SVD-free low-rank matrix factorization : wavefield reconstruction via jittered subsampling and reciprocity

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Abstract

Recently computationally efficient rank optimization techniques have been studied extensively to develop a new mathematical tool for the seismic data interpolation. So far, matrix completion problems have been discussed where sources are subsample according to a discrete uniform distribution. In this paper, we studied the effect of two different subsampling techniques on seismic data interpolation using rank-regularized formulations, namely jittered subsampling over uniform random subsampling. The other objective of this paper is to combine the fact of source-receiver reciprocity with the rank-minimization techniques to enhance the accuracy of missing-trace interpolation. We illustrate the advantages of jittered subsampling and reciprocity using a seismic line from Gulf of Suez to obtain high quality results for interpolation, a key application in exploration geophysics.

Introduction

Seismic data acquisition suffers from irregularly and/or coarsely sampled data. Interpolation is one of the crucial processing steps to get fully sampled data so that key techniques, such as Full Waveform Inversion and Migration, can be performed. Various methodologies have been proposed to perform interpolation via exploiting the sparse structure of seismic data in some transform domains, for example using the Fourier (Sacchi et al., 1998) and Curvelet (Herrmann and Hennenfent, 2008) transforms. Sparsity in not the only structure seismic data exhibit. When organized in a matrix, seismic data also exhibits low-rank structure, i.e. small number of nonzero singular values, or quickly decaying singular values. In recent years, the research community has proposed missing-trace interpolation via rank-minimization strategies (Oropeza and Sacchi, 2011; Kreimer and Sacchi, 2012; Kumar et al., 2013; Silva and Herrmann, 2013). Rank minimization shares basic principles with Compressive sensing (CS), as discussed by Aravkin et al. (2013) in the seismic context. Existing rank-minimization methods randomly subsample according to a discrete uniform distribution. Hennenfent and Herrmann (2008) showed that random undersampling using discrete uniform distribution is sub-optimal, because we can not control the maximum gap-size in the acquisition. Instead they proposed randomized subsampling, and in particular *jittered* subsampling to control the gap-size. The first objective of this paper is to compare the effect of different subsampling schemes on the seismic data interpolation in rank-minimization techniques. In order to exploit the rank structure of seismic data, we follow the strategy and rank-minimization formulation proposed by Aravkin et al. (2013). Seismic data obeys reciprocity within the noise level, with deviations from this property due to the acquisition geometry and source directivity (Fenati and Rocca, 1984). In case of sparsity-promoting based recovery techniques, Johnson (2013) showed the improvement in the interpolation via incorporating reciprocity. The second objective of this paper is to incorporate the physical principle of source-receiver reciprocity in the rank-minimization techniques to further improve the interpolation. Finally, we demonstrate the efficacy of the proposed method using a seismic line from the Gulf of Suez.

Matrix Factorization

Let X_0 be a matrix in $\mathbb{C}^{n\times m}$ and let $\mathscr{A}$ be a linear measurement operator that maps from $\mathbb{C}^{n\times m} \to \mathbb{C}^p$ with $p \ll n\times m$. Recht et al. (2010) showed that under certain general conditions on the operator $\mathscr{A}$, the solution to the rank minimization problem can be found by solving the following nuclear norm minimization problem:

$$\min_X \|X\|_* \quad \text{s.t. } \|\mathscr{A}(X)-b\|_2 \le \varepsilon, \qquad (\text{BPDN}_\varepsilon)$$

where b is a set of measurements, $\|X\|_* = \|\sigma\|_1$, and σ is the vector of singular values. The linear measurement operator is composed of the product of restriction operator $\mathbf{R}$, Measurement operator $\mathbf{M}$ and low-rankifying operator $\mathscr{S}^H$ such that $\mathscr{A} := \mathbf{RM}\mathscr{S}^H$, where ^{H} denotes the Hermitian transpose. In order to efficiently solve (BPDN$_\varepsilon$), we use an extension of the SPGℓ_1 solver (Berg and Friedlander, 2008) developed for the (BPDN$_\varepsilon$) problem in Aravkin et al. (2012). The SPGℓ_1 algorithm finds the solution to the (BPDN$_\varepsilon$) by solving a sequence of LASSO subproblems

