Fast imaging with multiples by sparse inversion

Ning Tu and Felix J. Herrmann

SLIM
University of British Columbia

Lin, Tu, and Herrmann, 2010; Verschuur and Berkhout, 2011; Whitmore et.al., 2010; Liu et.al., 2011

Motivation

- making use of primaries and multiples *simultaneously*
- avoiding imaging *artifacts* from multiples
- looking for a computationally *efficient* approach

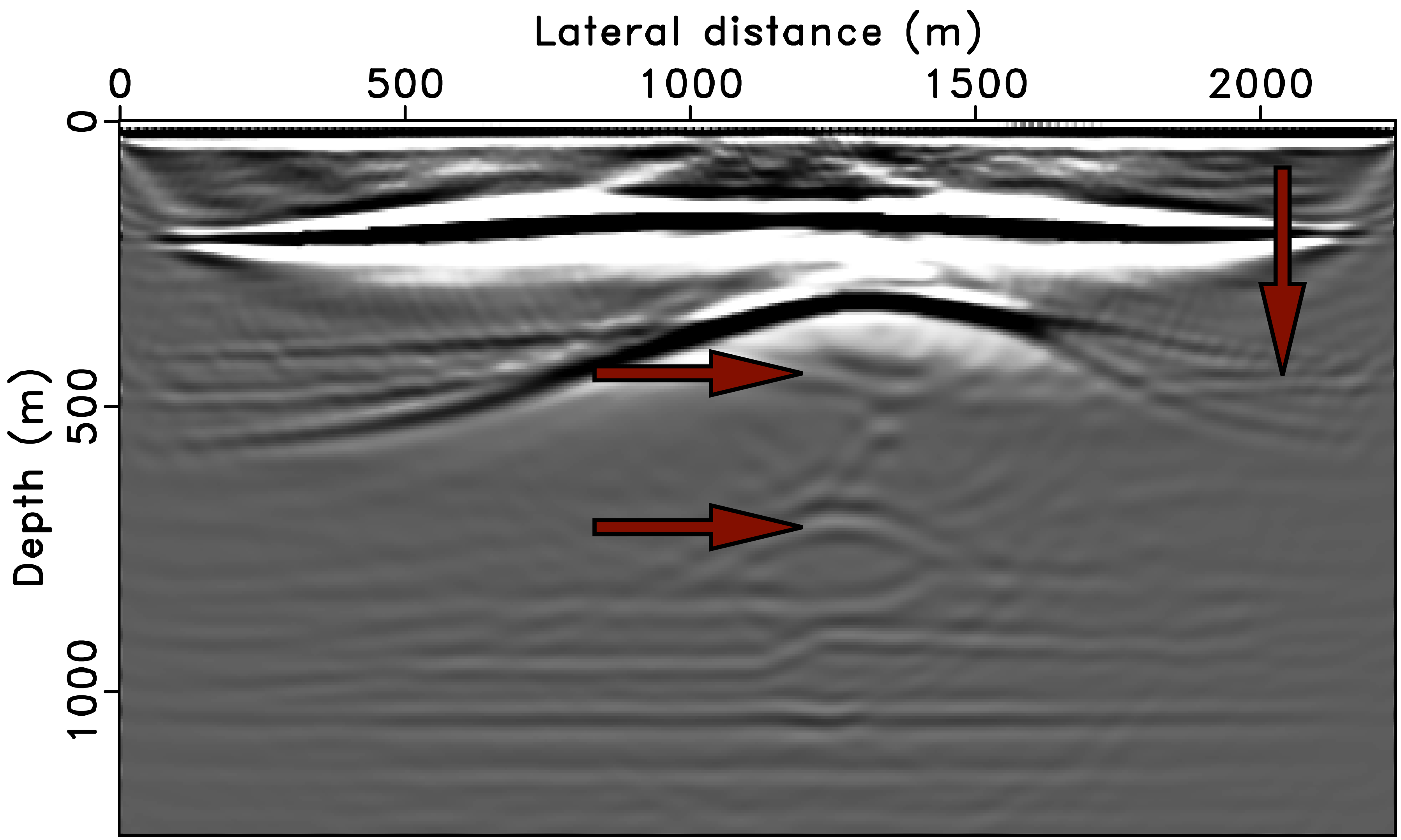
Tu and Herrmann, 2011; Verschuur and Berkhout, 2011; Lu et.al., 2011; Liu et.al., 2011

