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London '13

Seismic data interpolation and denoising using SVD-free low-rank matrix factorization

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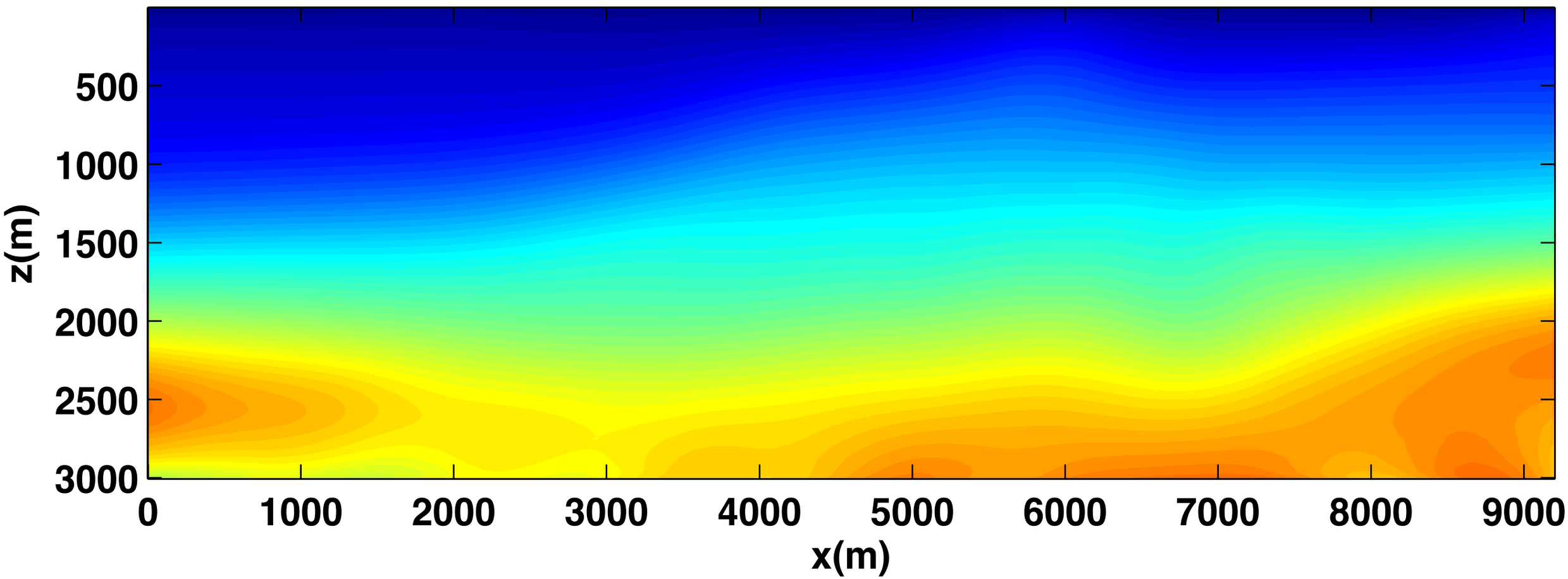
SLIM
University of British Columbia

Motivation

- acquisition challenges
  - missing data
  - noise
- fully sampled data
  - simultaneous shot based FWI & migration
  - estimation of primaries by sparse inversion & SRME
- exploit *low-rank* structure of seismic data
  - randomized sampling
  - *SVD-free* matrix factorization

Full waveform inversion

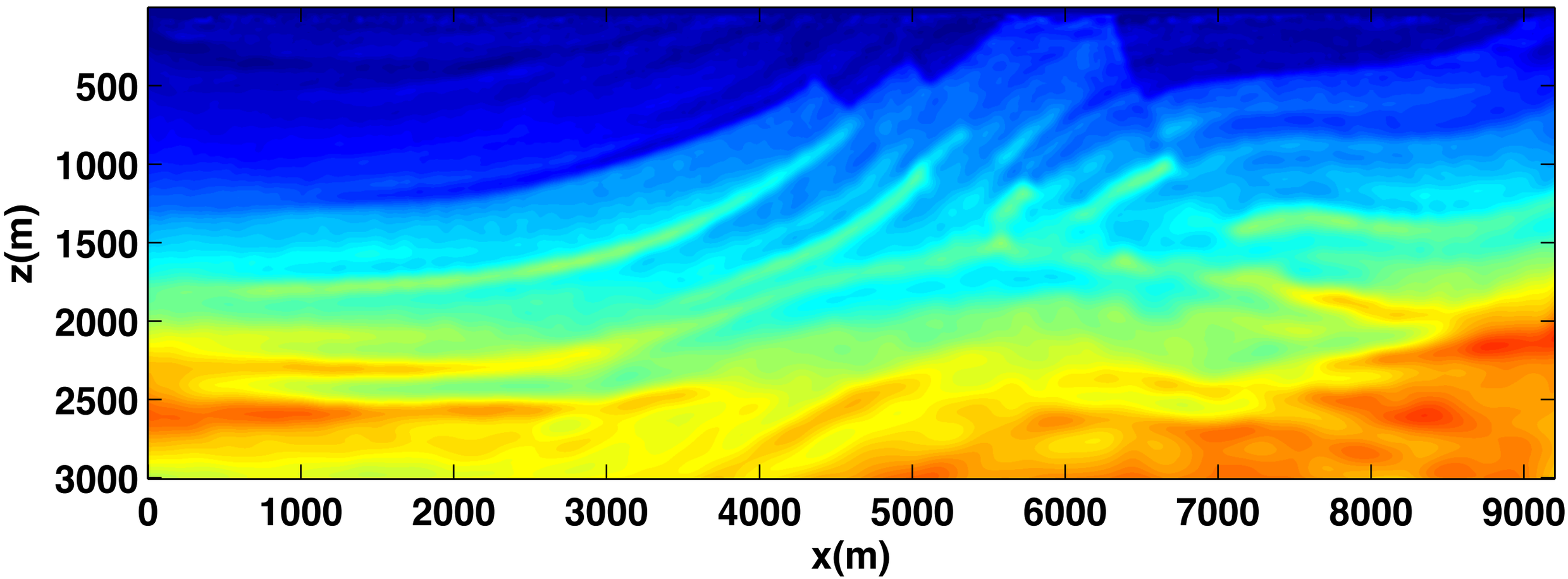
initial model



[Li et. al. 2012, van Leeuwen and Herrmann 2012]

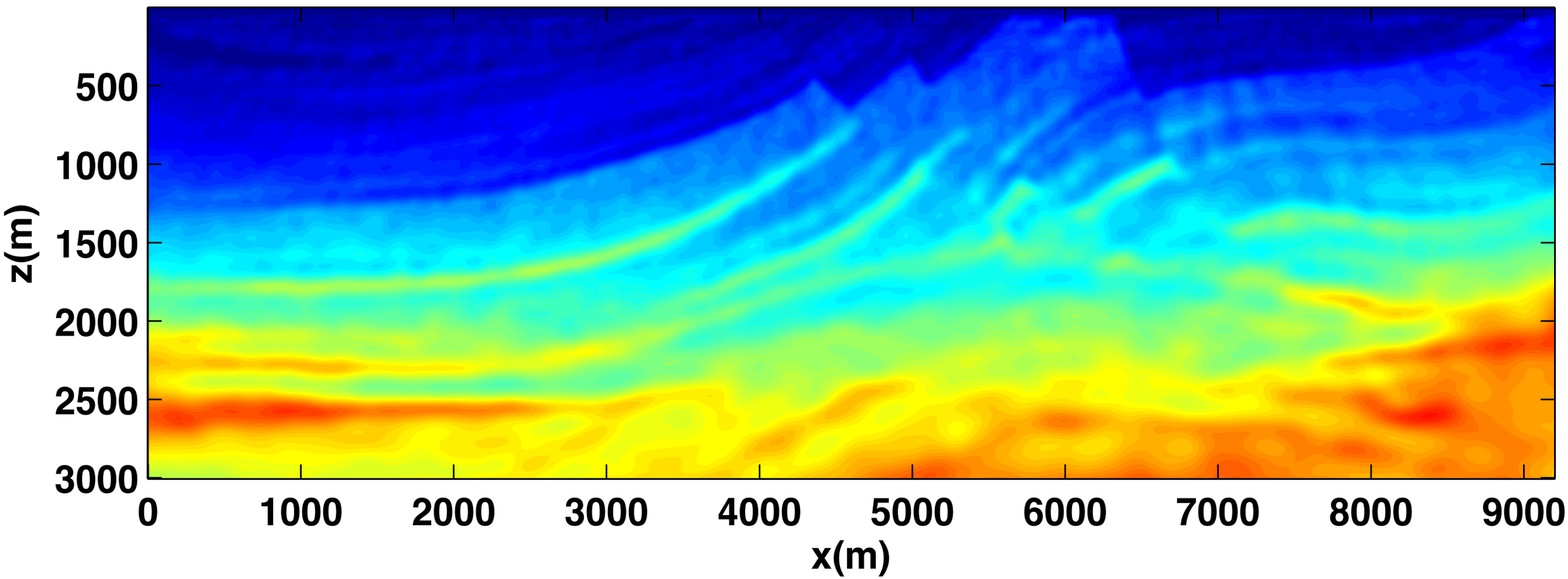
Full waveform inversion

[Ideal scenario, 60 sequential shots, redraw]



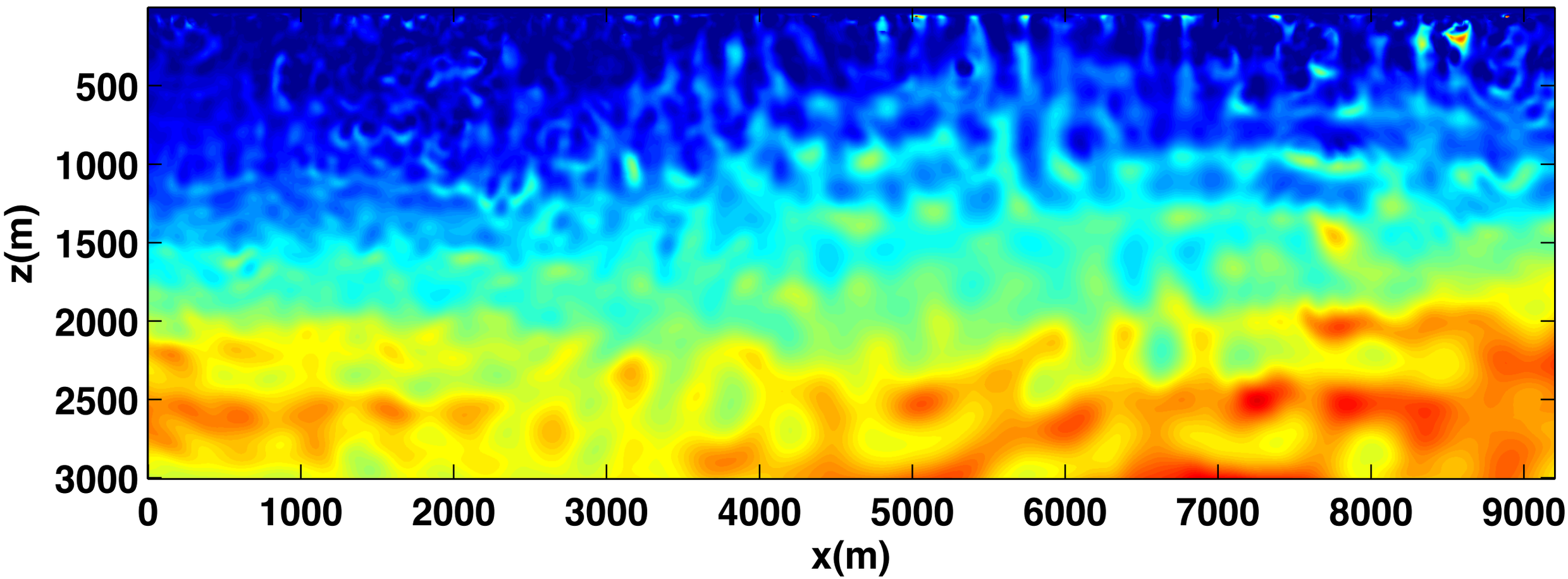
Full waveform inversion

[Ideal scenario, 20 simultaneous shots, redraw]



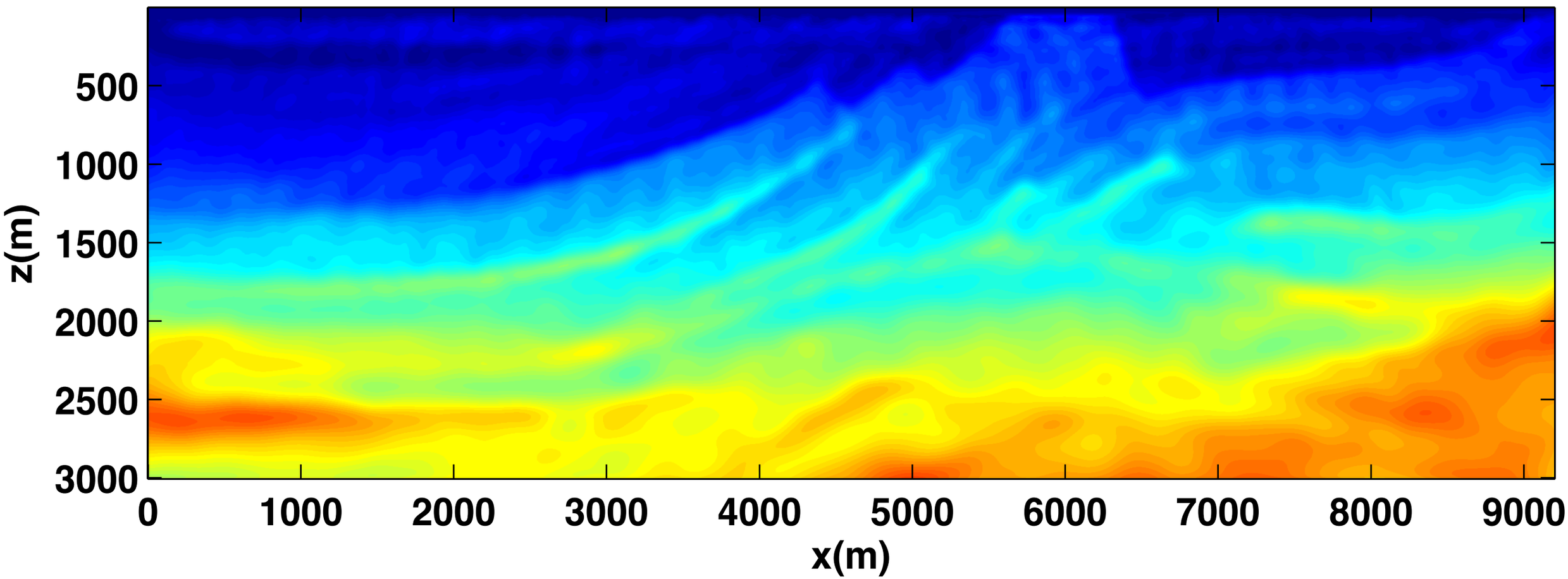
Full waveform inversion

[50% random missing shots, 60 sequential shots, redraw]



Full waveform inversion

[low-rank interpolation, 20 simultaneous shots, redraw]



[Candes and Donoho 2000, Herrmann 2008]

Compressive sensing

- signal structure
  - *sparse/compressible*
- sampling scheme
  - random missing traces make signal *less sparse* in transform domain
- recovery using *sparsity promoting* scheme

- **is *sparsity* the only inherent structure in seismic??**

[Candes and Plan 2010, Oropeza and Sacchi 2011]

Matrix completion

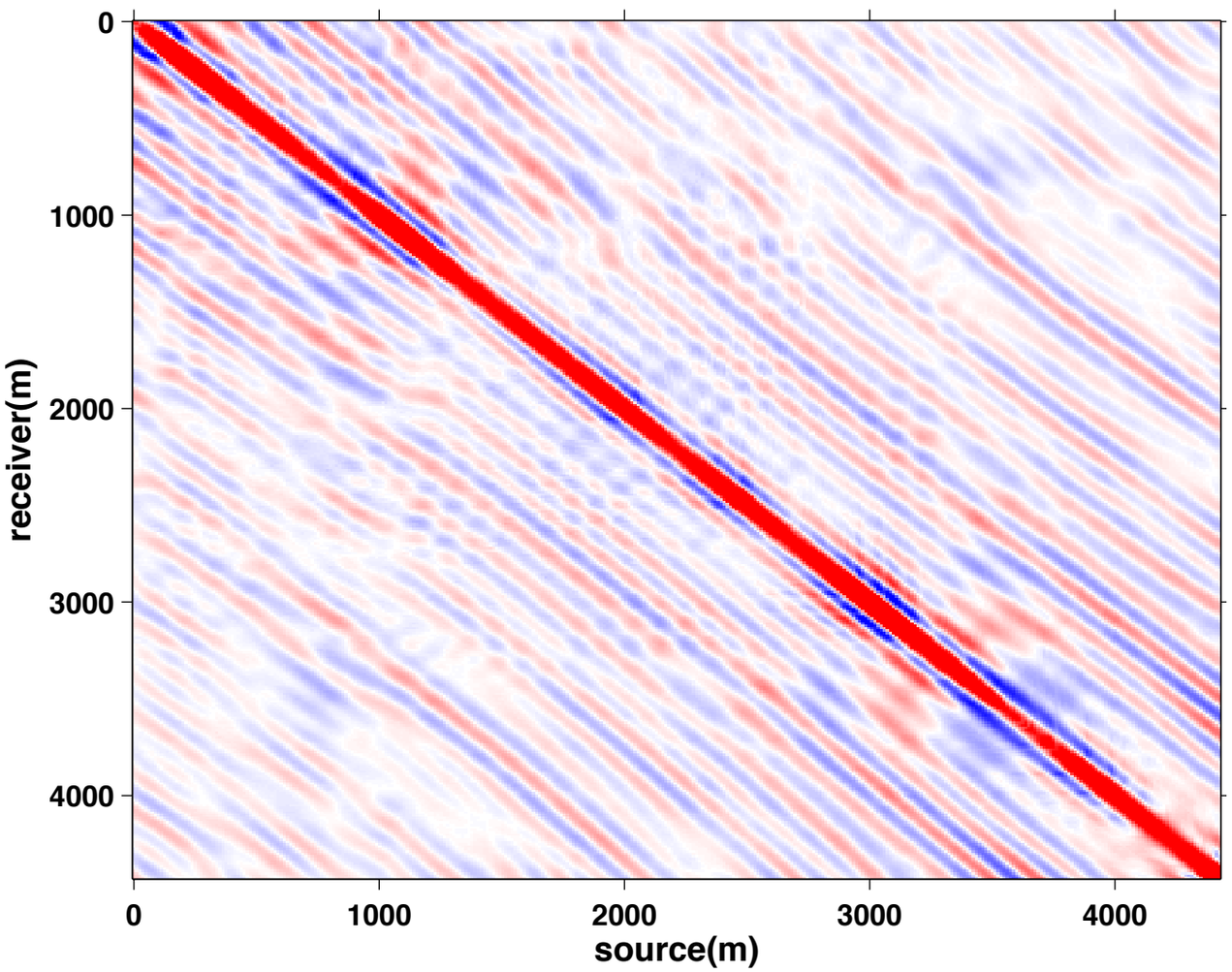
- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Low-rank structure

2-D acquisition

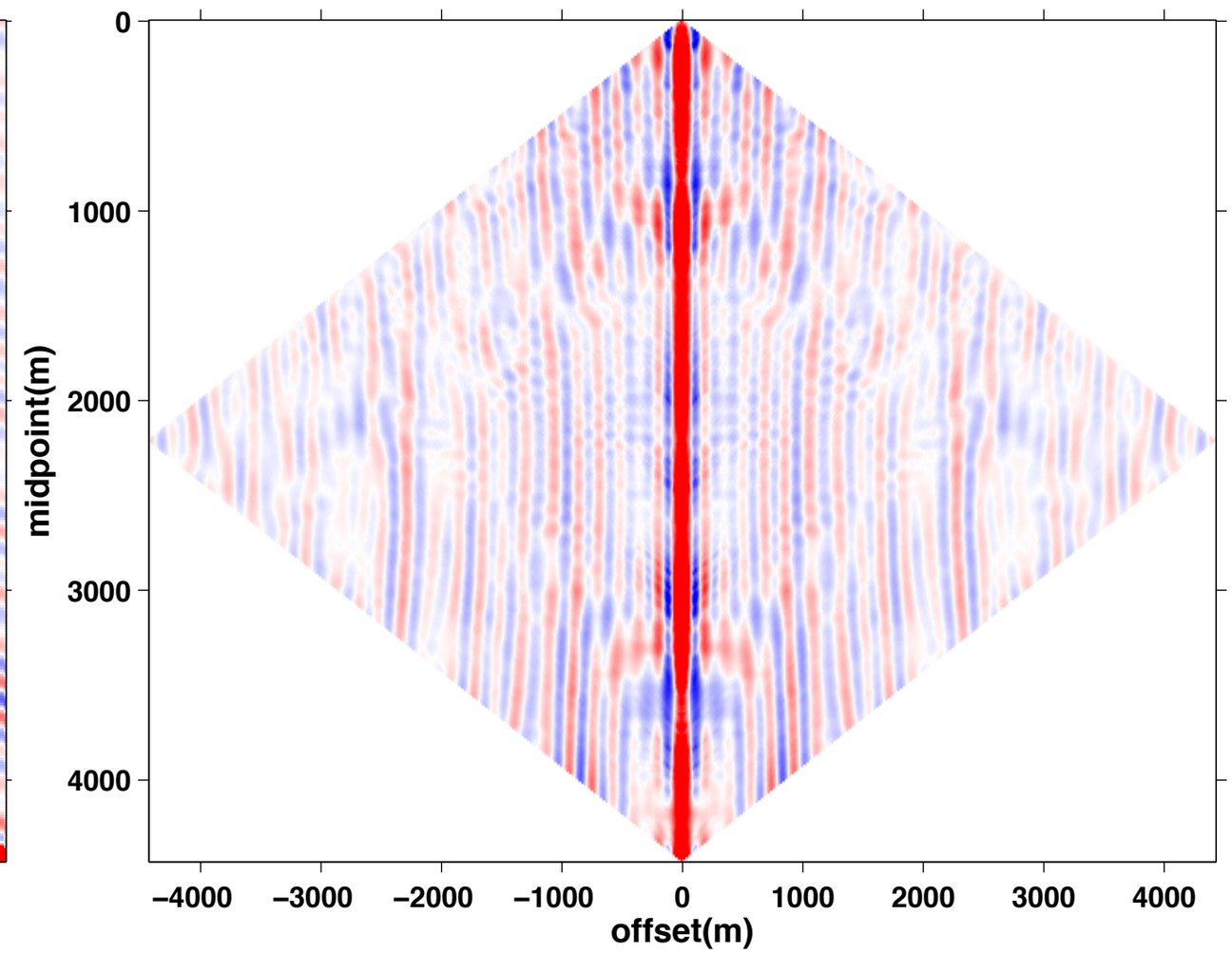
acquisition domain

[source-receiver]



