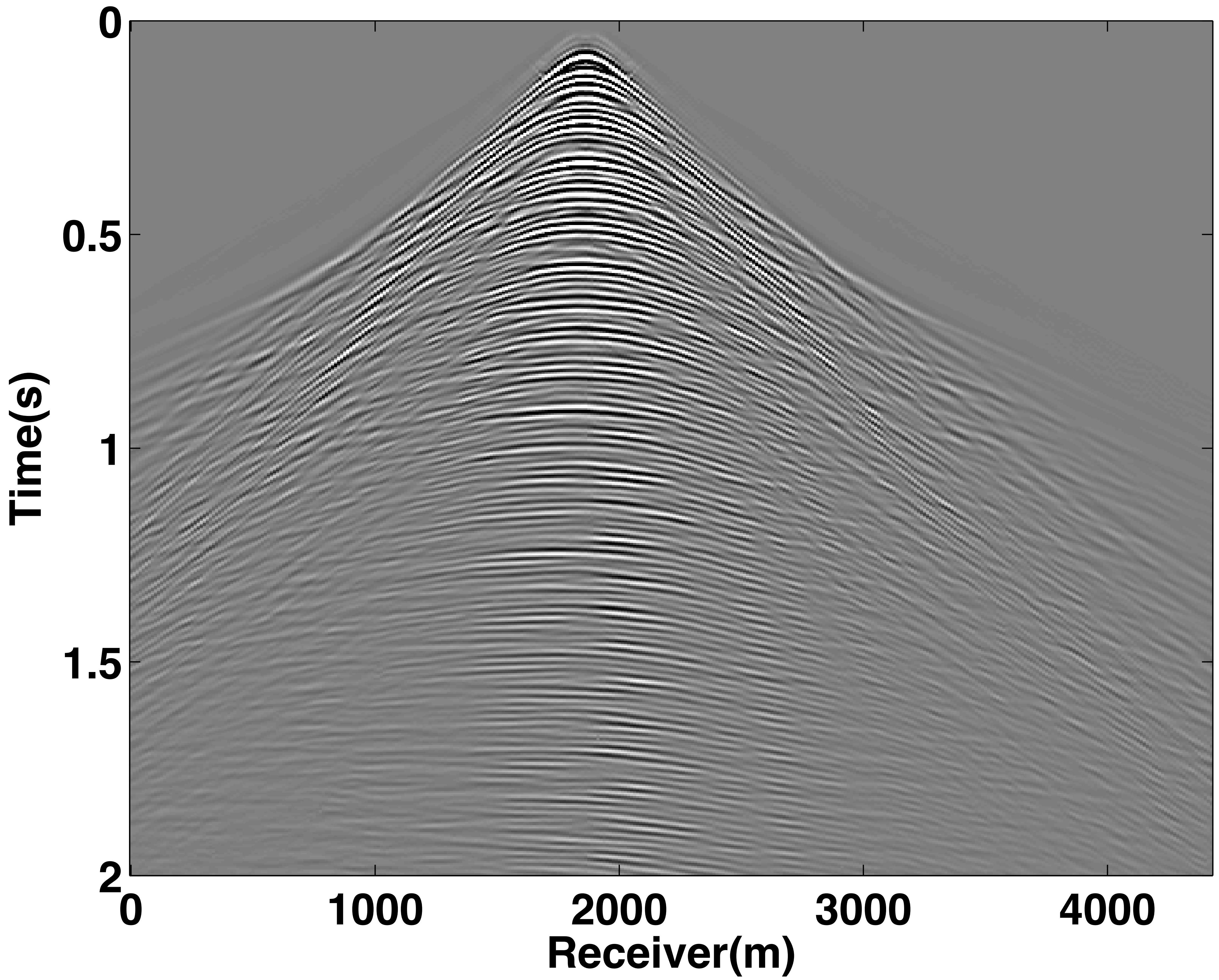
- incorporate reciprocity in transform-sampling operator

$$\mathcal{A} = \mathbf{RM}\frac{(\mathbf{I}+\mathbf{T})}{\mathbf{2}}\mathcal{S}^H$$