Primaries & multiples: not “OR” but “AND”

- primaries have a higher signal-to-noise ratio
- multiples provide extra illumination if correctly used

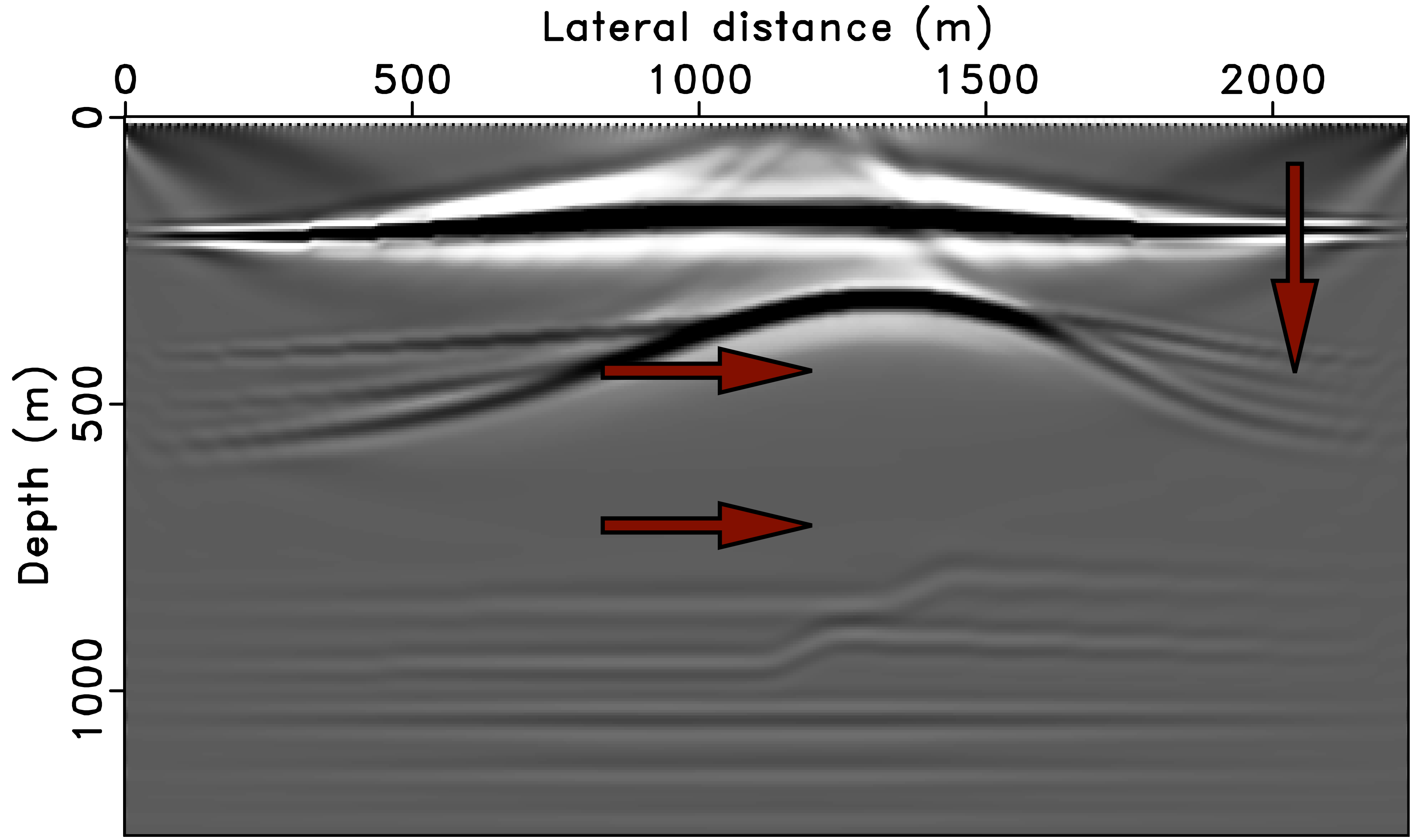
Whitmore et.al., 2010; Liu et.al., 2011

Artifacts from multiples



Reverse time migration of *multiples* using total data as “source”