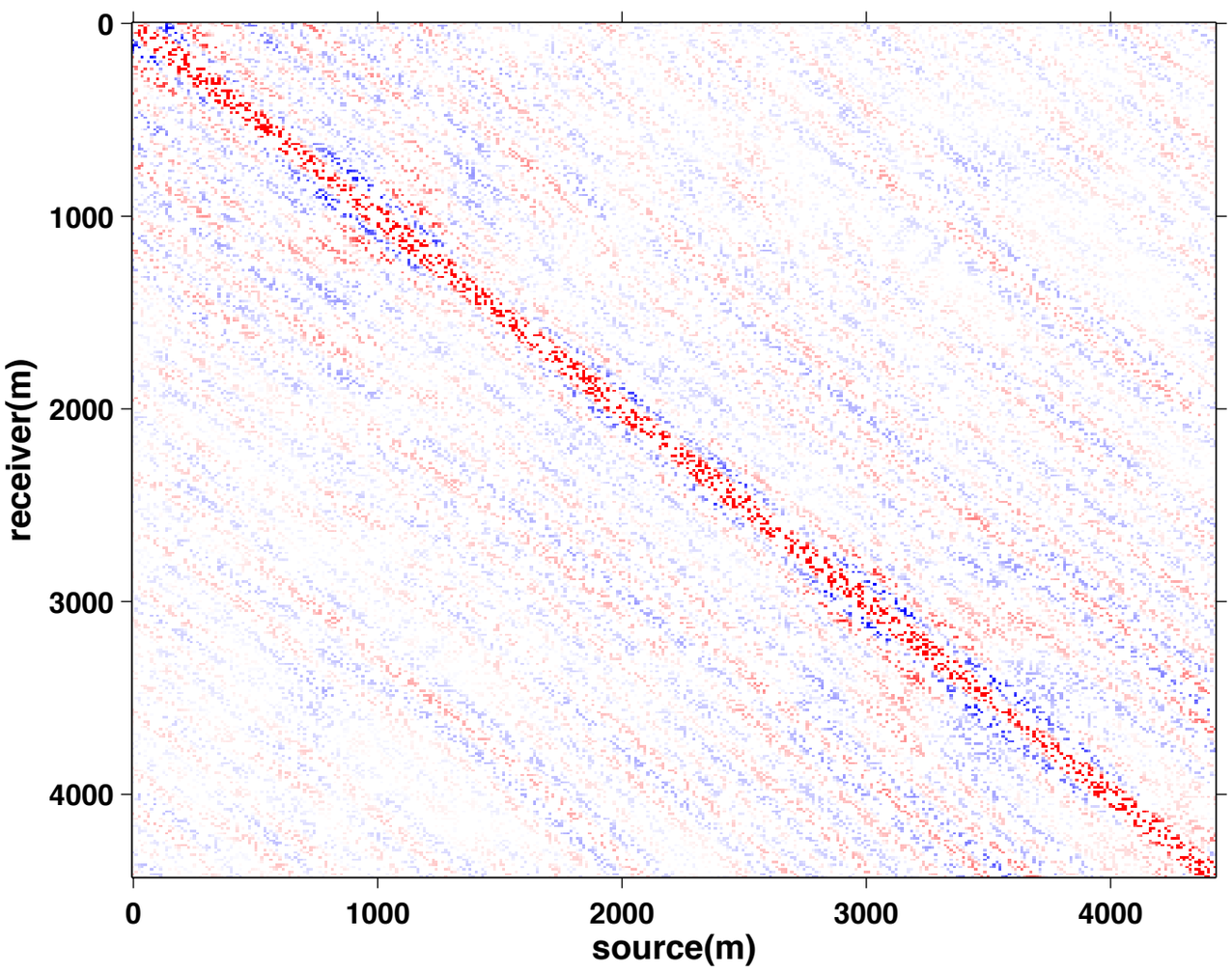
transform domain

[midpoint-offset]

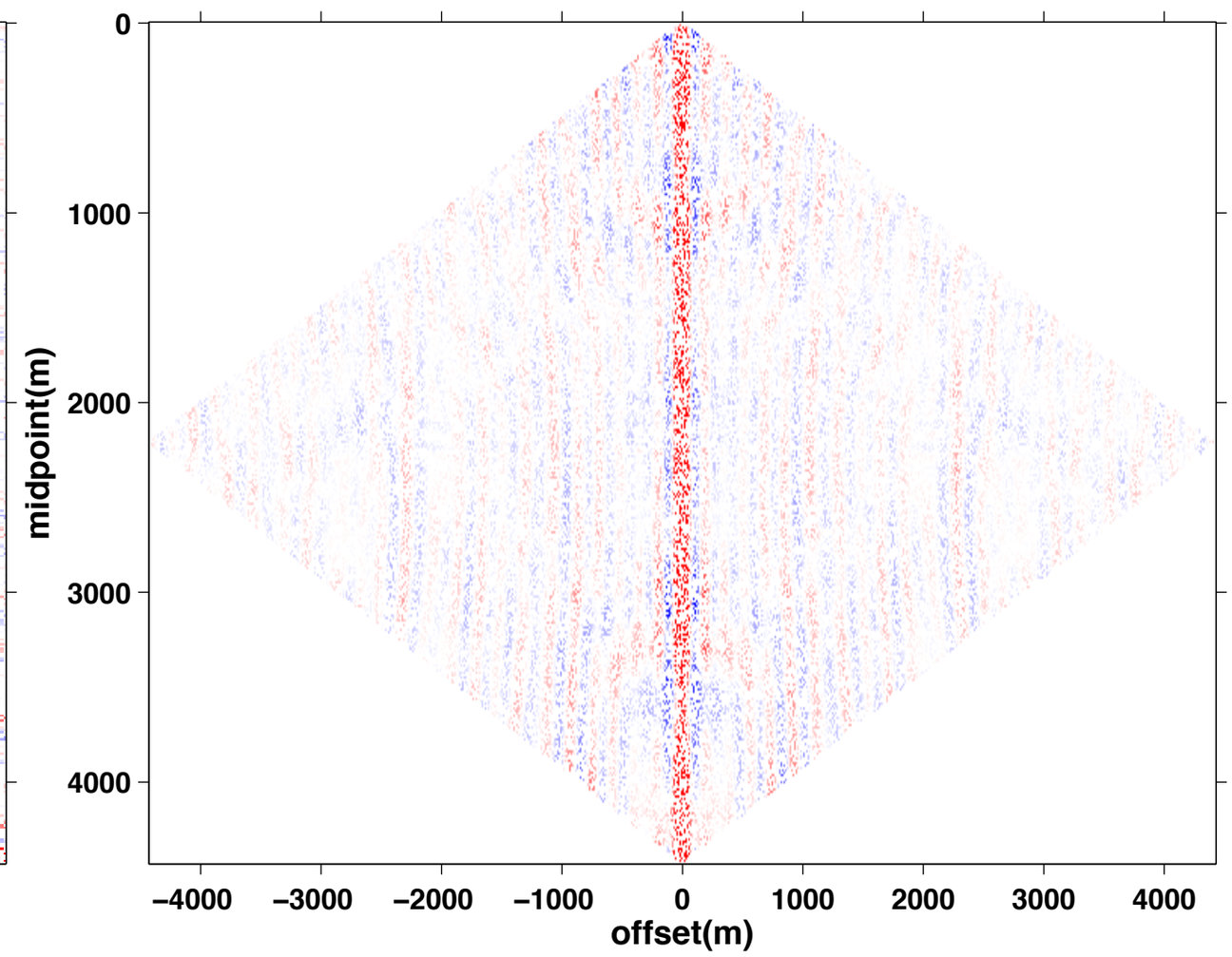


Matrix completion problem

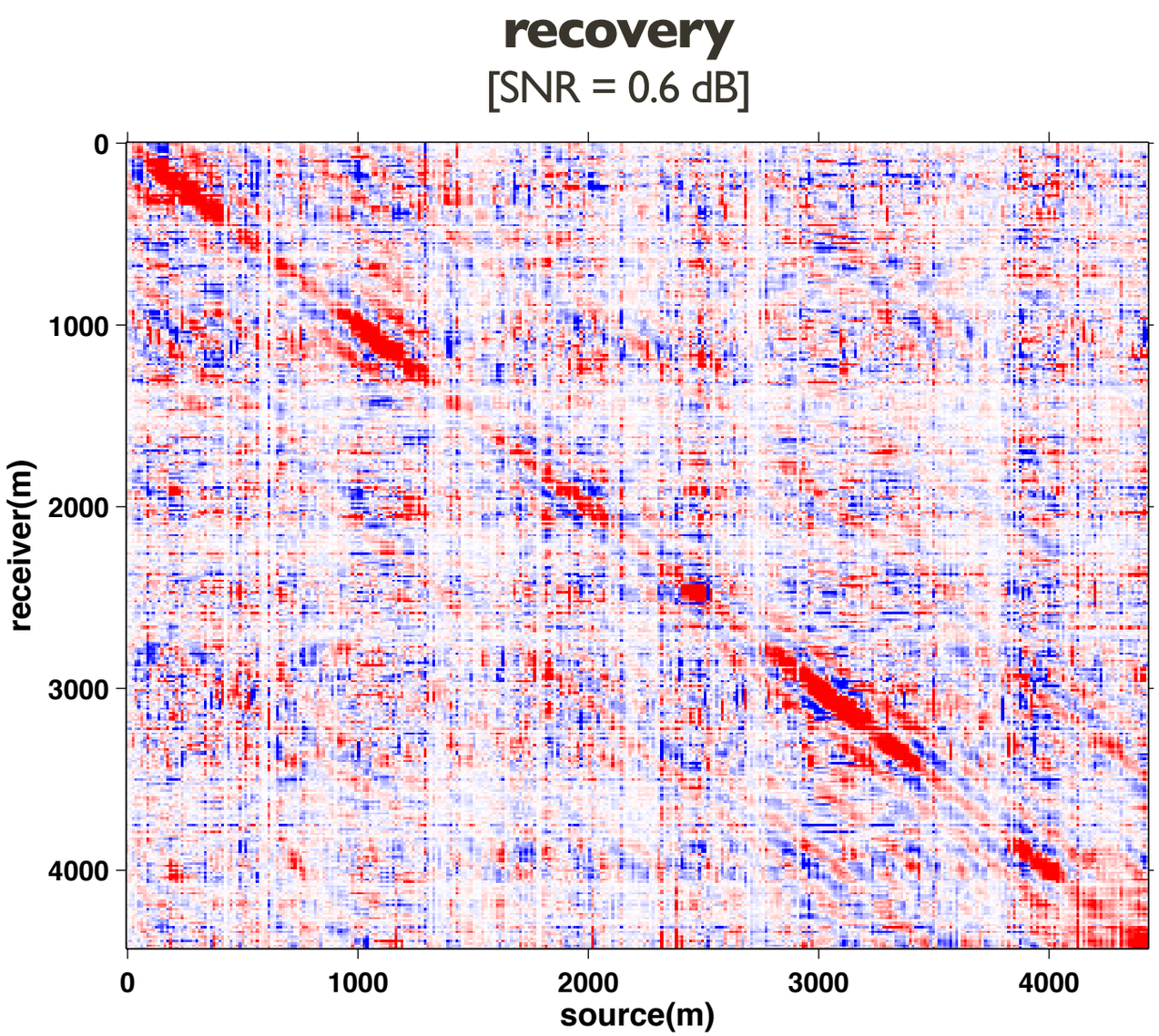
missing entries
[source-receiver]

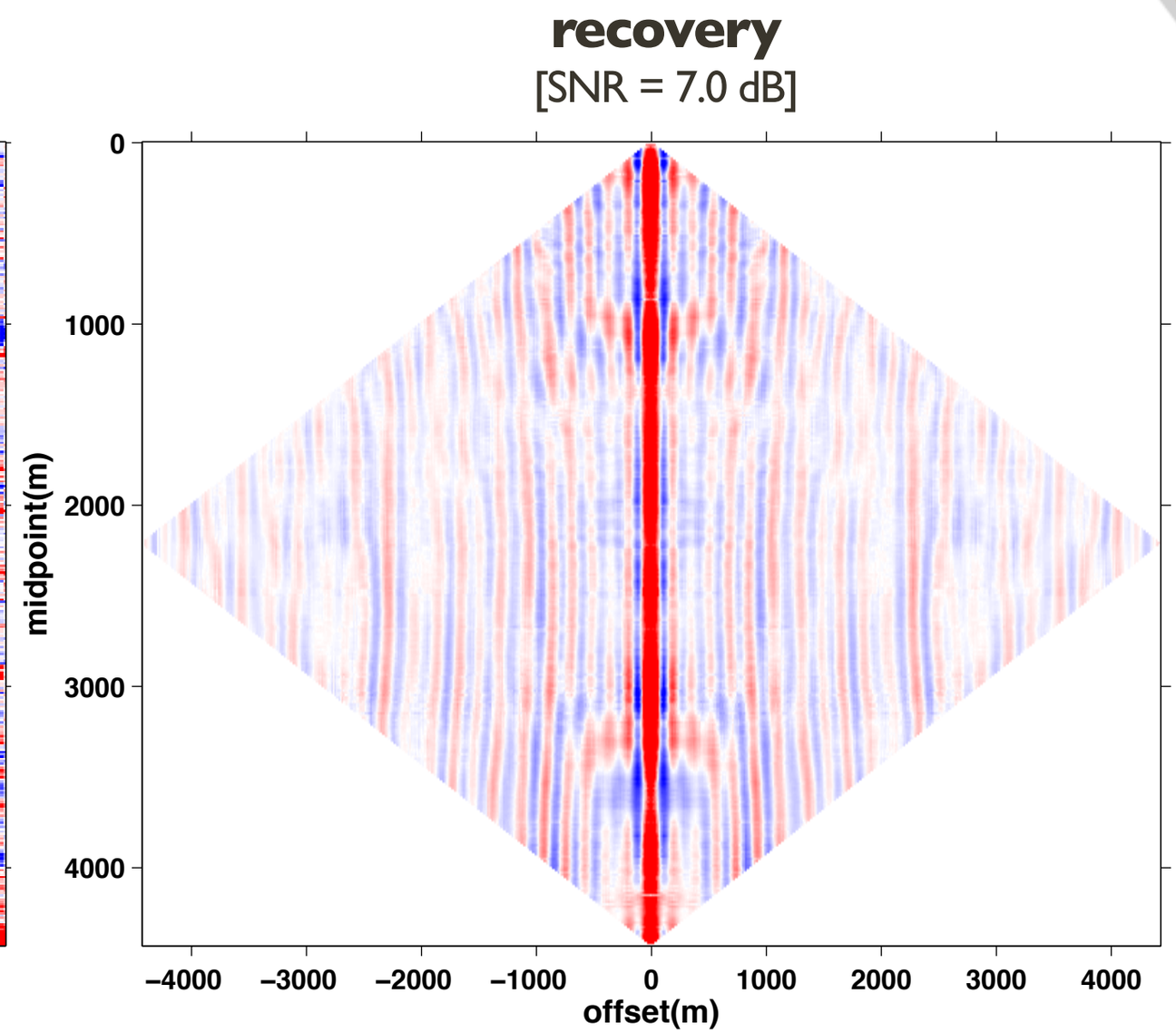


missing entries
[midpoint-offset]



