Low-rank Promoting Transformations and Tensor Interpolation - Applications to Seismic Data Denoising

Curt Da Silva and Felix J. Herrmann

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SLIM
University of British Columbia

Motivation

3D seismic experiments - 5D data

- expensive to acquire, store
- sample at *sub-Nyquist* rates

Data exhibits *low-rank* structure

- exploit structure for interpolation

Fully sampled data

- simultaneous sources in wave-equation based inversion
- mitigating multiples

Motivation

Unattenuated seismic noise can destroy the quality of a seismic image

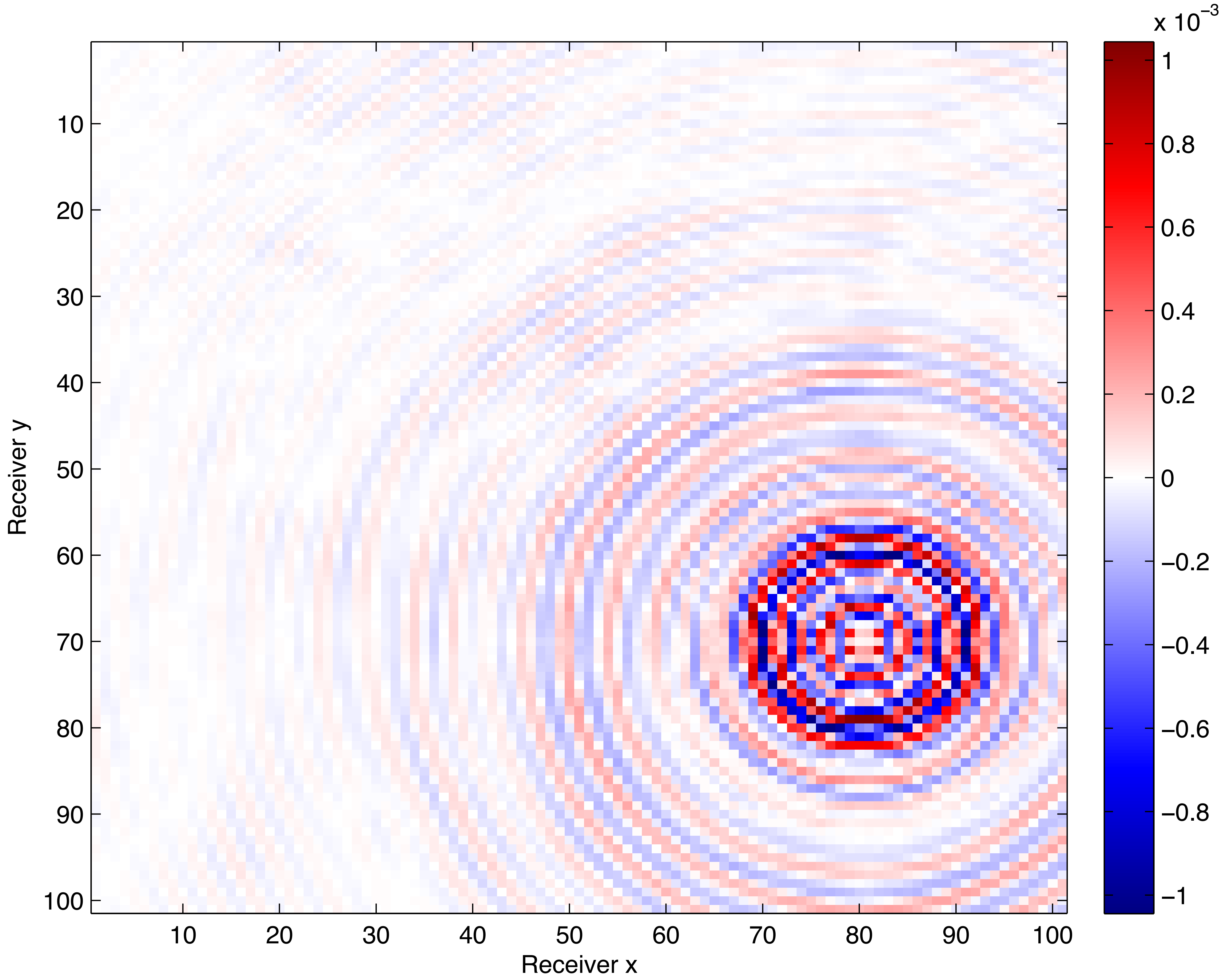
- garbage in, garbage out

Statistics of the noise can be unknown

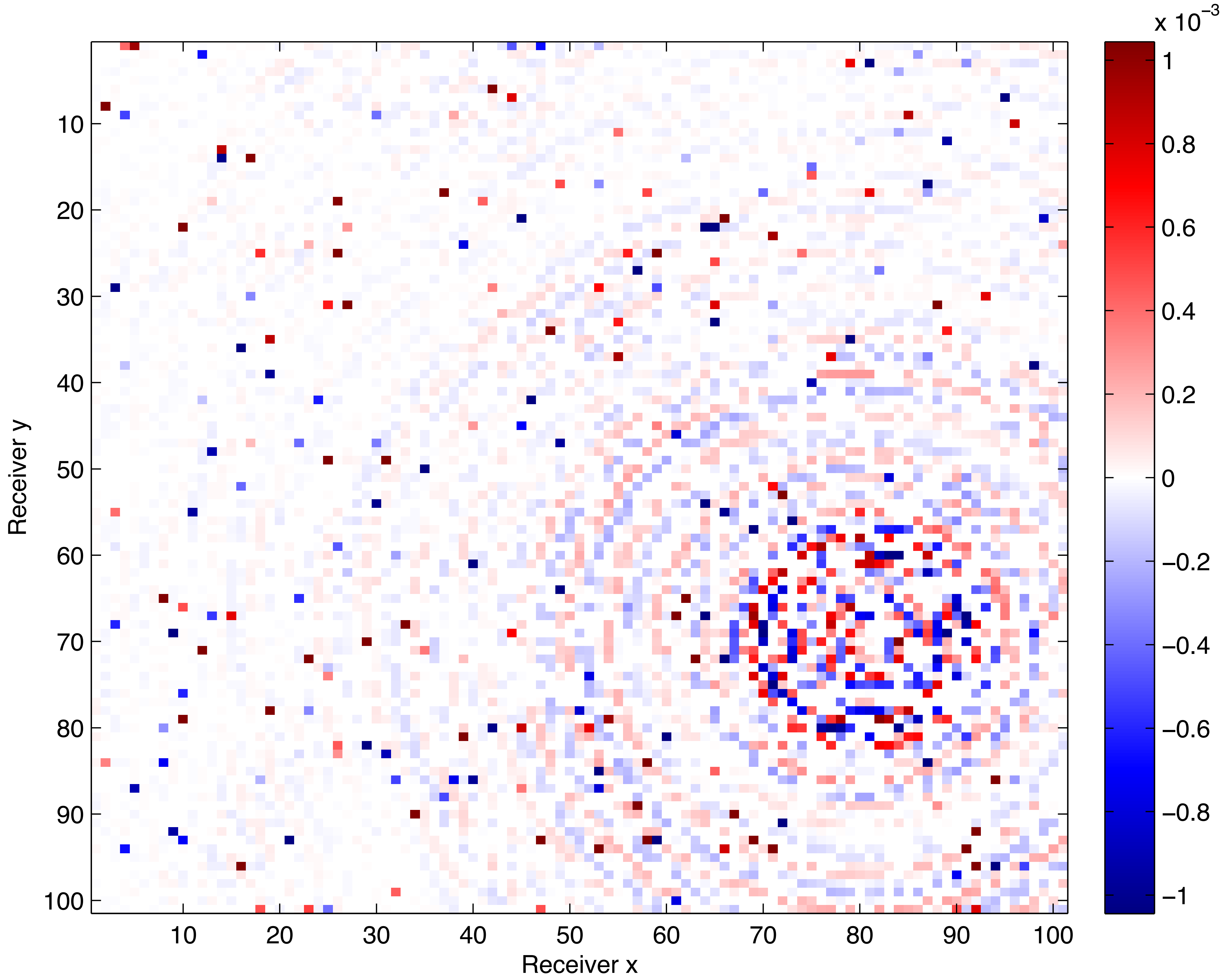
- high amplitude, localized
- caused by malfunctioning receivers, wildlife, ambient, unknown sources

7.34 Hz - 50% missing receivers - high noise

Common source gather



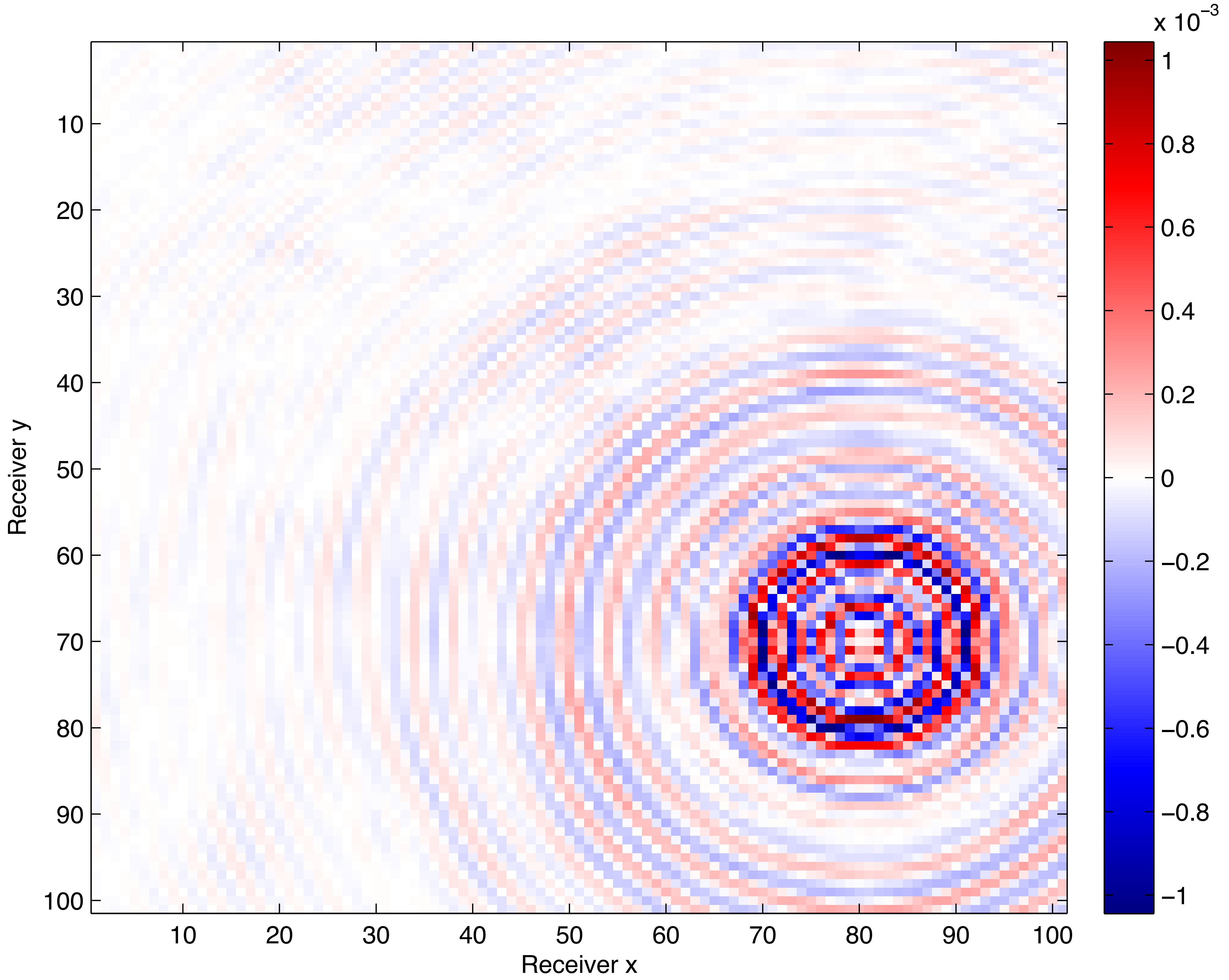
True data



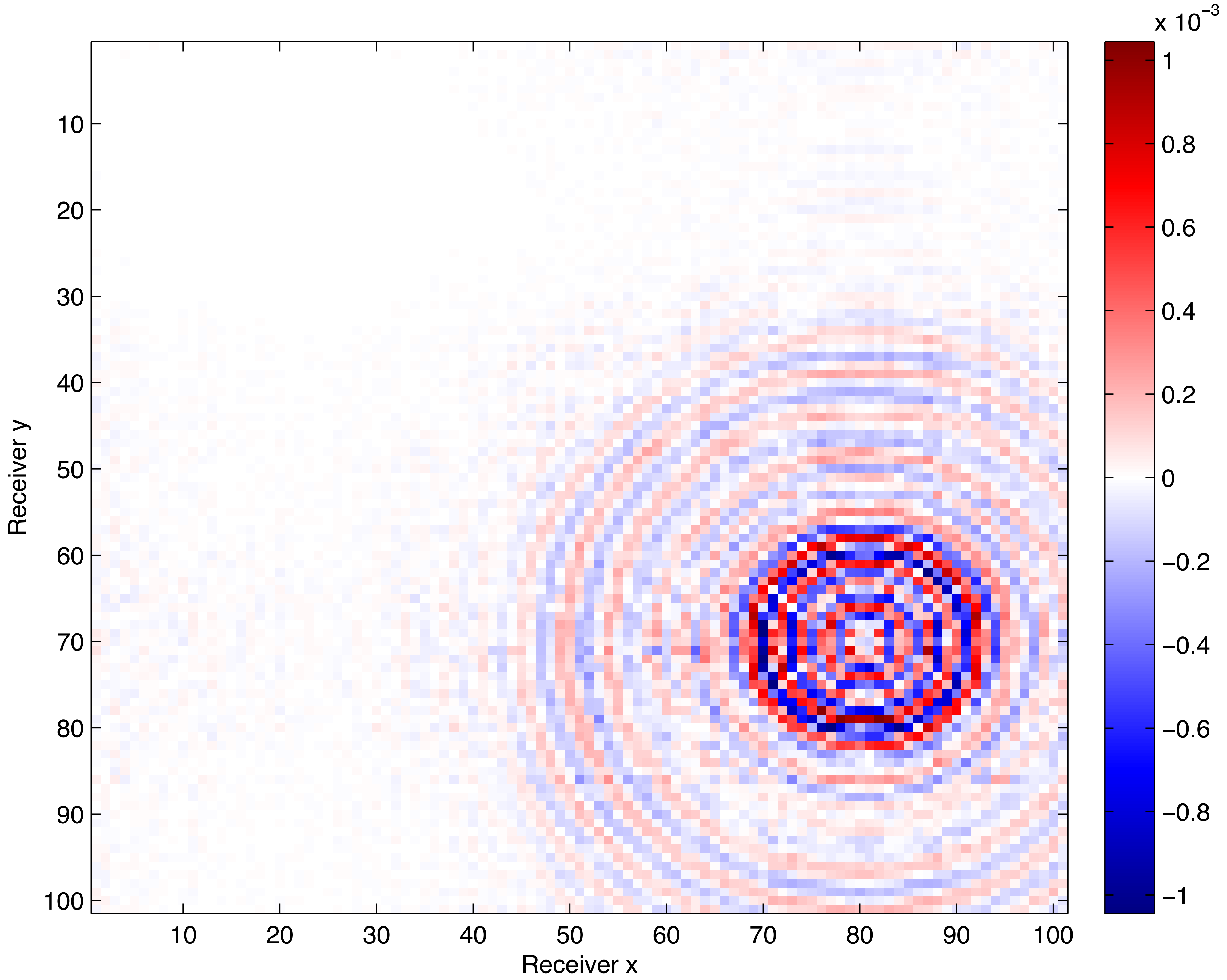
Subsampled data

7.34 Hz - 50% missing receivers - high noise

Common source gather



True data



MH Recovery - SNR 8.95 dB

[1] Kreimer and Sacchi, "A tensor higher-order singular value decomposition for prestack seismic data noise reduction and interpolation." (2012)
[2] Gao, Vicente, and Sacchi. "Evaluation of a fast algorithm for the eigen-decomposition of large block Toeplitz matrices with application to 5D seismic data interpolation." *(2011)*
[3] Da Silva, Kumar, et al, “” (2014)

Context

Low-rank matrix/tensor completion via *nuclear norm* projection [1]

- Require SVDs on huge data matrices
- Not scalable to large problem sizes

Data completion via Toeplitz embedding [2]

- Problem size - (# data points)^2
- Ad-hoc windowing - can degrade quality, as demonstrated in [3]

Goals

Review Hierarchical Tucker tensor format, principles of low-rank tensor recovery

Explore effect of transform domain on noise

- determine favourable recovery scenario

Compressive sensing
with sparsity promotion

Successful reconstruction scheme

Signal structure

- sparsity

Sampling

- subsampling decreases sparsity

Optimization

- look for sparsest solution

Multidimensional interpolation
with Hierarchical Tucker

Successful reconstruction scheme

Signal structure

- ***Hierarchical Tucker***

Sampling

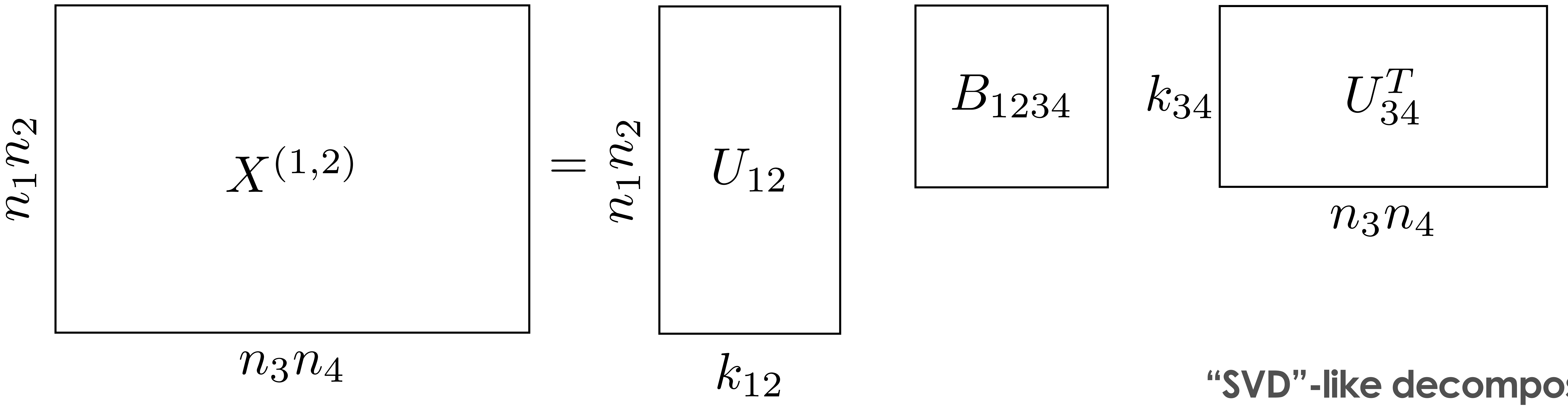
- subsampling, noise increases hierarchical rank

Optimization

- fit data in the Hierarchical Tucker format

Hierarchical Tucker format

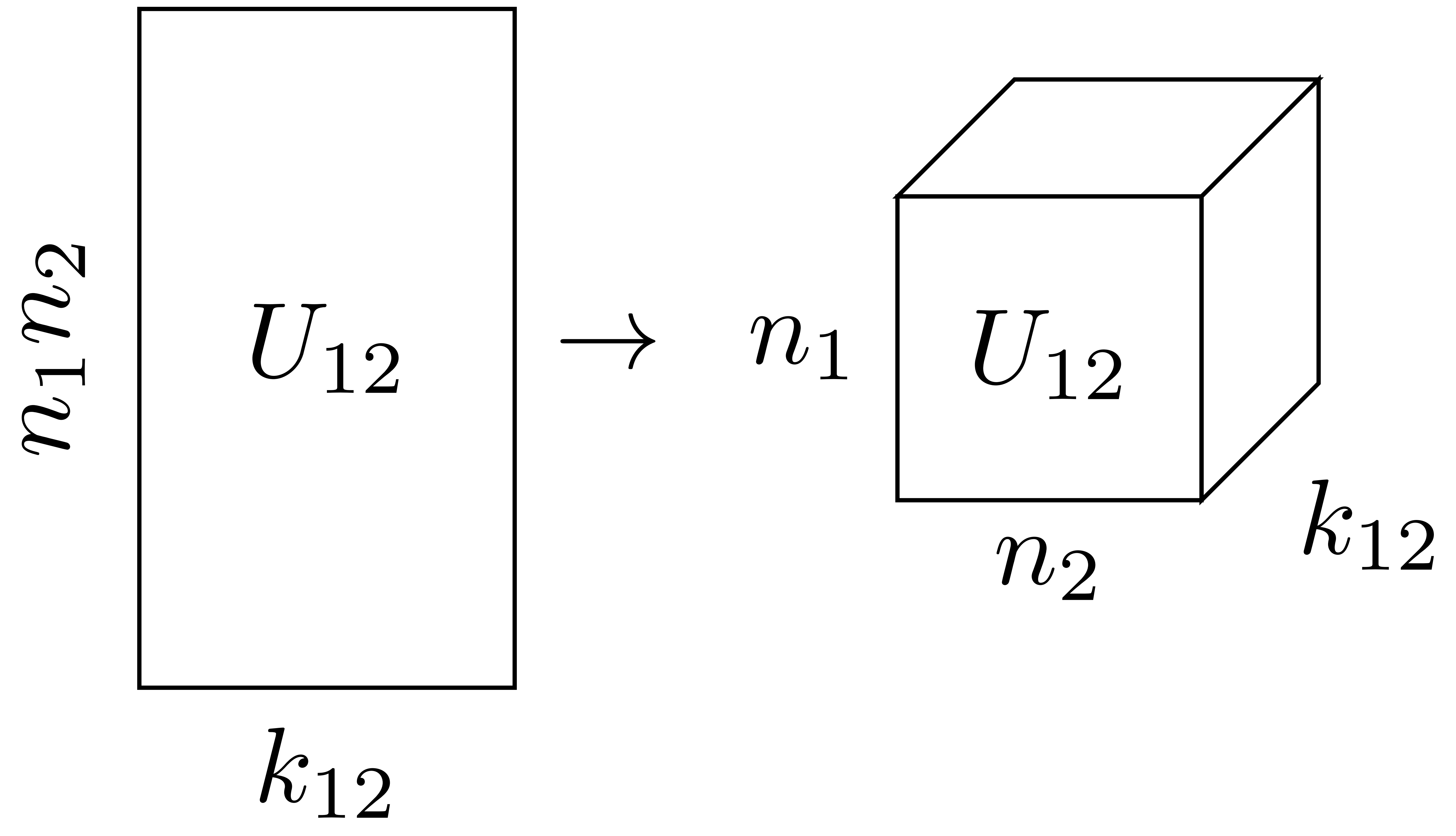
$X - n_1 \times n_2 \times n_3 \times n_4$ tensor



"SVD"-like decomposition

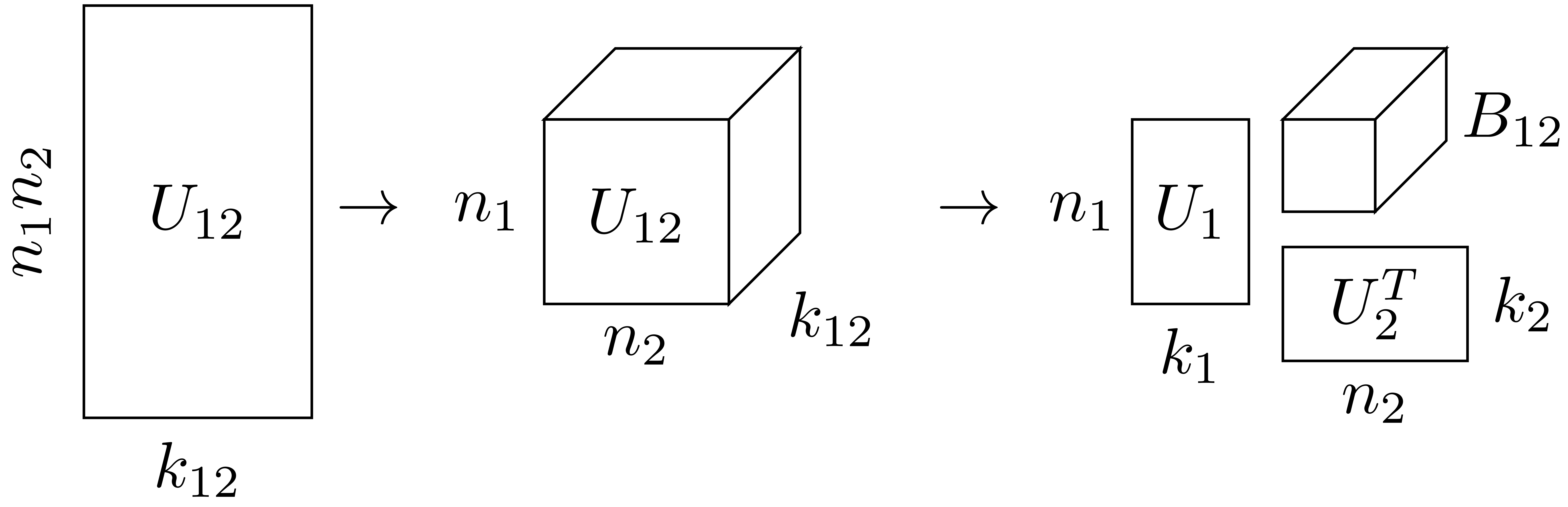
Hierarchical Tucker format

$X - n_1 \times n_2 \times n_3 \times n_4$ tensor



Hierarchical Tucker format

$X - n_1 \times n_2 \times n_3 \times n_4$ tensor



Hierarchical Tucker format

Intermediate matrices don't need to be stored

U_t, B_t - small parameter matrices

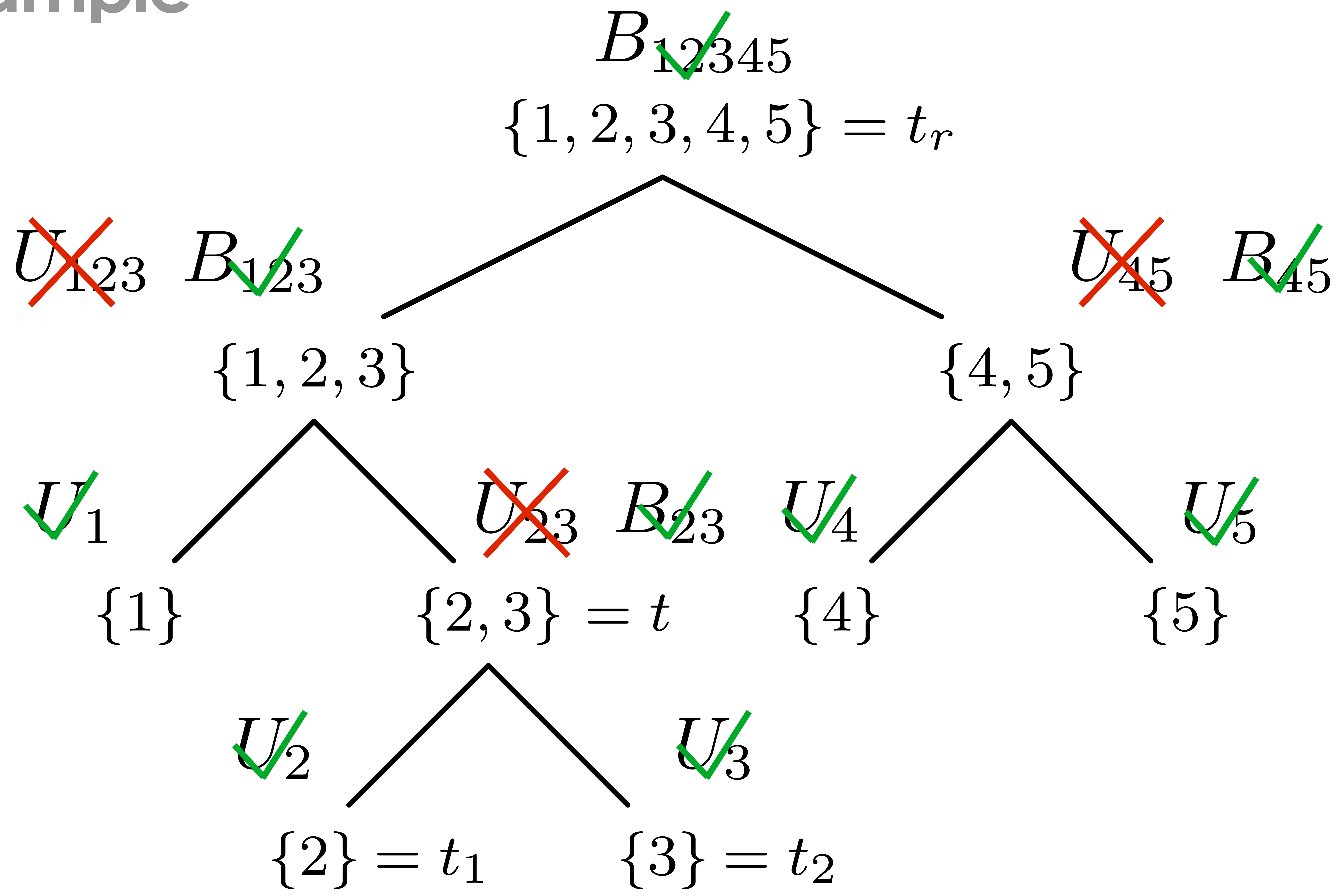
- specify the tensor completely

Separating groups of dimensions from each other

- dimension tree

A. Uschmajew, B. Vandereycken. The geometry of algorithms using hierarchical tensors. Linear algebra and its applications, 2013.

