Fast uncertainty quantification for 2D full-waveform inversion with randomized source subsampling

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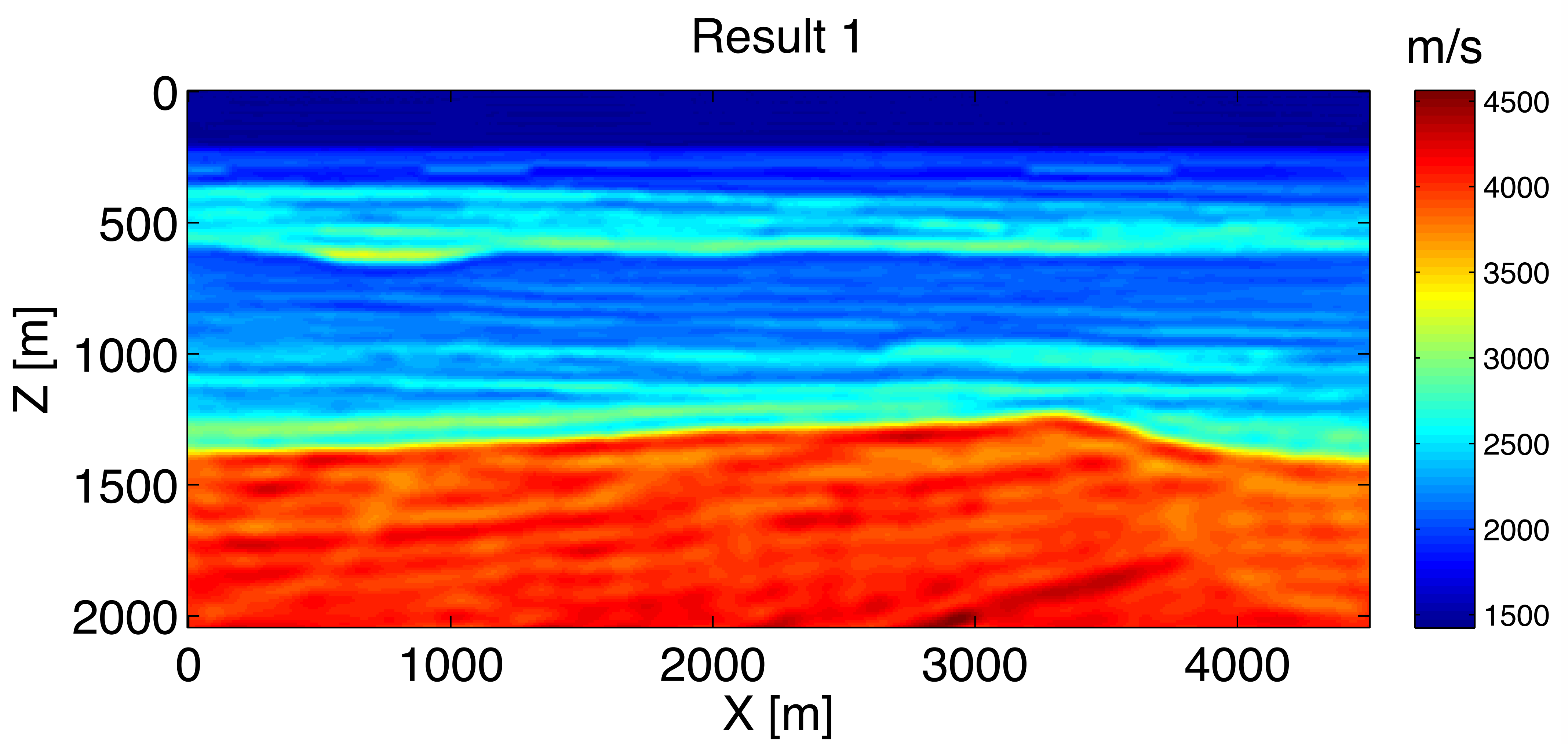
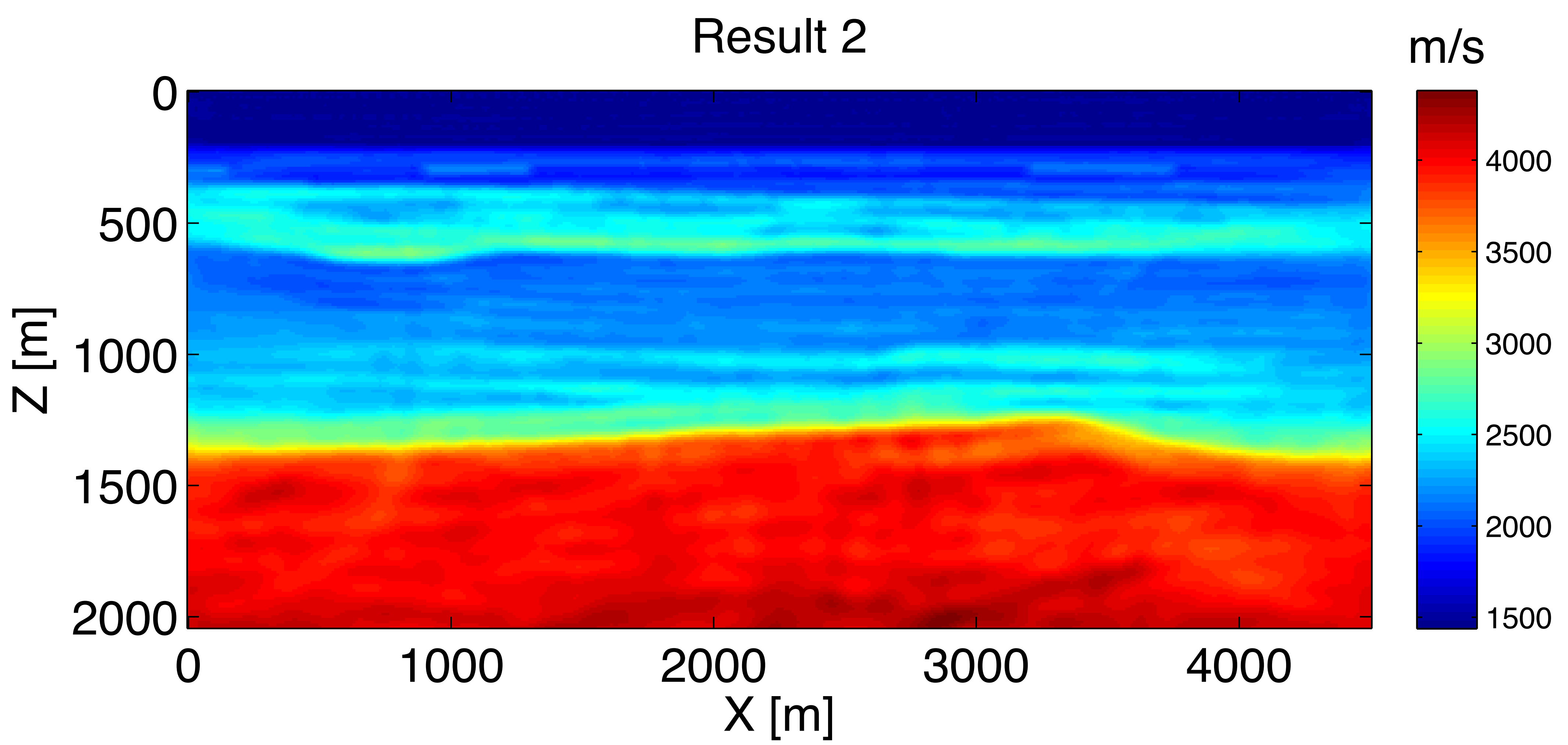
Motivation





Which one is better?

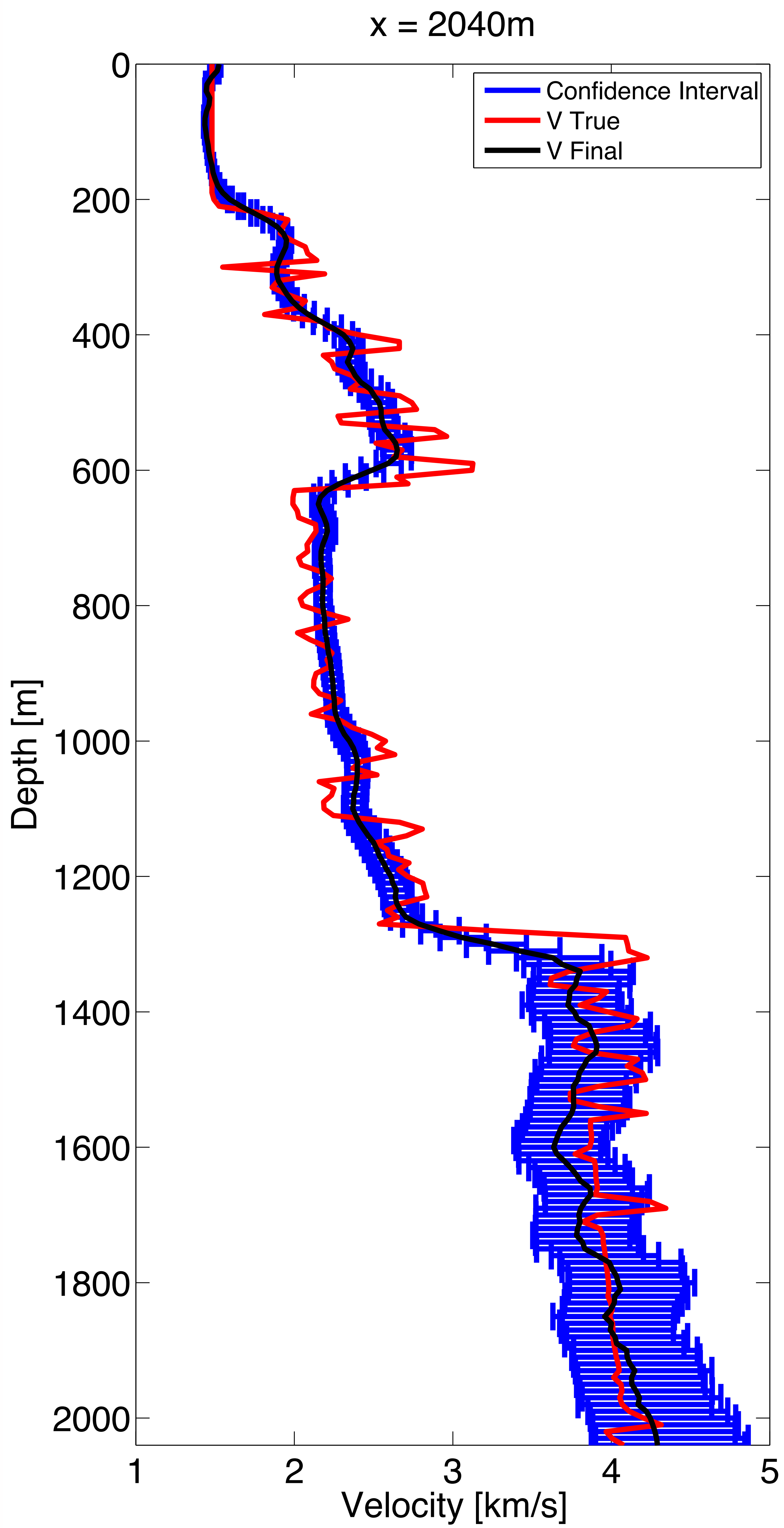
Misfit?

Eyeball norm?

Uncertainty?

Standard Deviation?

Confidence interval



Final Model

Confidence Interval

True Model

x = 2040m

Bayesian theory

Deterministic inverse problem:

$$\mathbf{m}^* = \arg\min\left(\frac{1}{2N_s}\sum_{1}^{N_s}\|f_i(\mathbf{m}) - \mathbf{d}_{i\text{obs}}\|_{W_i}^2 + \frac{1}{2}\|\mathbf{m} - \overline{\mathbf{m}}\|_R^2\right)$$

Statistical inverse problem with Bayesian theory:

$$\pi_{post}(\mathbf{m}) := \pi(\mathbf{m}|\mathbf{d}_{obs}) \propto \pi_{prior}(\mathbf{m})\pi(\mathbf{d}_{obs}|\mathbf{m})$$

where $\mathbf{m}$ is the model parameter, and $\mathbf{d}_{obs}$ is the observed data

(James Martin *et al*, 2012)

Bayesian theory

Assume:

$$\text{noise} \sim \mathcal{N}(0, \Gamma_{noise})$$
$$\text{prior model distribution} \sim \mathcal{N}(\mathbf{m}_{prior}, \Gamma_{prior}).$$

Negative log-posterior of the posterior pdf:

$$V(\mathbf{m}) := -\log \pi_{post}(\mathbf{m}) := \frac{1}{2N_s} \sum_{1}^{N_s} \|f_i(\mathbf{m}) - \mathbf{d}_{i\text{obs}}\|^2_{\Gamma^{-1}_{i\text{noise}}} + \frac{1}{2}\|\mathbf{m} - \mathbf{m}_{prior}\|^2_{\Gamma^{-1}_{prior}}$$
$$\mathbf{m}_{MAP} = \mathbf{m}^*$$

