Fast least-squares imaging with source estimation using multiples

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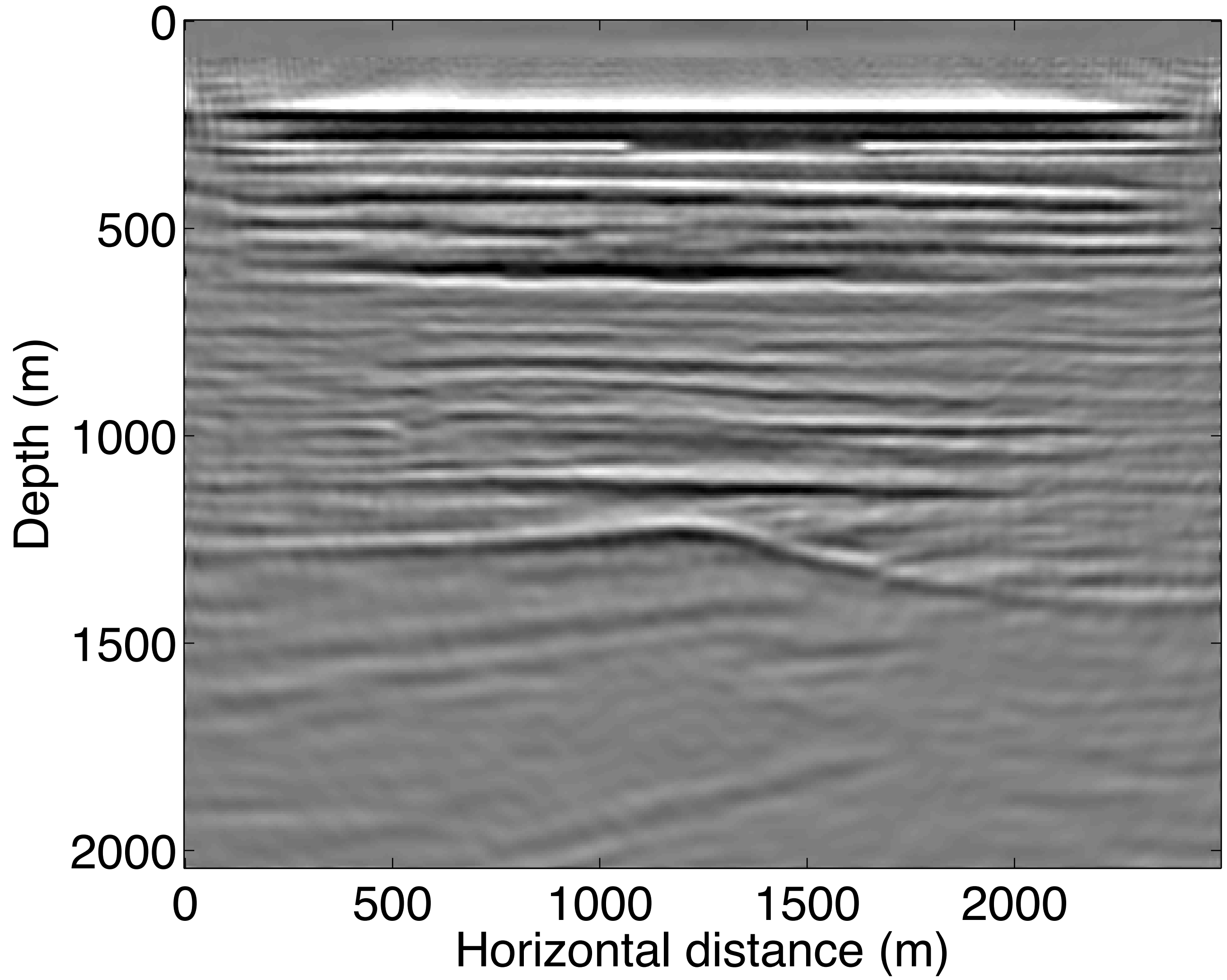
Motivation

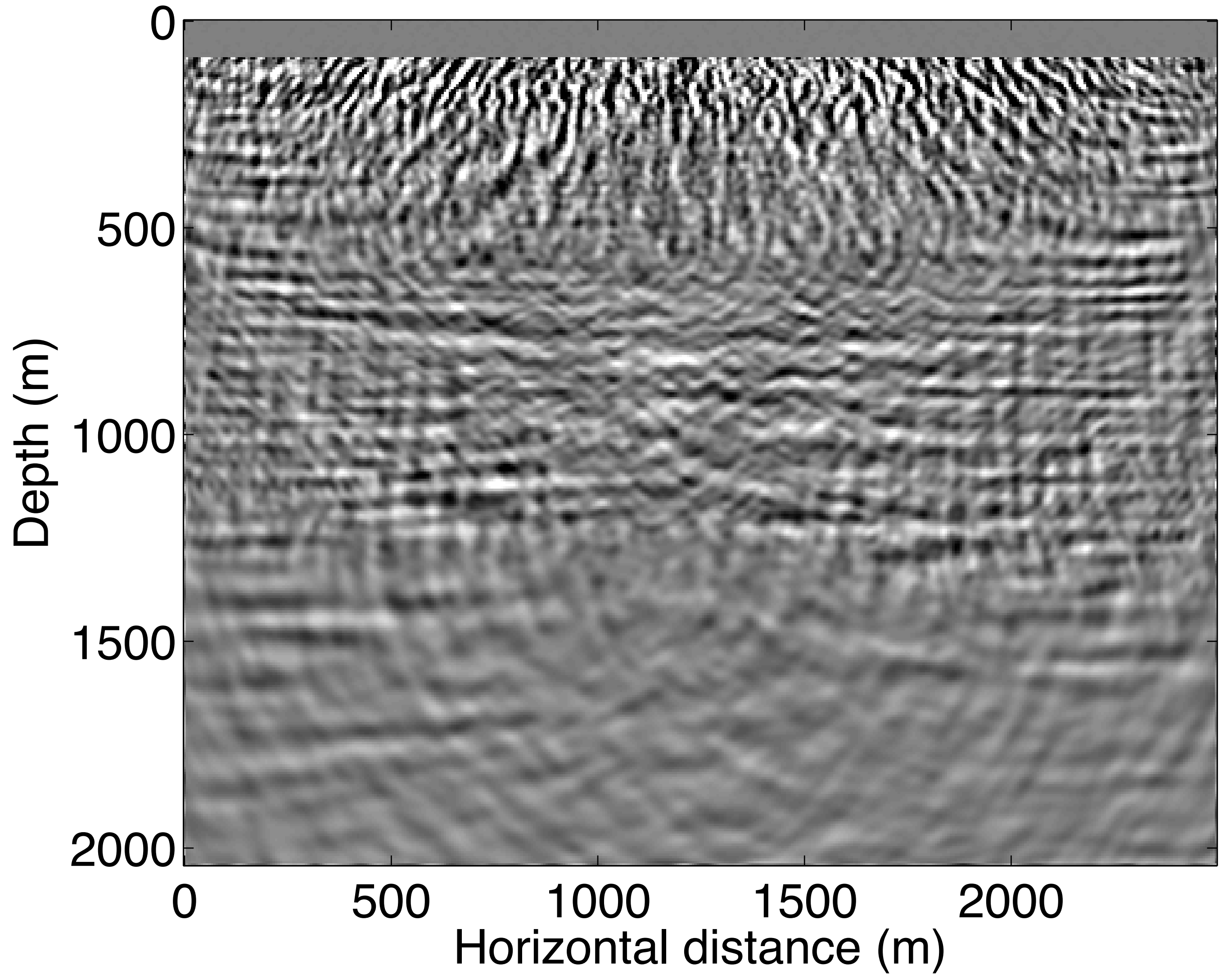
- high fidelity, true-amplitude seismic image by linearized inversion
- accurate source signature

How important is the source wavelet for linearized inversion?



