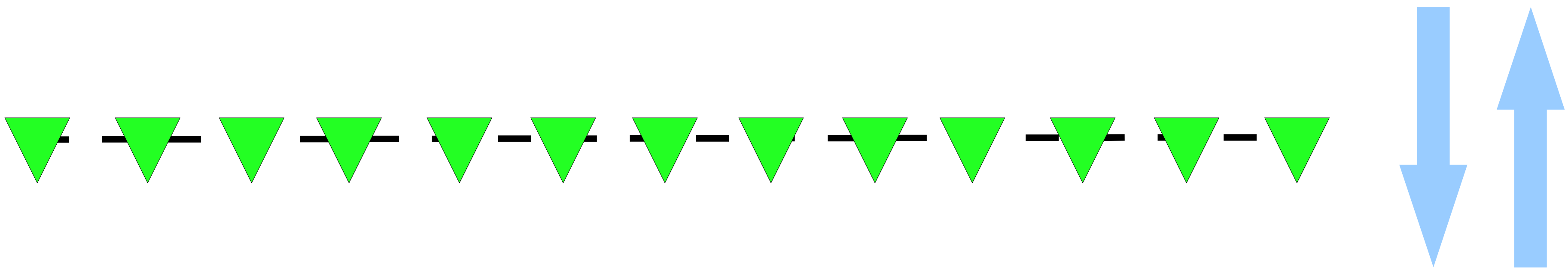
Up / down wavefield decomposition by sparse inversion



Joost van der Neut (*Delft University of Technology*)
Felix Herrmann (*University of British Columbia*)

The forward problem

Pressure

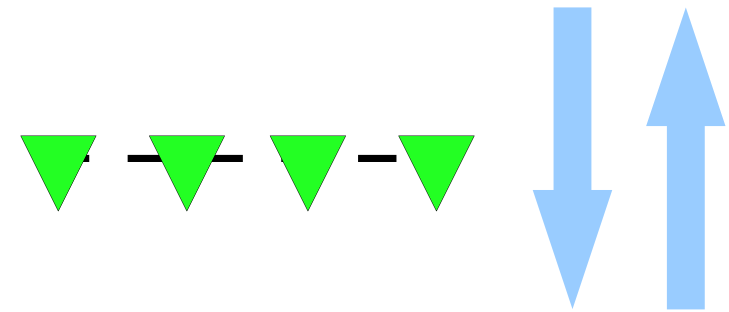
Particle velocity

Composition matrix

Downgoing field

Upgoing field

$$\begin{pmatrix}\hat{\mathbf{p}}\\ \hat{\mathbf{v}}\end{pmatrix} = \hat{\mathbf{L}}\begin{pmatrix}\hat{\mathbf{p}}^{+}\\ \hat{\mathbf{p}}^{-}\end{pmatrix}$$



Flux-normalized composition matrix (Wapenaar, 1998):

Angular frequency

Density

$$\hat{\mathbf{L}} = \frac{1}{\sqrt{2}}\begin{pmatrix}(\omega\varrho)^{\frac{1}{2}}\hat{\mathcal{H}}_1^{-\frac{1}{2}} & (\omega\varrho)^{\frac{1}{2}}\hat{\mathcal{H}}_1^{-\frac{1}{2}}\\ (\omega\varrho)^{-\frac{1}{2}}\hat{\mathcal{H}}_1^{\frac{1}{2}} & -(\omega\varrho)^{-\frac{1}{2}}\hat{\mathcal{H}}_1^{\frac{1}{2}}\end{pmatrix}$$

Square-root of the Helmholtz operator: $\hat{H}_1\hat{H}_1 = \hat{H}_2$

$$\hat{H}_2 = \left(\frac{\omega}{c}\right)^2 + \varrho\frac{\partial}{\partial x}\left(\frac{1}{\varrho}\frac{\partial}{\partial x}\cdot\right)$$

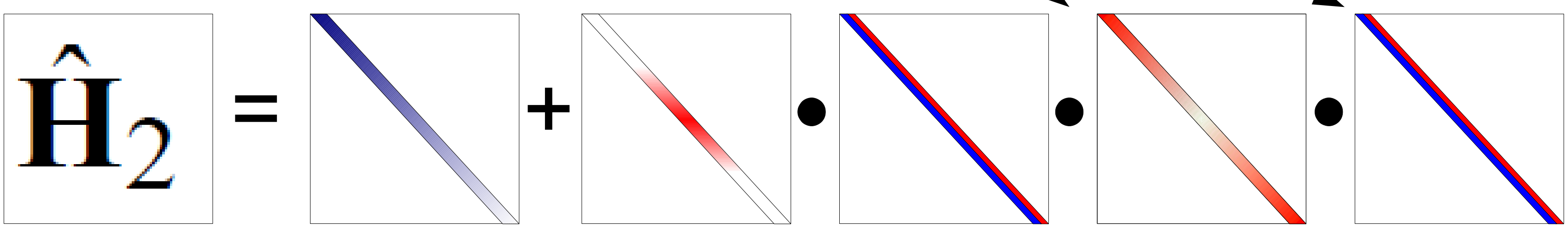
Velocity

Spatial derivatives along receiver array

Numerical computation of the square-root operator

Computation of the Helmholtz operator at each frequency:

$$\hat{H}_2 = \left(\frac{\omega}{c}\right)^2 + \varrho \frac{\partial}{\partial x}\left(\frac{1}{\varrho}\frac{\partial}{\partial x}\cdot\right)$$



Eigenvalue decomposition (Grimbergen et al., 1998):

$$\hat{\mathbf{H}}_2^{\frac{1}{n}} = \hat{\mathbf{L}}\hat{\Lambda}^{\frac{1}{n}}\hat{\mathbf{L}}^{-1}$$

n-th root

Inversion

Downgoing field

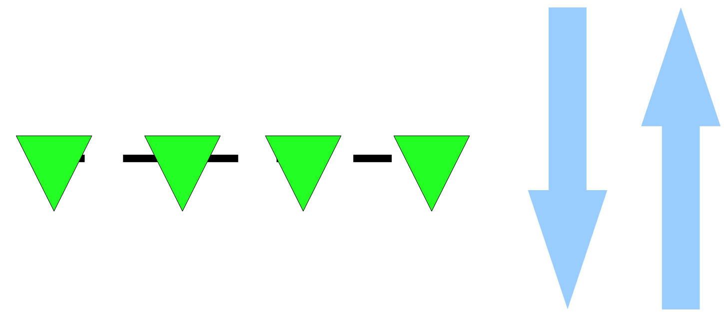
Upgoing field

Composition matrix

Pressure

Particle velocity

$$\begin{pmatrix} \hat{\mathbf{p}}^{+} \\ \hat{\mathbf{p}}^{-} \end{pmatrix} = \hat{\mathbf{L}}^{-1} \begin{pmatrix} \hat{\mathbf{p}} \\ \hat{\mathbf{v}} \end{pmatrix}$$



