Wave-equation extended images: computation and velocity continuation

Tristan van Leeuwen[1] and Felix J. Herrmann[1]
[1] Dept. of Earth and Ocean Sciences, University of British Columbia, 6339 Stores rd, Vancouver, BC, V6T1Z4

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Abstract

An extended image is a multi-dimensional correlation of source and receiver wavefields. For a kinematically correct velocity, most of the energy will be concentrated at zero offset. Because of the computational cost involved in correlating the wavefields for all offsets, such exteded images are computed for a subsurface offset that is aligned with the local dip. In this paper, we present an efficient way to compute extended images for all subsurface offsets without explicitly calculating the receiver wavefields, thus making it computationally feasible to compute such extended images. We show how more conventional image gathers, where the offset is aligned with the dip, can be extracted from this extended image. We also present a velocity continuation procedure that allows us to compute the extended image for a given velocity without recomputing all the source wavefields.

Introduction

An extended image is a multi-dimensional correlation of source and receiver wavefields, as a function of *all* subsurface offsets (see Sava and Vasconcelos, 2011, and references therein for a recent overview). In 2D, the extended image is then a 4-dimensional object (5-dimensional if we also consider temporal shifts). For the correct velocity model, the energy in this image volume is mainly focused at zero offset and this forms the basis of many migration velocity analysis schemes (Claerbout, 1985; Chauris et al., 2002; Sava and Biondi, 2004; Shen and Symes, 2008, and many others).
A notable advantage of obtaining the extended image for all subsurface offsets, rather than just horizontal or vertical offset, is that we do not need to estimate the dip (Biondi and Symes, 2004). Unfortunately, it is computationally infeasible to compute and store this extended image for all subsurface points. In this paper, we propose an alternative way of computing subsurface offset gathers for a single scattering point. By writing the multi-dimensional correlation as an outer product of the source and receiver wavefields we derive a new way to compute the subsurface offset gather without computing the whole receiver wavefield. This methodology gives us a way to prope the entire 4-dimensional image volume for information without ever explicitly computing or storing it. We discuss how more conventional image gathers can be extracted from this volume. We also present a velocity continuation procedure that enables us to compute the subsurface offset gathers for a given velocity at small additional cost without recomputing the source wavefields, given that we have the source wavefield for a reference wavefield. Finally, we discuss future research directions.

Extended images

The extended image is a multi-dimensional cross-correlation of the source and receiver wavefields

$$E(\mathbf{x},\mathbf{x}') = \sum_{i,\omega} v_i(\omega,\mathbf{x})u_i(\omega,\mathbf{x}')^*, \tag{1}$$

where $\mathbf{x} = (z,x)$ denotes the subsurface position and the index i runs over the sources. Note that we can incorporate a temporal shift via a simple phase factor $e^{\imath\omega\Delta t}$. If we organize s wavefield as a monochromatic matrix, where each column represents a source (i.e., a data matrix, see Berkhout, 1984), we can conveniently write this correlation as

$$E = \sum_{\omega} V_\omega U_\omega^*, \quad \text{with} \quad H_\omega(\mathbf{m})U_\omega = Q_\omega, \quad H_\omega^*(\mathbf{m})V_\omega = R_\omega, \tag{2}$$

where $H_\omega(\mathbf{m})$ is the Helmholtz matrix for frequency ω and model $\mathbf{m}$ and R_ω is the data-residual. The image volume is now an $N \times N$ matrix, where N is the number of gridpoints of the model. The entry (i,j) in the matrix captures the interaction between gridpoints i and j. The conventional image consists of the diagonal (zero-offset) elements of this matrix. Figure 1 illustrates how different subsurface-offset gathers are embedded in the 4-dimensional image volume and how they can be extracted from this matrix.
In practice it is not feasible to compute this whole image volume. We can, however, efficiently probe the image volume for information by 'matrix-free' multiplication with a vector: $\mathbf{y} = E\mathbf{x}$. The algorithm for this is as follows:

1. solve $H_\omega U_\omega = Q_\omega$ — *Compute the source wavefield for each source and frequency.*
2. let $\mathbf{w}_\omega = U_\omega^*\mathbf{x}$ — *Multiply the source wavefield with the input vector.*
3. solve $H_\omega^*\widetilde{V}_\omega = R_\omega\mathbf{w}_\omega$ — *Solve the adjoint equation for one simultaneous source per frequency.*
4. let $\mathbf{y} = \sum_\omega \widetilde{V}_\omega$ — *Stack over frequencies.*

In this manner, the product $\mathbf{y} = E\mathbf{x}$ can be computed from the source wavefield and one extra PDE solve per frequency. In case the input vector represents a single scattering point (i.e., $\mathbf{x} = [0,\ldots,0,1,0,\ldots,0]$), this procedure can be interpreted as an interferometric redatuming of the source to the location of the scatterer and the receivers to the whole subsurface (Vasconcelos et al., 2010; van Leeuwen and Herrmann, 2011). Instead of having to solve a PDE for each source to obtain the receiver wavefields, we need to solve a PDE for each subsufrace point at which we want to evaluate the extended image. This can be beneficial since we usually want to sample the image volume sparsly.

Extraction of offset-dip gathers

We can extract offset gathers at 'midpoint' (z_0, x_0) from the image volume via $E(z_0, x_0, z_0 + h_z, x_0 + h_x)$. If we transform (h_x, h_z) to polar coordinates (h, θ), so that h is the offset along a reflector with dip θ, the offset-dip gather is given by

$$I(z_0, x_0, h, \theta) = E(z_0, x_0, z_0 + h\cos\theta, x_0 + h\sin\theta). \tag{3}$$

The conventional horizontal offset gather at midpoint x_0, as a function of z and h, is then given by $I(z, x_0, h, 0)$. An example is shown in figure 4.

Velocity continuation

Velocity continuation is usefull for velocity model building because it allows to quickly scan over different velocity models without recomputing the source and receiver wavefields and the image volume. We discuss velocity continuation of a monochromatic image volume, with the understanding that we can repeat this procedure for each frequency and stack the end result over frequencies. From the definition of the image volume in eq. (2) we can write

$$H^*(\mathbf{m})E(\mathbf{m})H^*(\mathbf{m}) = RQ^* \tag{4}$$

This means that the extended image is the solution of a 'double' wave-equation. Since the right-hand-side is independent of the velocity, we may relate the extended images for two different models as

$$E(\mathbf{m}_2) = H^{-*}(\mathbf{m}_2)H^*(\mathbf{m}_1)E(\mathbf{m}_1)H^*(\mathbf{m}_1)H^{-*}(\mathbf{m}_2). \tag{5}$$

This continuation process is similar to the one described by Duchkov et al. (2008), where the continuation operator is a composition of a forward and inverse operator for the two models. We only gain computationally, however, if we can cheaply apply the composition $H^{-*}(\mathbf{m}_2)H^*(\mathbf{m}_1)$. If we are interested in the product $E(\mathbf{m}_2)\mathbf{x}$, however, we find

$$E(\mathbf{m}_2)\mathbf{x} = H(\mathbf{m}_2)^{-*}RU(\mathbf{m}_1)^*\widetilde{\mathbf{x}}, \tag{6}$$

where $\widetilde{\mathbf{x}} = H(\mathbf{m}_1)^*H(\mathbf{m}_2)^{-*}\mathbf{x}$. This means that we can compute the image volume for any velocity model –*without recomputing the source and receiver wavefields*– at the cost of two PDE solves (per frequency), given that we have access to the source wavefields for a reference model. While existing approaches (cf. Fomel, 2003; Schleicher et al., 2008; Chauris and Benjemaa, 2010; Guerra and Biondi, 2011) use the image gather for a reference velocity as starting point to compute the image gather in the updated velocity, we use the source wavefields for the reference model as starting point. As a result, our approach does not rely on a constant-velocity, geometric optics or small-perturbation approximation.