Example



Hierarchical Tucker format

Storage $\leq dNK + (d-2)K^3 + K^2$

Compare to N^d storage for the full tensor

Effectively breaking the curse of dimensionality when $K \ll N \quad d \geq 4$

Low frequency data compresses in HT

Seismic Hierarchical Tucker

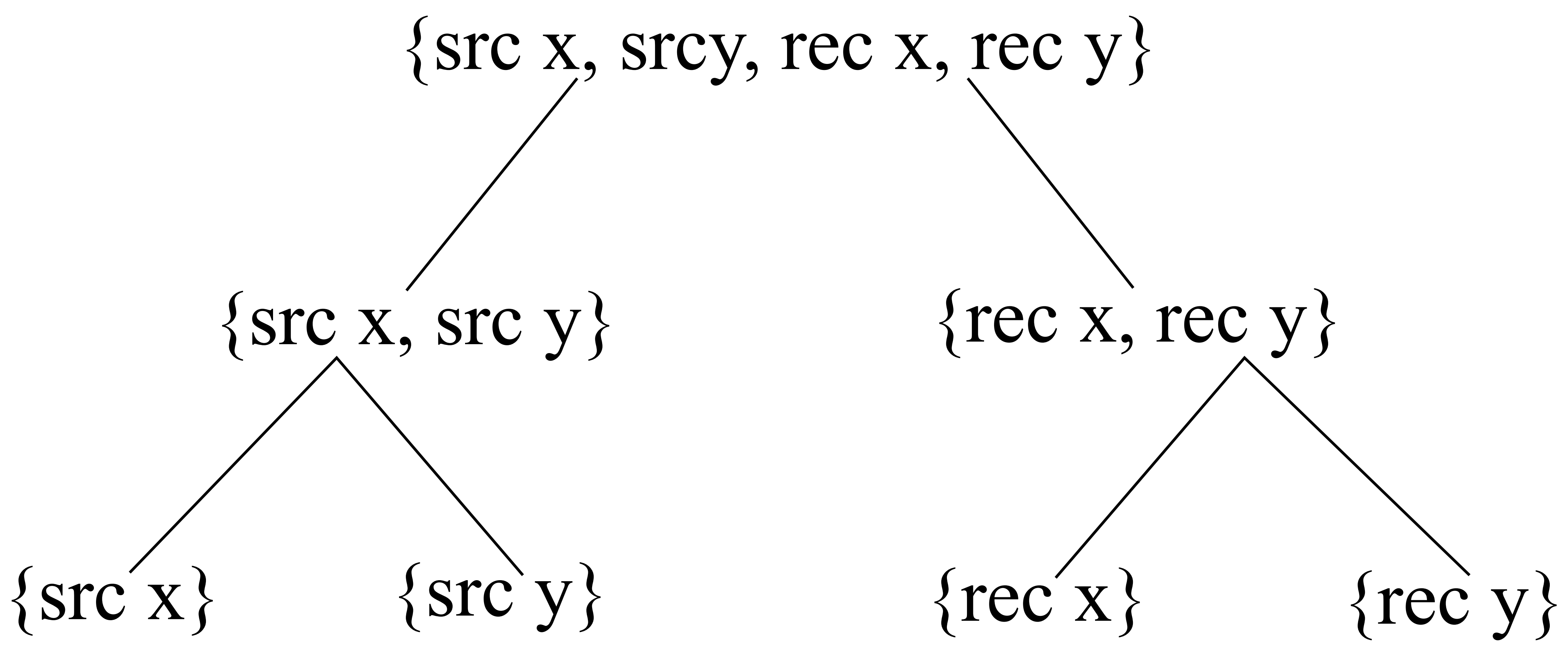
We consider a 3D seismic survey with coordinates
(src x, src y, rec x, rec y, time)

We take a Fourier transform in time and restrict ourselves to a single frequency slice

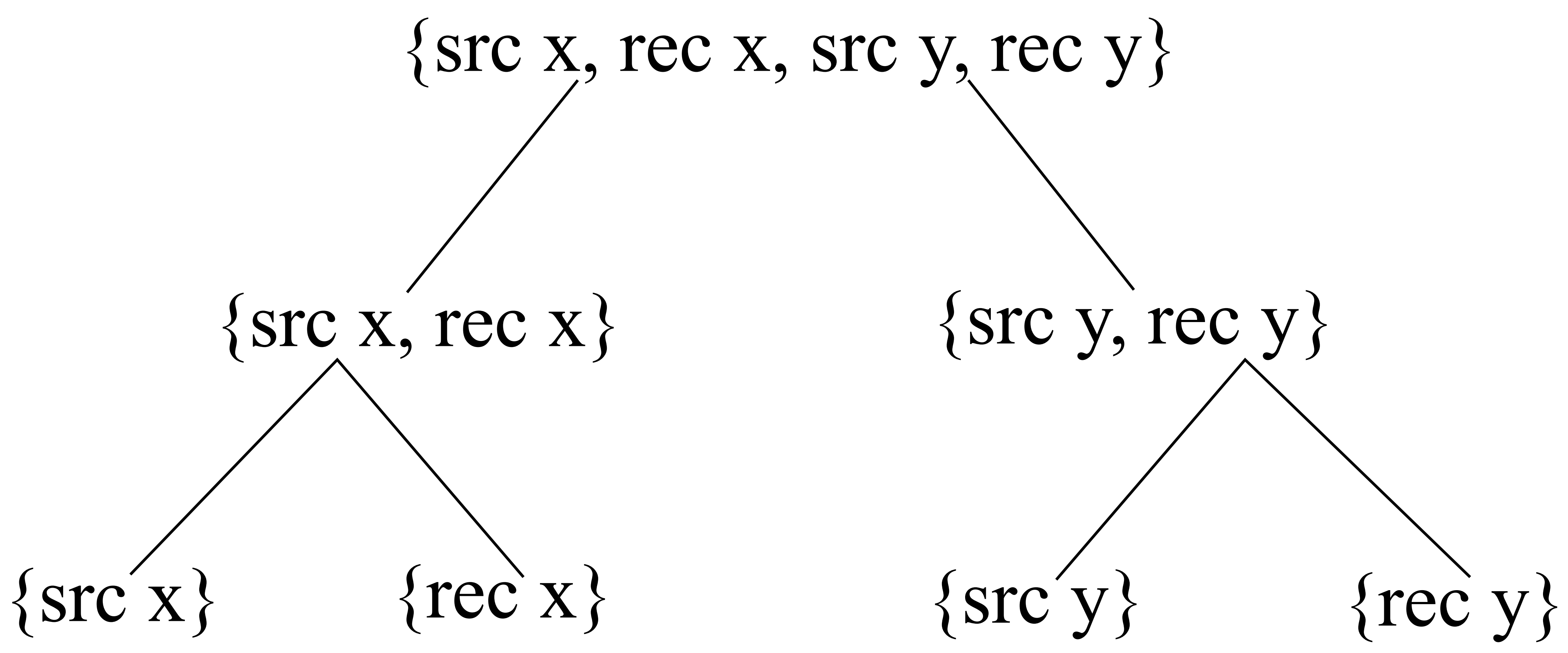
Seismic Hierarchical Tucker

For a frequency slice with coordinates (src x, src y, rec x, rec y),

there are essentially two choices of dimension splitting (by reciprocity)