Linearized inversion with *the true* wavelet

whereas...



Linearized inversion with *a wrong* wavelet

Theory

Least-squares migration with *unknown* source wavelet

$$\min_{\delta\mathbf{m},\mathbf{q}} \sum_{i=1}^{n_f} \|\mathbf{d}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \mathbf{Q}(q_i)]\delta\mathbf{m}\|_2^2$$

$\delta\mathbf{m}$: model perturbation

$\mathbf{q}$: source wavelet spectra $\mathbf{q} = [q_1, \cdots, q_{n_f}]$

$\mathbf{d}_i$: vectorized primary wavefield

$\nabla\mathbf{F}_i$: linearized demigration operator

$\mathbf{m}_0$: background model

$\mathbf{Q}(q_i)$: source wavefield $\mathbf{Q}(q_i) = q_i\mathbf{I}$

Major challenges

- preprocessing to remove coherent noise such as surface multiples
- expensive simulation cost
- nonlinearity with unknown source wavelet

Our solutions

- imaging *with active contributions* from surface multiples
- using *dimensionality reduction* techniques to speed up inversion
- estimating the source wavelet *on the fly*

Embracing surface multiples

- imaging primaries and multiples simultaneously
- removing amplitude/phase ambiguity using extra information from multiples
- exploiting higher-wavenumber components in multiples

Tu and Herrmann. 2012

$$\min_{\delta\mathbf{m},\mathbf{q}} \sum_{i=1}^{n_f} \|\mathbf{d}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \mathbf{Q}(q_i)]\delta\mathbf{m}\|_2^2$$

- $\mathbf{d}_i$: vectorized total up-going wavefield, primaries and surface multiples
- $\mathbf{Q}(q_i) = q_i\mathbf{I} - \mathbf{D}_i$: generalized source wavefield containing the total down-going wavefield

Herrmann and Li, 2012
Tu and Herrmann, 2012
Candes et. al., 2006

Dimensionality reduction with sparsity promotion

BPDN: $$\underset{\mathbf{x},\mathbf{q}}{\text{minimize}} \|\mathbf{x}\|_1$$
$$\text{subject to} \sum_{i\in\mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(q_i)]\mathbf{S}^*\mathbf{x}\|_2^2 \leq \sigma^2$$

frequency: select a random frequency subset

source: forming randomized source aggregates

$\mathbf{S}^*$: Curvelet synthesis operator

σ : tolerance for noise/modelling error, etc

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Aravkin et. al. 2012
van den Berg and Friedlander, 2008

Alternate formulation

LASSO: $$\min_{\mathbf{x},\mathbf{q}} \sum_{i\in\mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(q_i)]\mathbf{S}^*\mathbf{x}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau$$

τ : sparsity level

Herrmann and Li, 2012
Tu and Herrmann, 2012

Further acceleration by *rerandomization*

- We draw a new subsampling operator for each LASSO subproblem:
  - new random subset of frequencies
  - new randomized source aggregates
- faster convergence

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Source estimation

$$\min_{\mathbf{x},\mathbf{q}} \sum_{i\in\mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(q_i)]\mathbf{S}^*\mathbf{x}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau$$

- nonlinear by having two unknowns
- the two unknowns are separable
- alternating optimization

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Wavefield matching

Given an $\mathbf{x}$, a least squares solution for $\mathbf{q}$ can be determined:

- primaries only:

$$\tilde{q}_i(\mathbf{x}) = \frac{<\nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x}, \mathbf{d}_i>}{\|\nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x}\|_2^2}$$

- with multiples:

$$\tilde{q}_i(\mathbf{x}) = \frac{<\nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x}, \mathbf{d}_i + \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{D}}_i]\mathbf{S}^*\mathbf{x}>}{\|\nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x}\|_2^2}$$

Aravkin and van Leeuwen 2012

Variable projection

We now solve:

$$\min_{\mathbf{x}} \sum_{i \in \mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(\tilde{q}_i(\mathbf{x}))]\mathbf{S}^* \mathbf{x}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau$$

Example

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Courtesy of BG Group

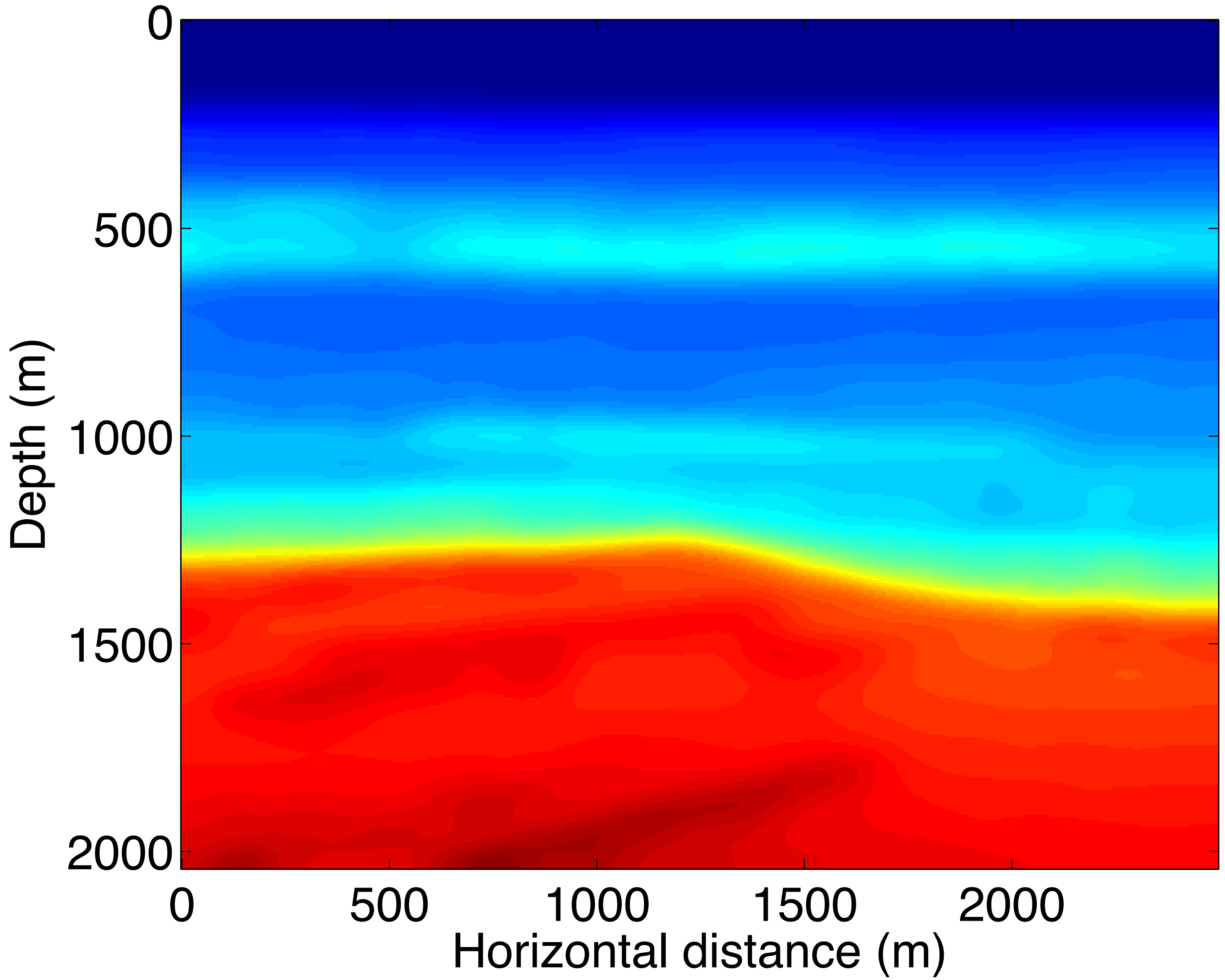
Experiments setup

- synthetic BG Compass model (cropped)
- 209 co-located sources/receivers, 12m spacing, 6m depth
- linearized data, i.e., $\mathbf{d} = \nabla \mathbf{F} \delta \mathbf{m}$
- Ricker wavelet w. 20Hz peak freq.
- 30 composite sources, 15 frequencies in the inversion, 74X subsampling