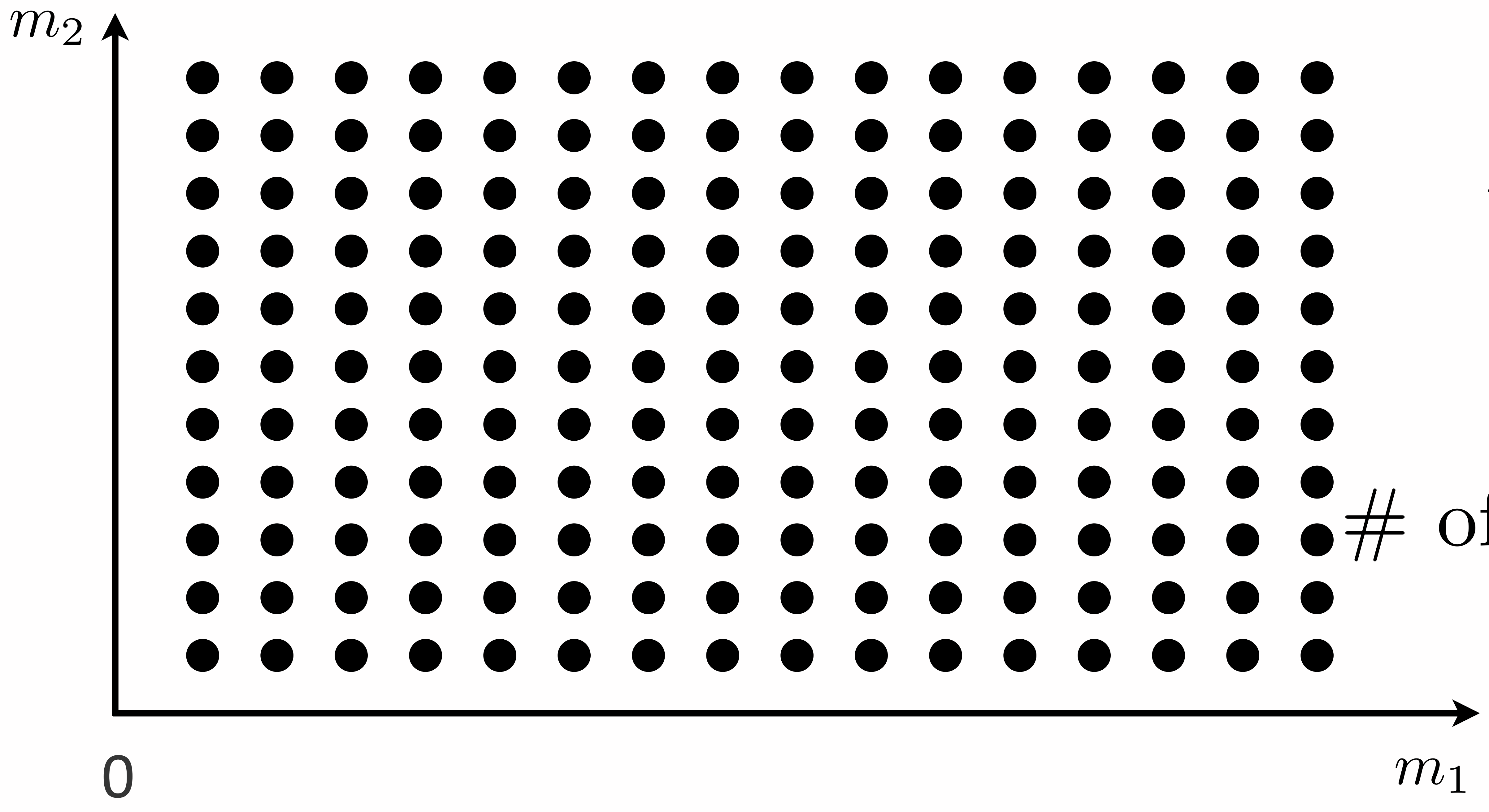
$$\|\mathbf{x}\|^2_{\Gamma^{-1}_{noise}} := \mathbf{x}^T \Gamma^{-1}_{noise} \mathbf{x}$$

Sampling the posterior pdf

How to obtain the posterior probability density function?

- Compute $\pi_{post}(\mathbf{m})$ by discretization?

$$N = N_{sub\_sample} * N_{para}$$



$$N_{para} >> 10^6$$
$$N_{sub\_sample} >> 10^2$$
$$N >> 10^8$$
$$N_s >> 10^2$$
$$\#\ \text{of PDE solvers} >> N * N_s$$

Sampling the posterior pdf

How to obtain the posterior probability density function?

- Markov chain Monte Carlo method ?   Metropolis - Hasting method

At sample $\mathbf{m}_k$

Draw sample $\mathbf{y}$ from the proposal distribution $\tilde{\pi}_k(\mathbf{m})$

$$\text{if } \min\left(1, \frac{\pi_{post}(\mathbf{y})\tilde{\pi}_y(\mathbf{m}_k)}{\pi_{post}(\mathbf{m}_k)\tilde{\pi}_k(\mathbf{y})}\right) > \alpha$$

set $\mathbf{m}_{k+1} = \mathbf{y}$

else

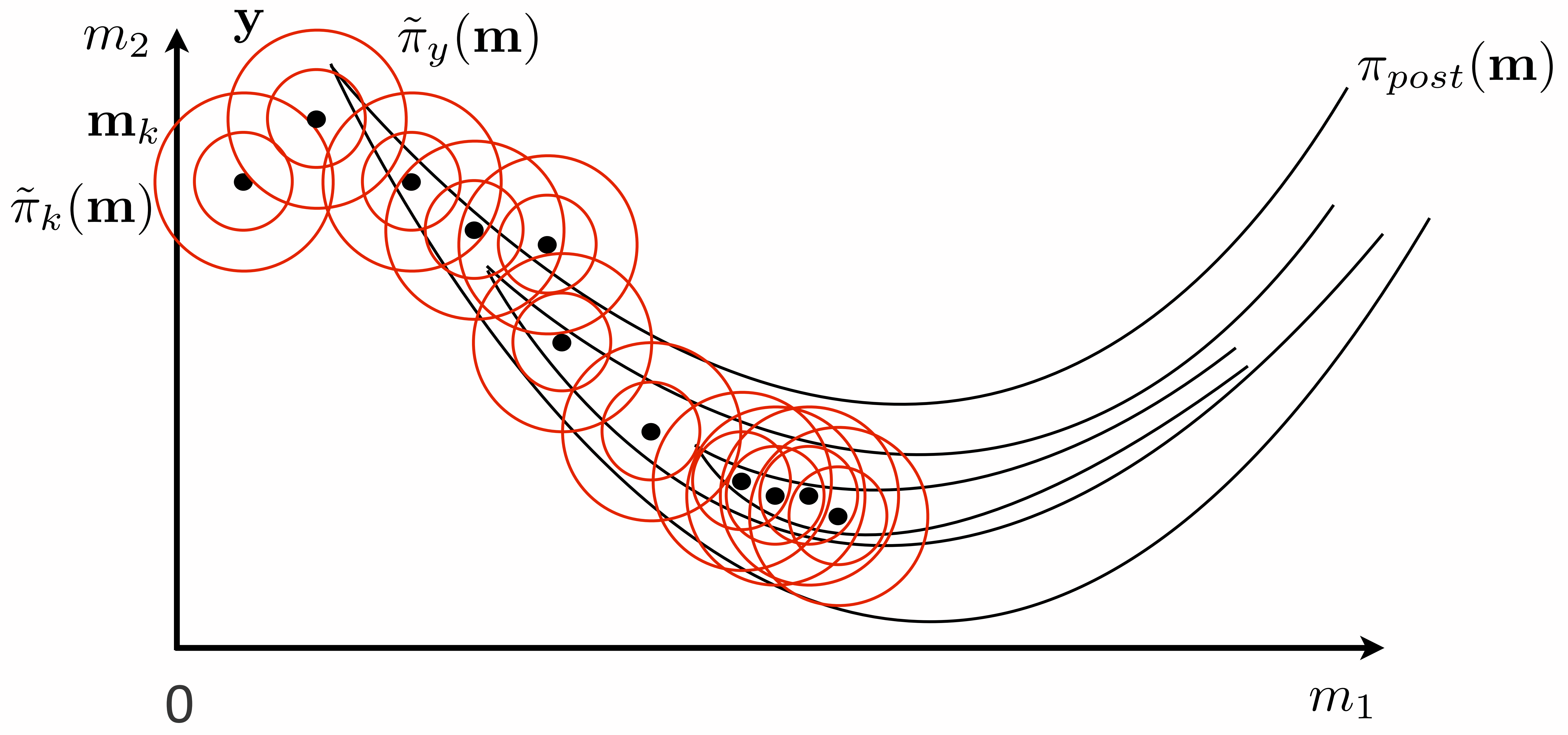
regenerate $\mathbf{y}$

end

(James Martin *et al*, 2012)

Sampling the posterior pdf

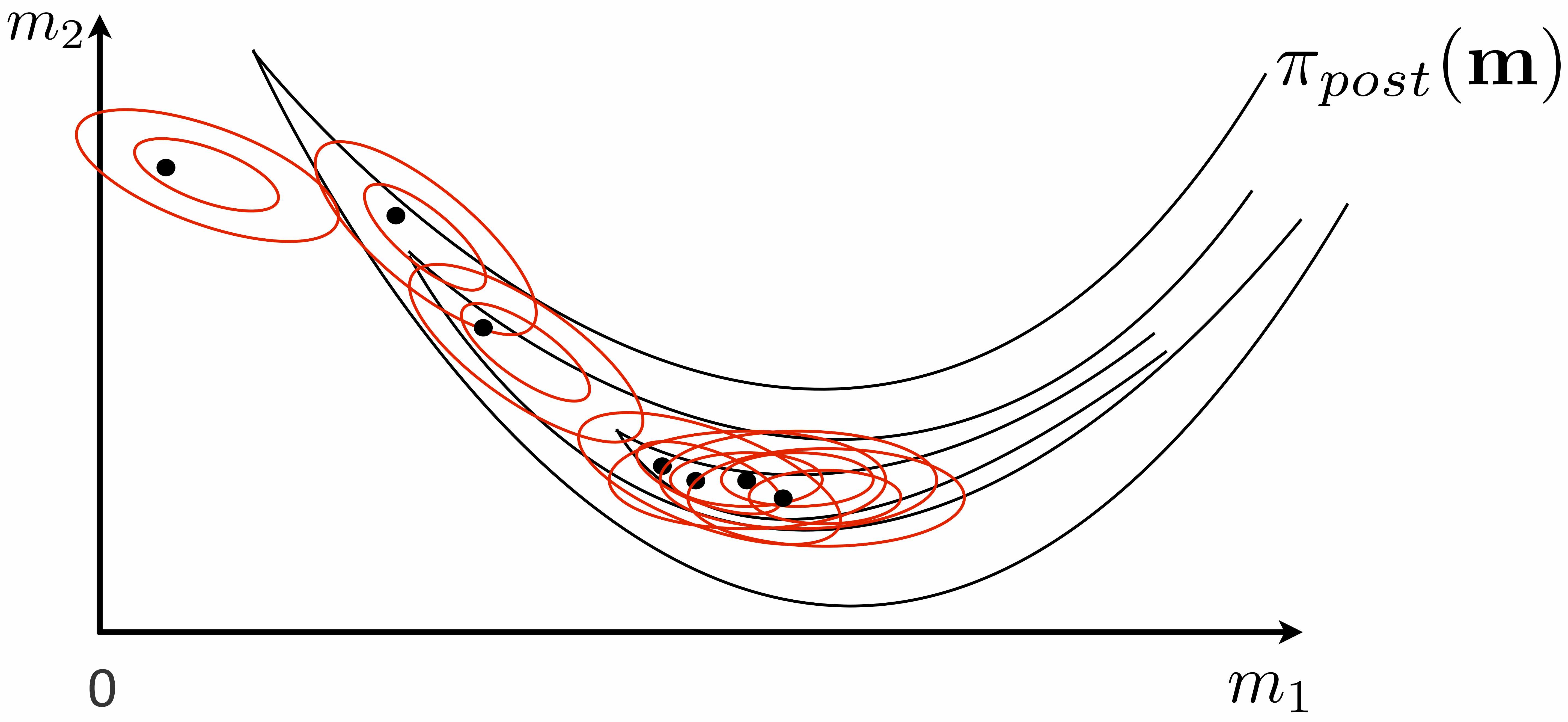
e.g. $\tilde{\pi}_k(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_k, \alpha\mathbf{I})$



Question: How to obtain a good $\tilde{\pi}_k(\mathbf{m})$?

Sampling the posterior pdf

$$\tilde{\pi}_k(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_k - \mathbf{H}_k^{-1}\mathbf{g}_k, \mathbf{H}_k^{-1})$$



(James Martin *et al*, 2012)

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Sampling the posterior pdf

How to obtain the posterior probability density function?

