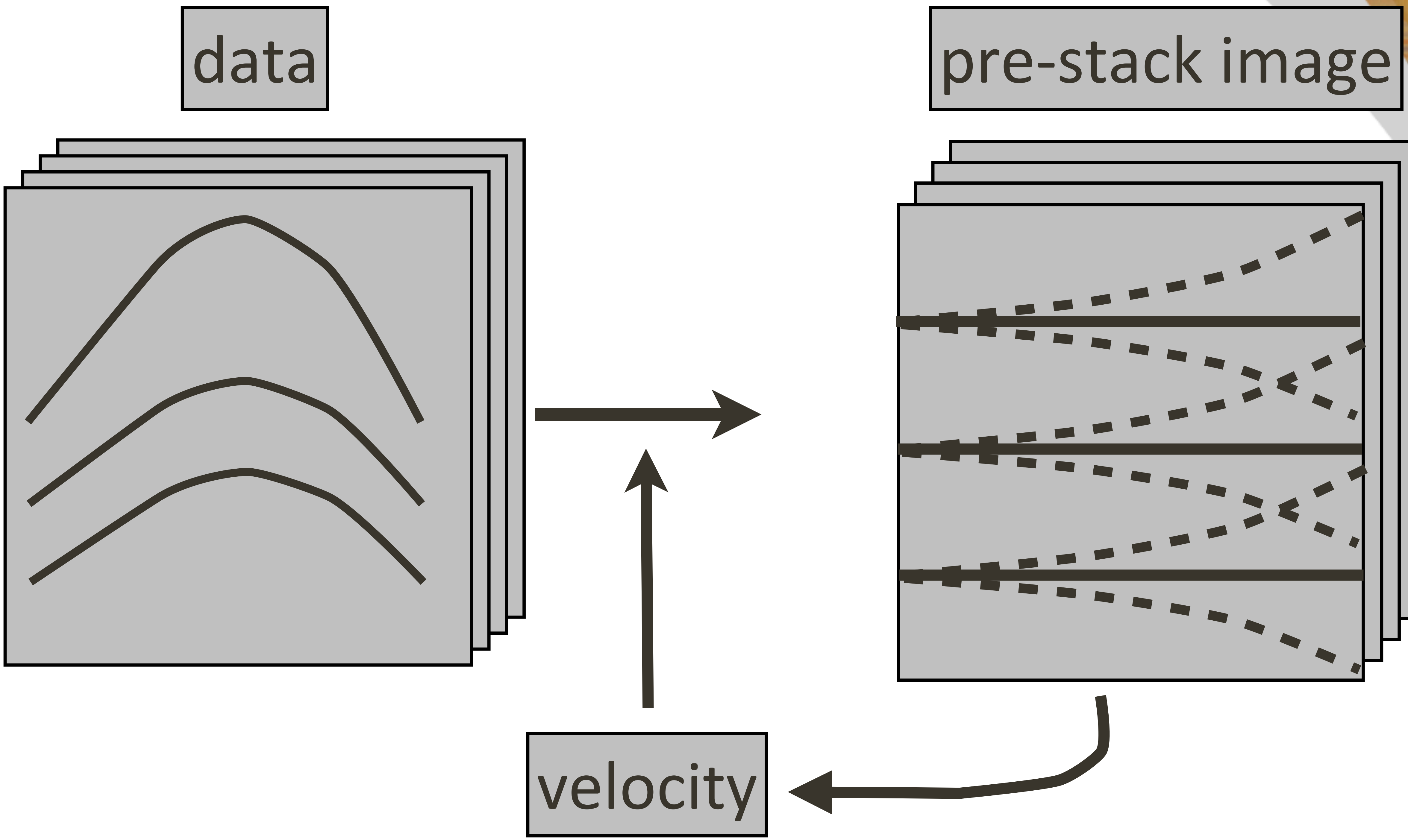
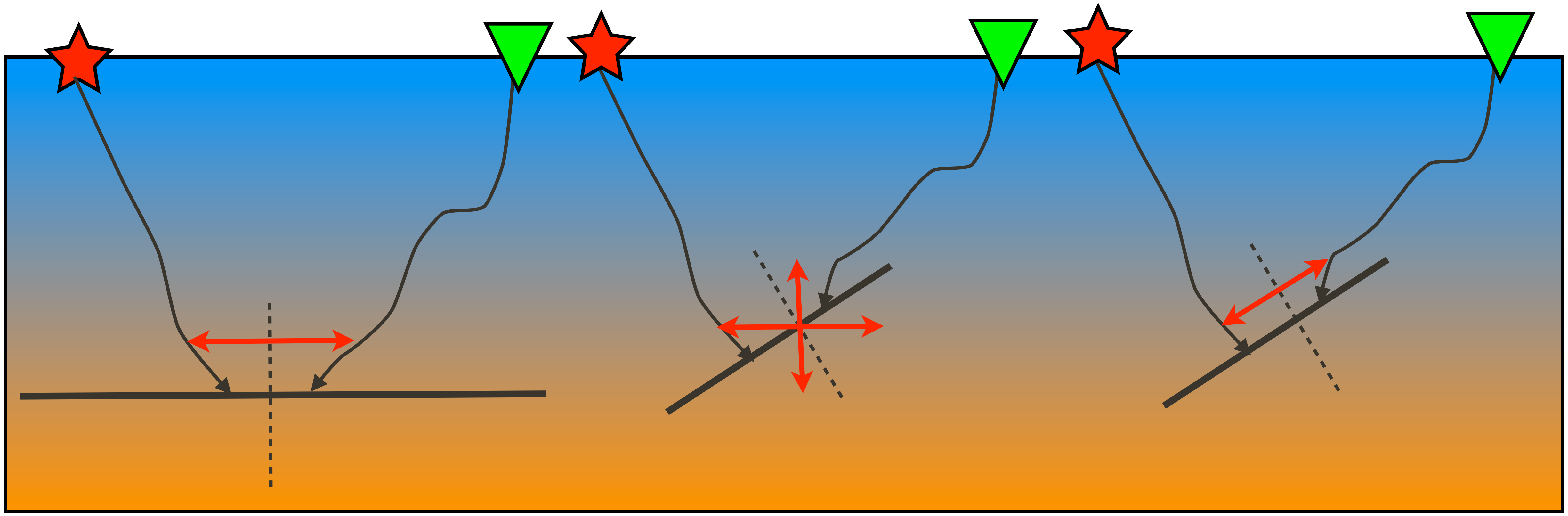
Wave-equation extended images: computation & velocity continuation

Tristan van Leeuwen

Felix J. Herrmann

SLIM
University of British Columbia


data
pre-stack image
velocity



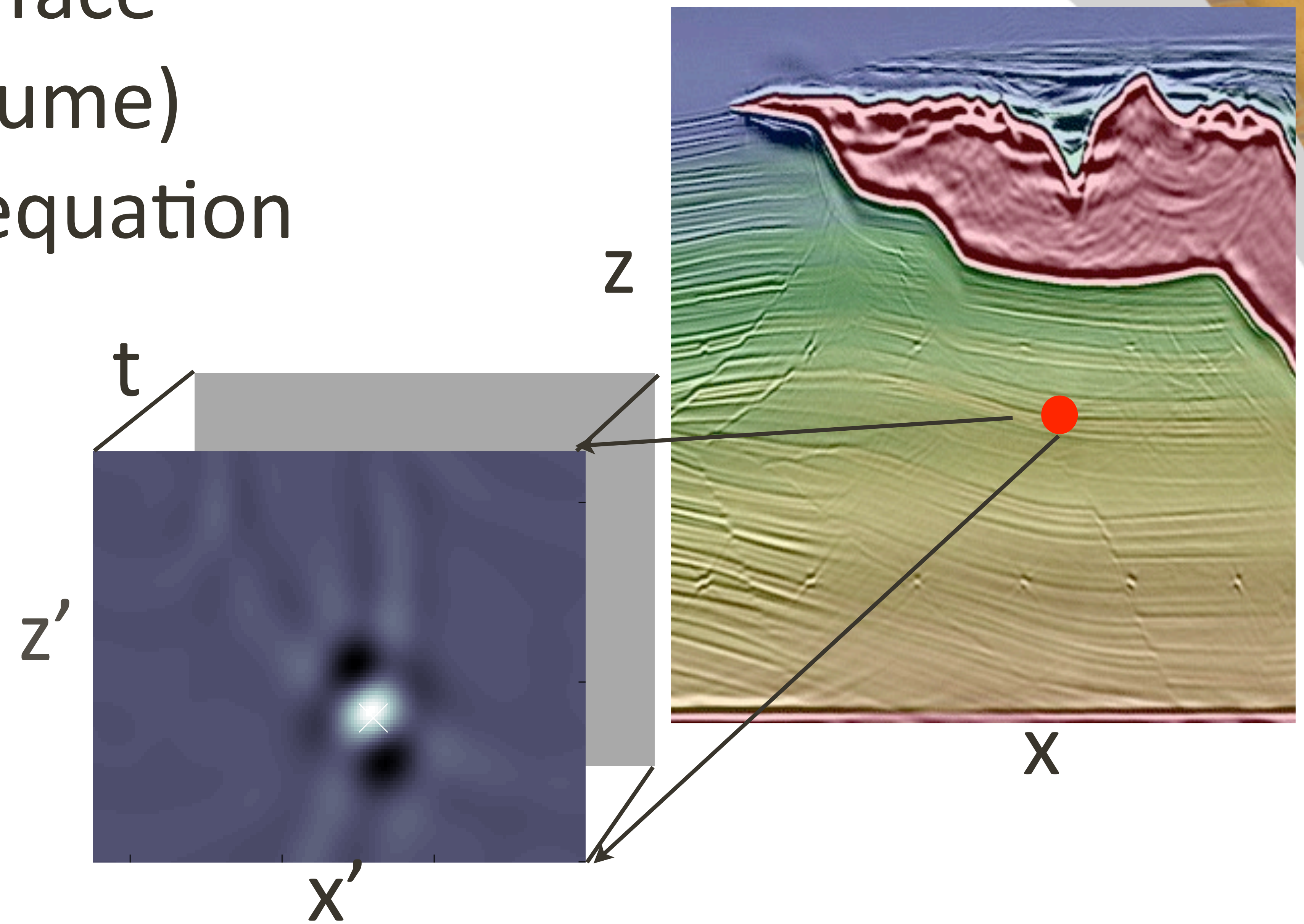
horizontal offset

horizontal +vertical offset

all offsets

[Biondo & Symes, ’04 ;Sava & Vasconcelos, ’11]

- use *all* subsurface offsets (5D volume)
- 2-way wave-equation



but.... we can never hope to compute or store such an image volume!

Can we work with the volume *implicitly* ?

Overview

- Anatomy
- Physics
- Computation
- MVA
- Conclusions

Extended images

Correlation of wavefields

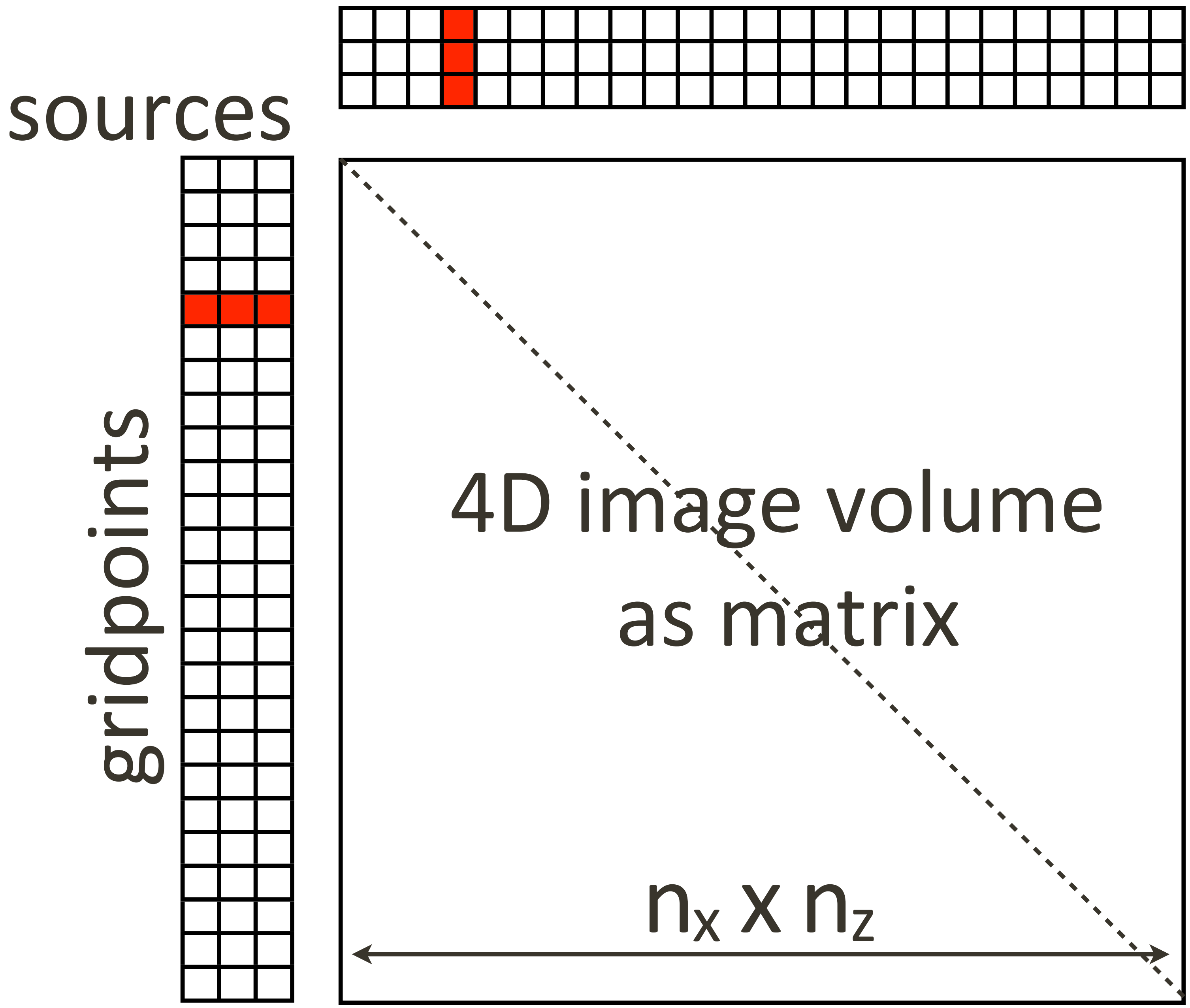
$$e(\omega, \mathbf{x}, \mathbf{x}') = \sum_{i,\omega} v_i(\omega, \mathbf{x}) u_i(\omega, \mathbf{x}')^*$$

in *data-matrix* notation:

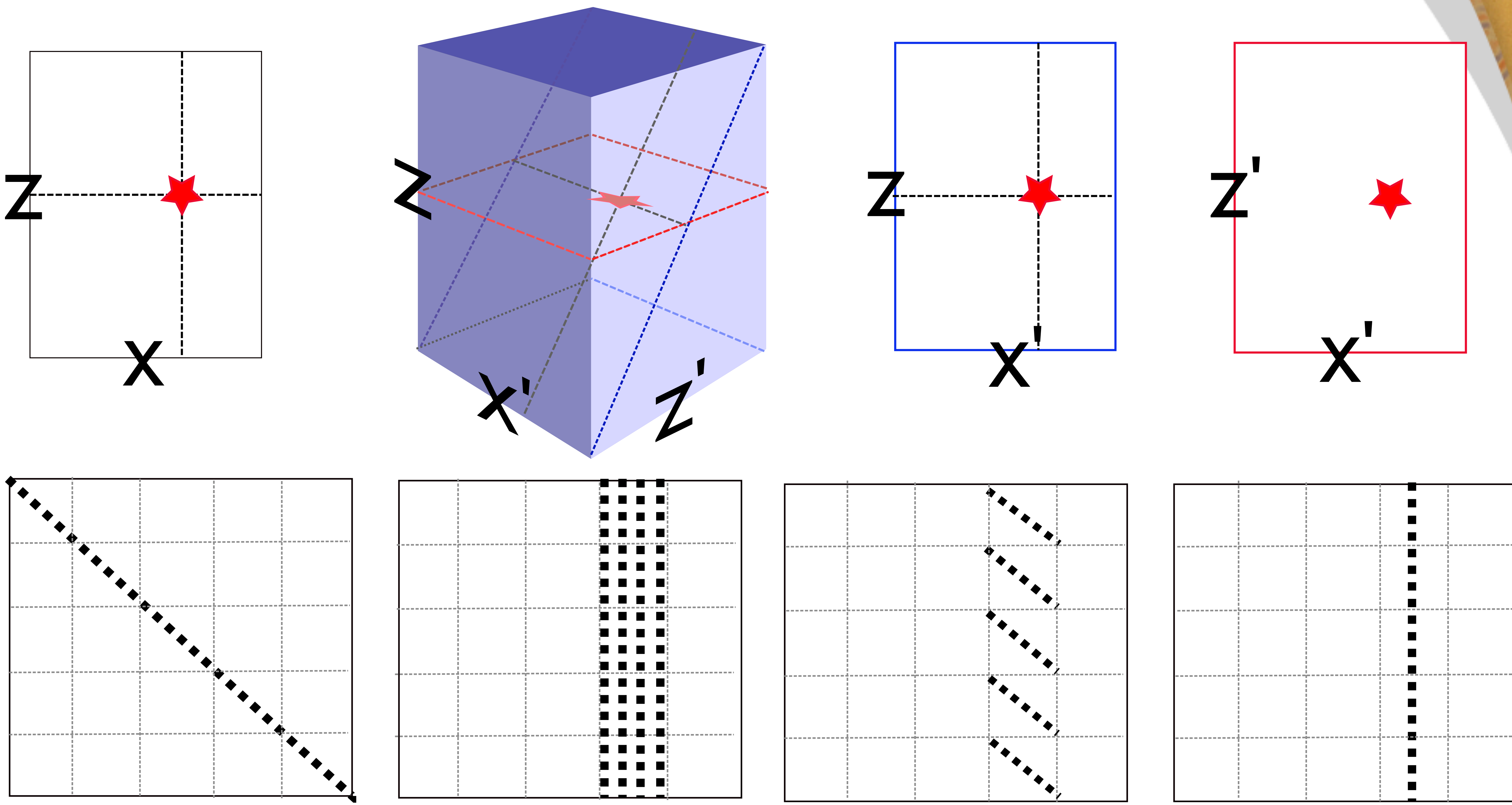
$$E(\omega) = \sum_{\omega} V(\omega) U(\omega)^*$$