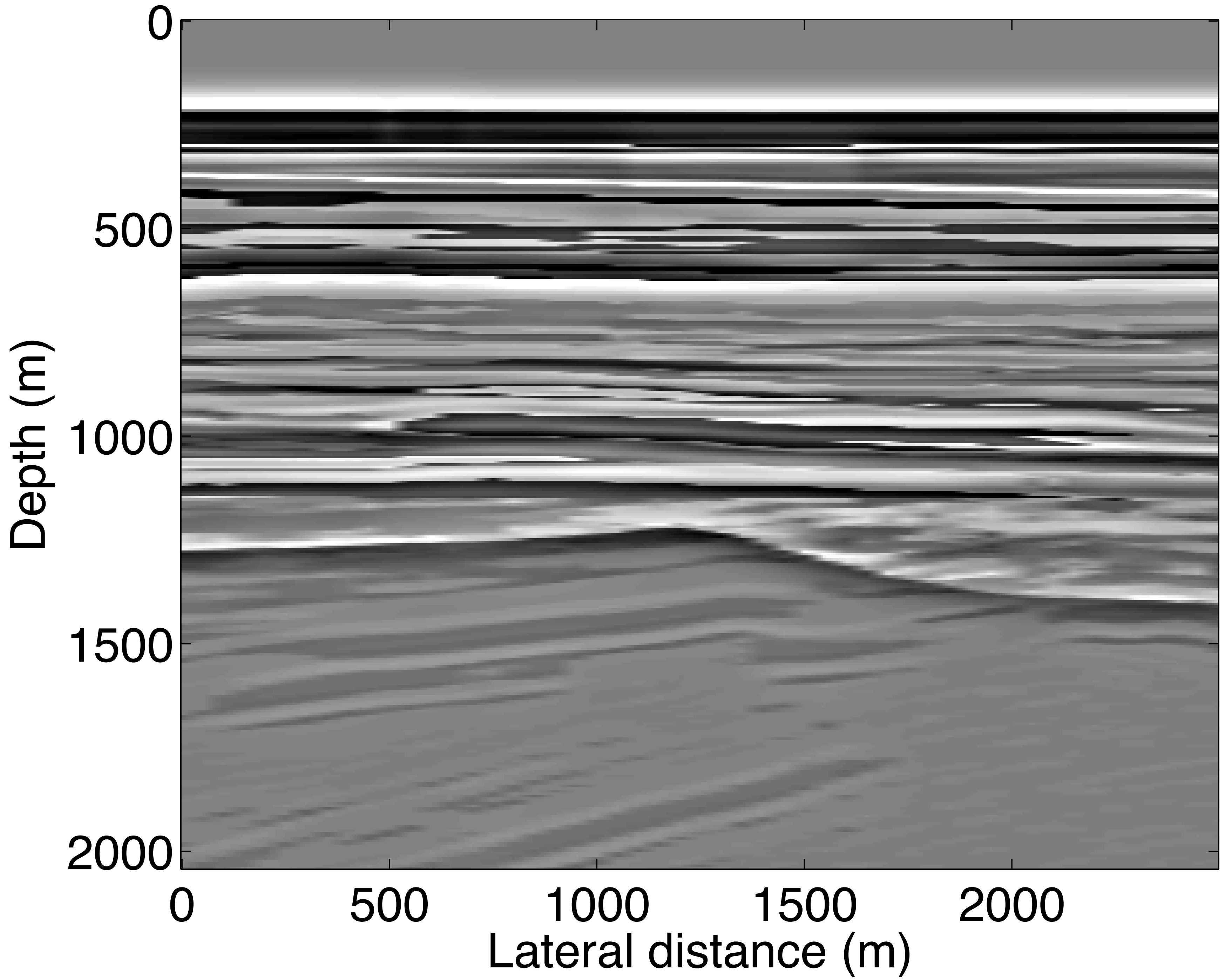
Experiments setup

<table>
<tr><td></td><td colspan="2">Source wavelet</td></tr>
<tr><td rowspan="2">Data type</td><td>Primary, True source</td><td>Primary, Source estimation</td></tr>
<tr><td>w. Multiples, True source</td><td>w. Multiples, Source estimation</td></tr>
</table>