Flux-normalized inverse composition matrix (Wapenaar, 1998):

Angular frequency

Density

$$\hat{\mathbf{L}}^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{\mathcal{H}}_1^{\frac{1}{2}} (\omega\varrho)^{-\frac{1}{2}} & \hat{\mathcal{H}}_1^{-\frac{1}{2}} (\omega\varrho)^{\frac{1}{2}} \\ \hat{\mathcal{H}}_1^{\frac{1}{2}} (\omega\varrho)^{-\frac{1}{2}} & -\hat{\mathcal{H}}_1^{-\frac{1}{2}} (\omega\varrho)^{\frac{1}{2}} \end{pmatrix}$$

Square-root of the Helmholtz operator: $\hat{H}_1 \hat{H}_1 = \hat{H}_2$

$$\hat{H}_2 = \left(\frac{\omega}{c}\right)^2 + \varrho \frac{\partial}{\partial x}\left(\frac{1}{\varrho}\frac{\partial}{\partial x}\cdot\right)$$

Velocity

Spatial derivatives along receiver array

Problems

1. Noise

$$\begin{pmatrix} \hat{\mathbf{p}}^+ \\ \hat{\mathbf{p}}^- \end{pmatrix} = \hat{\tilde{\mathbf{L}}}^{-1} \left[\begin{pmatrix} \hat{\mathbf{p}}_{signal} \\ \hat{\mathbf{v}}_{signal} \end{pmatrix} + \begin{pmatrix} \hat{\mathbf{p}}_{noise} \\ \hat{\mathbf{v}}_{noise} \end{pmatrix} \right]$$

2. Singularities

In fk-domain: $$\tilde{\mathbf{L}}^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{k_z}{\omega\varrho}} & \sqrt{\frac{\omega\varrho}{k_z}} \\ \sqrt{\frac{k_z}{\omega\varrho}} & -\sqrt{\frac{\omega\varrho}{k_z}} \end{pmatrix}$$

Vertical wavenumber:

$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$$

Horizontal wavenumber

At critical angles: $$\frac{\omega^2}{c^2} = k_x^2$$ → Numerical instability

Sparsity promotion

– Sparsity promotion: Candès et al. (2006), Donoho (2006)

– Applications in geophysics: Herrmann et al. (2008)

– The solution is assumed to be sparse in a transform domain:

Solution *Coefficients* *2D Curvelet Transform* *Kronecker Product* *Discrete Wavelet Transform*

$$\mathbf{p}^{\pm} = \mathbf{S}^{*}\mathbf{x}^{\pm} \qquad \mathbf{S} = \mathbf{C_2} \otimes \mathbf{W}$$

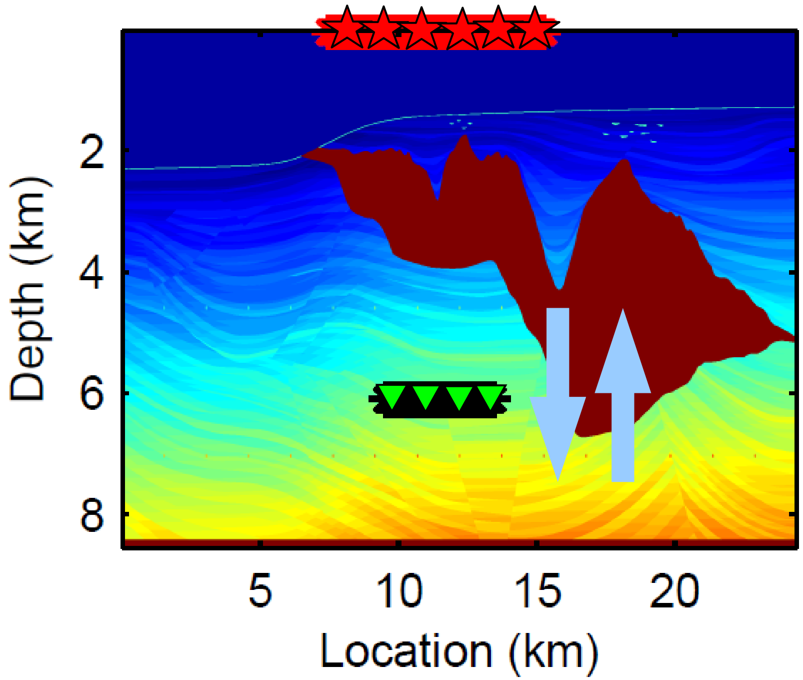
$$\underset{\mathbf{x}^{+},\mathbf{x}^{-}}{\text{minimize}} \left\| \begin{pmatrix} \mathbf{x}^{+} \\ \mathbf{x}^{-} \end{pmatrix} \right\|_1 \text{subject to} \left\| \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \end{pmatrix} - \mathbf{L} \begin{pmatrix} \mathbf{S}^{*} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{pmatrix} \begin{pmatrix} \mathbf{x}^{+} \\ \mathbf{x}^{-} \end{pmatrix} \right\|_2 \leq \sigma$$

Coefficients length *L1* *Residual* *L2* *Noise level*

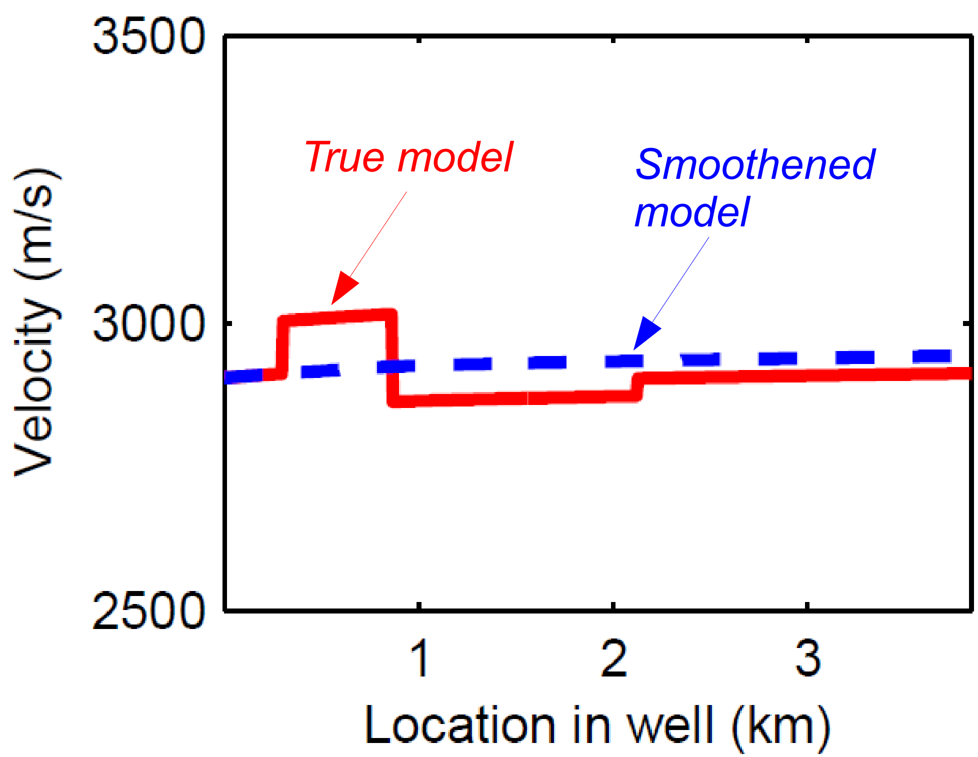
– spgl1 (Van den Berg and Friedlander, 2008)