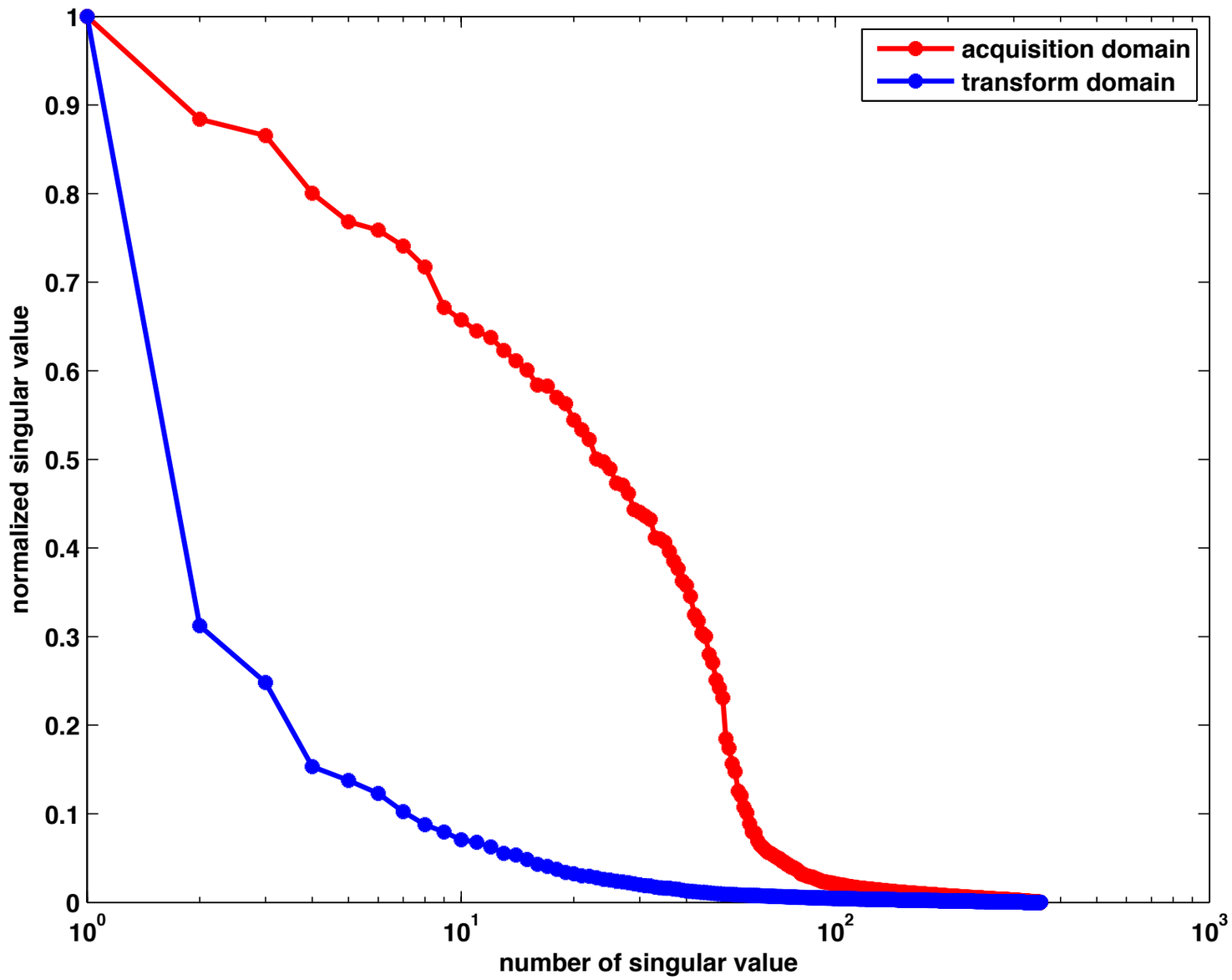
Low-rank interpolation





Singular value decay

2-D acquisition

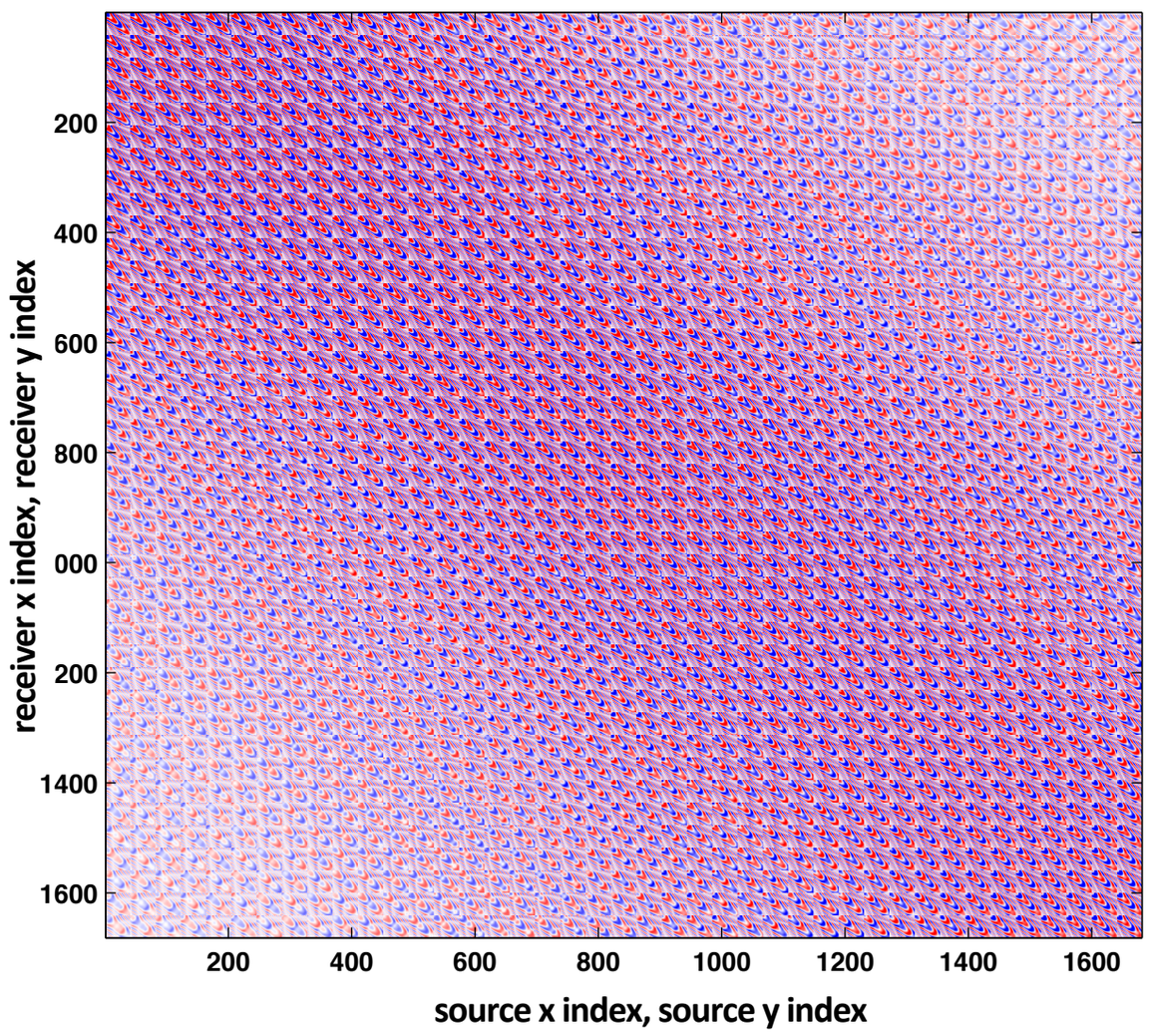


Low-rank structure

3-D acquisition

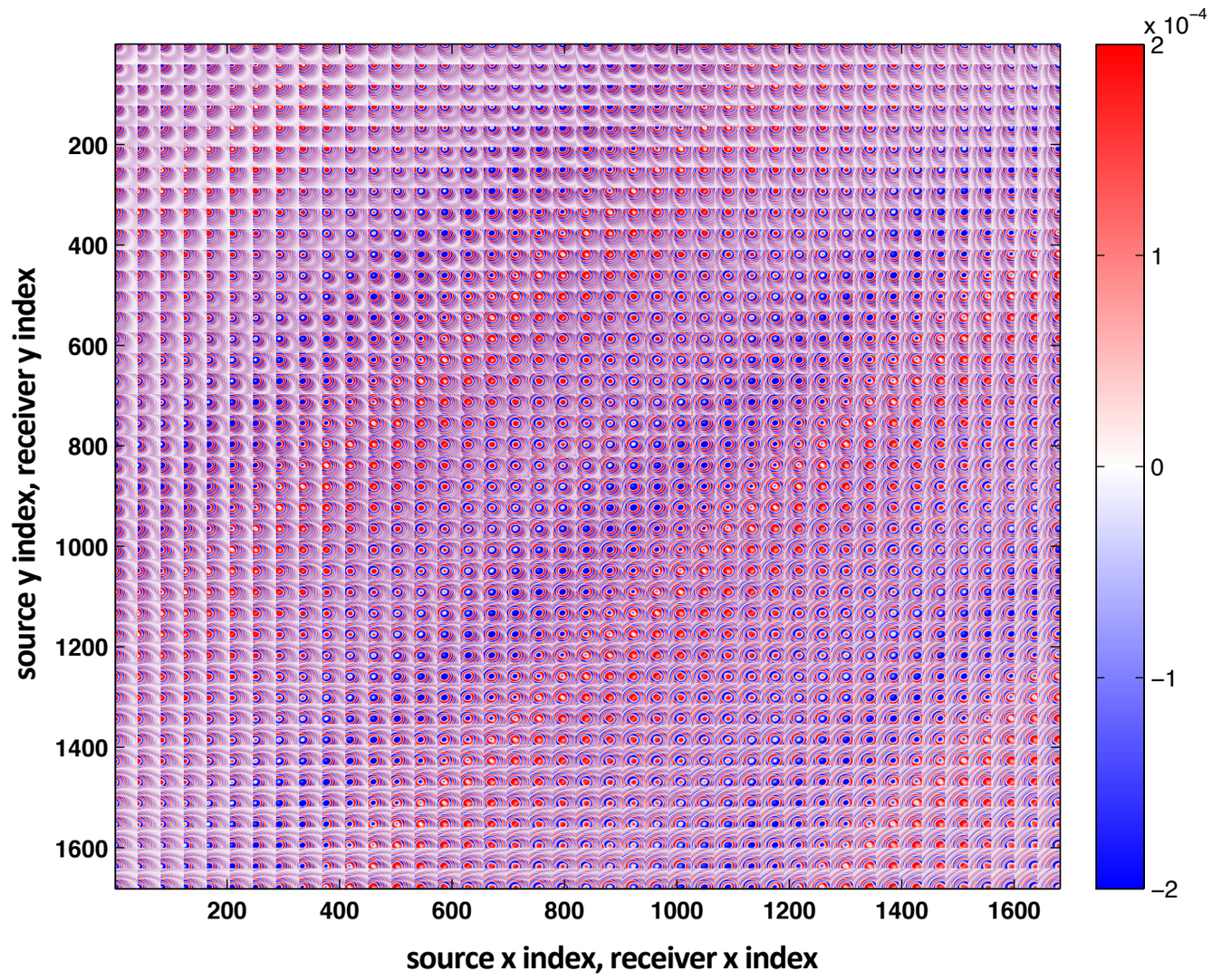
acquisition domain

[(source x,source y) & (receiver x, receiver y)]



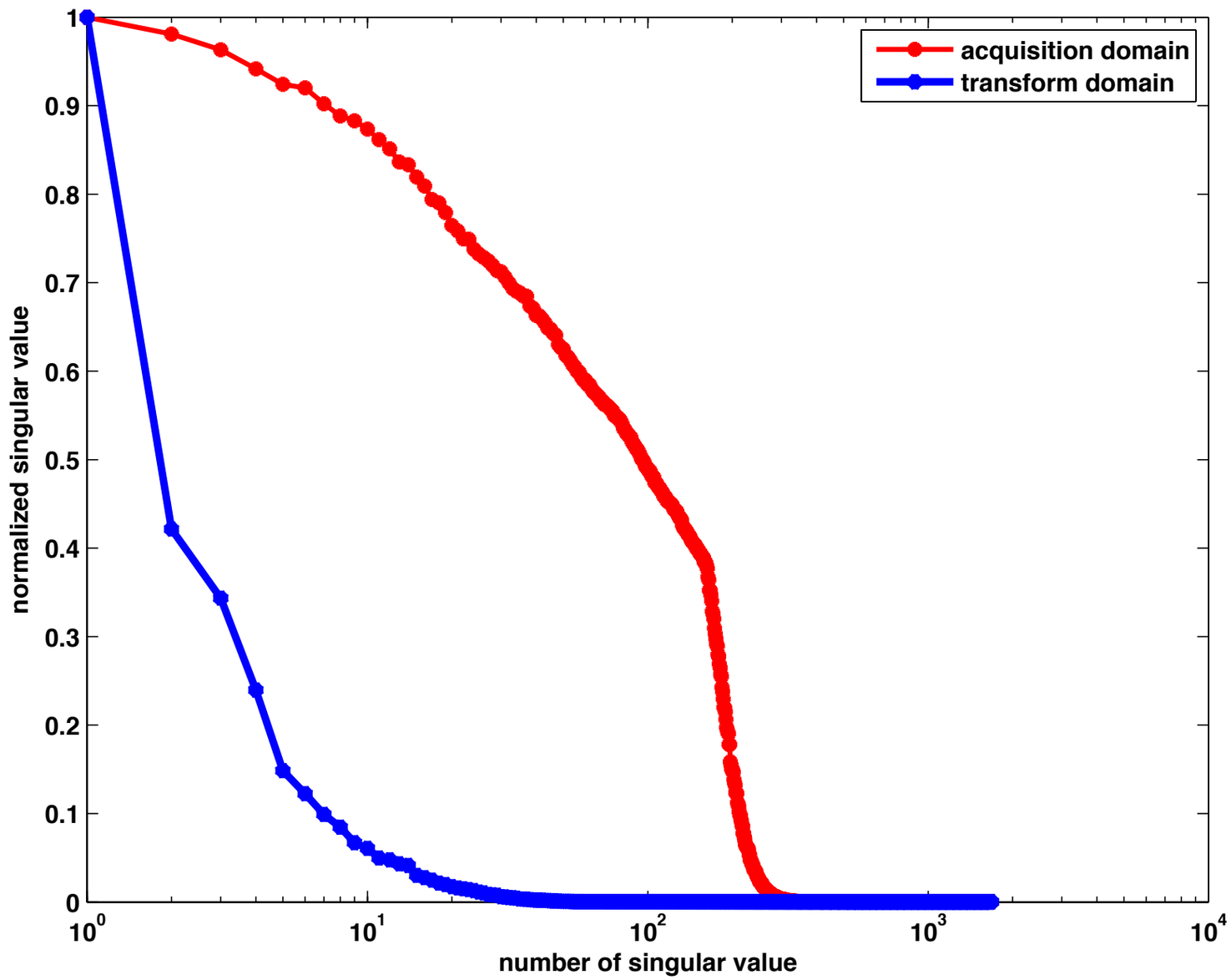
transform domain

[(source x,receiver x) & (source y, receiver y)]



Singular value decay

3-D acquisition



Matrix completion

- signal structure
  - *low rank/fast decay* of singular values

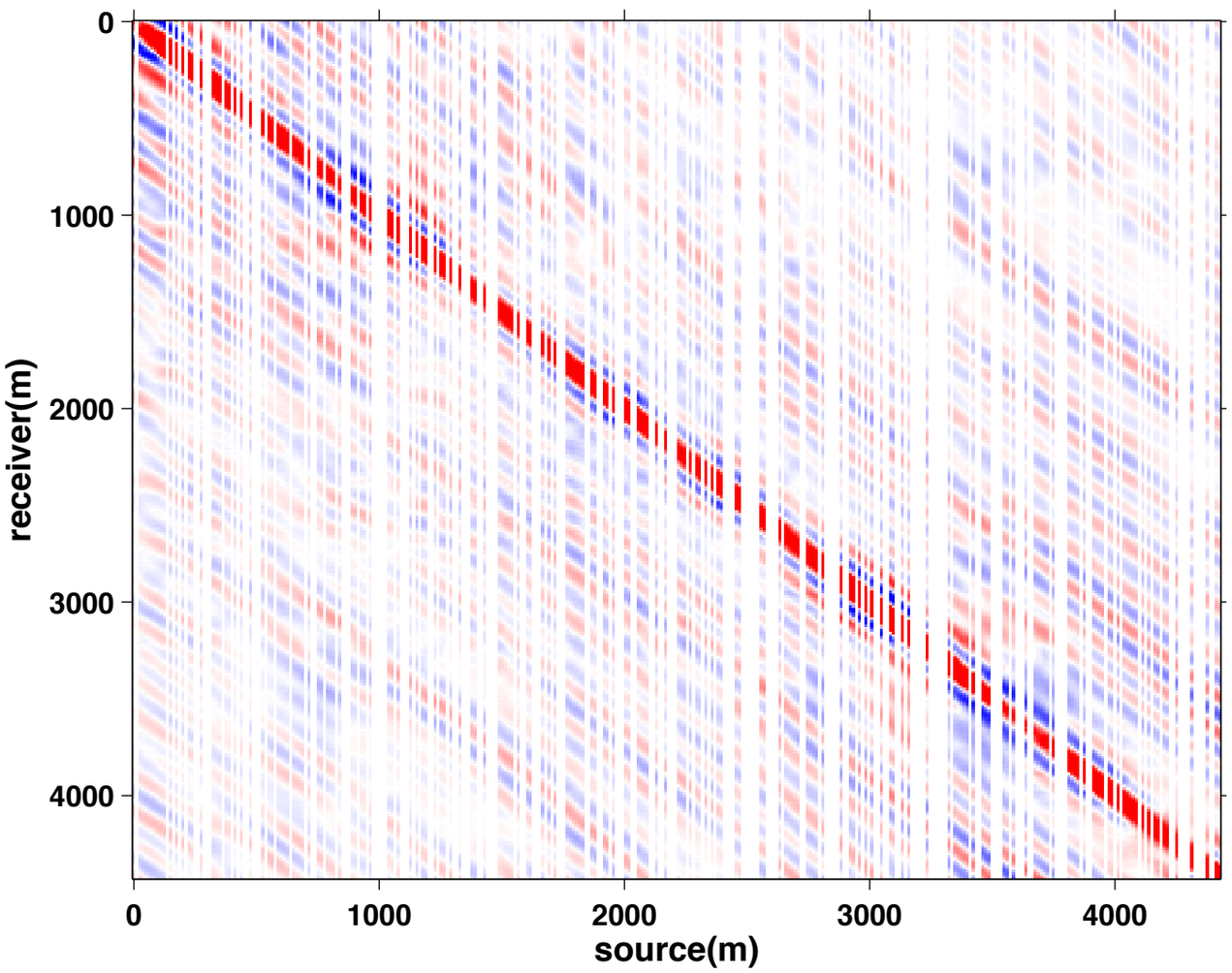
- sampling scheme
  - missing data *increase* rank in "transform domain"

- recovery using *rank penalization* scheme

2-D Acquisition
[randomized sampling]