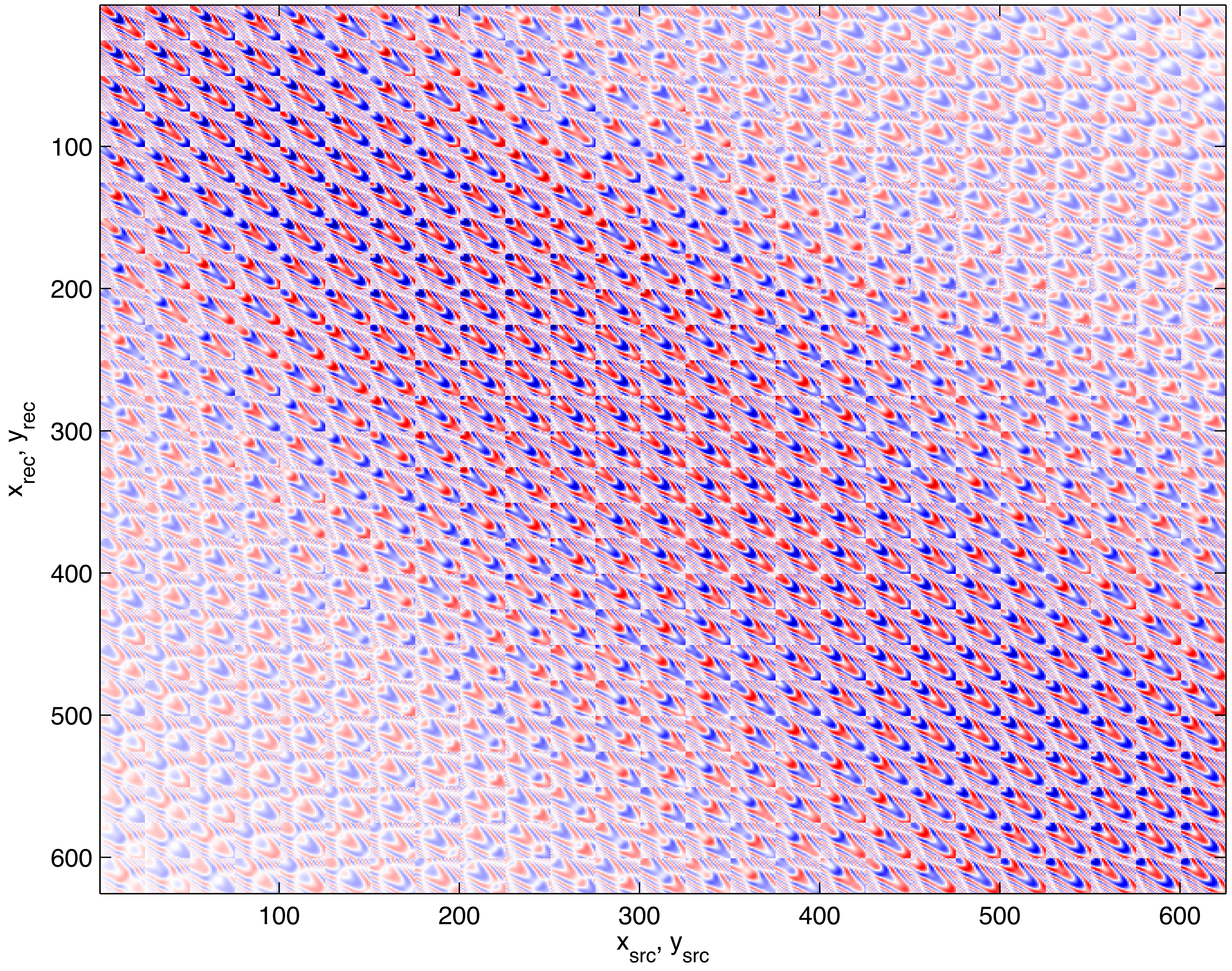


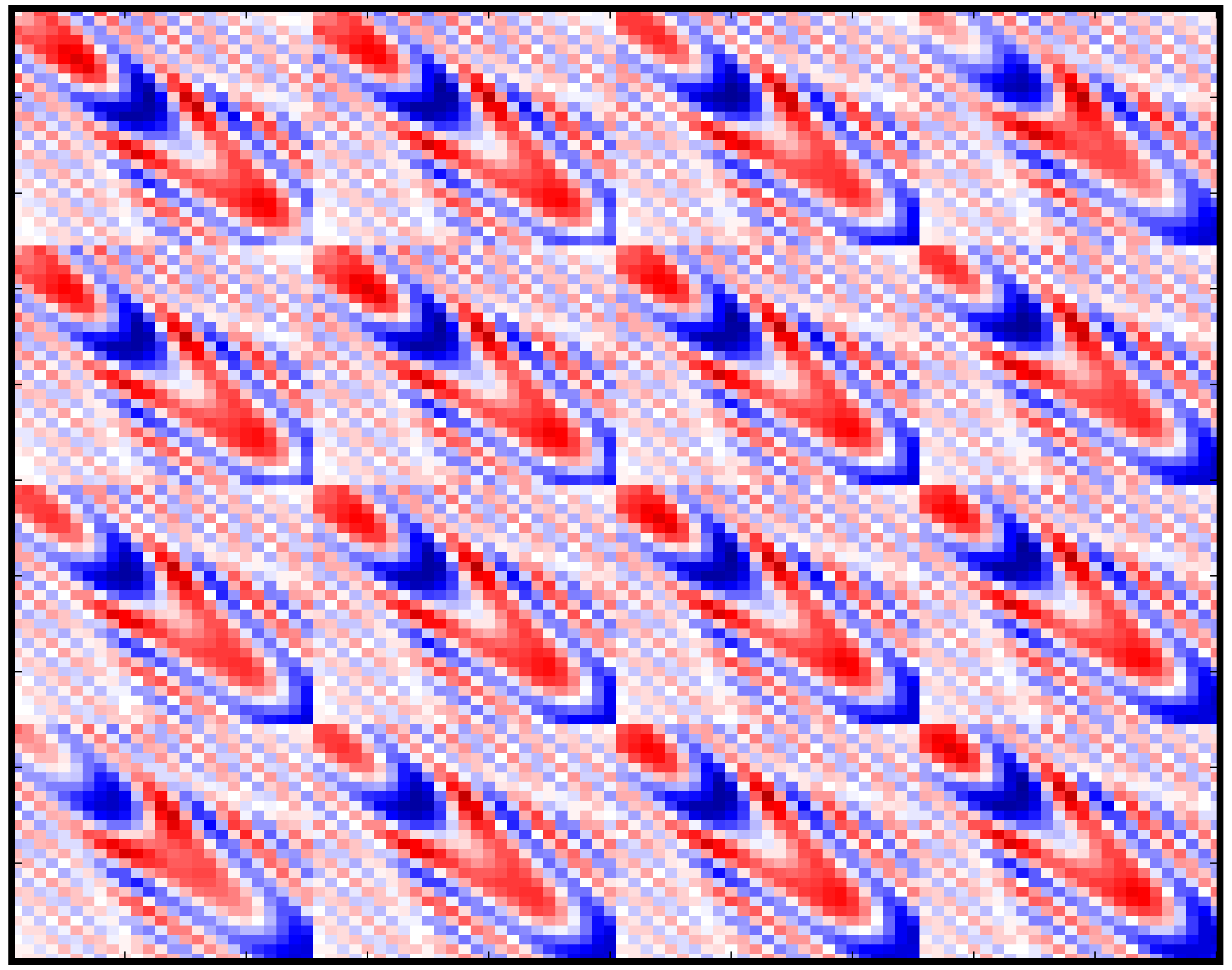
Canonical Decomposition

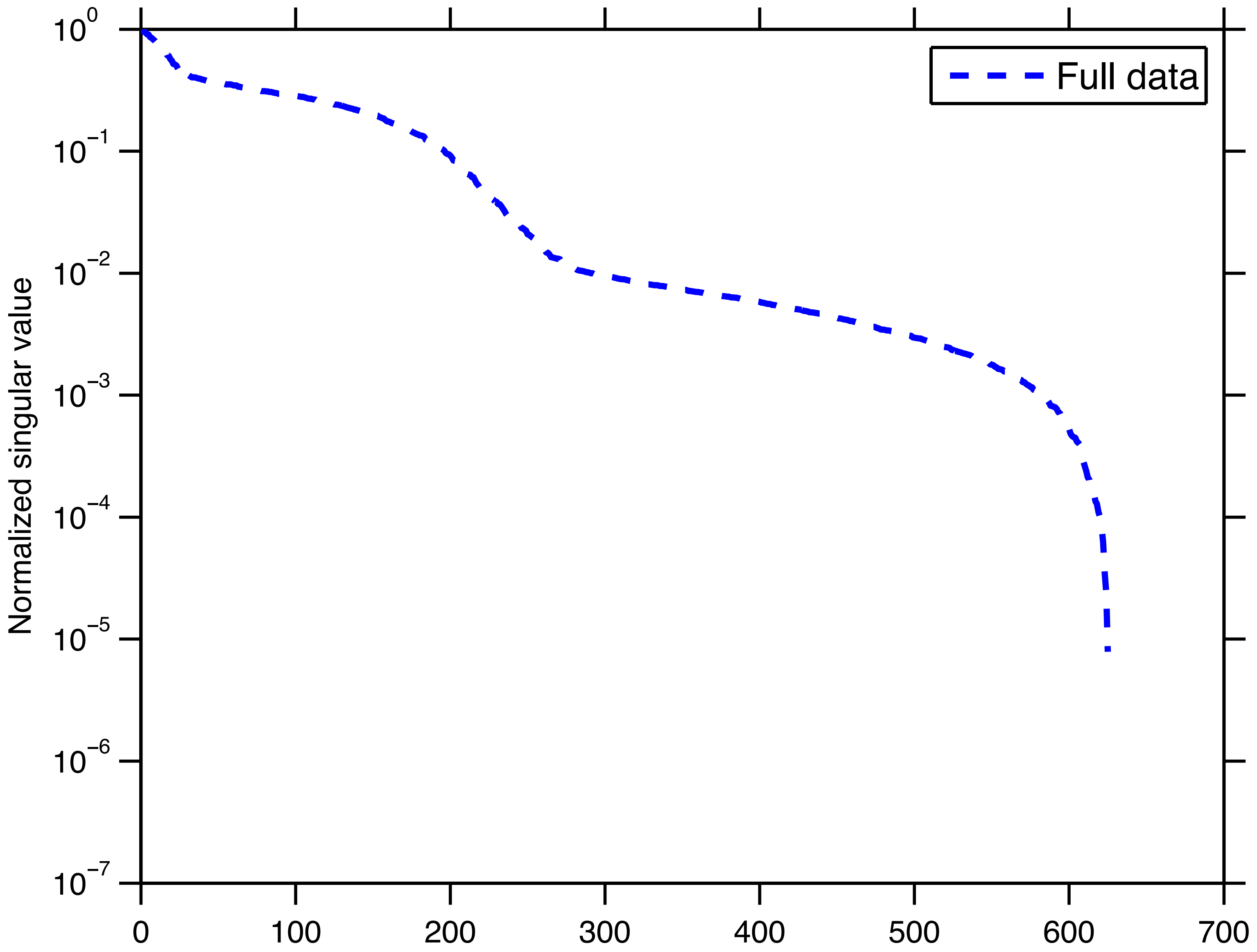


Non-canonical Decomposition

Matricizations

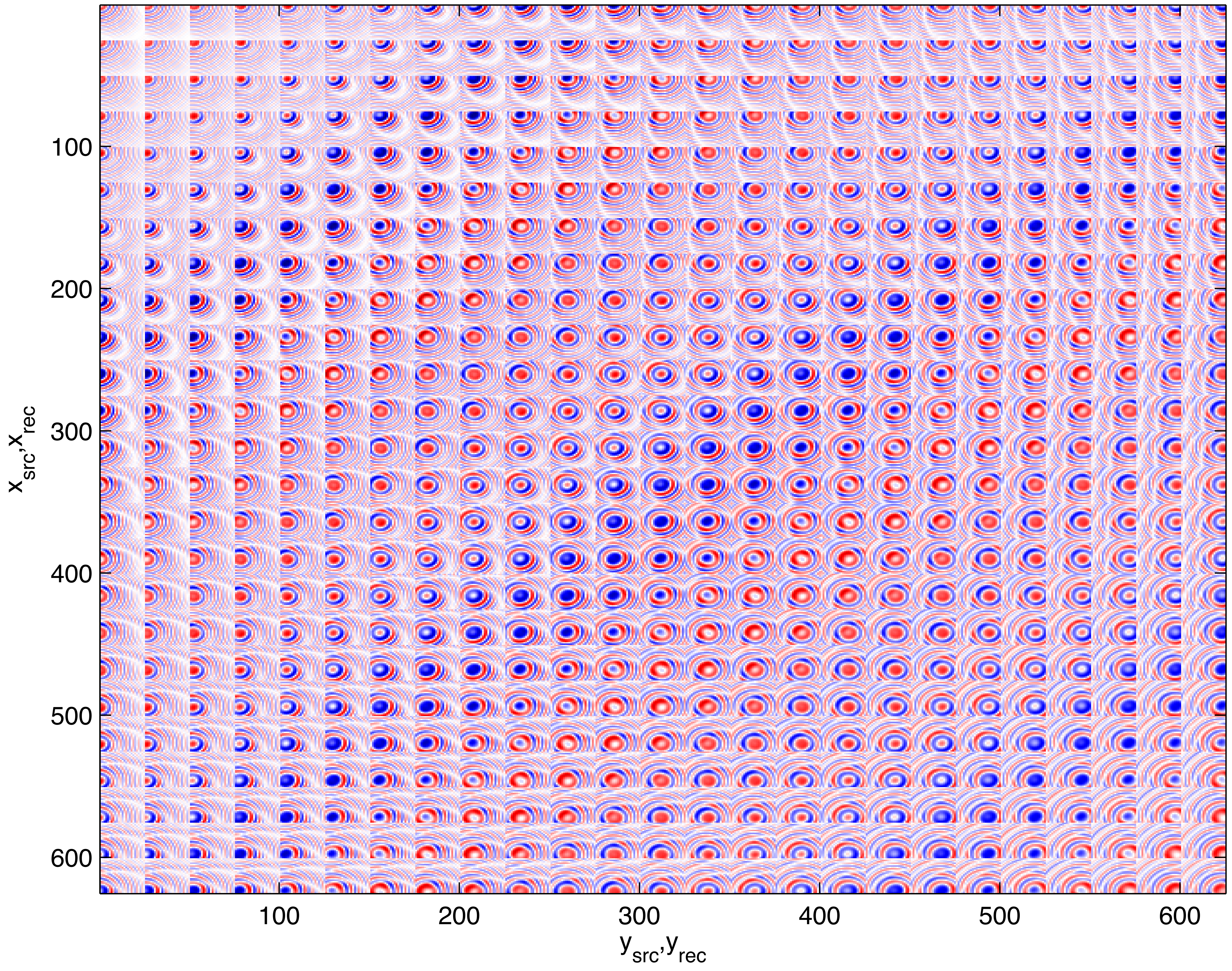


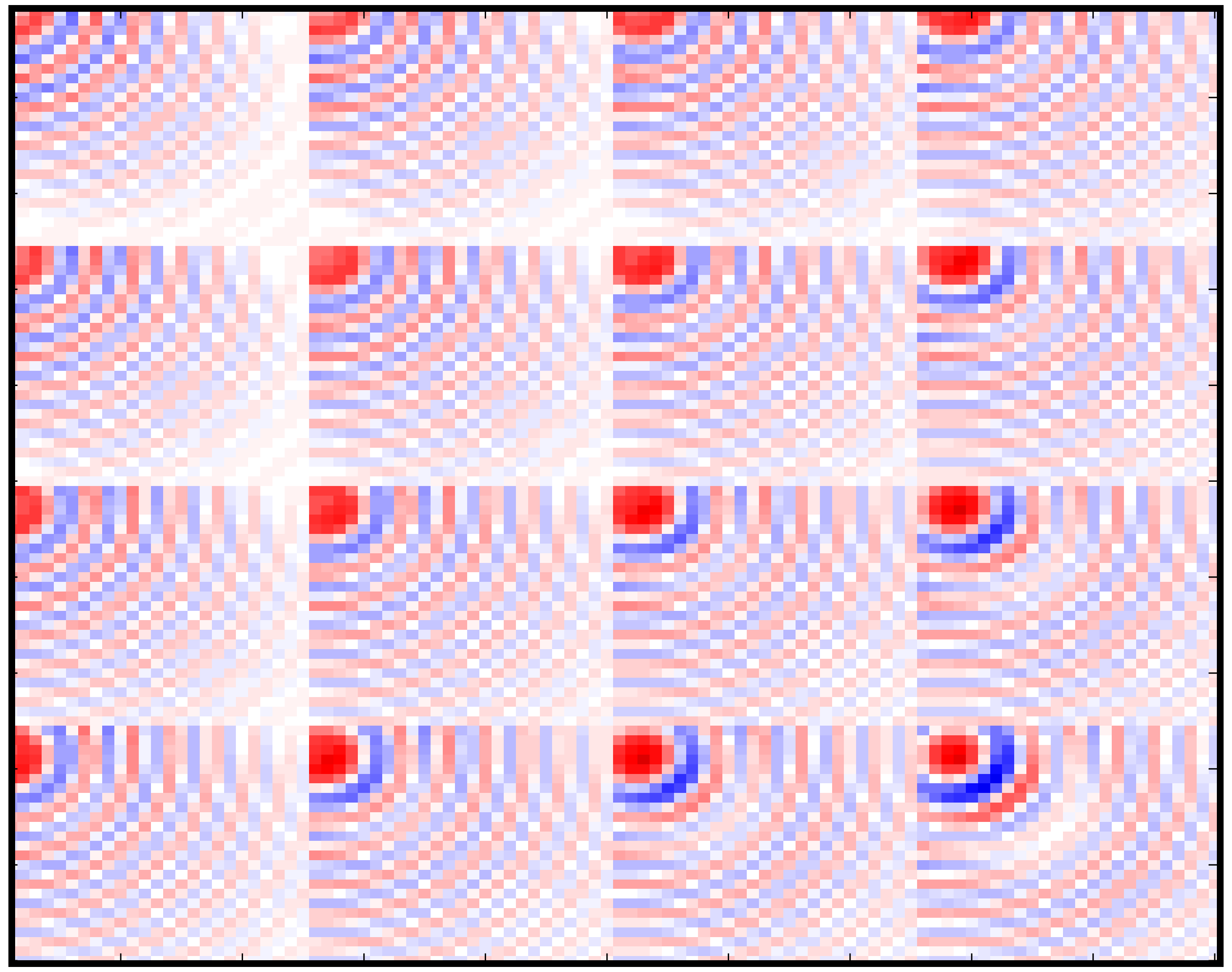


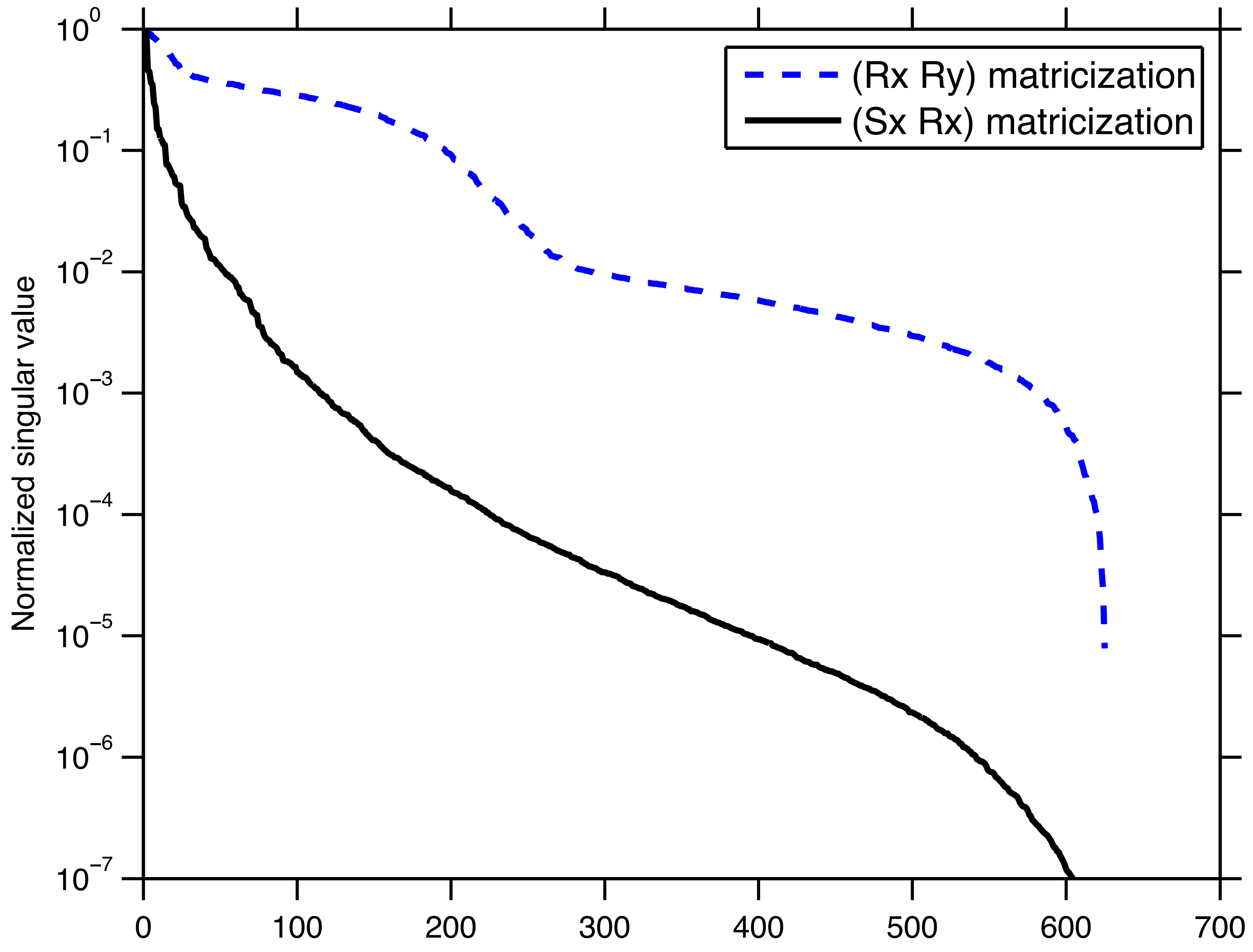


(Rec x, Rec y) matricization - Canonical ordering

Matricizations







(Src x, Rec x) matricization - Noncanonical ordering

Multidimensional interpolation

with Hierarchical Tucker

Successful reconstruction scheme

Signal structure

- Hierarchical Tucker

Sampling

- ***subsampling, noise increases hierarchical rank***

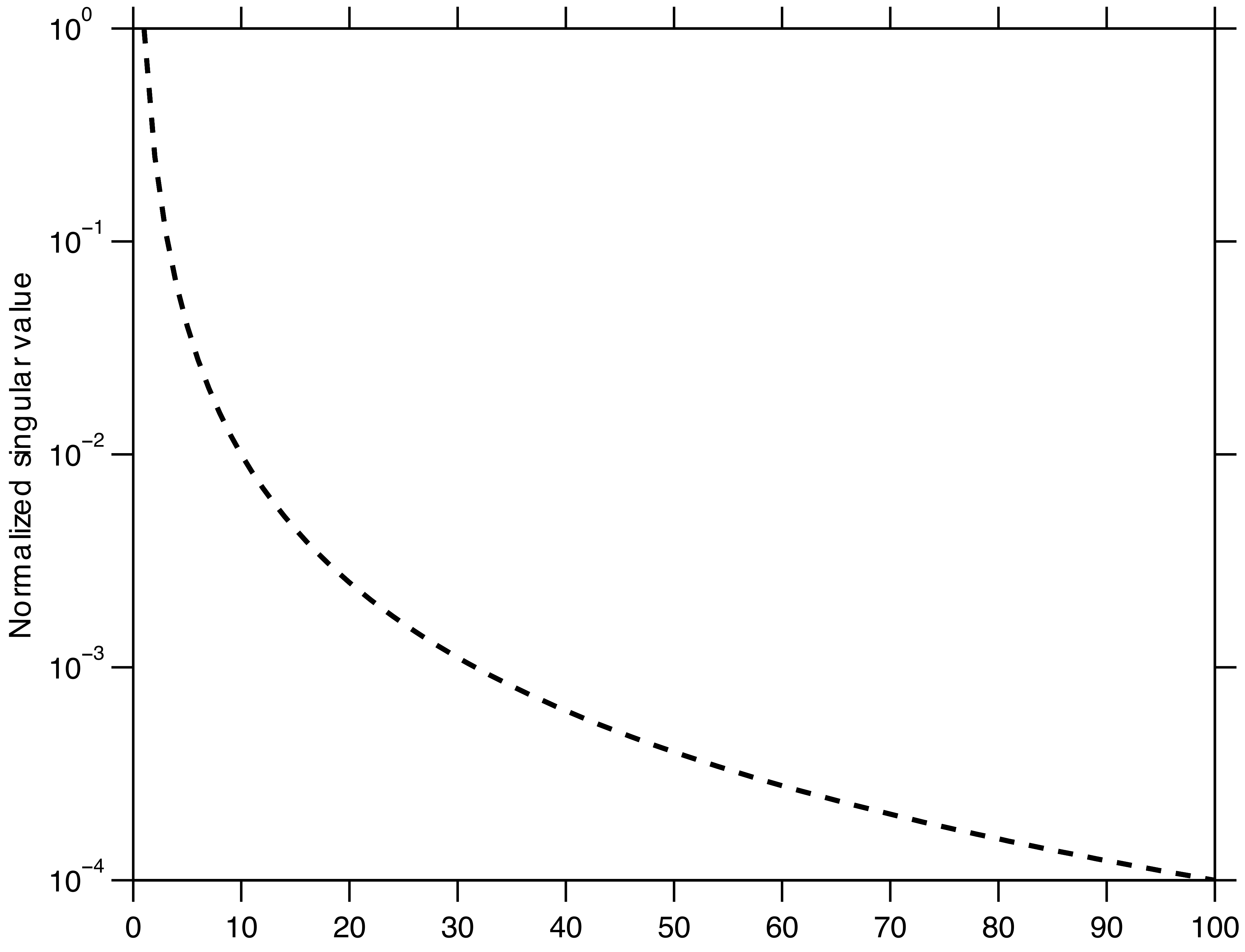
Optimization

- fit data in the Hierarchical Tucker format

Matrix Completion

$$\mathbf{X}$$

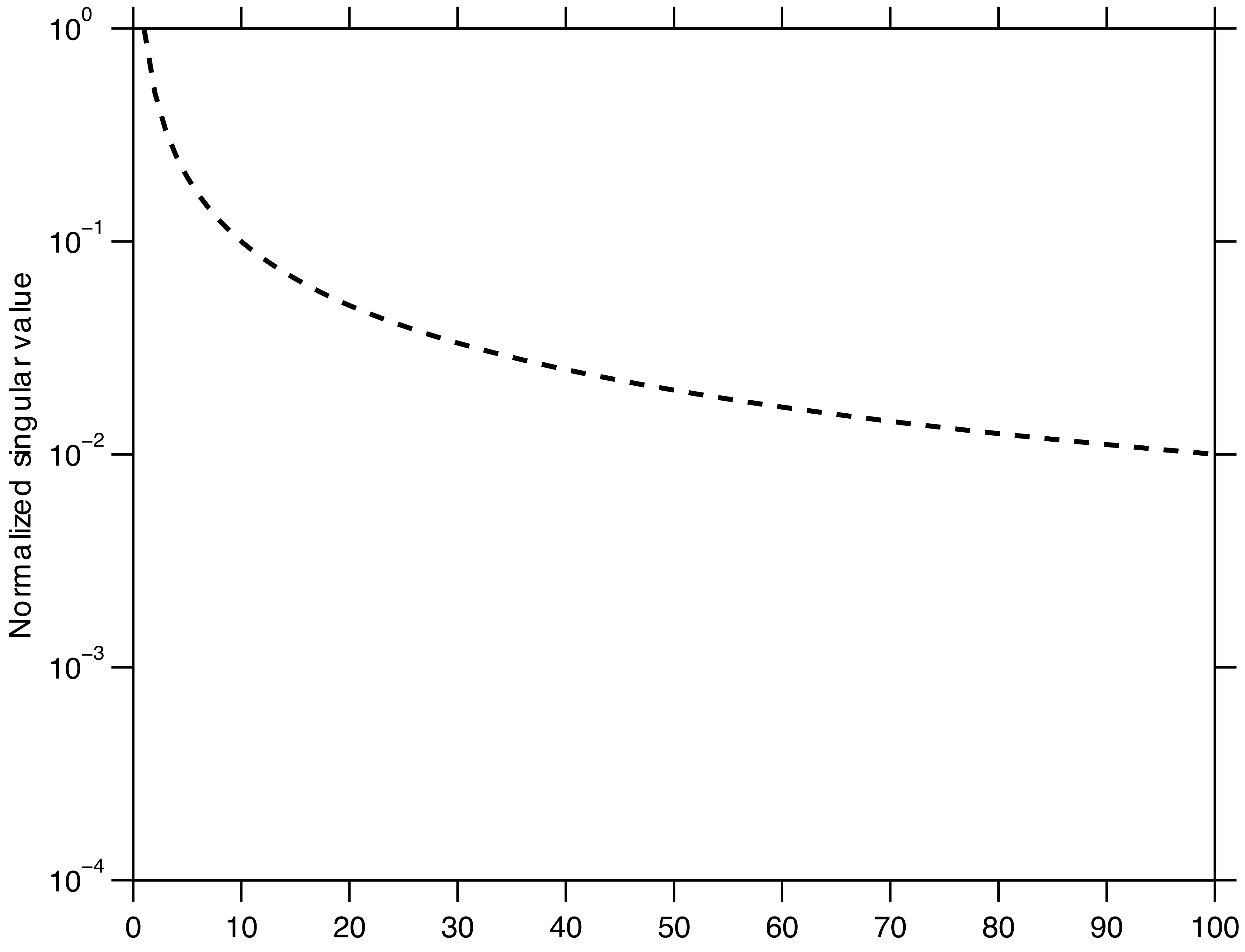
$$\begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix}$$



Matrix Completion

$$\mathcal{A}(\mathbf{X})$$

$$\begin{bmatrix} * & * & * & 0 & * \\ * & 0 & 0 & * & 0 \\ * & * & * & * & * \\ * & * & 0 & * & * \\ 0 & * & * & * & 0 \end{bmatrix}$$



Sampling

Sampling $(x_{\rm src}, y_{\rm src}, x_{\rm rec}, y_{\rm rec})$ points from the data

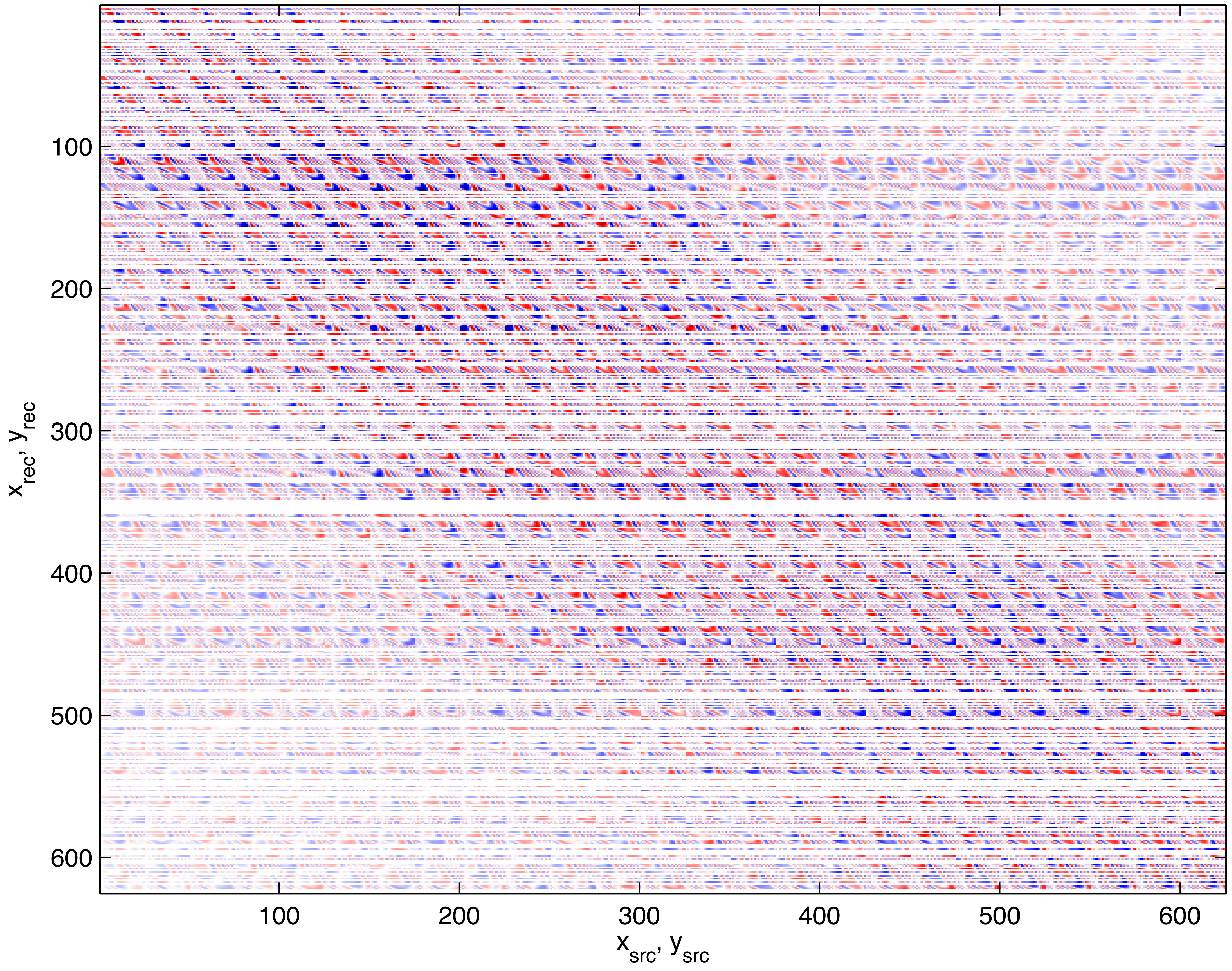
- idealized recovery
- impossible to physically implement

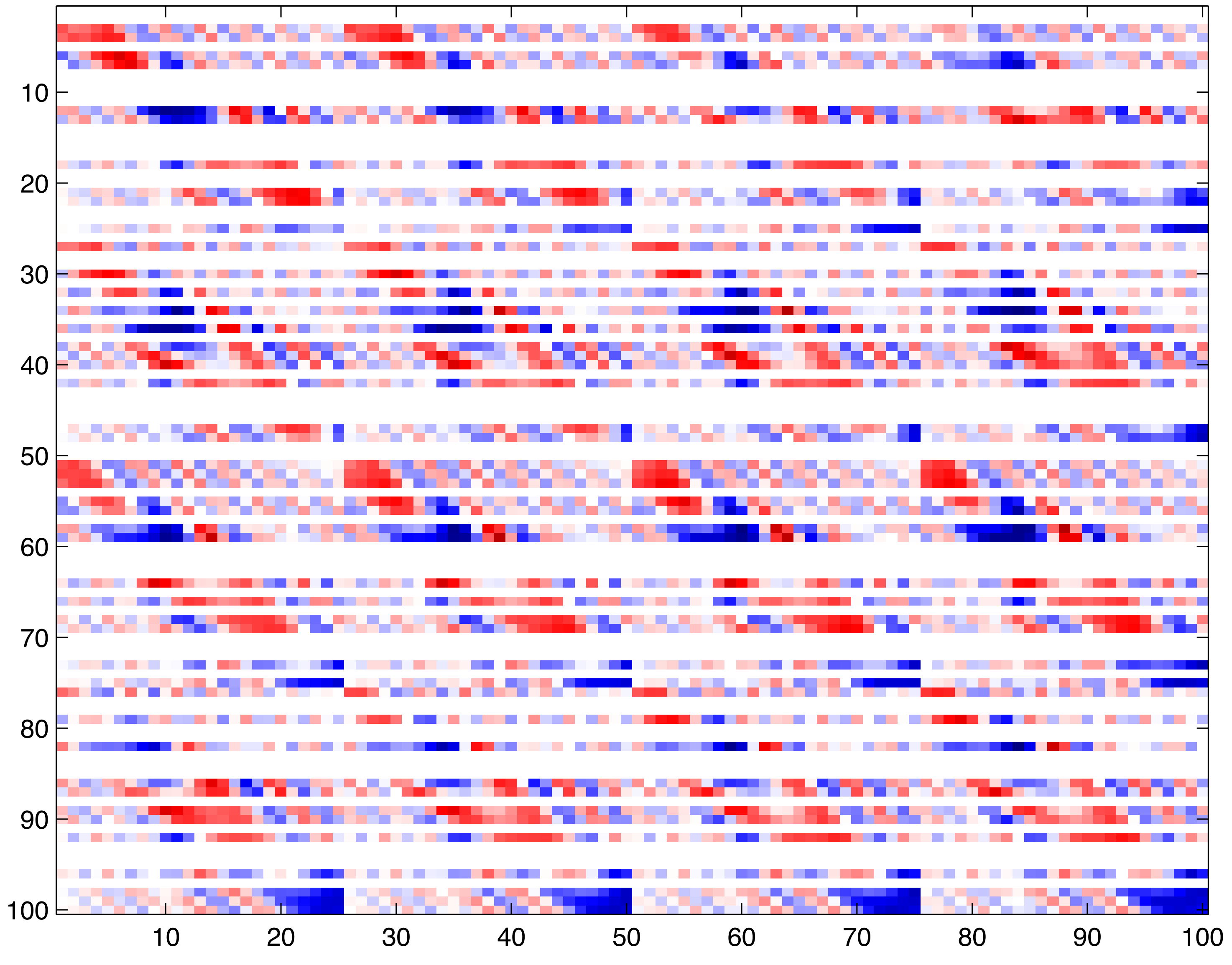
Sampling $(x_{\rm rec}, y_{\rm rec})$ points from the data

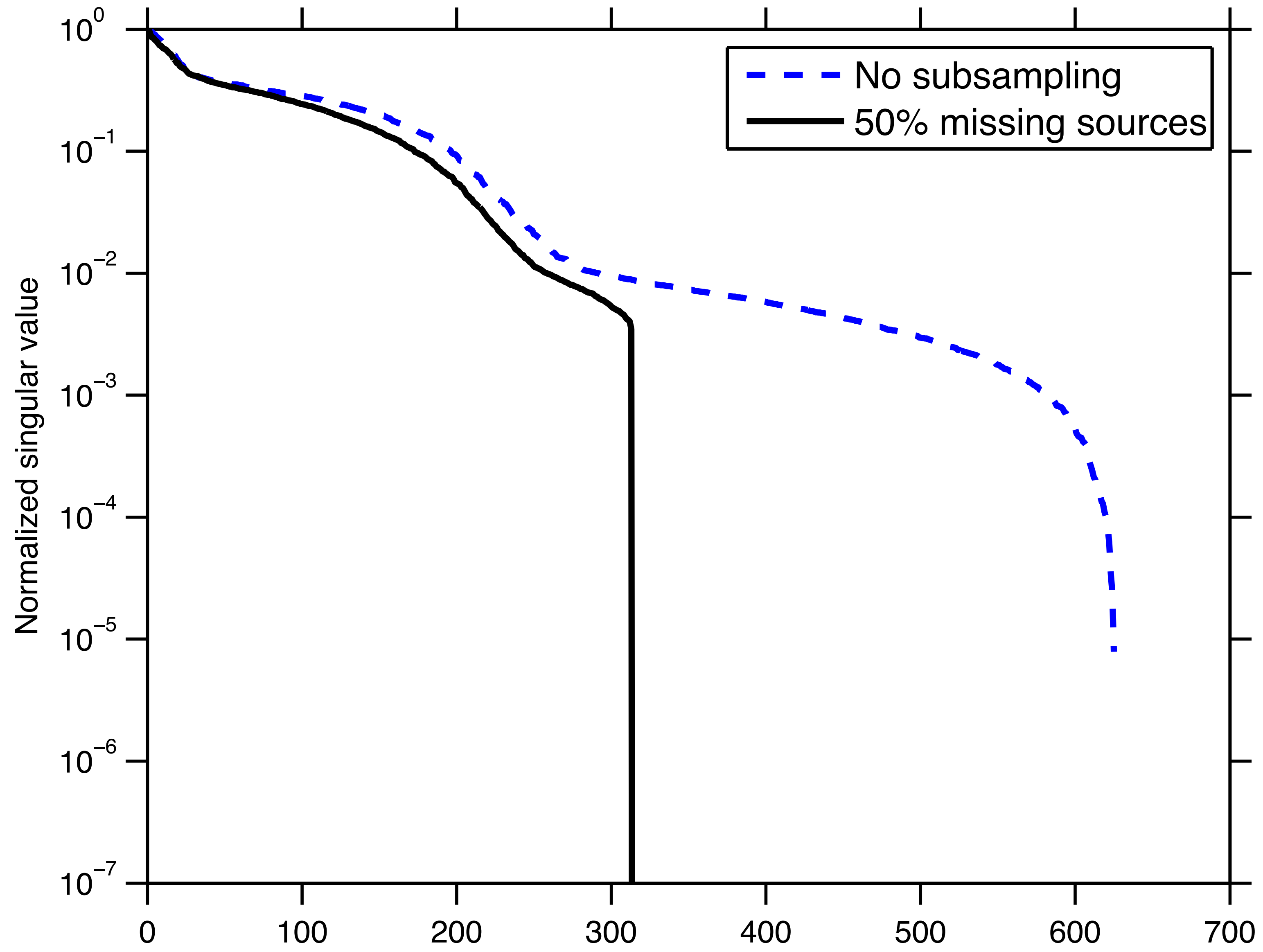
- less idealized
- possible to acquire data - e.g. ocean bottom nodes