Example

Velocity model:

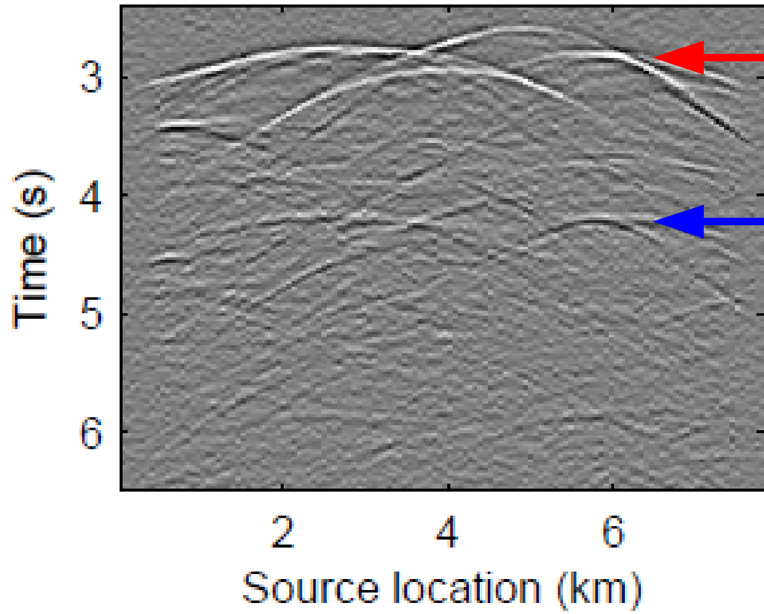


- 128 sources
- 128 receivers
- Noise added (SNR=5)



Pressure data

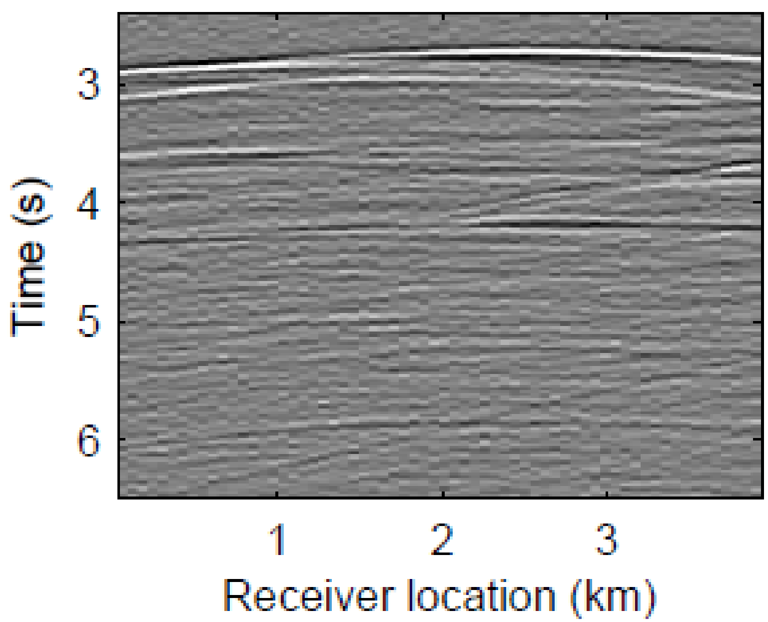
Common Receiver Gather



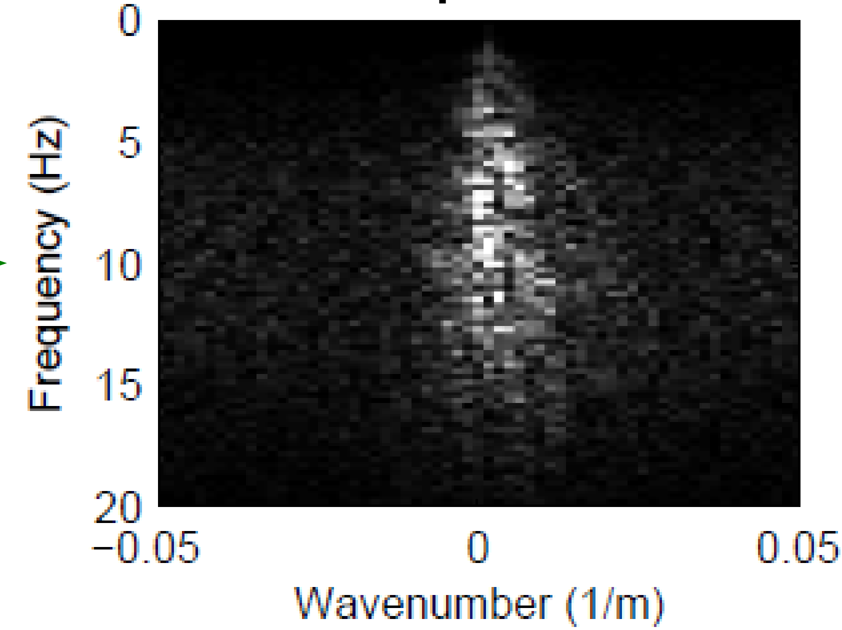
Downgoing field
(not changing polarity)

Upgoing field
(changing polarity)

