Extended images in action: efficient WEMVA via randomized probing

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Abstract

Image gathers as a function of subsurface offset are an important tool for velocity analysis in areas of complex geology. In this paper, we offer a new perspective on image gathers by organizing the extended image as a function of all subsurface offsets and all subsurface points into a matrix whose $(i, j)^{th}$ entry captures the interaction between gridpoints i and j. For even small problems, it is infeasible to form and store this matrix. Instead, we propose an efficient algorithm to glean information from the image volume via efficient matrix-vector products. We illustrate how this can be used to construct objective functions for automatic MVA.

Introduction

Wave-equation based migration velocity analysis (WEMVA) is an important tool for velocity estimation in complex geological settings. The objective of MVA is to measure the focusing of common-image gathers (CIG) by formulating an appropriate cost function (Biondi and Symes, 2004; Shen and Symes, 2008; Symes, 2008; Sava and Vasconcelos, 2011). In general, these CIG's (or extended images) are a function of all subsurface-offset and time-lags (or equivalently, frequencies) for all subsurface points using the full two-way wave equation. One of the difficulty in forming the extended image is that we can never hope to compute and store the full extended image volume. In this paper, we offer a new perspective to probe the image volume for information by computing the action of the image volume. We show that such actions can be computed cheaply and used in to formulate a cost efficient automatic MVA algorithm. The paper proceeds as follows. First, we write the extended image in terms of *data-matrices*. Then, we show how actions of the image-volume matrix on vectors can be computed followed by the construction of the objective function for automatic WEMVA. Finally, we show the efficacy of the proposed formulation on three different velocity models i.e. horizontal reflectors, horizontal reflectors model embedded with Gaussian anomaly (Lens model) and vertical gradient model.

Methodology

Following Berkhout (1993), we arrange the time-harmonic source and receiver wavefields into matrices U and V, where each column corresponds to a source experiment. Given these matrices, the extended image at a single frequency, for all subsurface offsets and for all subsurface points can be written as the outer product of these two matrices, i.e., we have

$$E(\omega) = U(\omega)V(\omega)^*, \tag{1}$$

where * denotes the Hermitian transpose. In 2D, the extended image is a 5-dimensional function of all subsurface offsets and temporal shifts, which is prohibitively expensive to compute and store, for all the subsurface points. We follow van Leeuwen and Herrmann (2012) to efficiently compute $\widetilde{E}$ by multiplying E with the tall matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_l]$ yielding

$$\widetilde{E} = EW = H^{-1}P_s^T QD^* P_r H^{-1} W, \tag{2}$$

where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents a single scattering point with the location of 1 corresponding to i^{th} grid location of the point scatterer. The computational cost of calculating $\widetilde{E}$ is $2l$ PDE solves plus the cost of correlating the source and data matrices. Thus, the cost of computing the CIG's does not depend on the number of sources or the number of subsurface offsets, as it does in the conventional methods (Sava and Vasconcelos, 2011). An overview of the computational complexity is shown in Table 1. In order to perform the automatic MVA, Shen and Symes (2008) consider penalizing the extended images as a function of subsurface offset with the lateral shift, which in our case is equivalent to demanding that our image matrix commutes with point-wise multiplication with the $\mathbf{x}$-position. For a focused image, we want to enforce that

$$E\mathrm{diag}(\mathbf{x}) \approx \mathrm{diag}(\mathbf{x})E. \tag{3}$$

The obvious way to achieve this is by formulating the optimization problem as

$$\min_{\mathbf{m}} \phi(\mathbf{m}) = ||E(\mathbf{m})\mathrm{diag}(\mathbf{x}) - \mathrm{diag}(\mathbf{x})E(\mathbf{m})||^2, \tag{4}$$

where $\|.\|$ is a matrix norm. If we choose the Frobenius norm as the matrix norm, then we can estimate this penalty via randomized trace estimation without explicitly forming the whole matrix. The basic idea is as follows. First, write the Frobenius norm as the matrix trace. Then, the trace can be estimated as

$$||A||_F^2 = \mathrm{trace}(A^T A) \approx \sum_{i=1}^{K} \mathbf{w}_i^T A^T A \mathbf{w}_i = \sum_{i=1}^{K} ||A\mathbf{w}_i||_2^2, \tag{5}$$