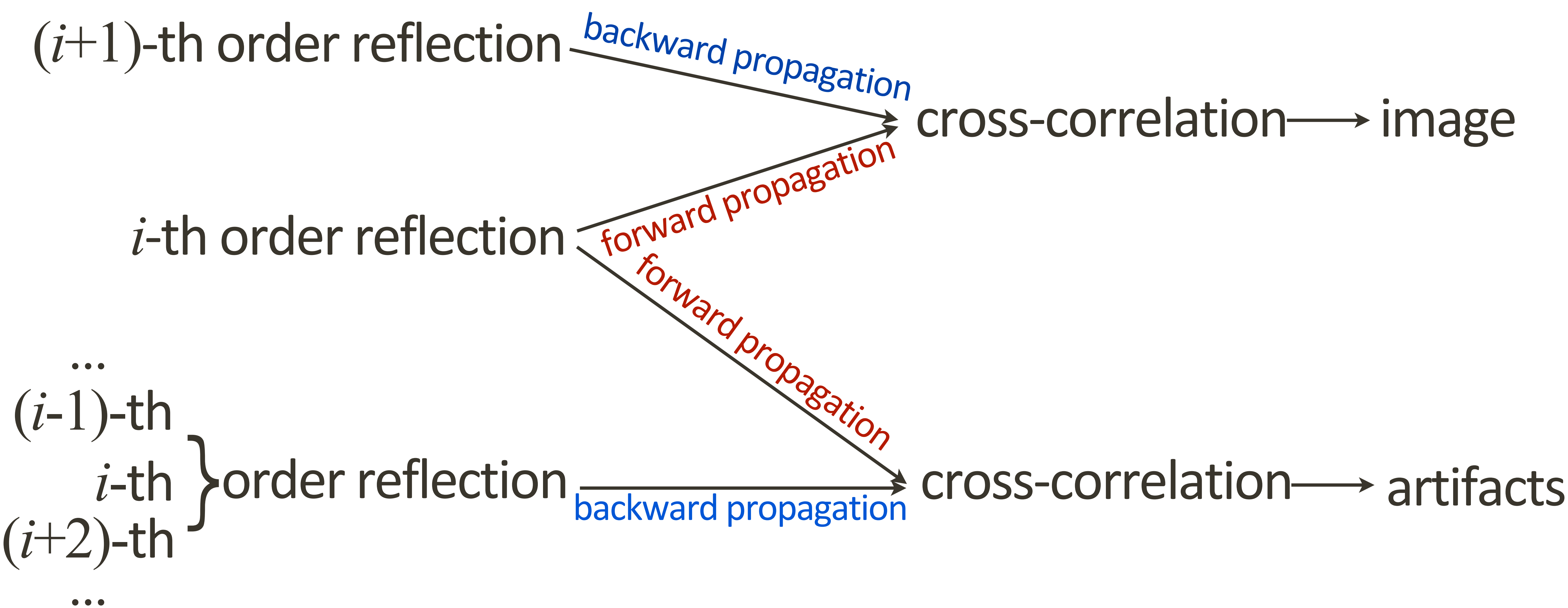
Not there for primaries



Reverse time migration of *primaries*

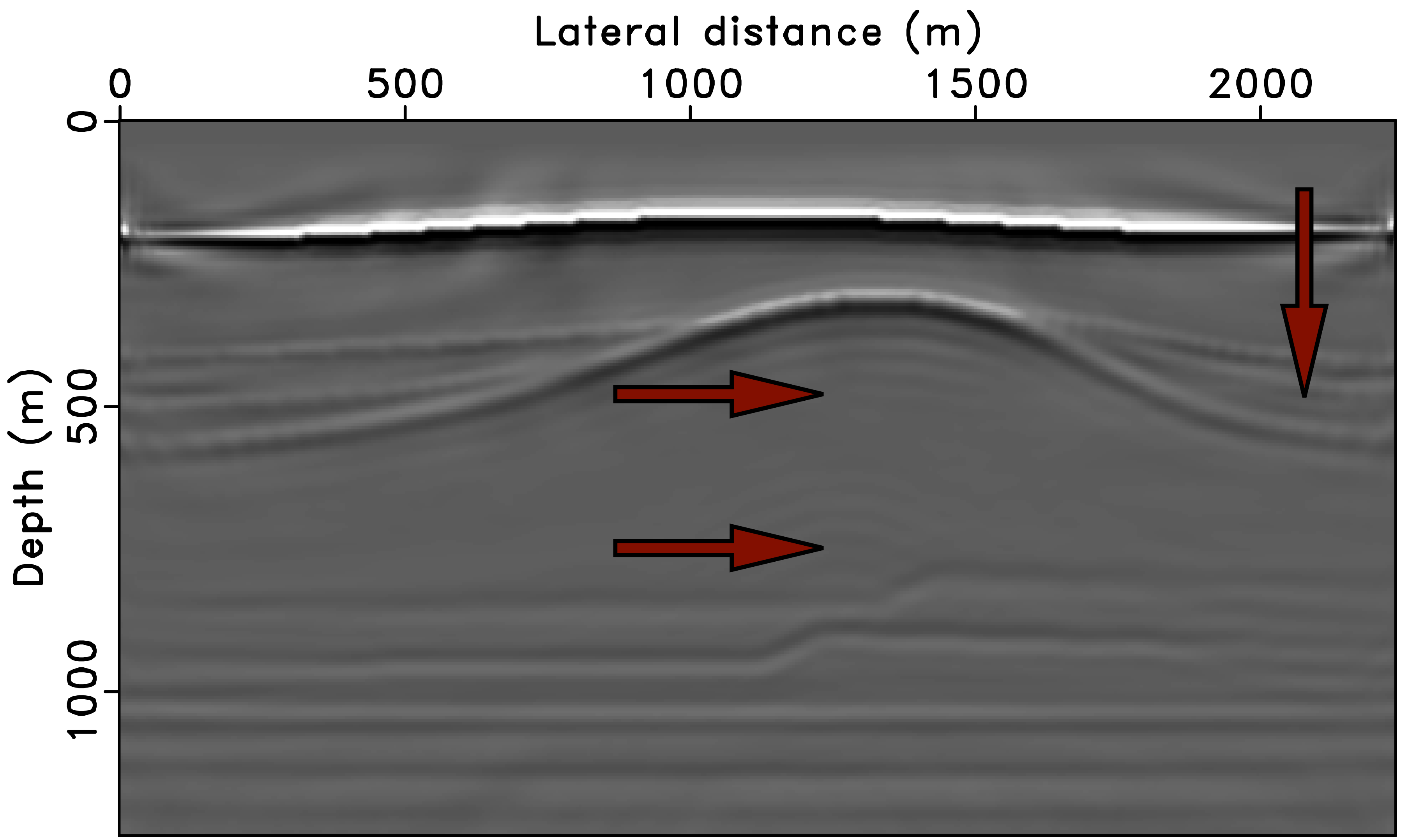
Liu et.al., 2011

When a free-surface presents



Lin et. al., 2010; Tu and Herrmann, 2011a

Want to avoid them?