Common Source Gather

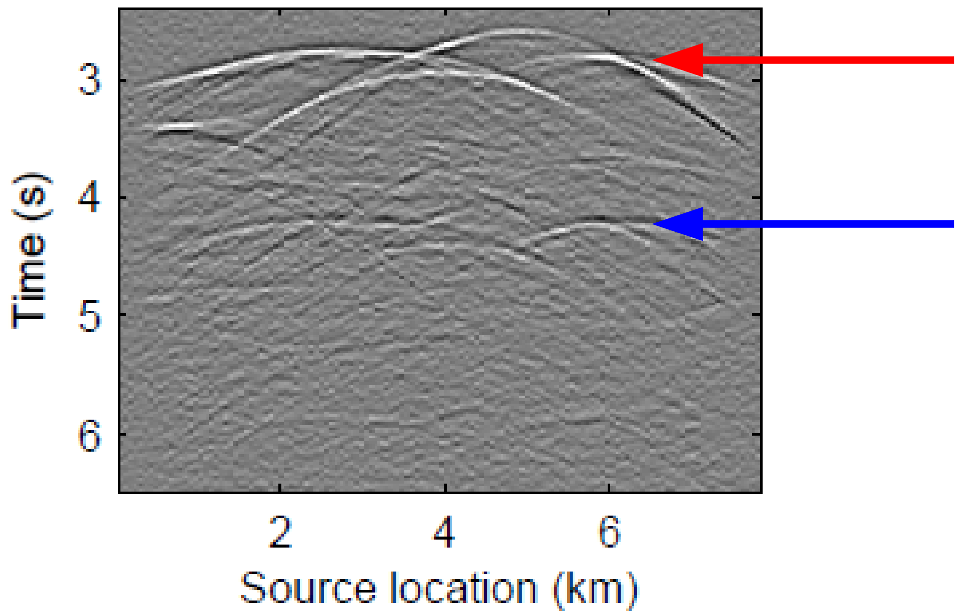


FK spectrum



Particle velocity data

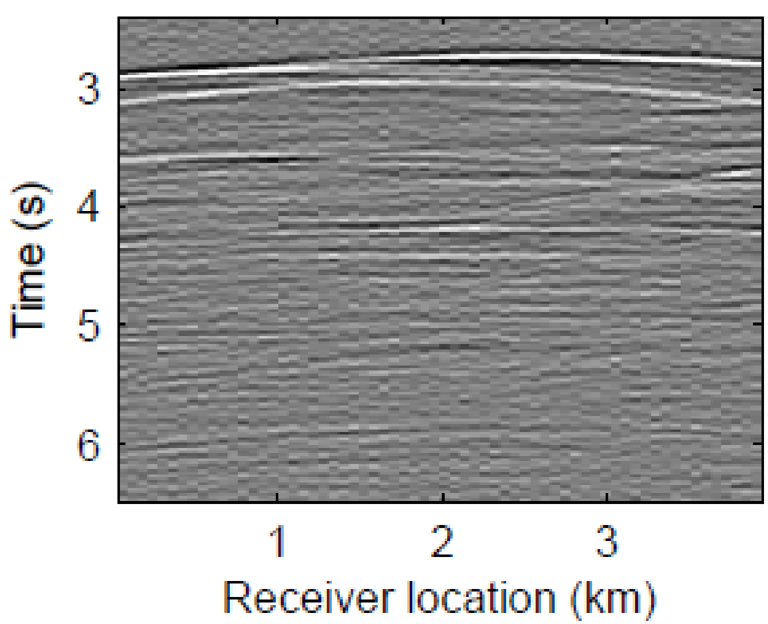
Common Receiver Gather



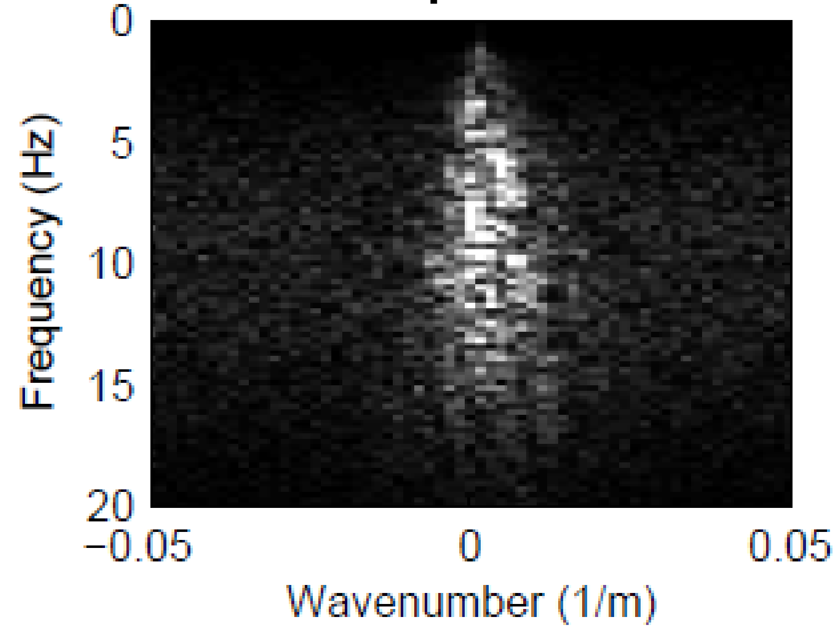
Downgoing field
(not changing polarity)

Upgoing field
(changing polarity)