imaging condition: $\sum_{\omega} \mathrm{diag}(E(\omega))$

Extended images

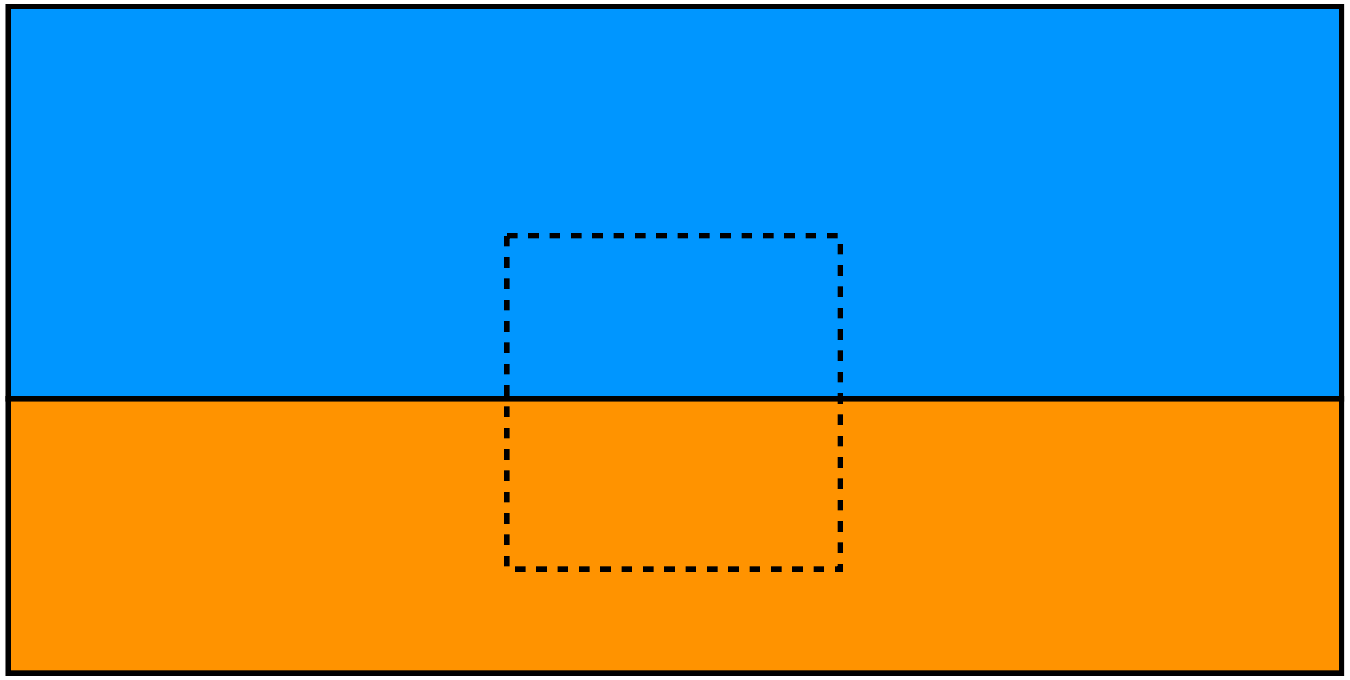


Extended images



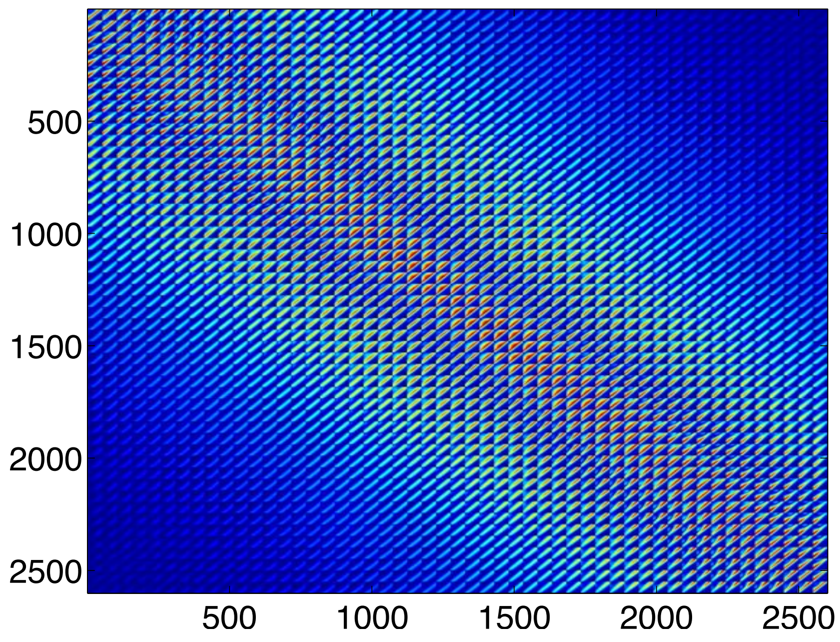
Extended images

example for one layer

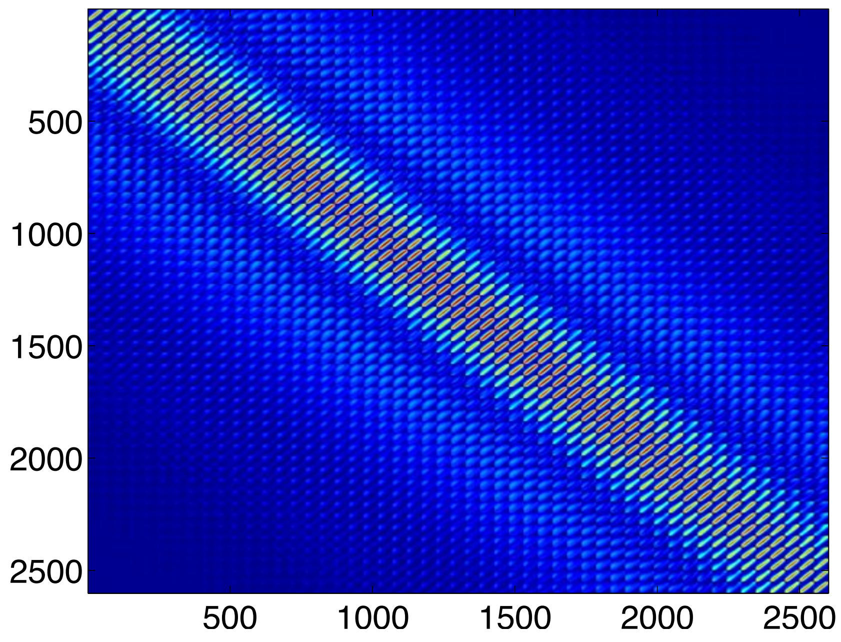


Extended images

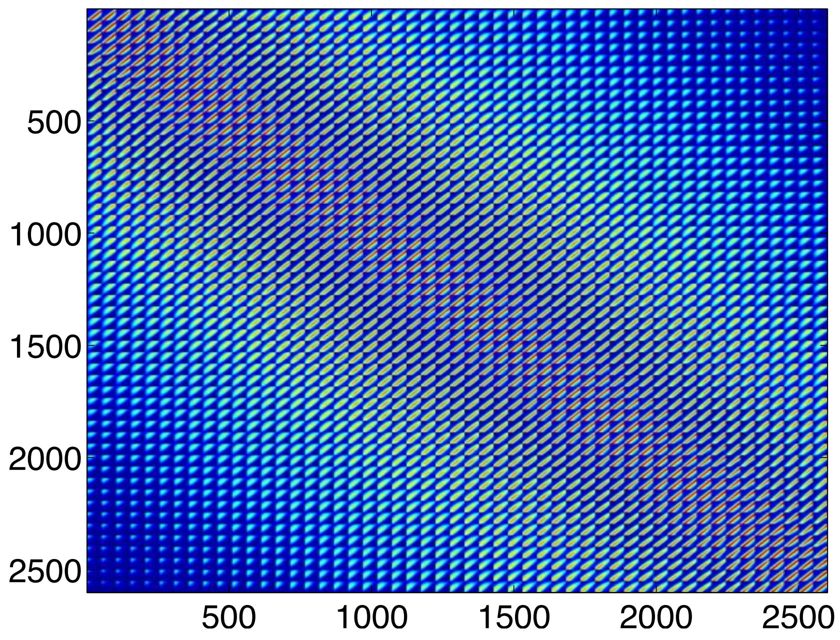
full matrix



low velocity



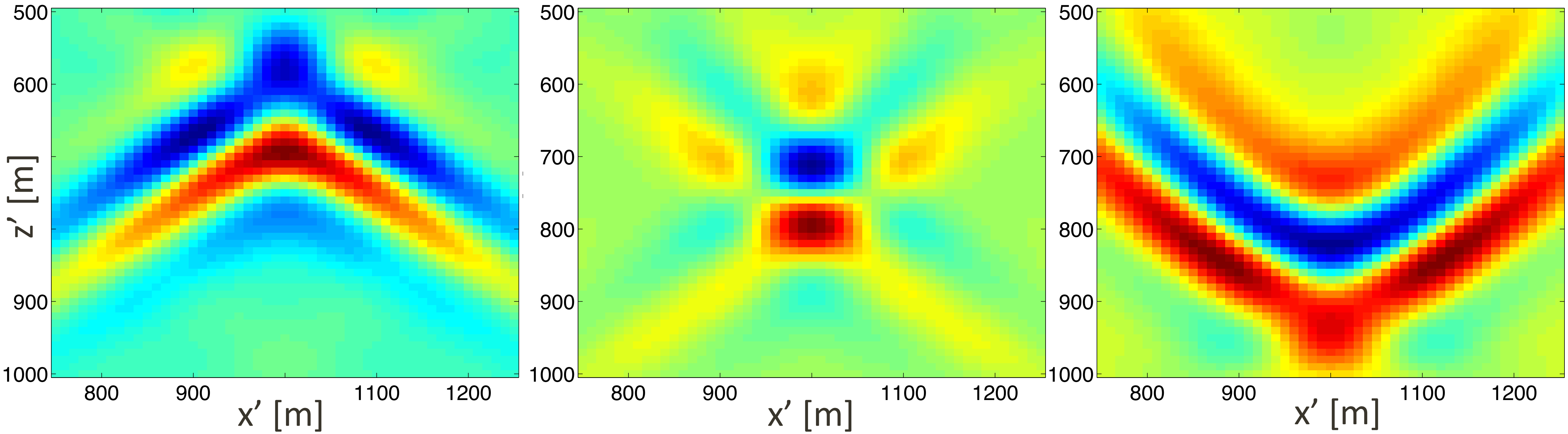
correct velocity



high velocity

Extended images

one column



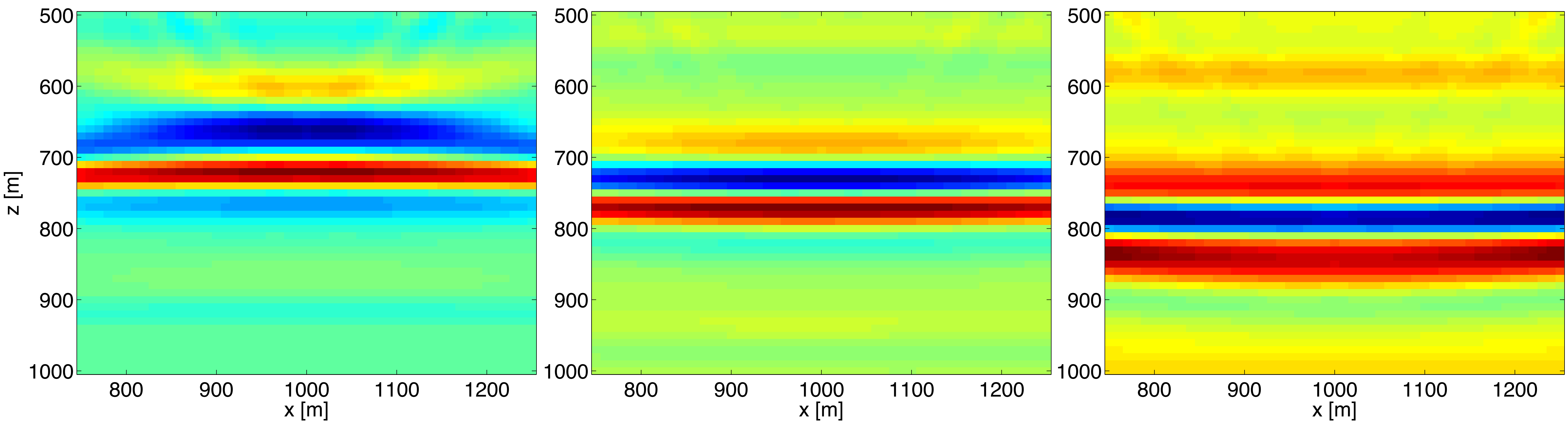
low velocity

correct velocity

high velocity

Extended images

diagonal



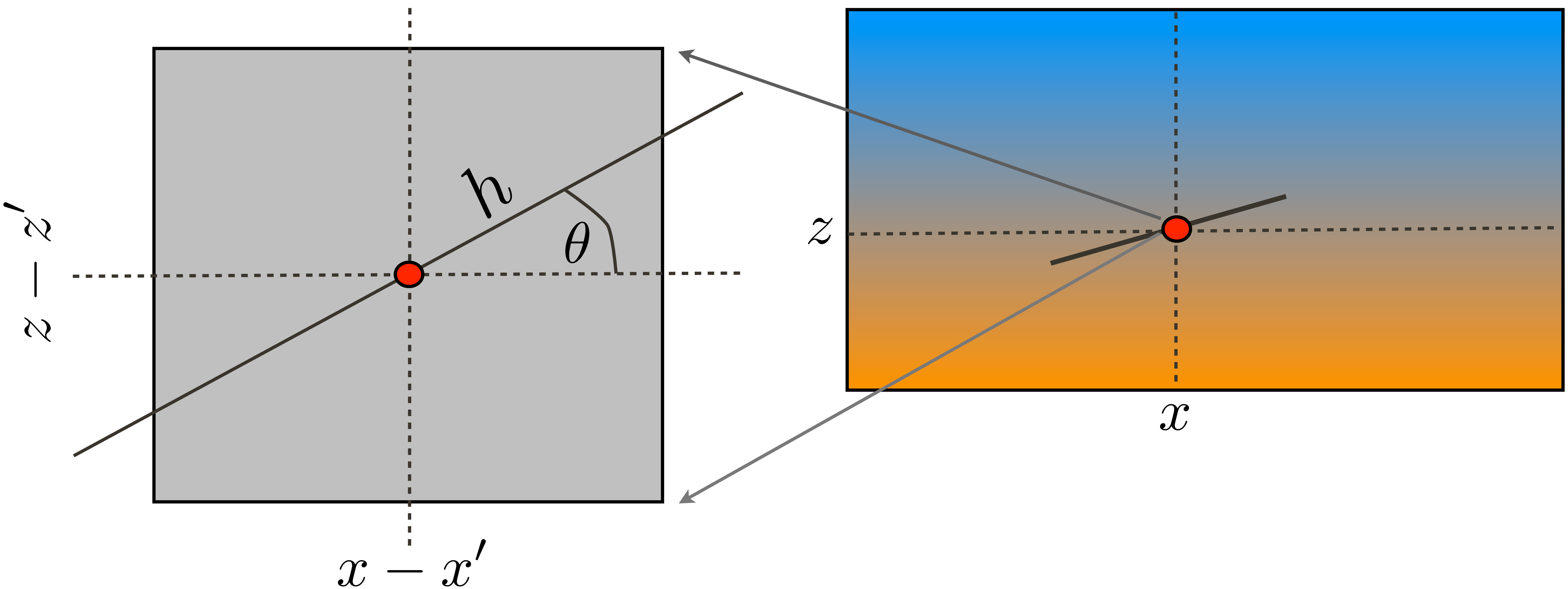
low velocity

correct velocity

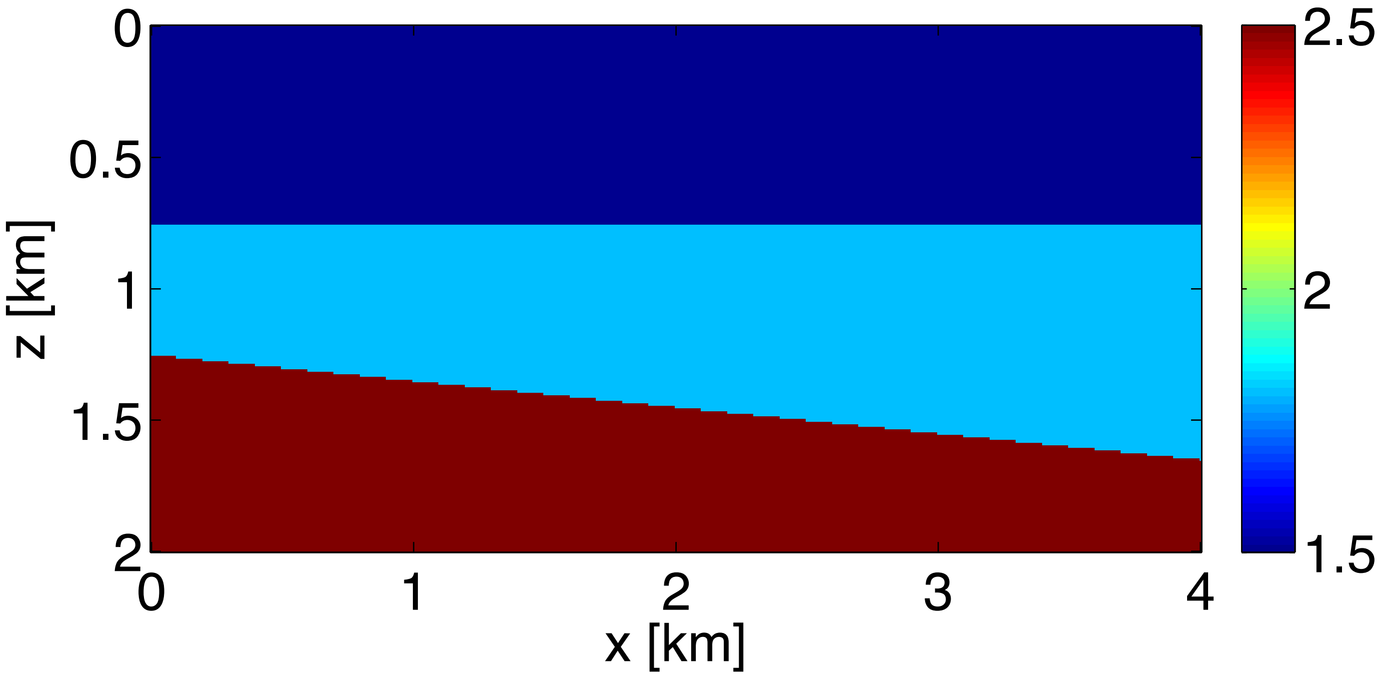
high velocity

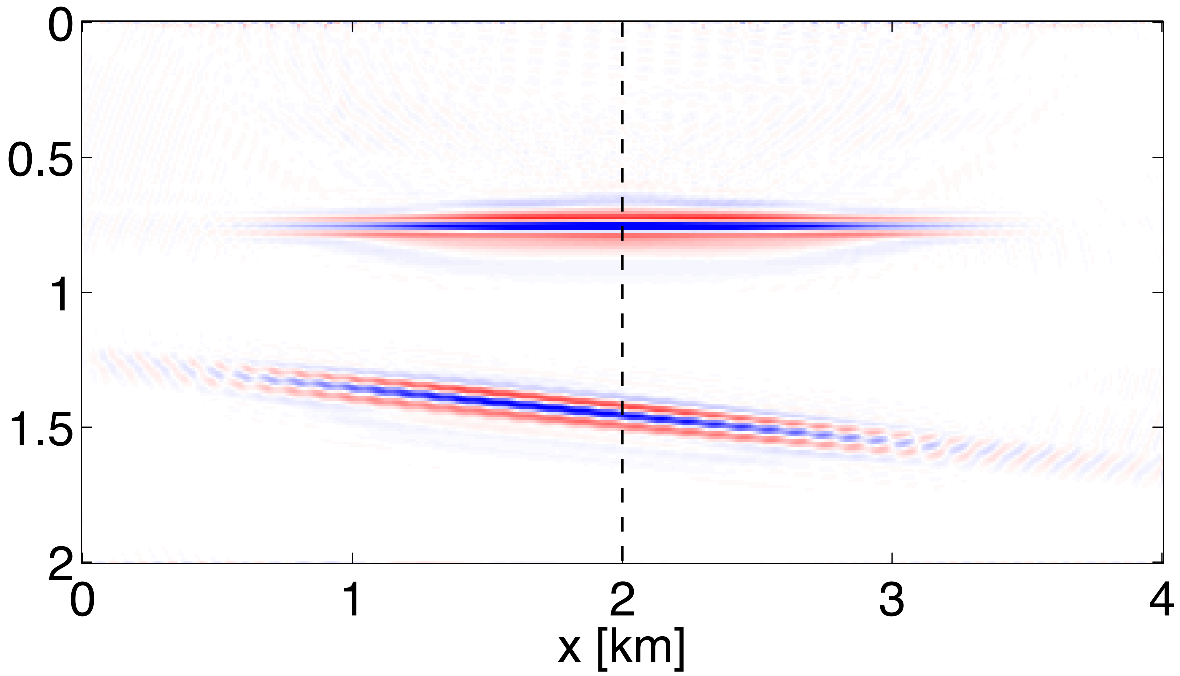
offset gathers

align subsurface offset with local dip

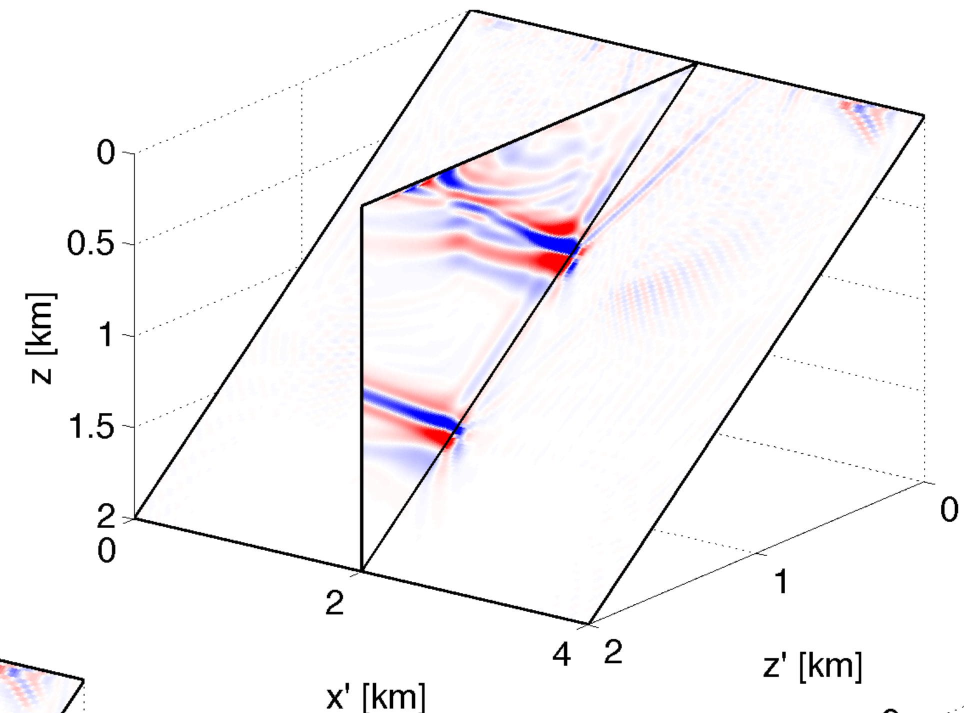


Example

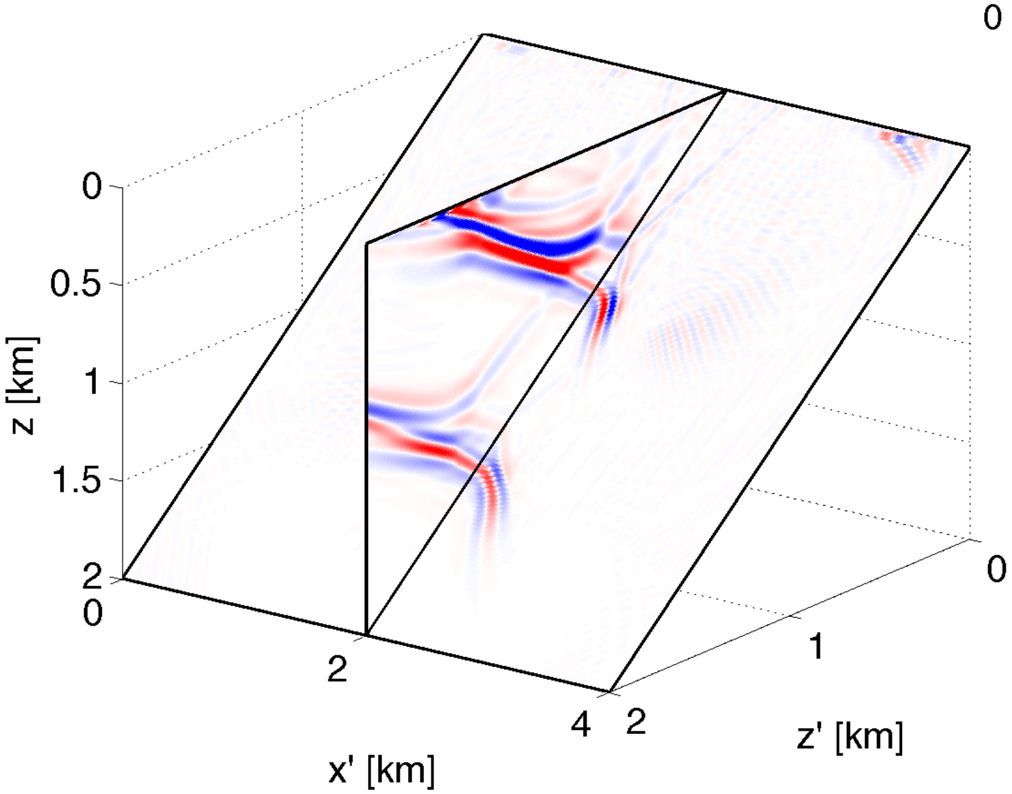




Example

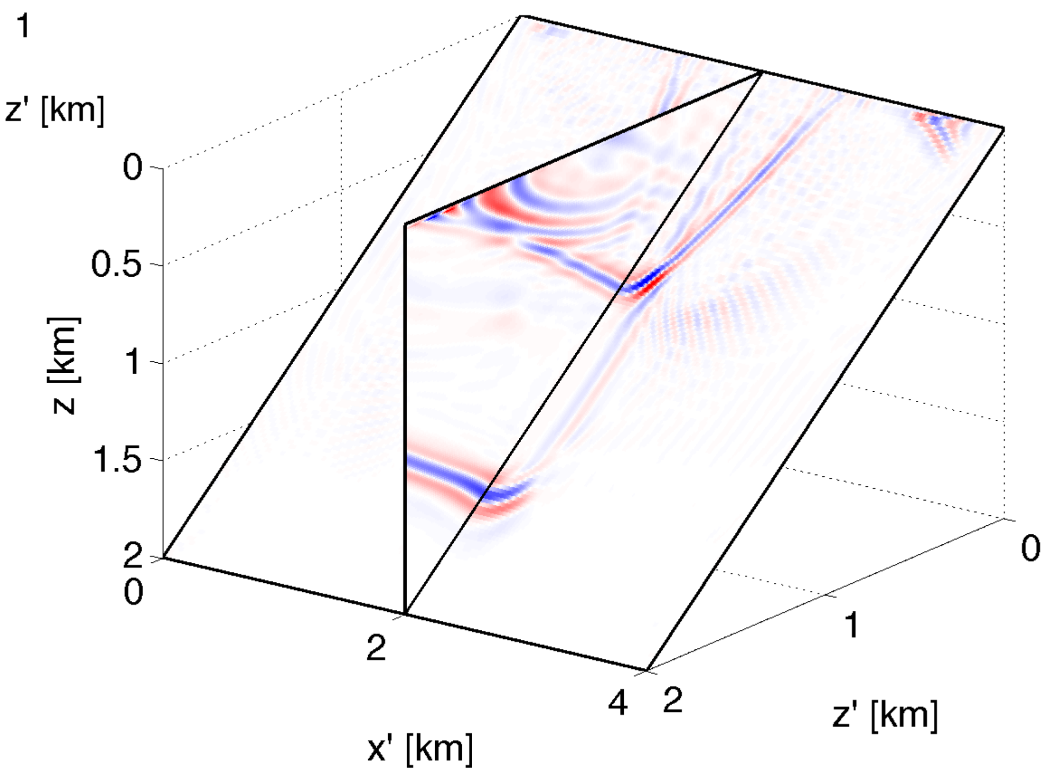


low velocity