- Stochastic Newton MCMC ?

$$V(\mathbf{m}) := -\log \pi_{post}(\mathbf{m}) := \frac{1}{2N_s}\sum_{1}^{N_s} \|f_i(\mathbf{m}) - \mathbf{d}_{i\mathrm{obs}}\|^2_{\Gamma^{-1}_{inoise}} + \frac{1}{2}\|\mathbf{m} - \mathbf{m}_{prior}\|^2_{\Gamma^{-1}_{prior}}$$

$$\tilde{\pi}_k(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_k - \mathbf{H}_k^{-1}\mathbf{g}_k, \mathbf{H}_k^{-1})$$

(James Martin *et al*, 2012)

Sampling the posterior pdf

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$$\tilde{\pi}_k(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_k - \mathbf{H}_k^{-1}\mathbf{g}_k, \mathbf{H}_k^{-1})$$

$$\tilde{\pi}(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_{\text{MAP}}, \mathbf{H}_{\text{MAP}}^{-1})$$

(James Martin *et al*, 2012) (Tan Bui-Thanh *et al*, 2013)

Sampling the posterior pdf

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$$\tilde{\pi}_k(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_k - \mathbf{H}_k^{-1}\mathbf{g}_k, \mathbf{H}_k^{-1})$$

$$\tilde{\pi}(\mathbf{m}) \backsim \mathcal{N}(\mathbf{m}_{\mathrm{MAP}}, \mathbf{H}^{-1}_{\mathrm{MAP}})$$

$$\text{Computational Cost } \sim \mathcal{O}(N_{\text{sample}} * N_s)$$

(James Martin *et al*, 2012) (Tan Bui-Thanh *et al*, 2013)

Randomized source subsampling

To speed up the computation of posterior pdf:

$$V(\mathbf{m}) := -\log \pi_{post}(\mathbf{m}) := \frac{1}{2N_s}\sum_{1}^{N_s} \|f_i(\mathbf{m}) - \mathbf{d}_{i\mathrm{obs}}\|^2_{\Gamma^{-1}_{inoise}} + \frac{1}{2}\|\mathbf{m} - \mathbf{m}_{prior}\|^2_{\Gamma^{-1}_{prior}}$$

Randomized source subsampling:

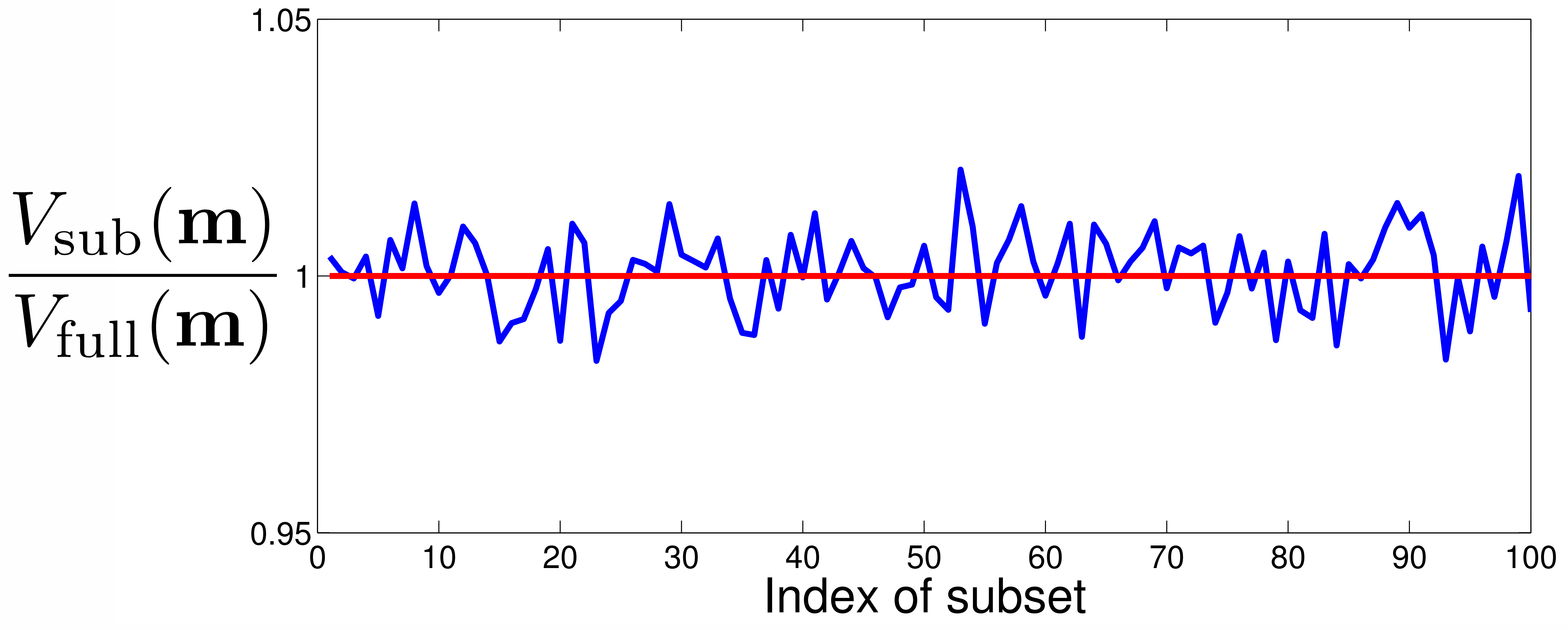
$$\frac{1}{N_s}\sum_{1}^{N_s} \|f_i(\mathbf{m}) - \mathbf{d}_{i\mathrm{obs}}\|^2_{\Gamma^{-1}_{inoise}} = \frac{1}{\|\mathcal{I}_s\|}\sum_{i\in\mathcal{I}_s} \|f_i(\mathbf{m}) - \mathbf{d}_{i\mathrm{obs}}\|^2_{\Gamma^{-1}_{inoise}} + \epsilon$$

(Farbod Roosta-Khorasani *et al*, 2013; Eldad *et al,* 2012; Michael P. Friedlander *et al*, 2013, van Leeuwen *et al*, 2013)

Randomized source subsampling

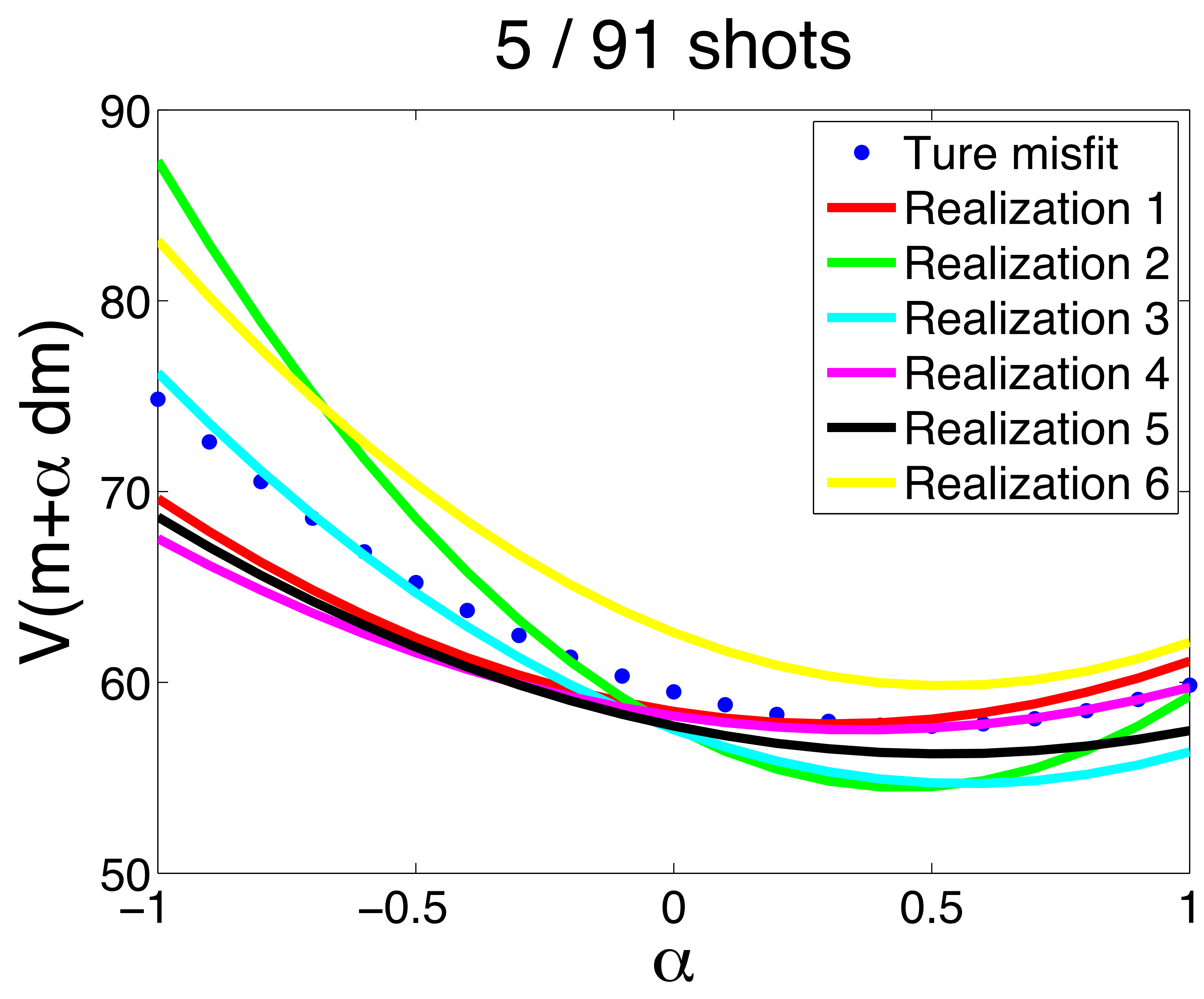
Computational Cost: $\mathcal{O}(N_s) \rightarrow \mathcal{O}(N_{rs}),\ N_{rs} << N_s$