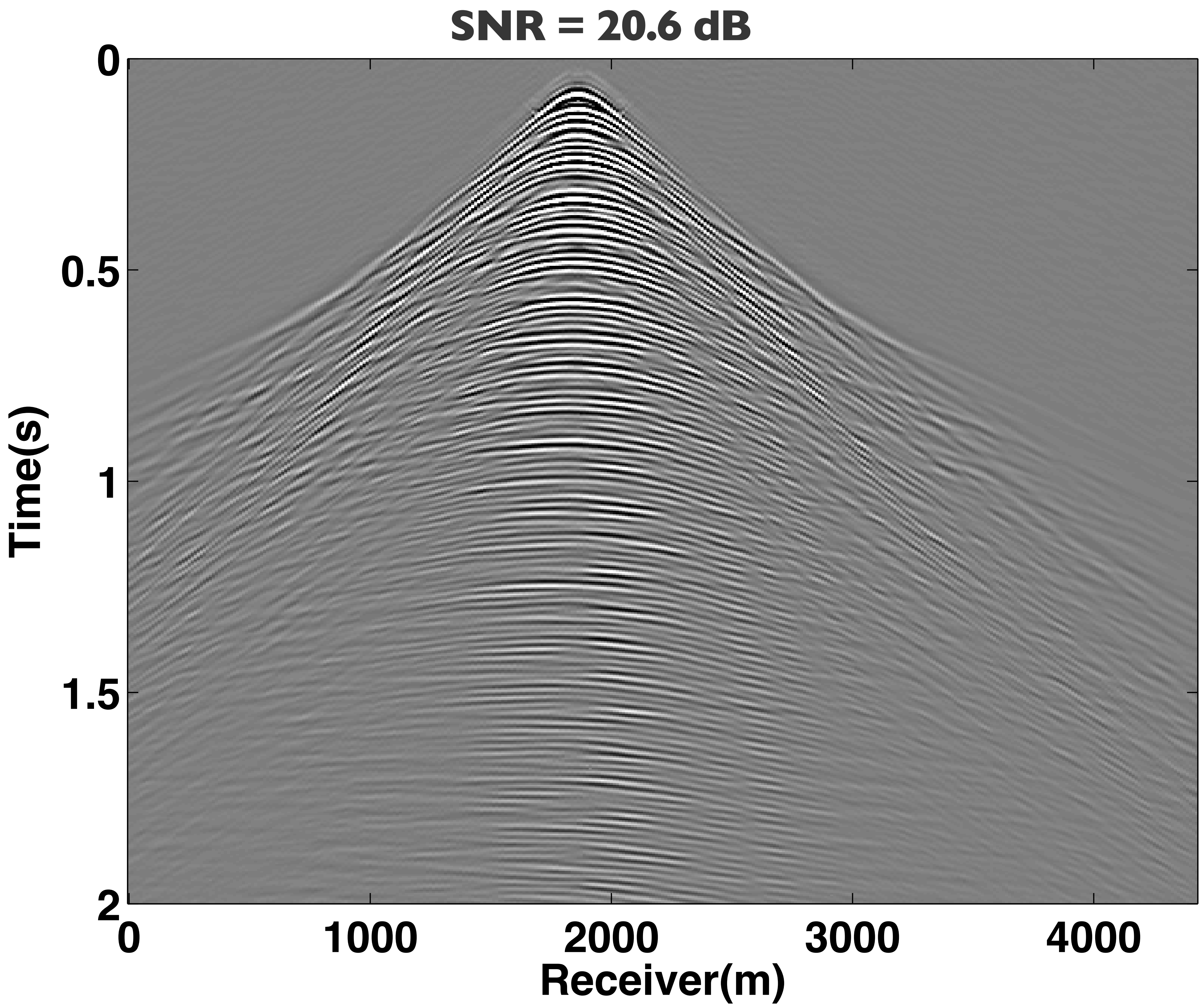
reciprocity term

Case 3

[jittered subsampling + reciprocity]