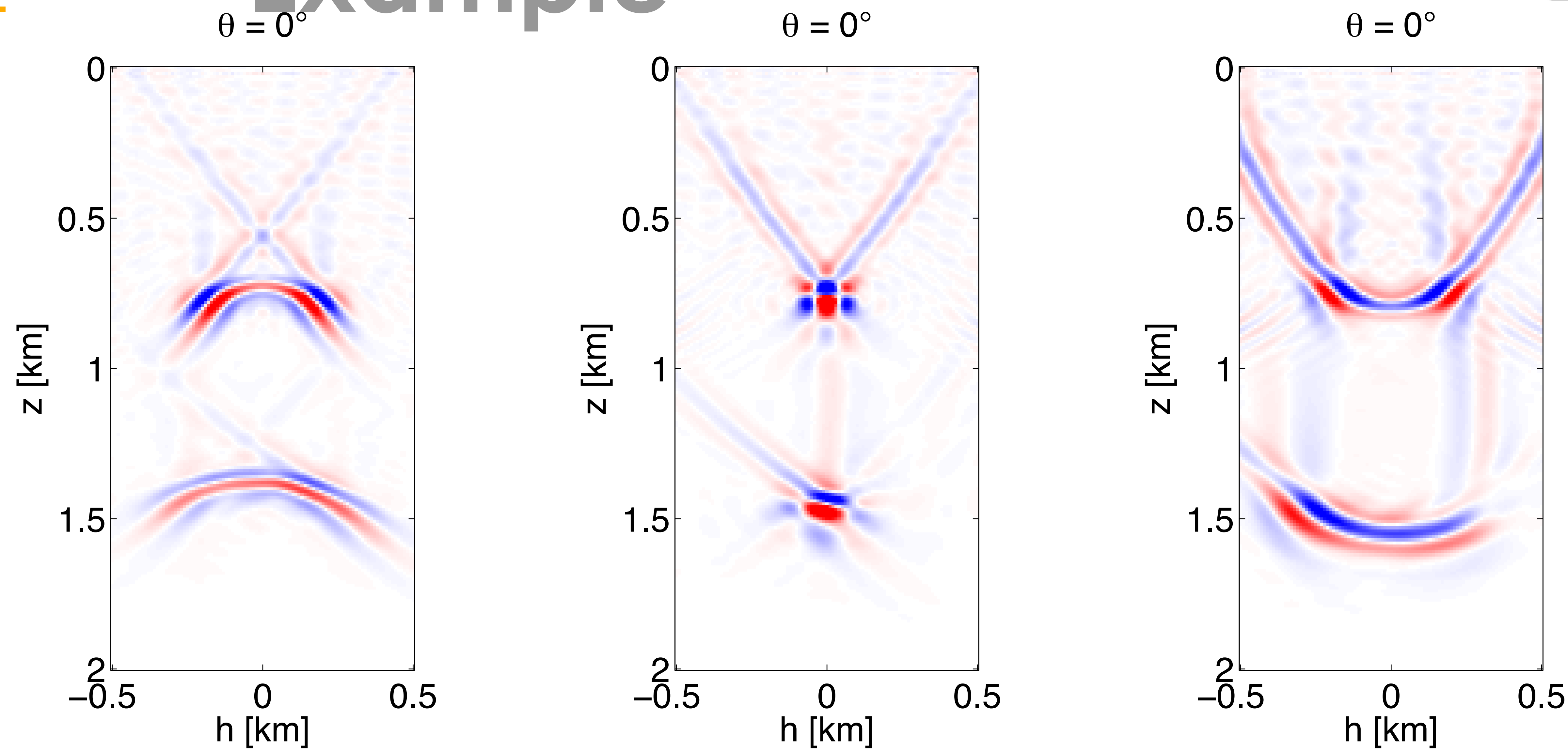


correct velocity

high velocity



Example



low velocity

correct velocity

high velocity

Double wave-equation

Helmholtz operator: $H = \omega^2 \text{diag}(\mathbf{m}) + \nabla^2$

source/receiver wavefields:

$$HU = P_s^T Q \qquad H^* V = P_r^T D$$

RTM extended image: $E = VU^*$

yields: $H^* E H = P_r^T D Q^* P_s$

Double wave-equation

$$Le(\omega, \mathbf{x}, \mathbf{x}') = \int \mathrm{d}\mathbf{s} \int \mathrm{d}\mathbf{r}\, d(\omega, \mathbf{s}, \mathbf{r}) \delta(\mathbf{x} - \mathbf{s}) \delta(\mathbf{x}' - \mathbf{r})$$

two-way:

$$L = \left[\omega^2/c(z,x)^2 + \partial_x^2 + \partial_z^2\right] \left[\omega^2/c(z',x')^2 + \partial_{x'}^2 + \partial_{z'}^2\right]$$

one-way (DSR):

$$L = \left[\partial_z - \imath\sqrt{\omega^2/c(z,x)^2 + \partial_x^2} - \imath\sqrt{\omega^2/c(z,x')^2 + \partial_{x'}^2}\right]$$

[Claerbout, '84; Stolk & de Hoop '01]

Velocity continuation

since r.h.s. is model-independent:

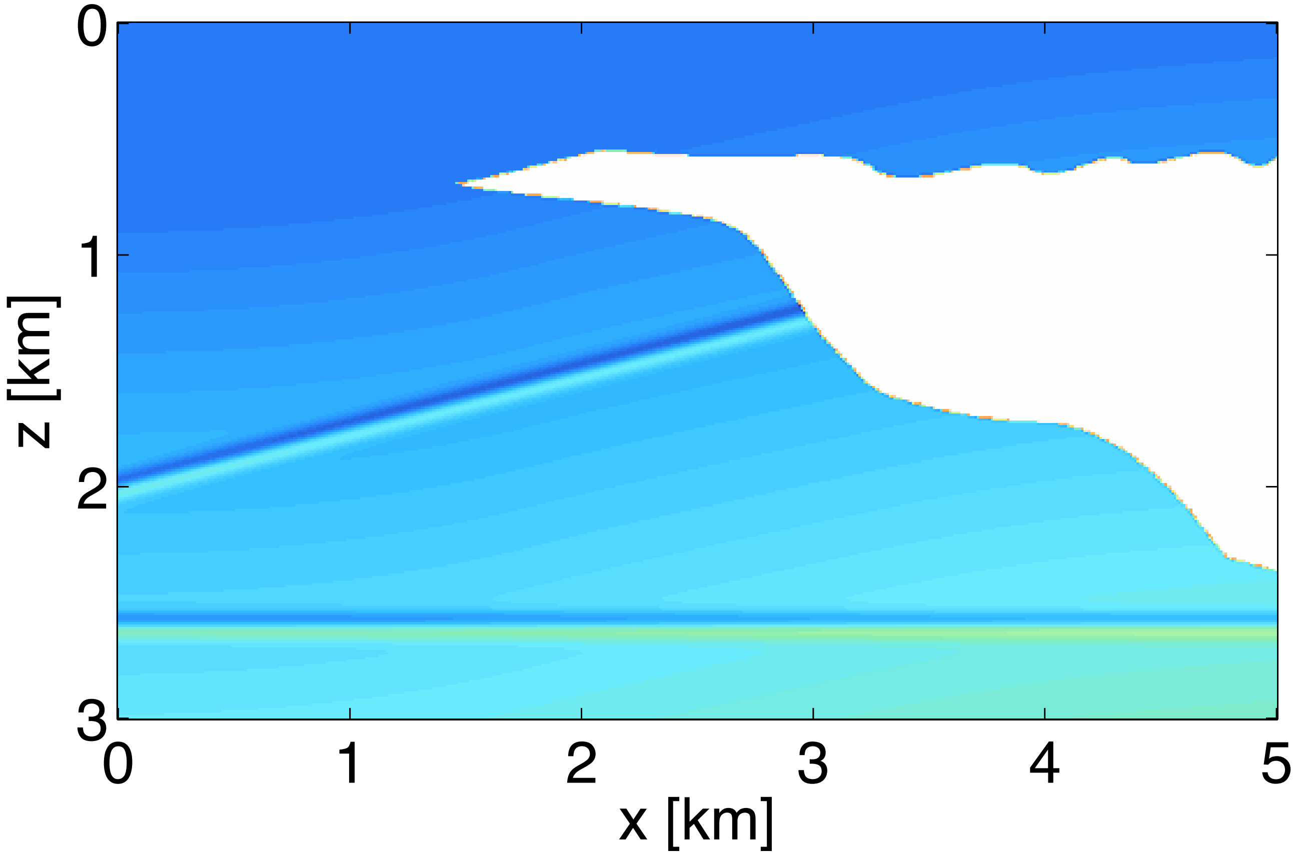
$$H_2^* E_2 H_2 = H_1^* E_1 H_1$$

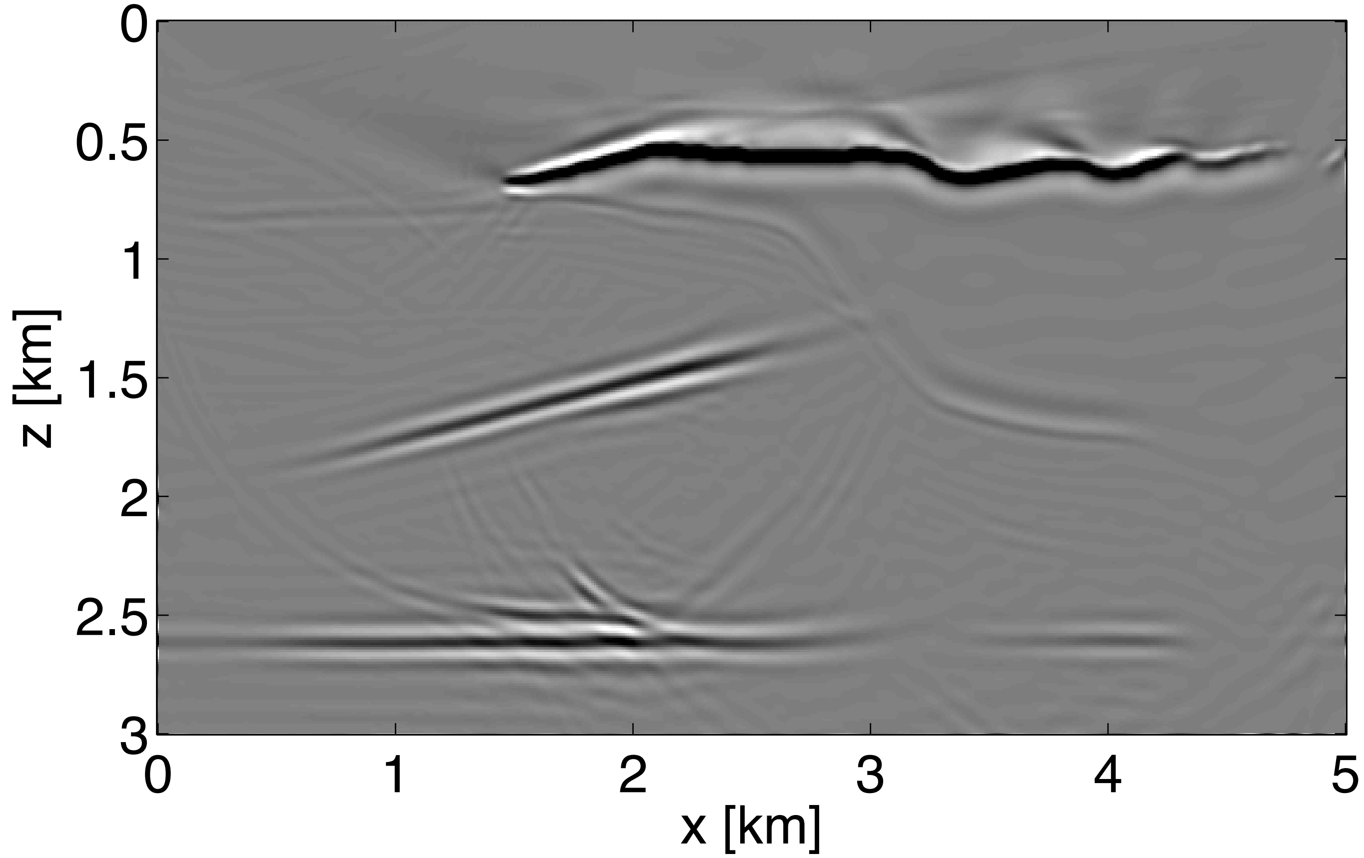
or

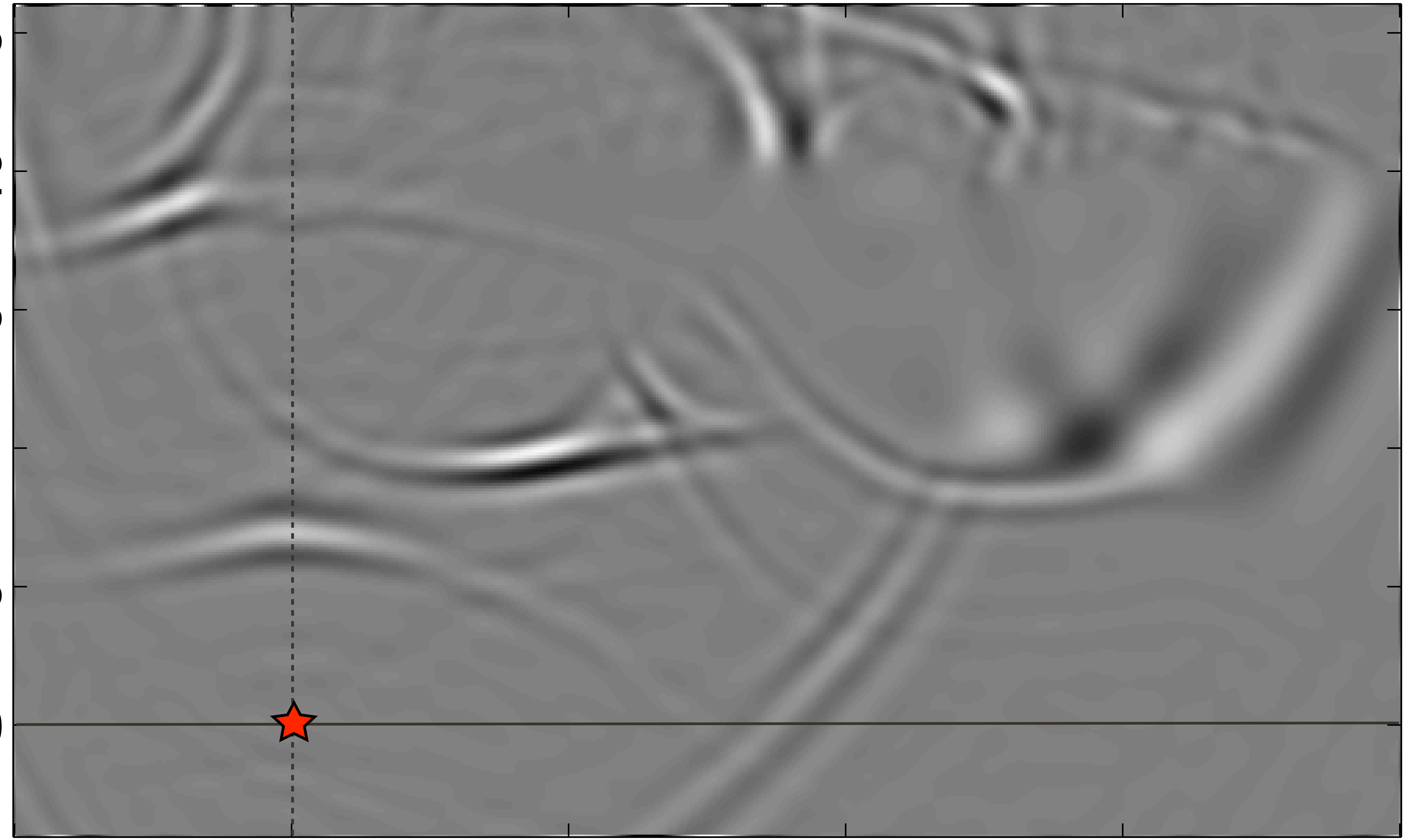
$$E_2 = H_2^{-*} H_1^* E_1 H_1 H_2^{-1}$$

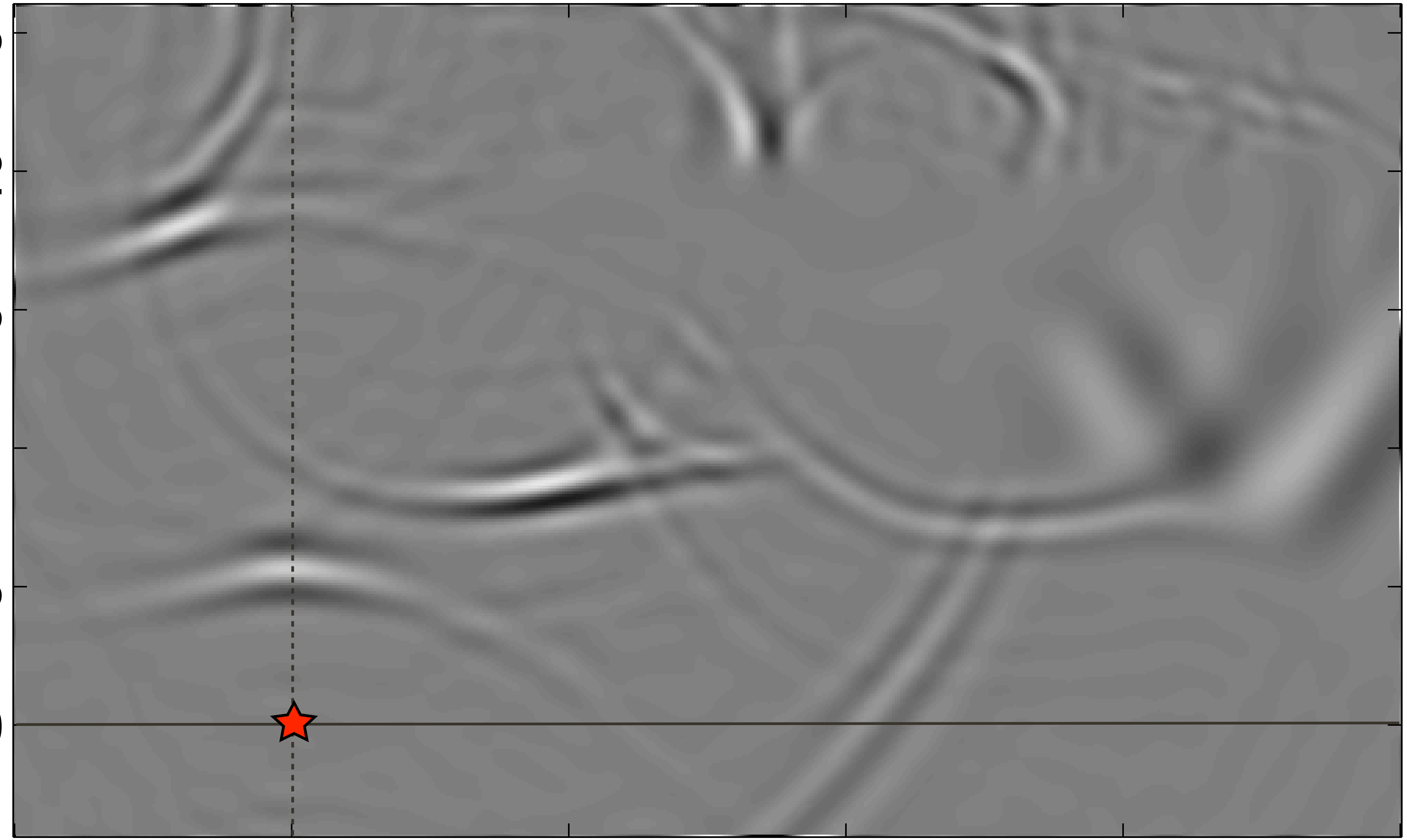
[Duchkov et al, '08]

