Only dither: efficient simultaneous marine acquisition

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January 17, 2012

Abstract

Simultaneous-source acquisition is an emerging technology that is stimulating both geophysical research and commercial efforts. The focus here is on simultaneous-source marine acquisition design and sparsity-promoting sequential-source data recovery. We propose a pragmatic simultaneous-source, randomized marine acquisition scheme where multiple vessels sail across an ocean-bottom array firing airguns at — sequential locations and randomly time-dithered instances. Within the context of compressive sensing, where the choice of the sparsifying transform needs to be incoherent with the compressive sampling matrix, we can significantly impact the reconstruction quality, and demonstrate that the compressive sampling matrix resulting from the proposed sampling scheme is sufficiently incoherent with the curvelet transform to yield successful recovery by sparsity promotion. Results are illustrated with simulations of "purely" random marine acquisition, which requires an airgun to be located at each source location, and random time-dithering marine acquisition with one and two source vessels. Size of the collected data volumes in all cases is the same. Compared to the recovery from the former acquisition scheme (SNR = 10.5dB), we get good results by dithering with only one source vessel (SNR = 8.06dB) in the latter scheme, which improve at the cost of having an additional source vessel (SNR = 9.44dB).

Introduction

Conventional marine acquisition is carried out as single-source experiments of the subsurface response, where the size of the field recording is a product of the number of source locations, the number of receiver locations active per source experiment, and the number of discretized time samples proportional to the length of the reflection series. Constrained by the Nyquist sampling rate, the increasing sizes of these already massive data volumes pose a fundamental shortcoming in the traditional sampling paradigm and make large area acquisition particularly expensive. Several works in the seismic acquisition literature have explored the concept of simultaneous or blended source activation to account for this situation (Beasley et al., 1998; de Kok and Gillespie, 2002; Beasley, 2008; Berkhout, 2008; Hampson et al., 2008). For simultaneous-source acquisition, the challenge is to estimate interference-free shot gathers and recover subtle late reflections that can be overlaid by interfering seismic responses from other shots. In this paper, we show that this challenge can be effectively addressed through a combination of tailored simultaneous-source acquisition design and curvelet-based recovery. This work follows up on our paper for the Geophysical Prospecting journal that is in revision.

Recently, "compressed sensing" (Donoho, 2006; Candès and Tao, 2006) has emerged as an alternate sampling paradigm in which randomized sub-Nyquist sampling is used to capture the structure of the data with the assumption that it is sparse or compressible in some transform domain. A signal is said to admit a sparse (or compressible) representation in a transform domain if only a small number k of the transform coefficients are nonzero (or if the signal can be well approximated by the k largest-in-magnitude transform coefficients). In seismic exploration, data consists of wavefronts that exhibit structure in multiple dimensions. With the appropriate data transformation, we capture this structure by a small number of significant transform coefficients resulting in a sparse representation of data. In this paper, we rely on the compressed sensing literature to analyze a physically realizable simultaneous-source marine acquisition technology, which we call *random time-dithering*, where acquisition related costs are no longer determined by the Nyquist sampling criteria, but by the transform-domain sparsity of the data. Under this paradigm, we continuously record over the whole acquisition process, collecting a single long "supershot" record that is acquired over a time interval shorter than the cumulative time of conventional marine acquisition (excluding downtime and overhead such as vessel turning). We then recover the canonical sequential single-source shot record by solving a sparsity promoting problem.

Compressed sensing overview

Compressive sensing (abbreviated as CS throughout the paper) is a process of acquiring random linear measurements of a signal and then reconstructing it by utilizing the prior knowledge that the signal is sparse or compressible in some transform domain. One of the main advantages of CS is that it combines sampling and compression in a single linear step, thus reducing the cost of traditional Nyquist sampling followed by dimensionality reduction through data compression. This feature of CS is beneficial to seismic acquisition where the acquisition costs are now quantified by the transform-domain sparsity of seismic data instead of the grid size.