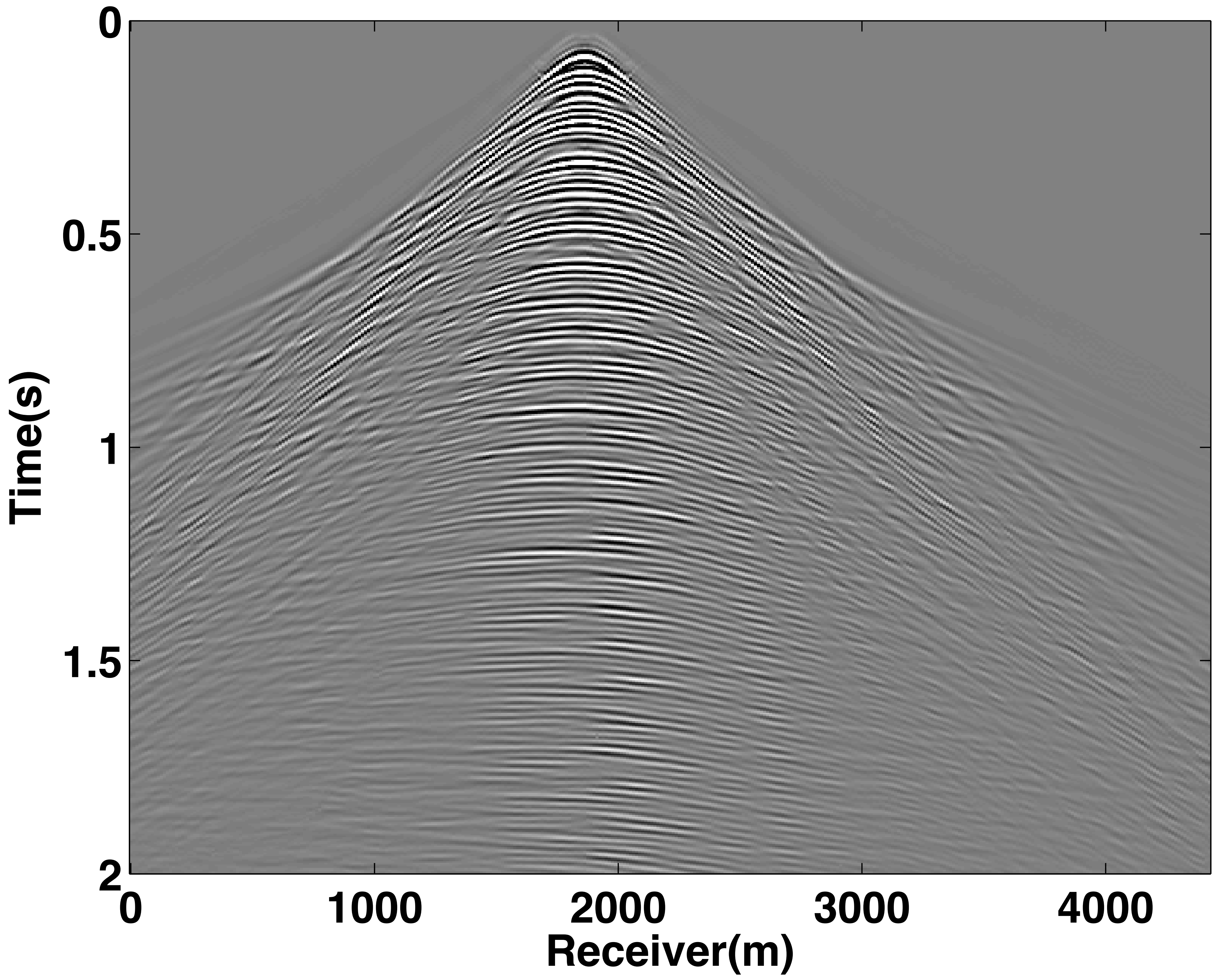
Ground Truth



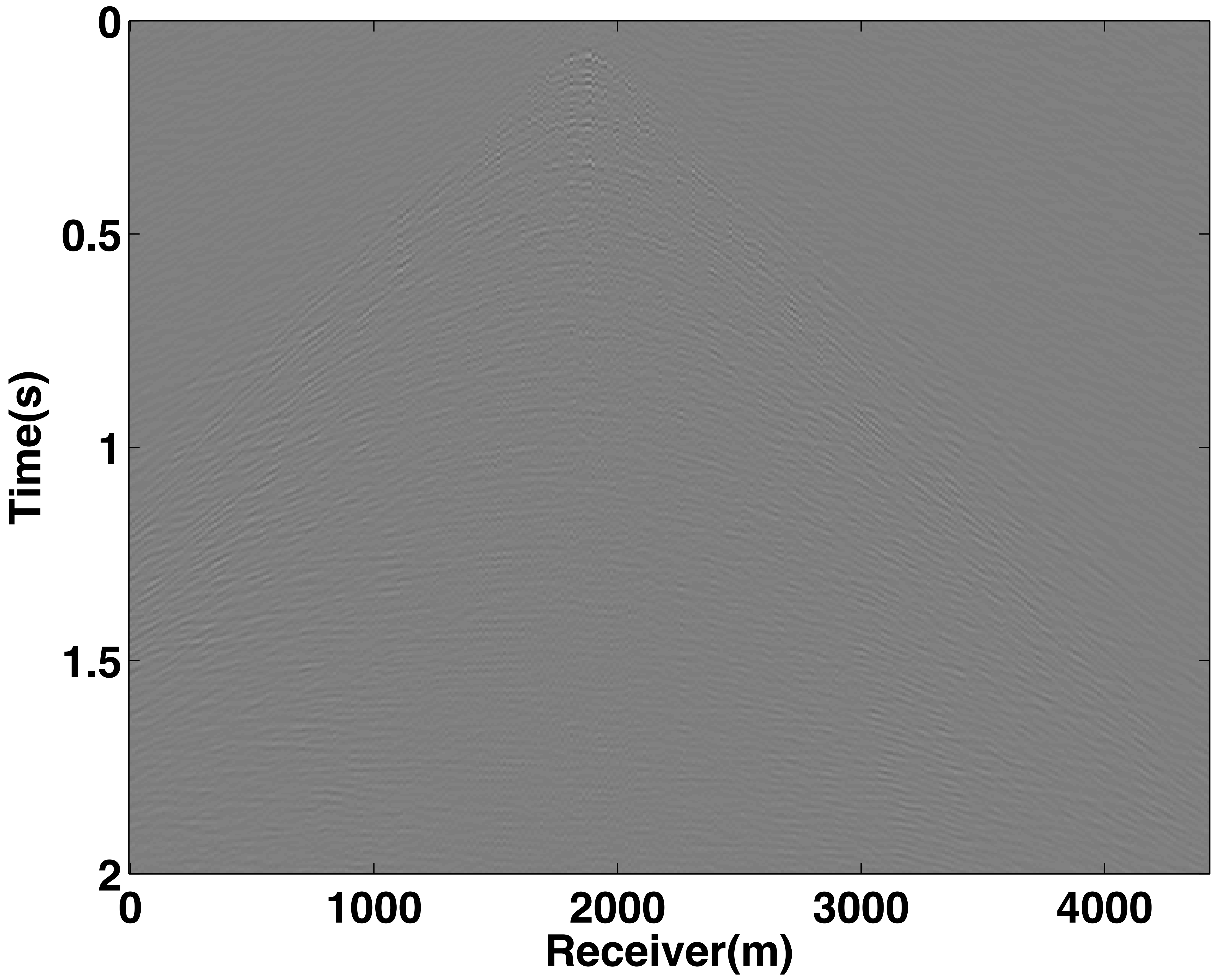
Recovery

Case 3

[jittered subsampling + reciprocity]



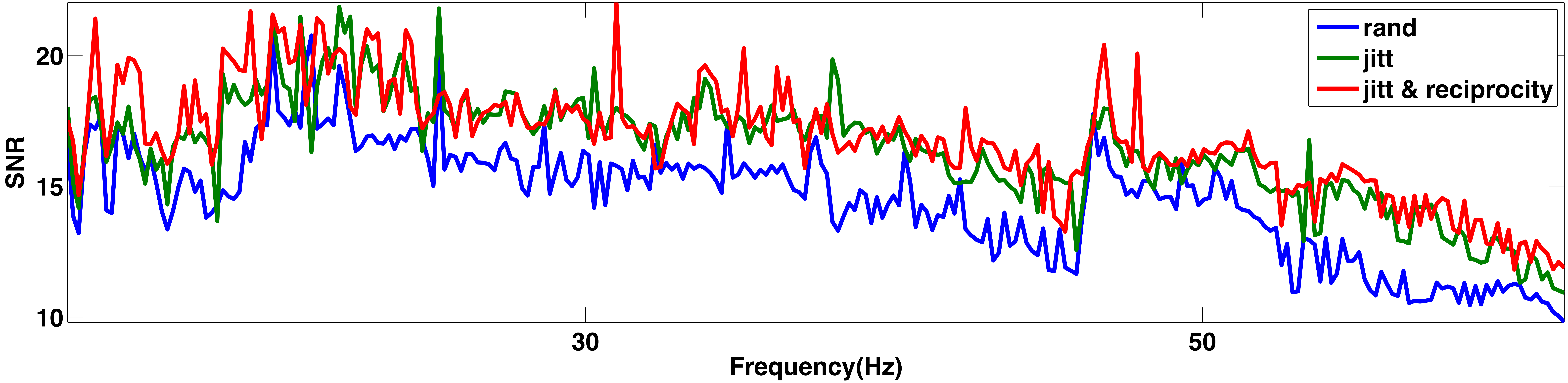
Ground Truth



Diference

Qualitative interpretation

random v/s jittered v/s jittered+reciprocity



Conclusion

- jittered sampling gives advantage of controlling the gap size
- matrix factorization allows SVD-free low-rank methods that work fast on large data
- memory and computationally efficient
- reciprocity enhance the interpolation results

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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