Realistic recovery

50% random receivers removed

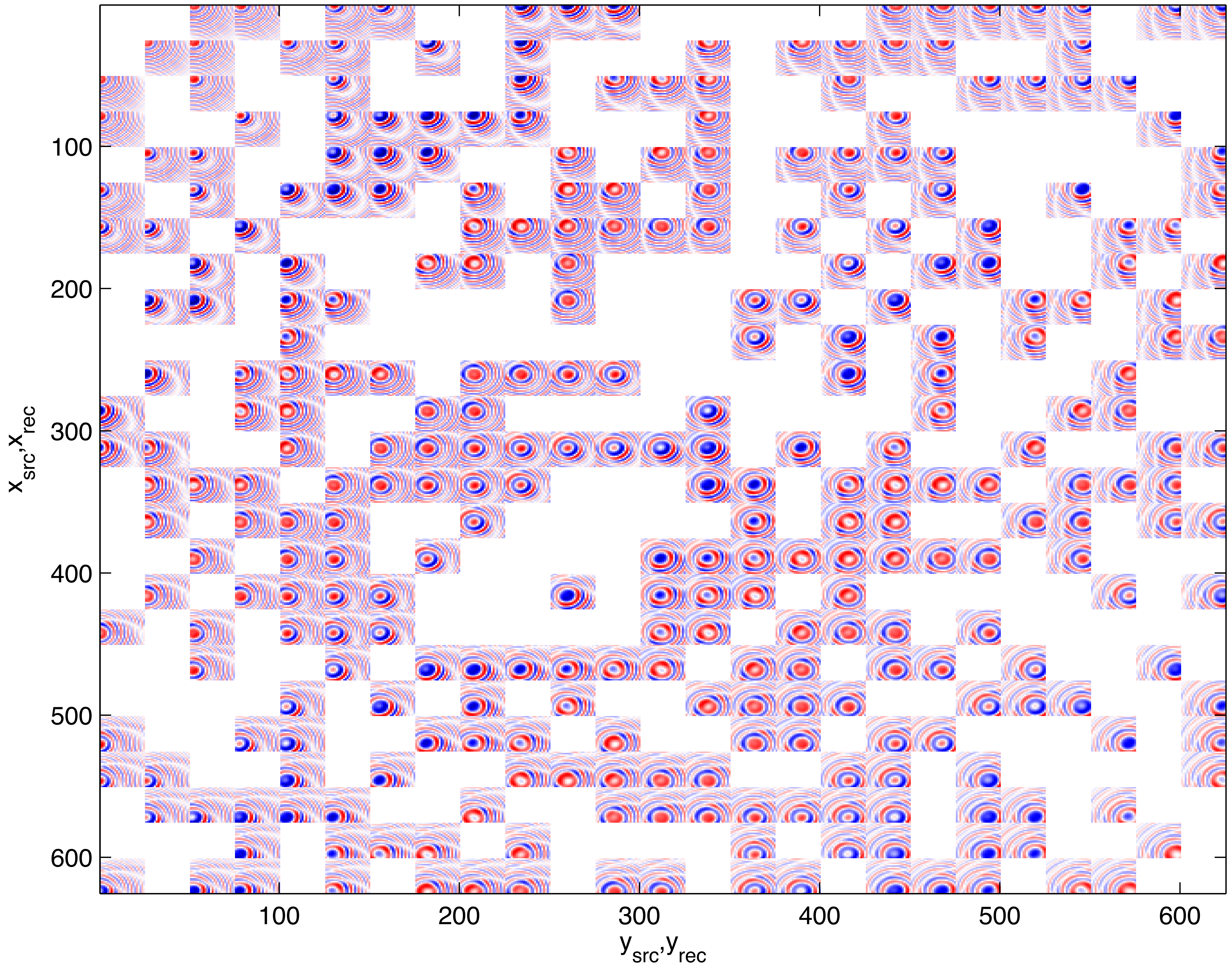


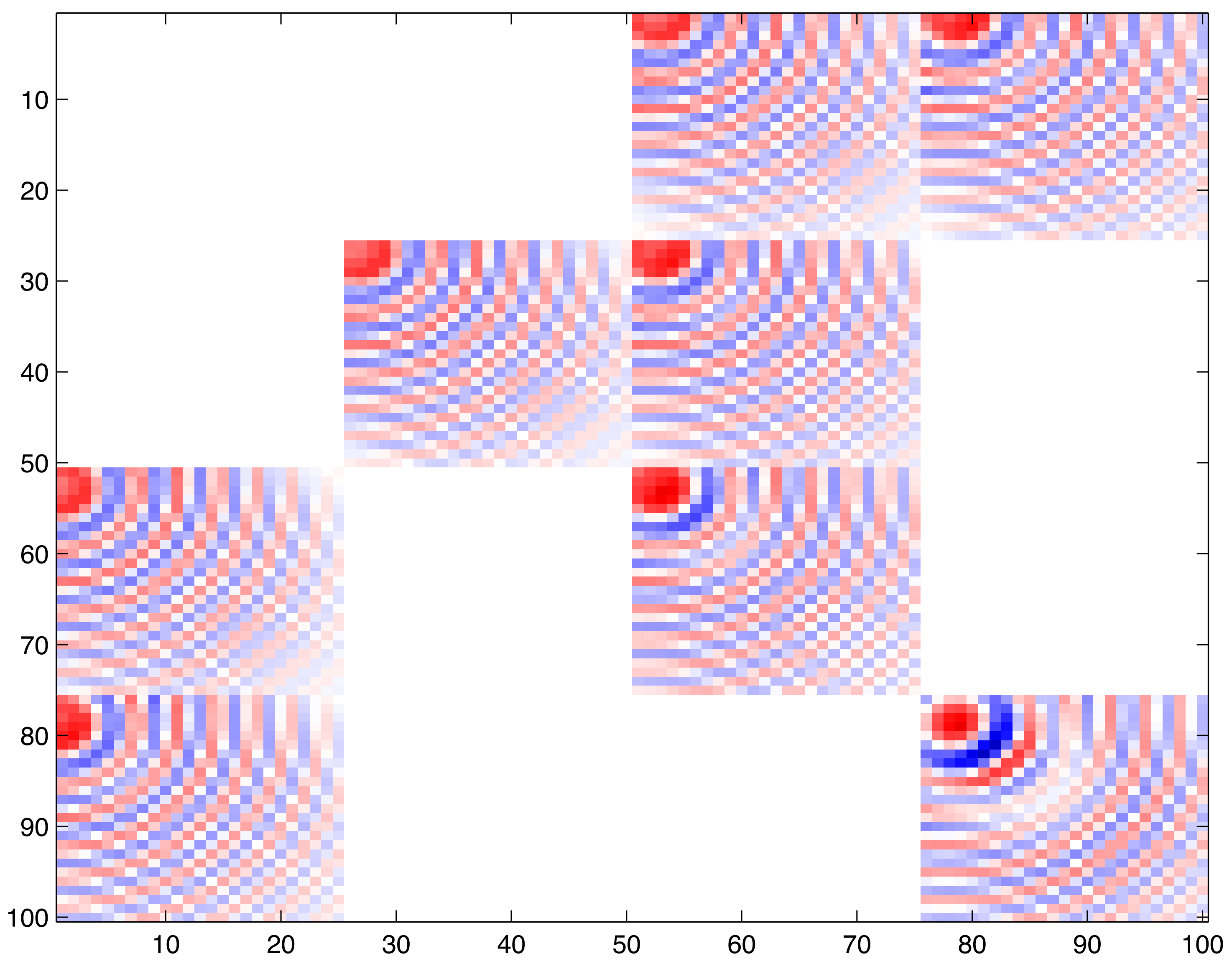


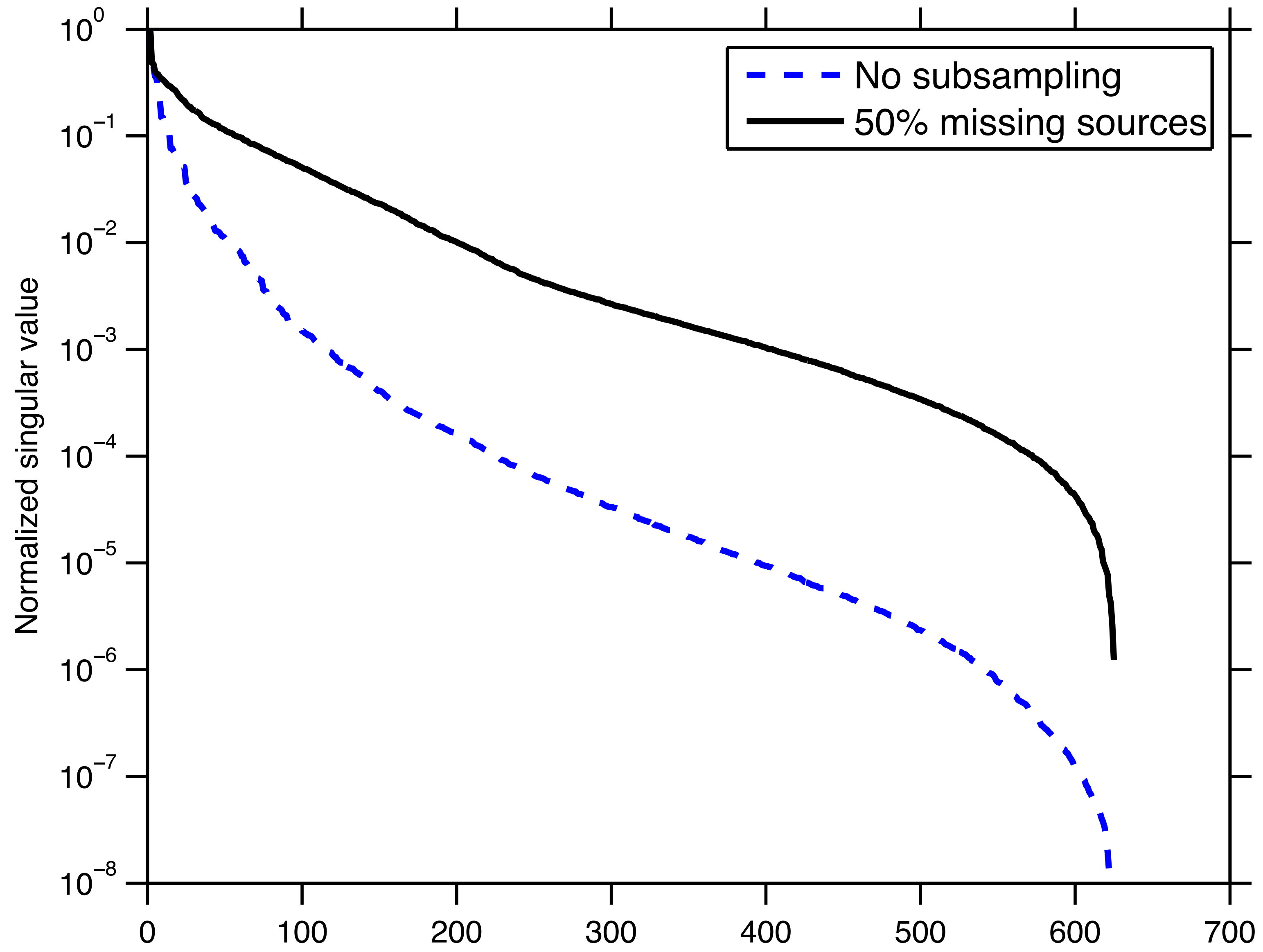


(Rec x, Rec y) matricization - Canonical ordering

Realistic recovery
50% random receivers removed







(Src x, Rec x) matricization - Noncanonical ordering

Data organization

(rec x, rec y) organization

- High rank
- Missing sources operator - removes columns
- Poor recovery scenario

(src x, rec x) organization

- Low rank
- Missing sources operator - removes blocks
- Closer to ideal recovery scenario

Interpolation with noise

Malfunctioning receivers

- unknown, malfunctioning receivers generating Gaussian noise

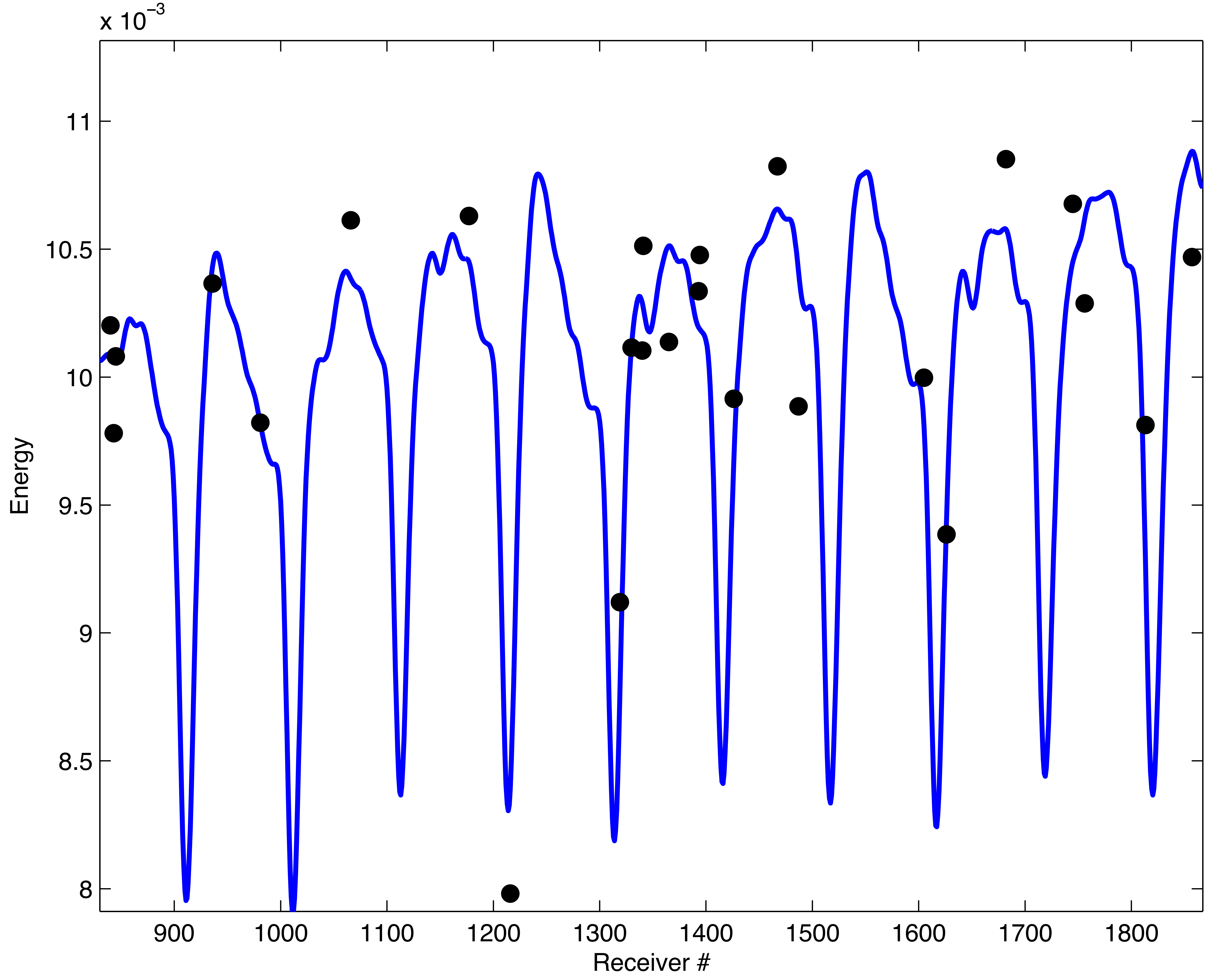
Low noise

- energy scaled to energy of removed receivers

High noise

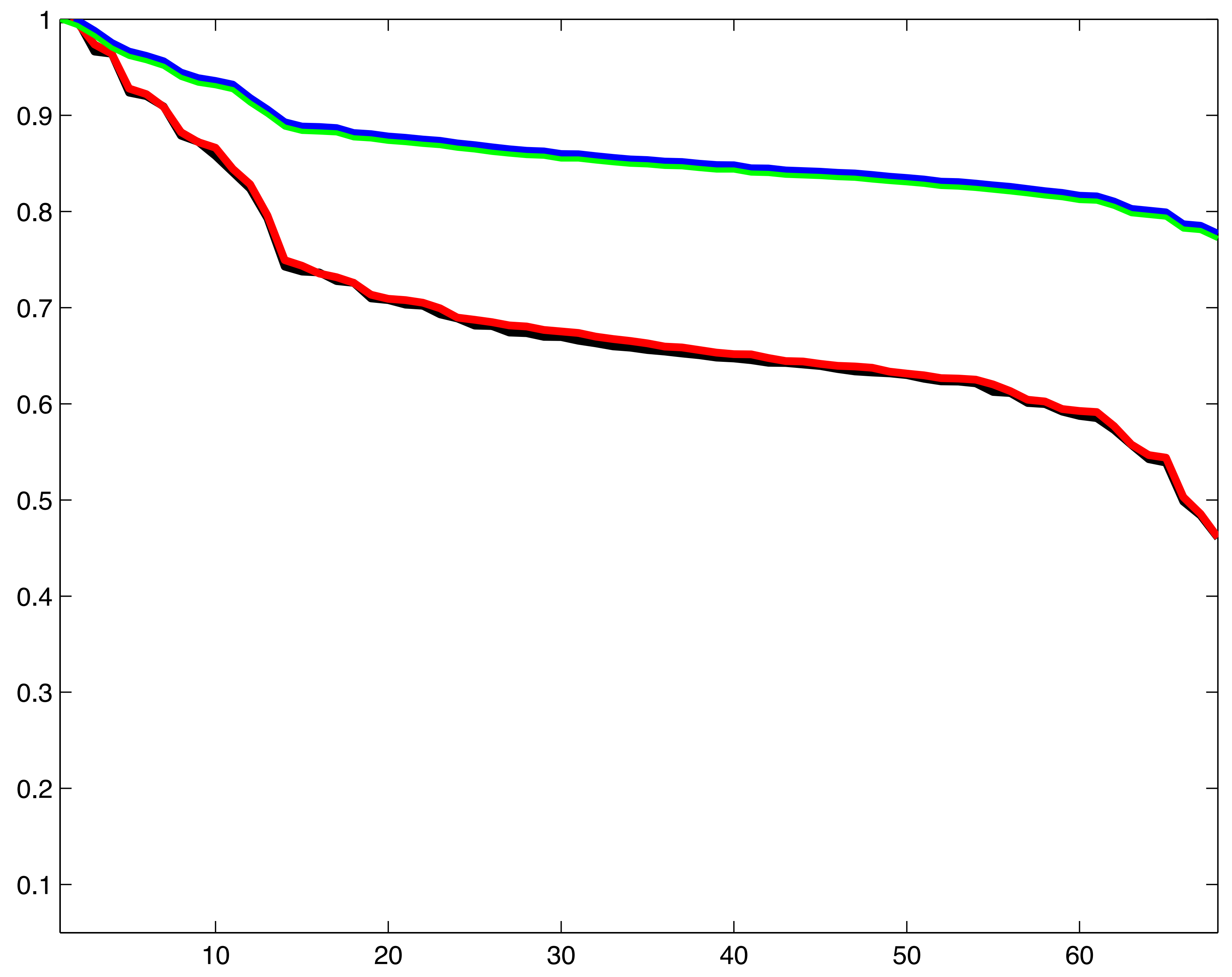
- total noise energy scaled to entire data energy

Receiver energy - low noise

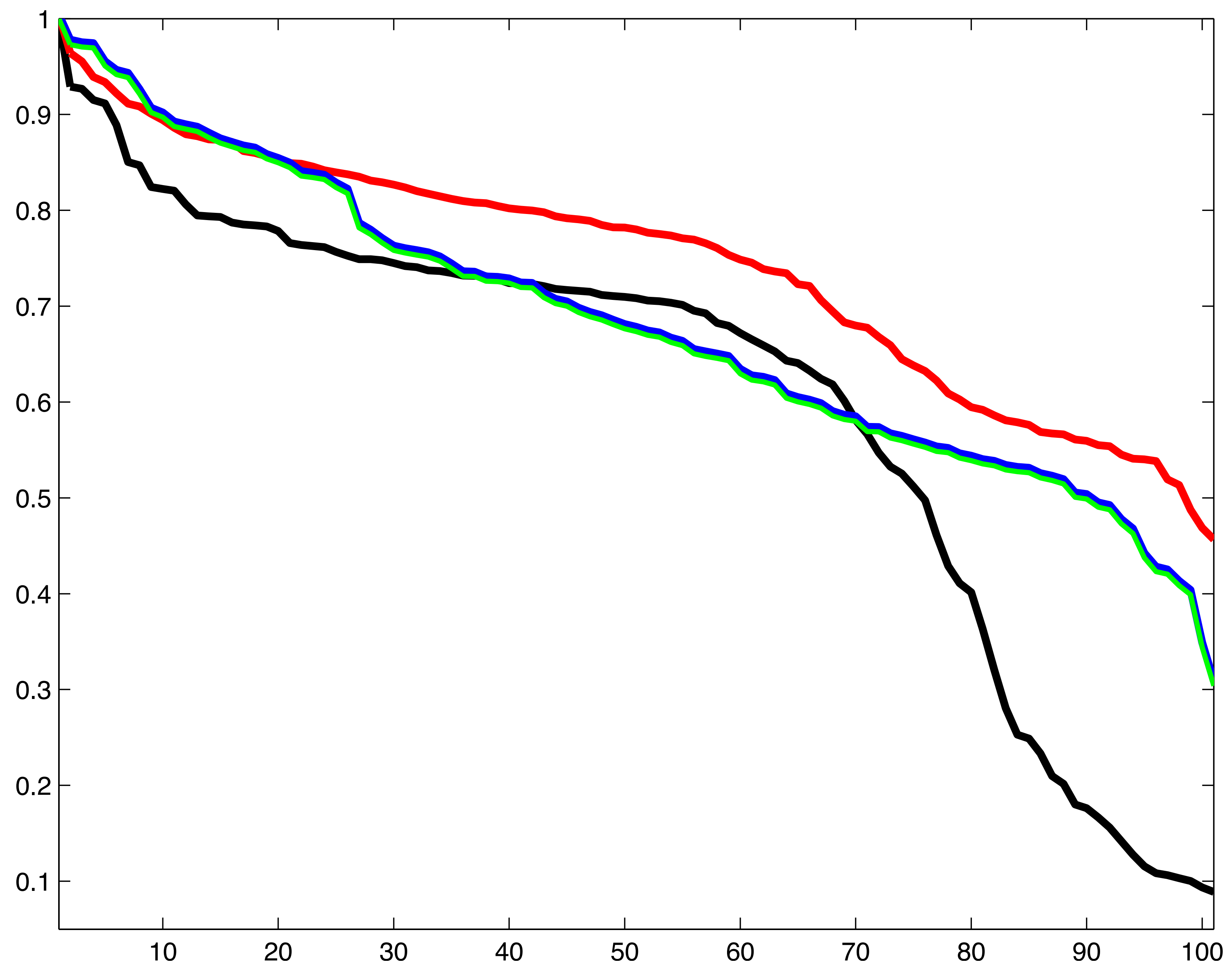


Interpolation with noise

Black - original
Red - subsampled
Blue - low noise
Green - high noise



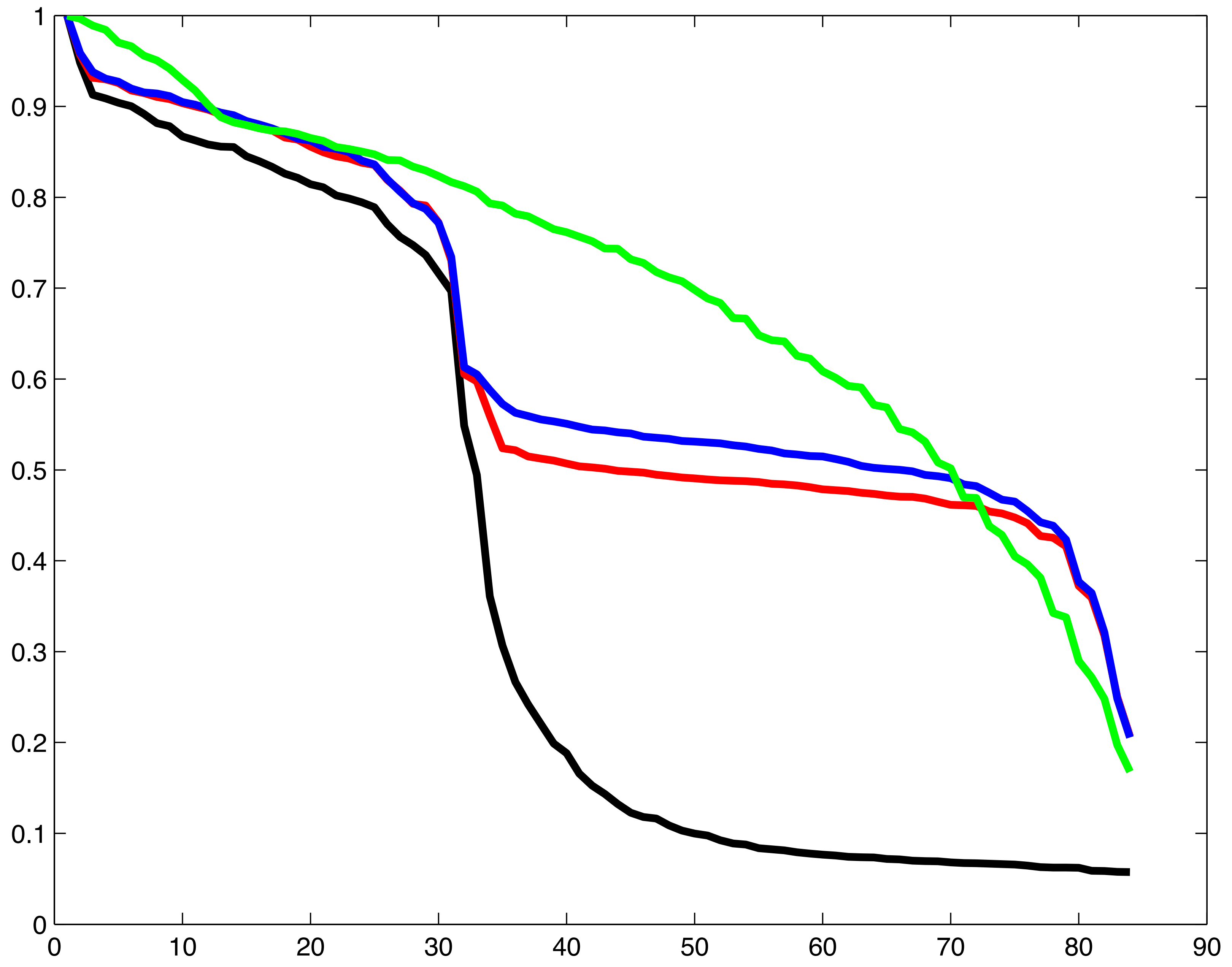
x_{src} singular values



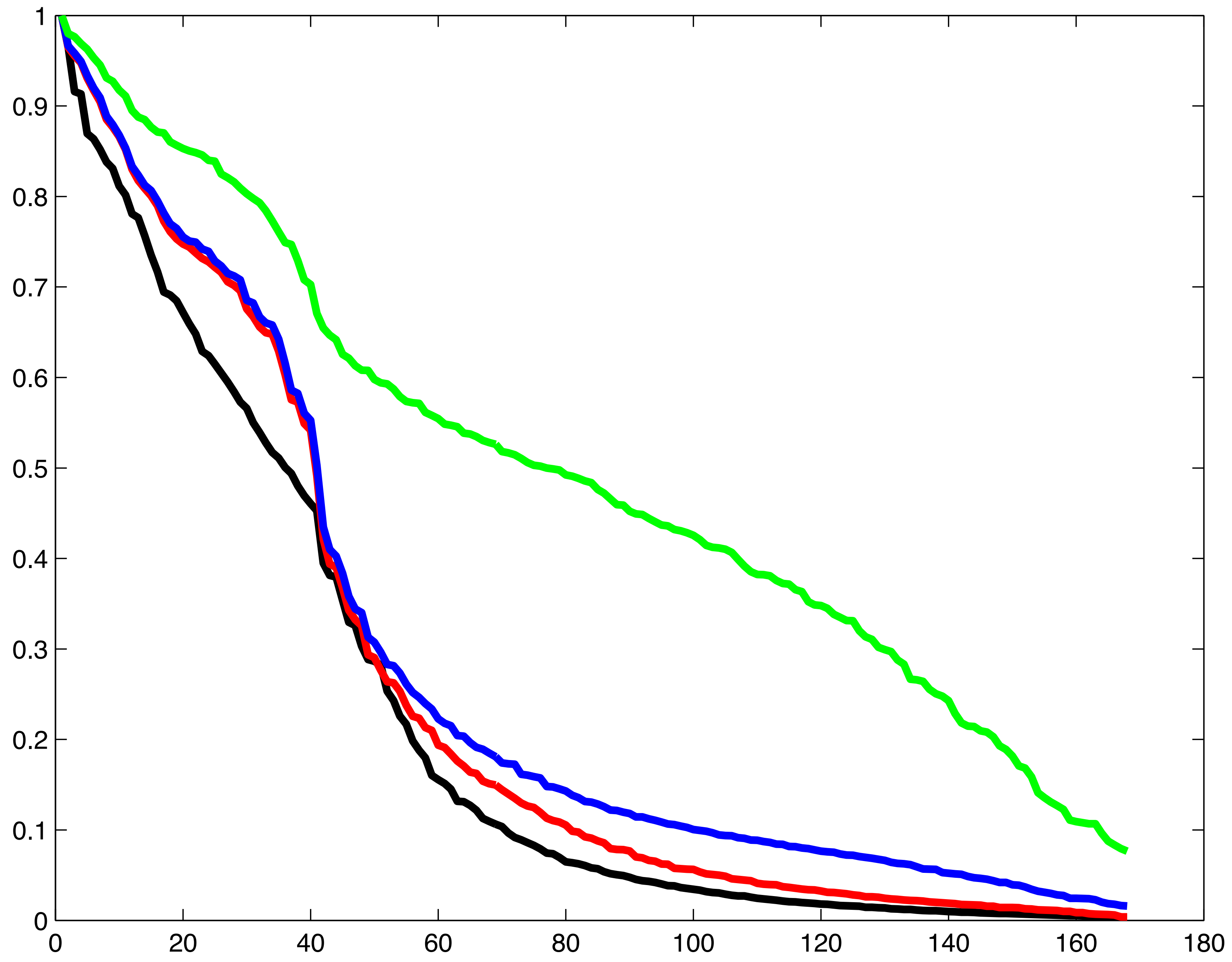
x_{rec} singular values

Interpolation with noise

Black - original
Red - subsampled
Blue - low noise
Green - high noise



$x_{midpoint}$ singular values



x_{offset} singular values

Interpolation with noise

Source-receiver domain

- subsampling ***increases*** the singular values in all dimensions
- noise ***does not change*** source-side singular values, *decreases* receiver-side singular values

Conclusion, *in this domain*

- Low-rank HT optimization ***will*** interpolate values in noiseless case
- Low-rank HT optimization ***cannot*** distinguish between noise & signal in noisy case

Interpolation with noise

Midpoint-offset domain

- subsampling ***increases*** the singular values in all dimensions
- noise ***increases*** the singular values in both the midpoint and offset dimensions