For high resolution data represented by the N-dimensional vector $\mathbf{f_0} \in \mathbb{R}^N$, which admits a sparse representation $\mathbf{x_0} \in \mathbb{C}^P$ in some transform domain characterized by the operator $\mathbf{S} \in \mathbb{C}^{P\times N}$ with $P \geq N$, the sparse recovery problem involves solving an underdetermined system of equations

$$\mathbf{b} = \mathbf{A}\mathbf{x_0}, \tag{1}$$

where $\mathbf{b} \in \mathbb{C}^n$, $n \ll N \leq P$, represents the compressively sampled data of n measurements, and $\mathbf{A} \in \mathbb{C}^{n\times P}$ represents the measurement matrix. We denote by $\mathbf{x_0}$ a sparse synthesis coefficient vector of $\mathbf{f_0}$. Note that $\mathbf{A}$ can be composed of the product of a restriction operator (subsampling matrix) $\mathbf{R} \in \mathbb{R}^{n\times N}$, an $N \times N$ mixing matrix $\mathbf{M}$, and the sparsifying operator $\mathbf{S}$ such that

$$\mathbf{A} := \mathbf{R}\mathbf{M}\mathbf{S}^H, \tag{2}$$

(here ^{H} denotes the Hermitian transpose) and $\mathbf{A}\mathbf{x_0} = \mathbf{R}\mathbf{M}\mathbf{f_0}$. When $\mathbf{x_0}$ is strictly sparse (i.e., only $k < n$ nonzero entries in $\mathbf{x_0}$), sparsity-promoting recovery can be achieved by solving the ℓ_0 minimization problem, which is a combinatorial problem and quickly becomes intractable as the dimension increases. Instead, the basis pursuit (BP) convex optimization problem

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{C}^P} \|\mathbf{x}\|_1 \quad \text{subject to } \mathbf{b} = \mathbf{A}\mathbf{x}, \tag{3}$$

where $\tilde{\mathbf{x}}$ represents the estimate of $\mathbf{x_0}$, and the ℓ_1 norm $\|\mathbf{x}\|_1$ is the sum of absolute values of the elements of a vector $\mathbf{x}$, can be used to recover $\tilde{\mathbf{x}}$. The BP problem typically finds a sparse or (under some conditions) the sparsest solution that explains the data exactly.

Simultaneous marine acquisition as a CS problem

Consider marine data organized in a seismic line with N_s sources, N_r receivers, and N_t time samples. For simplicity, we assume that all sources see the same receivers, which makes our method applicable to marine acquisition with ocean-bottom cables (OBC). The seismic line can be reshaped into an N dimensional vector $\mathbf{f}$, where $N = N_sN_rN_t$. We wish to recover a sparse approximation $\tilde{\mathbf{f}}$ of the discretized wavefield $\mathbf{f}$ from measurements $\mathbf{b} = \mathbf{RMf}$ (applying $\mathbf{RM}$ to $\mathbf{f}$ reduces it to a single long "supershot" of length $n \ll N$ that consists of a superposition of N_s impulsive shots). The sparse approximation $\tilde{\mathbf{f}}$ is obtained by inverting the compressive sampling matrix in Eq. 2 with the basis pursuit sparsity-promoting program in Eq. 3, yielding $\tilde{\mathbf{f}} = \mathbf{S}^H\tilde{\mathbf{x}}$. To solve this one-norm optimization problem, we use the SPGℓ_1 solver (Berg and Friedlander, 2008).

The design of the sampling operator $\mathbf{RM}$, is critical to the success of the recovery algorithm. In the simultaneous marine acquisition scenario, the classic sequential acquisition with a single airgun is replaced with continuous acquisition with multiple airguns firing at random times and at random locations that span the entire survey area. This "ideal" simultaneous-source sampling (or random dithering) scheme is illustrated in Fig. 1(a), where the stars indicate the firing times and locations of the multiple sources. Such simultaneous acquisition schemes require an airgun to be located at each source location throughout the survey, which is infeasible. In this paper we present a new alternative which requires a small number of vessels that map the entire survey area while firing sequential shots at randomly time-dithered instances. In the random time-dithered marine acquisition scheme a single airgun or multiple airguns are fired sequentially with random lag intervals between shots. This scheme is illustrated in Fig. 1(b), where, similar to the simultaneous source scheme, the firing times remain random but the source positions are sorted with respect to survey time. For random time-dithering with two source vessels (Fig. 1(c)), the first vessel starts at the first source location and the following vessel starts at a random source location and moves sequentially thereon (the blue arrow marks the starting point of each vessel).

