Wave-equation extended images: computation & velocity continuation

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data

velocity

pre-stack image
[Biondo & Symes, ’04; Sava & Vasconcelos, ’11]
• use all subsurface offsets (5D volume)
• 2-way wave-equation

but.... we can never hope to compute or store such an image volume!
Can we work with the volume *implicitly*?
Overview

- Anatomy
- Physics
- Computation
- MVA
- Conclusions
Extended images

Correlation of wavefields

\[ e(\omega, x, x') = \sum_{i, \omega} v_i(\omega, x) u_i(\omega, x')^* \]

in data-matrix notation:

\[ E(\omega) = \sum_{\omega} V(\omega) U(\omega)^* \]

imaging condition: \[ \sum_{\omega} \text{diag}(E(\omega)) \]
Extended images

4D image volume as matrix

\( n_x \times n_z \)
Extended images

x

z

X

Z

x'

z'

X'

Z'

X'

Z'

x'

z'

X'

Z'

x'

z'

X'

Z'

x'

z'

X'

Z'

x'

z'

X'

Z'

x'

z'

X'

Z'
Extended images

example for one layer
Extended images

full matrix

low velocity  correct velocity  high velocity
Extended images

one column

low velocity
correct velocity
high velocity
Extended images

diagonal

low velocity

correct velocity

high velocity
offset gathers

align subsurface offset with local dip

$x - x'$

$z - z'$

$\theta$

$h$

$\Sigma$

$x$

$z$

$0$
Example
Example

low velocity

correct velocity

high velocity
Example

\[ h \text{ [km]} \quad z \text{ [km]} \quad e = 0^\circ \]

\[ \theta = 0^\circ \]

- low velocity
- correct velocity
- high velocity
Double wave-equation

Helmholtz operator: \[ H = \omega^2 \text{diag}(m) + \nabla^2 \]

source/receiver wavefields:

\[ HU = P_s^T Q \quad H^*V = P_r^T D \]

RTM extended image: \[ E = VU^* \]

yields: \[ H^*EH = P_r^T DQ^*P_s \]
Double wave-equation

\[ Le(\omega, x, x') = \int ds \int dr \ d(\omega, s, r) \delta(x - s) \delta(x' - r) \]

two-way:

\[ L = \left[ \frac{\omega^2}{c(z, x)^2} + \partial_x^2 + \partial_z^2 \right] \left[ \frac{\omega^2}{c(z', x')^2} + \partial_{x'}^2 + \partial_{z'}^2 \right] \]

one-way (DSR):

\[ L = \left[ \partial_z - i \sqrt{\frac{\omega^2}{c(z, x)^2} + \partial_x^2} - i \sqrt{\frac{\omega^2}{c(z, x')^2} + \partial_{x'}^2} \right] \]

[Clarebout, ’84; Stolk & de Hoop ’01]
Velocity continuation

since r.h.s. is model-independent:

\[ H_2^* E_2 H_2 = H_1^* E_1 H_1 \]

or

\[ E_2 = H_2^{-*} H_1^* E_1 H_1 H_2^{-1} \]

[Duchkov et al, ’08]
Examples
Extended images

• complete image volume too large to form: $(n_x \times n_z)^2$
• instead, probe volume for information via mat-vecs $E_y$
• $y$ can be interpreted as subsurface source function
Computation

mat-vec with extended image:

$$e = Ey = H^{-*}P_r^T DQ^* P_s H^{-1}y$$

- \(d = P_s H^{-1}y\) (one subsurface source)
- \(w = Q^*d\) (source weights)
- \(e = H^{-*}P_r^T (Dw)\) (one source)
MVA

focusing penalty:  \[ f(m) = \|W \odot E(m)\|_F^2 \]

[Shen & Symes, ’08]
MVA

Use techniques from randomized trace estimation:

\[
||A||_F^2 = \text{trace}(A^TA) \approx \sum_{i=1}^{K} w_i^T A^T A w_i = \sum_{i=1}^{K} ||A w_i||_2^2
\]

where

\[
\sum_{i=1}^{K} w_i w_i^T \approx I
\]

[Avron & Toledo, ’11]
Vectors can be interpreted as subsurface source functions.
\[ f(\alpha) = \| W \odot E(\alpha m_{\text{true}}) \|_F^2 \]

true

estimated \( K = 10 \)
Conclusions

• image volume for all offsets easily expressed in terms of data matrices
• two-way equivalent of DSR equation
• work with easy-to-compute mat-vecs
• use techniques from randomized trace estimation to compute focusing penalty
Future work

- automatically detect dip
- use trace estimation ideas in MVA inversion
- AVA
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