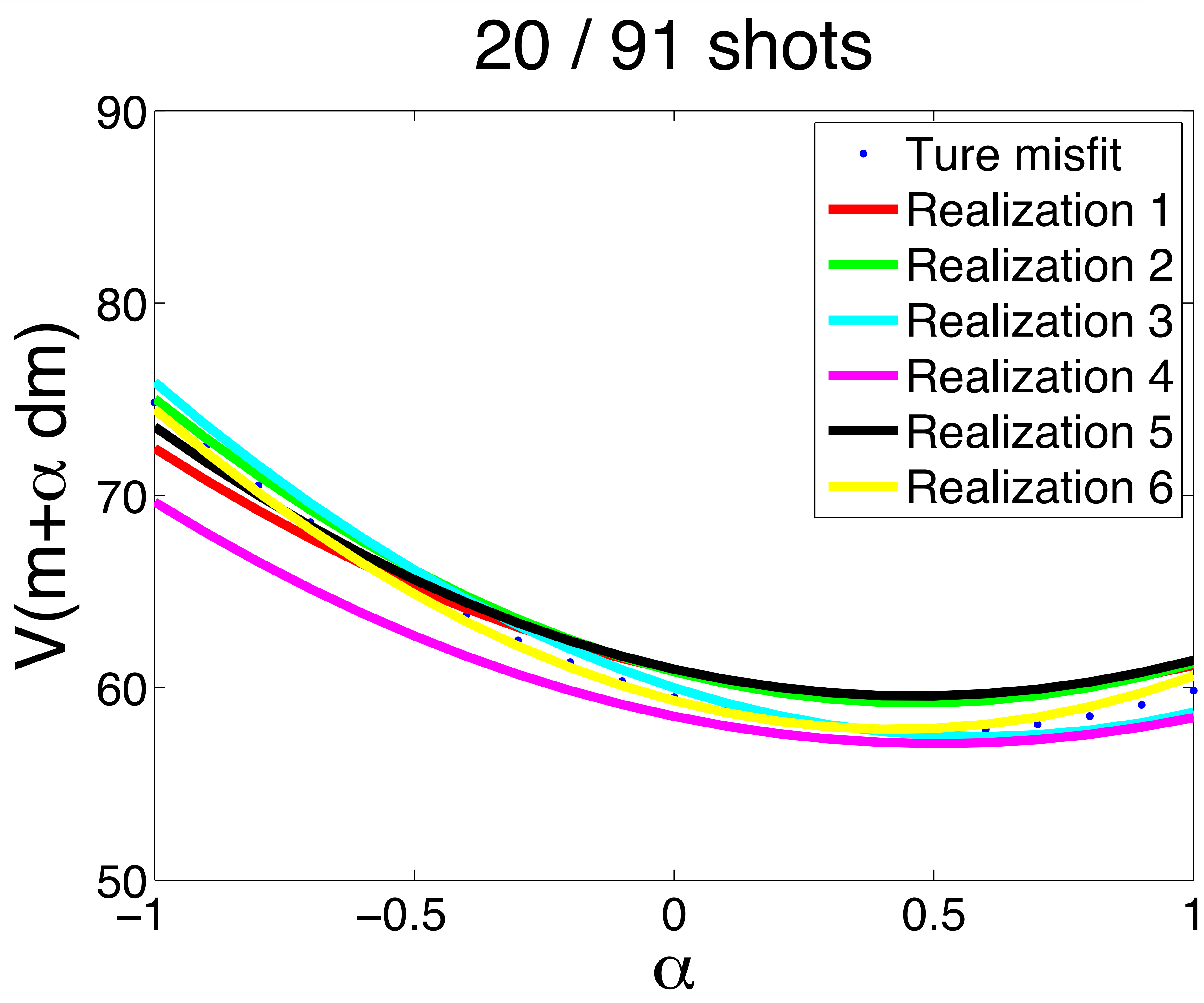
Subset misfit *vs* Full misfit



Randomized source subsampling

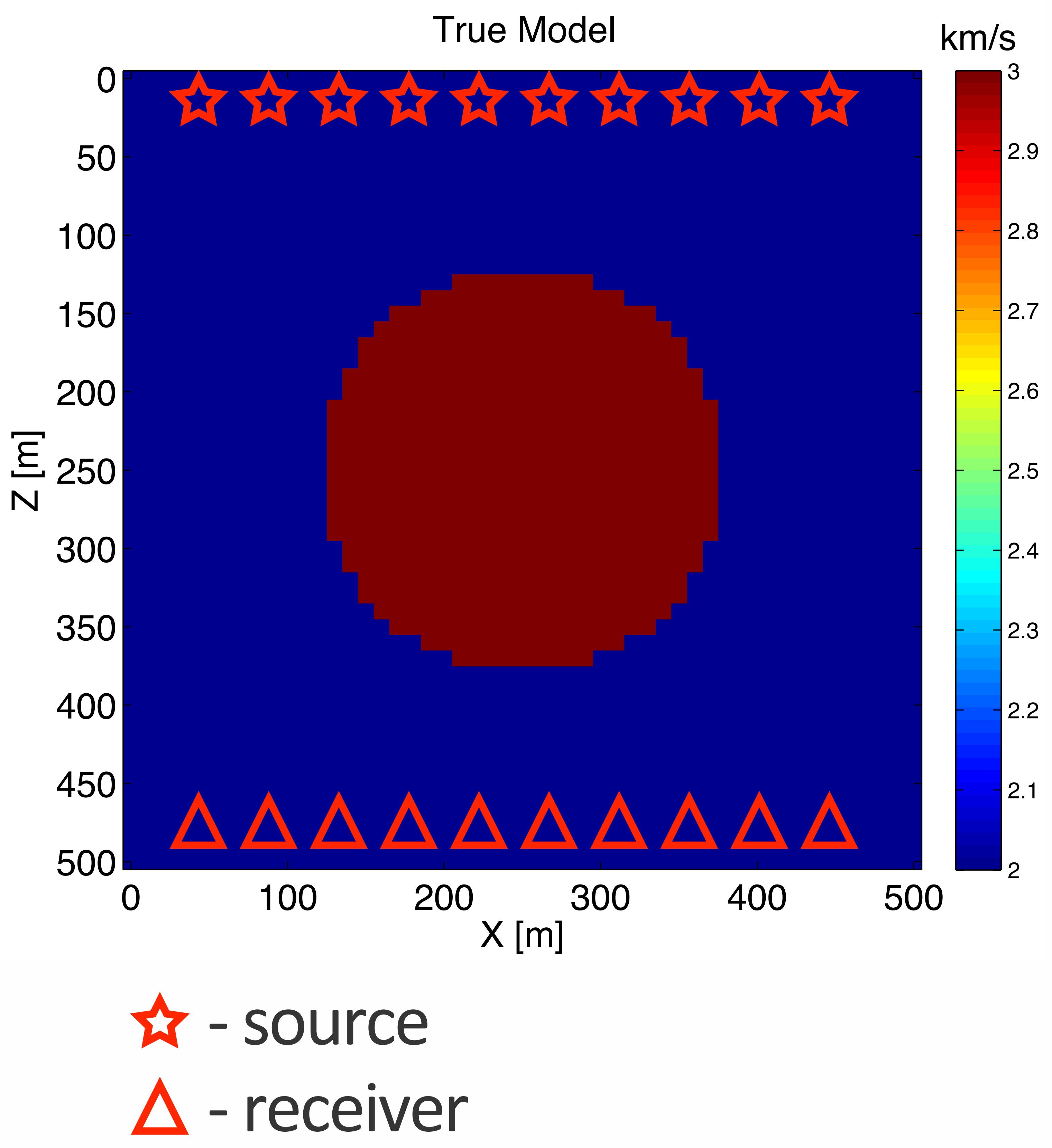
Computational Cost: $\mathcal{O}(N_s) \rightarrow \mathcal{O}(N_{rs})$





Numerical Experiments

Camambert model



Acquisition Geometry:

26 shots

51 receivers

10 frequencies

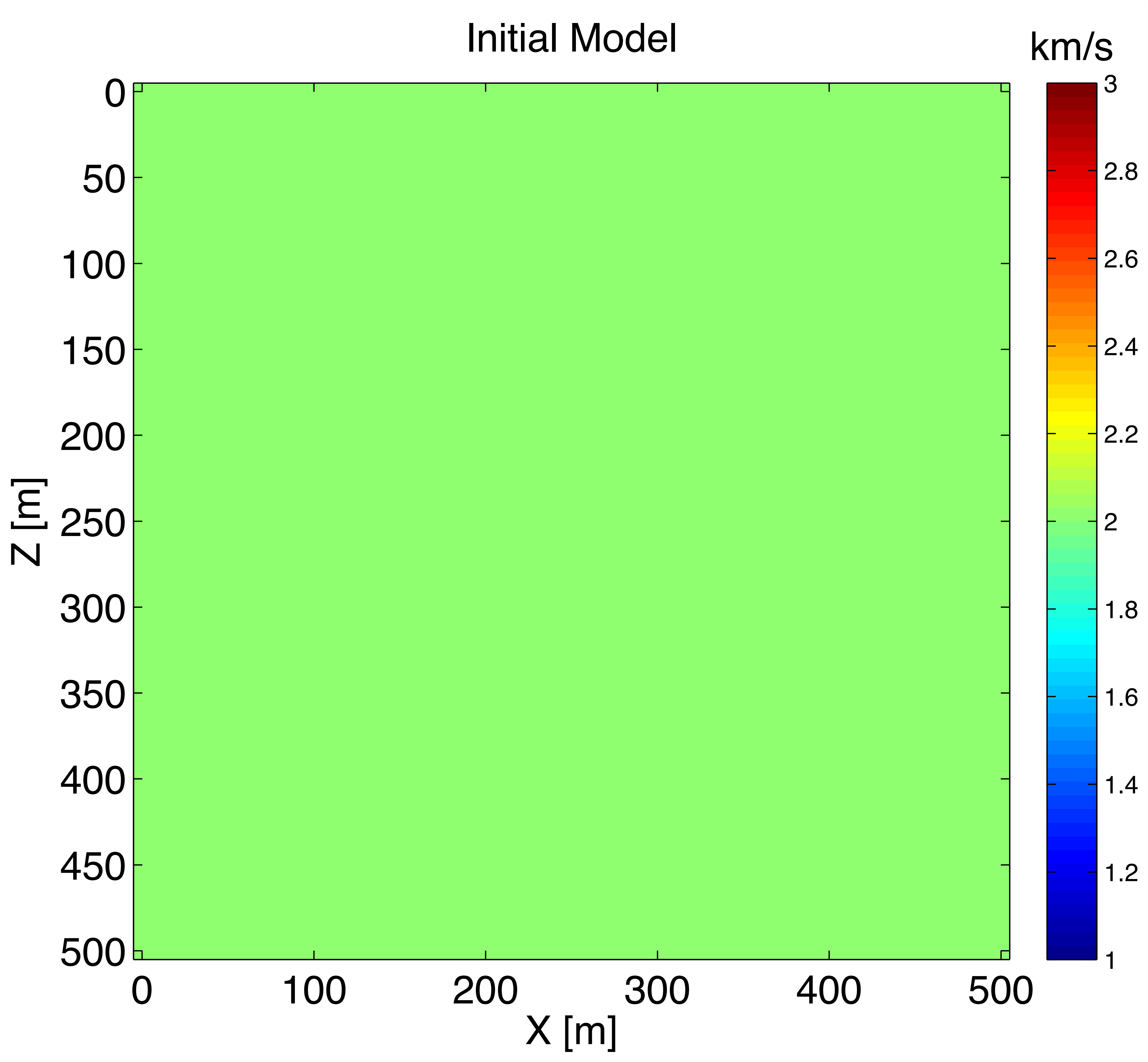
Numerical Experiments

Statistical parameter to be inverted:

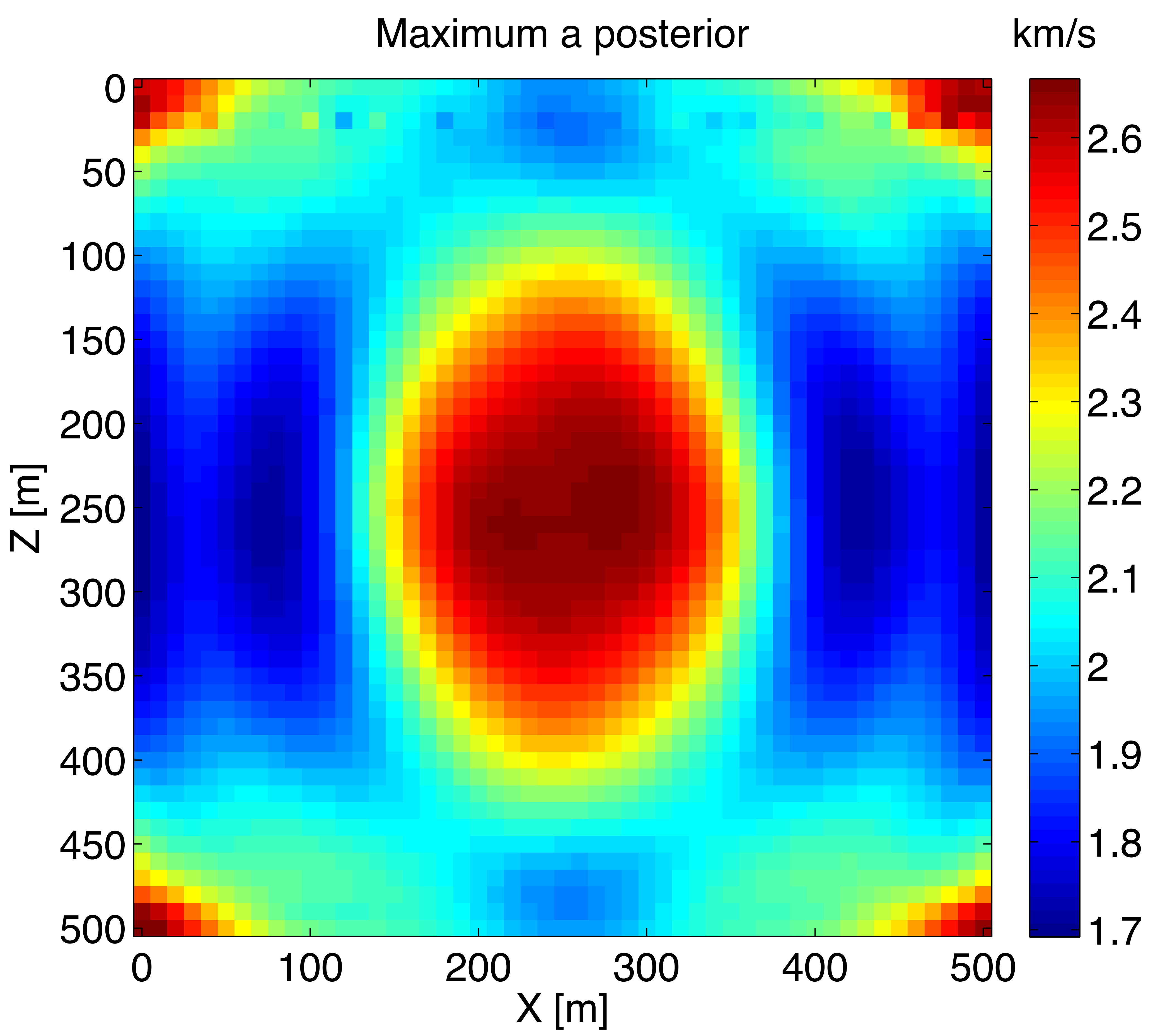
Standard deviation : σ

Confidence interval: $P(\mathbf{m} \in \mathrm{I}_{ci}) \geq \alpha$

Number of randomly selected shots: 5 , 10 ,26

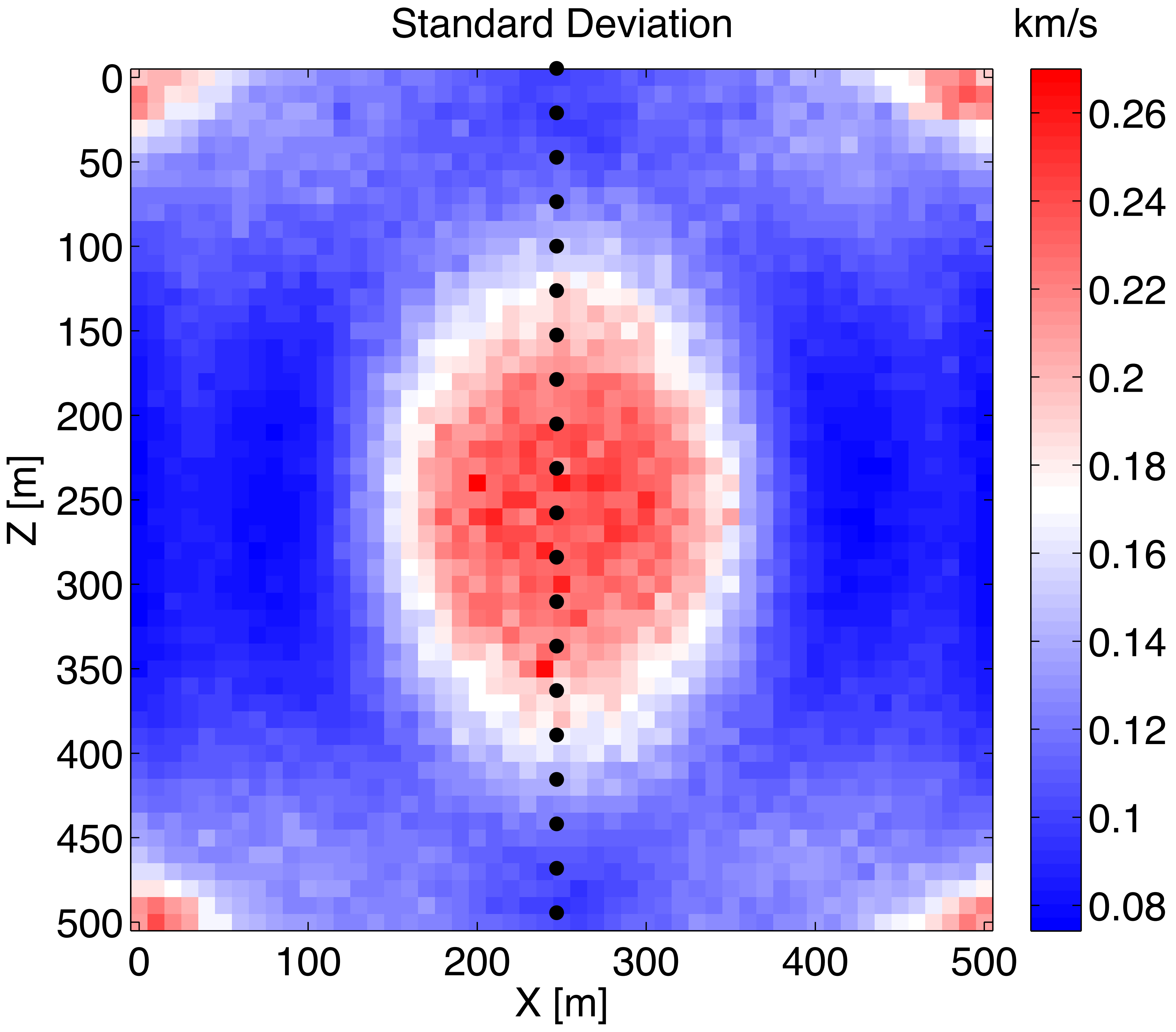


Initial model

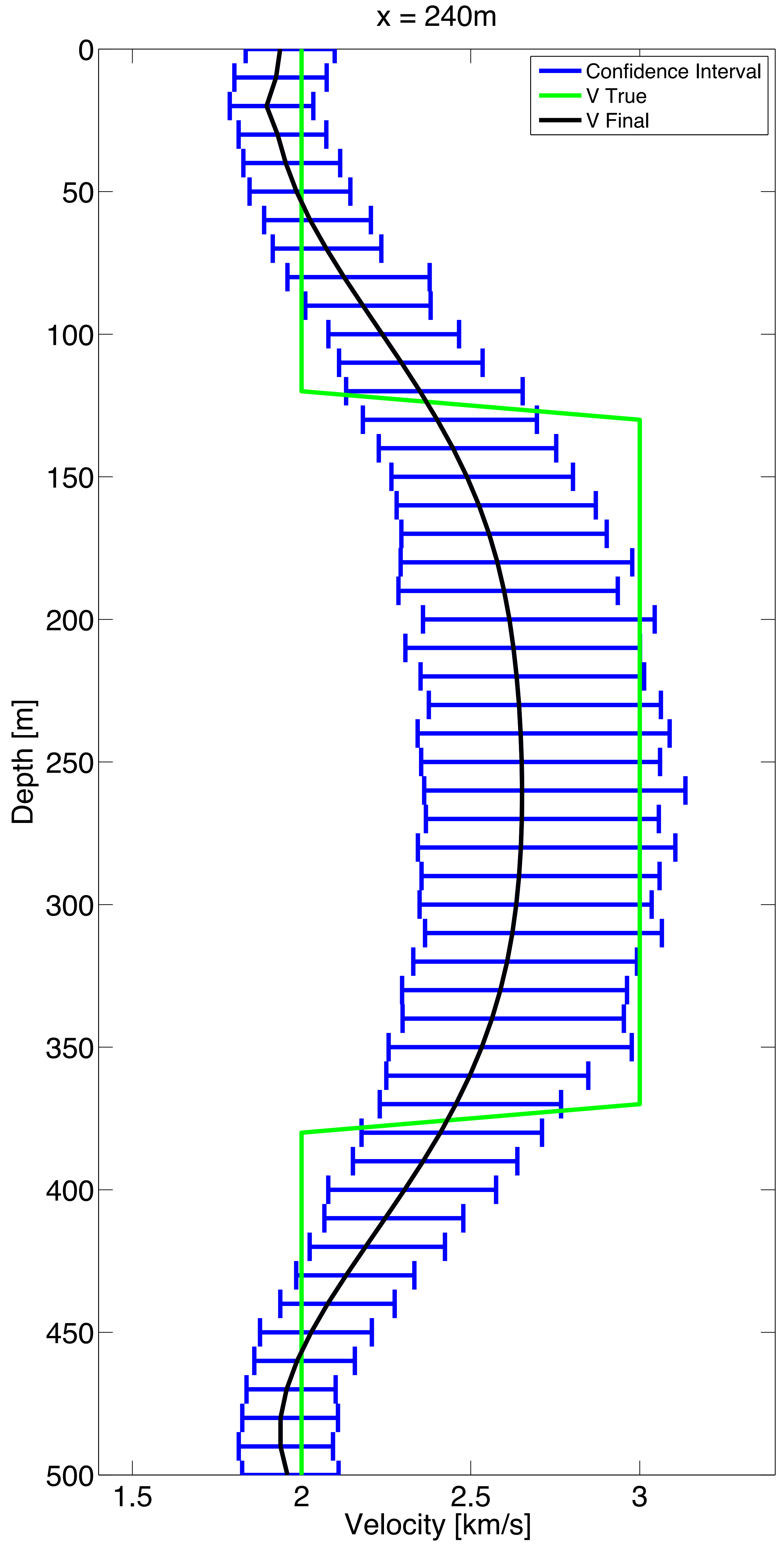


MAP

Nrs = 5



Standard Deviation

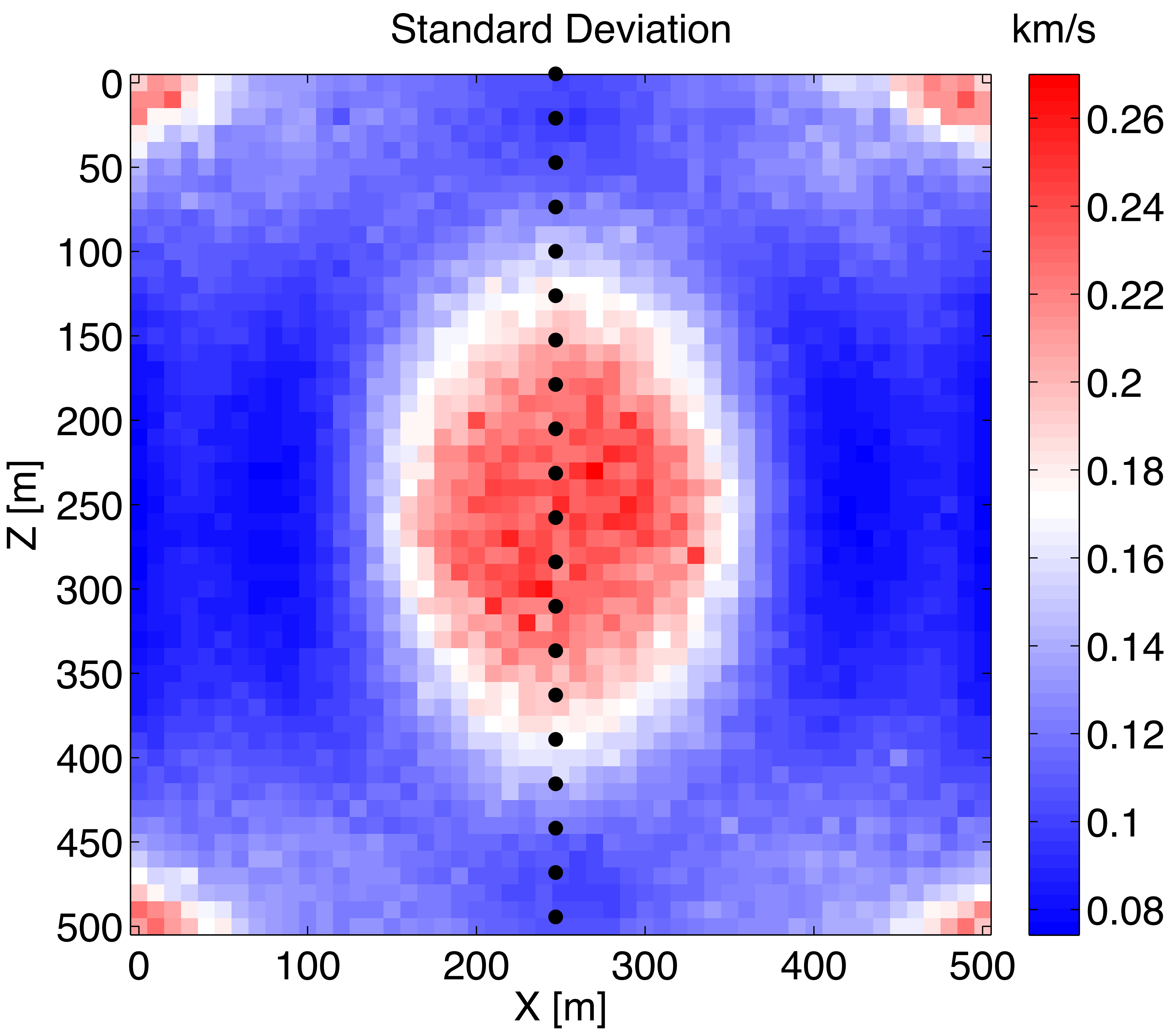


$\alpha = 0.9$

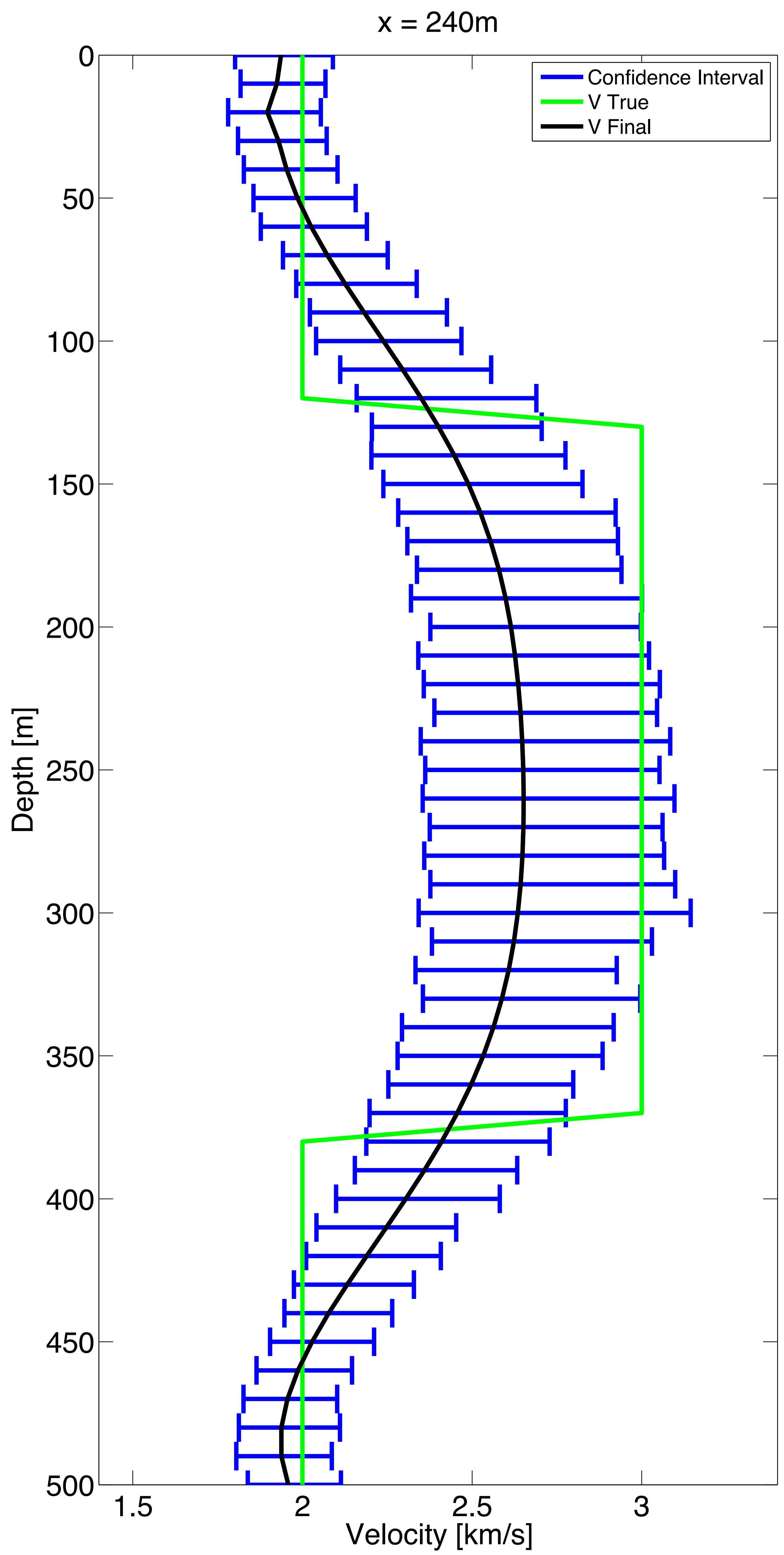
- Final Model
- Confidence Interval
- True Model