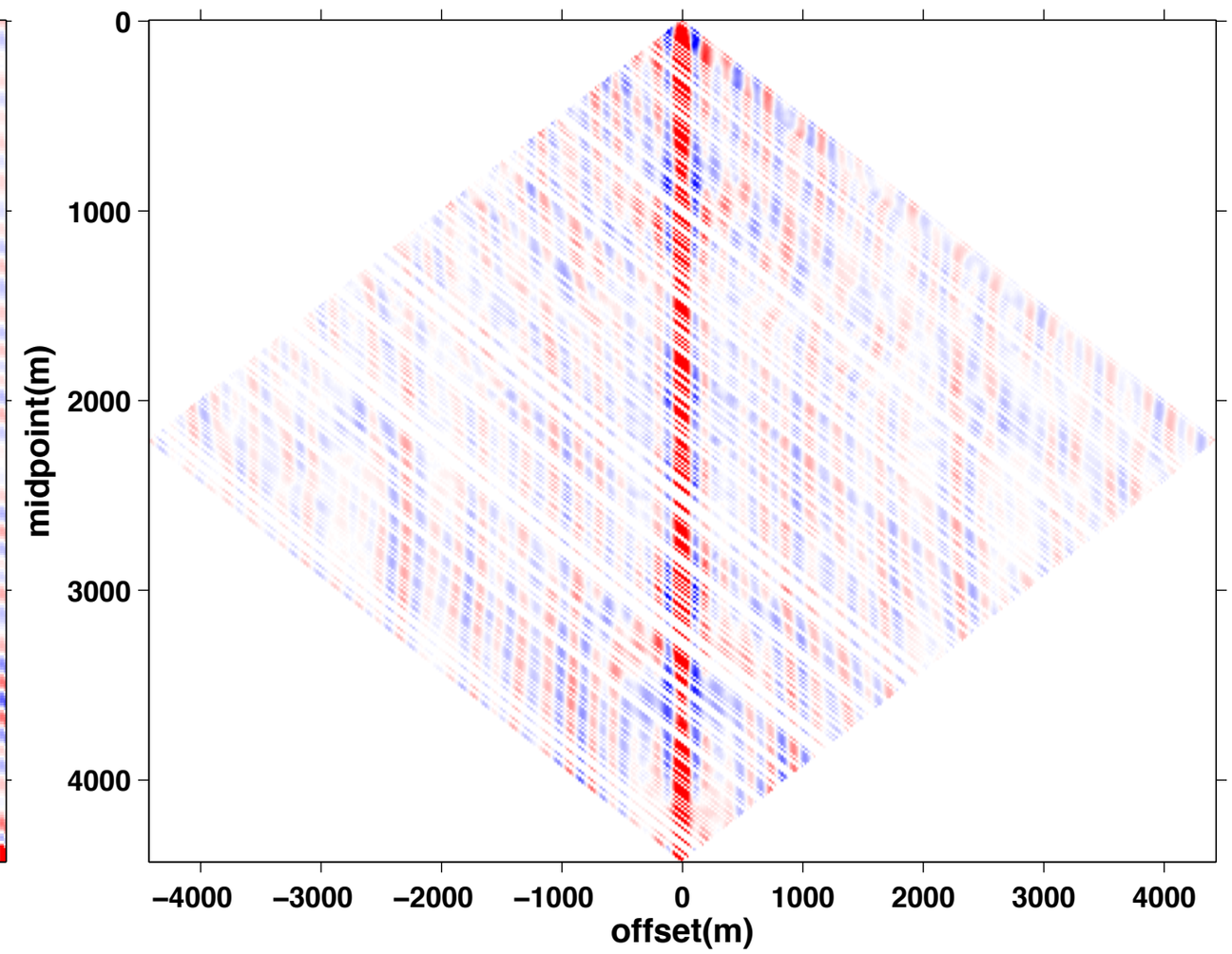
acquisition domain

missing columns *do not* increase rank

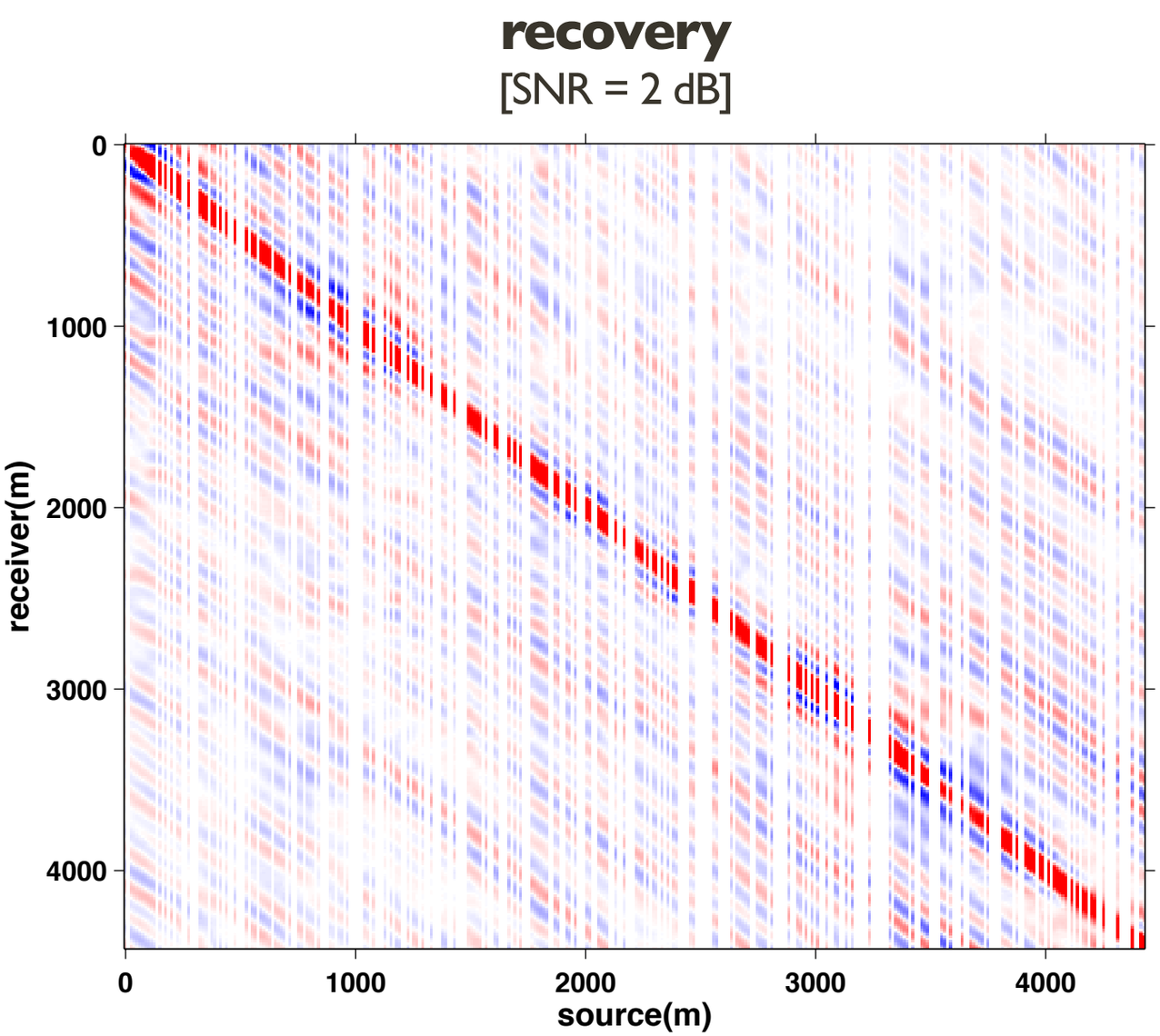


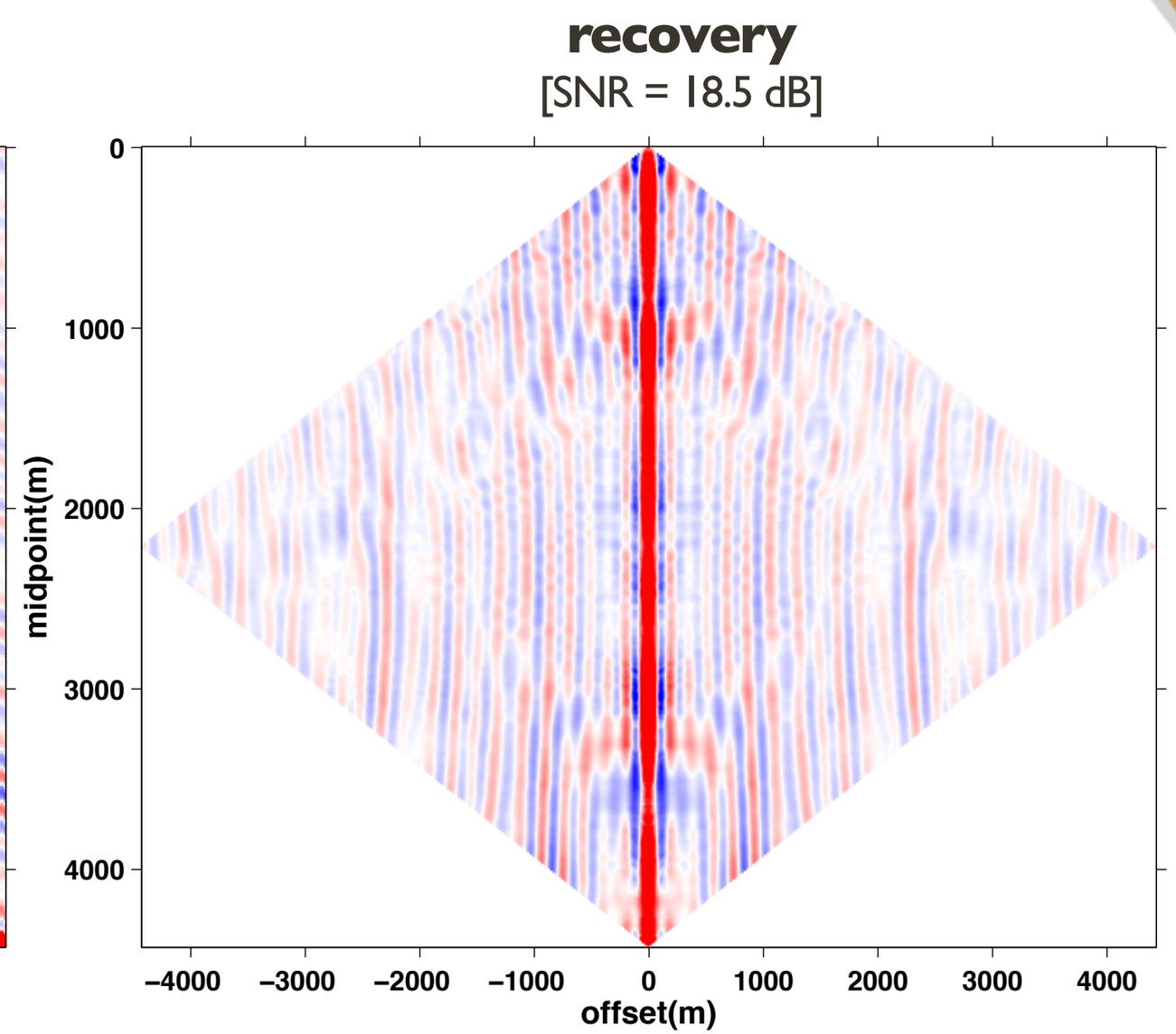
transform domain

missing columns *do* increase rank



Low-rank interpolation

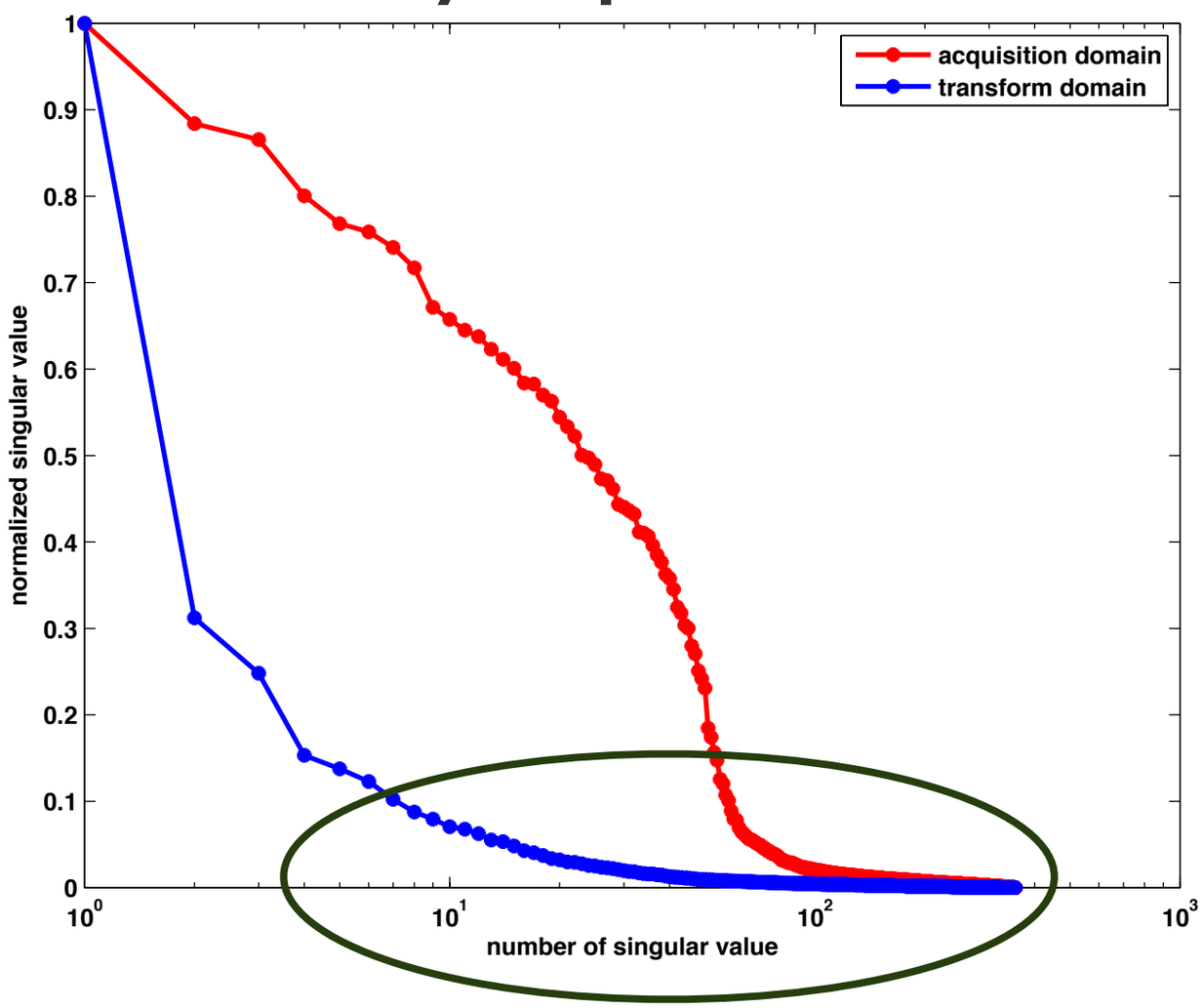




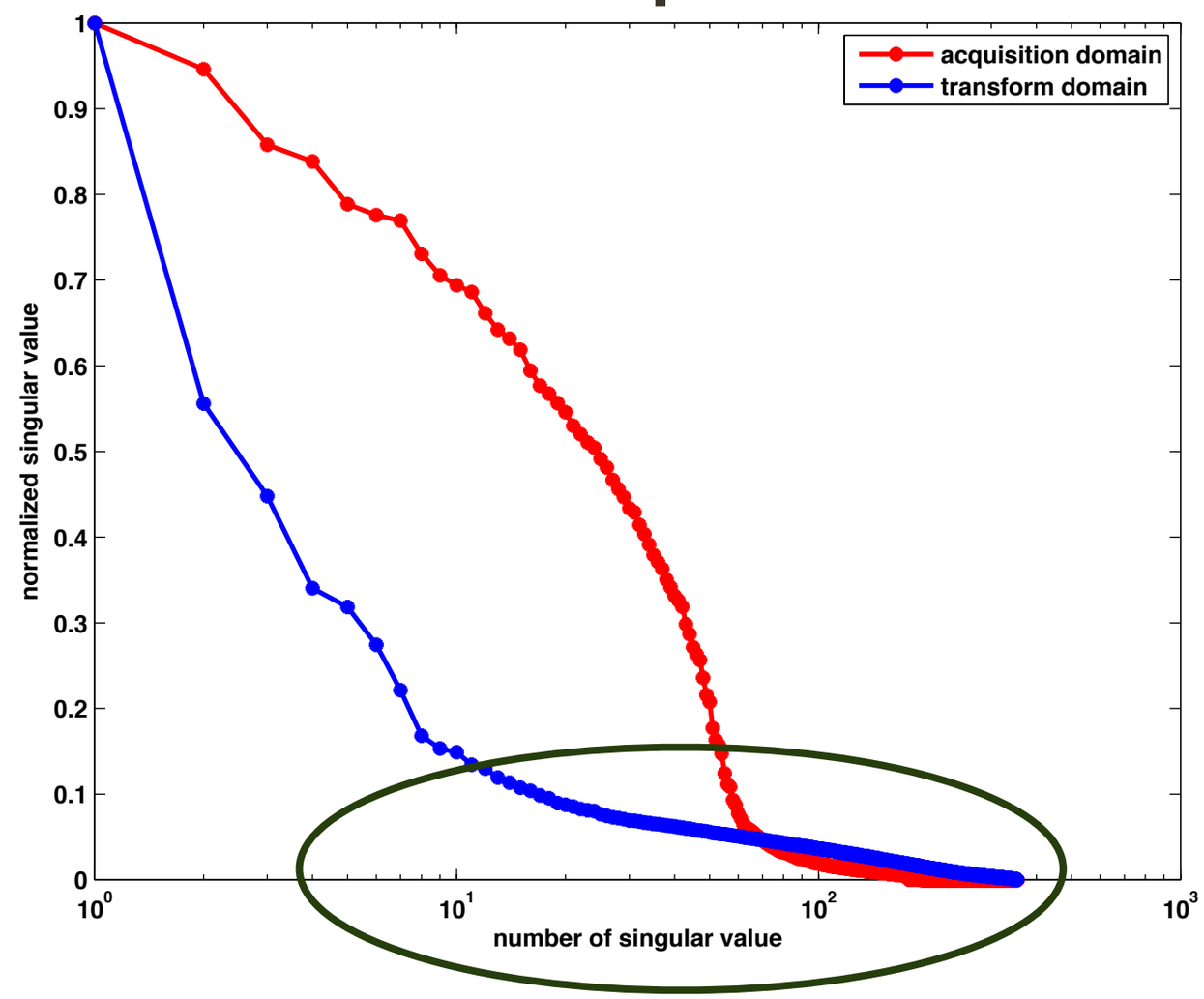
Randomized sampling

singular value decay

fully sampled data



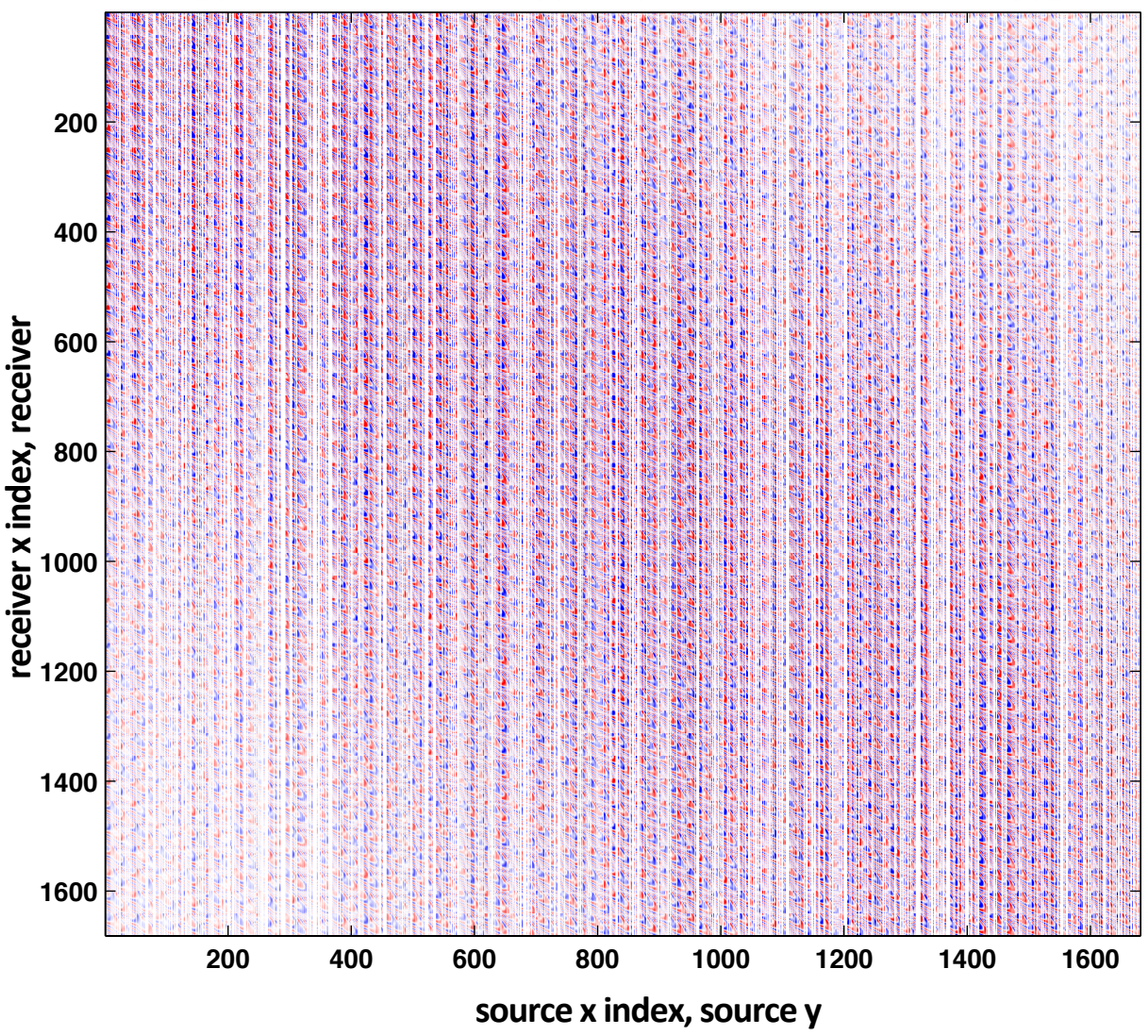
random sampled data



3-D Acquisition
[randomized sampling]

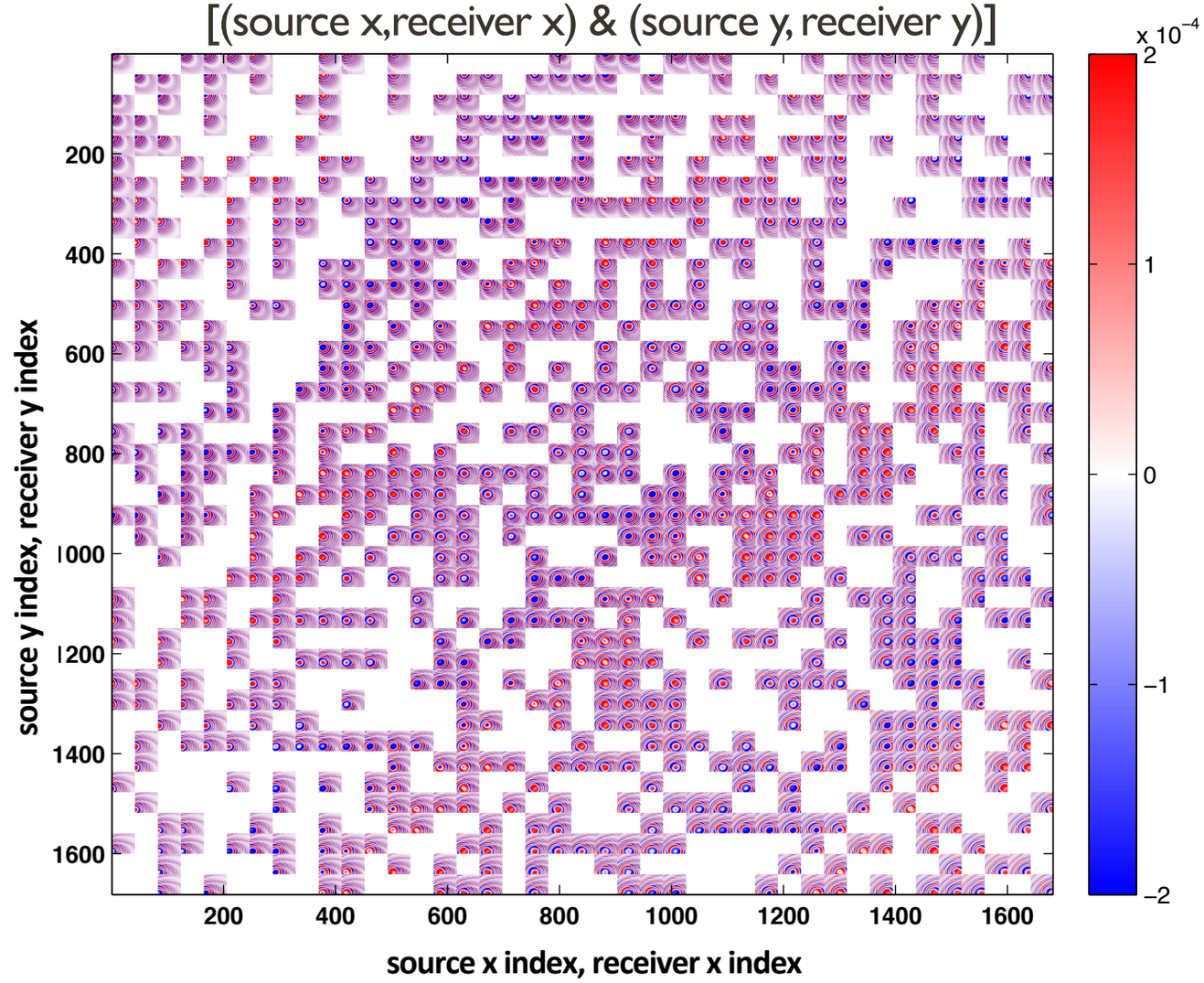
acquisition domain

[(source x,source y) & (receiver x, receiver y)]