Common Source Gather

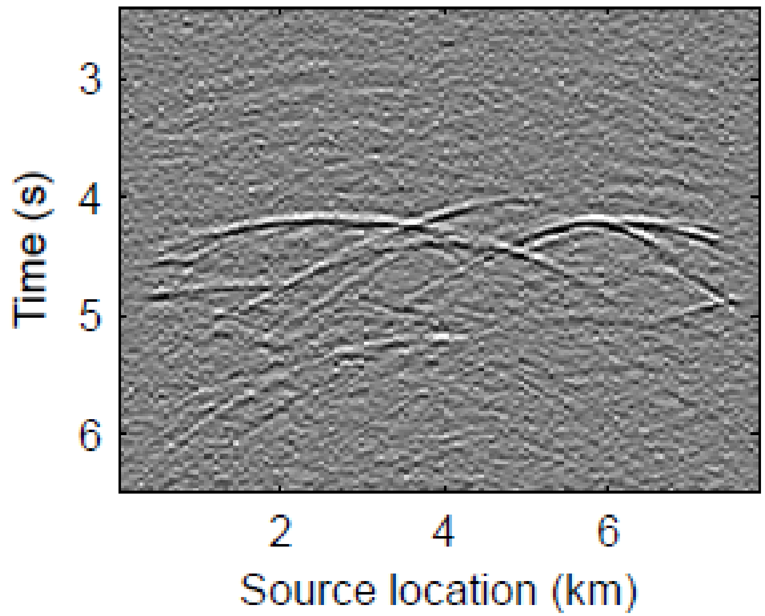


FK spectrum



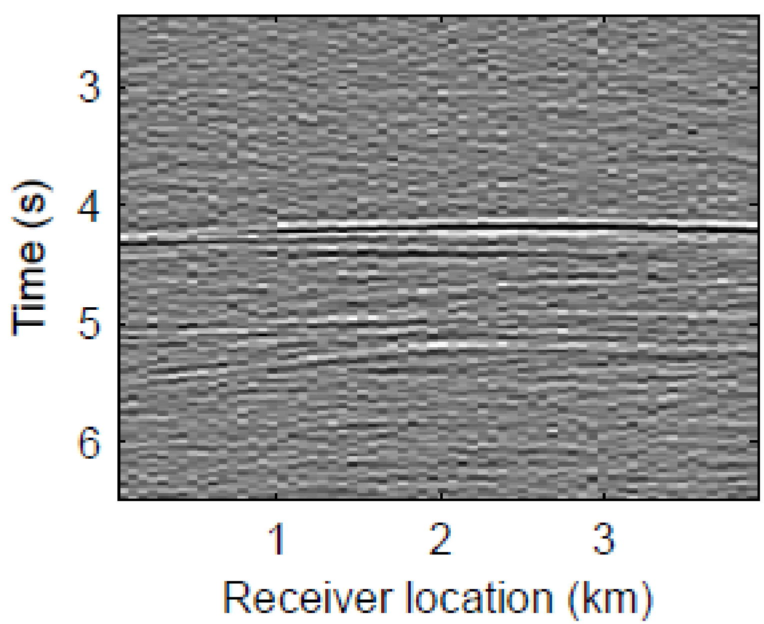
Upgoing field retrieved by exact inversion