$$\min_X \|\mathscr{A}(X)-b\|_2 \quad \text{s.t. } \|X\|_* \le \tau, \qquad (\text{LASSO}_\tau)$$

where τ is updated by traversing the Pareto curve. Solving each LASSO subproblem requires a projection onto the nuclear norm ball $\|X\|_* \le \tau$ in every iteration by performing a singular value decomposition and then thresholding the singular values. In the case of large scale seismic problems, it becomes prohibitively expensive to carry out such a large number of SVDs. Instead, we adopt a recent factorization-based approach to nuclear norm minimization (Rennie and Srebro, 2005; Lee et al., 2010; Recht and Ré, 2011). The factorization approach parametrizes the matrix $X \in \mathbb{C}^{n\times m}$ as the product of two low rank factors $L \in \mathbb{C}^{n\times k}$ and $R \in \mathbb{C}^{m\times k}$, such that,

$$X = LR^H. \qquad (1)$$

The optimization scheme can then be carried out using the factors L and R instead of X, thereby significantly reducing the size of the decision variable from nm to $k(n+m)$ when $k \ll m,n$. Rennie and Srebro (2005) showed that the nuclear norm obeys the relationship

$$\|X\|_* = \|LR^T\|_* \le \frac{1}{2}\left\|\begin{bmatrix} L \\ R \end{bmatrix}\right\|_F^2 =: \Phi(L,R), \qquad (2)$$

where $\|\cdot\|_F^2$ is Frobenius norm of the matrix (sum of the squared entires). Consequently, the LASSO subproblem can be replaced by

$$\min_{L,R} \|\mathscr{A}(LR^T) - b\|_2 \quad \text{s.t. } \Phi(L,R) \leq \tau\,, \tag{3}$$

where the projection onto $\Phi(L,R) \leq \tau$ is easily achieved by multiplying each factor L and R by the scalar $2\tau/\Phi(L,R)$. By equation (2), we are guaranteed that $\|LR^T\|_* \leq \tau$ for any solution of (3).

Seismic Data Interpolation

We implement the proposed formulation on a seismic line from Gulf of Suez with $N_s = 355$ sources, $N_r = 355$ receivers, $N_t = 1024$ time samples and a sampling interval of 0.004s. Most of the energy of the seismic line is concentrated in the 8-65Hz frequency band. In order to interpolate, we apply a sub-sampling mask that randomly removes 50% of the receivers. In this example, we use the transformation from the source-receiver (s-r) domain to the midpoint-offset (m-h) domain. To show that the m-h transformation is a good choice, we plot (figure 1) the singular values decay of monochromatic frequency slices at 12Hz and 60Hz in the s-r and m-h domain. Notice that the decay of singular values in both frequency slices are faster in the m-h domain. Sub-sampling does not noticeably change the decay of singular value in the (s-r) domain; but destroys the fast decay of singular values in the (m-h) domain, an essential feature for interpolation using nuclear-norm minimization. In each example, we use 150 iterations of SPGℓ_1 for all frequency slices.

Jittered Undersampling

Success of CS hinges on the fact that sampling should destroy the structure of data in some transform domain. Random subsampling according to a discrete uniform distribution is one such favourable sampling scheme (Herrmann and Hennenfent, 2008). The obvious question is the extent of randomness we can allow in the subsampling system. Hennenfent and Herrmann (2008) showed that random subsampling according to a discrete uniform distribution creates large gap in the data and proposed to use the jittered subsampling, which shares the benefit of random subsampling and also controls the maximum gap size (Figure 2). In case of rank minimization techniques, jittered sampling shares the benefit of controlling the average amount of information per row in the transform domain as evident in Figure 3, whereas in case of uniform random sampling the chances of missing the information is much higher. We solve the interpolation problem to compare the effect of uniform random versus jitter subsampling and clearly see that SNR increased by a factor of 3-4 dB (Figure 4), which highlights the benefit of jittered subsampling.