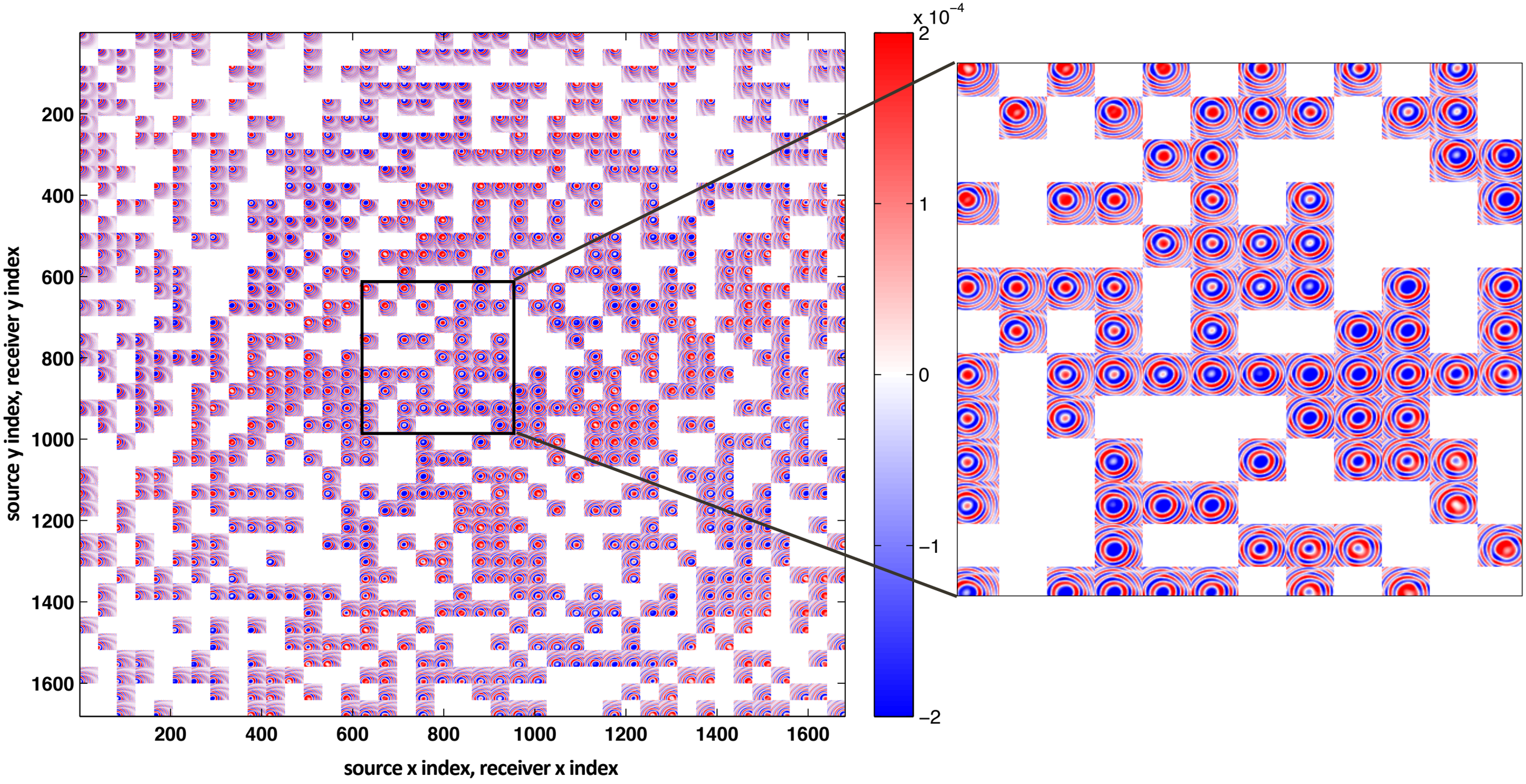
transform domain

[(source x,receiver x) & (source y, receiver y)]



3-D Acquisition

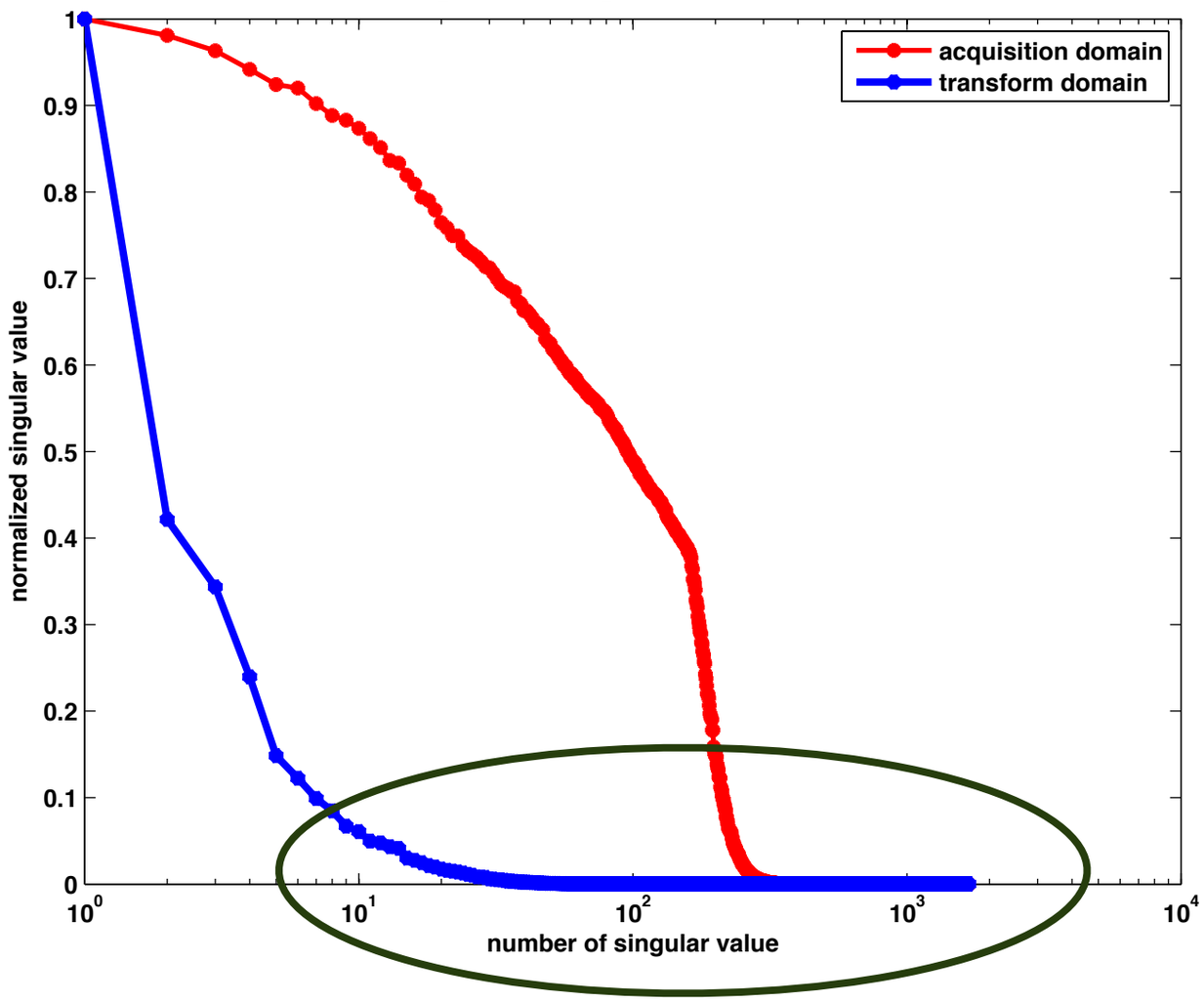
[randomized sampling - “transform” domain]



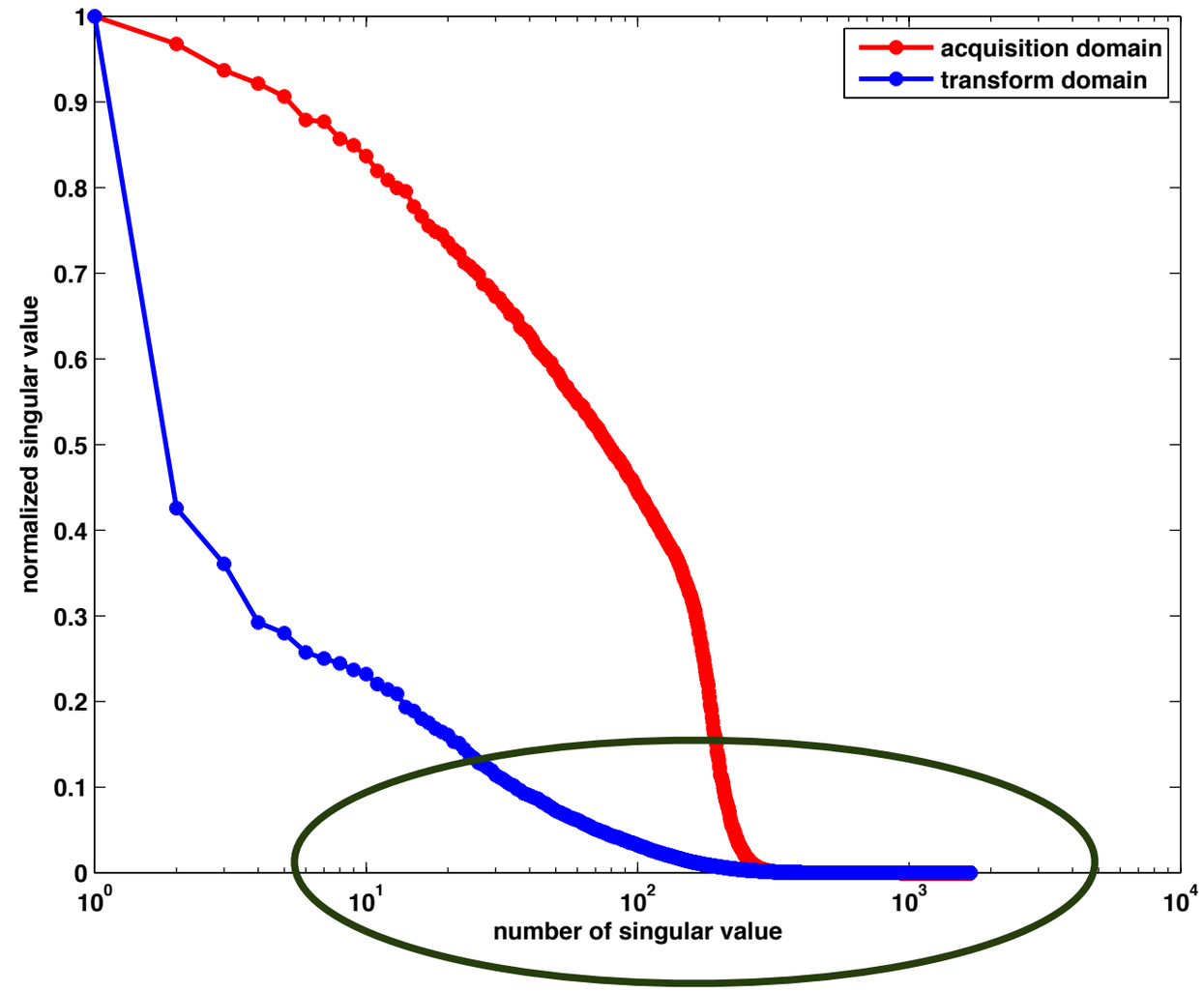
Randomized sampling

singular value decay

fully sampled data



random sampled data



Low-rank domain

- 2-D acquisition
  - midpoint-offset

- 3-D acquisition
  - (source x, receiver x) and (source y, receiver y)
  - midpoint-offset-azimuth

Observations

- sampling become *incoherent* in "transform" domain
- *slow decay* of singular values in "transform" domain

Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Rank minimization

- given a set of measurements $\mathbf{b}$, aim is to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} \quad rank(\mathbf{X}) \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$rank(\mathbf{X}) =$ number of singular values of $\mathbf{X}$

- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator
$\mathbf{M}$: measurement operator
$\mathcal{S}^H$: transform operator

Rank minimization

- prohibitively *expensive*
  - do not know rank value in advance
  - search over all possible values of rank

- instead solve nuclear-norm minimization
  - convex relaxation of rank-minimization

[Recht et. al. 2010]

[Recht et. al. 2010]

SLIM

Nuclear-norm minimization

- we want to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

where λ_i are the *singular* values

Challenges

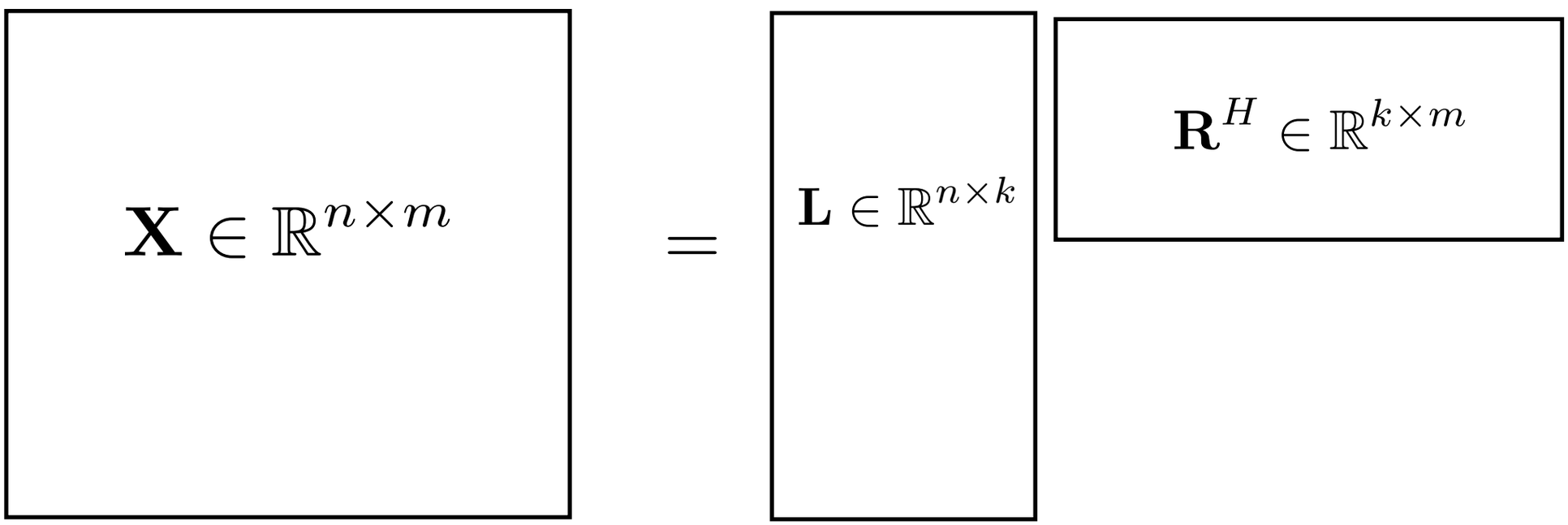
- requires repeated application of *SVD* for projection

- expensive to compute for large system
  - curse of dimensionality

- can we exploit rank structure "*SVD* free"

[Rennie and Srebro 2005, Lee et. al. 2010, Recht and Re 2011]

Factorized formulation



$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$

[Berg and Friedlander 2008, Aravkin et al. 2012b]

Factorized formulation

- reformulate $BPDN_\sigma$ formulation

$$\min_{\mathbf{L},\mathbf{R}} ||\mathbf{L}\mathbf{R}^H||_* \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \quad \text{s.t.} \ \|\mathbf{L}\mathbf{R}^H\|_* \leq \tau$$

where τ is a rank regularization parameter

[Rennie and Srebro 2005]

Factorized formulation

- nuclear norm is then define as

$$\|\mathbf{L}\mathbf{R}^H\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2$$

where $\|\cdot\|_F^2$ is sum of squares of all entries

- choose rank k *explicitly* & avoid costly *SVD's*

Denoising

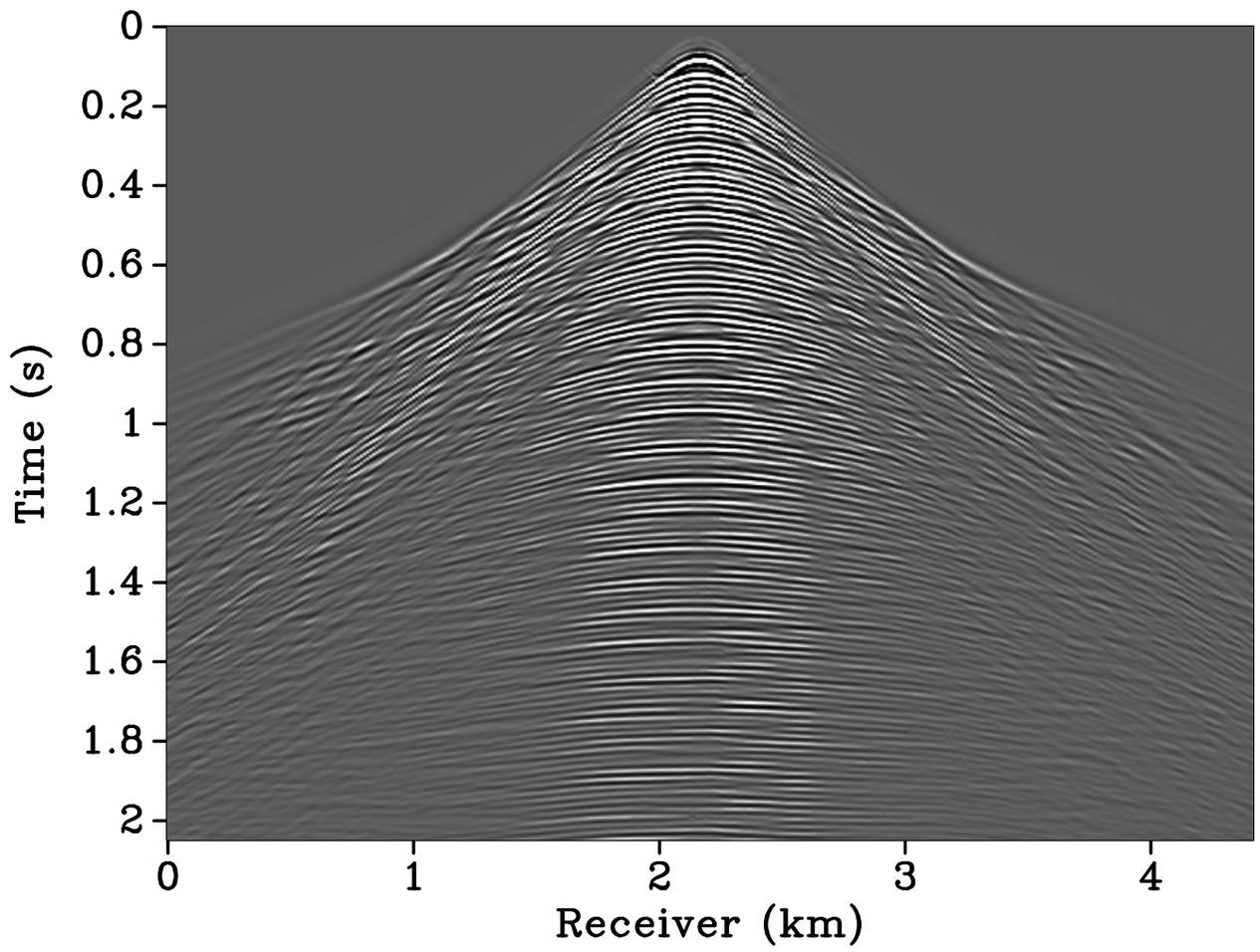
- Gulf of Suez
  - 2-D seismic line
  - 50% missing
  - 10% high amplitude noise

- Gulf of Mexico
  - monochromatic data slice at 2 Hz
  - low-amplitude noise

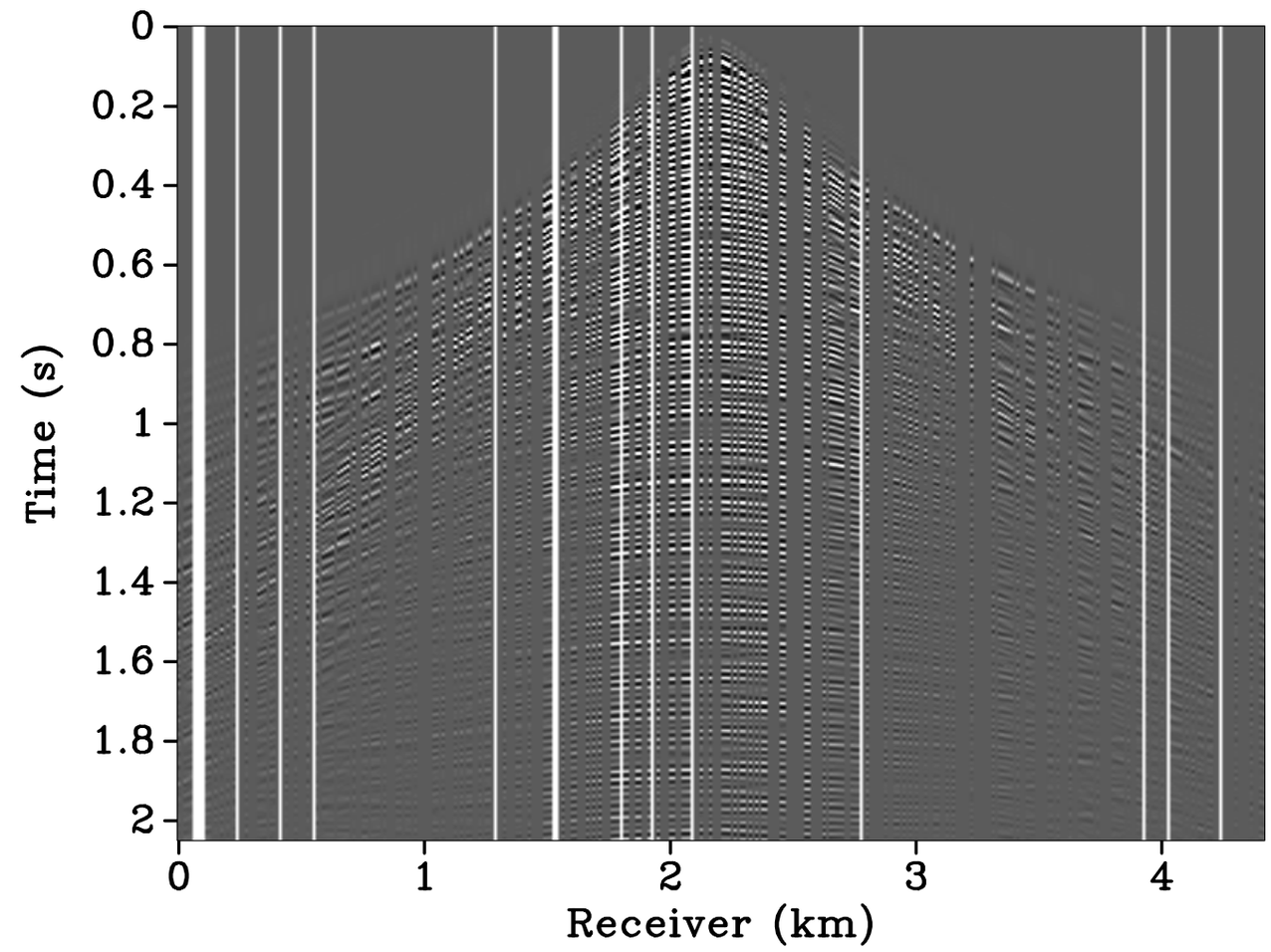
Gulf of Suez
[erratic gaussian noise]