Discussion and Conclusion

When we organize the extended image volume as a matrix, we can conveniently express it as the outer product of the source and receiver wavefields. We can then exploit this special structure to efficiently calculate the action of this matrix on a given vector, without explicitly constructing the whole image volume. An immediate application is the calculation of subsurface offset gathers for a single scattering point. The extended image matrix contains all possible subsurface offsets (horizontal, vertical, diagonal) and hence we do not need to estimate reflector dips before computing the gathers.

We find that the extended image for *all offsets* is the solution of a 'double' wave-equation, and this extends the fact that the extended image for *horizontal* offset can be produced by downward continuation

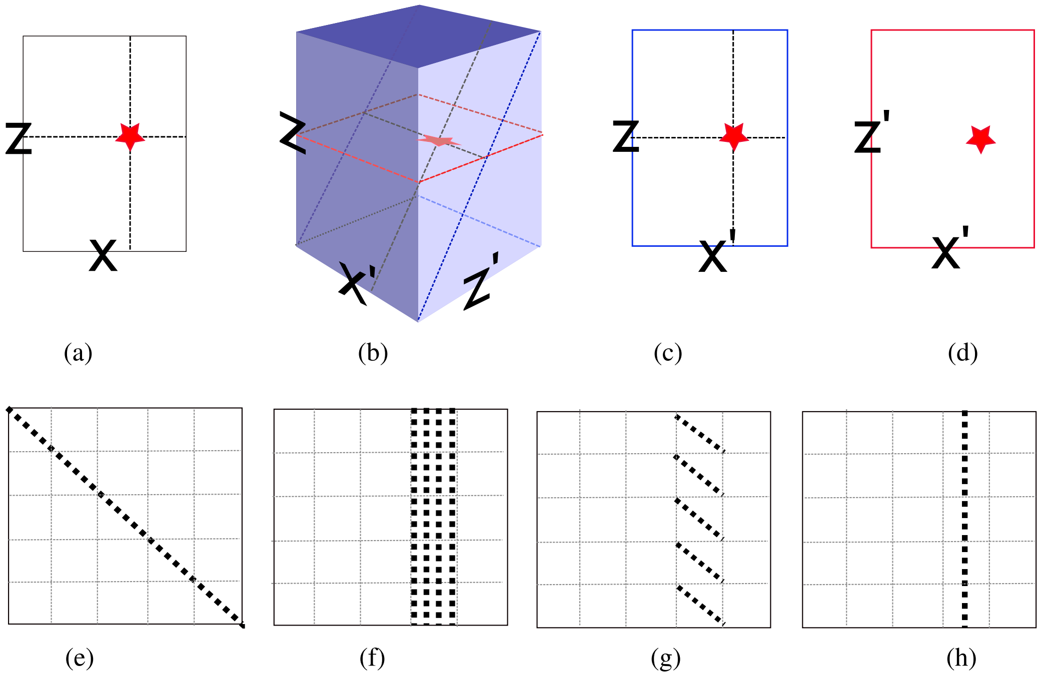

Figure 1 *Different slices through the 4-dimensional image volume* $E(z,z',x,x')$ *around* $z = z_0$ *and* $x = x_0$*. (a) Conventional image* $E(z,z,x,x)$*, (b) Image gather for horizontal and vertical offset* $E(z,z',x_0,x')$*, (c) Image gather for horizontal offset* $E(z,z,x_0,x')$ *and (d) Image gather for a single scattering point* $E(z_0,z',x_0,x')$*. (e-g) shows how these slices are organized in the matrix representation of* E*.*