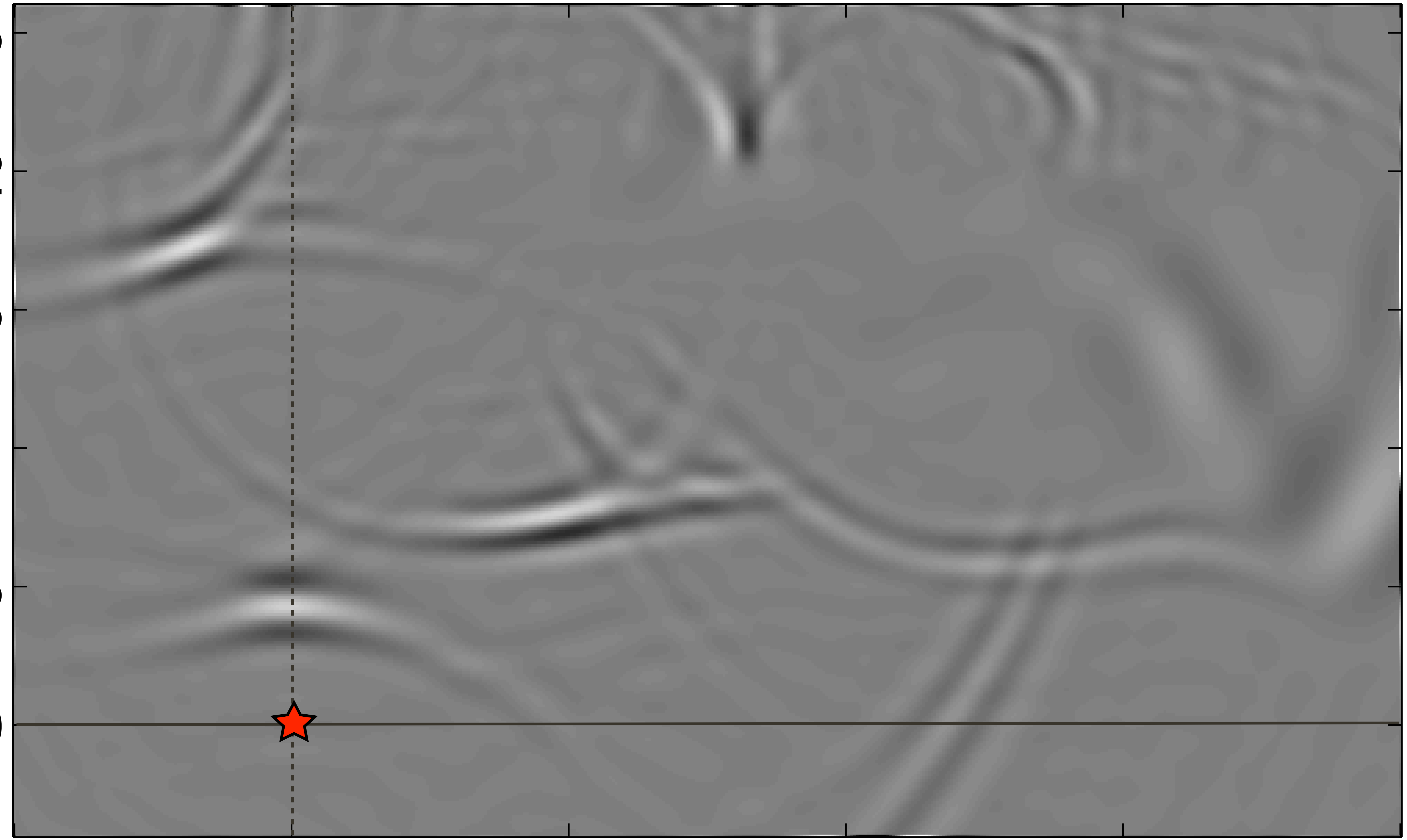
Examples

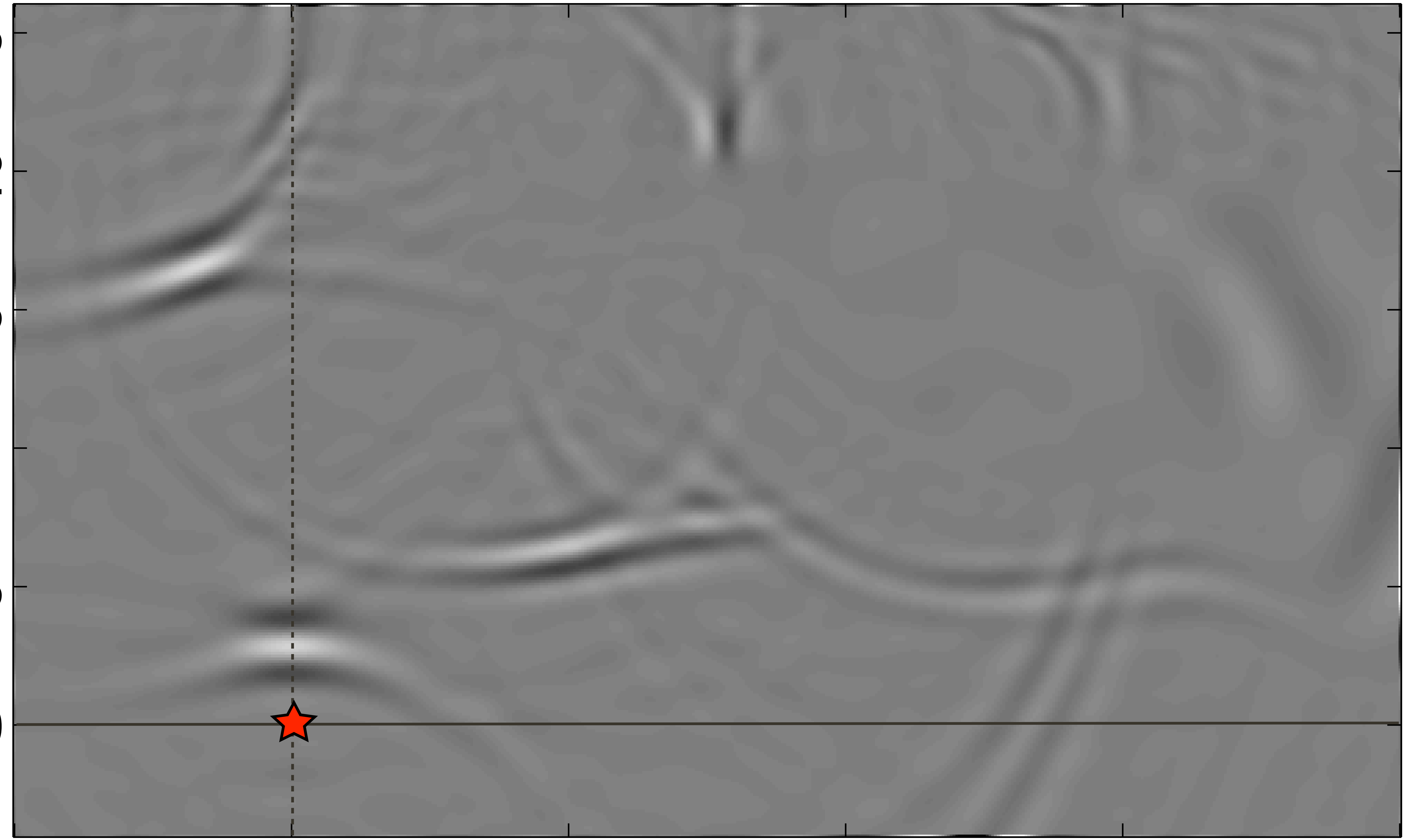


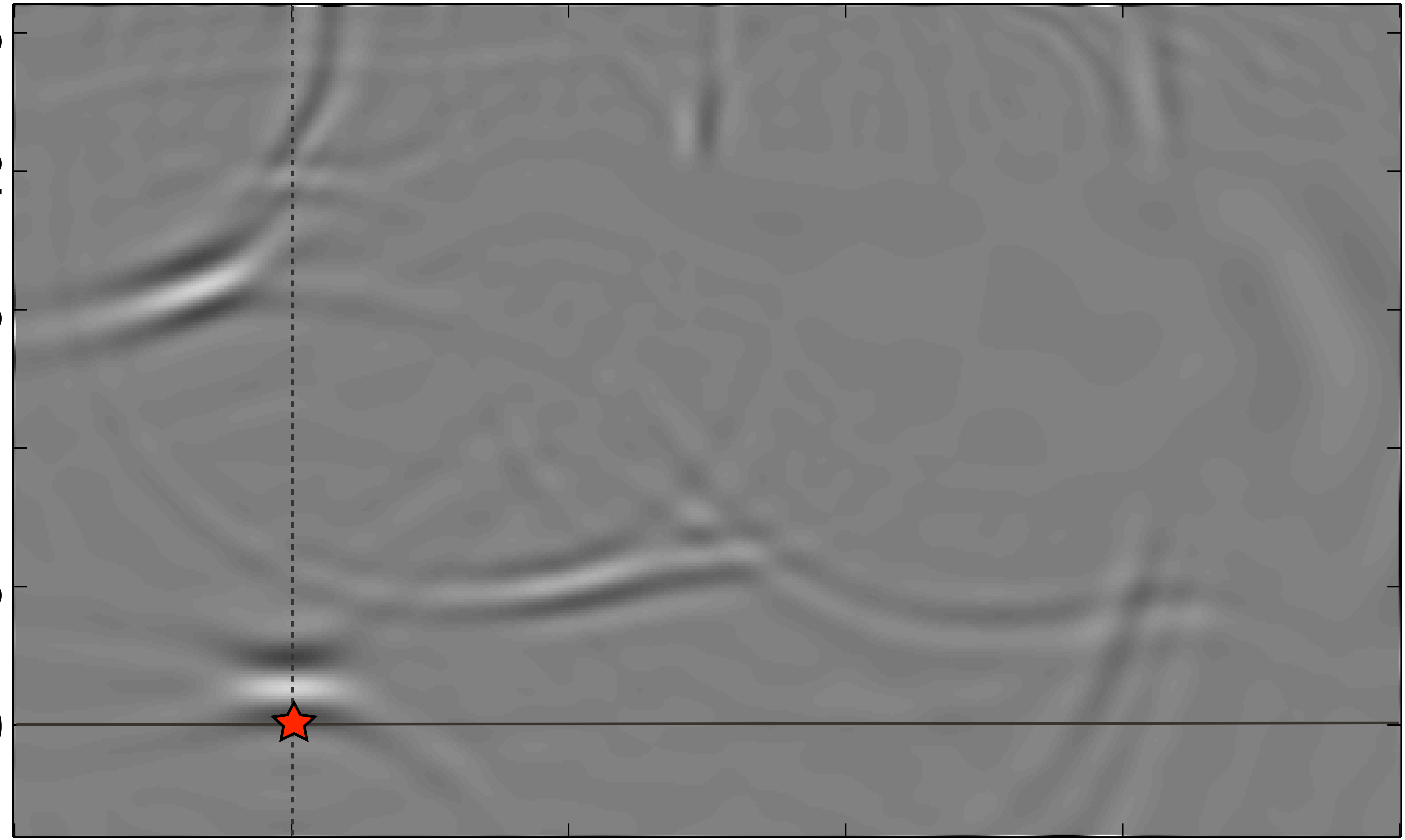


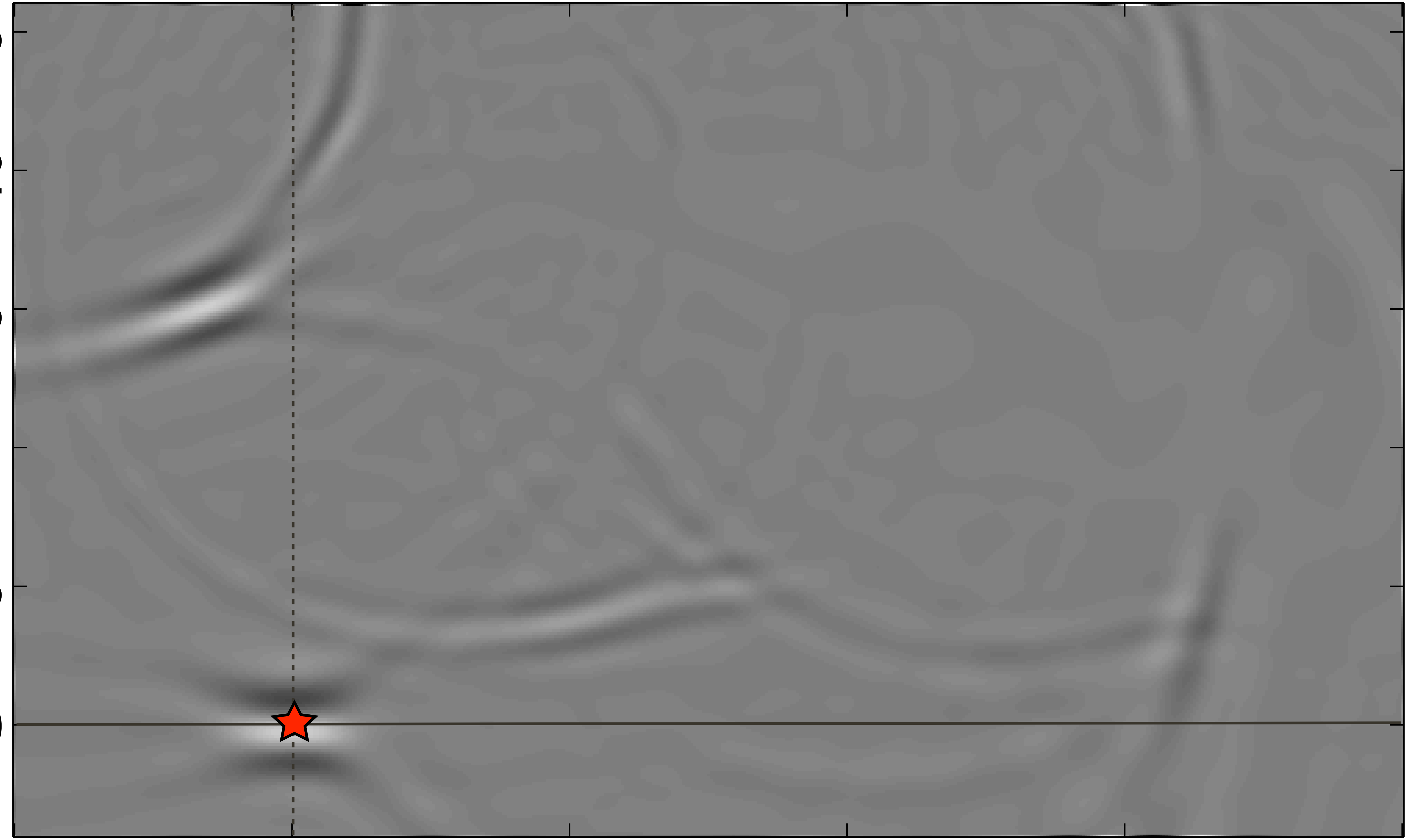


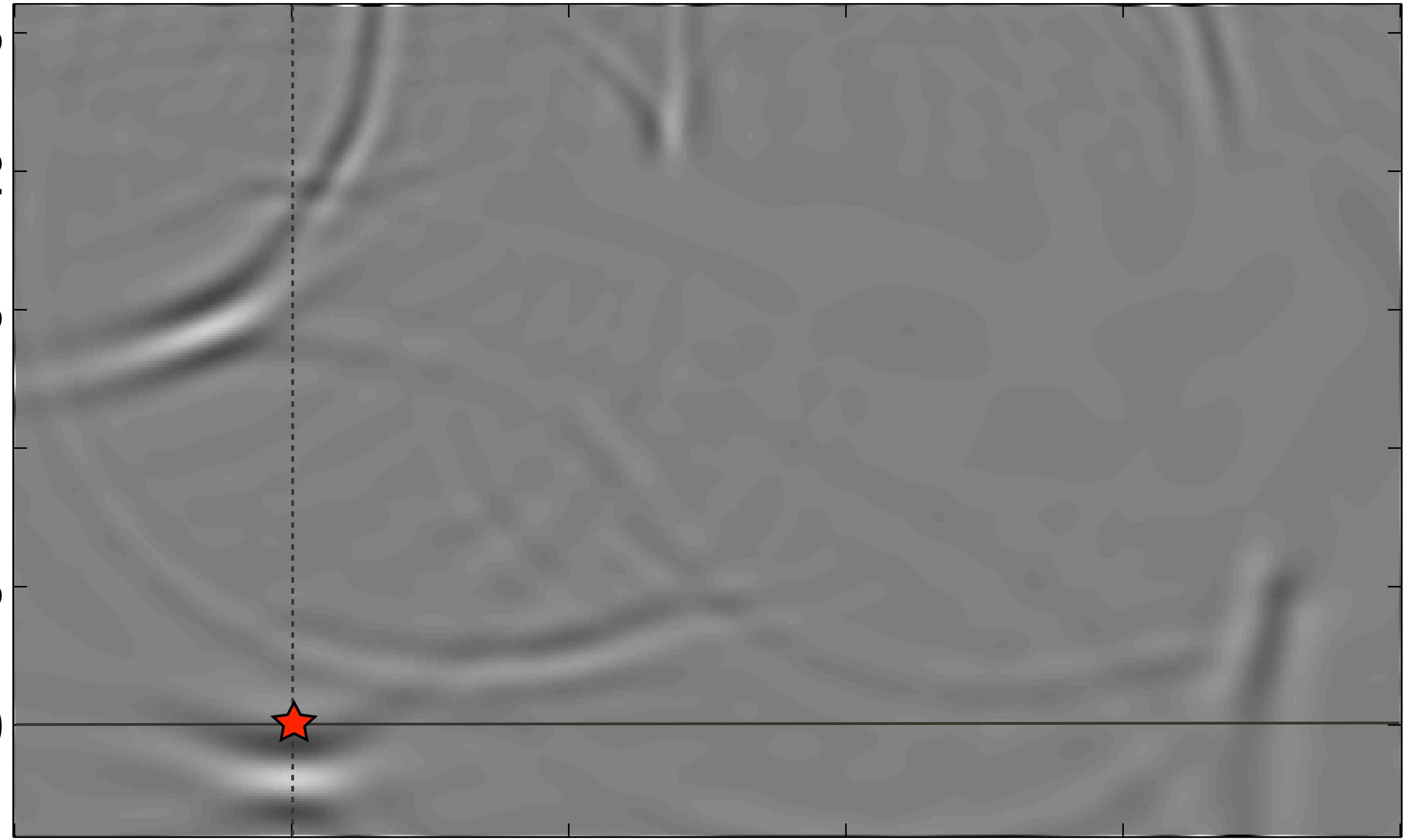


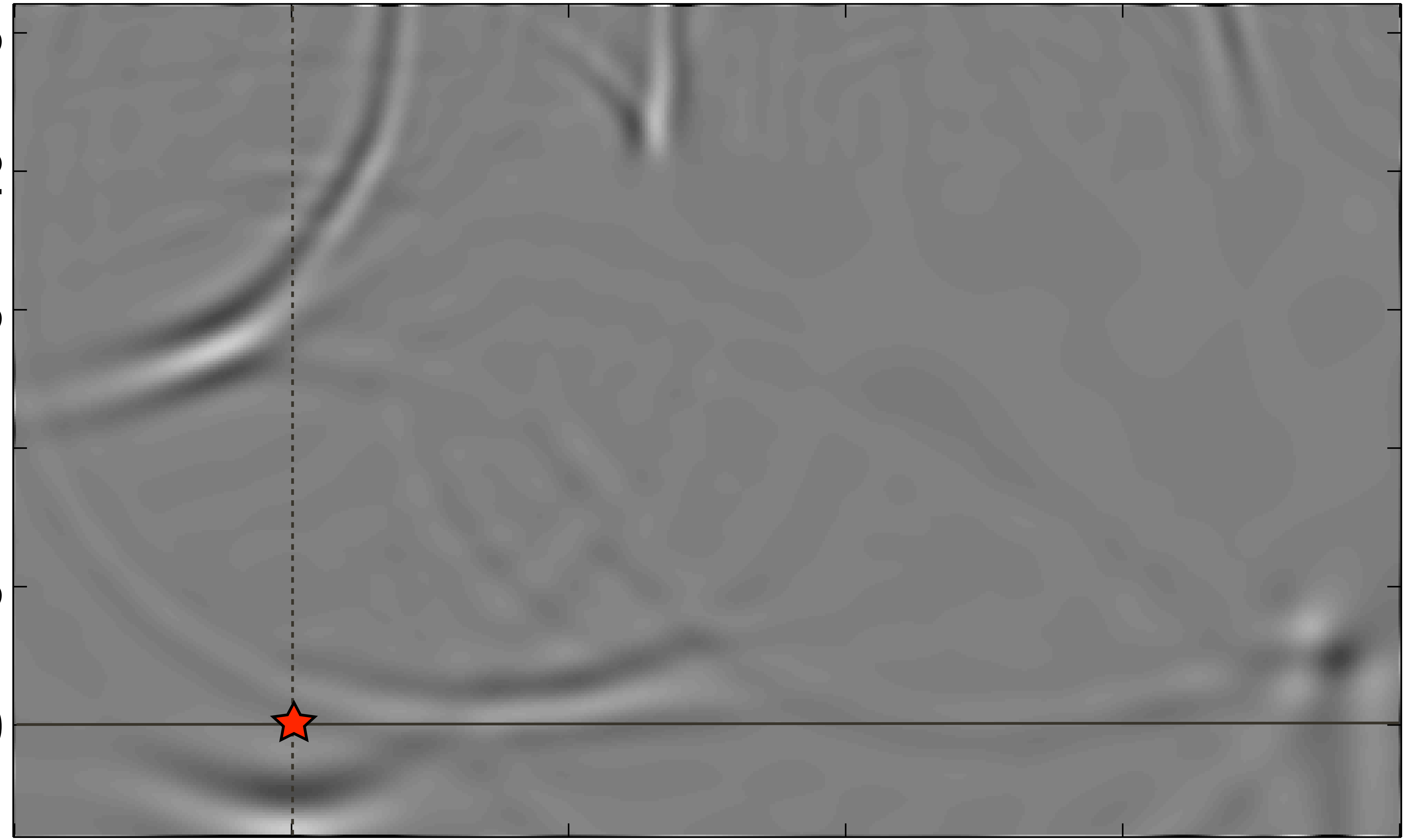


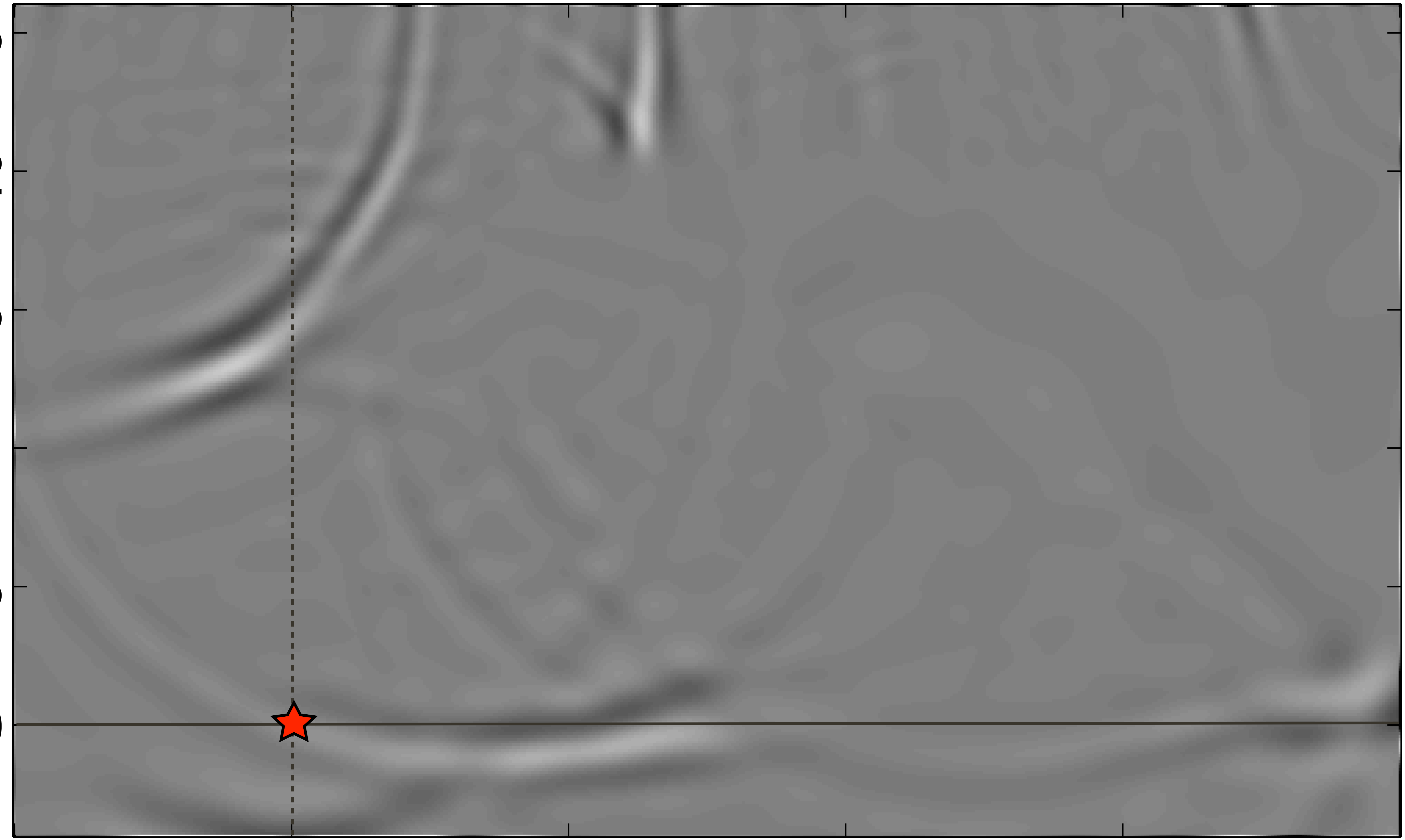


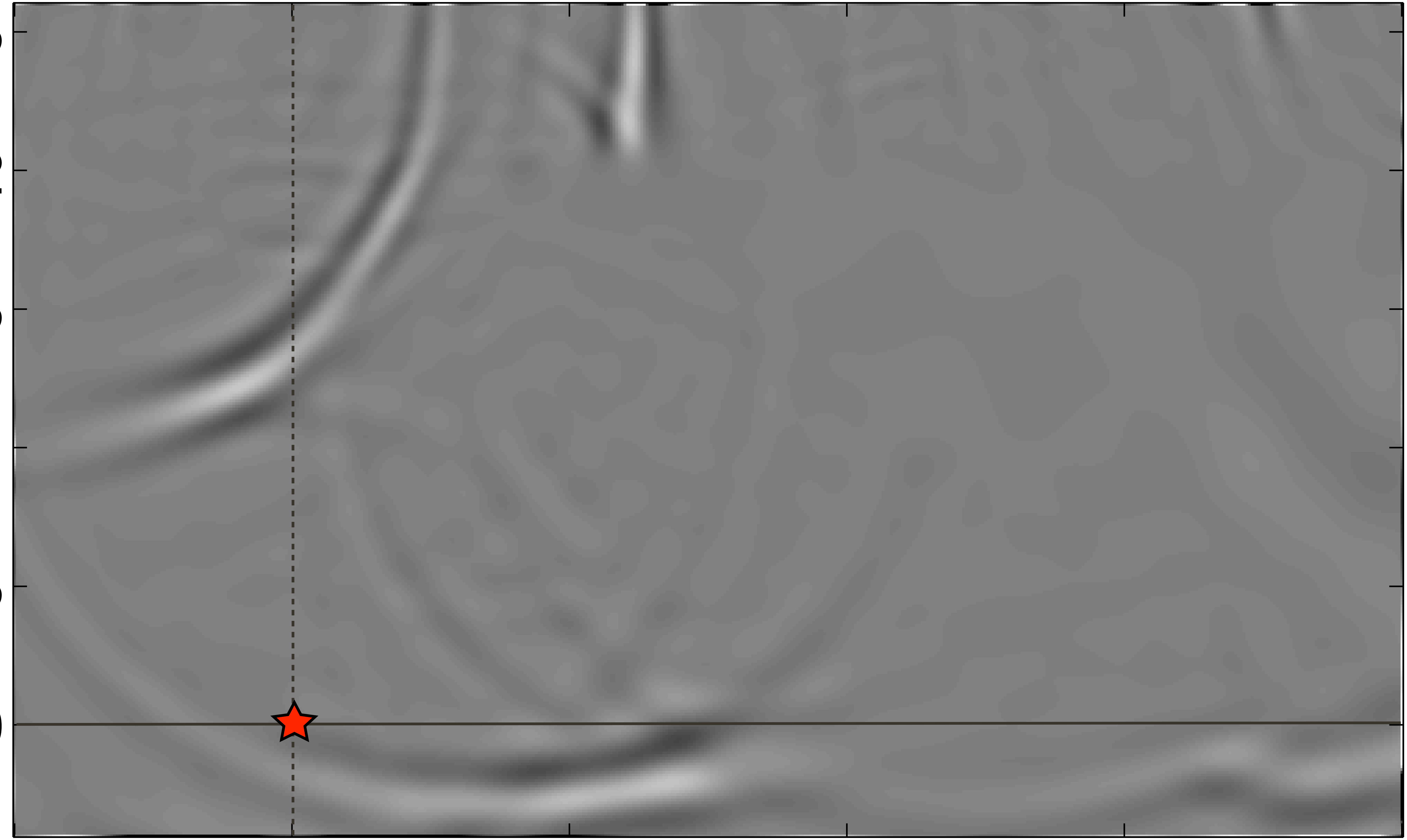


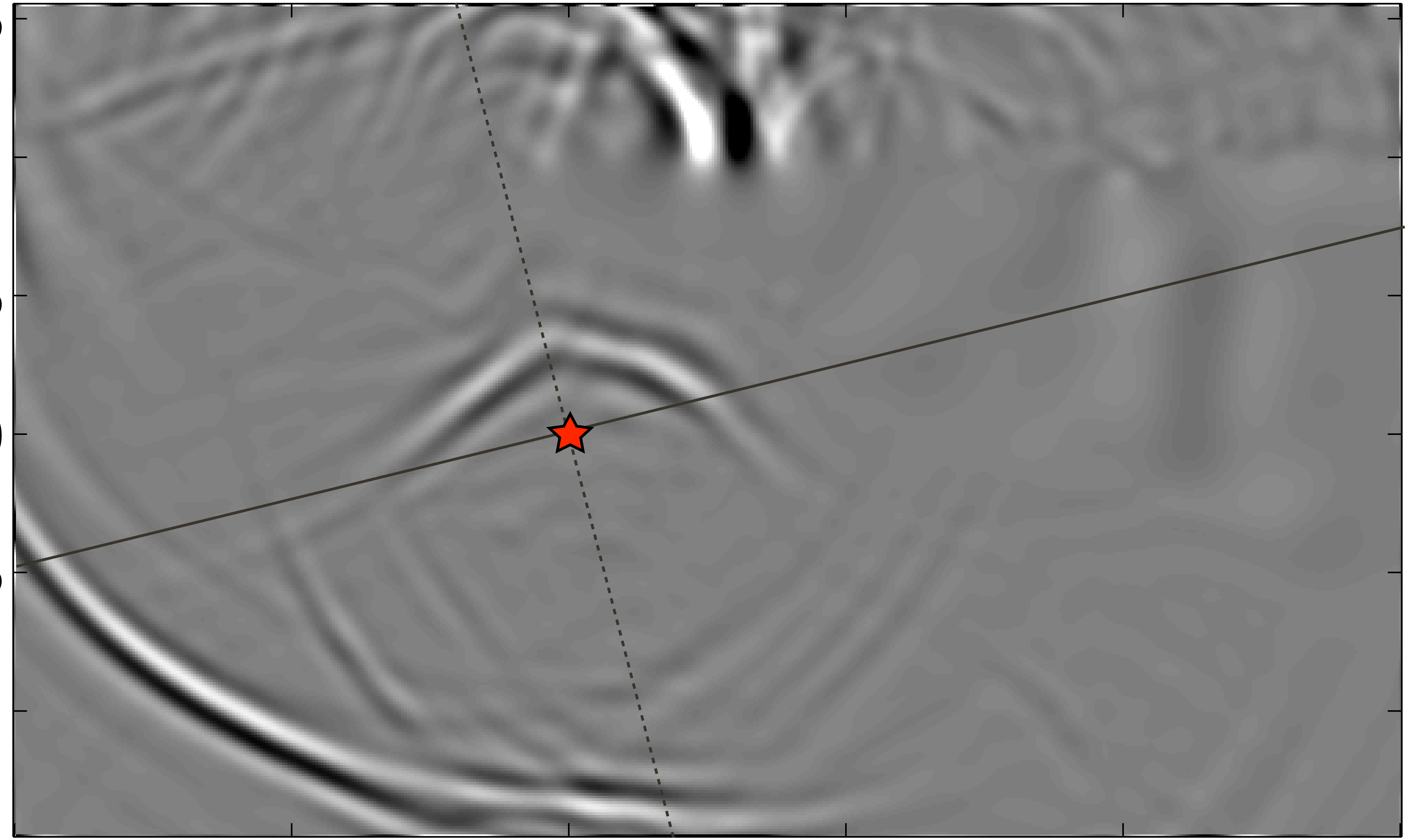


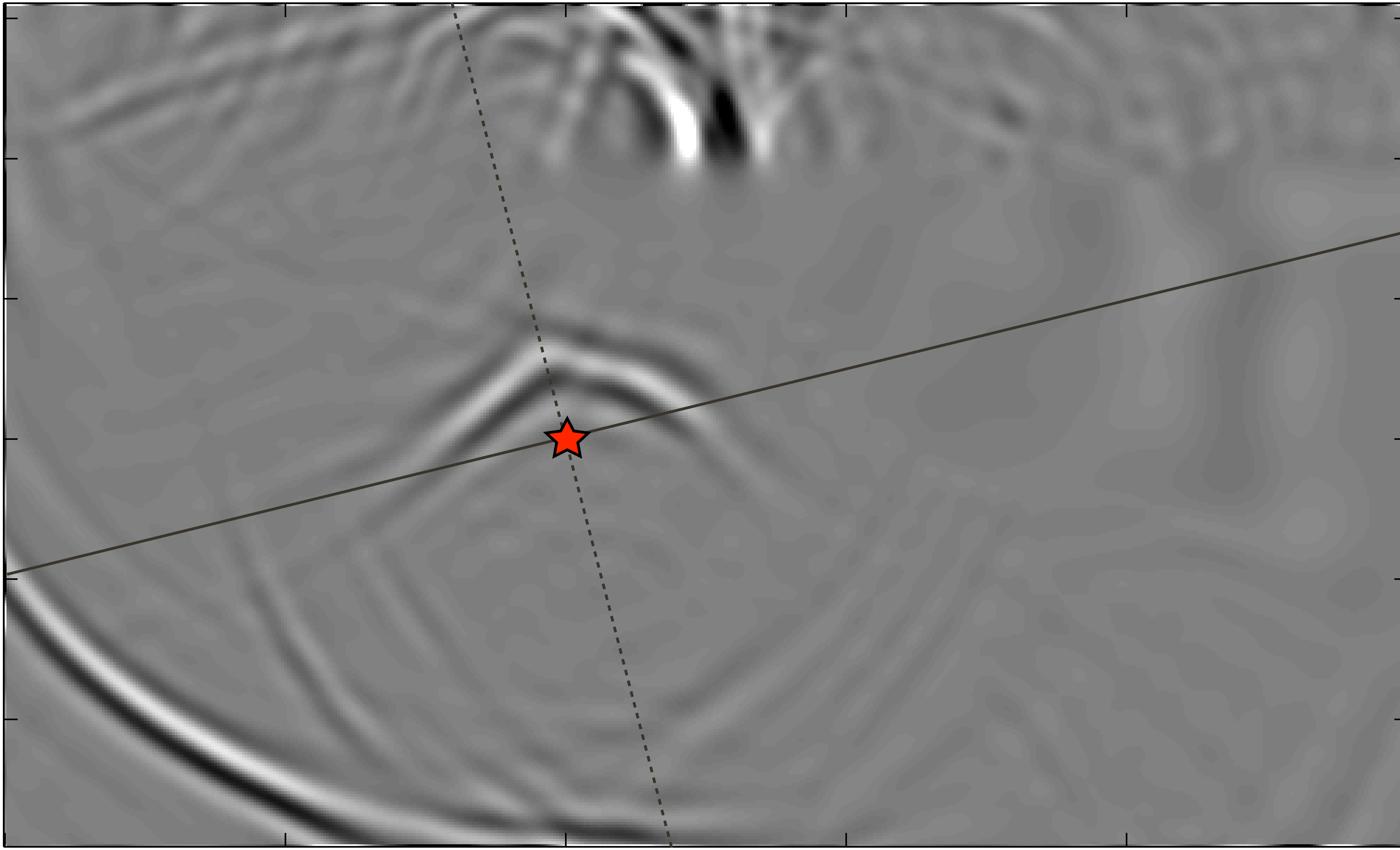


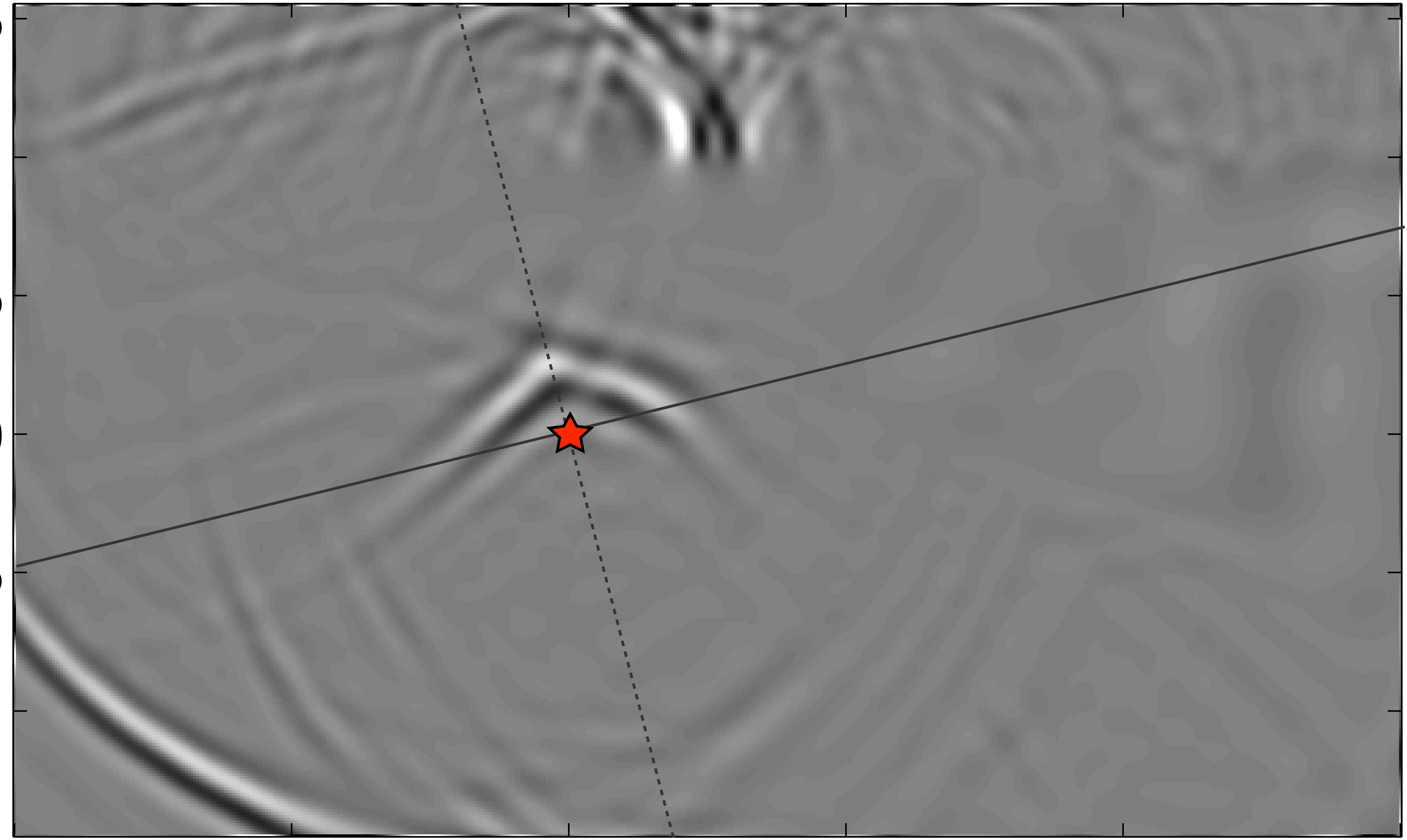


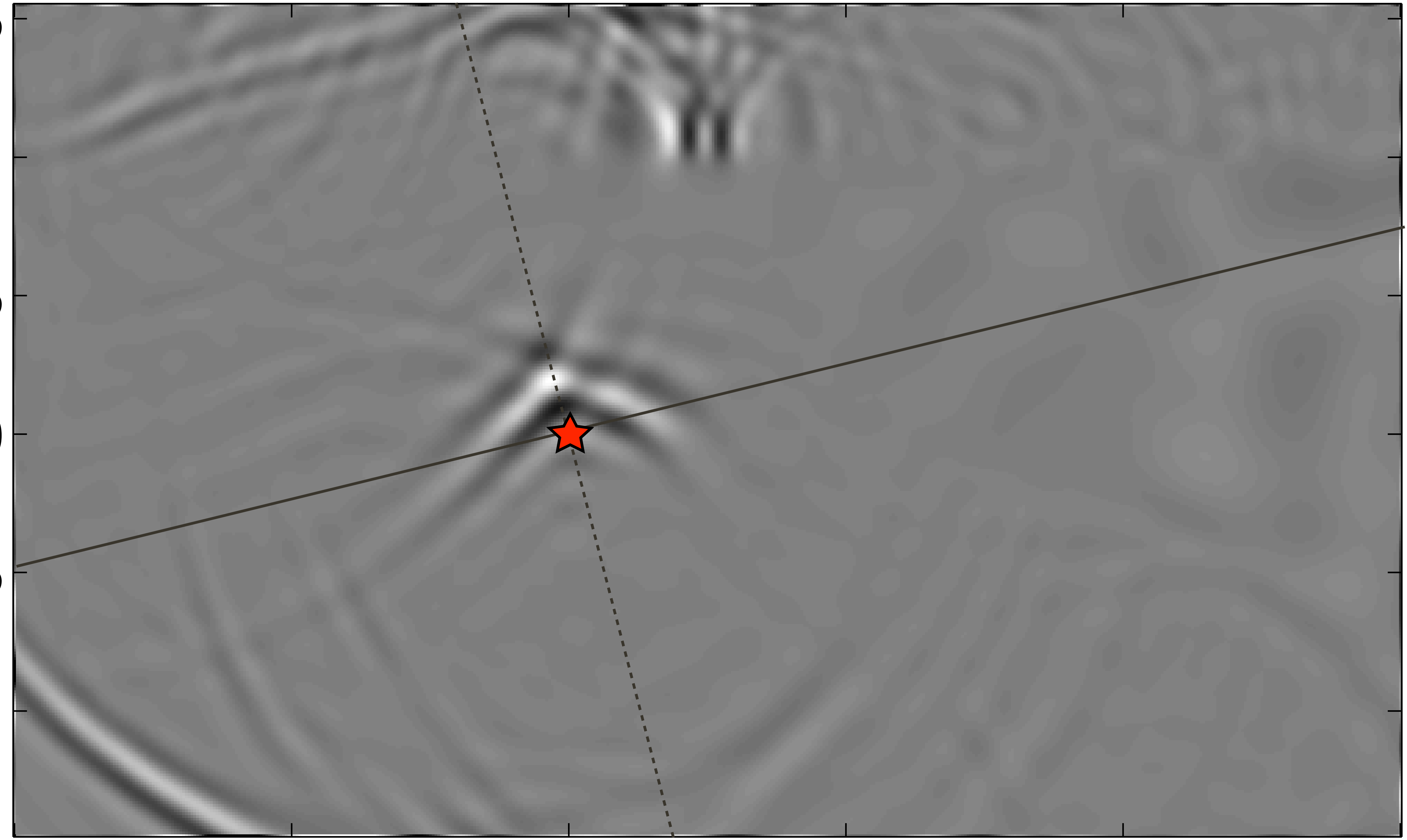


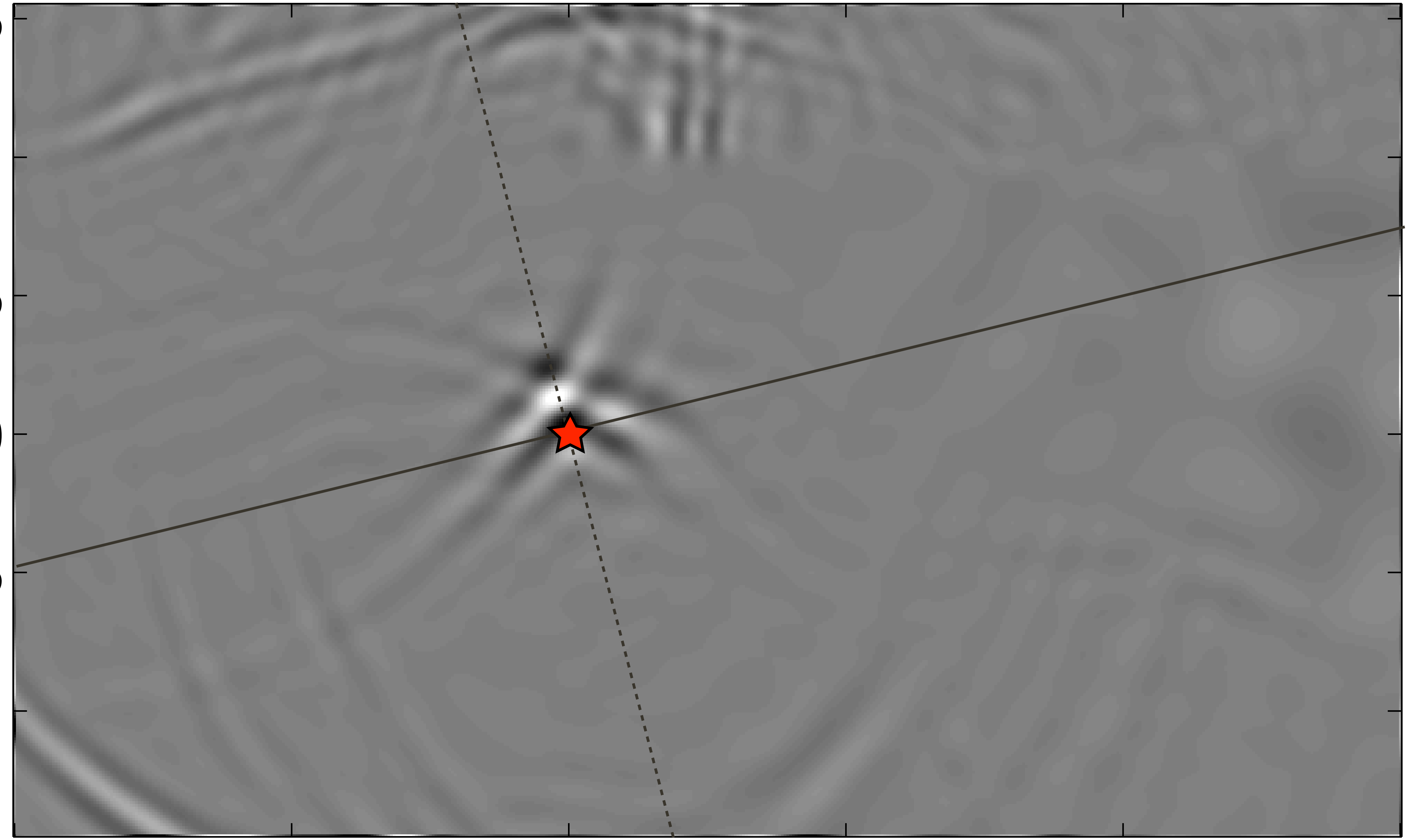


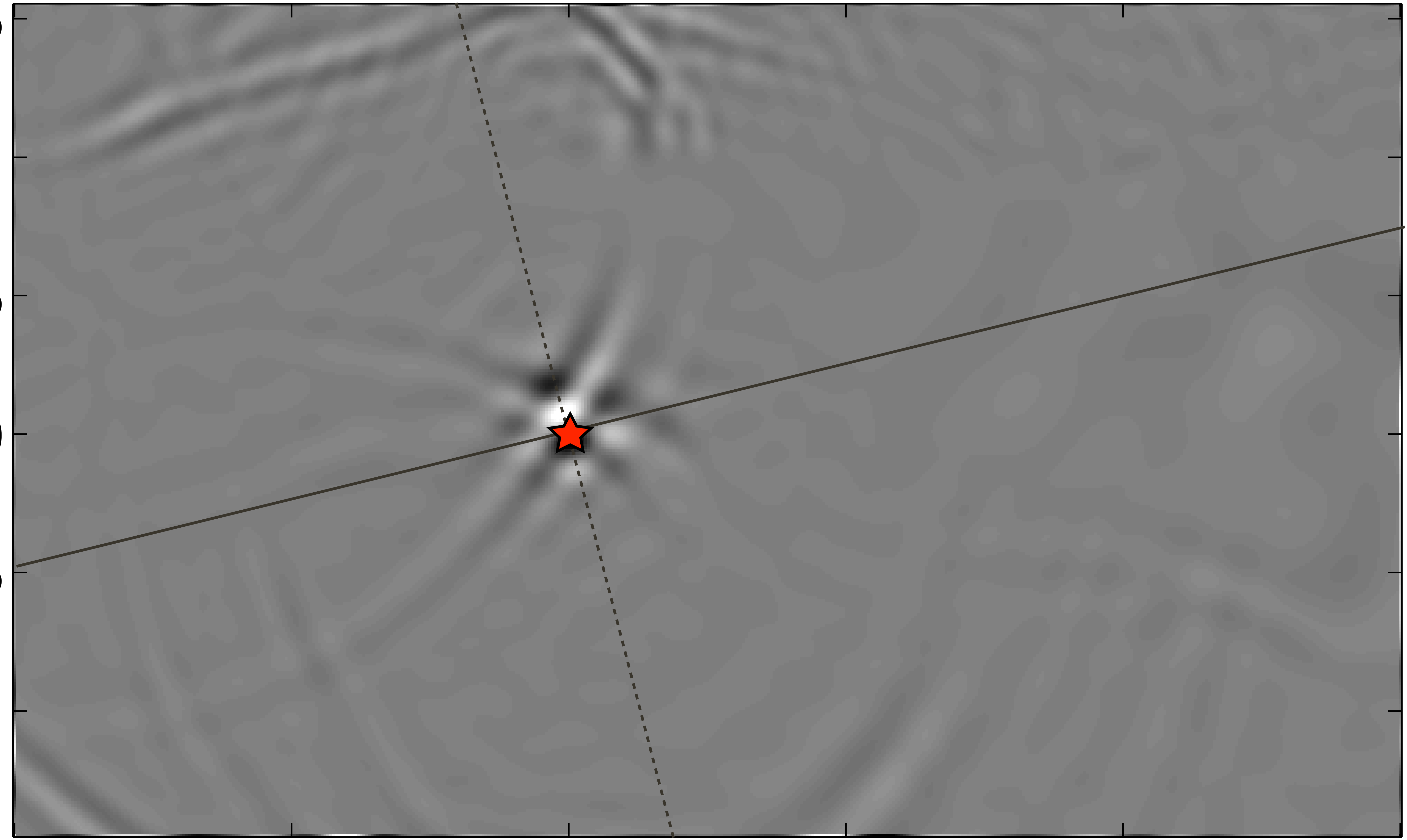