Background velocity



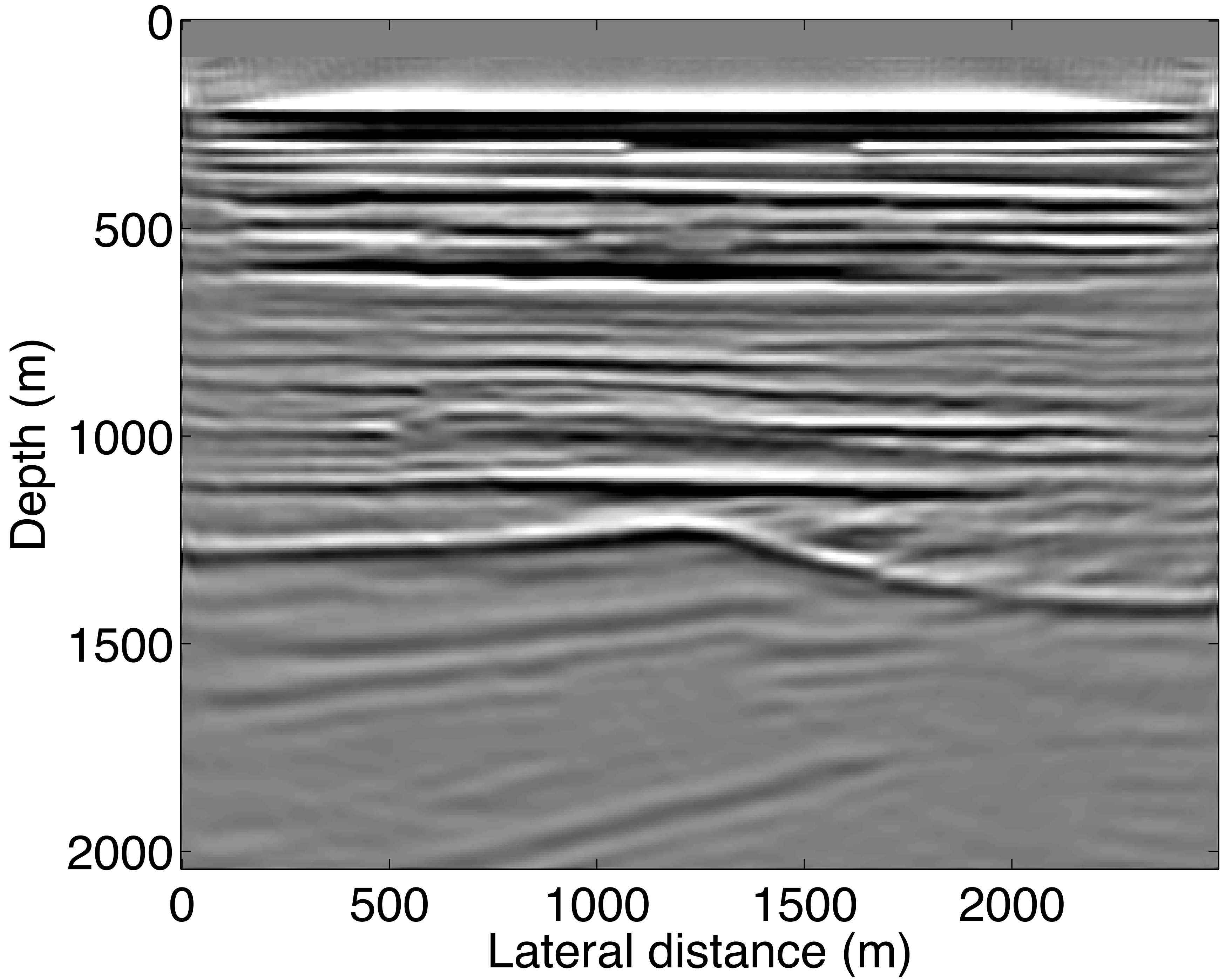
True perturbation



Using primaries

w. true source

Primary, True source	Primary, Source estimation
w. Multiples, True source	w. Multiples, Source estimation



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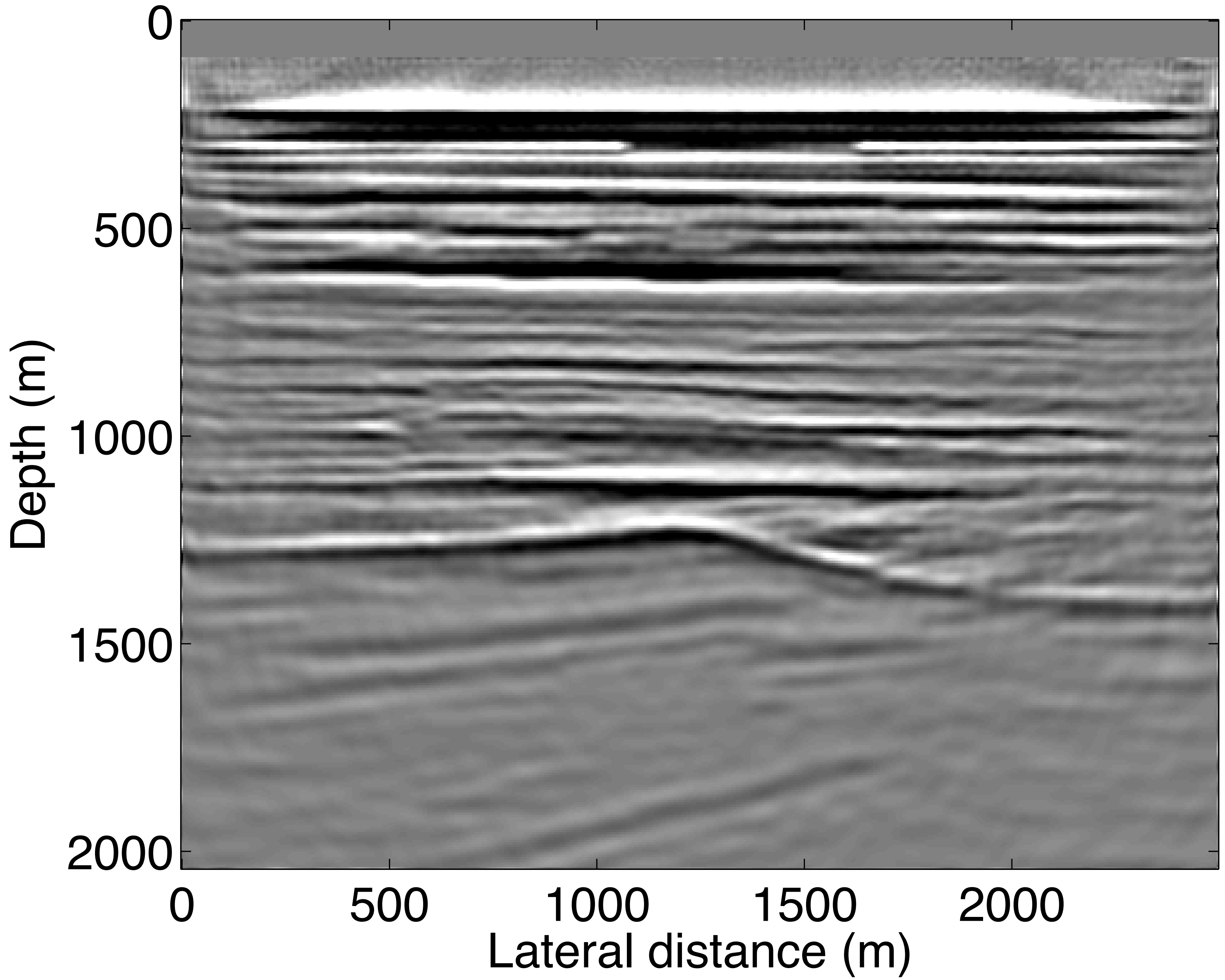
Using primaries

w. source estimation

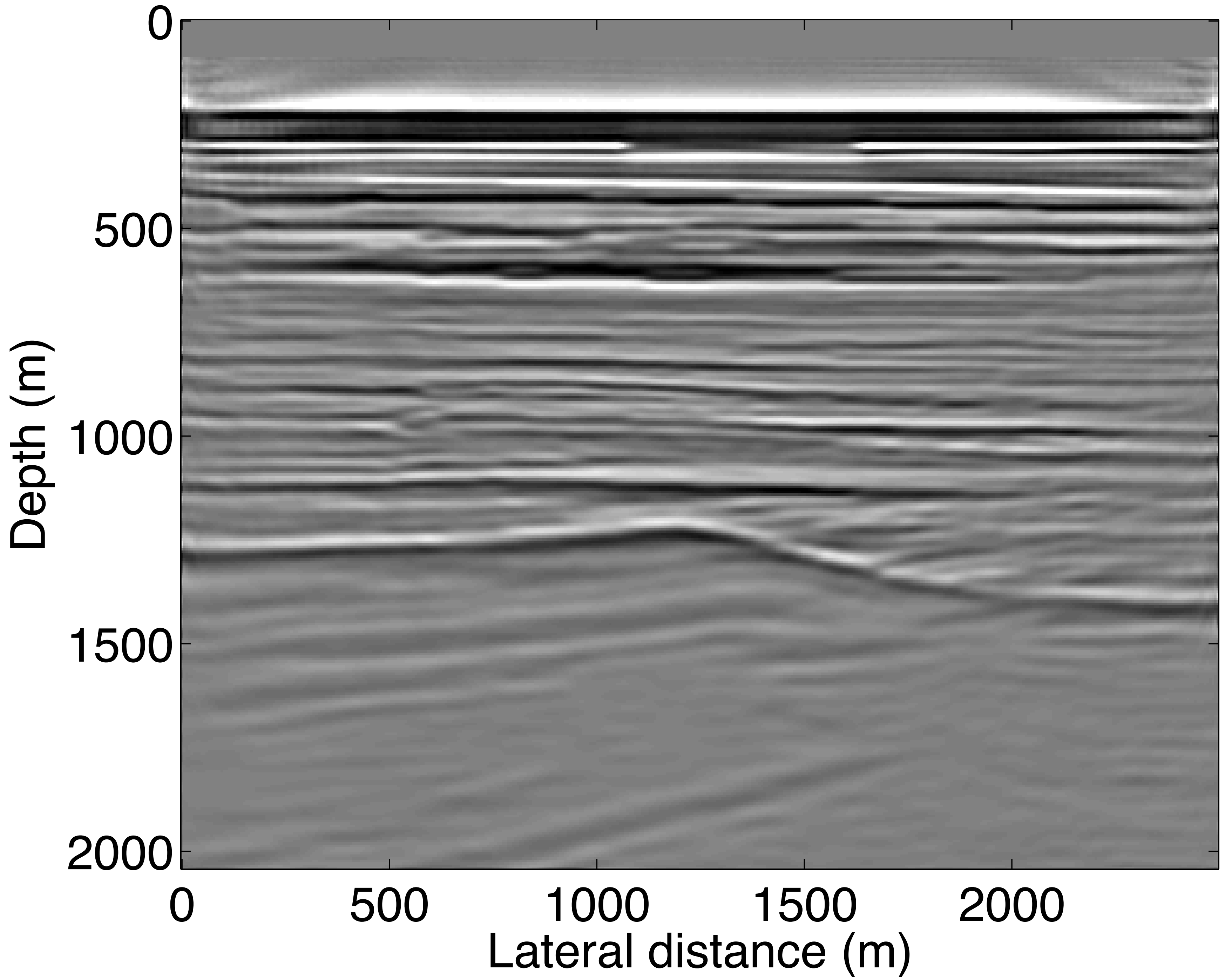
Primary, True source	Primary, Source estimation
w. Multiples, True source	w. Multiples, Source estimation