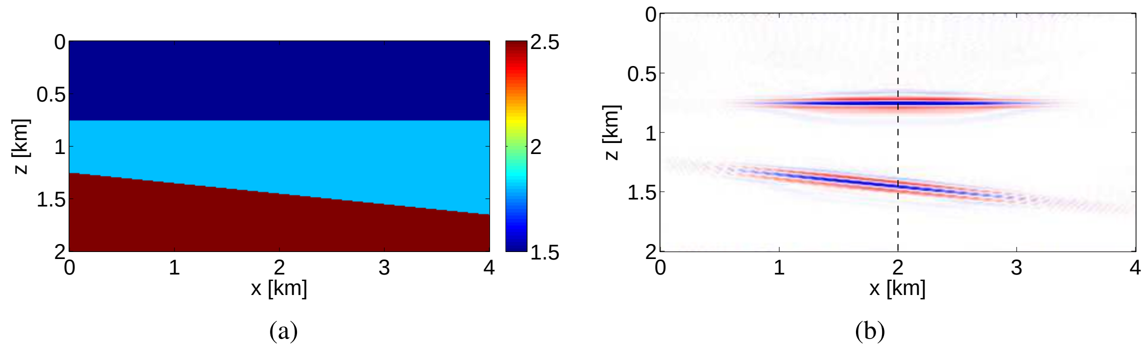

Figure 2 *(a) Simple velocity model used for examples, (b) image for correct velocity.*

with the double-square-root equation (Berkhout, 1984; Stolk and de Hoop, 2001; Biondi, 2003).
Velocity continuation of these image gathers can be done efficiently, given that we have access to the source wavefields for a reference velocity. In this case we do not need to recompute either source or receiver wavefields in the target velocity and need only two PDE solves per frequency to calculate the new image gather.
To reap the full benefits of the discussed techniques we need a misfit criterion or focusing penalty that that can be evaluated using only multiplications of the image volume with given vectors.

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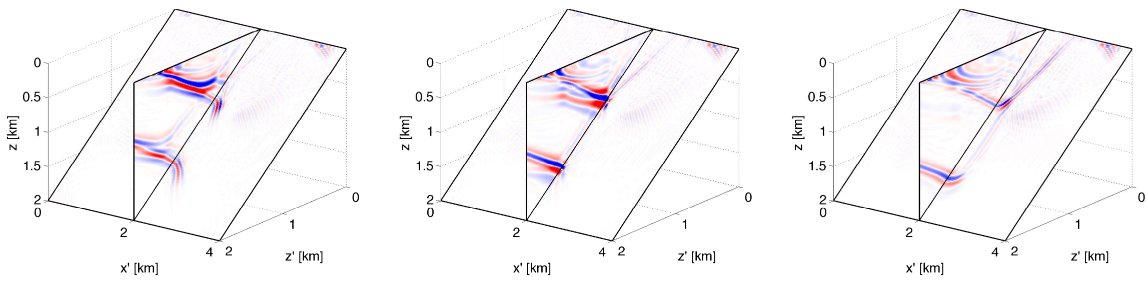
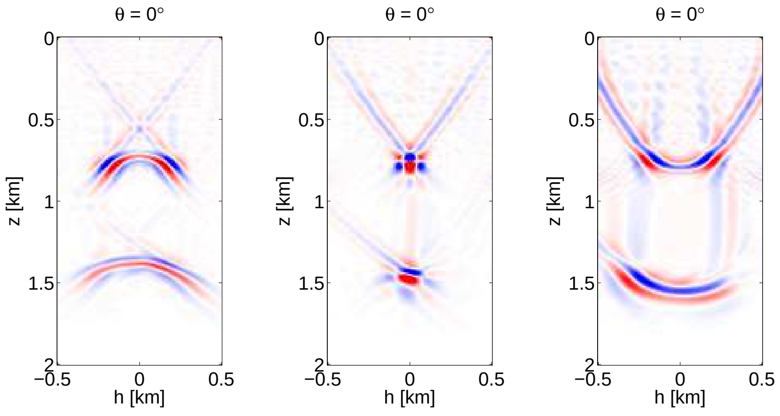


(a) (b) (c)

Figure 3 *Slices through the image volume along x_0=2 km for (a) low, (b) correct and (c) high velocity.*



(a) (b) (c)

Figure 4 *Image gathers at x_0=2 km and $\theta = 0^\circ$ for (a) low, (b) correct and (c) high velocity.*

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