Since the subsampling is performed in the source-time domain, the sampling operator is defined as

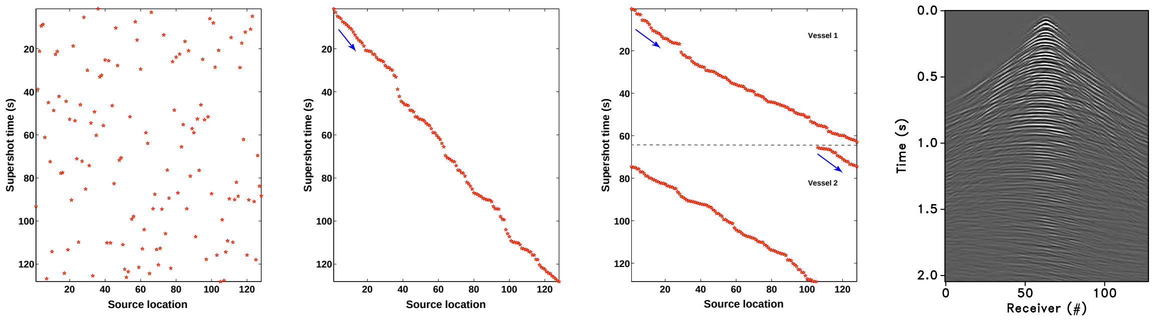
$$\mathbf{RM} := [\mathbf{I} \otimes \mathbf{T}], \tag{4}$$

where $\otimes$ is the Kronecker product, $\mathbf{I}$ is an $N_r \times N_r$ identity matrix, and $\mathbf{T}$ is a combined random shot selector and time shifting operator. This structure decouples the receiver axis from the source-time axis in the sampling operator allowing $\mathbf{T}$ to operate on the vectorized common receiver gathers. This Kronecker product simply repeats the operation of $\mathbf{T}$ on every available receiver. The operator $\mathbf{T}$ turns the sequential-source recordings into continuous recordings with N_s impulsive sources, and firing at time instances selected uniformly at random from $\{1, \ldots (N_s - 1)N_t\}$ discrete times. Consequently, the operator $\mathbf{T}$ subsamples the N_sN_t samples recorded at each receiver to $m \ll N_sN_t$ samples resulting in a total number of compressive samples $n = mN_r$. Note, it is also possible to subsample the receiver axis or equivalently randomize positions of the OBC transducers.

Experimental results

We illustrate the performance of our simultaneous source acquisition approach by comparing two different schemes; random dithering, and random time-dithering with one and two source vessels, on a seismic line from the Gulf of Suez (Fig. 1(d) shows a common-shot gather). The size of the collected data volume, and hence the cost are the same in all cases. The fully sampled sequential data is composed of $N_s = 128$ shots, $N_r = 128$ receivers and $N_t = 512$ time samples. Prestack data from sequential sources is recovered using ℓ_1 minimization with 3D curvelets as the sparsifying transform (it is well known that seismic data admit sparse representations by curvelets that capture "wavefront sets" efficiently (see e.g. Smith, 1998; Ying et al., 2005; Candès and Demanet, 2005; Candès et al., 2006; Herrmann et al., 2008, and the references therein)). The recovery performance is evaluated in terms of the *signal-to-noise ratio* (SNR), which (for a signal $\mathbf{x}$ and its estimate $\tilde{\mathbf{x}}$) is computed as:

$$\mathrm{SNR}(\mathbf{x}, \tilde{\mathbf{x}}) = -20 \log_{10} \frac{\|\mathbf{x} - \tilde{\mathbf{x}}\|_2}{\|\mathbf{x}\|_2}. \tag{5}$$



(a) (b) (c) (d)