With multiples

w. true source

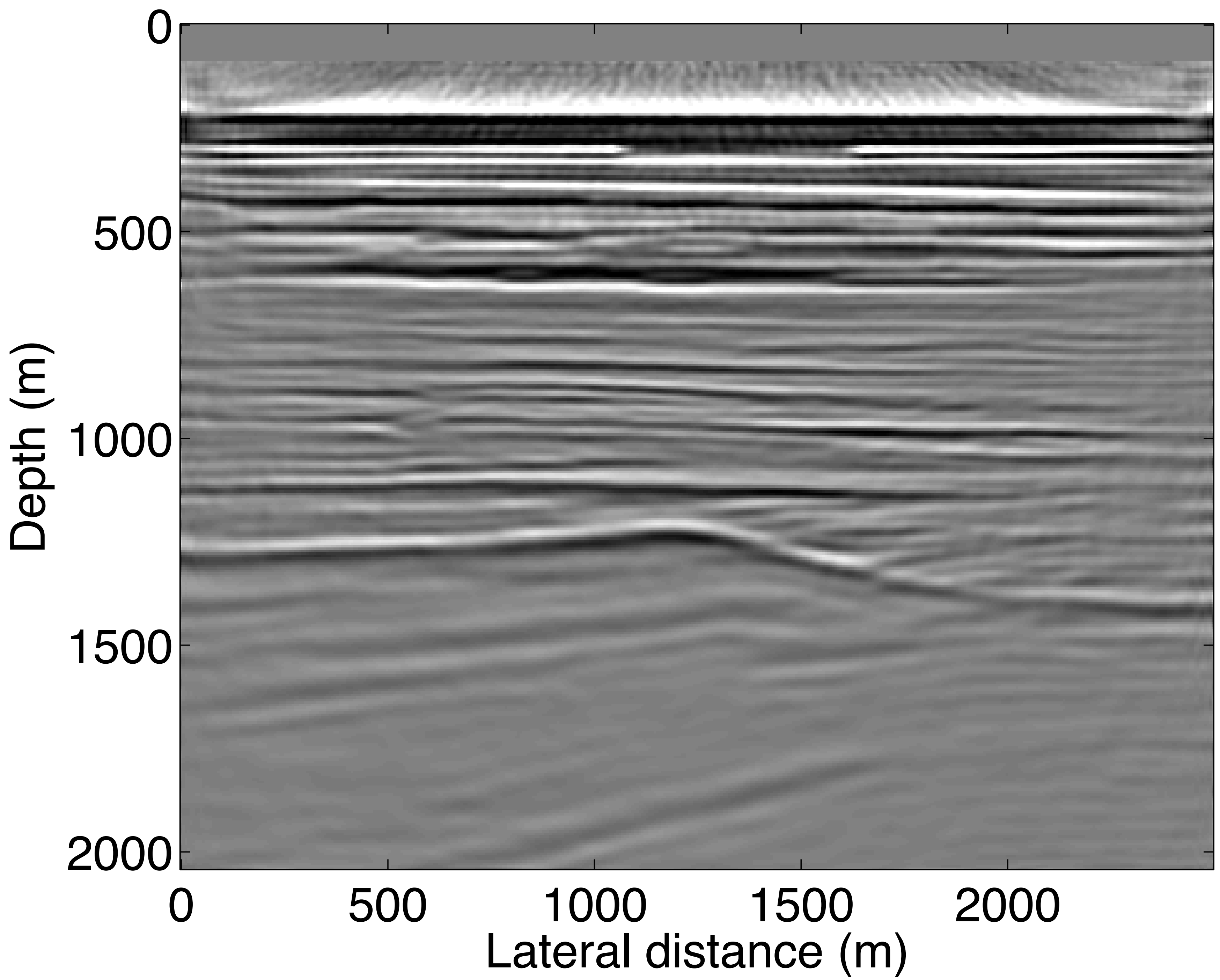
Primary, True source	Primary, Source estimation
w. Multiples, True source	w. Multiples, Source estimation