Reciprocity

We restrict the solution of the inversion problem to obey reciprocity. This method is viewed as projection of data into a symmetry subspace (Johnson, 2013). Given a monochromatic data slice, under the assumption of reciprocity, we can rewrite $\mathscr{A} = \mathbf{R}\frac{(\mathbf{I}+\mathbf{T})}{\mathbf{2}}\mathscr{S}^H$, where $\mathbf{T}$ is the matrix transpose on each monochromatic data slice. Figure 3 shows the benefit of incorporating the reciprocity in the interpolation problem. We are able to enhance the SNR by a factor of 0.6 - 1 dB.

Conclusions

We compared the effect of two different subsampling schemes on the seismic data interpolation in the rank-minimization techniques. We also incorporated source-receiver reciprocity to enhance the interpolation results. It is evident that jittered sampling performs better then the uniform randomized sampling, since we have control over the average amount of information needed to interpolate the missing traces of seismic data. To solve the nuclear-norm minimization problem, we combined the Pareto curve approach for optimizing (BPDN$_\varepsilon$) formulations with the SVD-free matrix factorization methods, following Aravkin et al. (2013). The technique is very promising, since it is SVD-free and therefore may be used for very large-scale systems. The experimental results on a seismic line from the Gulf of Suez demonstrate the potential benefit of the methodology.

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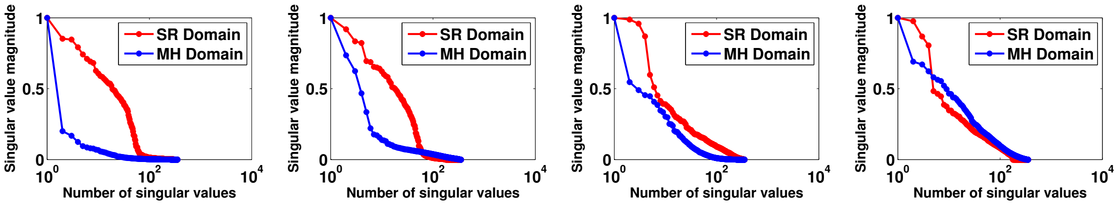
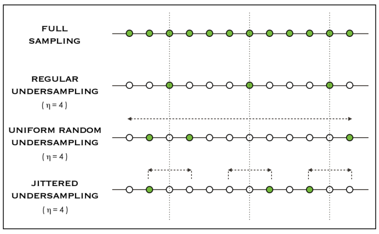


(a) (b) (c) (d)

Figure 1 *Singular value decay of fully sampled (a) low frequency slice at 12 Hz and (c) high frequency slice at 60 Hz in (s-r) and (m-h) domains. Singular value decay of* 50% *subsampled (b) low frequency slice at 12 Hz and (d) high frequency data at 60 Hz in (s-r) and (m-h) domains.*



(a)

Figure 2 *Schematic comparison between different undersampling schemes. η is the undersampling factor. The full sampling scheme represents the spatial grid on which the signal is alias-free. In subsequent sampling scheme, solid circle represents the grid location on which data is acquired.*