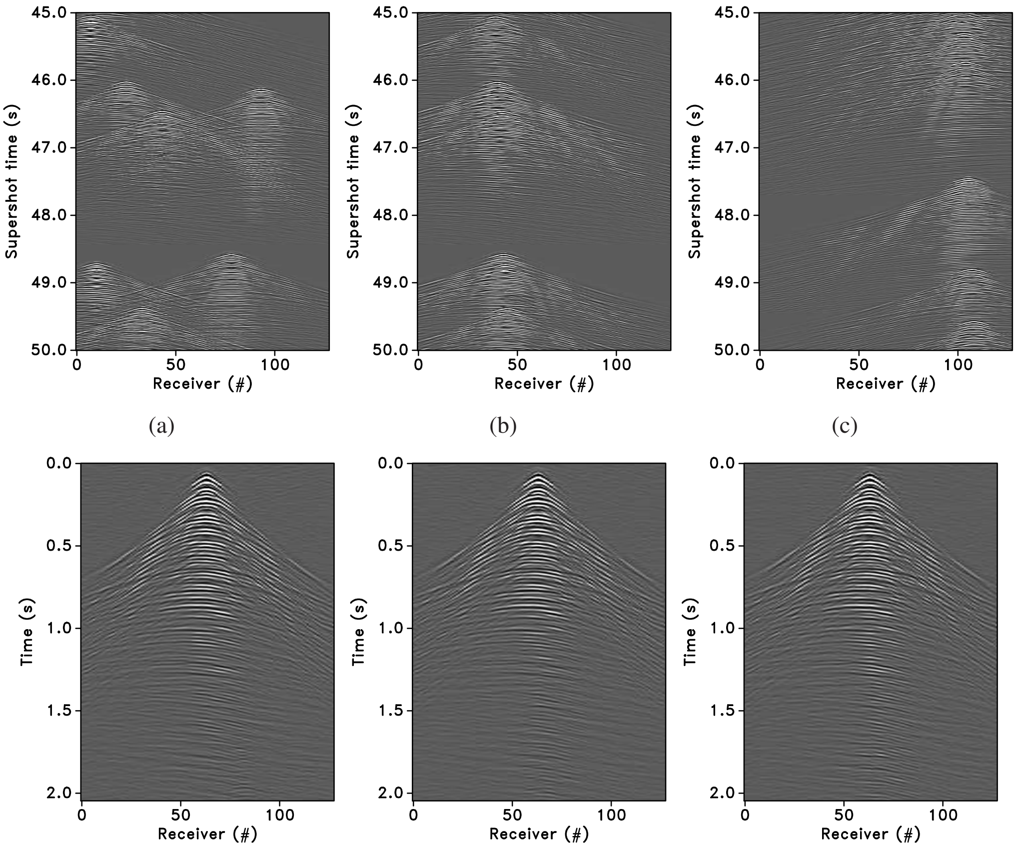
Figure 1 *(a) Random dithered (or "ideal" simultaneous-source) acquisition. Random time-dithered acquisition with (b) one, and (c) two source vessels. (d) Common-shot gather from Gulf of Suez data set.*

Random dithering: We simulate simultaneous-source marine acquisition by randomly selecting 128 shots from the total survey time $t = N_s \times N_t$ with a subsampling ratio $\gamma = \frac{m}{N_s \times N_t} = 0.5$. The subsampling is performed only in time so that the length of the "supershot" is half the length of the conventional survey time of sequential-source data. Fig. 2(a) represents the "supershot" plotted as conventional survey by applying the sampling operator **RM** to sequential-source data. Notice that this type of "marine" acquisition is physically realizable only with a limited number of simultaneous sources, although truly random positioning of sources may still prove impractical depending on the manoeuvrability of source vessels. To exploit continuity of the wavefield along all three coordinate axes, we use the 3D curvelet transform. Fig. 2(d) shows the recovered data with an SNR of 10.5 dB, obtained with 200 iterations of solving the BP problem using SPGℓ_1.

Random time-dithering: To overcome the limitation in the number of simultaneous sources required by the "ideal" simultaneous-source approach, we propose the random time-dithering scheme. Under this scheme, we allow all 128 shots to be fired sequentially with adjacent shots firing before the previous shot fully decays. Therefore, the start time of each shot is chosen uniformly at random between the starting time of the previous shot and the time by which the previous shot record decays. A section of the "supershot" obtained by random time-dithering with one source vessel is shown in Fig. 2(b). With $\gamma = 0.5$ and the 3D curvelet transform, a recovery of 8.06 dB is achieved (Fig. 2(e)). This demonstrates the effectivenes of the random time-dithering scheme because it's performance approaches that of the "ideal" simultaneous-source acquisition, with a single source vessel. Furthermore, the sparsity-promoting recovery from data collected by random time-dithering with two source vessels (Fig. 2(c) shows a section of the "supershot") improves significantly, at the cost of the additional source vessel, as shown in Fig. 2(f) (SNR = 9.44 dB).

Conclusions

Recovering single-source prestack data volumes from simultaneously acquired marine data essentially involves removing noise-like crosstalk from coherent seismic responses. We identify simultaneous marine acquisition as a linear subsampling system, and also propose a randomized time-dithering acquisition scheme which can match with few multiple vessels the performance of an "ideal" simultaneous-source acquisition. By comparing sparsity-promoting reconstructions on a real seismic marine line



(a) (b) (c)

(d) (e) (f)

Figure 2 *(a), (b), and (c) show simultaneous marine data as a section between 45 and 50 seconds of the "supershot" for random dithered acquisition, random time-dithered acquisition with one and two source vessels, respectively. (d), (e), and (f) show the corresponding sparse recovery results with SNR = 10.5dB, 8.06dB, and 9.44dB, respectively.*

sampled with different acquisition schemes, where the size of the collected data volumes is the same in all cases, we quantitatively verified the above statement.

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