With multiples

w. source estimation

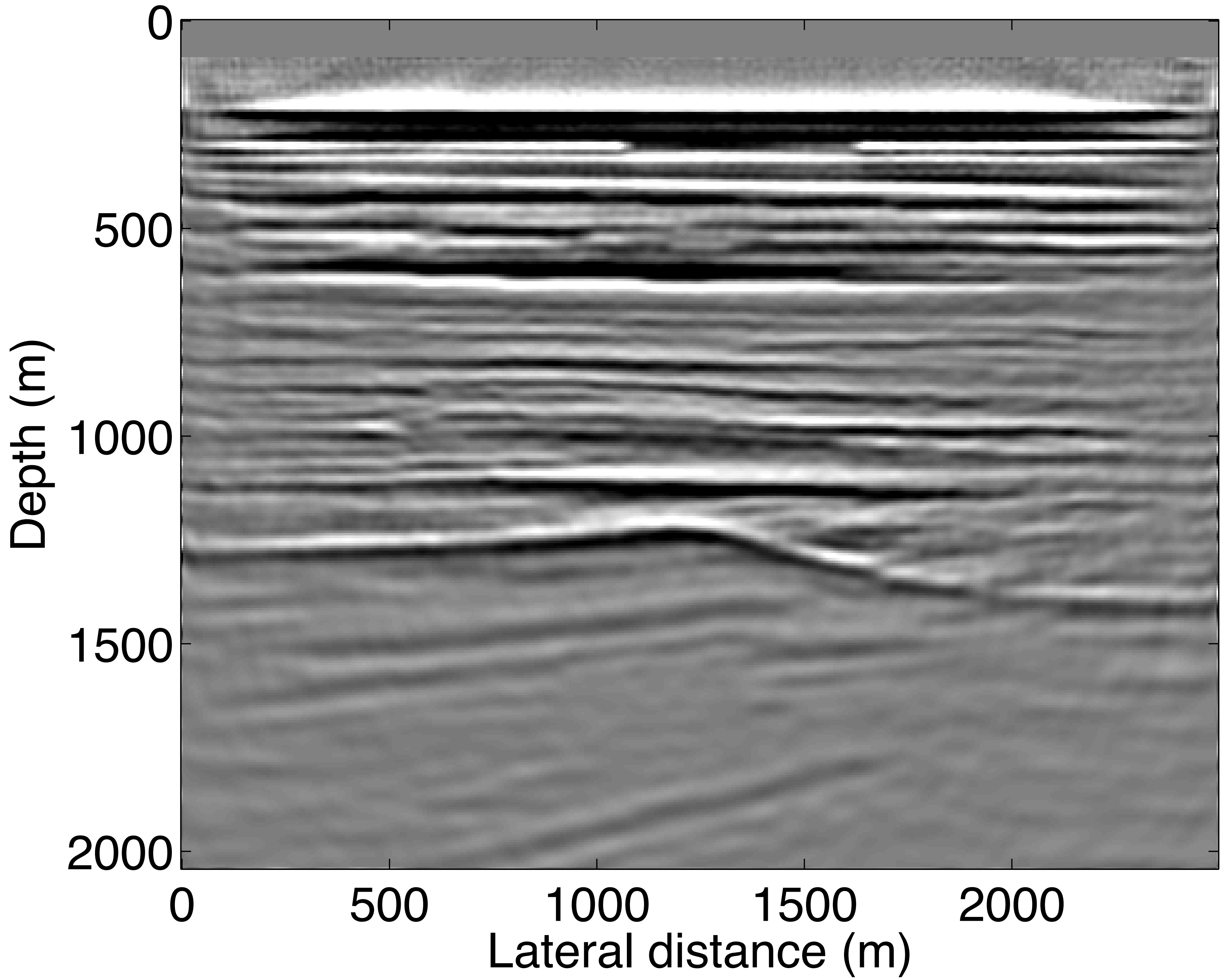
Primary, True source	Primary, Source estimation
w. Multiples, True source	w. Multiples, Source estimation



Using primaries

w. source estimation

PRIMARY, TRUE SOURCE	PRIMARY, SOURCE ESTIMATION
W. MULTIPLES, TRUE SOURCE	W. MULTIPLES, SOURCE ESTIMATION



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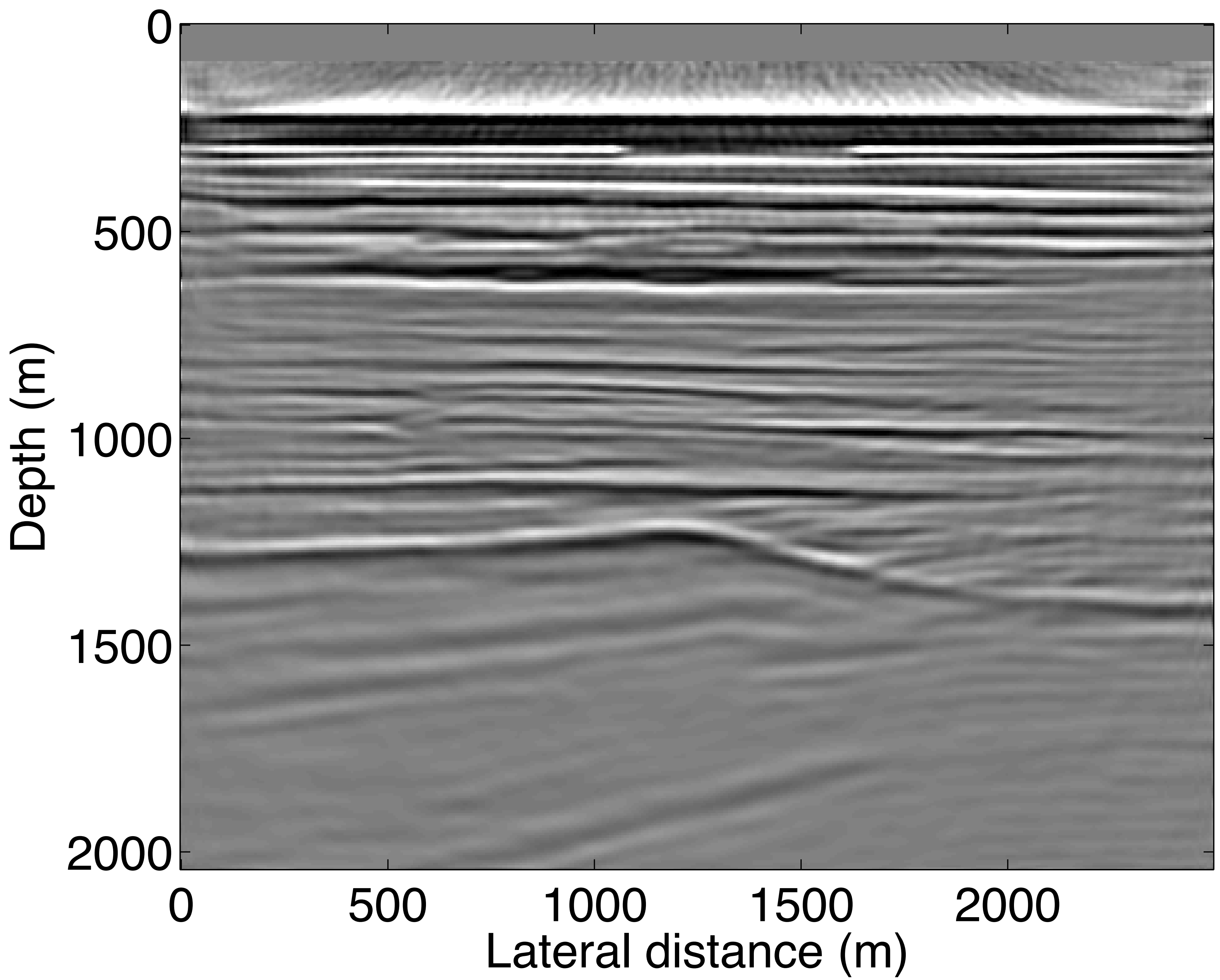
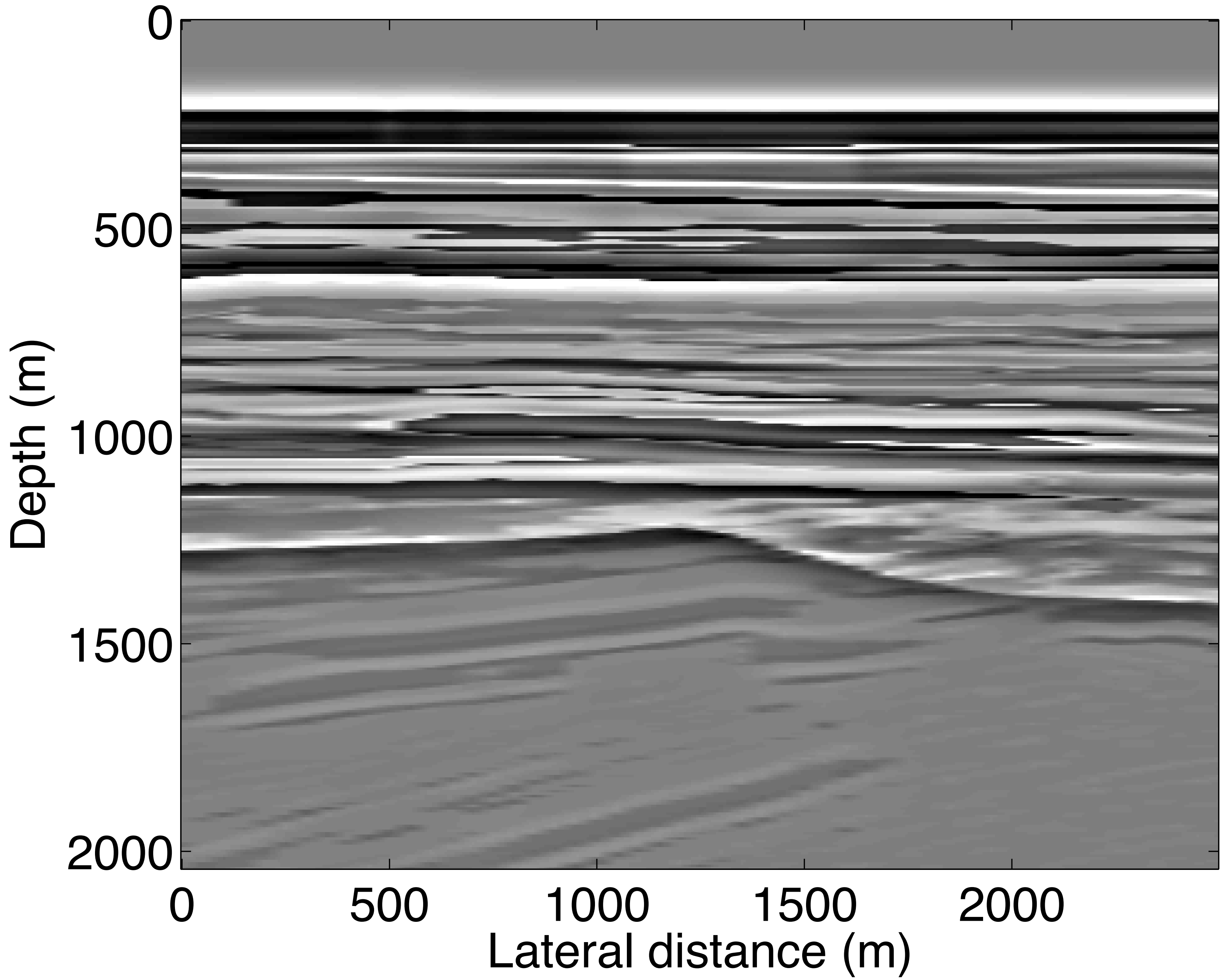
With multiples