Common Receiver Gather

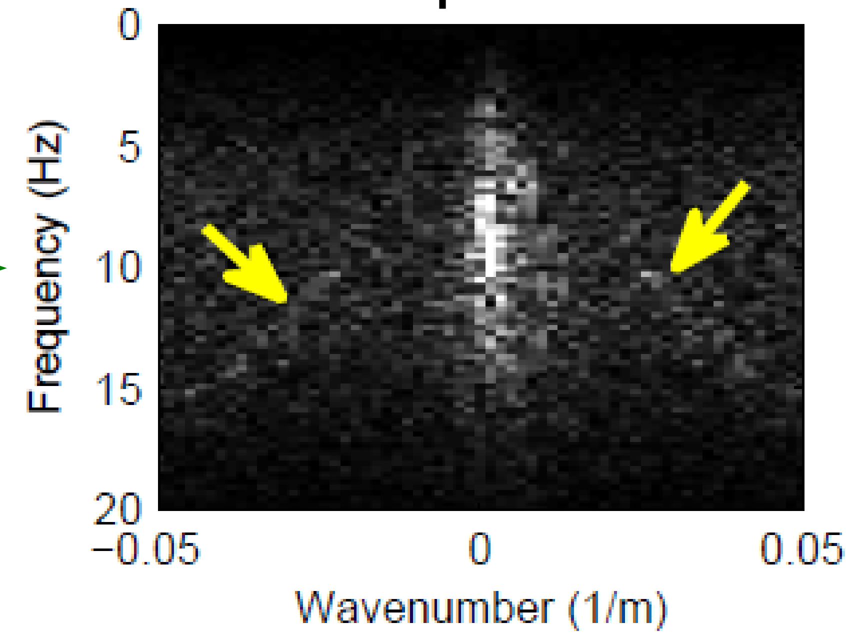


- Results are noisy
- Instabilities at singularities

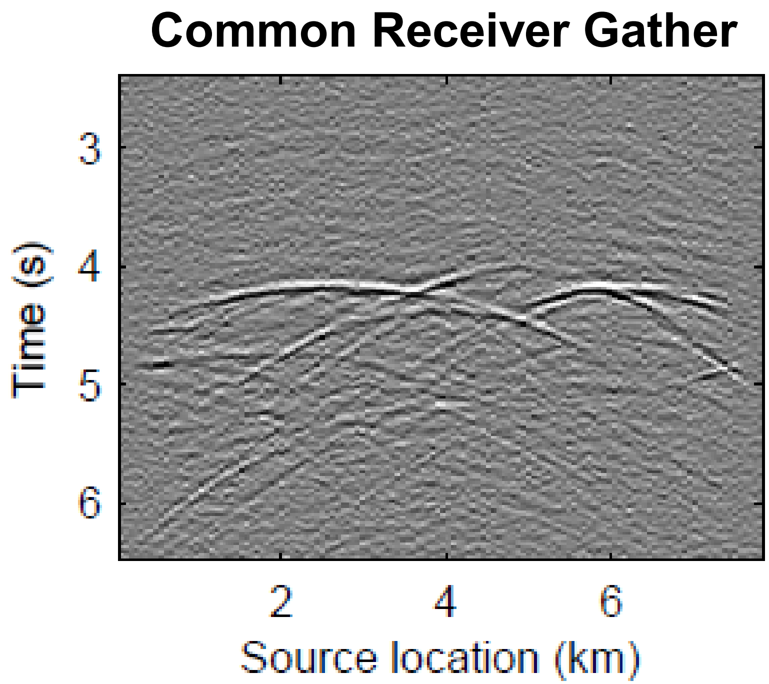
Common Source Gather



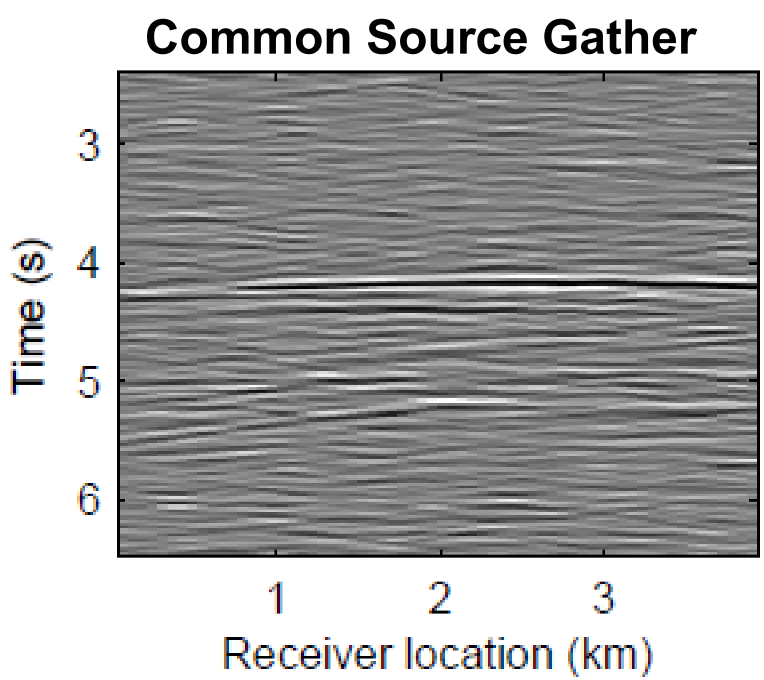
FK spectrum

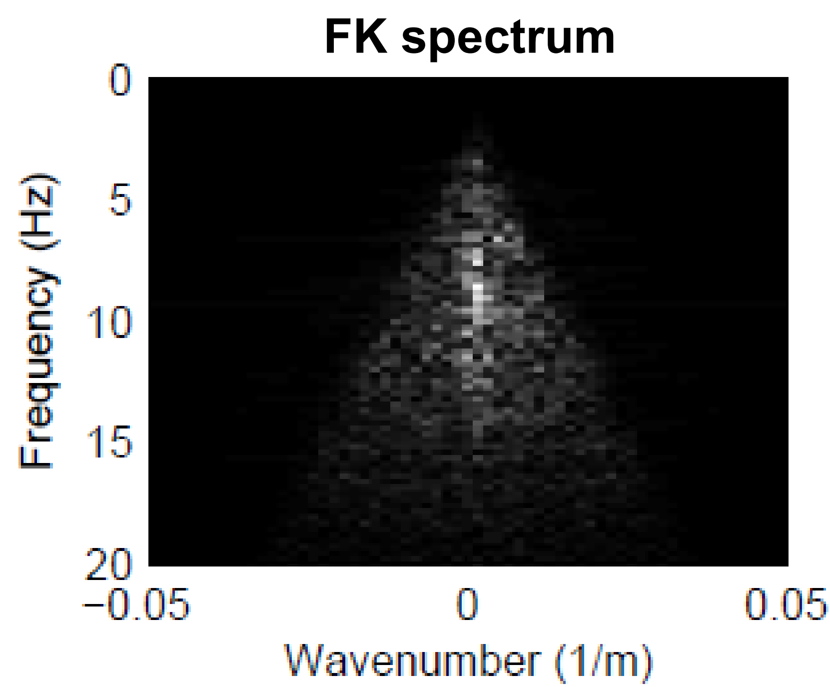


Upgoing field retrieved by exact inversion



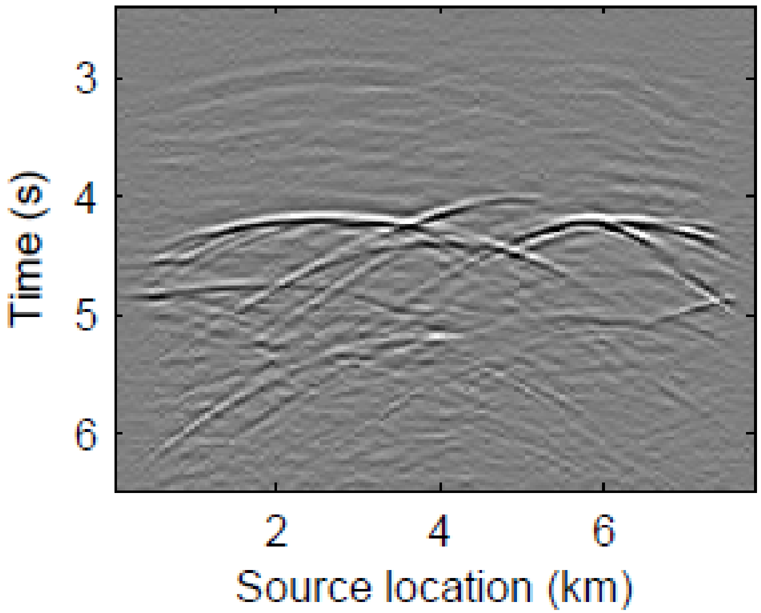
– After FK-filtering





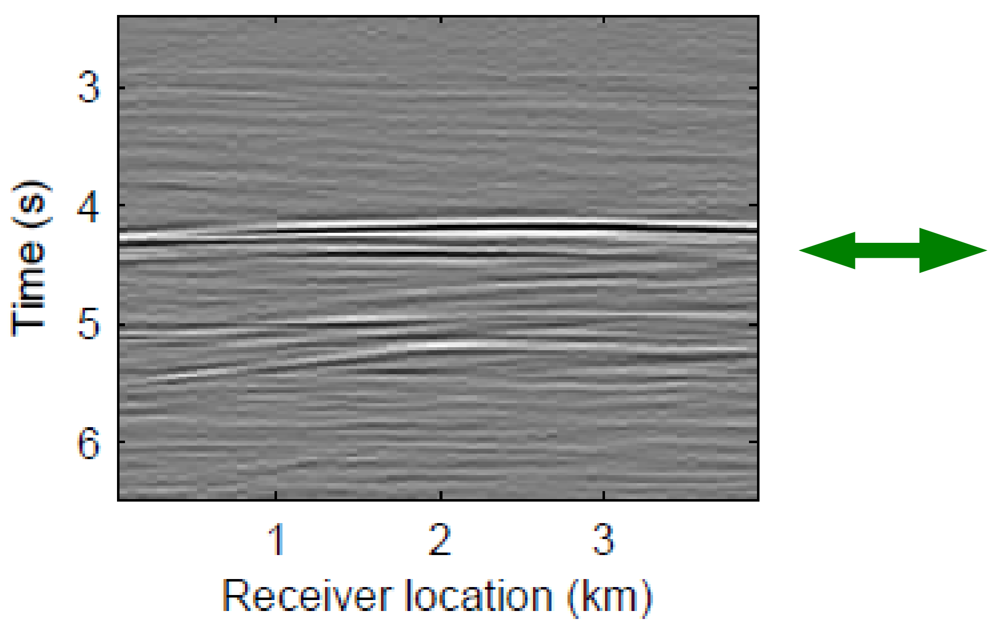
Upgoing field retrieved by sparse inversion