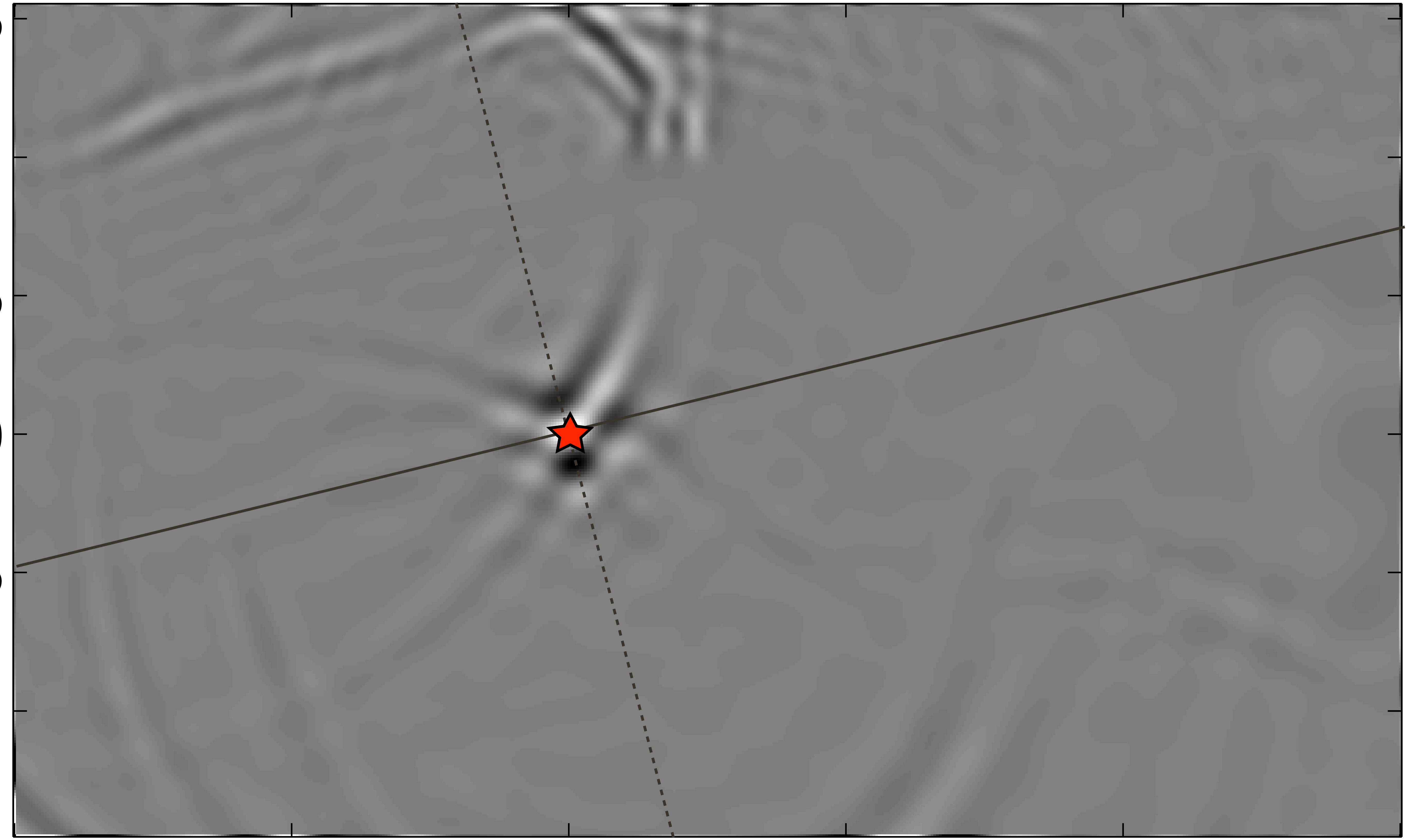


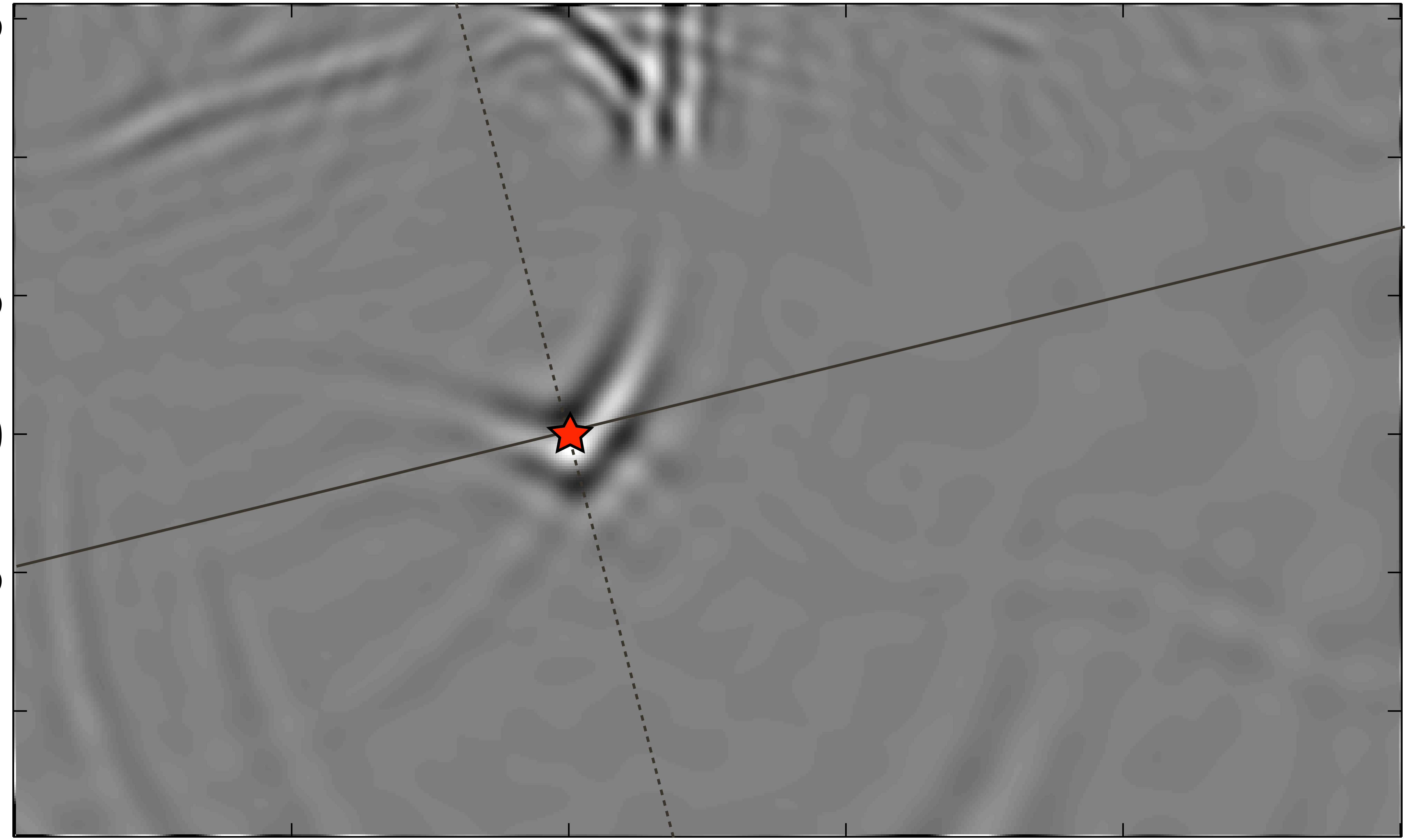


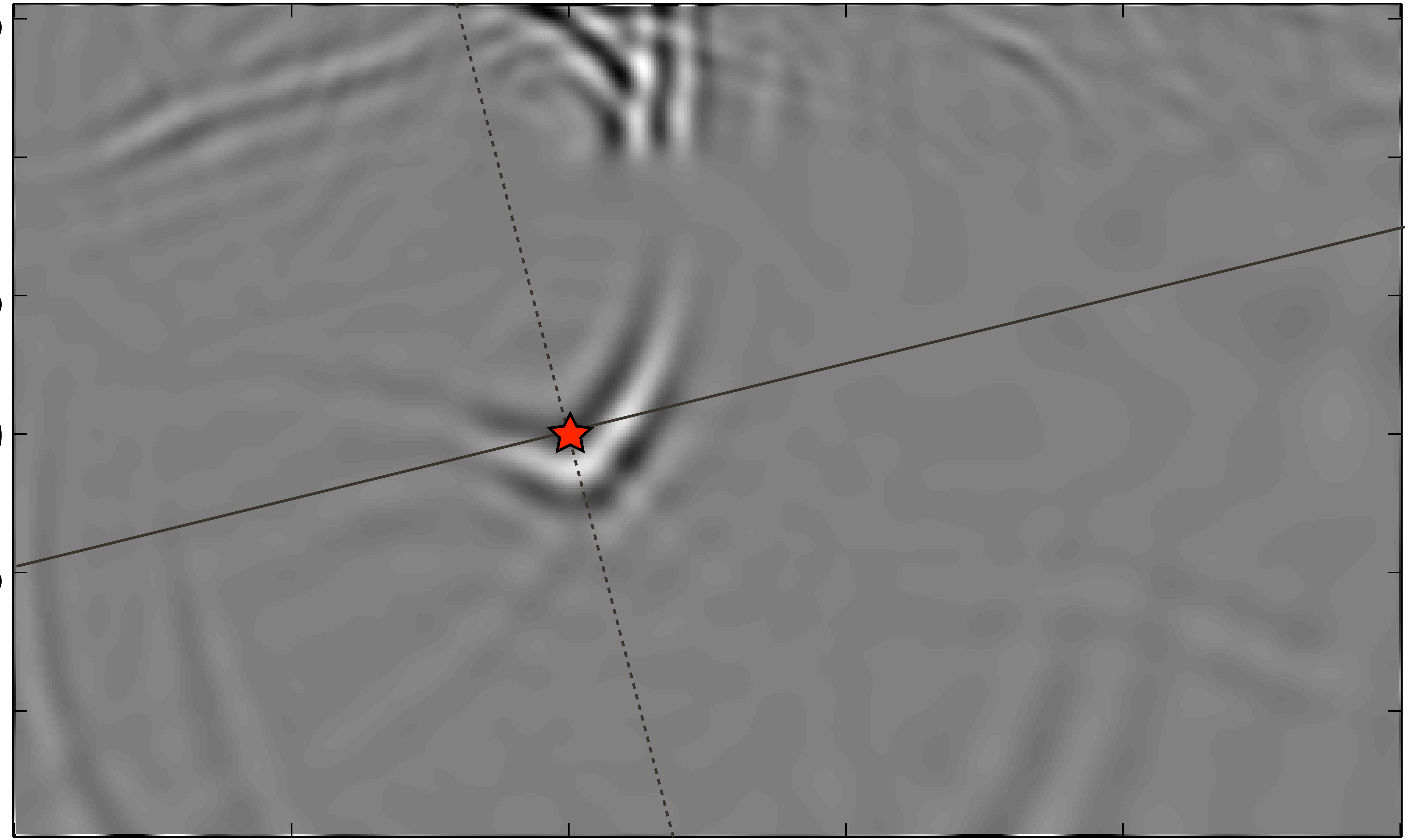


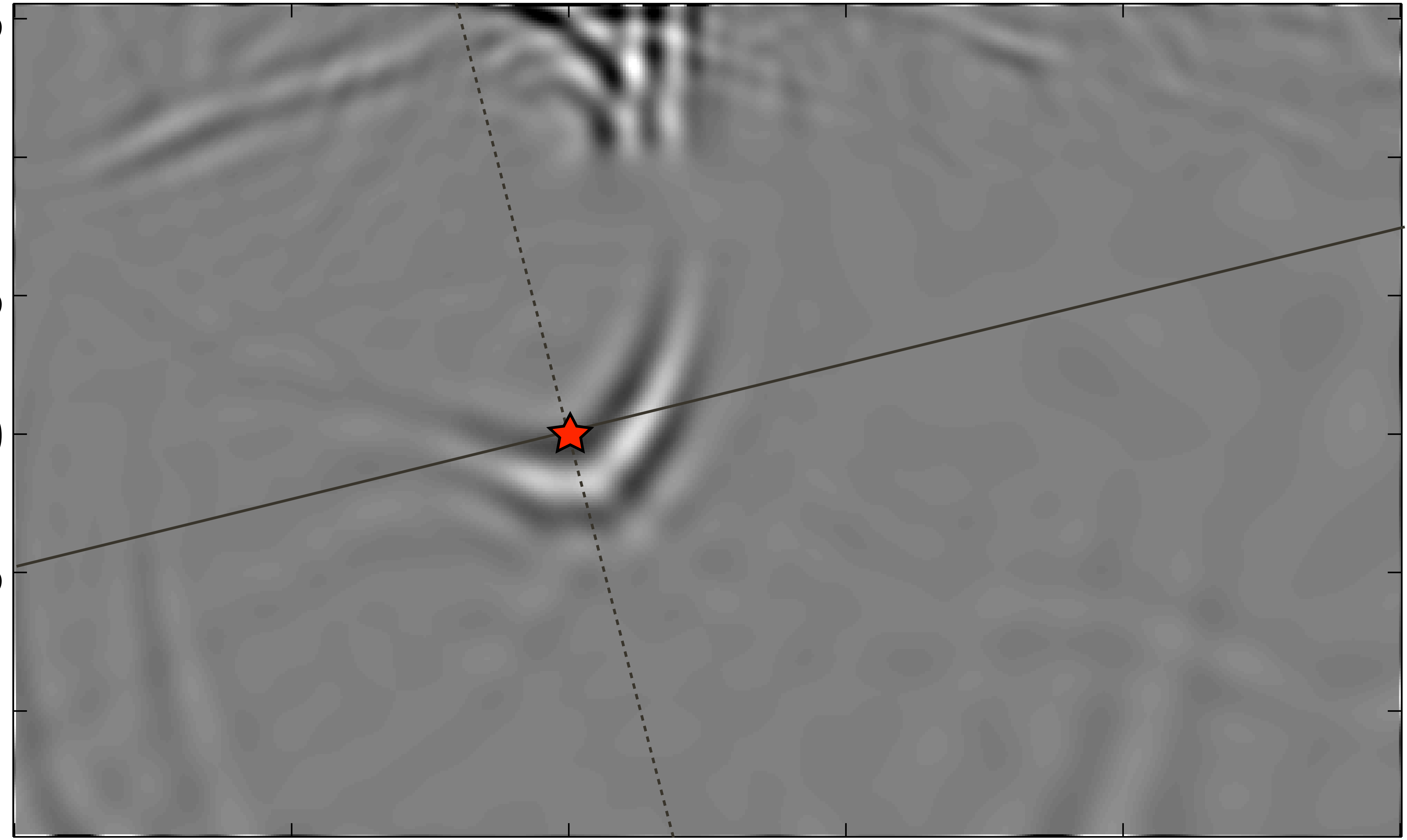


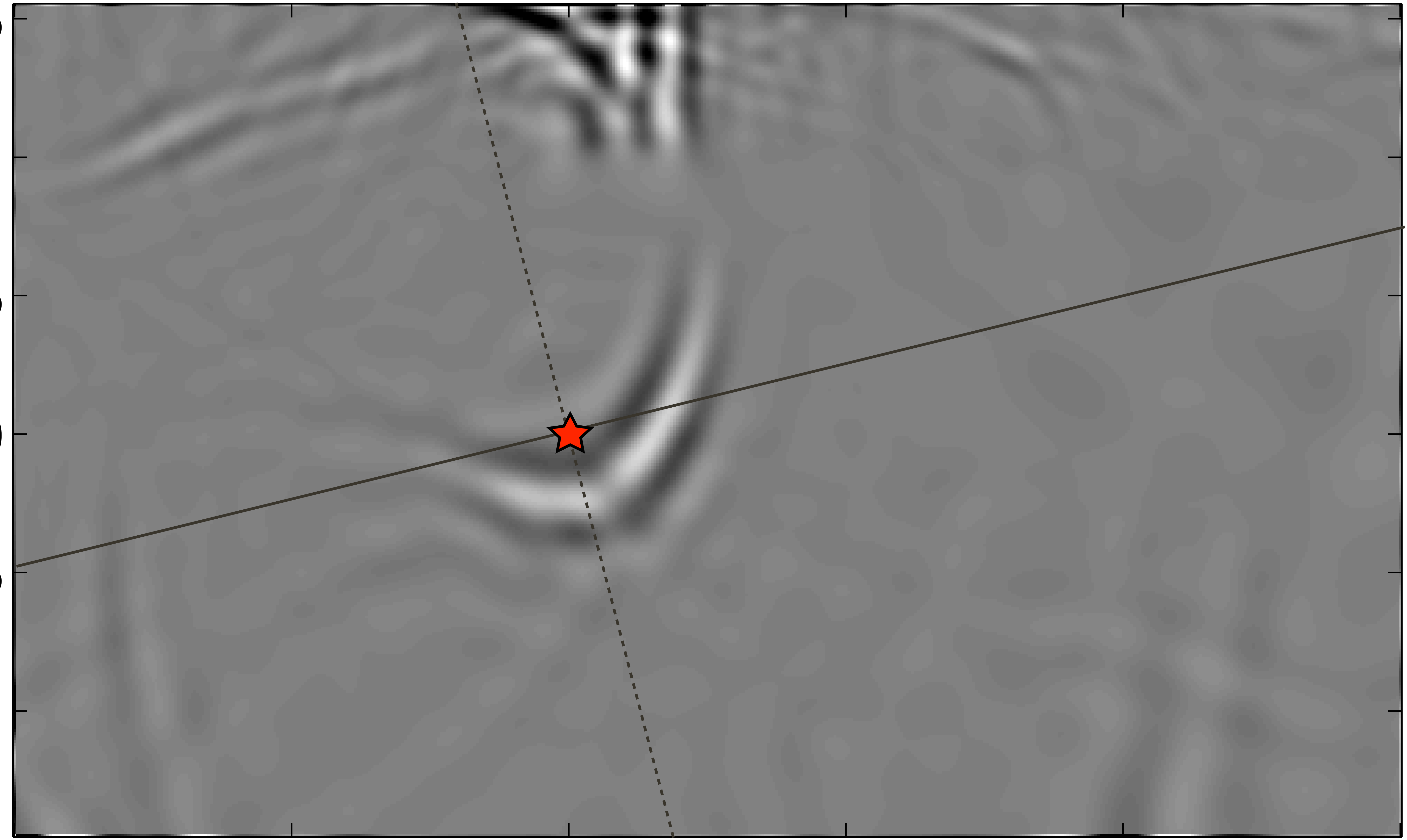












Extended images

- complete image volume too large to form: $(n_x \times n_z)^2$
- instead, probe volume for information via mat-vecs $E\mathbf{y}$
- $\mathbf{y}$ can be interpreted as subsurface source function

Computation

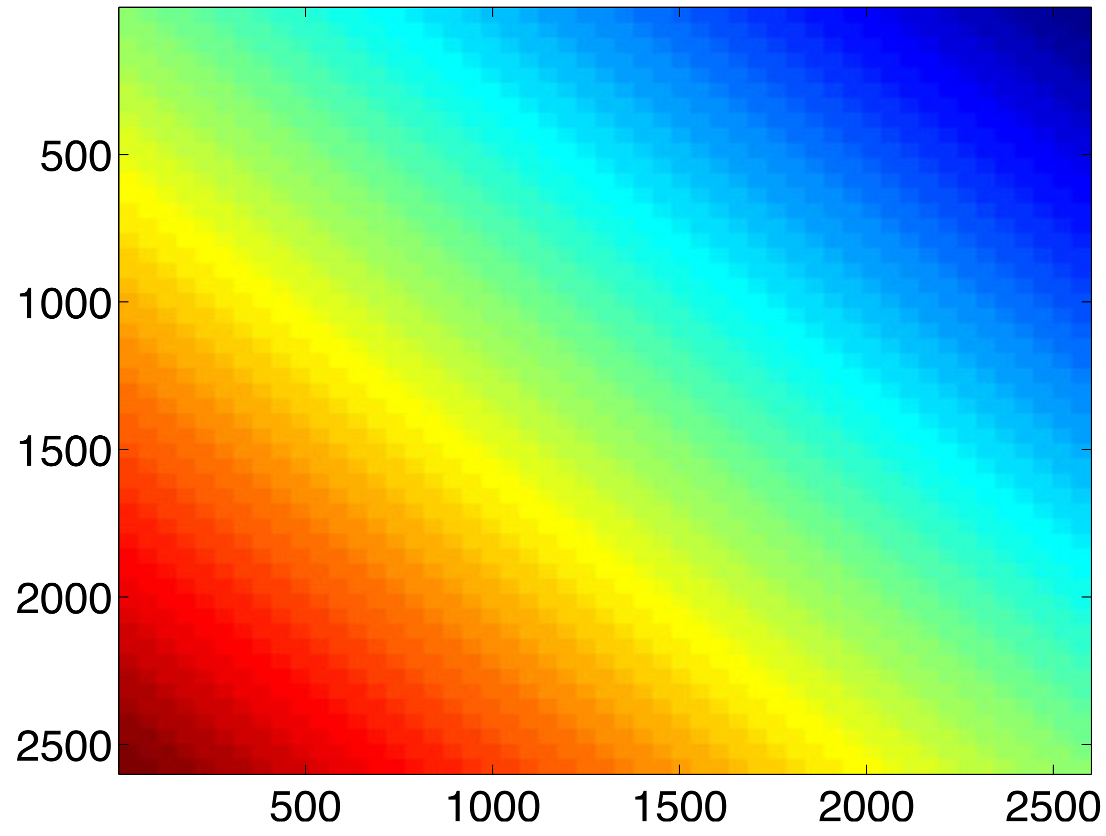
mat-vec with extended image:

$$\mathbf{e} = E\mathbf{y} = H^{-*}P_r^T D Q^* P_s H^{-1}\mathbf{y}$$

- $\mathbf{d} = P_s H^{-1}\mathbf{y}$ *(one subsurface source)*