- **noise level = 1000 times the maximum amplitude in data**

Regular sampled data



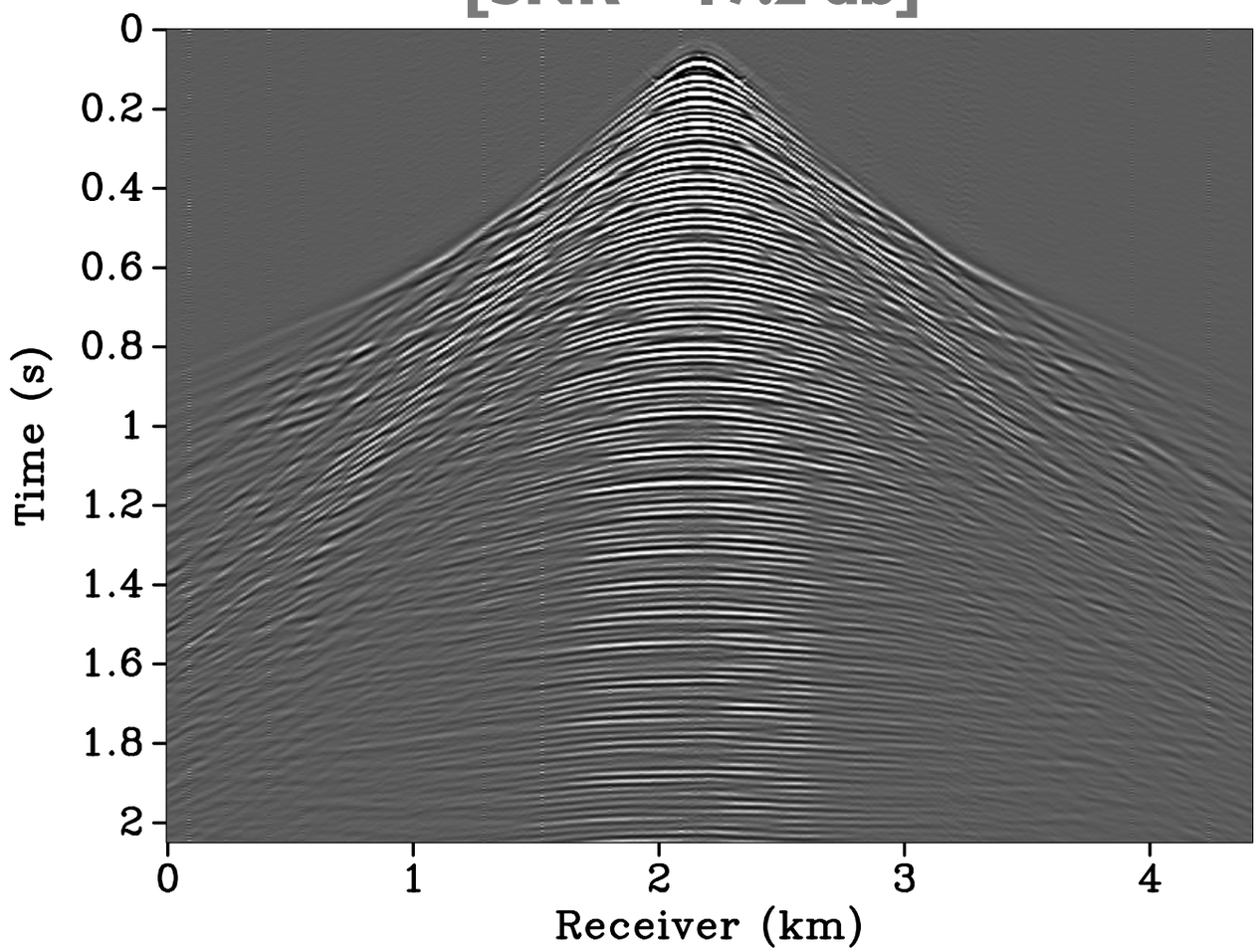
50% missing and 10% erratic noise



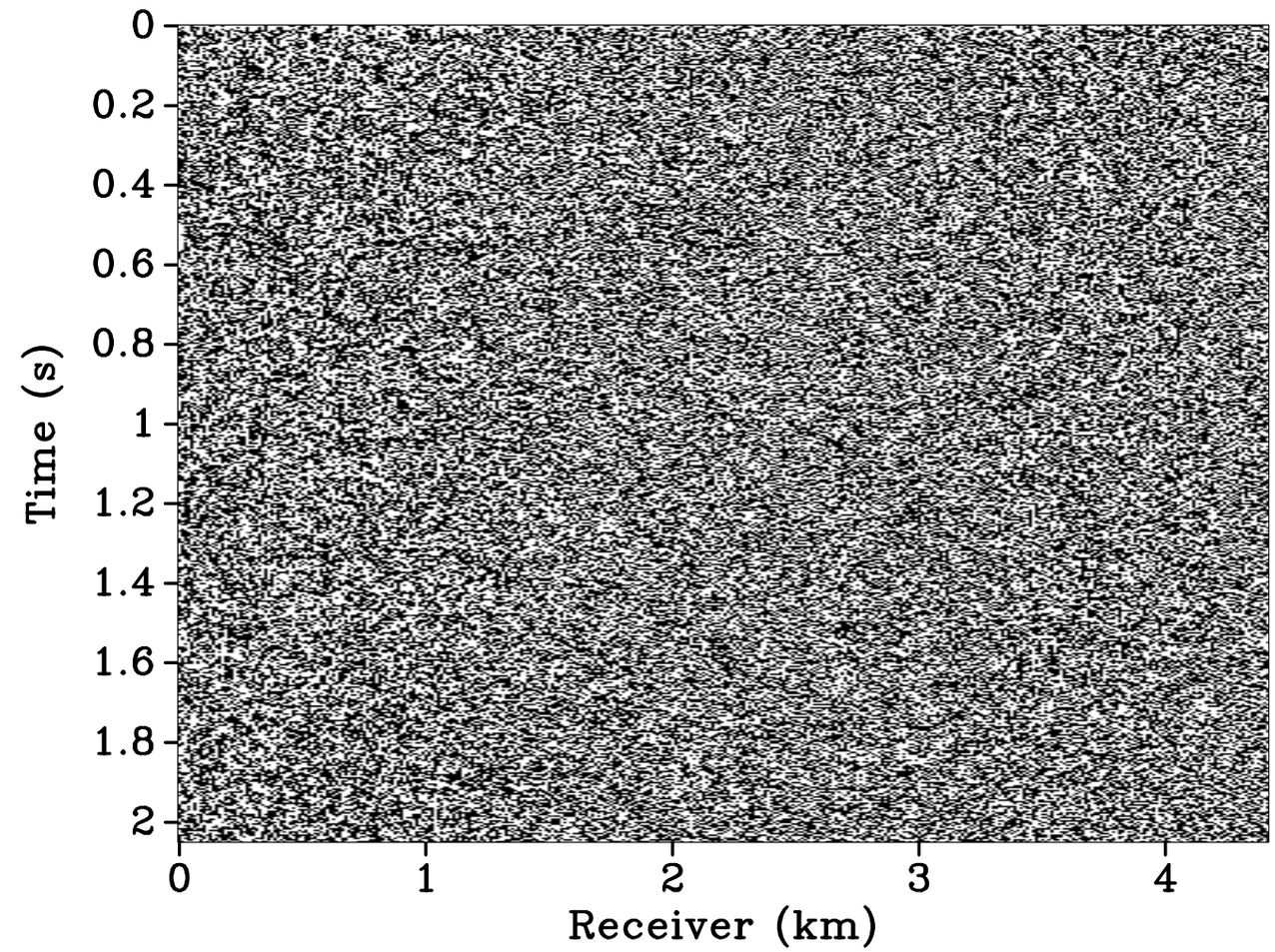
[Aravkin et. al. 2013]

Recovery - Shot Gather

Robust Penalty
[SNR = 17.2 db]

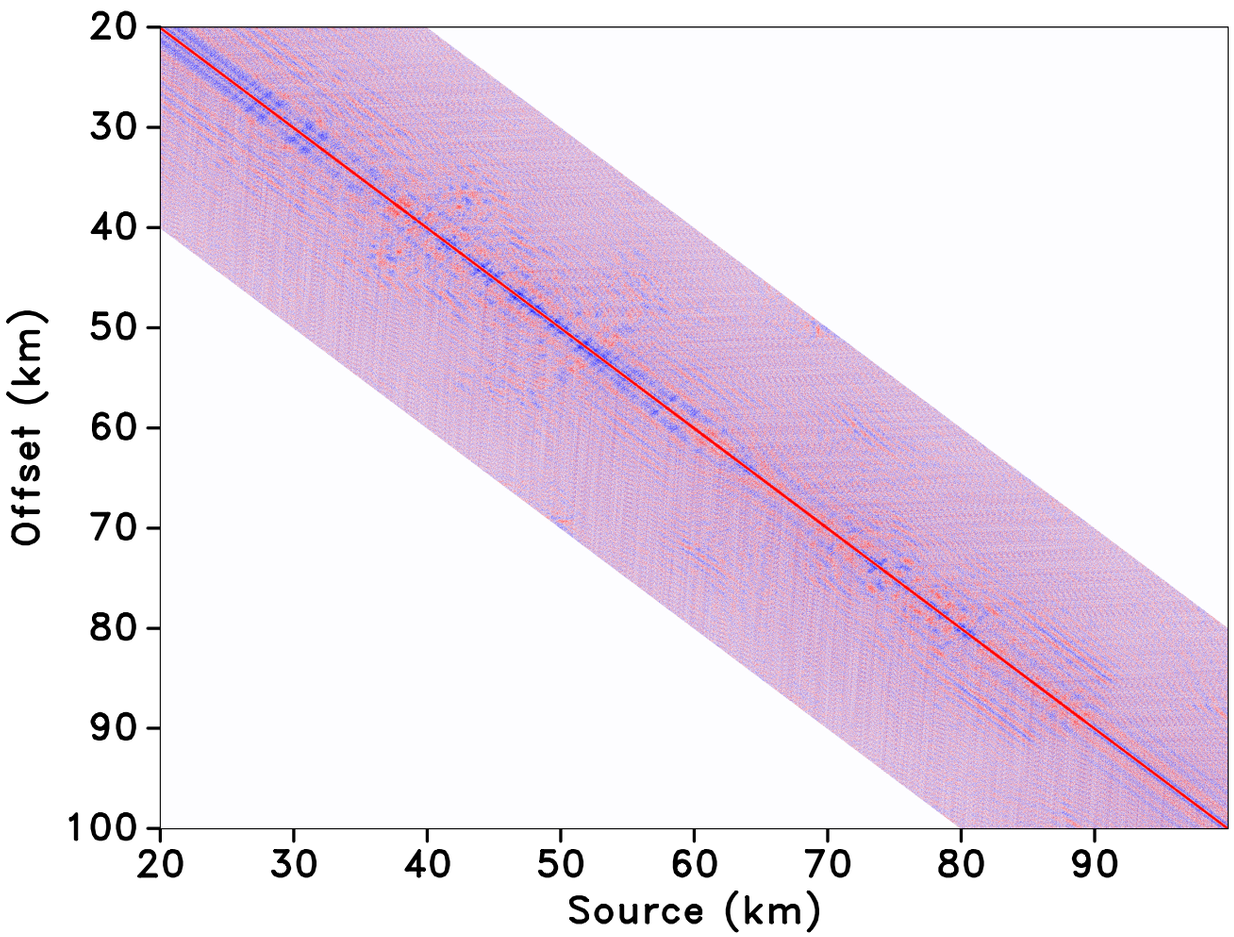


Least-squares Penalty

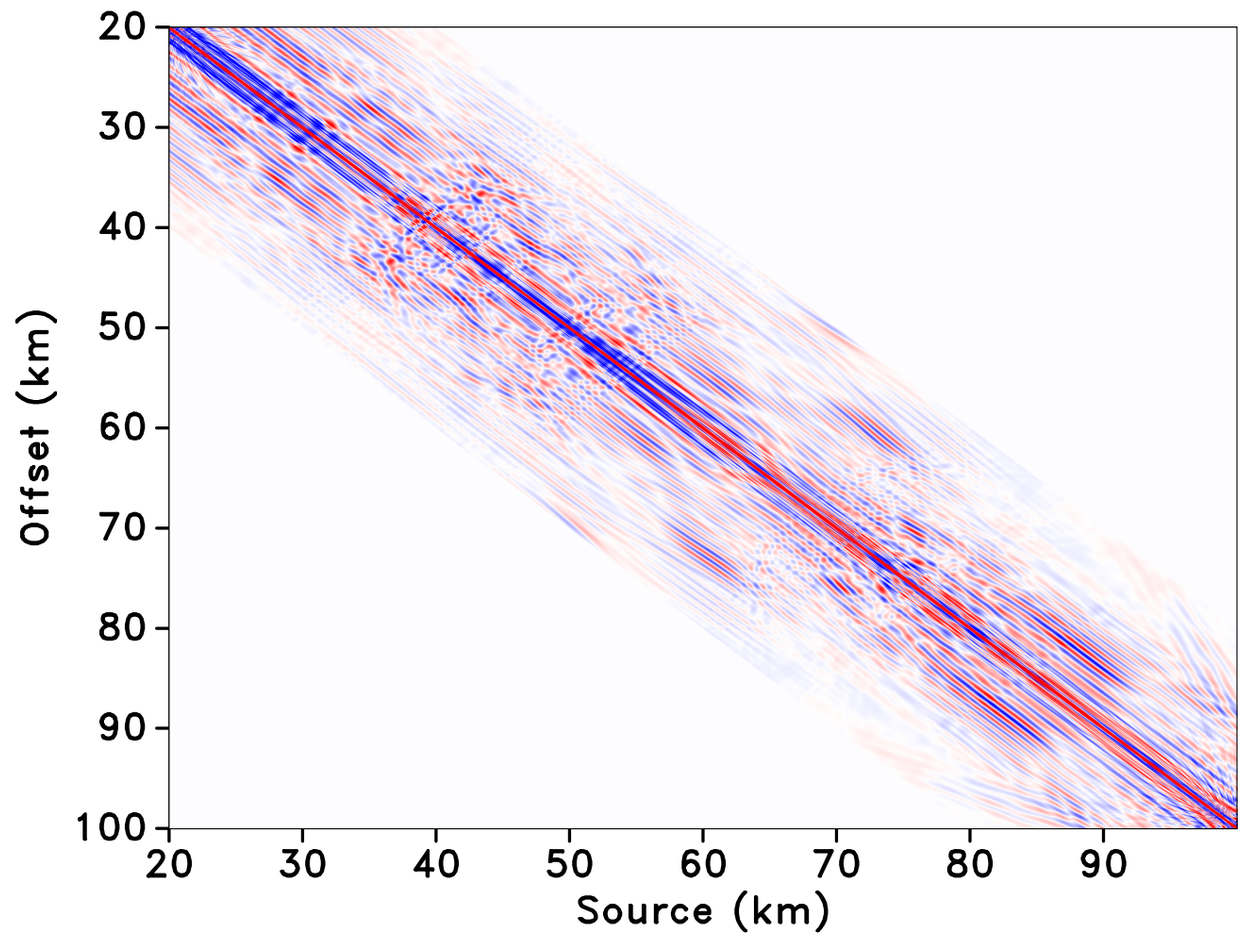


Gulf of Mexico
[2 Hz]

before denoising

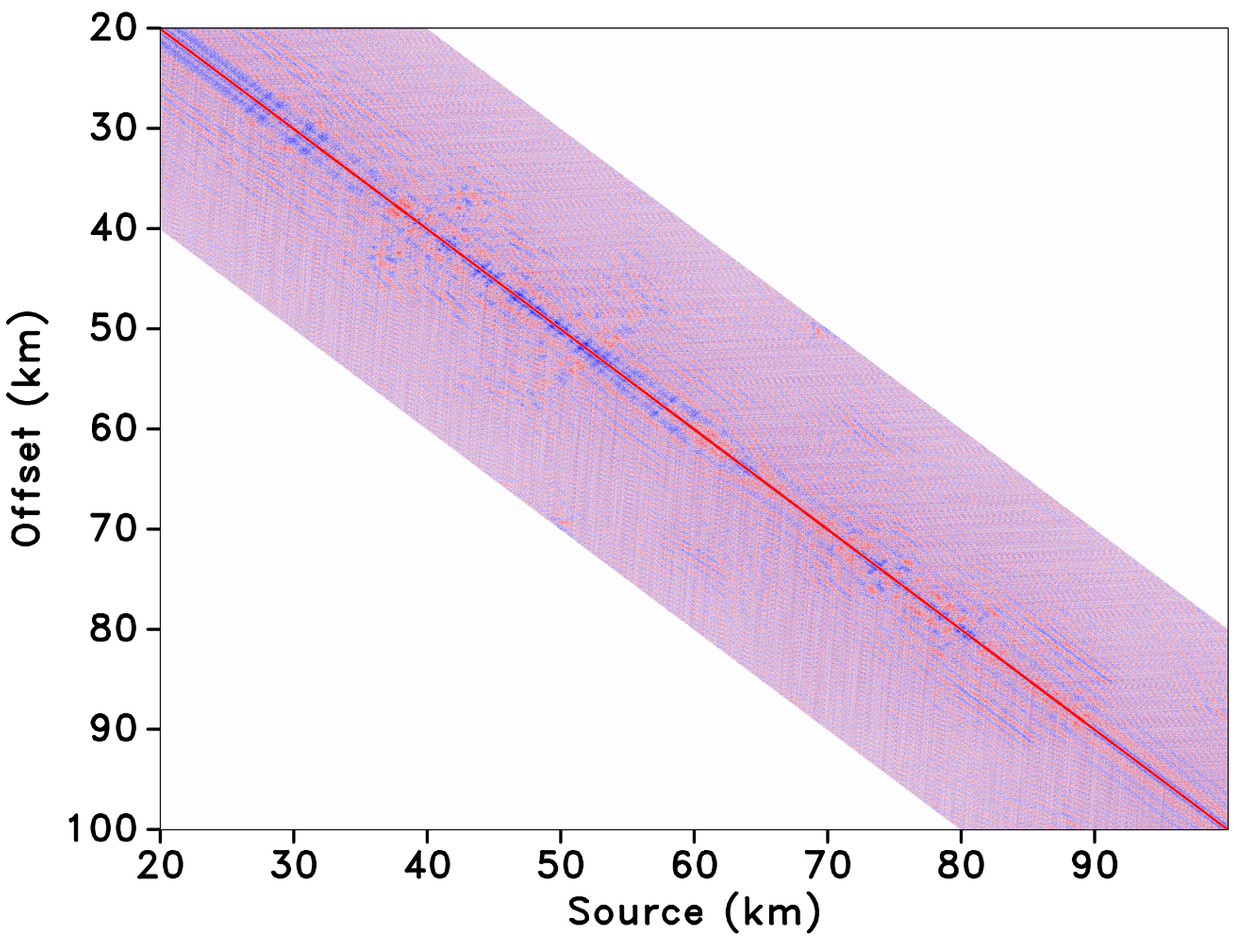


after denoising
[curvelet]