where $\sum_{i=1}^{K} \mathbf{w}_i \mathbf{w}_i^T \approx I$. Thus, the optimization can be written as

$$\min_{\mathbf{m}} \widetilde{\phi}(\mathbf{m}) = \sum_{i=1}^{K} ||R(\mathbf{m})\mathbf{w}_i||^2, \tag{6}$$

where

$$R(\mathbf{m}) = E(\mathbf{m})\mathrm{diag}(\mathbf{x}) - \mathrm{diag}(\mathbf{x})E(\mathbf{m}). \tag{7}$$

The gradient of this objective is given by

$$\nabla\widetilde{\phi}(\mathbf{m}) = \sum_{i=1}^{K}\left(DE(\mathbf{m},\mathrm{diag}(\mathbf{x})\mathbf{w}_i) - \mathrm{diag}(\mathbf{x})DE(\mathbf{m},\mathbf{w}_i)\right)^{*}R(\mathbf{m})\mathbf{w}_i, \tag{8}$$

where

$$DE(\mathbf{m},\mathbf{w}) = \frac{\partial E(\mathbf{m})\mathbf{w}}{\partial \mathbf{m}}, \tag{9}$$

is the Jacobian and its action on a vector can be calculated as

$$DE(\mathbf{m},\mathbf{w})\delta\mathbf{m} = -\omega^2\left(E(\mathbf{m})\mathrm{diag}(\widetilde{\mathbf{w}}) + H(\mathbf{m})^{-1}\widetilde{E}\right)\delta\mathbf{m}, \tag{10}$$

where $\widetilde{\mathbf{w}} = H(\mathbf{m})^{-1}\mathbf{w}$ and $\widetilde{E} = E(\mathbf{m})\mathbf{w}$.

Shen (2013) showed that we can use the same focusing principle to perform the automatic velocity analysis in case of diving waves, which is one of the main driving criteria in Full Waveform Inversion. In this paper, we test the proposed formulation on both the reflected and diving wave cases.

Experiment and Results

We first demonstrate the advantage of randomized trace estimation using a subset of Marmousi model. Figure 1 shows the true and approximate penalties (in the Frobenius norm) as a function of velocity perturbation for reflected waves. We can clearly see that we only need a few probing vectors to approximate the true objective function. Next, we perform the automatic velocity analysis on three different velocity models. We start with the simplest example of horizontal reflectors model. We perform 15 L-BFGS iterations with $K = 90$ in this case. Figure 2a,e show the corresponding true velocity model and the image gather. The initial model (Figure 2b) used for WEMVA is a vertical gradient model. The corresponding image gathers (Figure 2f) are defocused. Figure 2c,g show the estimated velocity model and the corresponding image gathers. We get a good reconstruction of velocity model upto the depth of 1400m, which is well illuminated by the reflected waves. The next example is the Lens model. 30 L-BFGS iterations are performed with $K = 100$ in this case. Figure 3a,e show the corresponding true velocity model and the image gather. The initial model (Figure 3b) used for WEMVA is a vertical gradient model. The corresponding image gathers (Figure 3f) are defocused. Note that in practice, probably a more accurate starting model can be used, so this can be considered a worst case scenario. Figure 3c,g show the estimated velocity model and the corresponding image gather. The focused image gathers indicate that we get a good reconstruction of the Gaussian anomaly which is not present in the starting model. As mentioned before, one of the driving criteria of MVA is diving waves, so we use the same vertical gradient example (Shen, 2013) to test the proposed formulation for the case of diving waves. We perform 7 L-BFGS iterations with $K = 120$ in this case. Figure 4a,d show the corresponding true velocity model and the image gather. The initial model (Figure 4b) used for WEMVA is an incorrect starting model. Hence, the corresponding image gathers (Figure 4e) are defocused. As illustrated in Figure 4c, we get a good reconstruction of the velocity model upto the depth of 600m, which is well illuminated by the diving waves. Figure 4f shows the corresponding image gather.