Common Receiver Gather

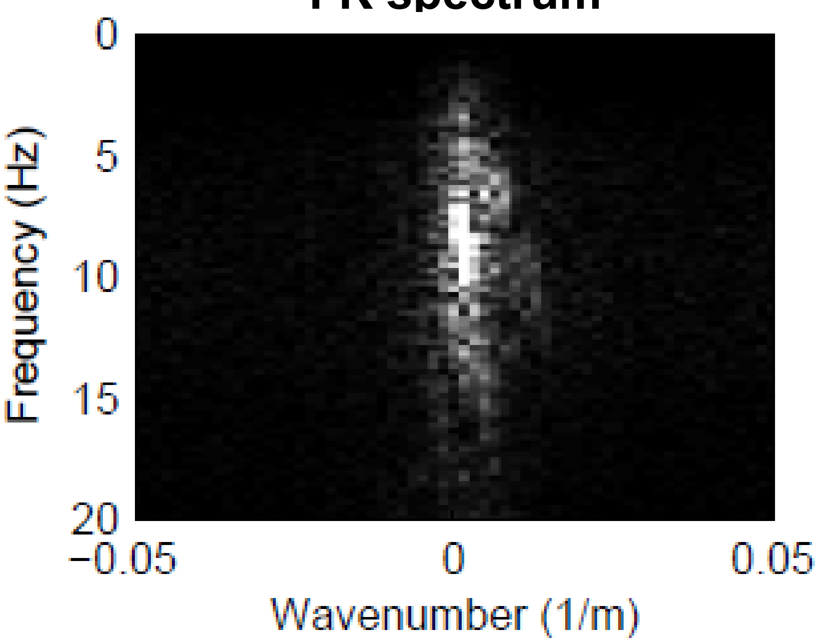


– Result less noisy
– No instablities at singularities

Common Source Gather

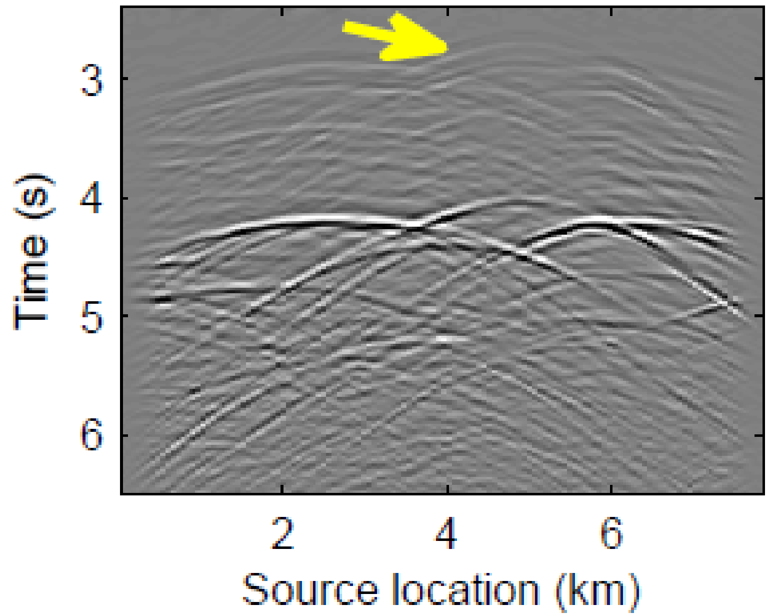


FK spectrum



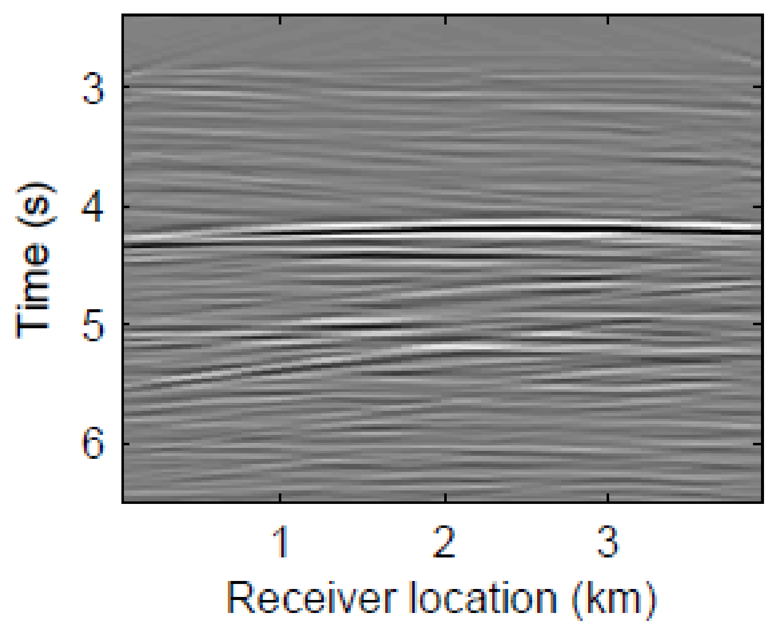
Upgoing field retrieved without noise

Common Receiver Gather

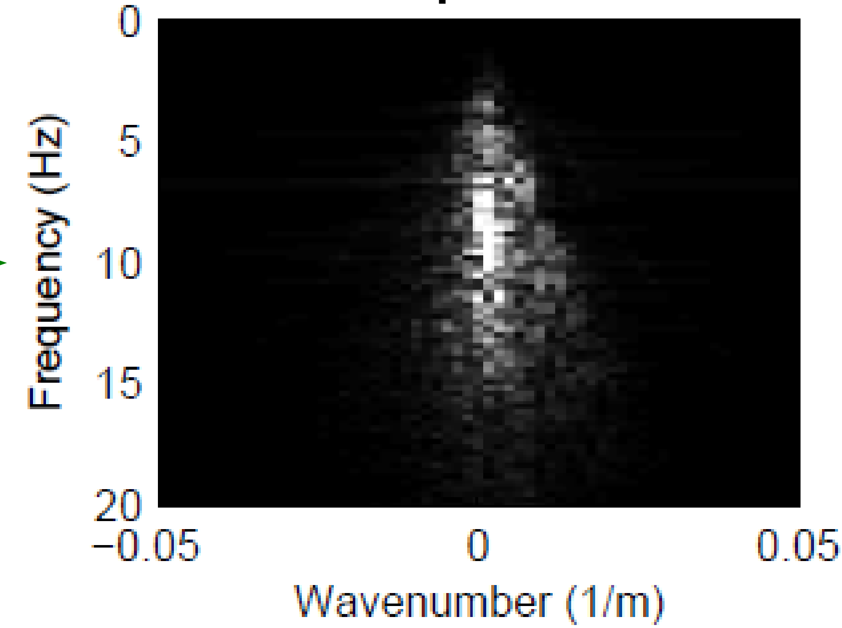


– Detailed information can be lost if noise level σ is set too high
– Instabilities can occur if noise level σ is set too low

Common Source Gather



FK spectrum



Extension to elastic media

Shear traction
Shear traction
Normal traction
Horizontal particle velocity
Horizontal particle velocity
Vertical particle velocity

