Up / down wavefield decomposition by sparse inversion

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The forward problem

Flux-normalized composition matrix (Wapenaar, 1998):

\[ \hat{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\omega \mathcal{Q})^{1/2} \hat{\mathcal{H}}^{-1/2}_1 & (\omega \mathcal{Q})^{1/2} \hat{\mathcal{H}}^{-1/2}_1 \\ (\omega \mathcal{Q})^{-1/2} \hat{\mathcal{H}}^{1/2}_1 & -(\omega \mathcal{Q})^{-1/2} \hat{\mathcal{H}}^{1/2}_1 \end{pmatrix} \]

Square-root of the Helmholtz operator:

\[ \hat{H}_2 = \left( \frac{\omega}{c} \right)^2 + \mathcal{Q} \frac{\partial}{\partial x} \left( \frac{1}{\mathcal{Q}} \frac{\partial}{\partial x} \right) \]
Numerical computation of the square-root operator

Computation of the Helmholtz operator at each frequency:

\[ \hat{H}_2 = \left( \frac{\omega}{c} \right)^2 + \varrho \frac{\partial}{\partial x} \left( \frac{1}{\varrho} \frac{\partial}{\partial x} \right) \]

Eigenvalue decomposition (Grimbergen et al., 1998):

\[ \hat{H}_2^{\frac{1}{n}} = \hat{L} \hat{\Lambda}^{\frac{1}{n}} \hat{L}^{-1} \]

\( n \)-th root
Flux-normalized inverse composition matrix (Wapenaar, 1998):

\[
\hat{L}^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\hat{\mathcal{H}}_{1}^{\frac{1}{2}} (\omega q)^{-\frac{1}{2}} & \hat{\mathcal{H}}_{1}^{-\frac{1}{2}} (\omega q)^{\frac{1}{2}} \\
\hat{\mathcal{H}}_{1}^{\frac{1}{2}} (\omega q)^{-\frac{1}{2}} & -\hat{\mathcal{H}}_{1}^{-\frac{1}{2}} (\omega q)^{\frac{1}{2}}
\end{pmatrix}
\]

Square-root of the Helmholtz operator:

\[
\hat{H}_{2} = \left( \frac{\omega}{c} \right)^{2} + q \frac{\partial}{\partial x} \left( \frac{1}{q} \frac{\partial}{\partial x} \cdot \right)
\]
Problems

1. Noise

\[
\begin{pmatrix}
\hat{p}^+ \\
\hat{p}^-
\end{pmatrix} = \hat{L}^{-1} \left[ \begin{pmatrix}
\hat{p}_{\text{signal}} \\
\hat{v}_{\text{signal}}
\end{pmatrix} + \begin{pmatrix}
\hat{p}_{\text{noise}} \\
\hat{v}_{\text{noise}}
\end{pmatrix} \right]
\]

2. Singularities

In \( \text{fk-domain} \):

\[
\hat{L}^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{\frac{k_z}{\omega Q}} & -\sqrt{\frac{\omega Q}{k_z}} \\
\sqrt{\frac{k_z}{\omega Q}} & \sqrt{\frac{\omega Q}{k_z}}
\end{pmatrix}
\]

Vertical wavenumber:

\[
k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}
\]

At critical angles:

\[
\frac{\omega^2}{c^2} = k_x^2
\]

Numerical instability
Sparsity promotion


– Applications in geophysics: Herrmann et al. (2008)

– The solution is assumed to be sparse in a transform domain:

\[
p^\pm = S^* x^\pm
\]

\[
S = C_2 \otimes W
\]

\[
\text{minimize } \| (x^+, x^-) \|_1 \text{ subject to } \| (p^v) - L \begin{pmatrix} S^* & 0 \\ 0 & S \end{pmatrix} \begin{pmatrix} x^+ \\ x^- \end{pmatrix} \|_2 \leq \sigma
\]

– spgl1 (Van den Berg and Friedlander, 2008)
Example

Velocity model:
- 128 sources
- 128 receivers
- Noise added (SNR=5)
Pressure data

Common Receiver Gather

Dowgoing field (not changing polarity)

Upgoing field (changing polarity)

Common Source Gather

FK spectrum
Particle velocity data

Common Receiver Gather

Downgoing field (not changing polarity)

Upgoing field (changing polarity)

Common Source Gather

FK spectrum
Upgoing field retrieved by exact inversion

- Results are noisy
- Instabilities at singularities
Upgoing field retrieved by exact inversion

Common Receiver Gather

Time (s)

Source location (km)

Common Source Gather

Time (s)

Receiver location (km)

FK spectrum

– After FK-filtering
Upgoing field retrieved by sparse inversion

- Result less noisy
- No instabilities at singularities
Upgoing field retrieved without noise

- Detailed information can be lost if noise level $\sigma$ is set too high
- Instabilities can occur if noise level $\sigma$ is set too low
Extension to elastic media

\[
\begin{pmatrix}
\hat{\tau}_{xz} \\
\hat{\tau}_{yz} \\
\hat{\tau}_{zz} \\
\hat{v}_x \\
\hat{v}_y \\
\hat{v}_z
\end{pmatrix}
= \hat{L}
\begin{pmatrix}
\hat{p}_P^+ \\
\hat{p}_S^+ \\
\hat{p}_S^- \\
\hat{p}_P^- \\
\hat{p}_{Sv}^- \\
\hat{p}_{Sh}^-
\end{pmatrix}
\]

– Composition matrix – Wapenaar et al. (2008)

– Potential application: OBC (Ocean Bottom Cable) decomposition (Schalkwijk et al., 2003)
Application: Up / down deconvolution

- OBC data (Amundsen, 2001) → Free-surface multiple elimination
- Streamer data (Majdanski et al., 2011) → Free-surface multiple elimination
- Downhole receivers (Wapenaar et al., 2008) → Interferometric redatuming / virtual source method
Conclusion

* Instabilities caused by singularities of the square-root operator and noise are avoided by sparse inversion

* Applications: dual-sensor streamer data, OBC data, downhole receivers (virtual source method)
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