Conclusions

We have discussed an efficient way of gleaning information from extended image volumes via computing its action on input vectors. In the proposed formulation, the extended image for a single frequency, as a function of all subsurface offsets, evaluated at all subsurface points is organized in a matrix whose $(i,j)^{\text{th}}$ entry captures the interaction between gridpoints i and j. We can extract all conventional image gathers from this matrix. The dominant computational cost of one matrix-vector product is 2 PDE solves and does not depend on the desired sampling of the subsurface offsets nor the number of sources and receivers. We have shown how to use such matrix-vector products to form conventional image gathers and how they can be used to formulate a cost efficient function for automatic MVA.

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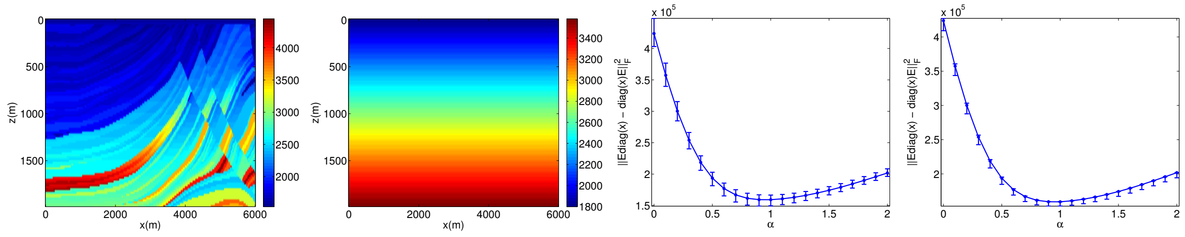
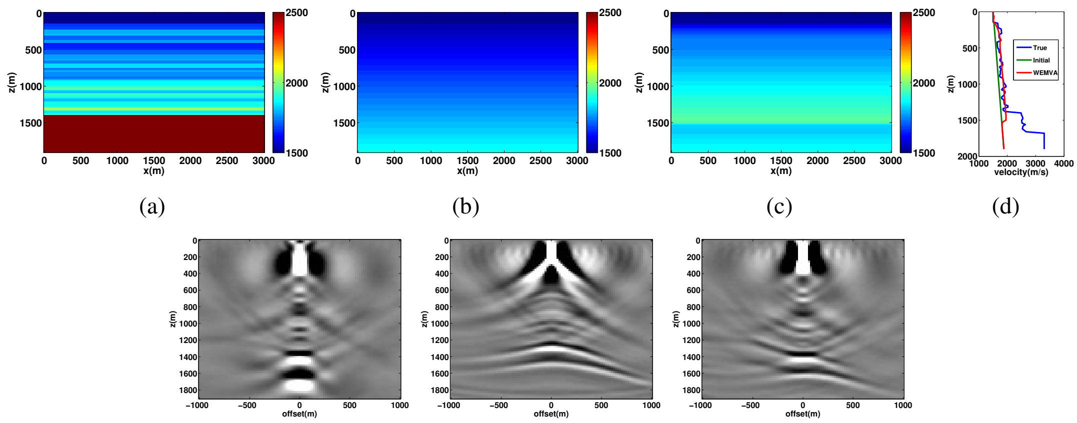
	# of PDE solves	flops
conventional	$2N_s$	$N_s N_{h_x} N_{h_y} N_{h_z}$
this paper	$2N_x$	$N_r N_s$

Table 1 *Computational complexity of the two schemes in terms of the number of sources* N_{s}*, receivers* N_{r}*, sample points* N_x *and desired number of subsurface offsets in each direction* $N_{h_{\{x,y,z\}}}$*.*



(a) (b) (c) (d)