Conclusion, in this domain

- Low-rank HT optimization ***will*** interpolate values in noiseless case
- Low-rank HT optimization ***will*** interpolate and distinguish between noise and signal

Multidimensional interpolation
with Hierarchical Tucker

Successful reconstruction scheme

Signal structure

- Hierarchical Tucker

Sampling

- subsampling, noise increases hierarchical rank

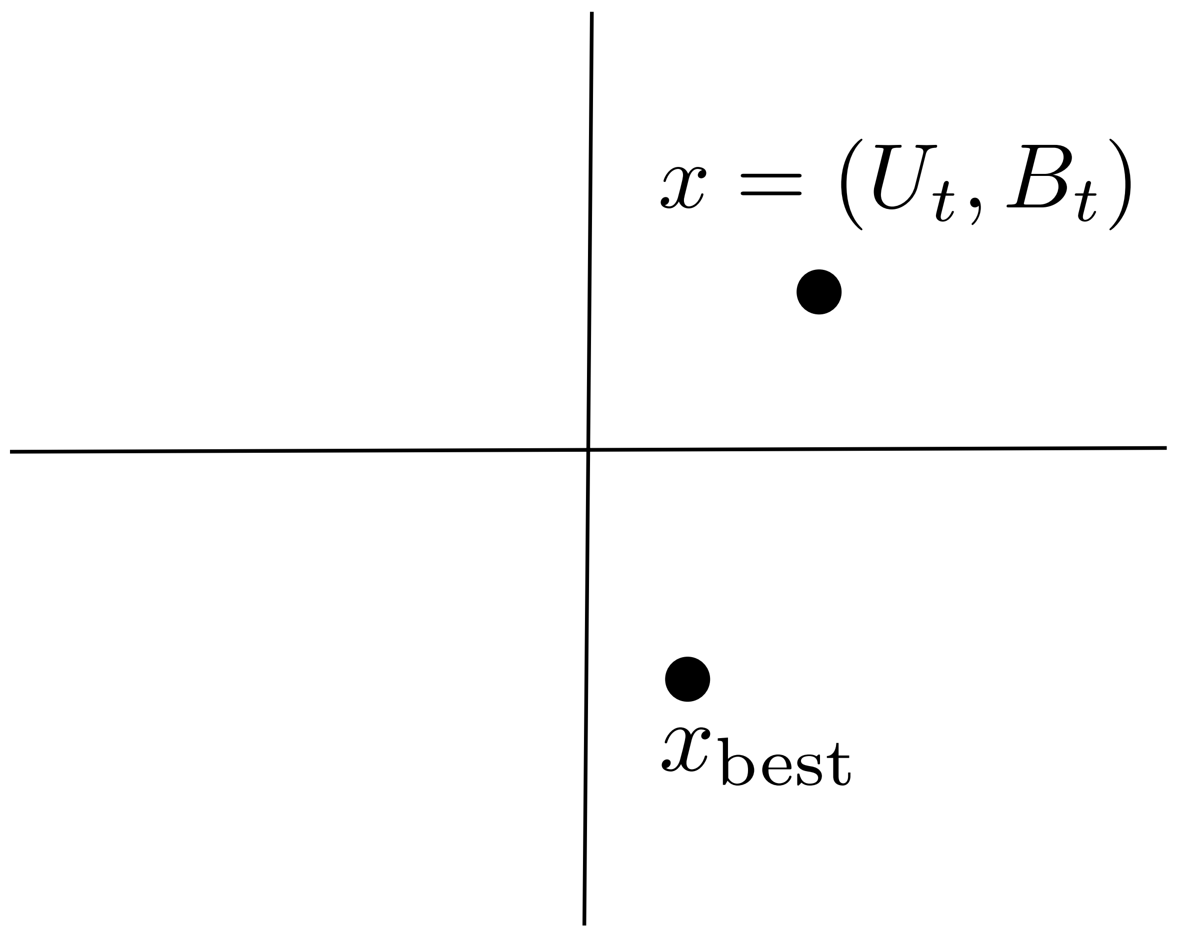
Optimization

- ***fit data in the Hierarchical Tucker format***

Optimization program

$\mathbb{C}^D$

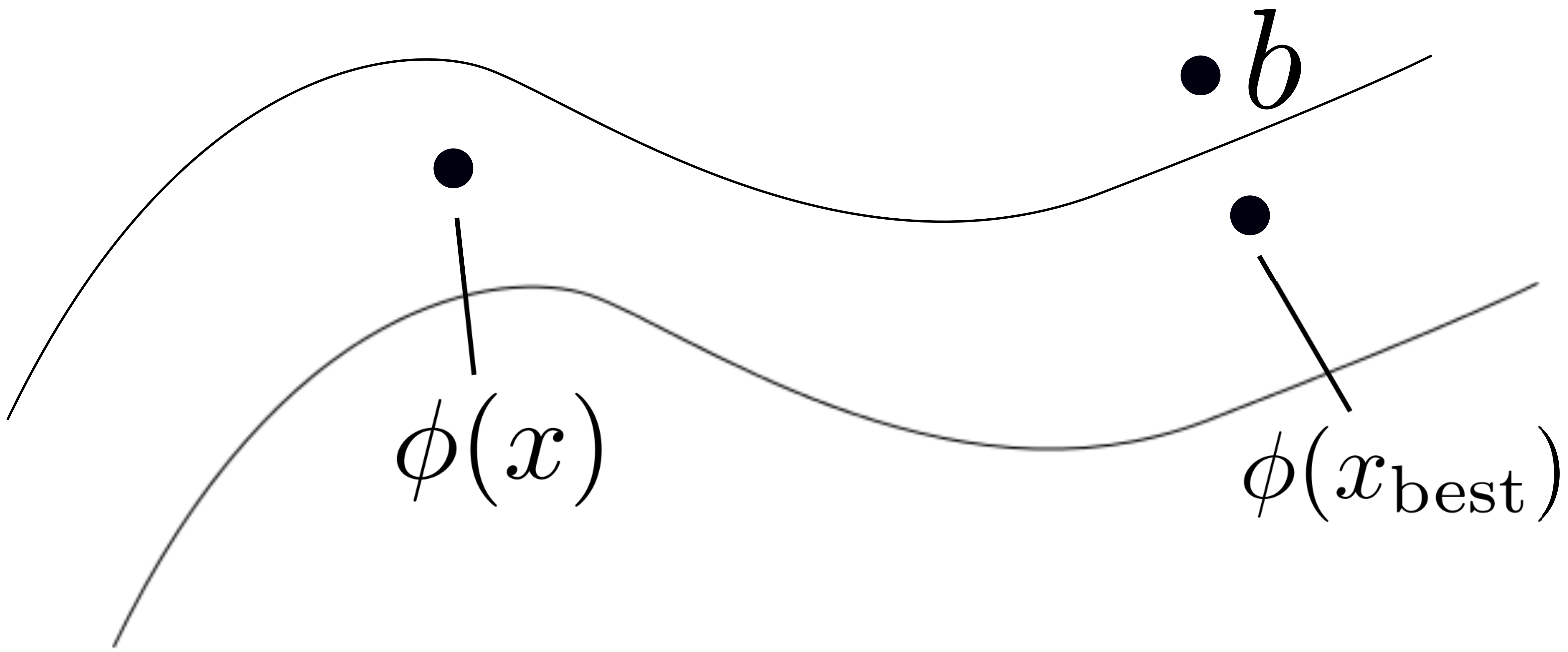
$\phi(x)$



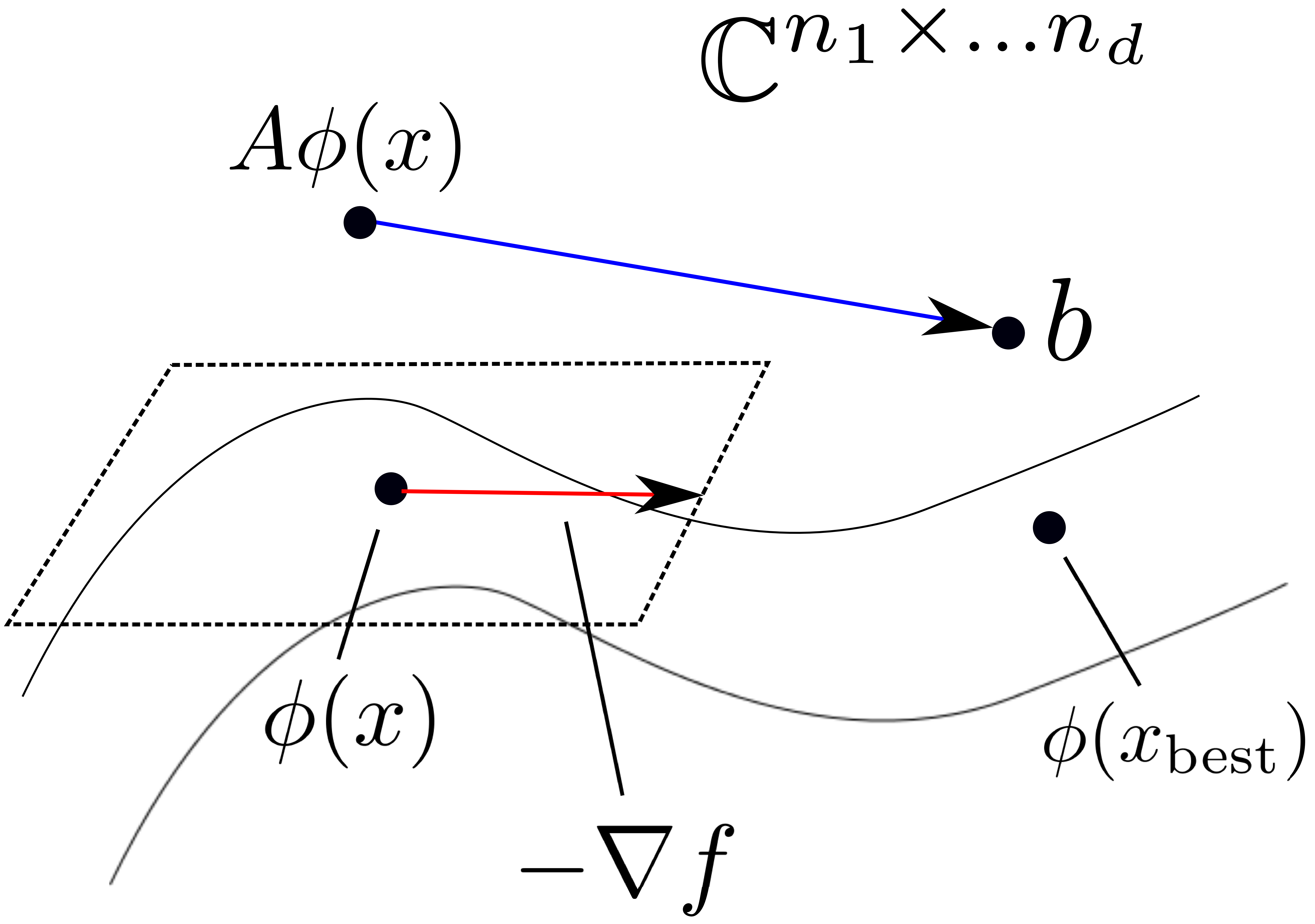
Parameter space

Full-tensor space

$\mathbb{C}^{n_1 \times \dots n_d}$



Optimization program



C. Da Silva and F. J. Herrmann, *Optimization on the Hierarchical Tucker manifold - applications to tensor completion*, 2014

Derivatives

Only involves matrix-matrix multiplications of small matrices compared to the full-tensor space

Parallelizable - multilinear product can be done in parallel

SVD-free - no large-scale SVDs, unlike nuclear norm-based methods

Results

Synthetic BG Group data

Unknown model

- 68 x 68 sources with 401 x 401 receivers, data at 7.34 Hz

Receivers subsampled to 101 x 101

Recovered with Gauss-Newton

Noise

Removed 50% of receivers randomly

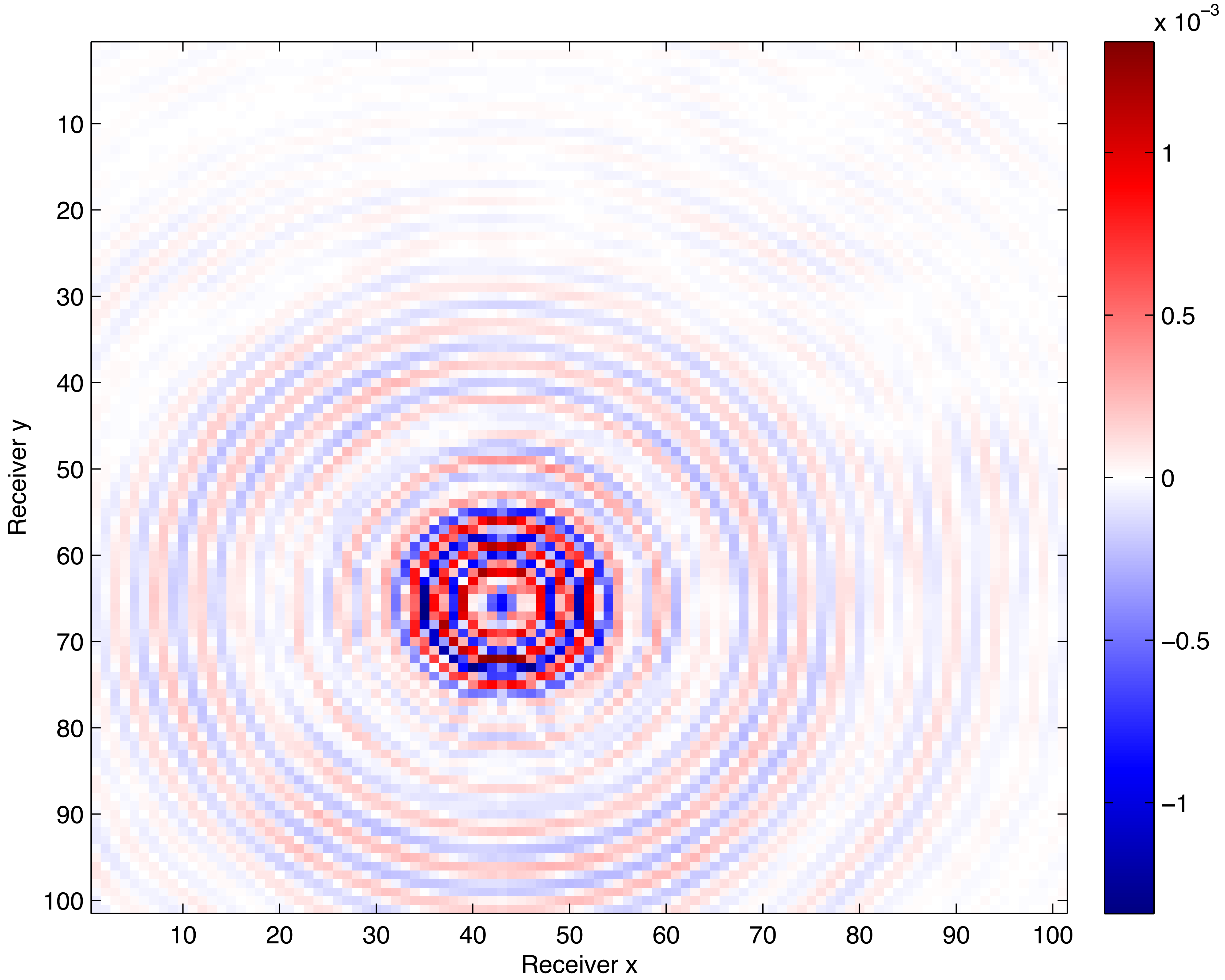
5% of remaining receivers replaced with random Gaussian noise

Low noise - energy scaled to energy of removed receivers

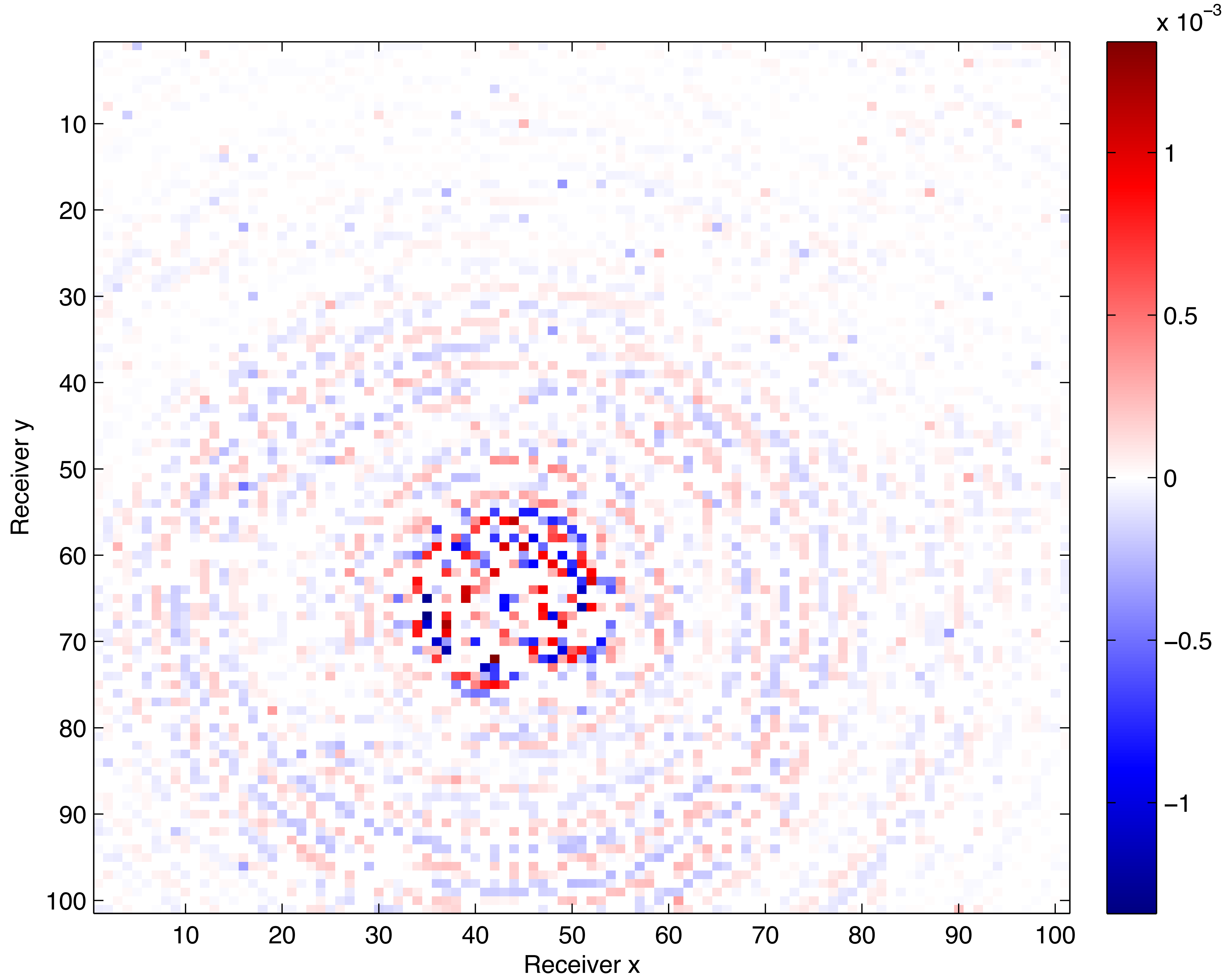
High noise - total noise energy scaled to entire data energy