w. source estimation

Primary, True source	Primary, Source estimation
w. Multiples, True source	w. Multiples, Source estimation

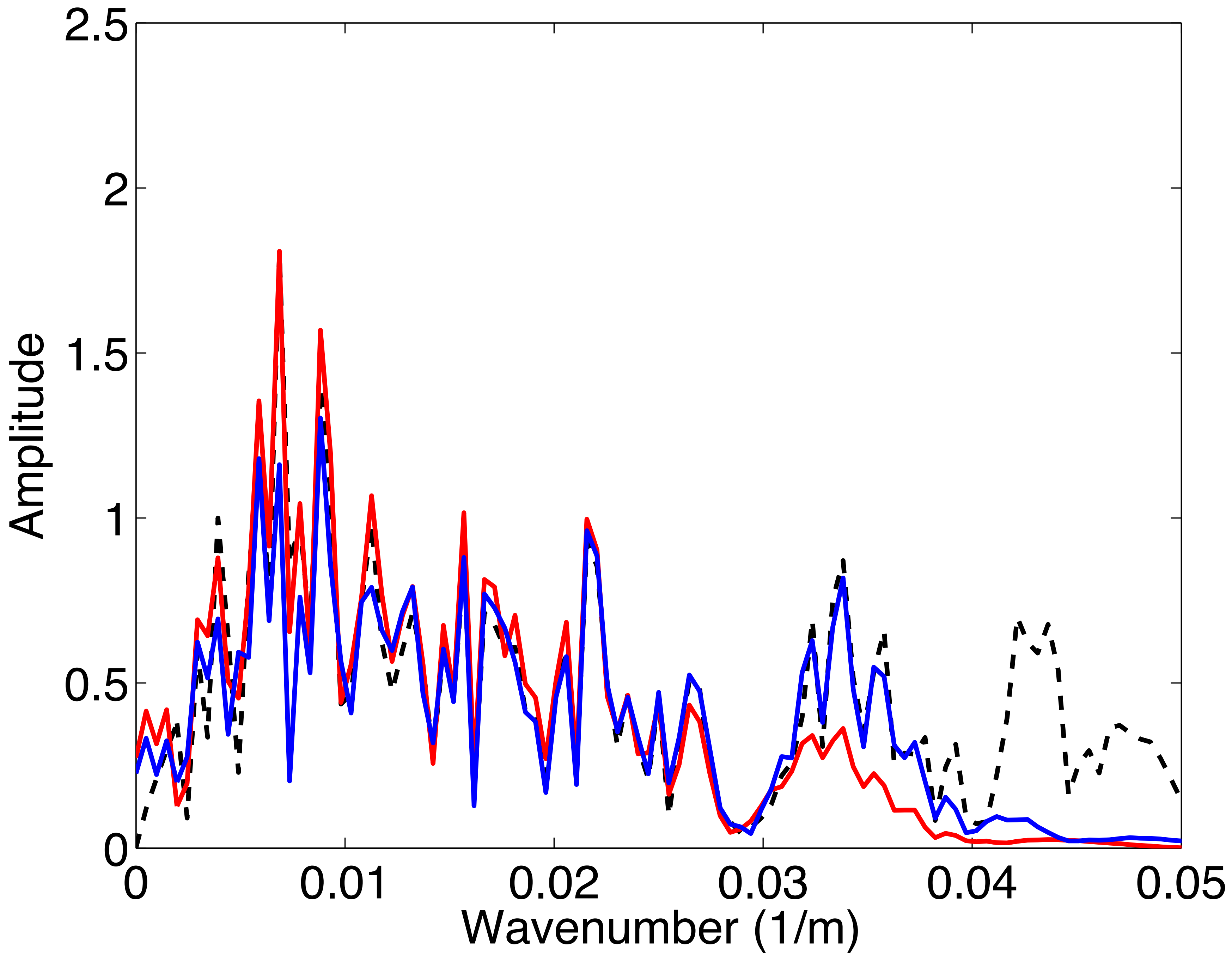


True perturbation



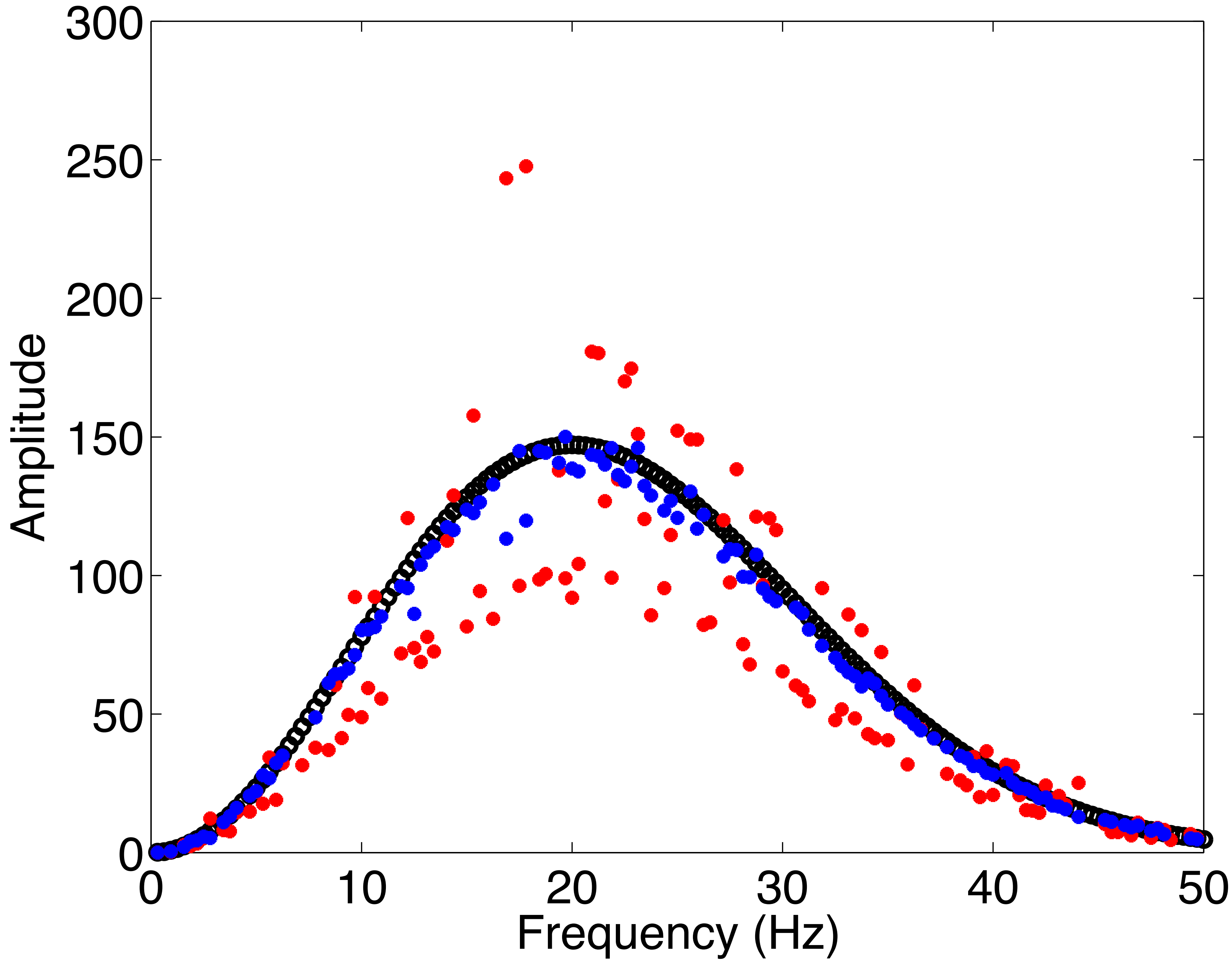
Wavenumber contents

[of traces from images w. source estimation]



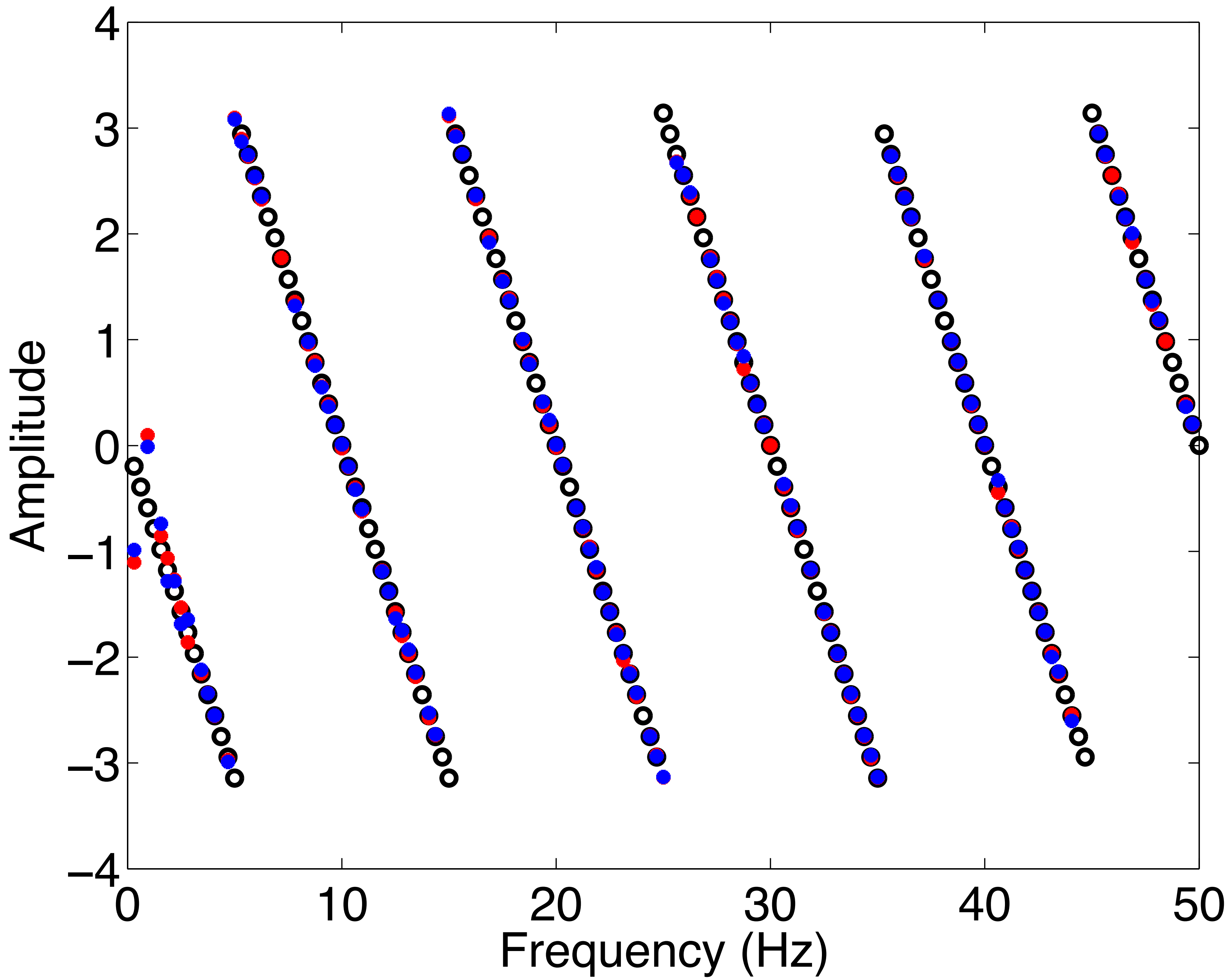
Black dashed: true; Blue: w. multiple; Red: primaries only (rescaled)

Estimated wavelet
[amplitude spectrum]



Black dashed: true; Blue: w. multiple; Red: primaries only (rescaled)

Estimated wavelet
[phase spectrum]



Black: true; Blue: w. multiple; Red: primaries only (rescaled)

Conclusion

- The use of surface-related multiples improves both the image resolution, and the accuracy of estimated source wavelet.
- With sparse constraint and rerandomization, we greatly reduce the dimensionality of the system without compromising the image quality.
- The proposed source estimation works well in the linearized sparse inversion framework.

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Acknowledgements

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Thank you for your attention!

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