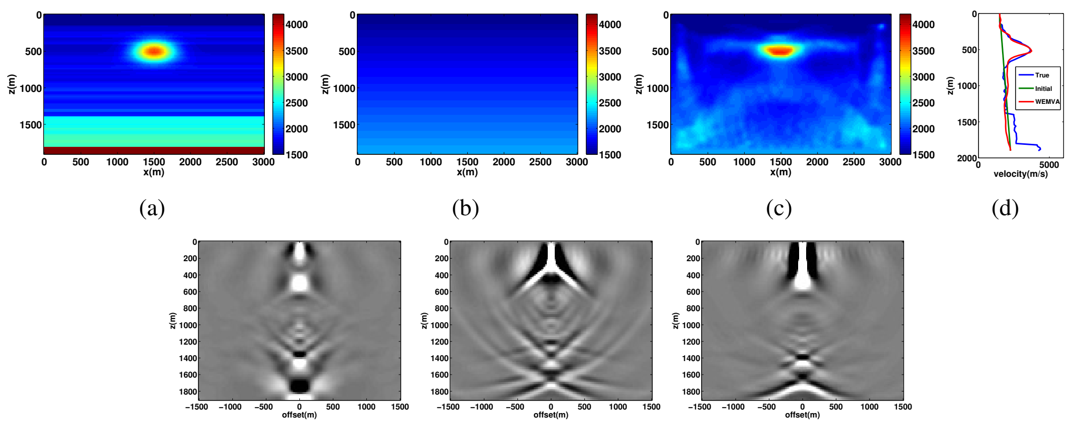
Figure 1 *Randomized trace estimation. (a,b) True and initial velocity model. Objective functions for WEMVA based on the Frobenius norm, as a function of velocity perturbation using the complete matrix (blue line) and error bars of approximated objective function evaluated via 5 different random probing with (c) K=80 and (d) K = 500 for the Marmousi model.*



(a) (b) (c) (d)

(e) (f) (g)

Figure 2 *Horizontal reflectors model. (a,b,c) True model, initial model and inversion results after 15 iterations. (d) Vertical trace profile extracted along x=1000m. (e,f,g) CIG along x=1000 and z for true model, initial model and inversion results.*



(a) (b) (c) (d)

(e) (f) (g)

Figure 3 *Lens model. (a,b,c) True model, initial model and inversion results after 30 iterations. (d) Vertical trace profile extracted along x=1500m. (e,f,g) CIG along x=1500 and z for true model, initial model and inversion results.*

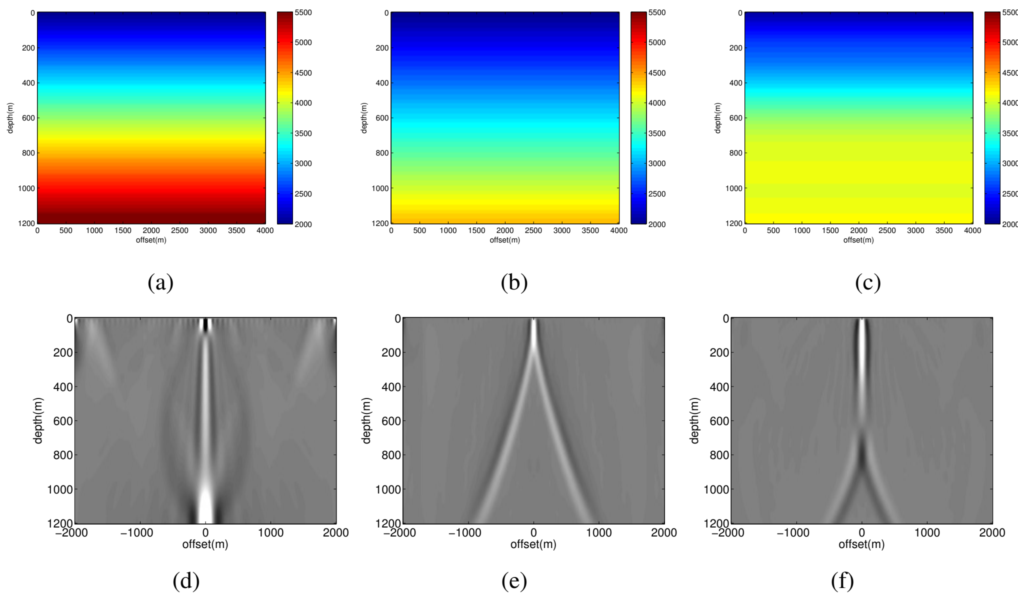


Figure 4 *Diving wave case. (a,b,c) True, initial and inversion results after 7 iterations. (d,e,f) CIG along x=2000 and z for true, initial and inversion results.*

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