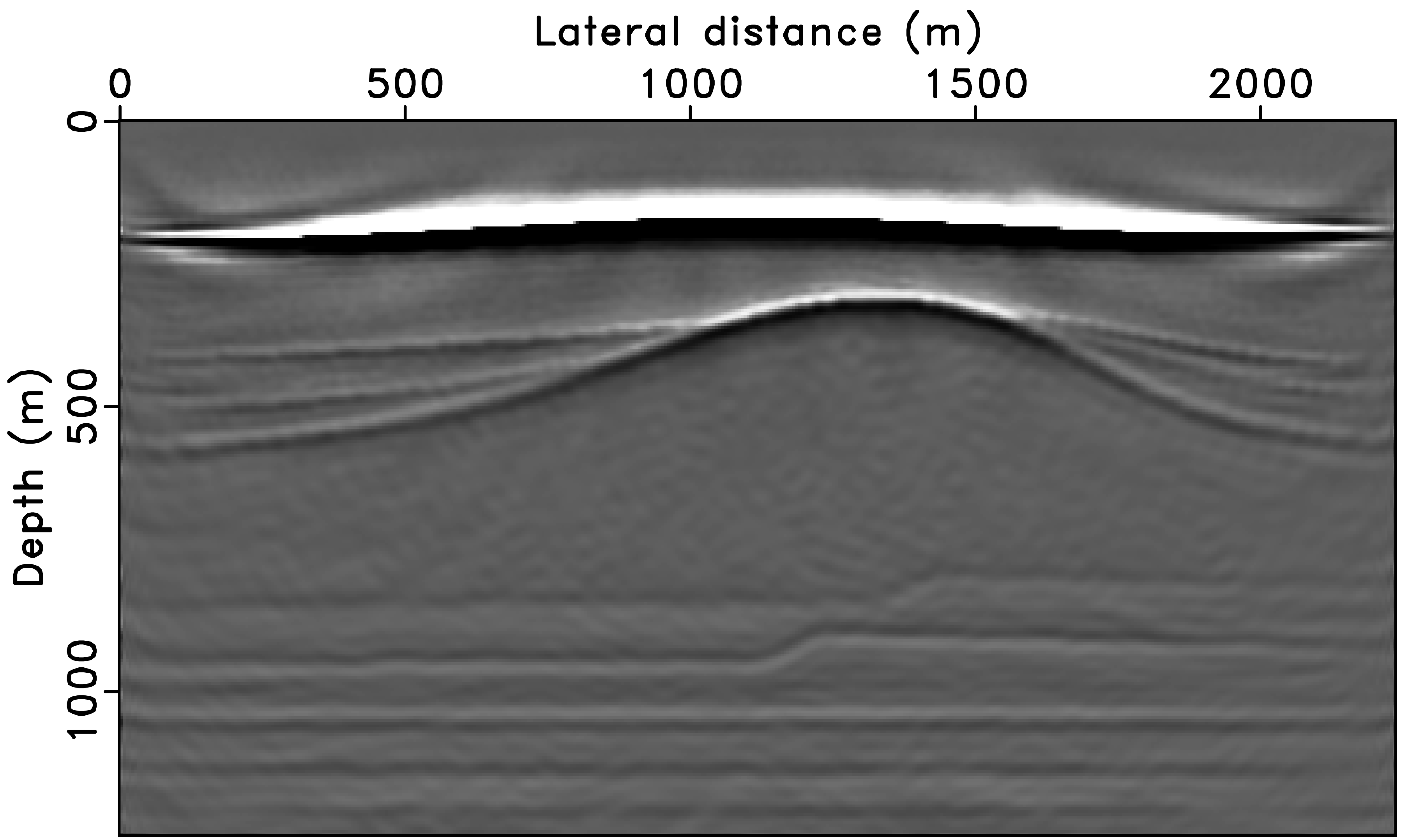
Imaging of multiples by *inversion*

Inversion? Sounds expensive...