Confidence Interval

Nrs = 10



Standard Deviation

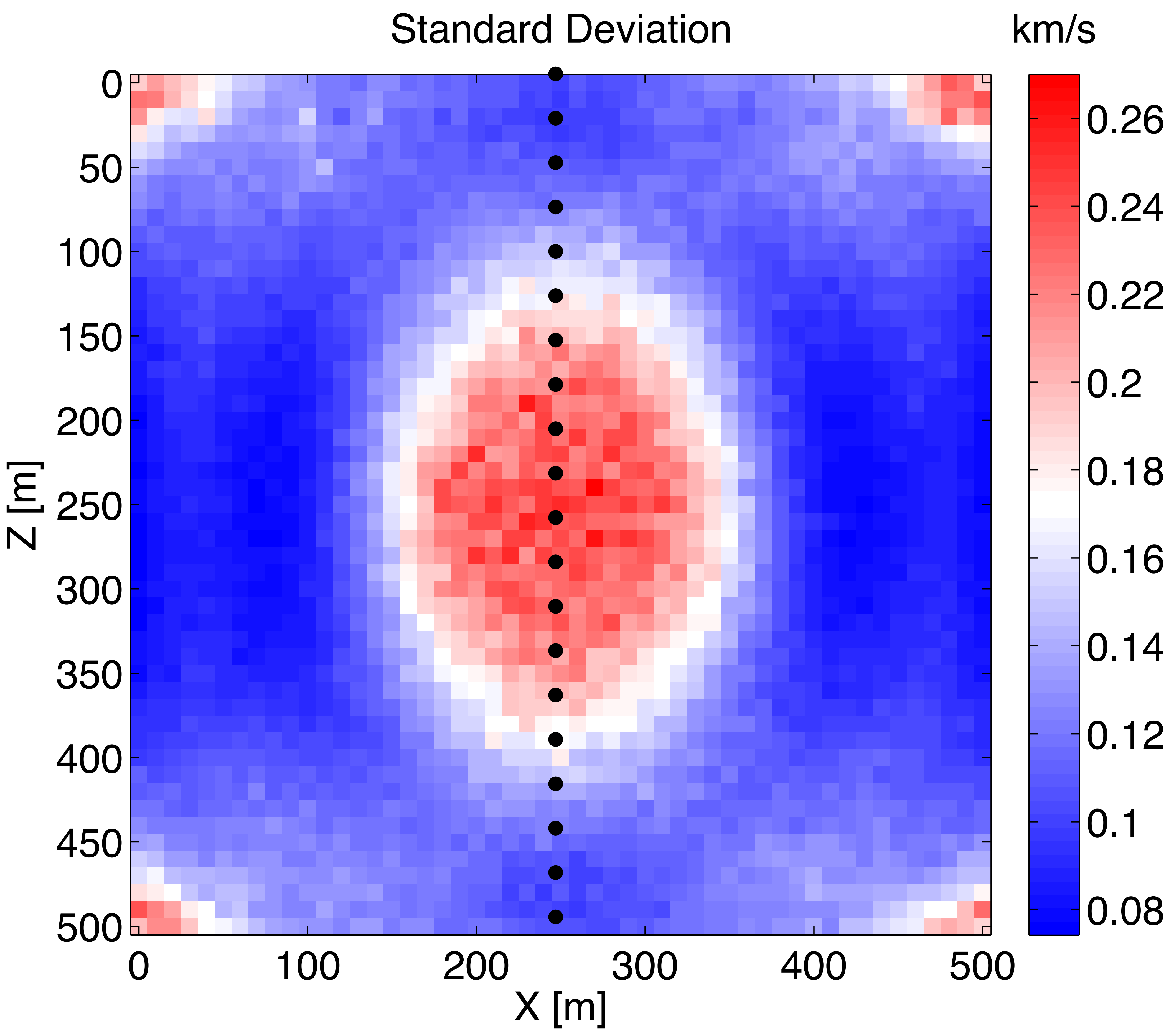


$\alpha = 0.9$

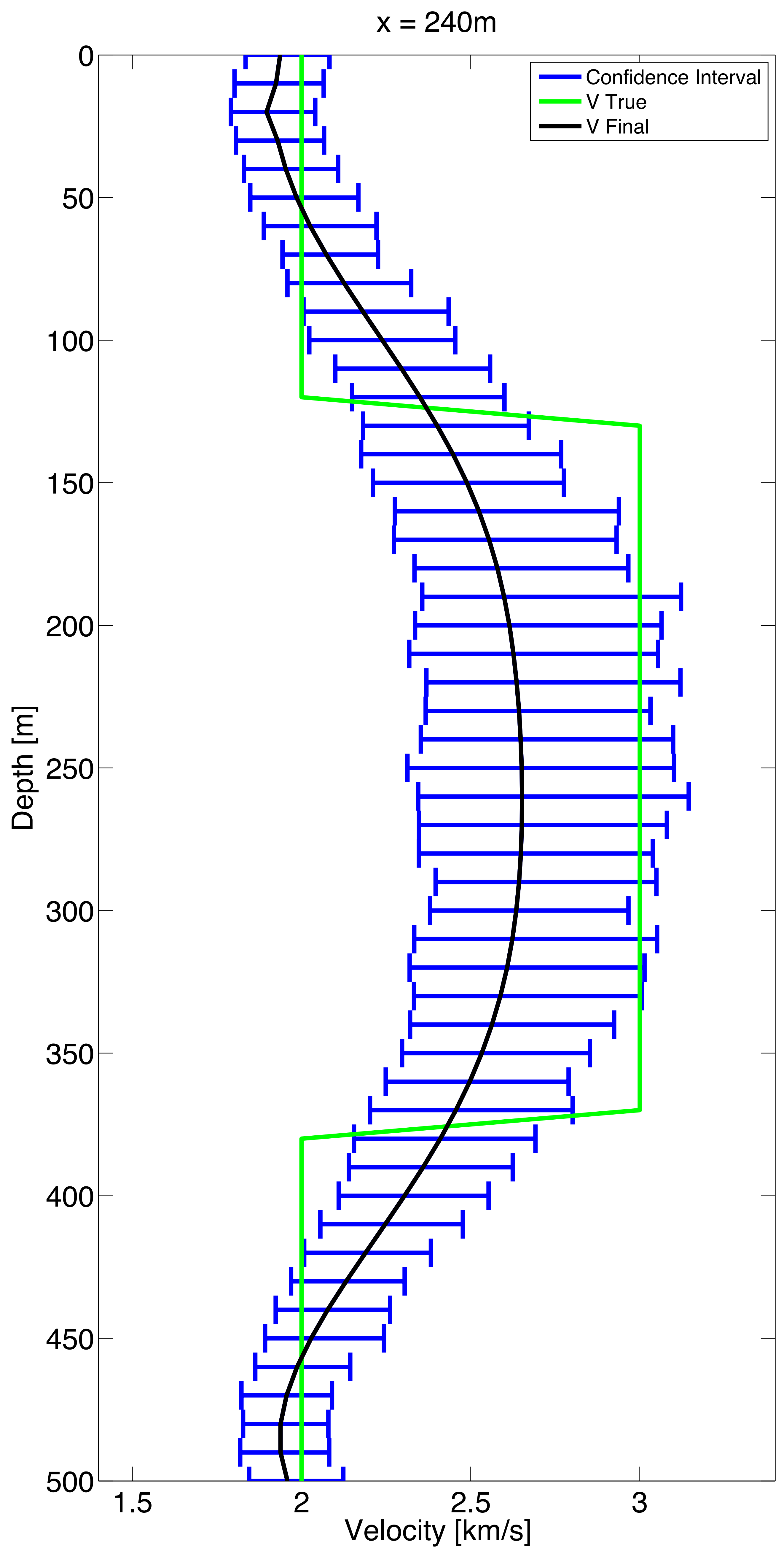
Final Model
Confidence Interval
True Model

Confidence Interval

Nrs = 26



Standard Deviation



$\alpha = 0.9$

Final Model

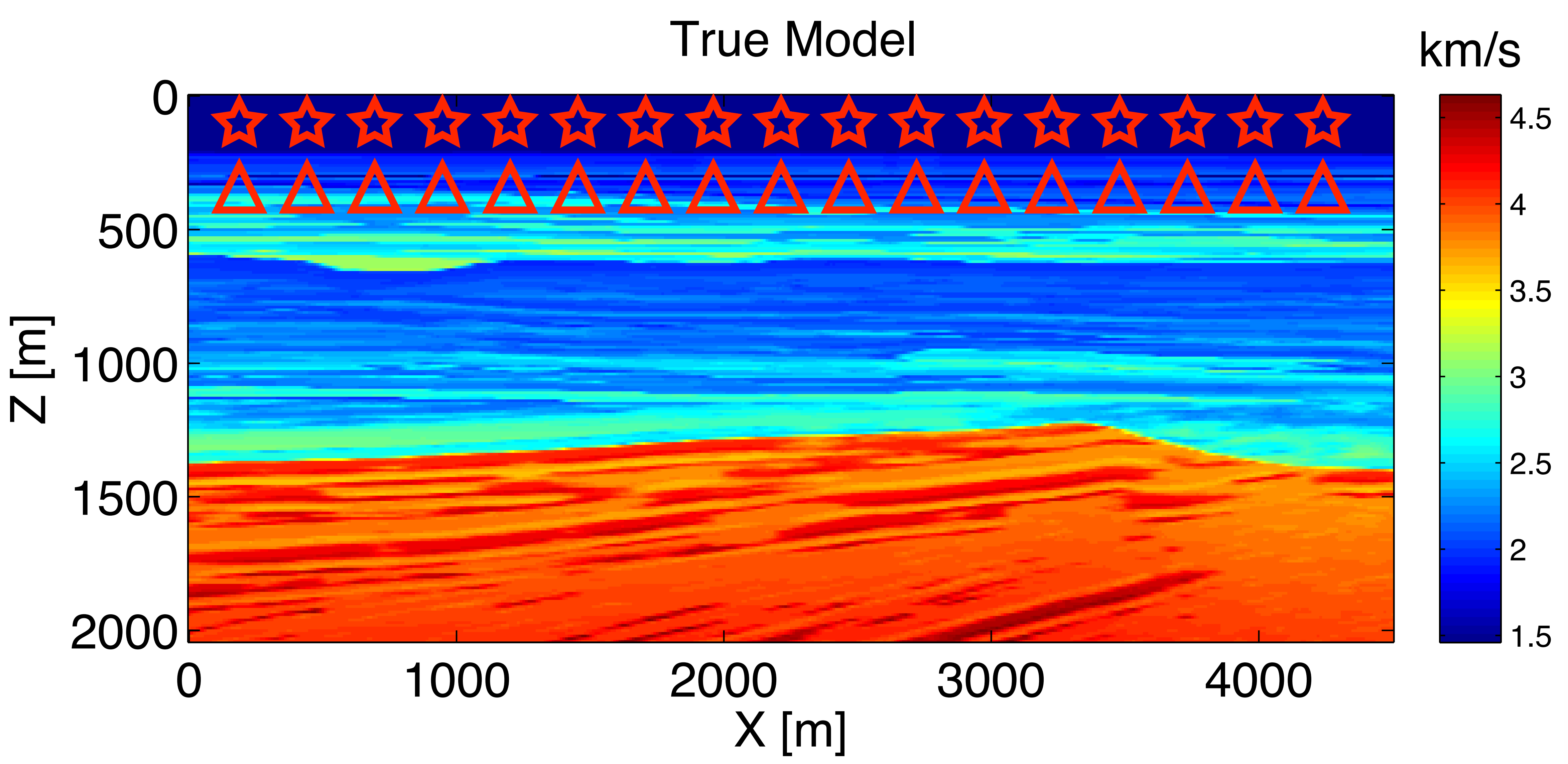
Confidence Interval

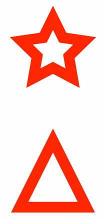
True Model

Confidence Interval

Numerical Experiments

BG model





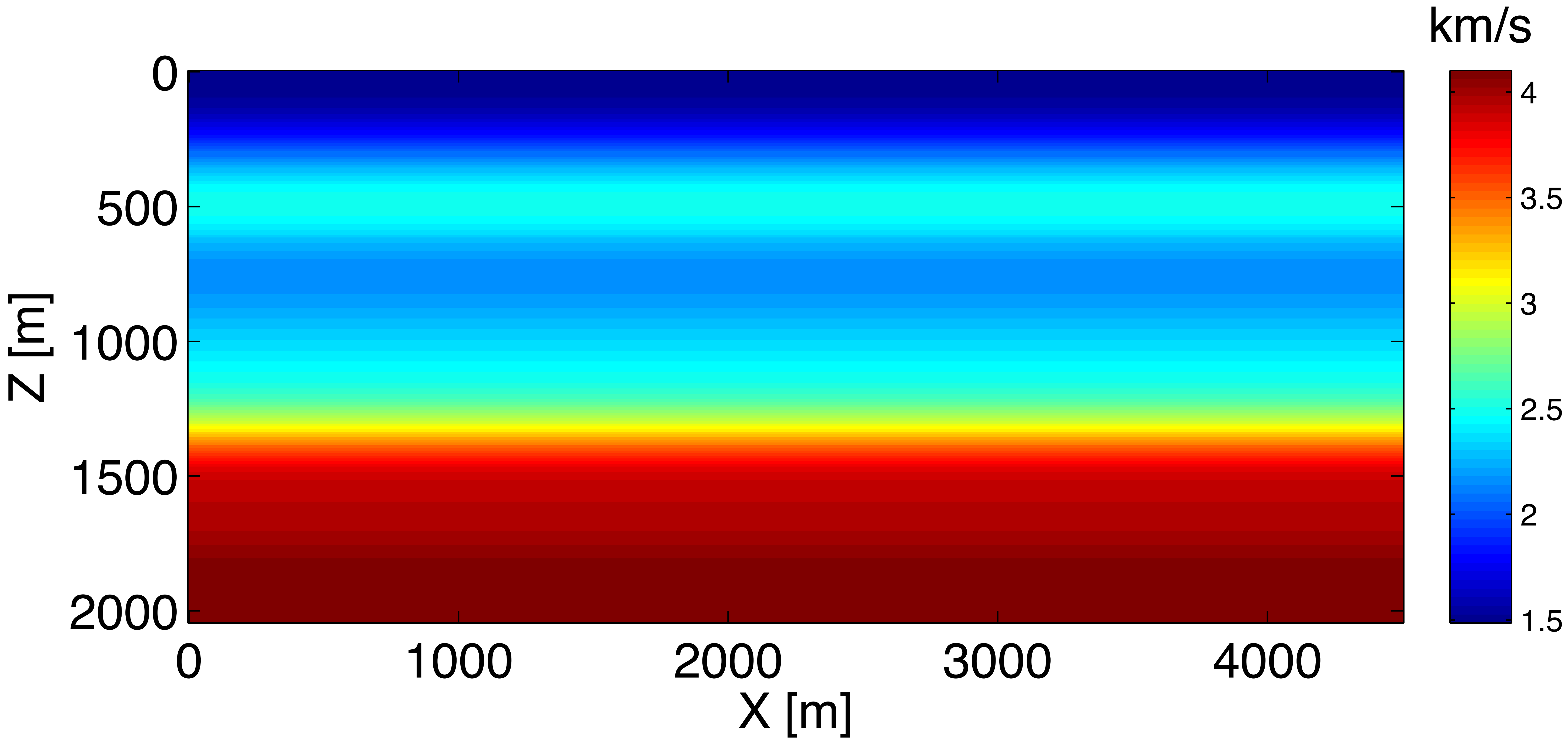
Acquisition Geometry:

91 shots

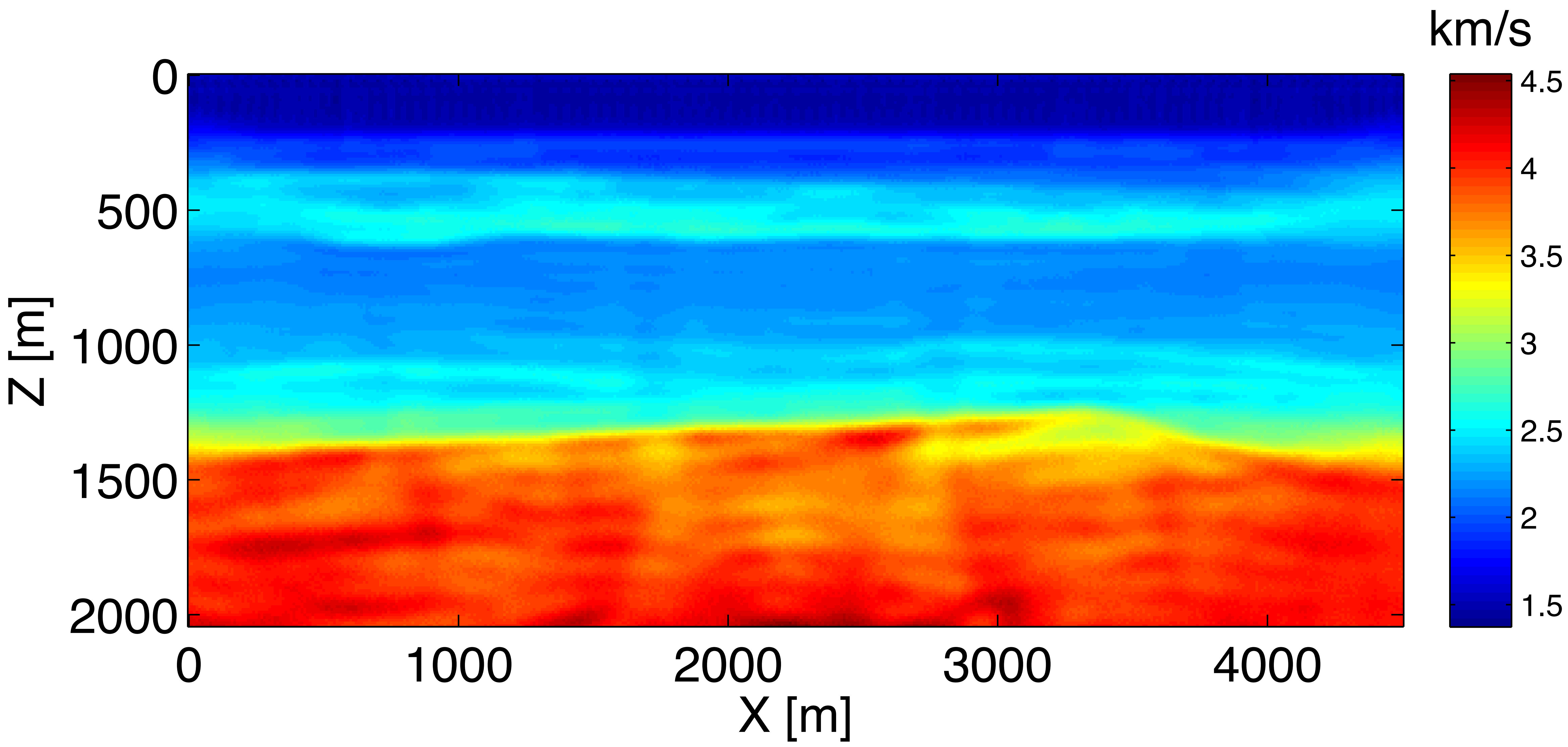
451 receivers

15 frequencies from 3Hz to 17Hz

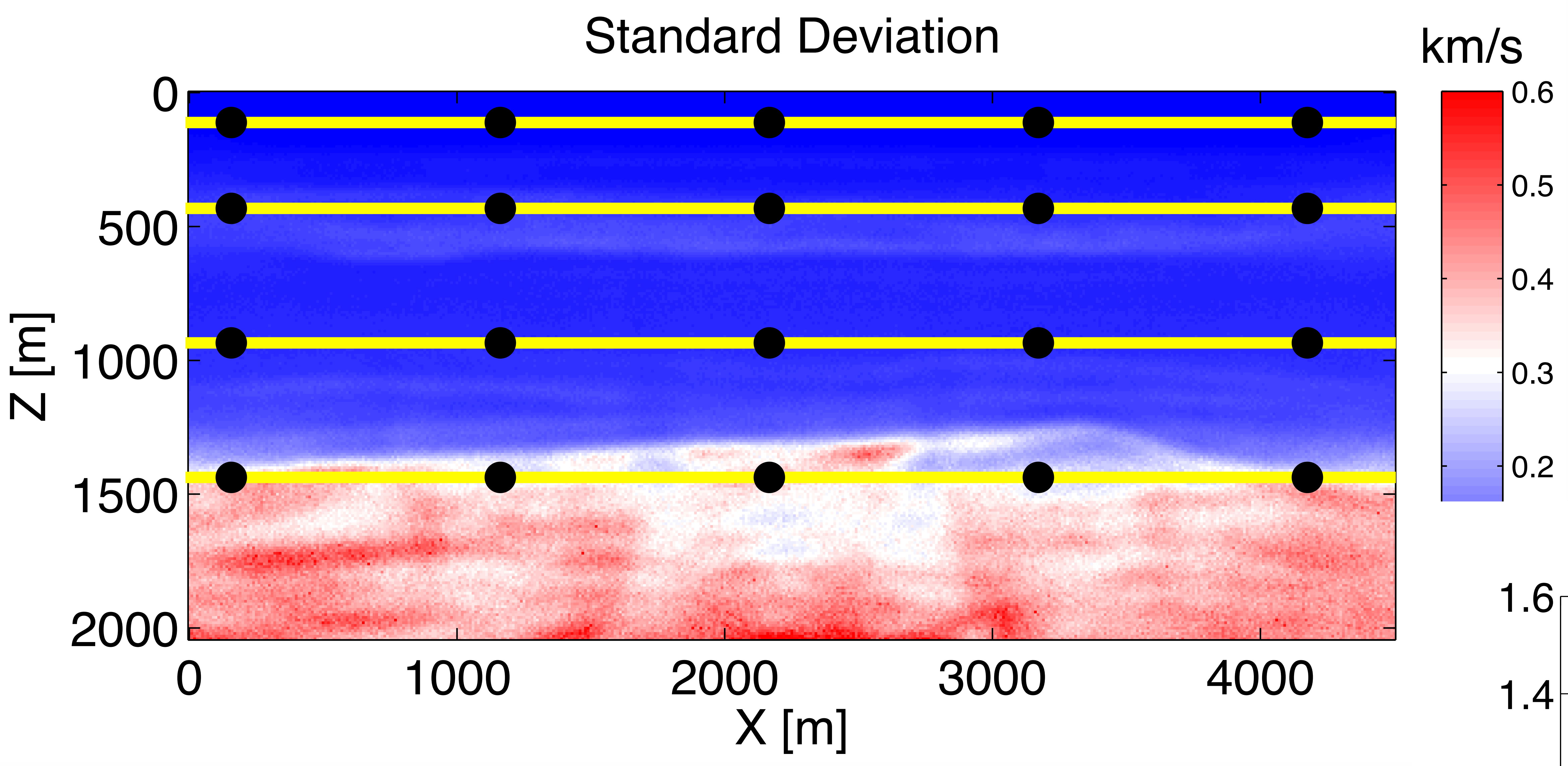
Number of randomly selected shots: 5



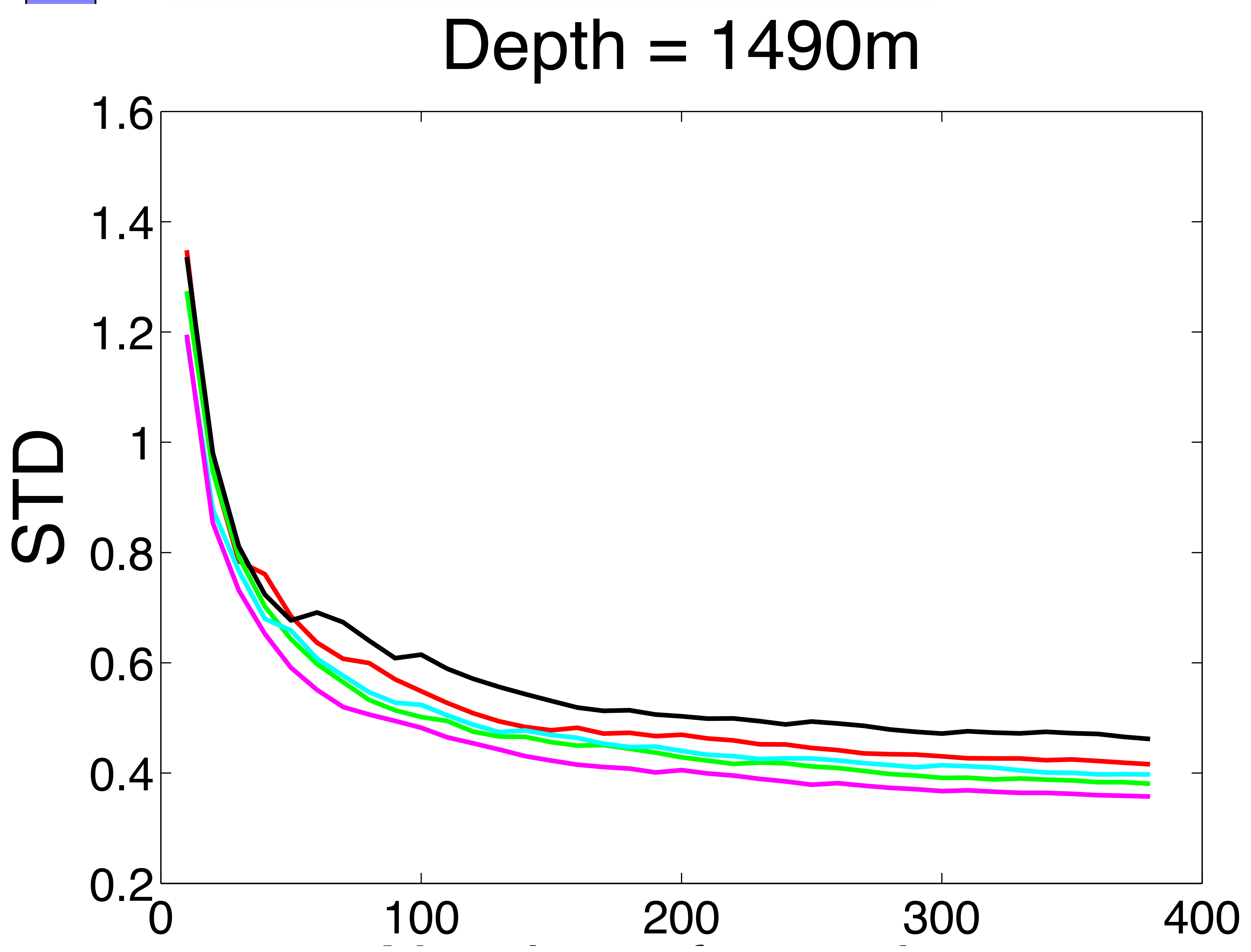
Initial model



MAP



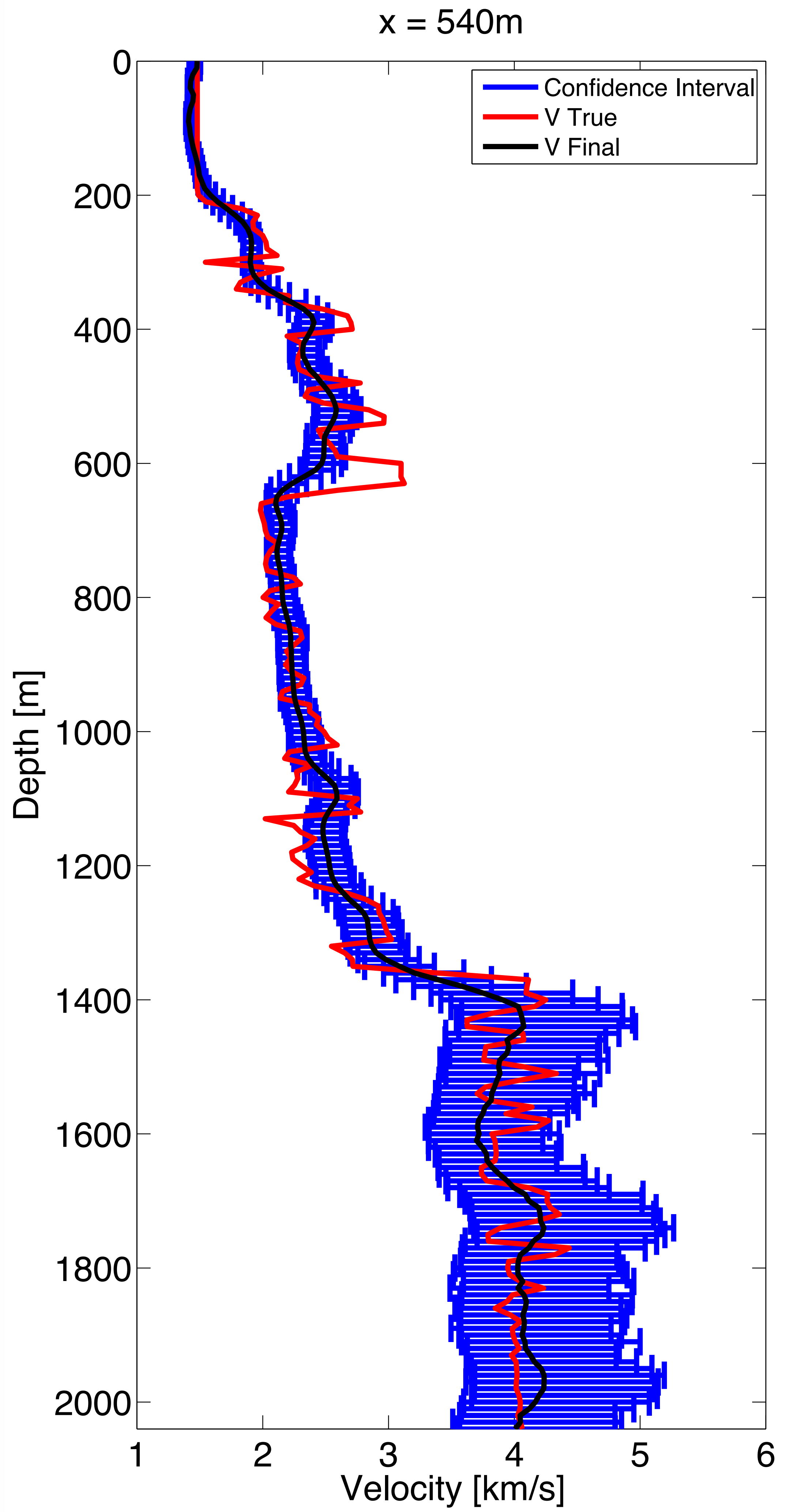
Standard deviation



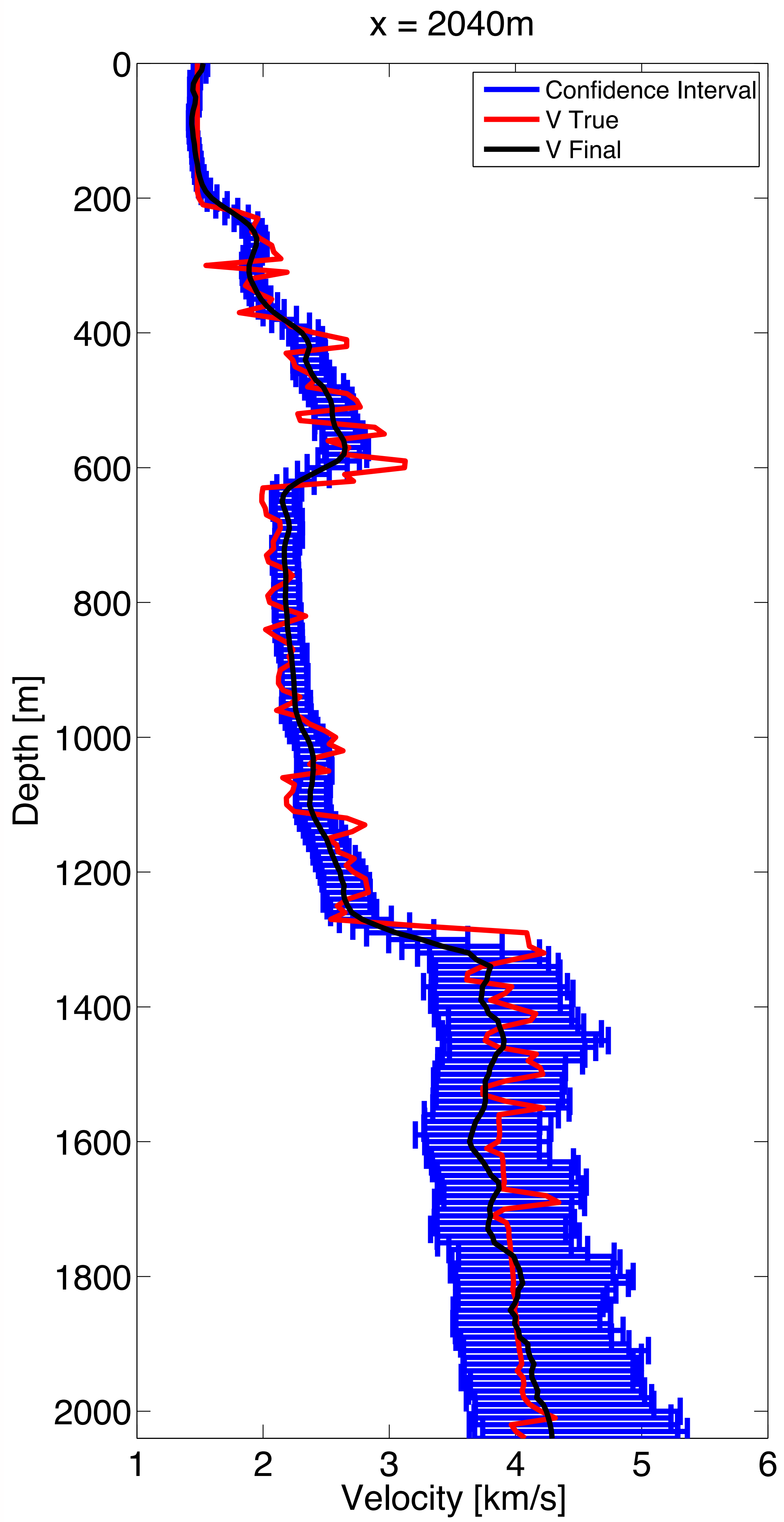
Number of random realization of velocity model

Confidence Interval

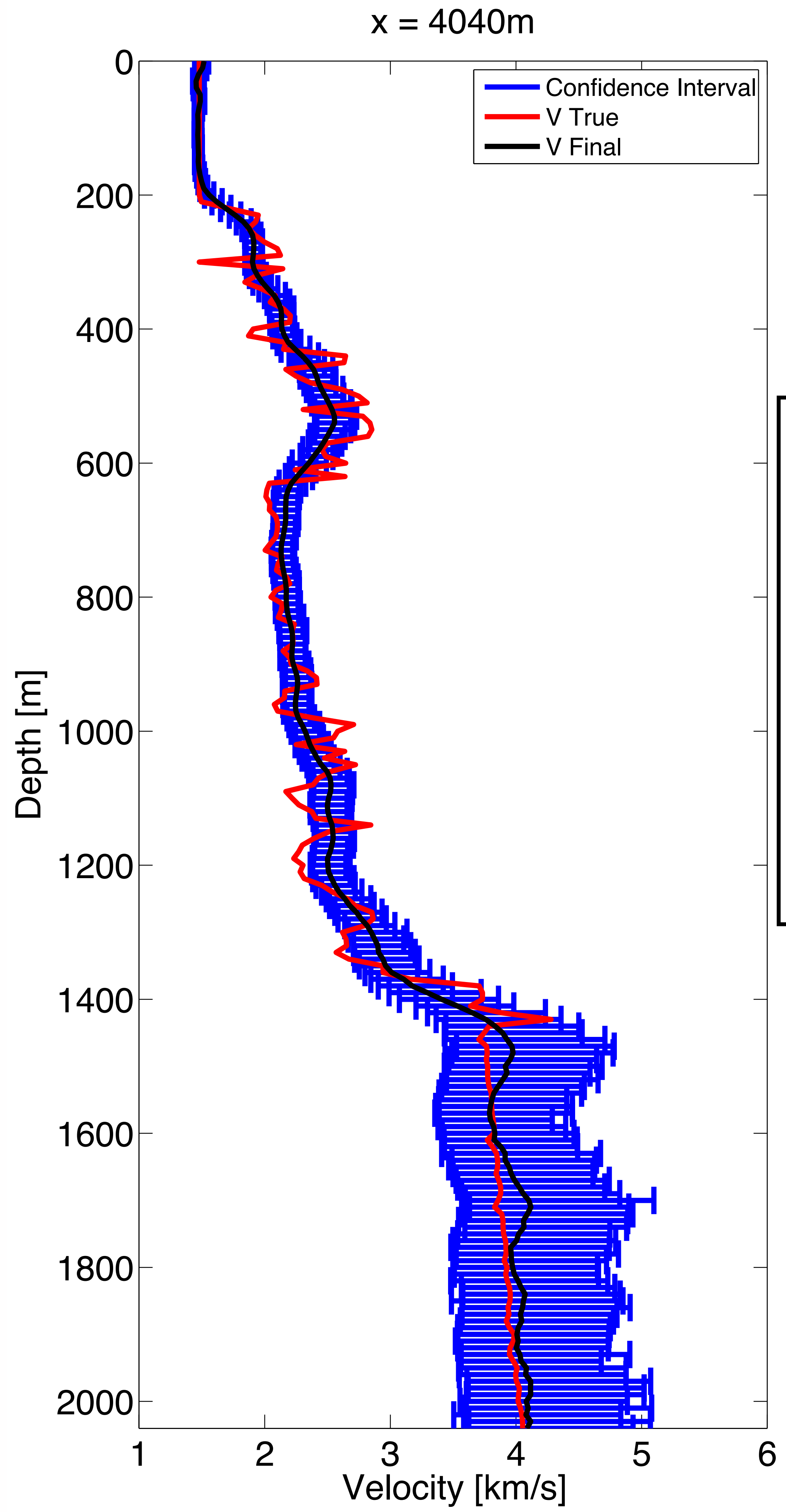
$\alpha = 0.9$