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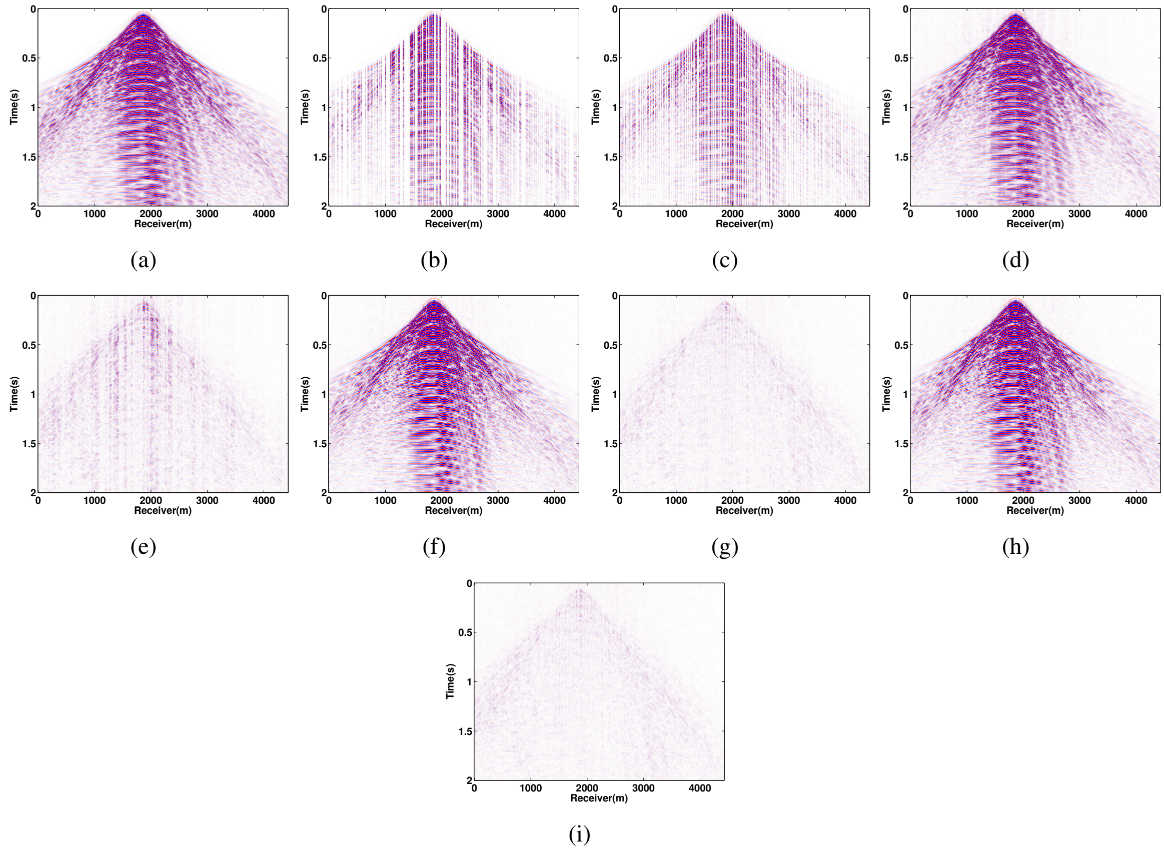
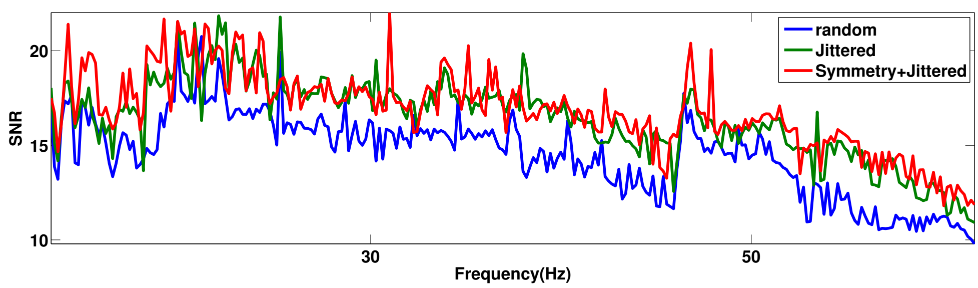


Figure 3 *Comparison of the interpolation results in case of uniform random sampling, jittered sampling and reciprocity . (a) Ground truth (common receiver gather). (b,c)* 50% *subsampled common receiver gather in case of uniform random and jittered subsampling. (d,e) Recovery and difference in case of uniform random subsampling with a SNR of 16.5 dB. (f,g) Recovery and difference in case of jittered subsampling with a SNR of 20.0 dB. (h,i) Recovery and difference in case of reciprocity incorporated along with jittered subsampling with a SNR of 20.6 dB.*



(a)

Figure 4 *Qualitative measurement of the seismic data interpolation for 8-65 Hz frequency band. We can see the significant improvements in the SNR by incorporating the Jittered subsampling and Reciprocity.*