- repeated evaluations of the Born scattering operator and its adjoint
- each evaluation requires solving 4*(#source)*(#frequencies) PDEs

Sneak peek of our result

[with a computational budget of a single RTM with all data]



Fast imaging of *total up-going wavefield* by sparse inversion

Method

Incorporating the free surface

Total data and the surface-free Green's function can be related by the SRME formulation:

$$\mathbf{P}_{\omega_\mathrm{i}} = \mathbf{G}_{\omega_\mathrm{i}}(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}})$$

$\mathbf{P}$: total up-going wavefield
$\mathbf{G}$: surface-free Green's function
$\mathbf{Q}$: source wavelet
$\mathbf{R}$: surface reflectivity

Expressed in model space

$$
\begin{aligned}
\mathbf{P}_{\omega_\mathrm{i}} &= \mathrm{vec}^{-1}(\mathbf{F}_{\omega_\mathrm{i}}[\mathbf{m},\mathbf{I}])(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}) \\
&= \mathbf{D}_\mathrm{r}\mathbf{H}_{\omega_\mathrm{i}}^{-1}[\mathbf{m}](\mathbf{D}_\mathrm{s}^{*}\mathbf{I})(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}) \\
&= \mathbf{D}_\mathrm{r}\mathbf{H}_{\omega_\mathrm{i}}^{-1}[\mathbf{m}](\mathbf{D}_\mathrm{s}^{*}(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}})) \\
&\doteq \mathrm{vec}^{-1}(\mathbf{F}_{\omega_\mathrm{i}}[\mathbf{m},\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}])
\end{aligned}
$$

$\mathbf{F}$: modelling operator

$\mathbf{m}$: true velocity/density model

$\mathbf{I}$: impulsive source array

$\mathbf{D}$: detection operator at receiver/source locations

Linearization

$$\mathbf{p}_{\omega_\mathrm{i}} = \nabla\mathbf{F}_{\omega_\mathrm{i}}[\mathbf{m}_0, \mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}]\delta\mathbf{m}$$

$\nabla\mathbf{F}$: Born scattering operator
$\mathbf{m_0}$: background model
$\delta\mathbf{m}$: model perturbation
$\mathbf{p}_{\omega_\mathrm{i}}$: vectorized wavefield

Stacking over frequencies

$$
\begin{aligned}
\mathbf{p} &= \begin{bmatrix} \nabla \mathbf{F}_{\omega_1}(\mathbf{m}_0, \mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}} \mathbf{P}_{\omega_\mathrm{i}}) \\ \vdots \\ \nabla \mathbf{F}_{\omega_\mathrm{nf}}(\mathbf{m}_0, \mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}} \mathbf{P}_{\omega_\mathrm{i}}) \end{bmatrix} \delta \mathbf{m} \\
&\doteq \nabla \mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{R}\mathbf{P}] \delta \mathbf{m}
\end{aligned}
$$

$$
\delta \mathbf{m} = \nabla \mathbf{F}^{\dagger}[\mathbf{m}_0, \mathbf{Q} + \mathbf{R}\mathbf{P}] \mathbf{p}
$$

Sparse inversion

We use a sparsity-promotion formulation:

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\mathbf{p} - \nabla\mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