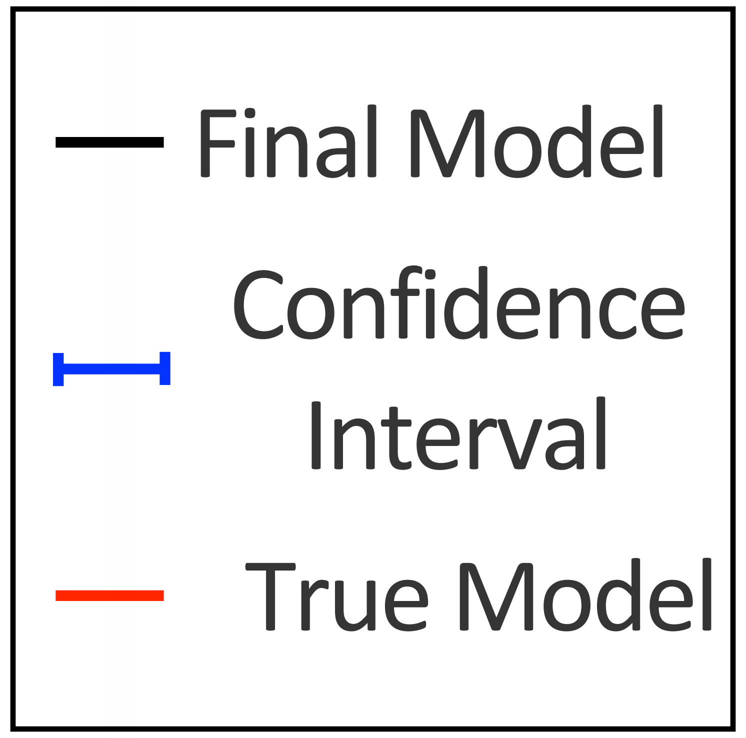
x = 540m



x = 2040m



x = 4040m



Conclusions

- Using the randomized source sub-sampling method decreases the computational cost of the McMC.
- The low-rank assumption of the Hessian may not be suitable for the seismic exploration problem.
- Look at other framework to formulate the posterior pdf, such as the penalty method.

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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