7.34 Hz - 50% missing receivers - low noise

Common source gather



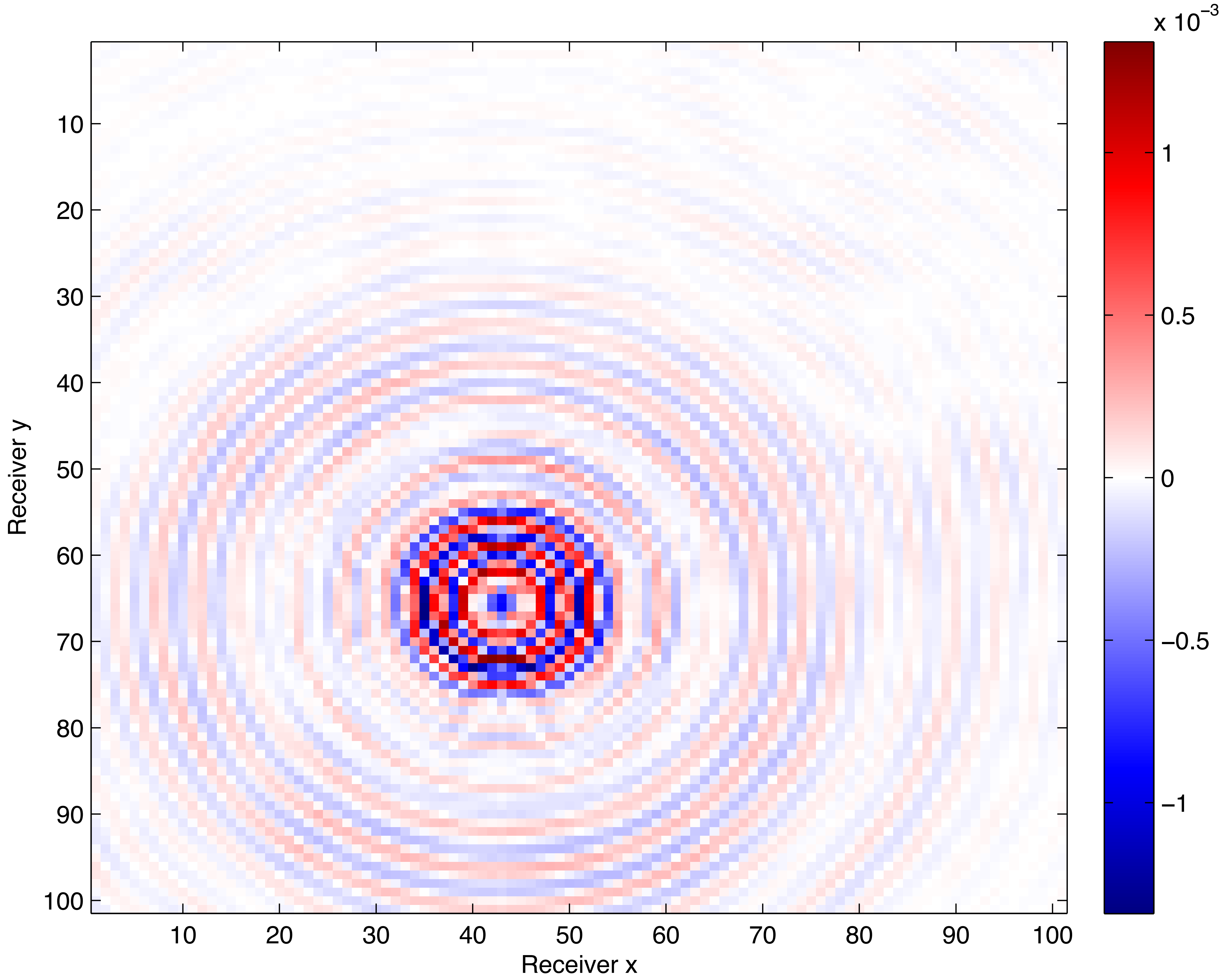
True data



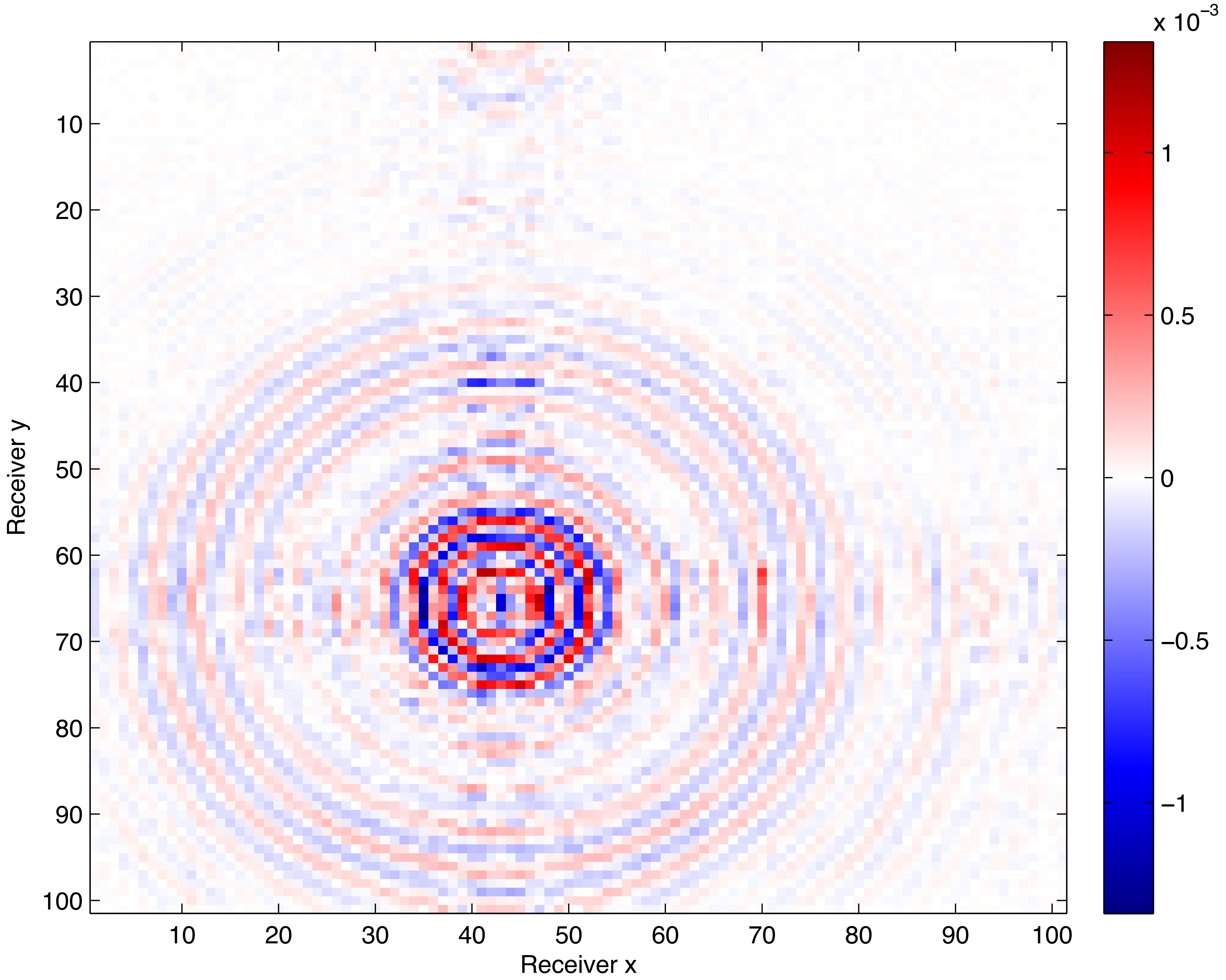
Subsampled data

7.34 Hz - 50% missing receivers - low noise

Common source gather



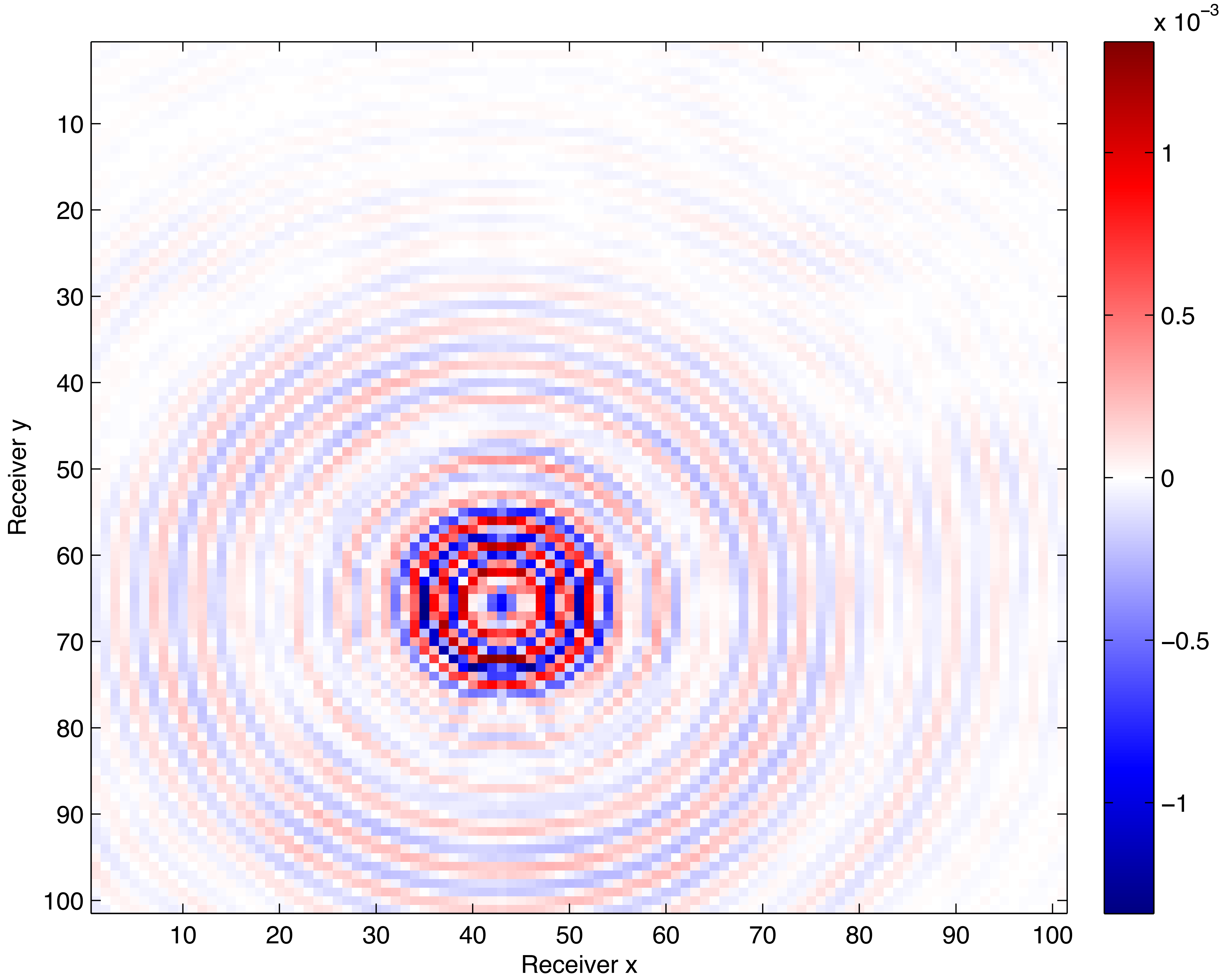
True data



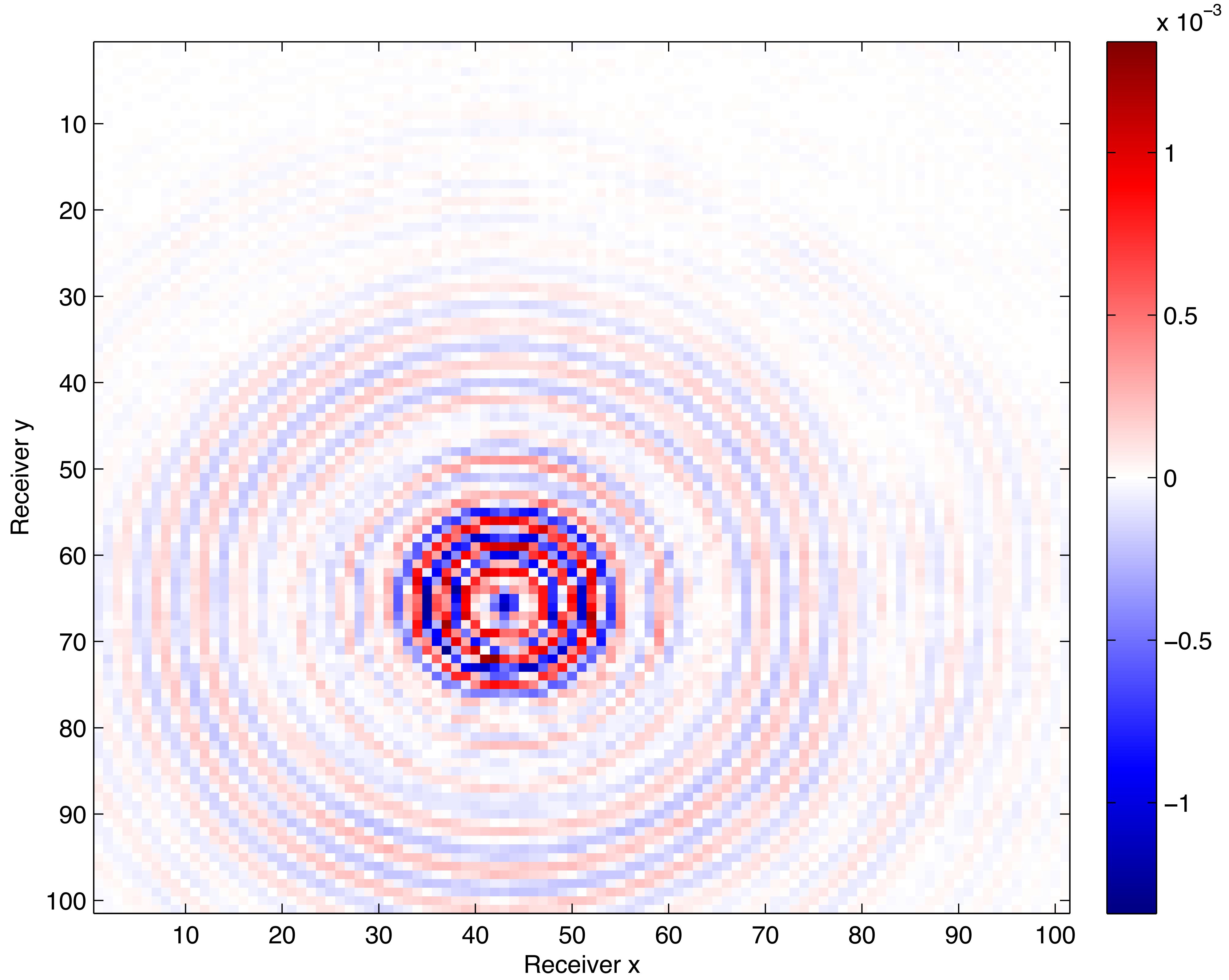
SR Recovery - SNR 7.8 dB

7.34 Hz - 50% missing receivers - low noise

Common source gather



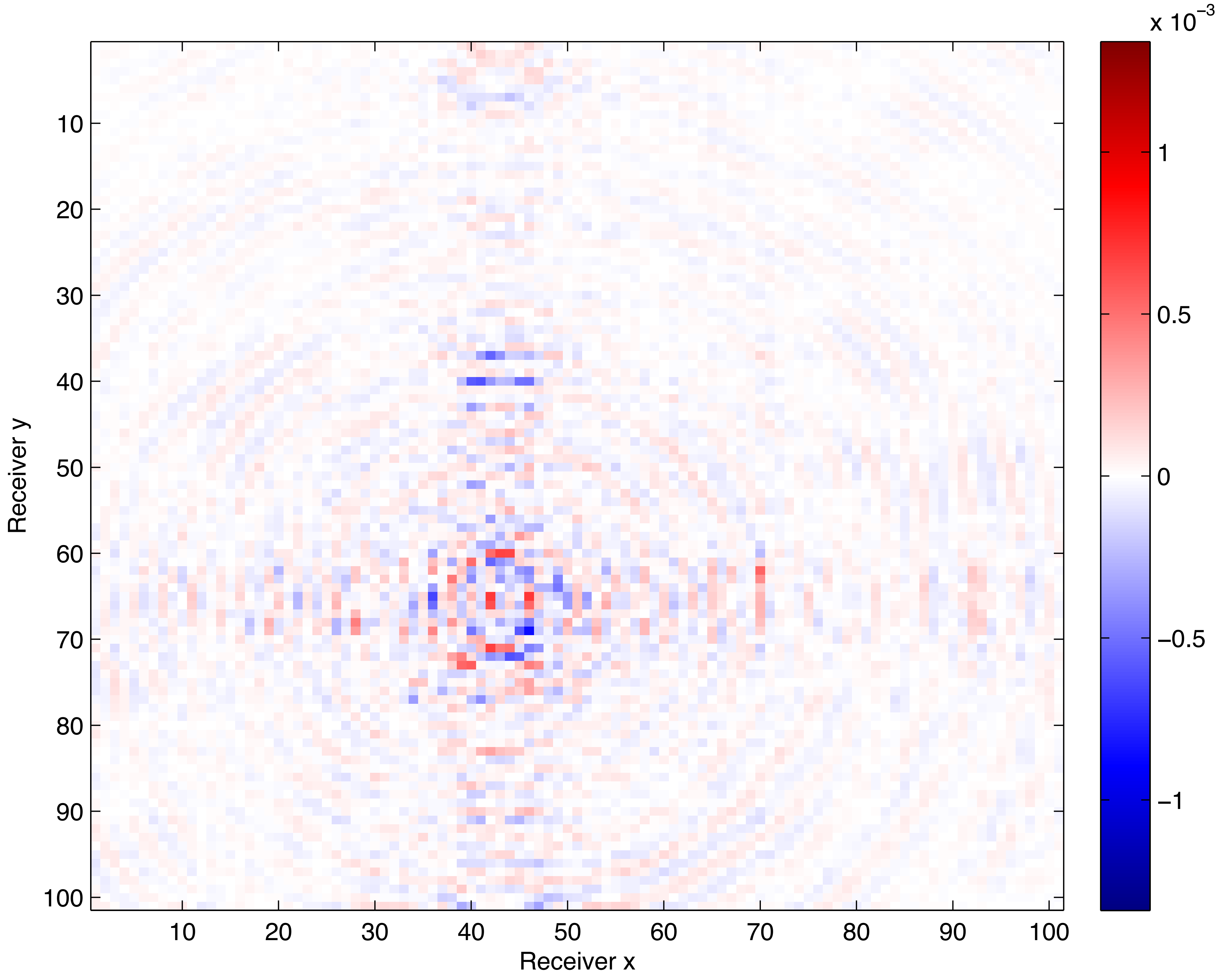
True data



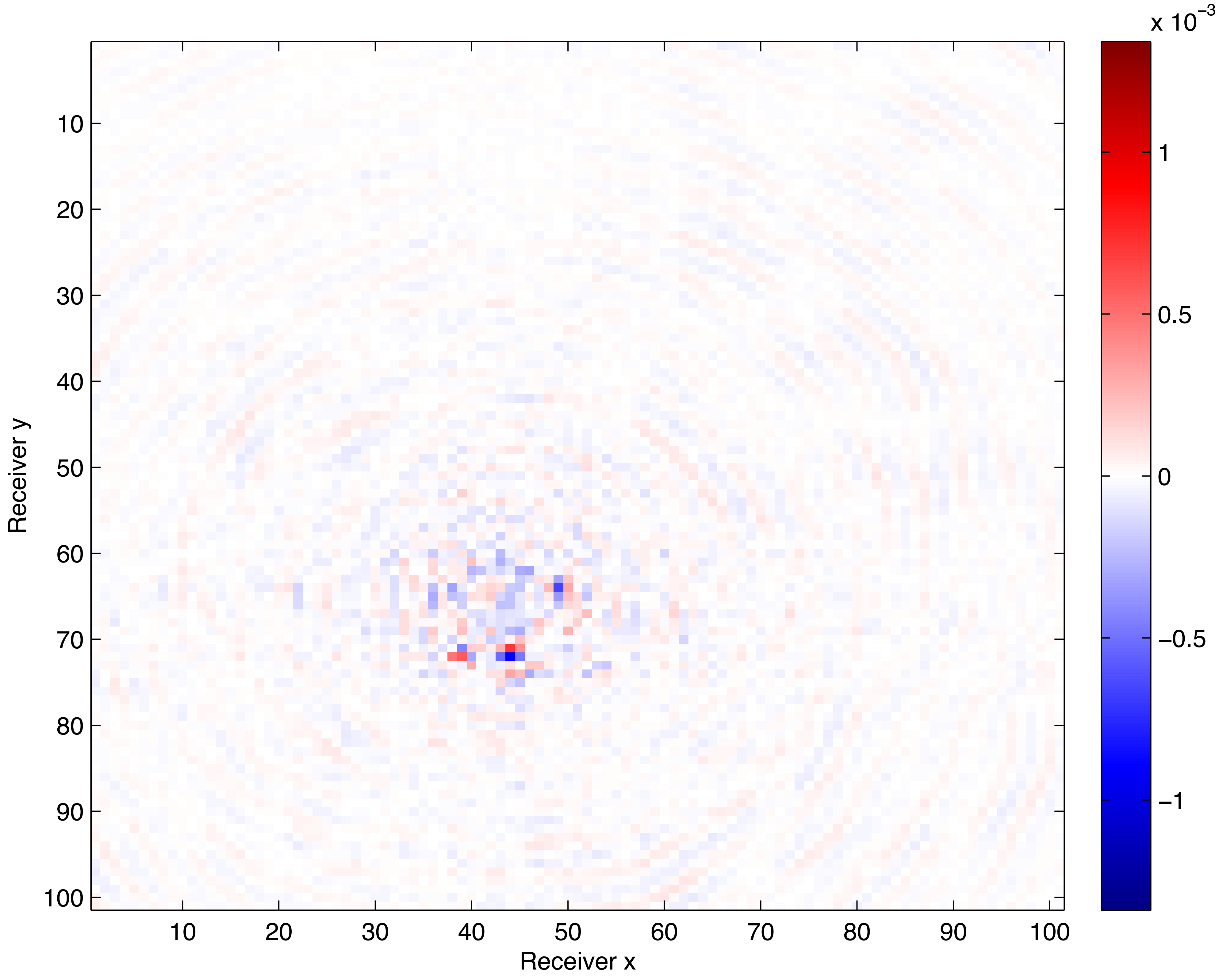
MH Recovery - SNR 12.6 dB

7.34 Hz - 50% missing receivers - low noise

Common source gather



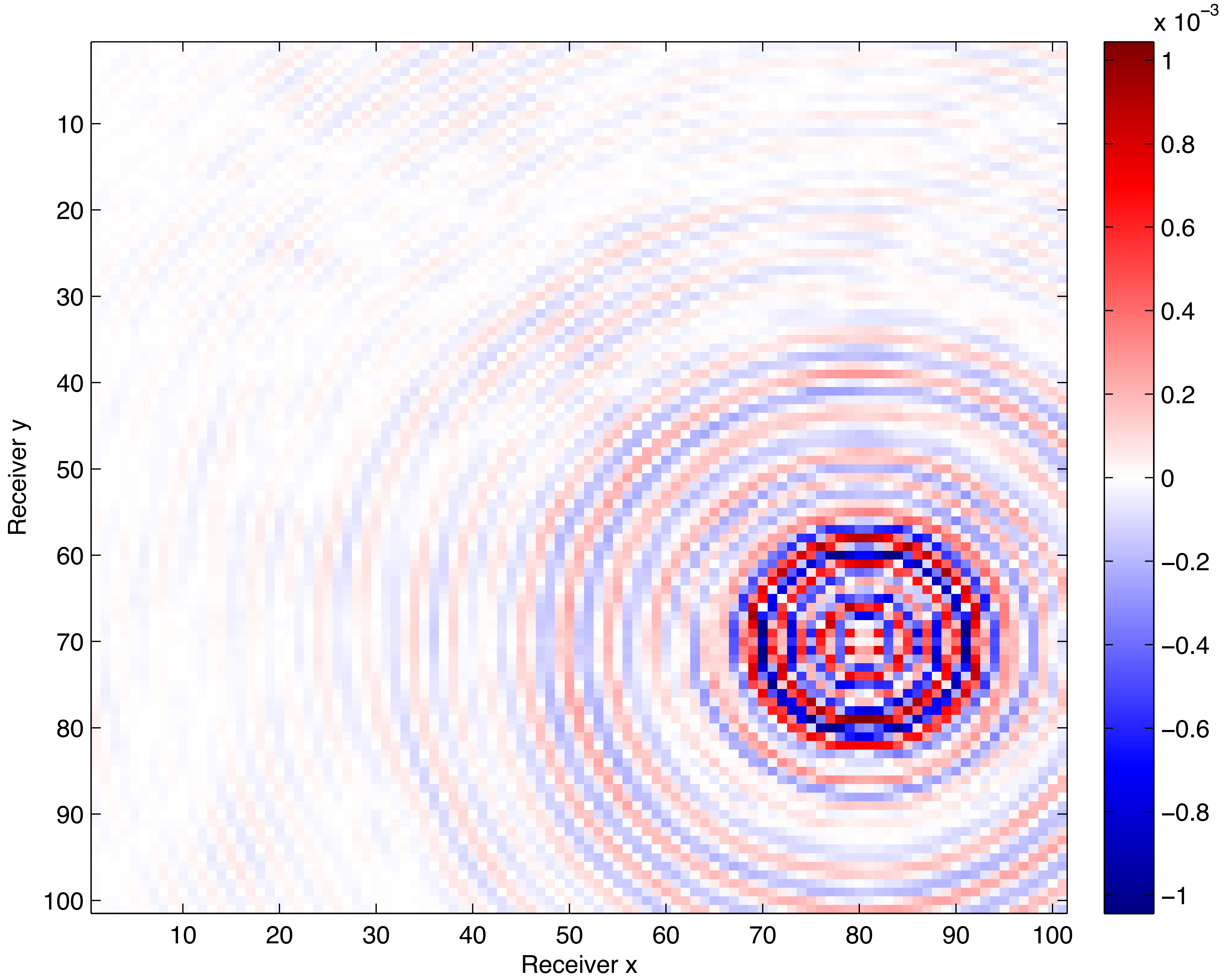
SR Difference



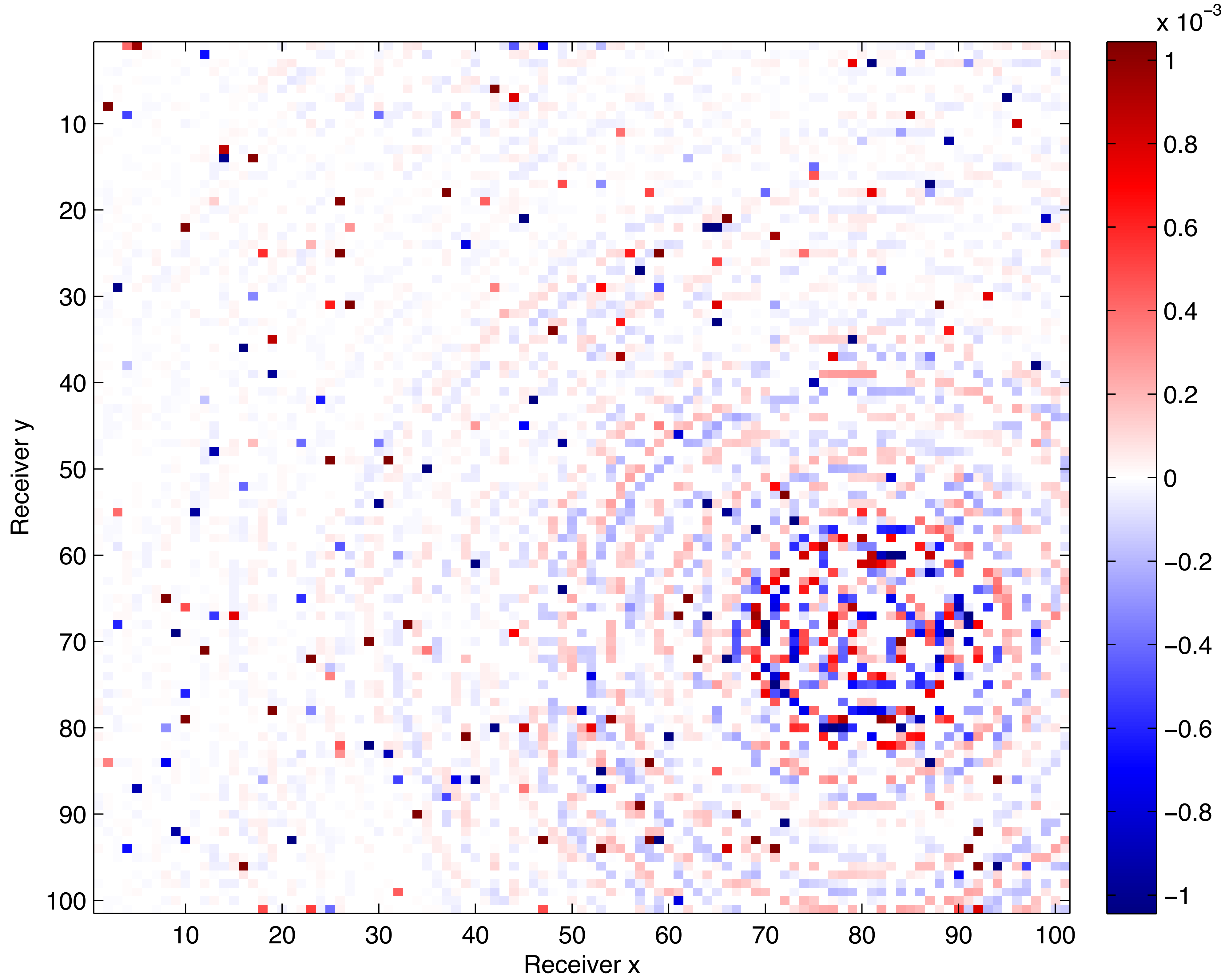
MH Difference

7.34 Hz - 50% missing receivers - high noise

Common source gather



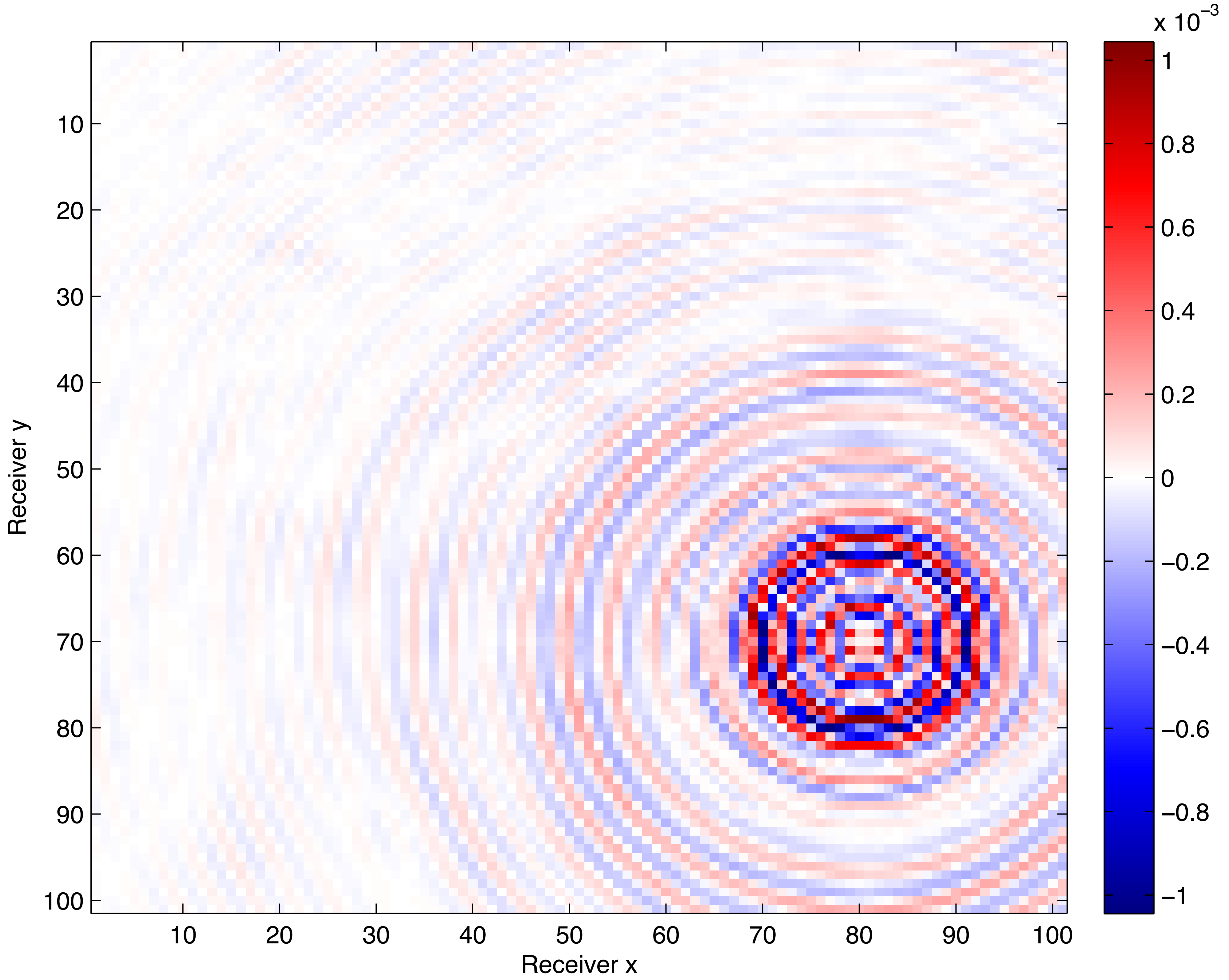
True data