- $\mathbf{w} = Q^*\mathbf{d}$ *(source weights)*
- $\mathbf{e} = H^{-*}P_r^T(D\mathbf{w})$ *(one source)*

MVA

focusing penalty: $f(\mathbf{m}) = ||W \odot E(\mathbf{m})||_F^2$



[Shen & Symes, '08]

SLIM

MVA

Use techniques from randomized trace estimation:

$$||A||_F^2 = \text{trace}(A^T A) \approx \sum_{i=1}^{K} \mathbf{w}_i^T A^T A \mathbf{w}_i = \sum_{i=1}^{K} ||A\mathbf{w}_i||_2^2$$

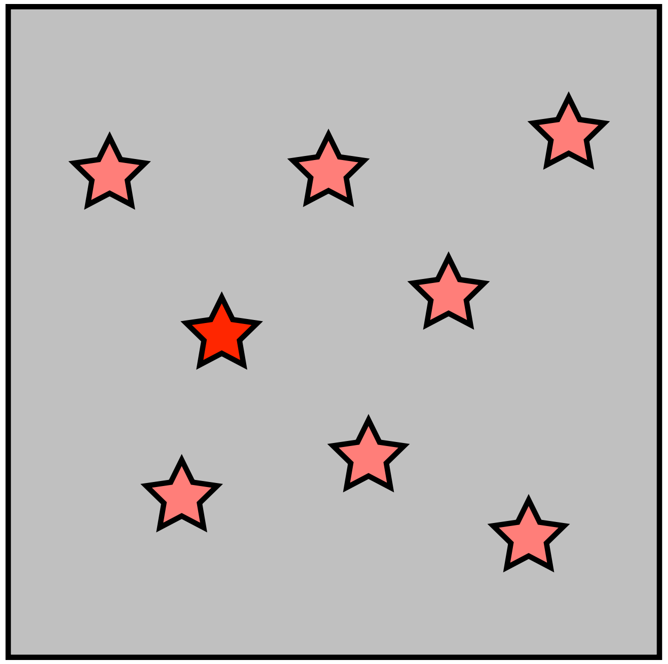
where

$$\sum_{i=1}^{K} \mathbf{w}_i \mathbf{w}_i^T \approx I$$

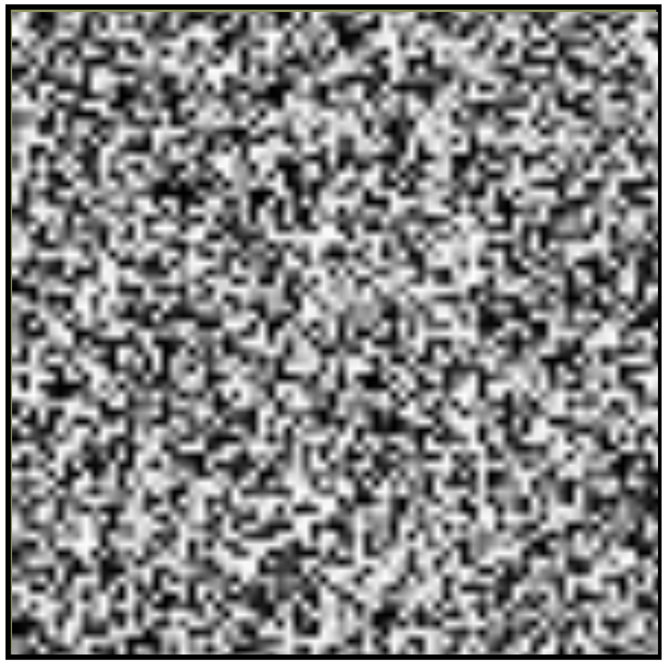
[Avron & Toledo, '11]

MVA