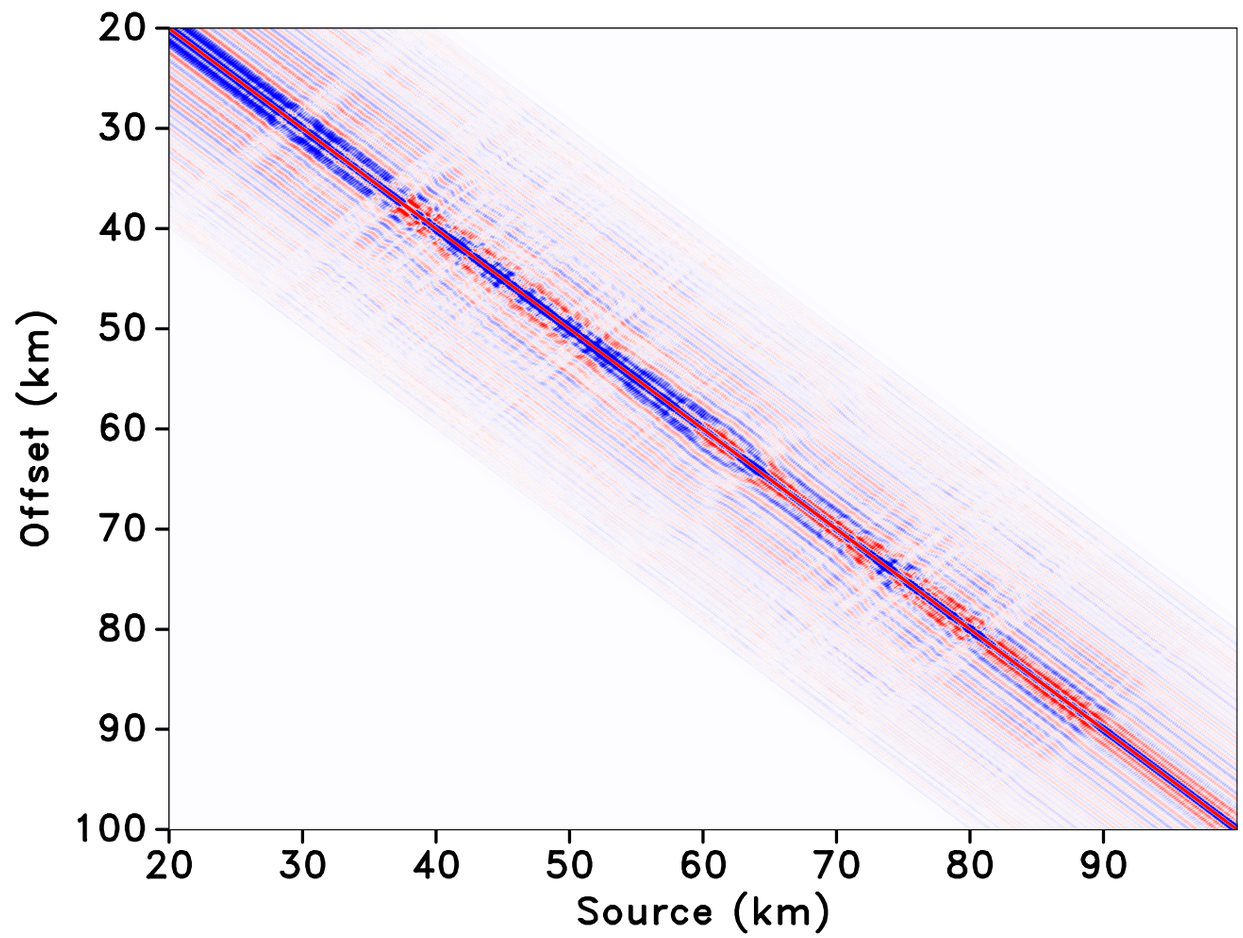


Gulf of Mexico
[2 Hz]

before denoising

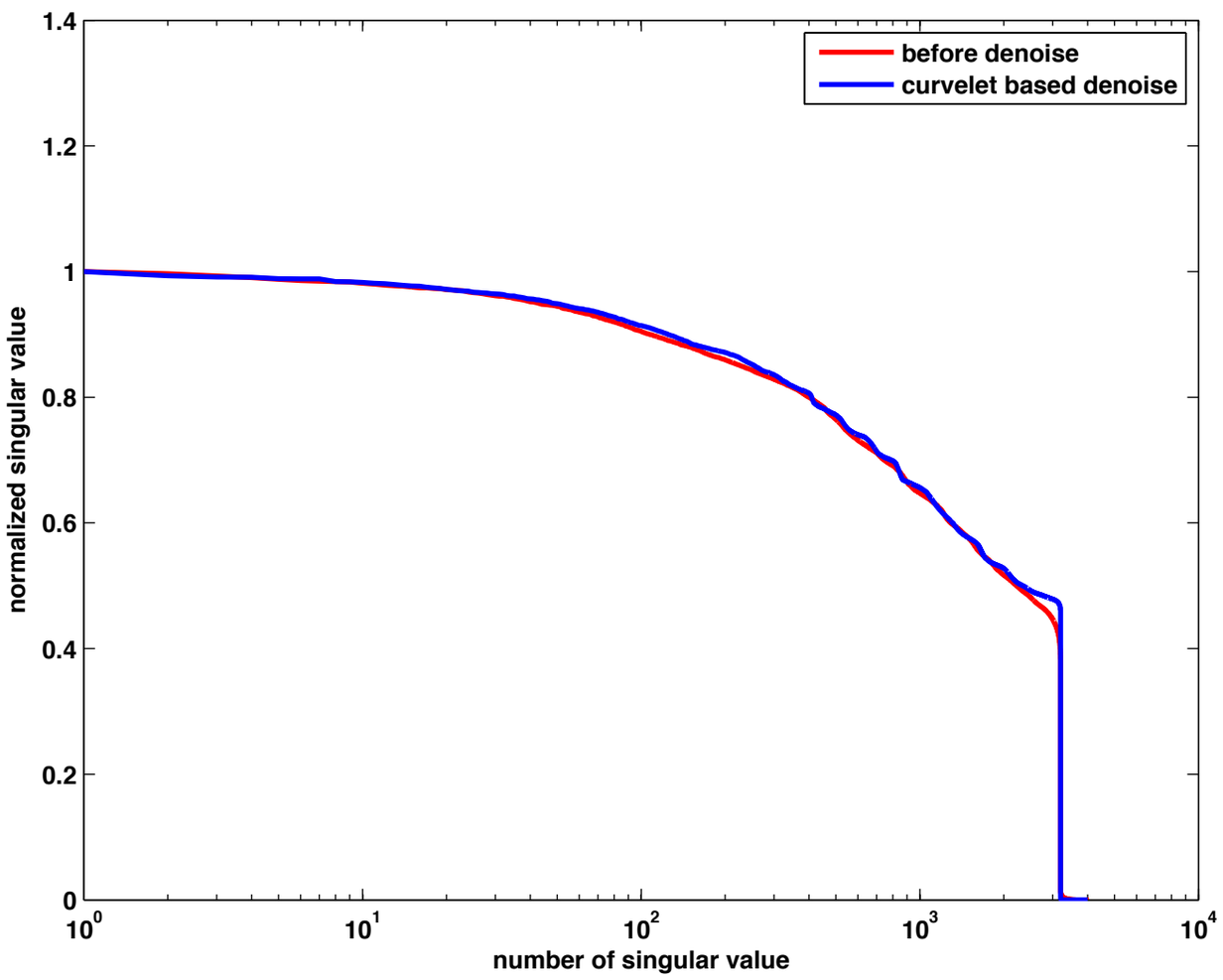


after denoising
[low-rank]

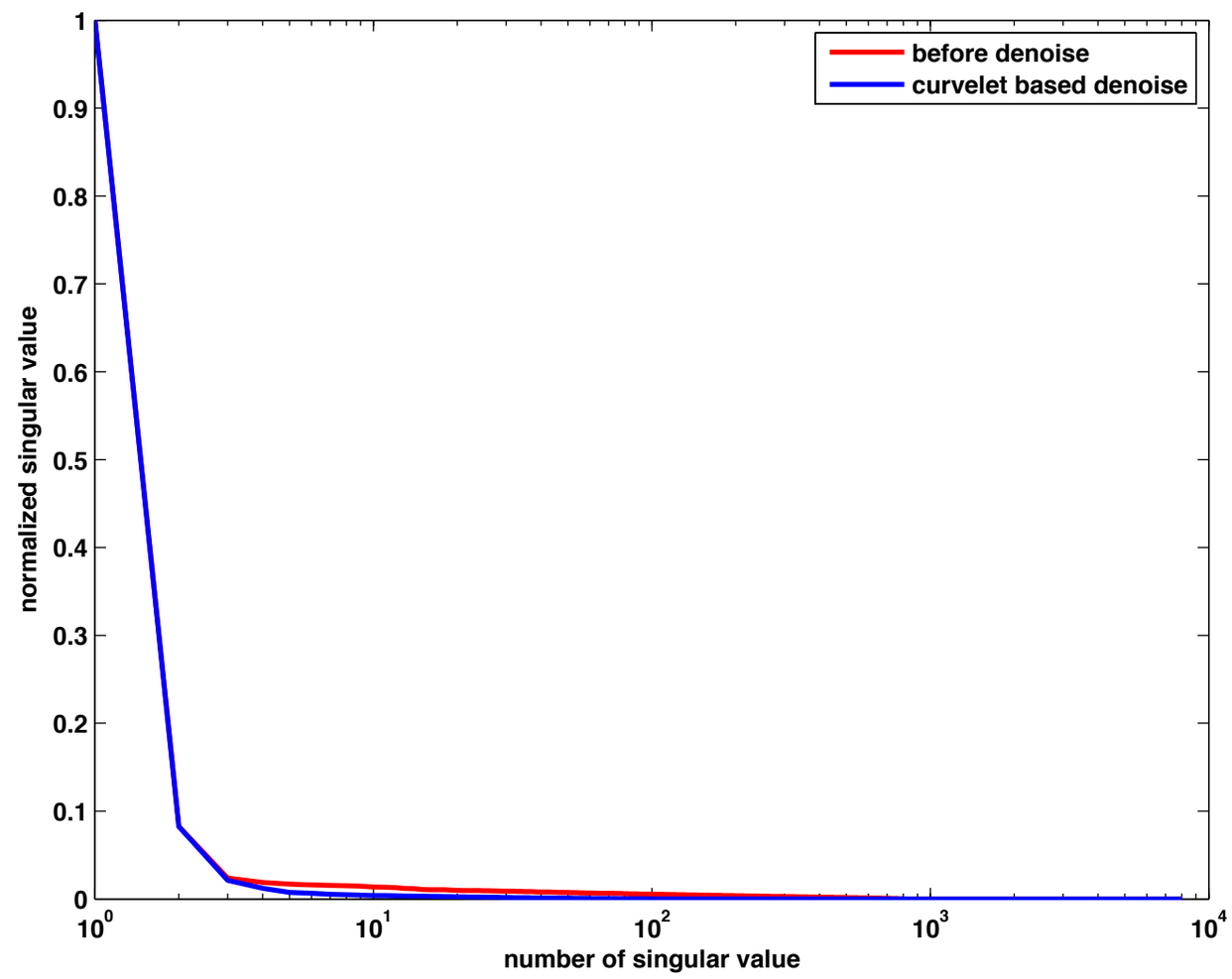


No Free Lunch

acquisition domain
[source-receiver]



transform domain
[midpoint-offset]



Comparison

	Low-rank	Curvelet
computational time	$O(minutes)$	$O(hours)$
storage	$k \times (n+m)$	$8 \times nm$

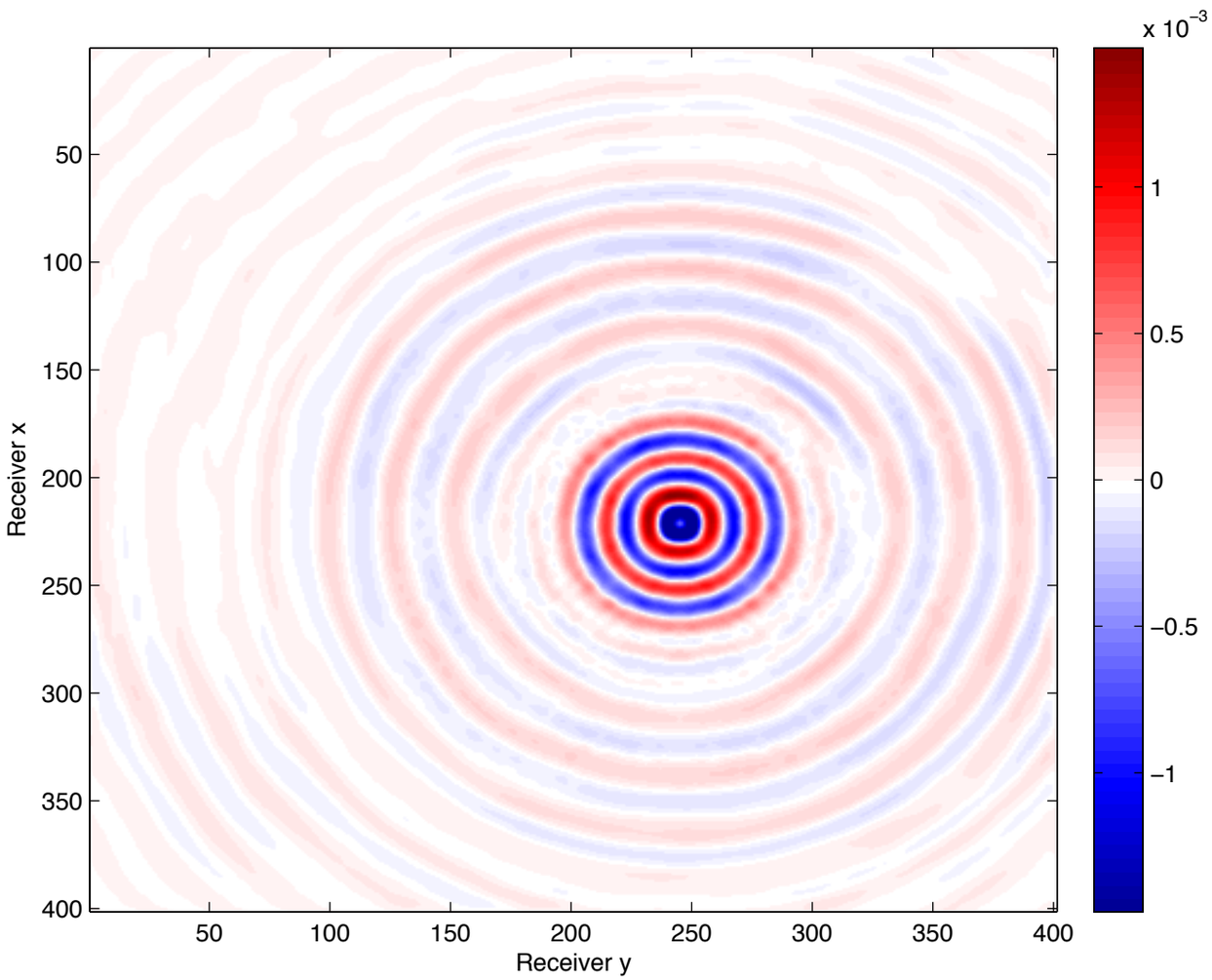
Interpolation

- 3-D synthetic BG data set
- 4624 shots spaced by 150m
- 401 x 401 receivers spaced by 25m
- 25% missing shots
- monochromatic data slices at 4.8 Hz

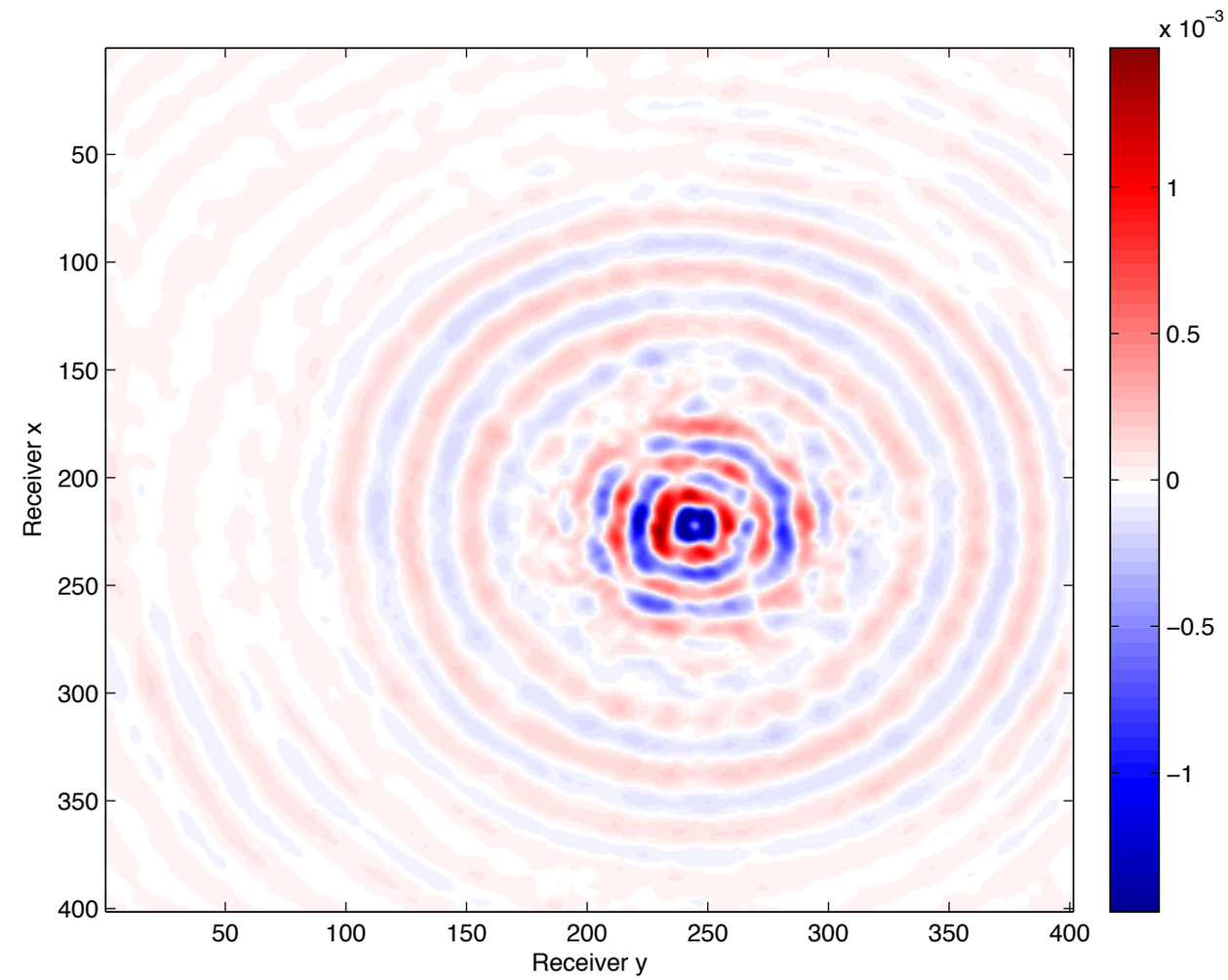
Recovery - Shot Gather

[Curvelet, 1000 iterations, 4.8 Hz]

true data
(src x, src y = 38,42)