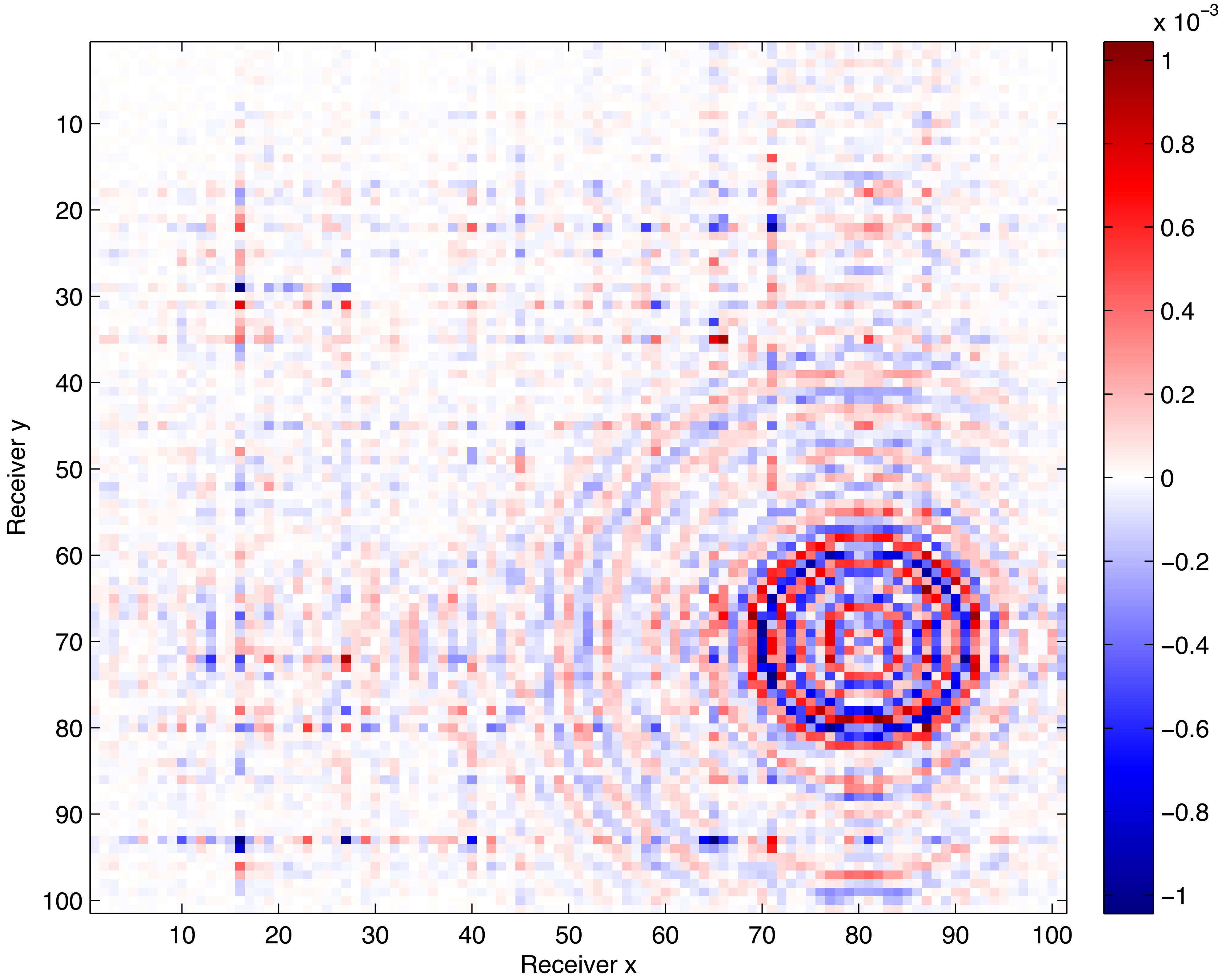
Subsampled data

7.34 Hz - 50% missing receivers - high noise

Common source gather



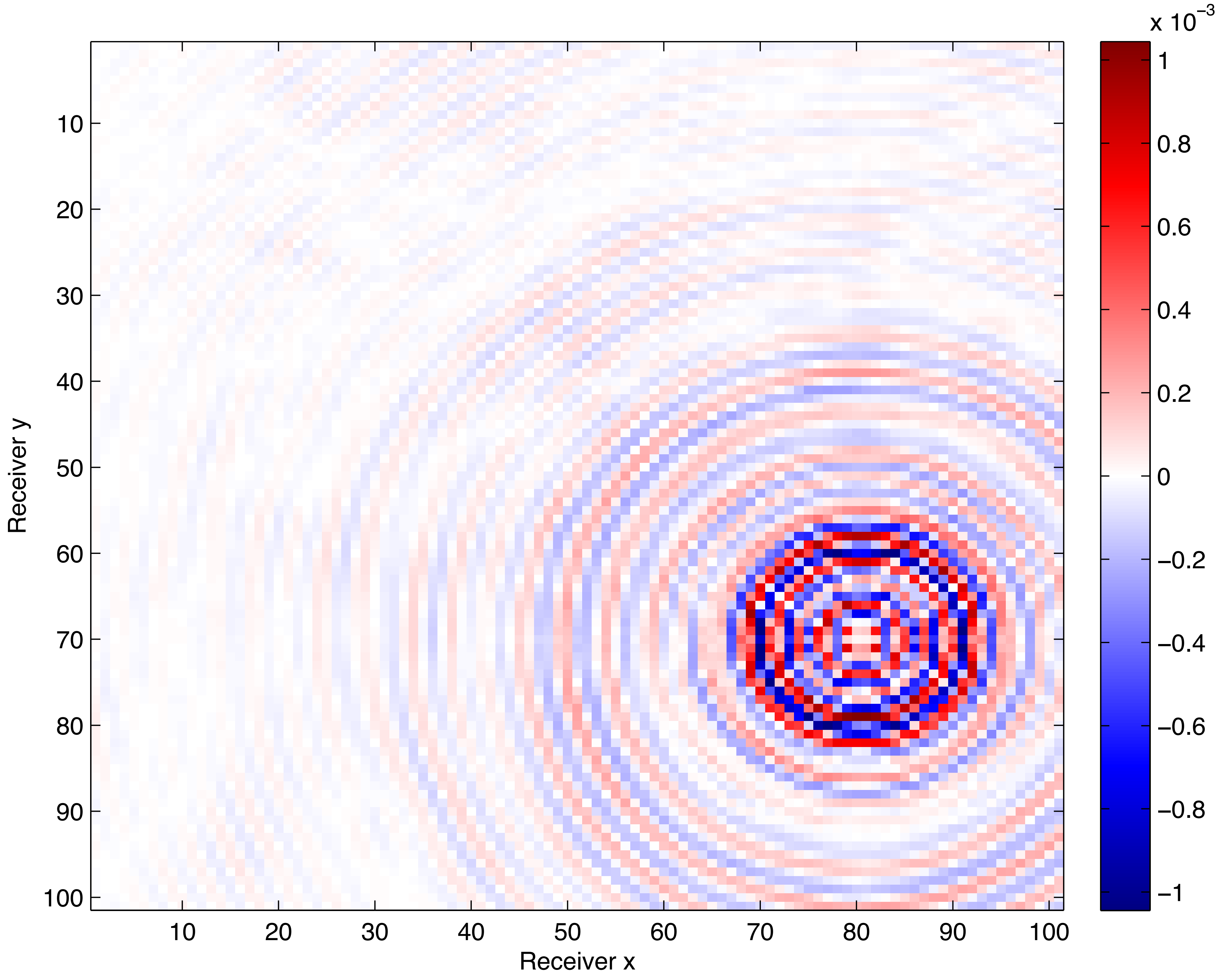
True data



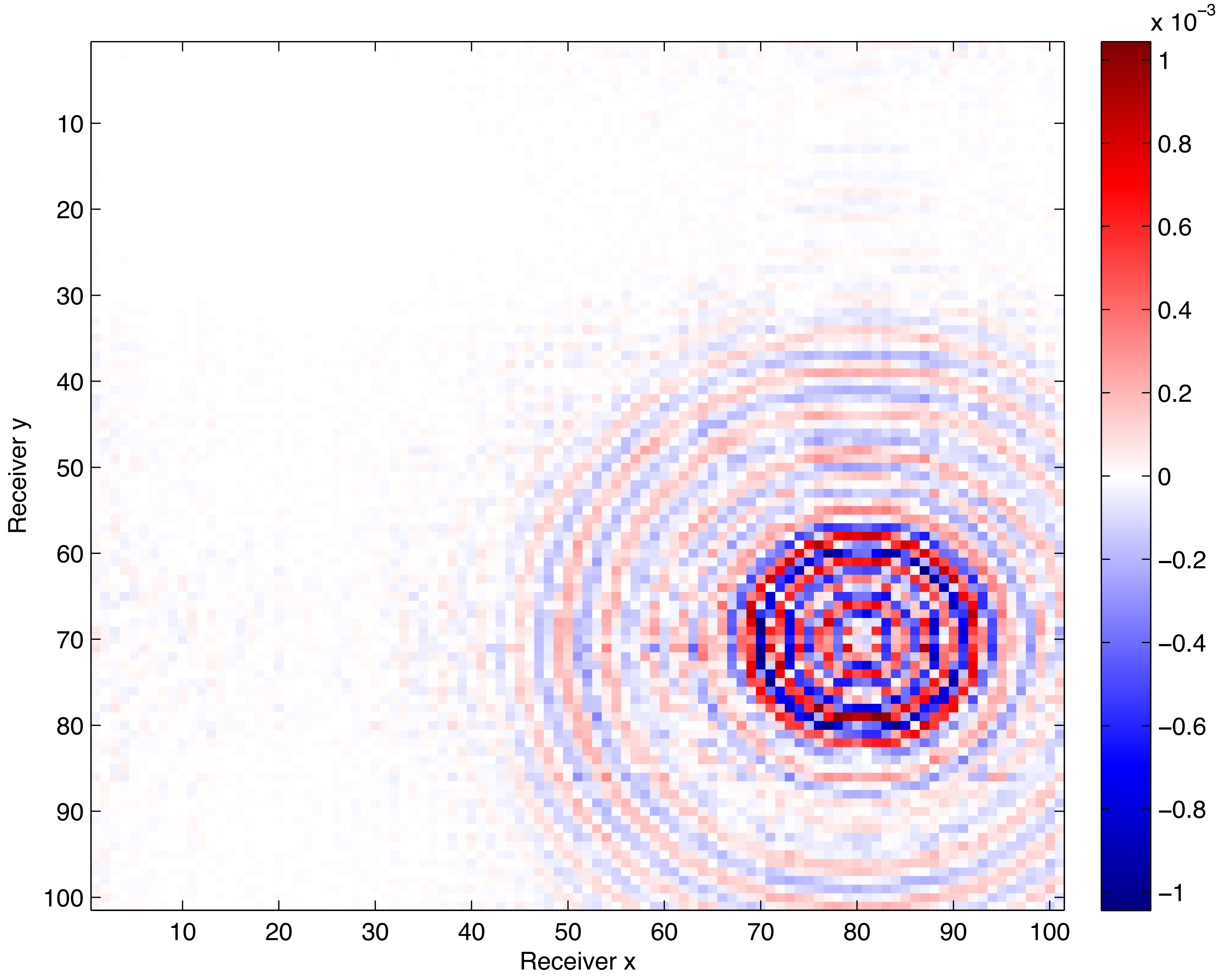
SR Recovery - SNR 3.05 dB

7.34 Hz - 50% missing receivers - high noise

Common source gather



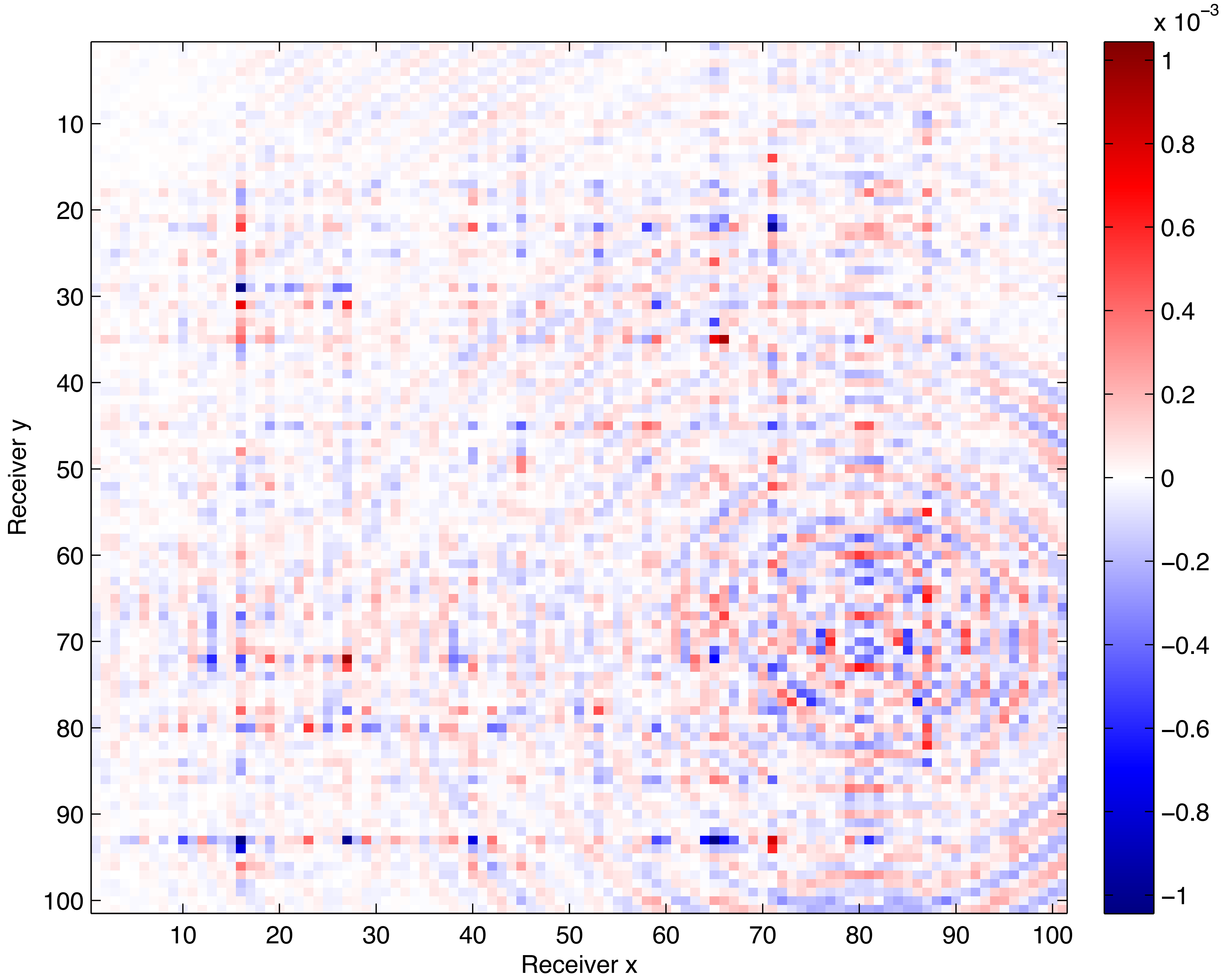
True data



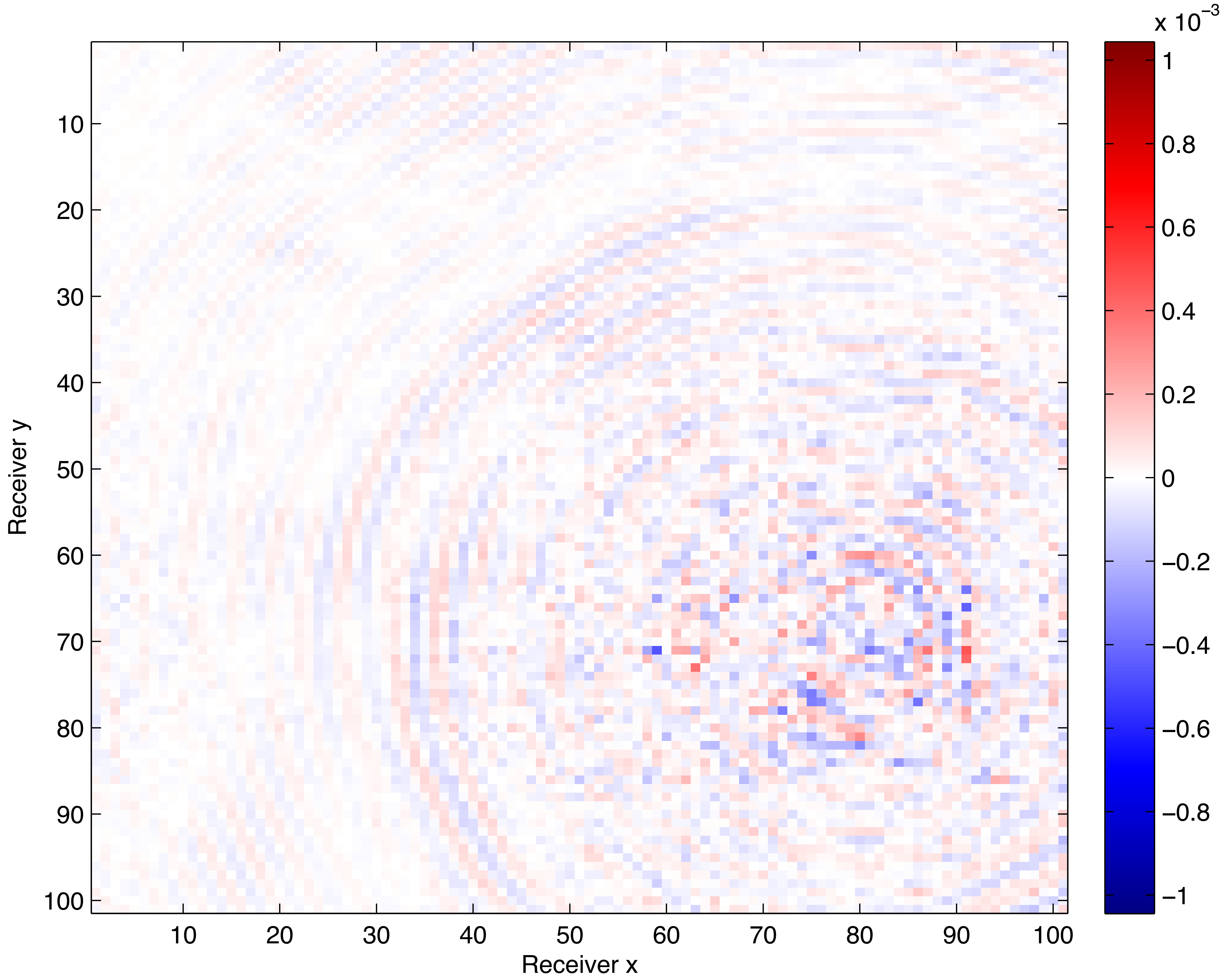
MH Recovery - SNR 8.95 dB

7.34 Hz - 50% missing receivers - high noise

Common source gather

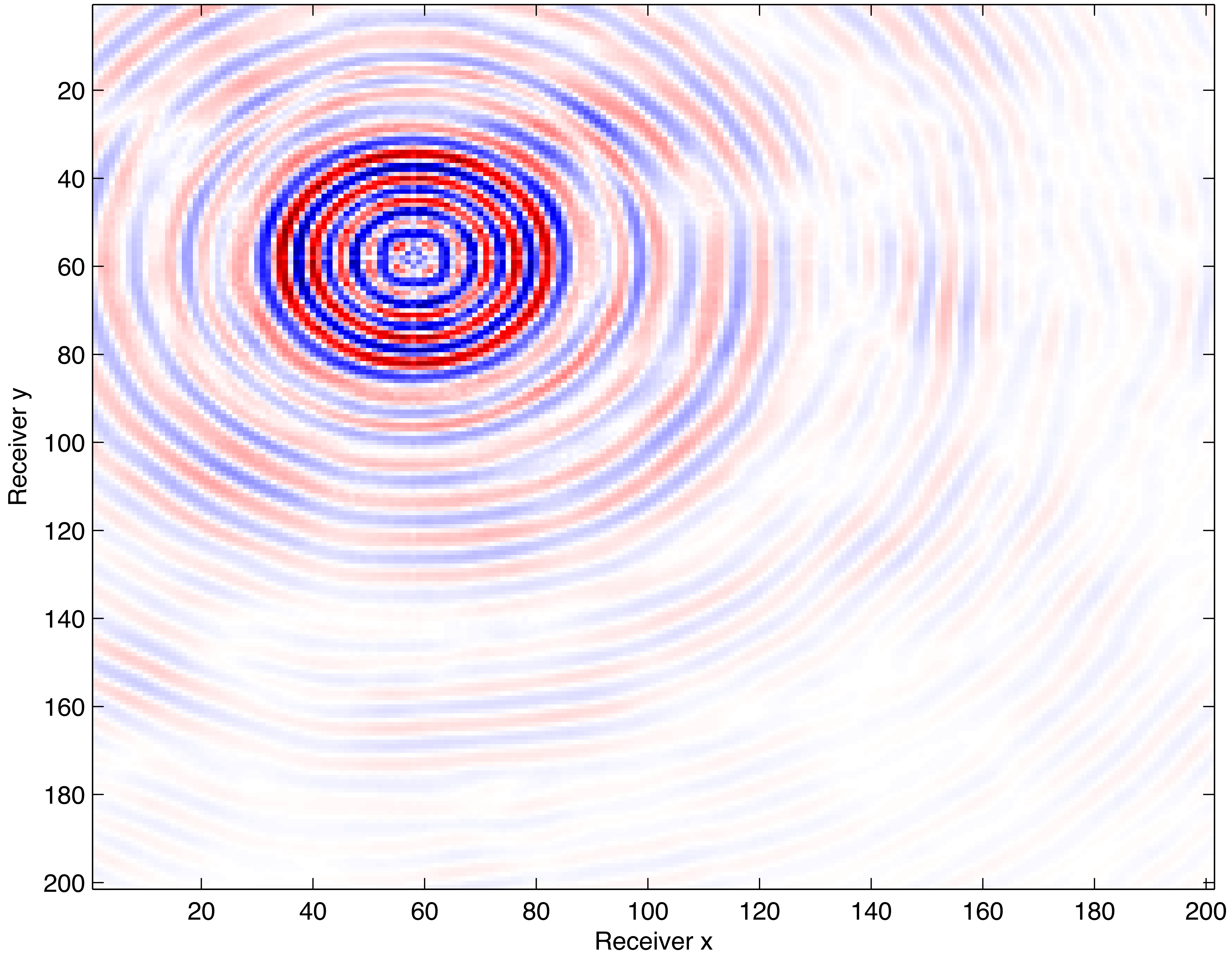


SR Difference

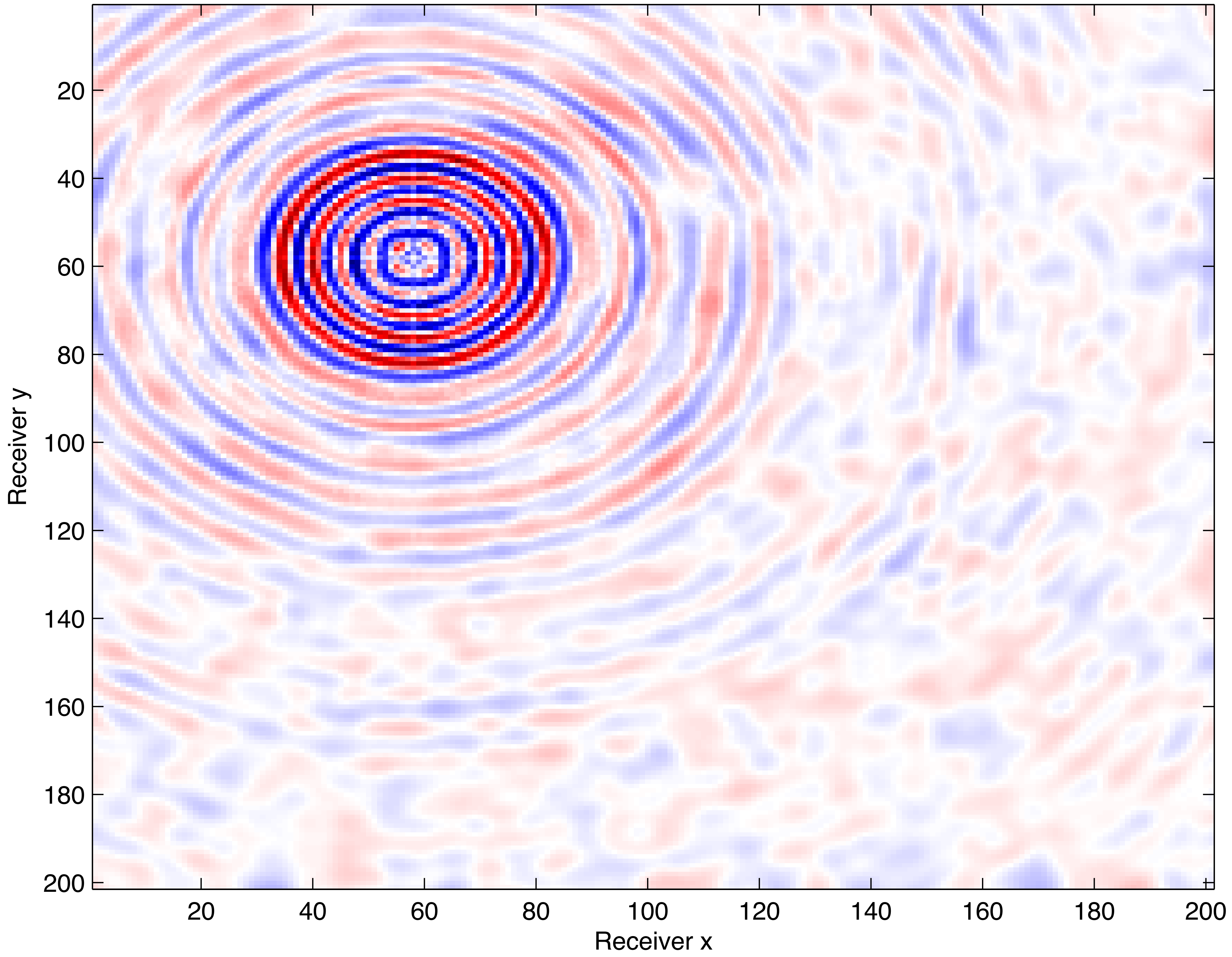


MH Difference

7.34 Hz - Denoising

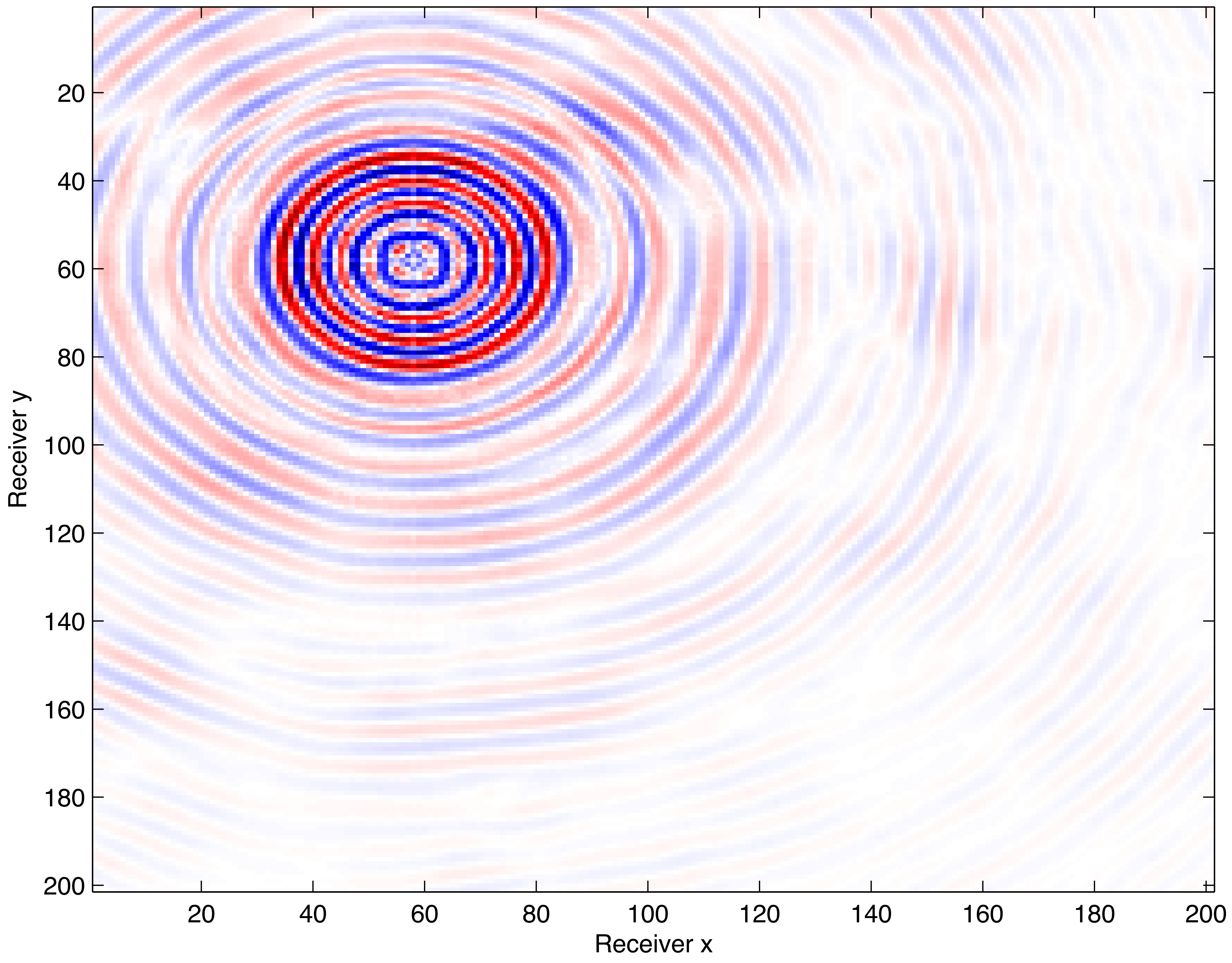


True data

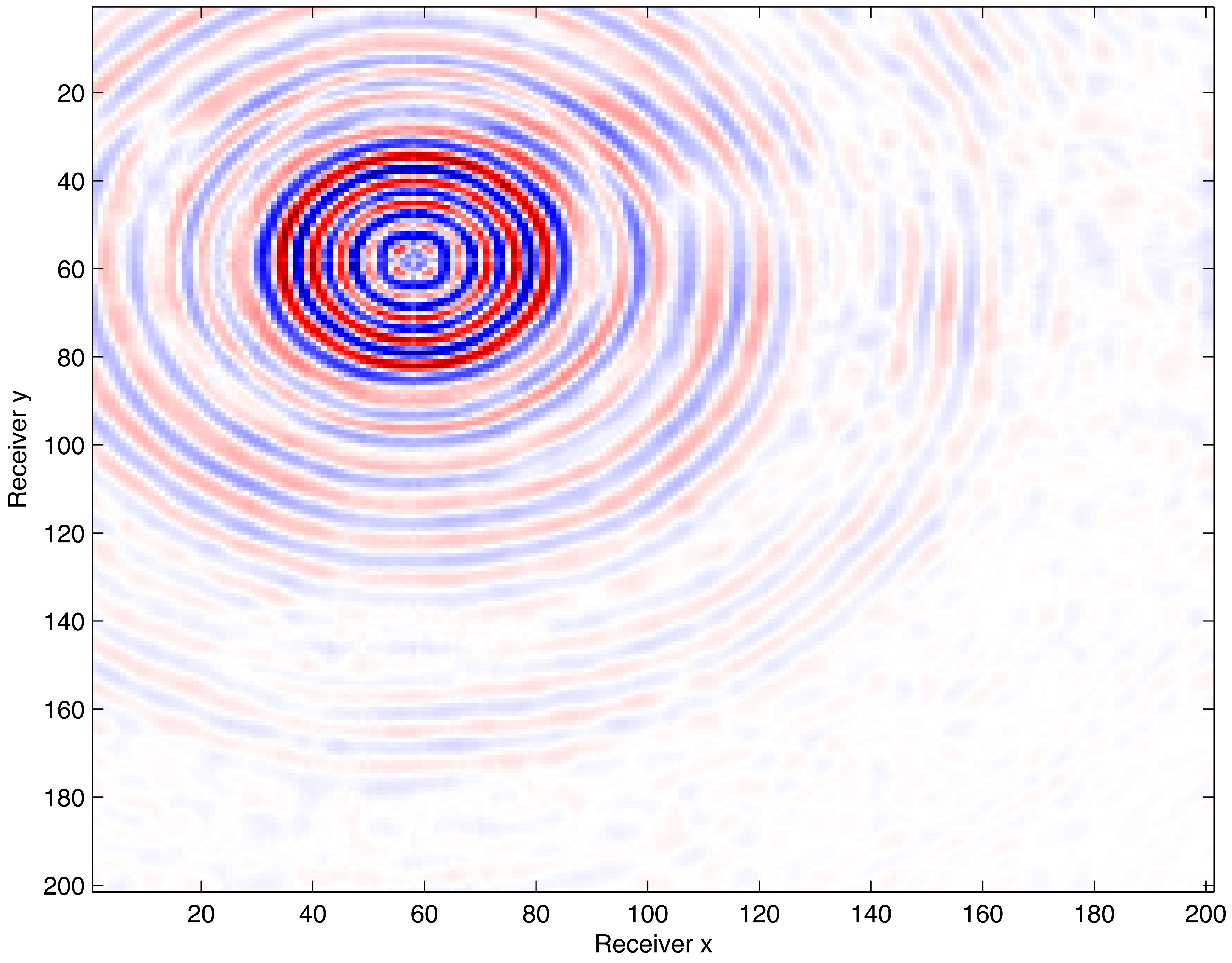


Noisy input - SNR 14.9 dB

7.34 Hz - Denoising

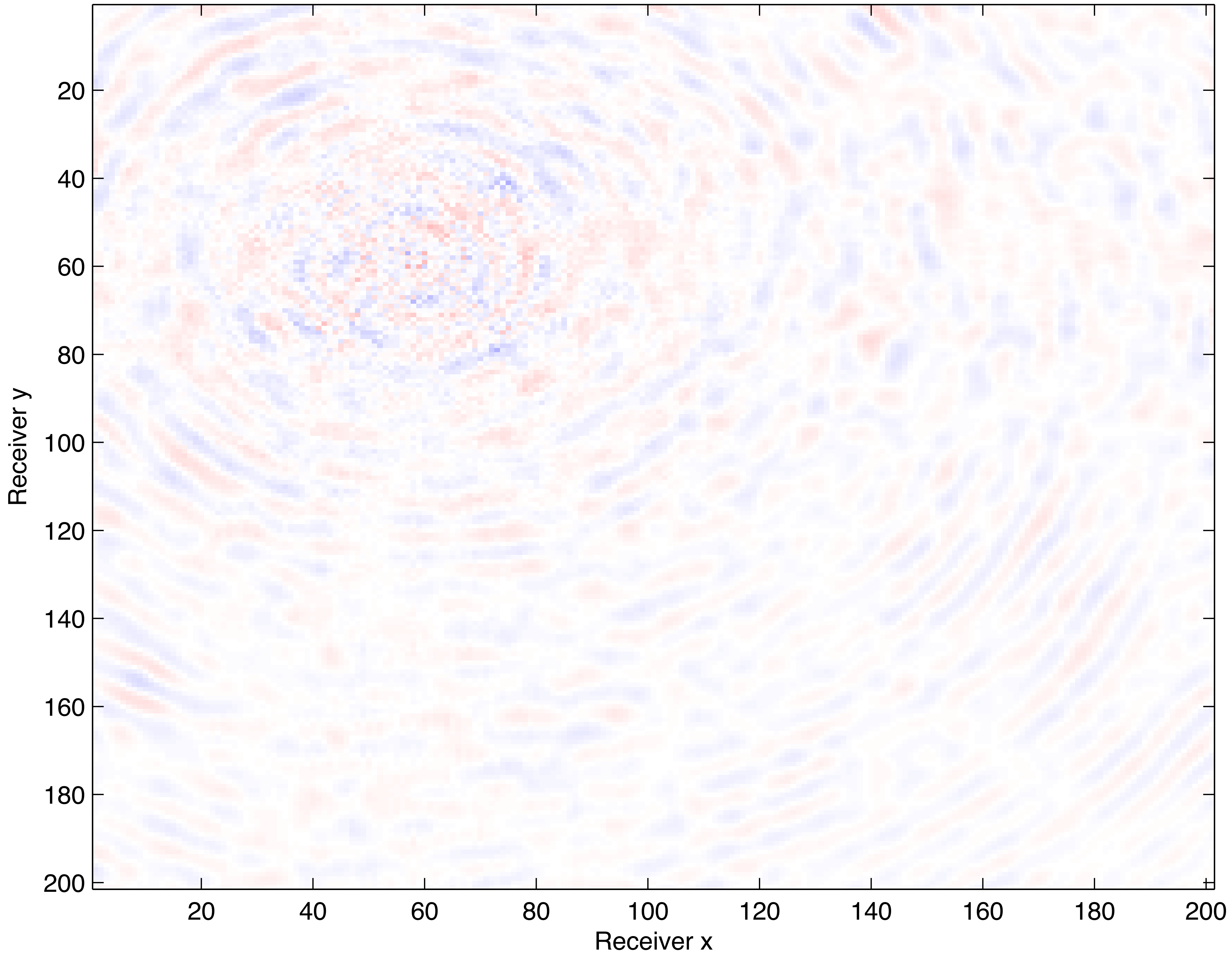


True data

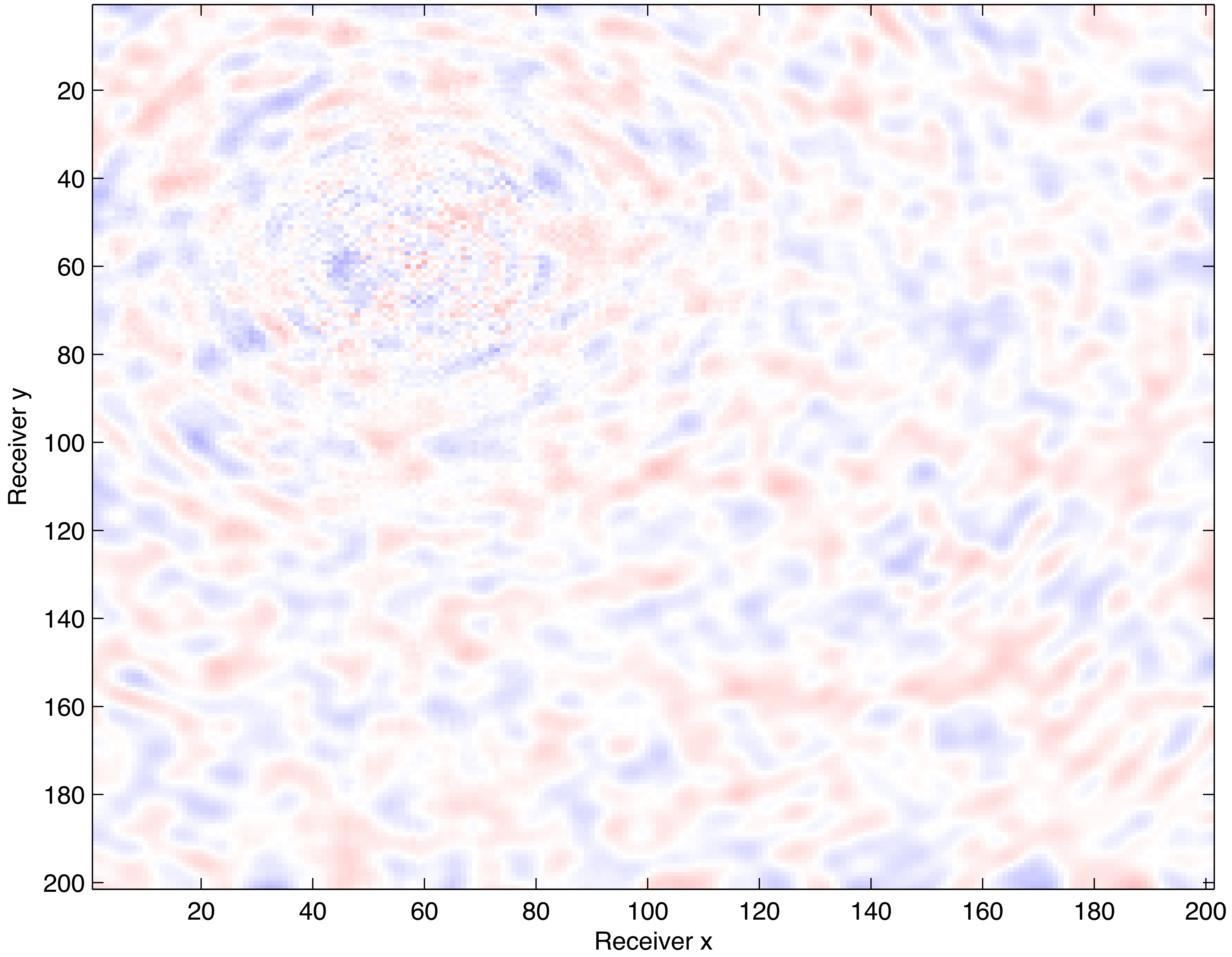