Vectors can be interpreted as subsurface source functions



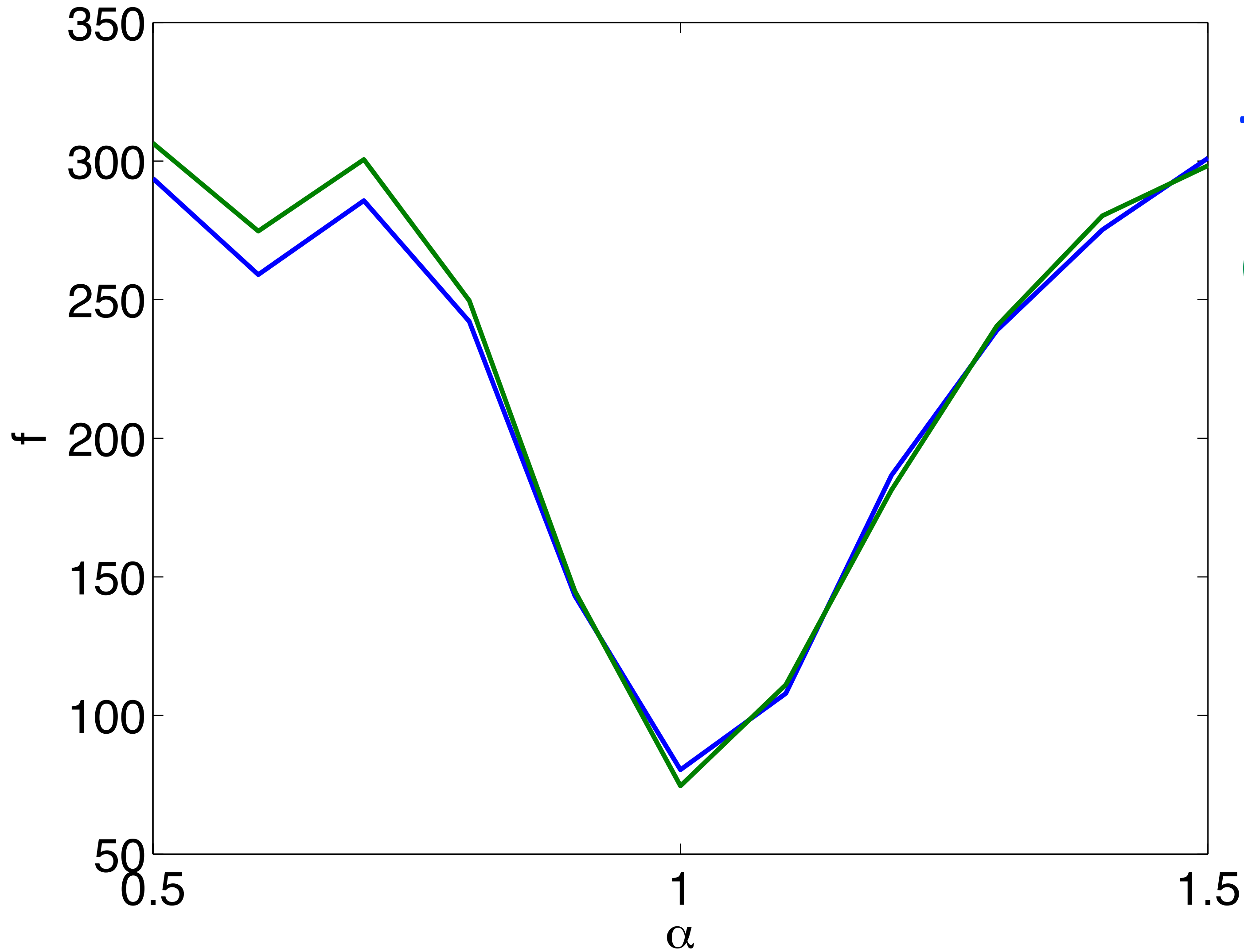
randomly chosen seq. sources



sim. sources

MVA

$$f(\alpha) = ||W \odot E(\alpha \mathbf{m}_{\text{true}})||_F^2$$



true

estimated $K = 10$

Conclusions

- image volume for *all* offsets easily expressed in terms of data matrices
- two-way equivalent of DSR equation
- work with easy-to-compute mat-vecs
- use techniques from randomized trace estimation to compute focussing penalty

Future work

- automatically detect dip
- use trace estimation ideas in MVA inversion
- AVA

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