approximated data
SNR=3.69 db

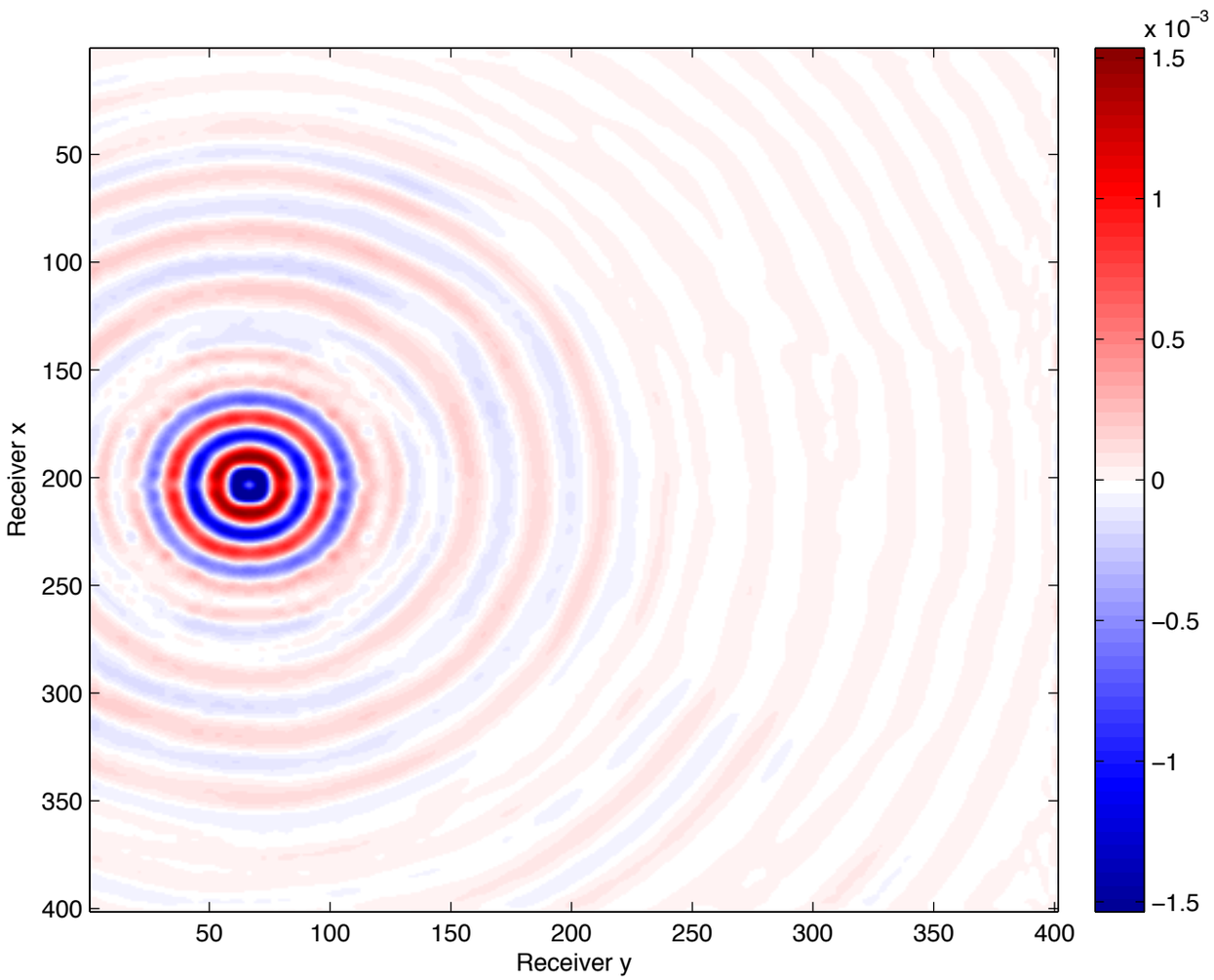


- average SNR = 3.12 dB

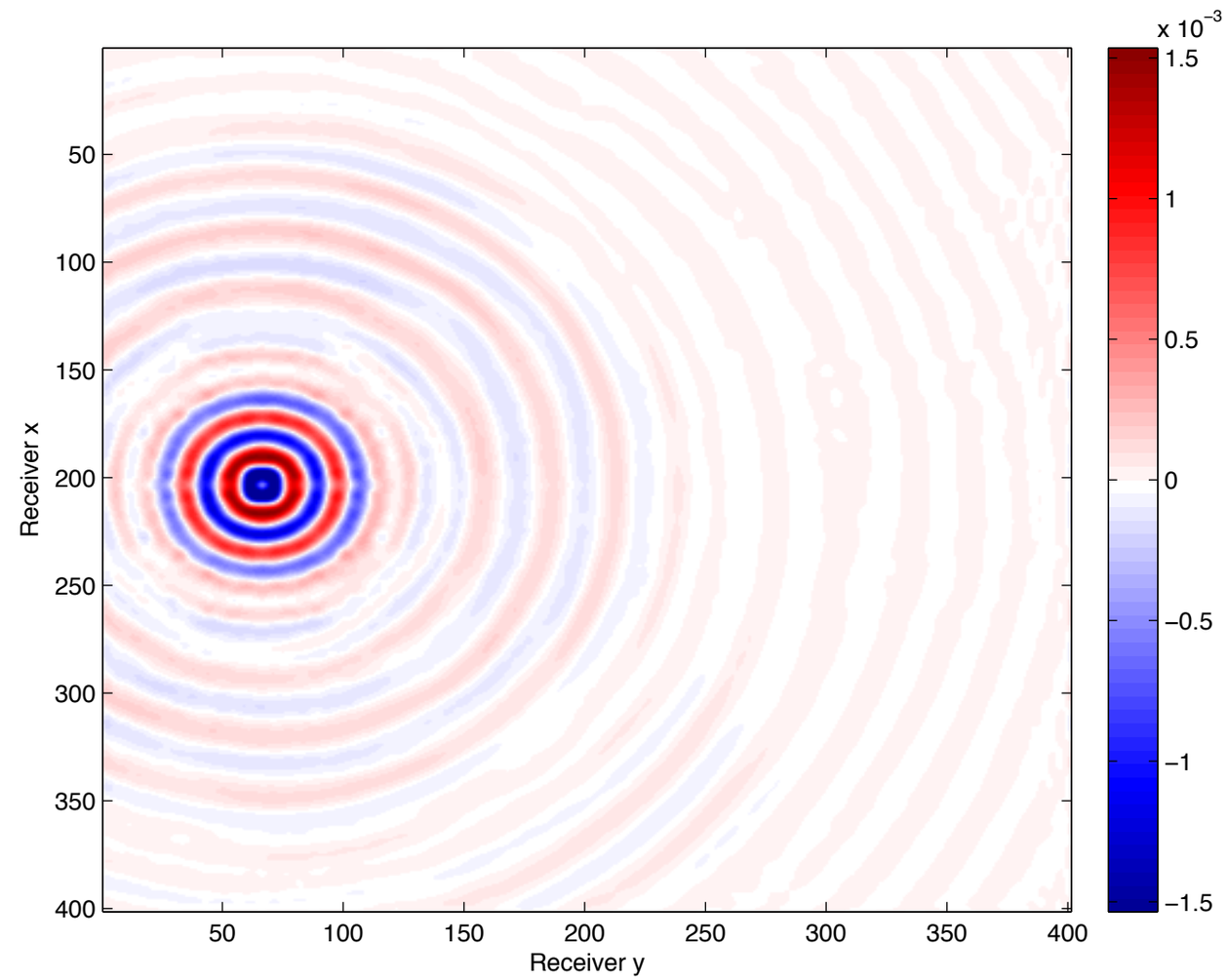
Recovery - Shot Gather

[low-rank, rank=120, 1000 iterations, 4.8 Hz]

true data
(src x, src y = 35,12)



approximated data
SNR=24.0 db

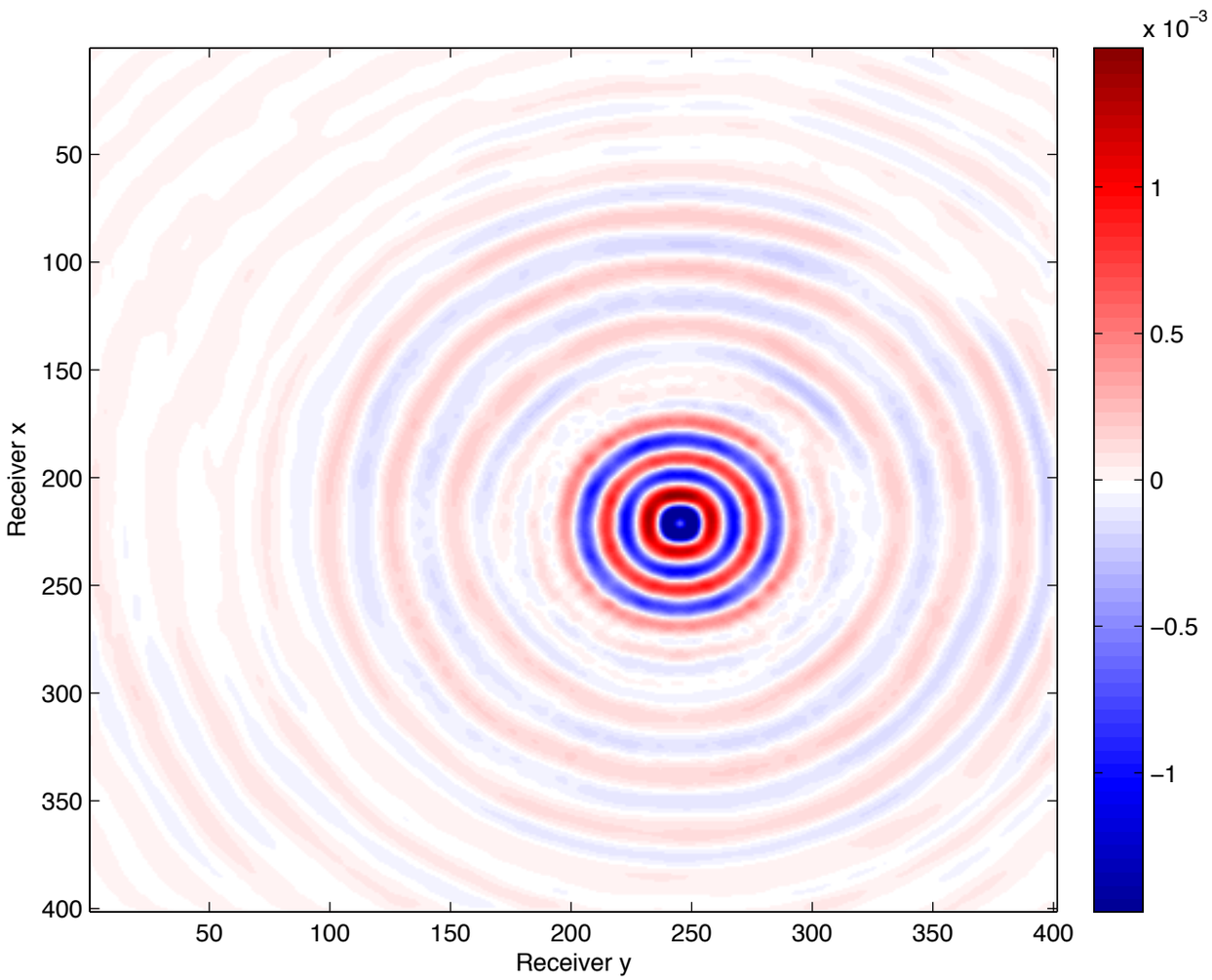


- average SNR = 21.5 dB

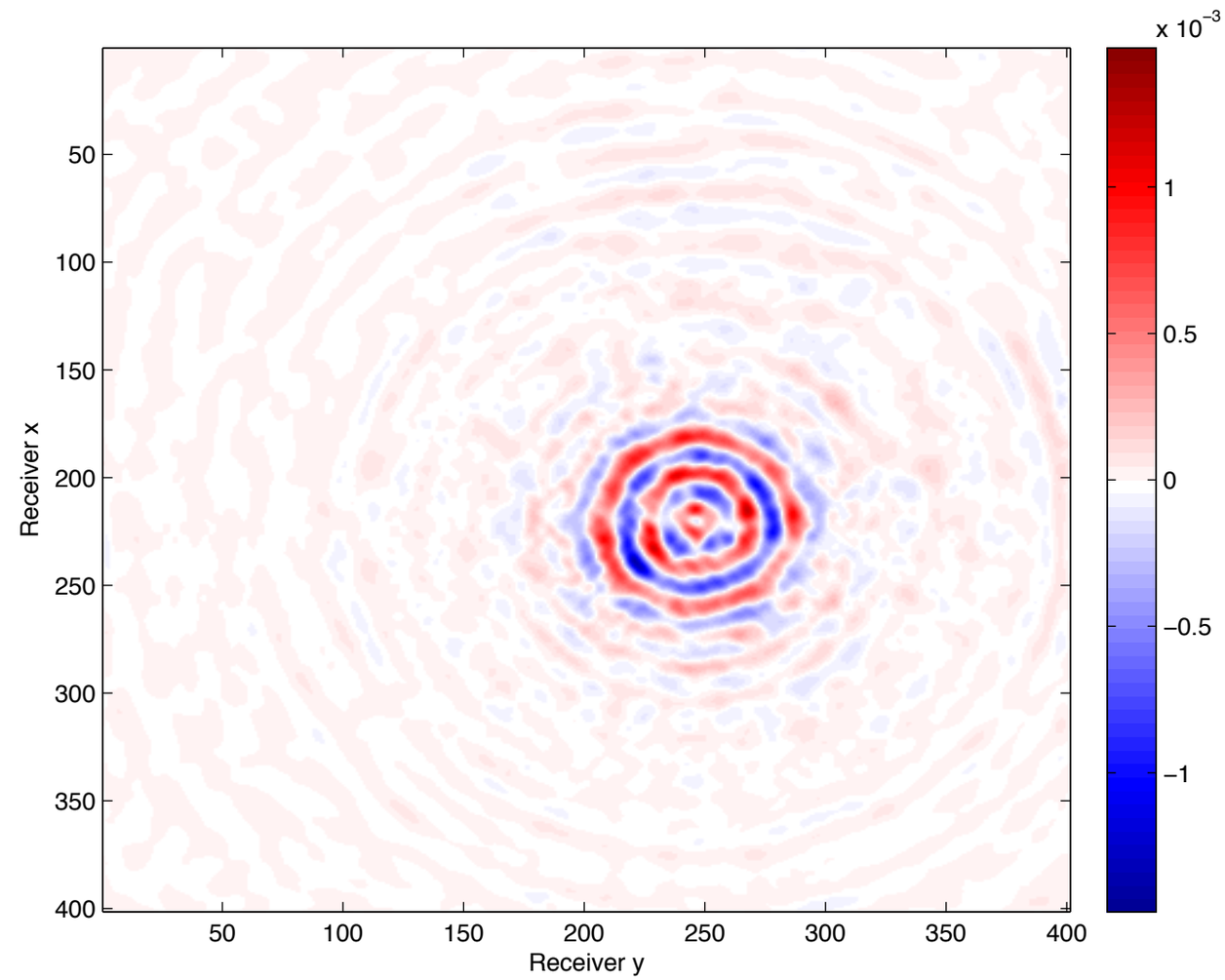
Recovery - Shot Gather

[Curvelet, 1000 iterations, 4.8 Hz]

true data
(src x, src y = 38,42)



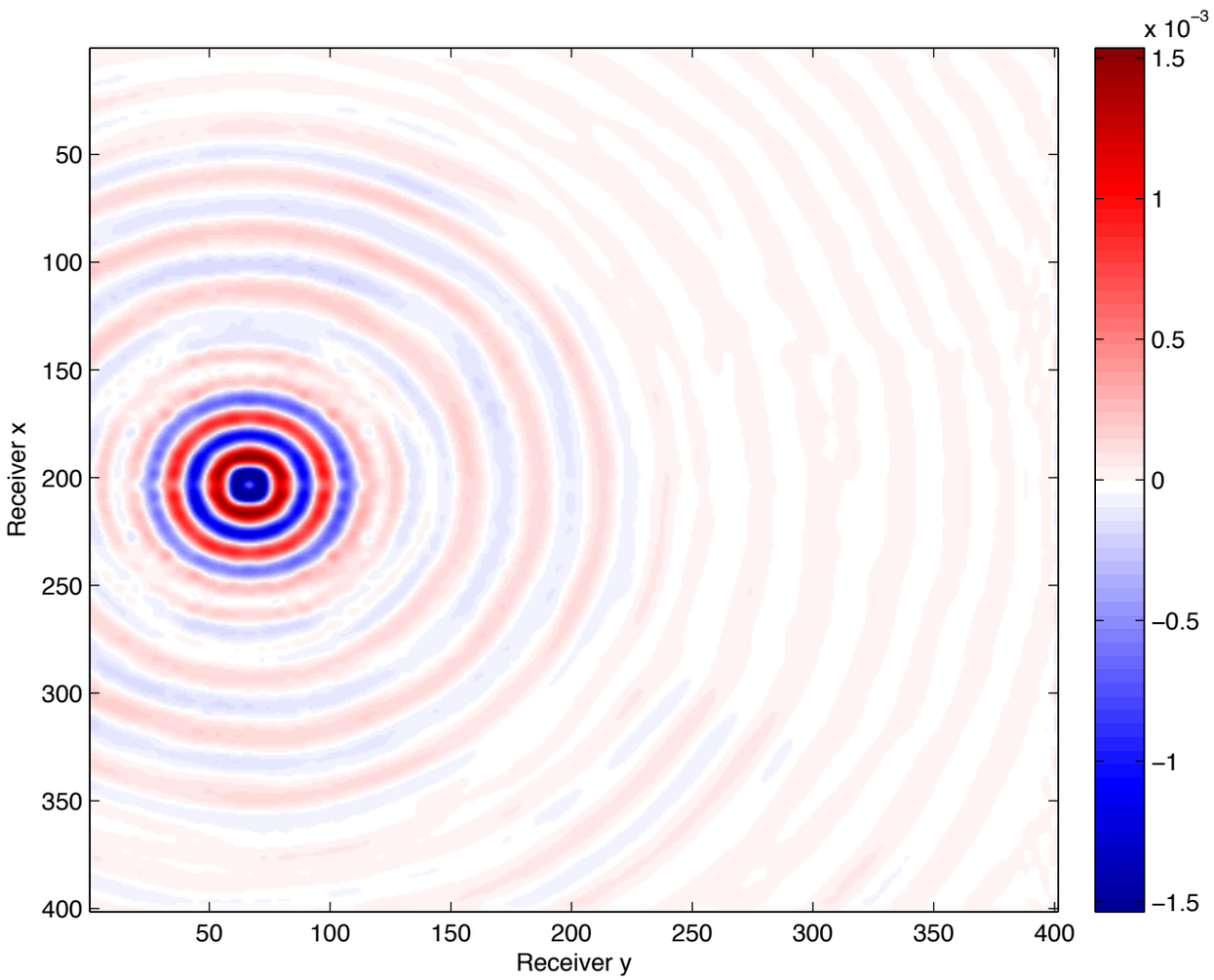
difference



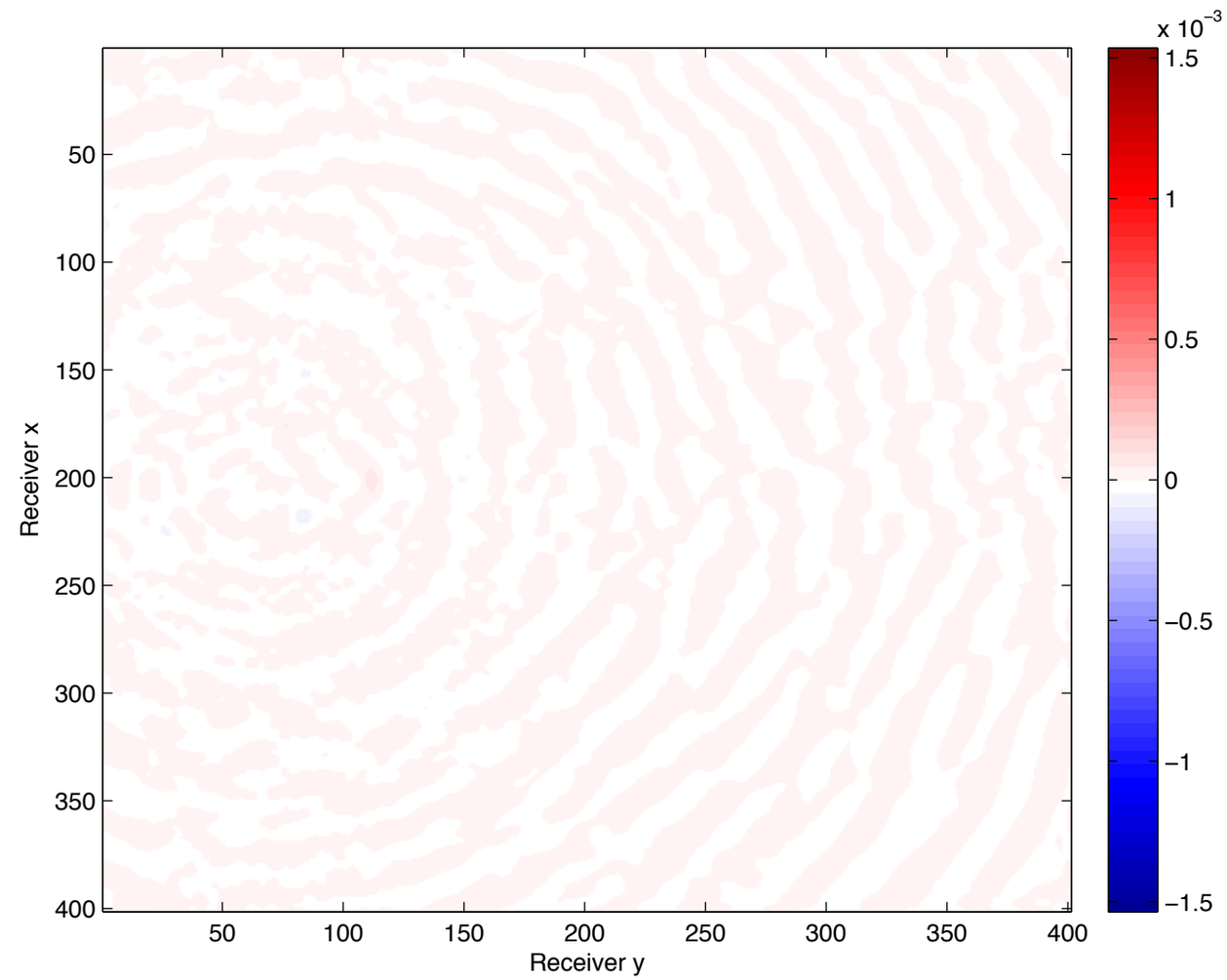
Recovery - Shot Gather

[low-rank, rank=120, 1000 iterations, 4.8 Hz]

true data
(src x, src y = 35,12)



difference



Comparison

	Low-rank	Curvelet
computational time	$O(hours)$	$O(days)$
storage	$k \times (n+m)$	$27 \times nm$

Conclusion

- matrix factorization allows *SVD-free* robust rank penalty methods that work fast on large data
- robust formulations are able to ignore *LARGE* unexplained artifacts in the data
- low-rank structure holds promise for data recovery and more compact representation.

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