Estimated - SNR 20.1 dB

7.34 Hz - Denoising

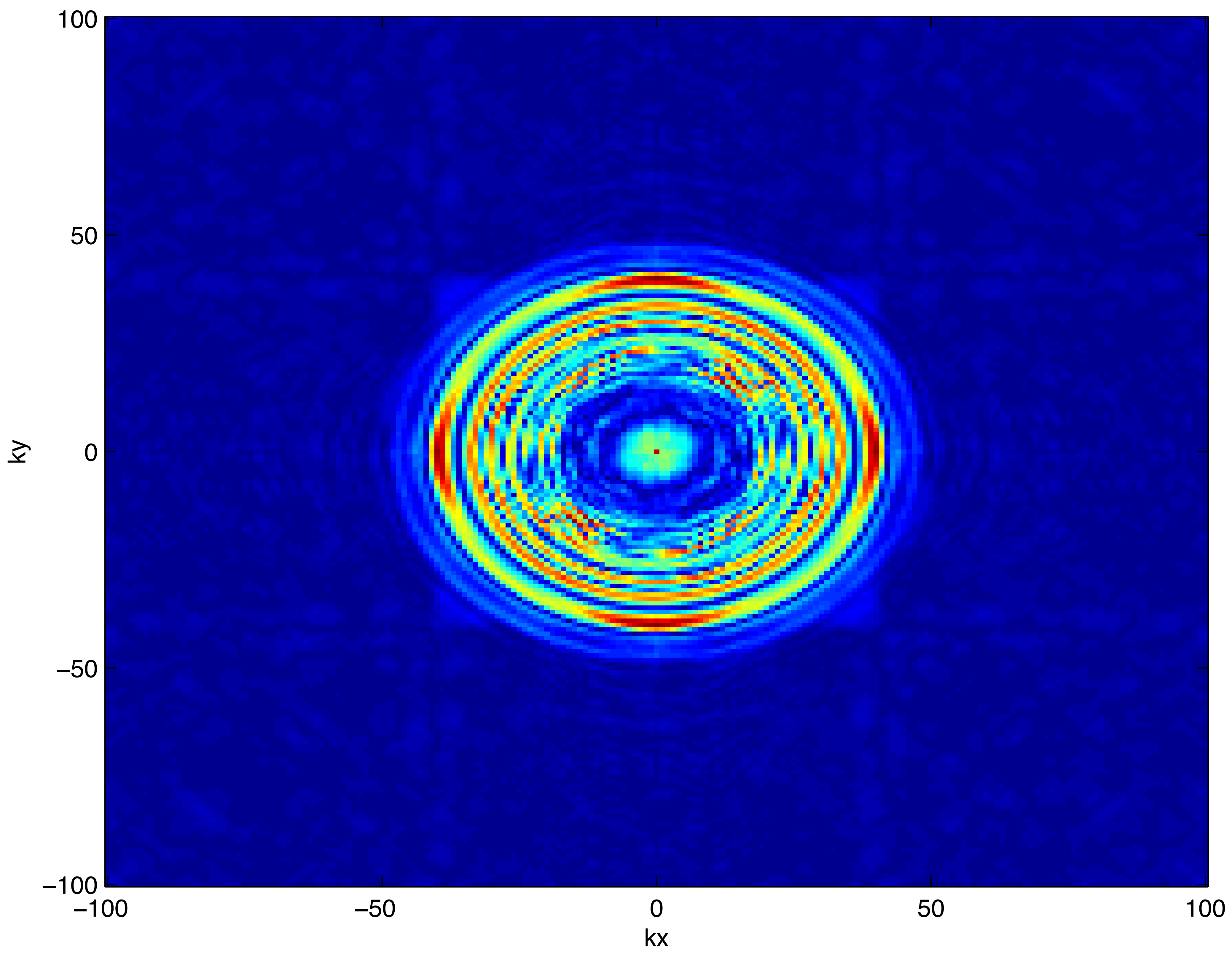


Difference

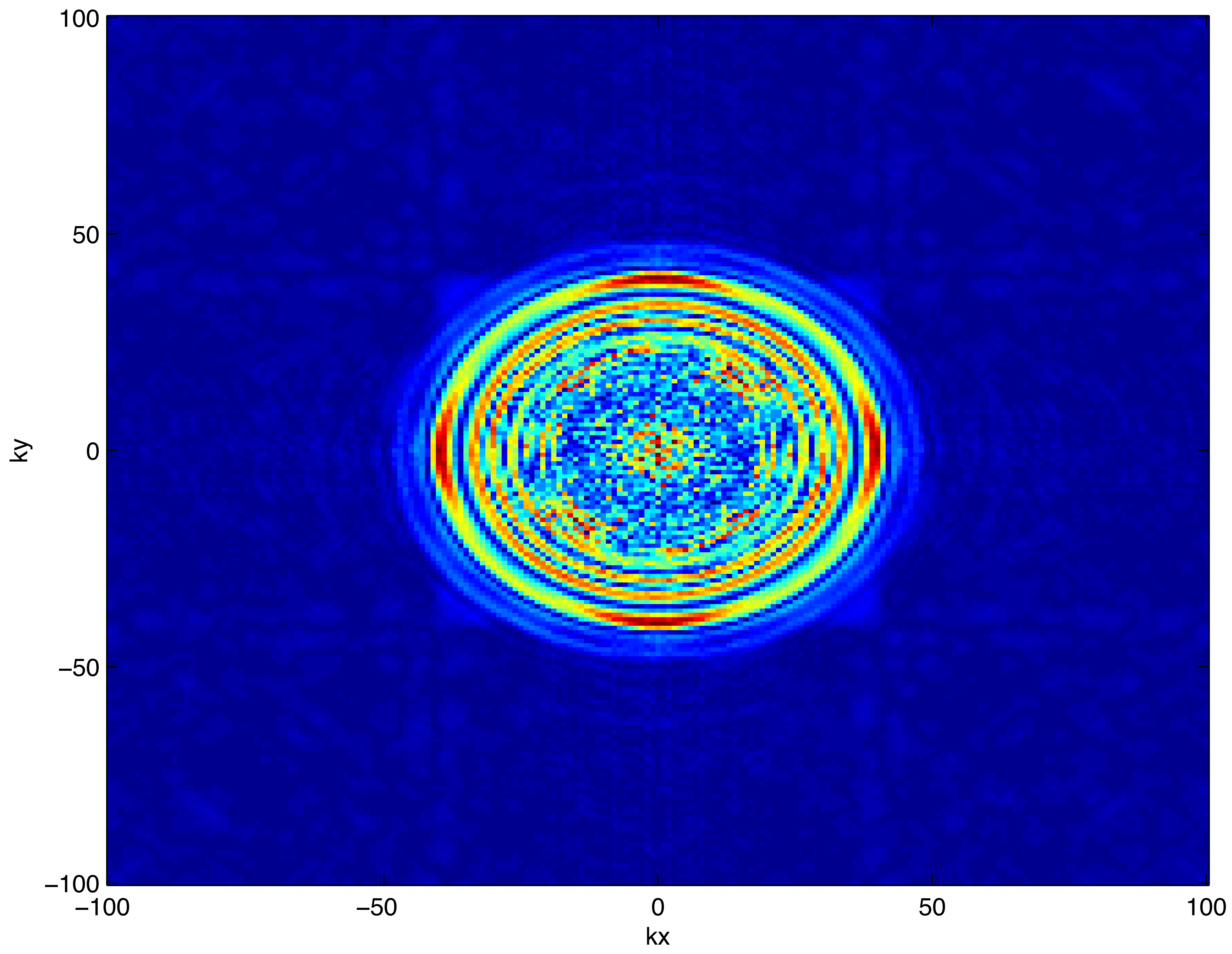


Input + output difference

7.34 Hz - Denoising

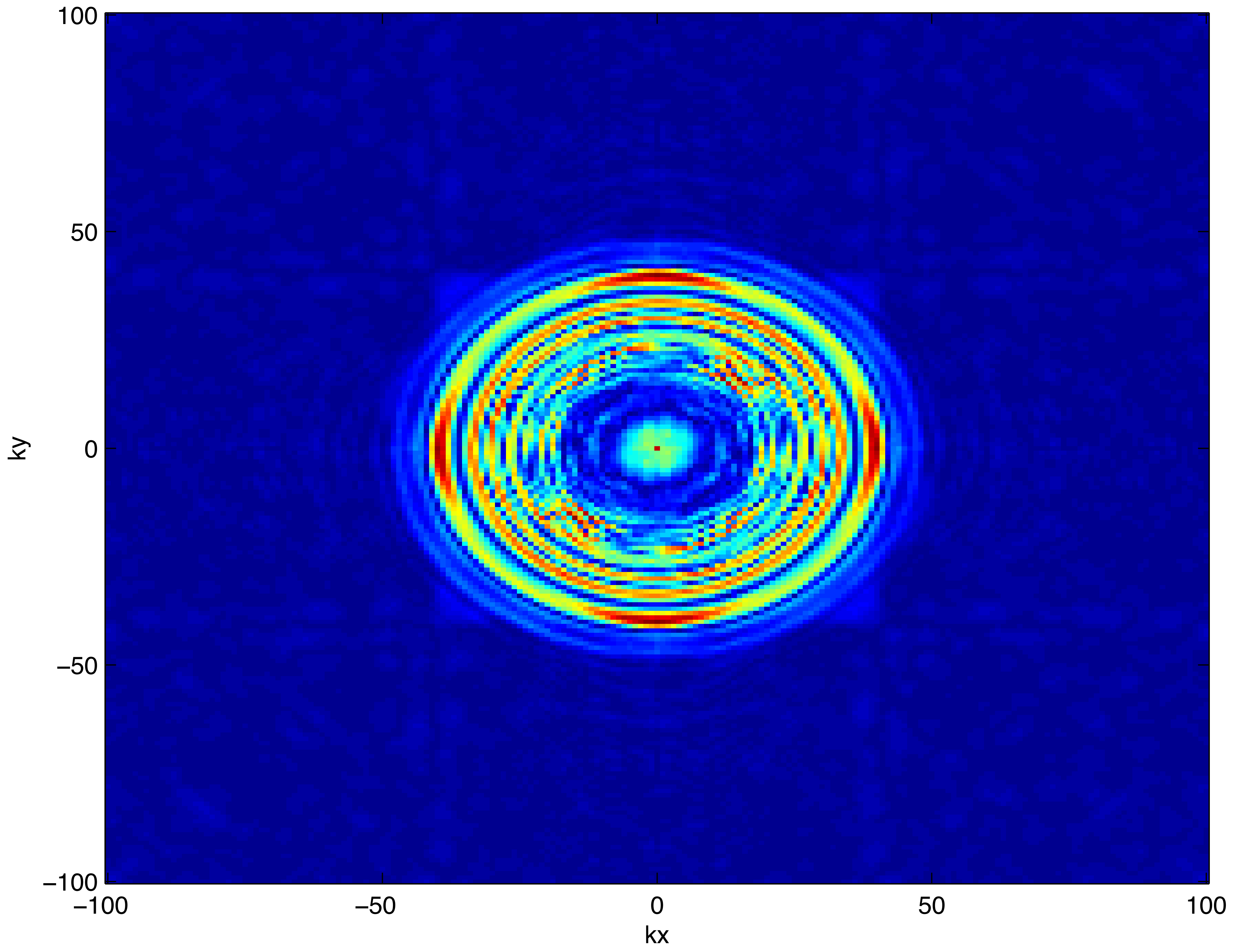


True spectrum

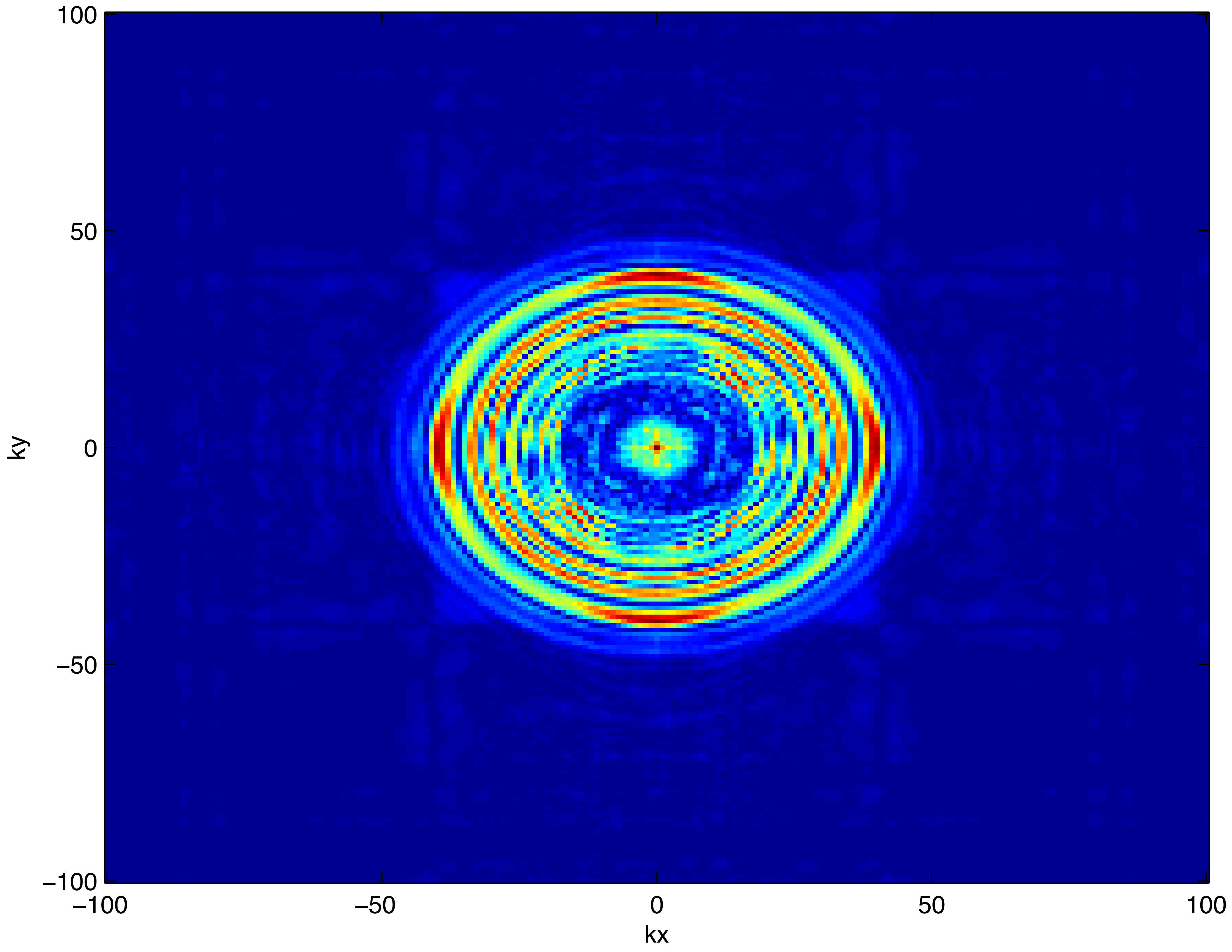


Noisy spectrum

7.34 Hz - Denoising



True spectrum



Estimated spectrum

Conclusion

3D seismic data has an underlying structure that we can exploit for interpolation (Hierarchical Tucker format)

Different schemes for organizing data - important for recovery

Conclusion

We can interpolate HT tensors with missing entries using the Riemannian manifold structure of the HT format

Important to use an appropriate *transform domain* (e.g. midpoint offset) so that sampling + noise *increase* the singular values in that domain

Acknowledgements

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