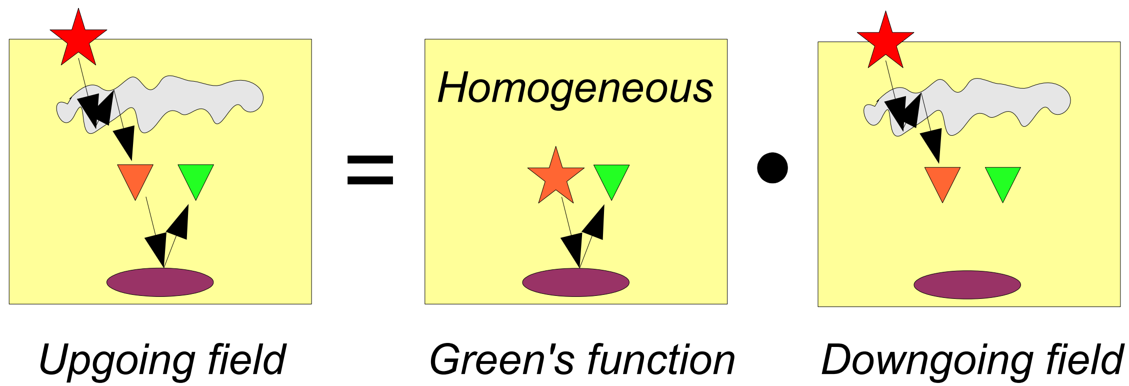
$$\begin{pmatrix} -\hat{\tau}_{\mathbf{xz}} \\ -\hat{\tau}_{\mathbf{yz}} \\ -\hat{\tau}_{\mathbf{zz}} \\ +\hat{\mathbf{v}}_{\mathbf{x}} \\ +\hat{\mathbf{v}}_{\mathbf{y}} \\ +\hat{\mathbf{v}}_{\mathbf{z}} \end{pmatrix} = \hat{\mathbf{L}} \begin{pmatrix} \hat{\mathbf{p}}_{P}^{+} \\ \hat{\mathbf{p}}_{Sv}^{+} \\ \hat{\mathbf{p}}_{Sh}^{+} \\ \hat{\mathbf{p}}_{P}^{-} \\ \hat{\mathbf{p}}_{Sv}^{-} \\ \hat{\mathbf{p}}_{Sh}^{-} \end{pmatrix}$$

Downgoing P-wave field
Downgoing Sv-wave field
Downgoing Sh-wave field
Upgoing P-wave field
Upgoing Sv-wave field
Upgoing Sh-wave field

– Composition matrix – Wapenaar et al. (2008)

– Potential application: OBC (Ocean Bottom Cable) decomposition (Schalkwijk et al., 2003)

Application: Up / down deconvolution

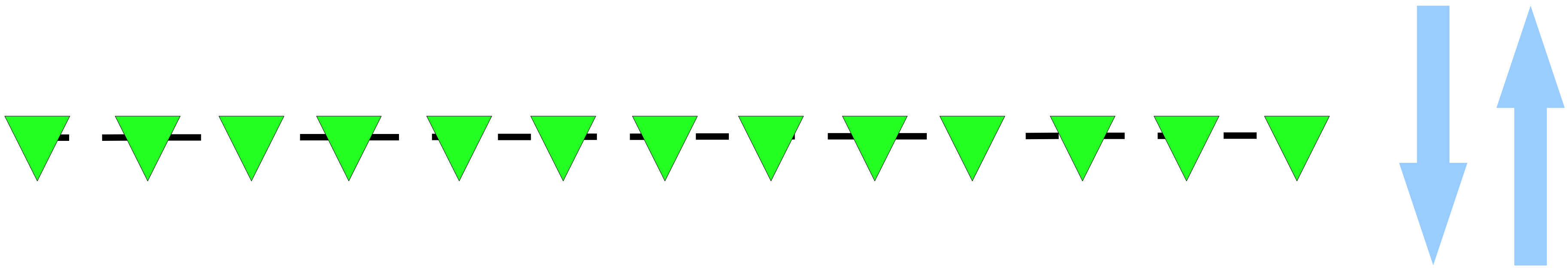


– OBC data (Amundsen, 2001)
→ Free-surface multiple elimination

– Streamer data (Majdanski et al., 2011)
→ Free-surface multiple elimination

– Downhole receivers (Wapenaar et al., 2008)
→ Interferometric redatuming / virtual source method

Conclusion



* Instabilities caused by singularities of the square-root operator and noise are avoided by sparse inversion

* Applications: dual-sensor streamer data, OBC data, downhole receivers (virtual source method)

Acknowledgements & References

- We would like to thank Tristan van Leeuwen, Tim Lin, Ning Tu , Xiang Li and Ian Hanlon (University of British Columbia) for helping with implementations and for discussions.
- This work was supported by the Dutch Technology Foundation STW (stw), applied science division of NWO and the Technology Program of the Ministry of Economic Affairs (grant DCB.7913).

- Amundsen, L., 2001, Elimination of free-surface related multiples without need of the source wavelet: Geophysics, 66, 327-341.

- Candès, E., J. Romberg, and T. Tao, 2006, Stable signal recovery from incomplete and inaccurate measurements: Communications on Pure and Applied Mathematics, 59, 1207–1223.

- Donoho, D., 2006, For most large underdetermined systems of linear equations the minimal l(1)-norm solution is also the sparsest solution: Communications on Pure and Applied Mathematics, 59, 797–829.

- Grimbergen, J. L. T., F. J. Dessing, and K. Wapenaar, 1998, Modal expansion of one-way operators in laterally varying media: Geophysics, 63, 995-1005.

- Herrmann, F. J., D. Wang, G. Hennenfent, and P. P. Moghaddam, 2008, Curvelet-based seismic data processing: a multiscale and nonlinear approach: Geophysics, 73, A1-A5.

- Majdanski, M., C. Kostov, E. Kragh, I. Moore, M. Thompson, and J. Mispel, 2011, Attenuation of free-surface multiples by up/down deconvolution for marine towed-streamer data: Geophysics, 76, V129-V138.

- Schalkwijk, K.M., Wapenaar, C.P.A., and Verschuur, D.J., 2003, Adaptive decomposition of multicomponent ocean-bottom seismic data into downgoing and upgoing P- and S-waves: Geophysics, Vol. 68, 1091-1102.

- Wapenaar, K., 1998, Reciprocity properties of one-way propagators: Geophysics, 63, 1795-1798.

- Wapenaar, K., E. Slob, and R. Snieder, 2008, Seismic and electromagnetic controlled-source interferometry in dissipative media: Geophysical Prospecting, 56, 419-434.