$\mathbf{C}$: curvelet transform

solver: $\mathrm{SPG}\ell_1$

SLIM

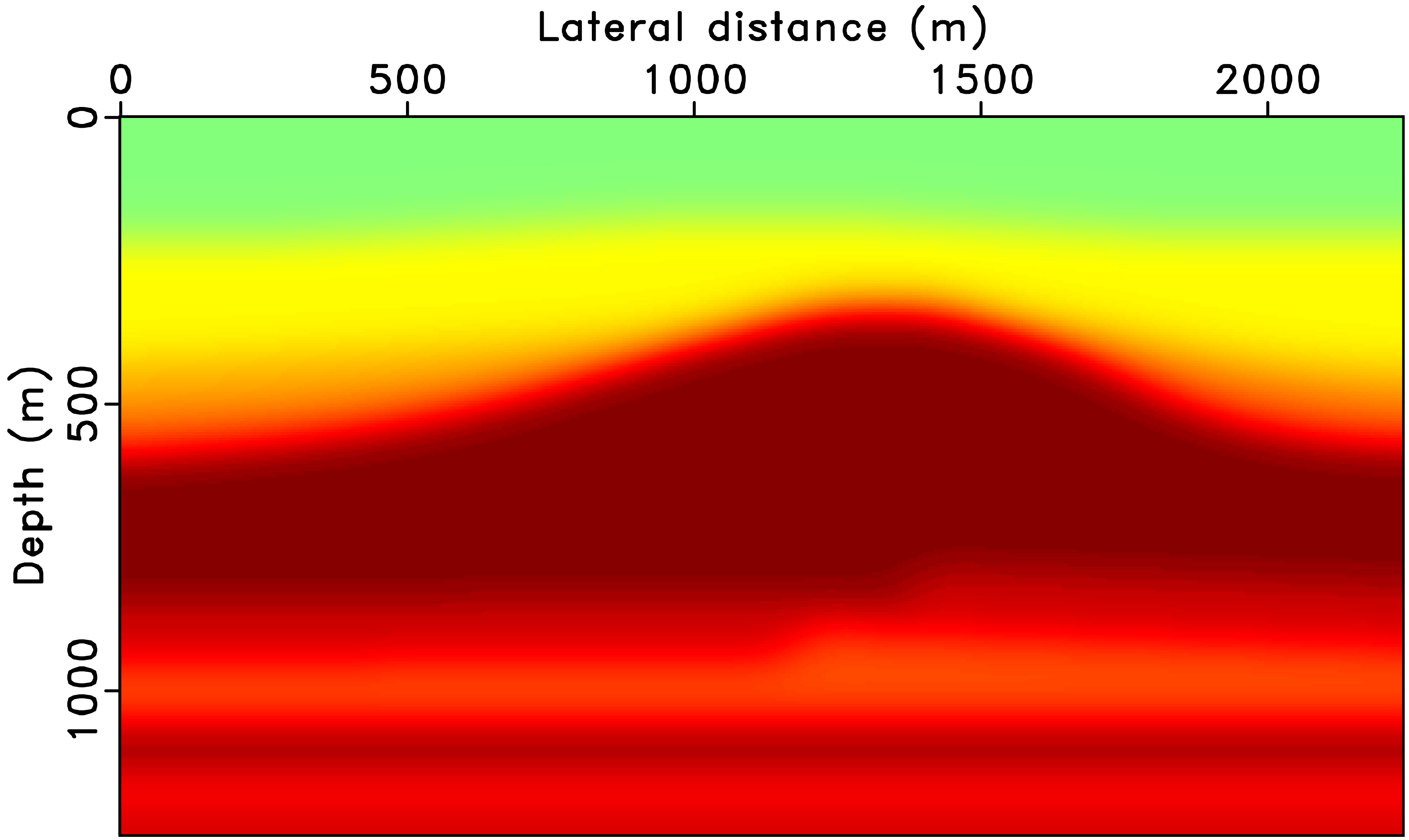
Demonstrative examples

- model grid spacing: 5 meters
- using linearized data:

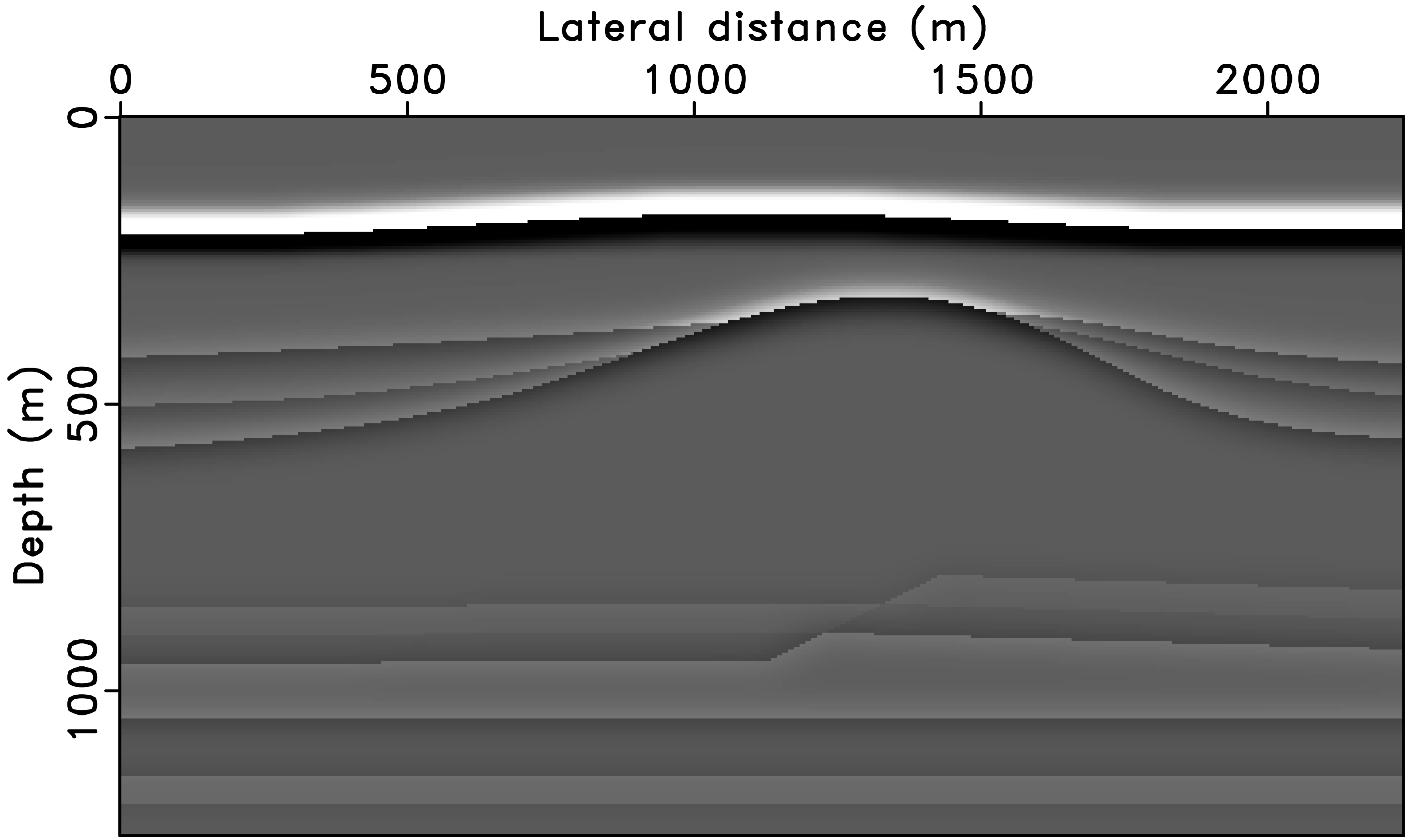
$$\nabla\mathbf{F}[\mathbf{m_0}, \mathbf{Q}+\mathbf{RP}]\delta\mathbf{m}$$

- 150 collocated sources/receivers
- 122 frequencies in 0-60Hz range

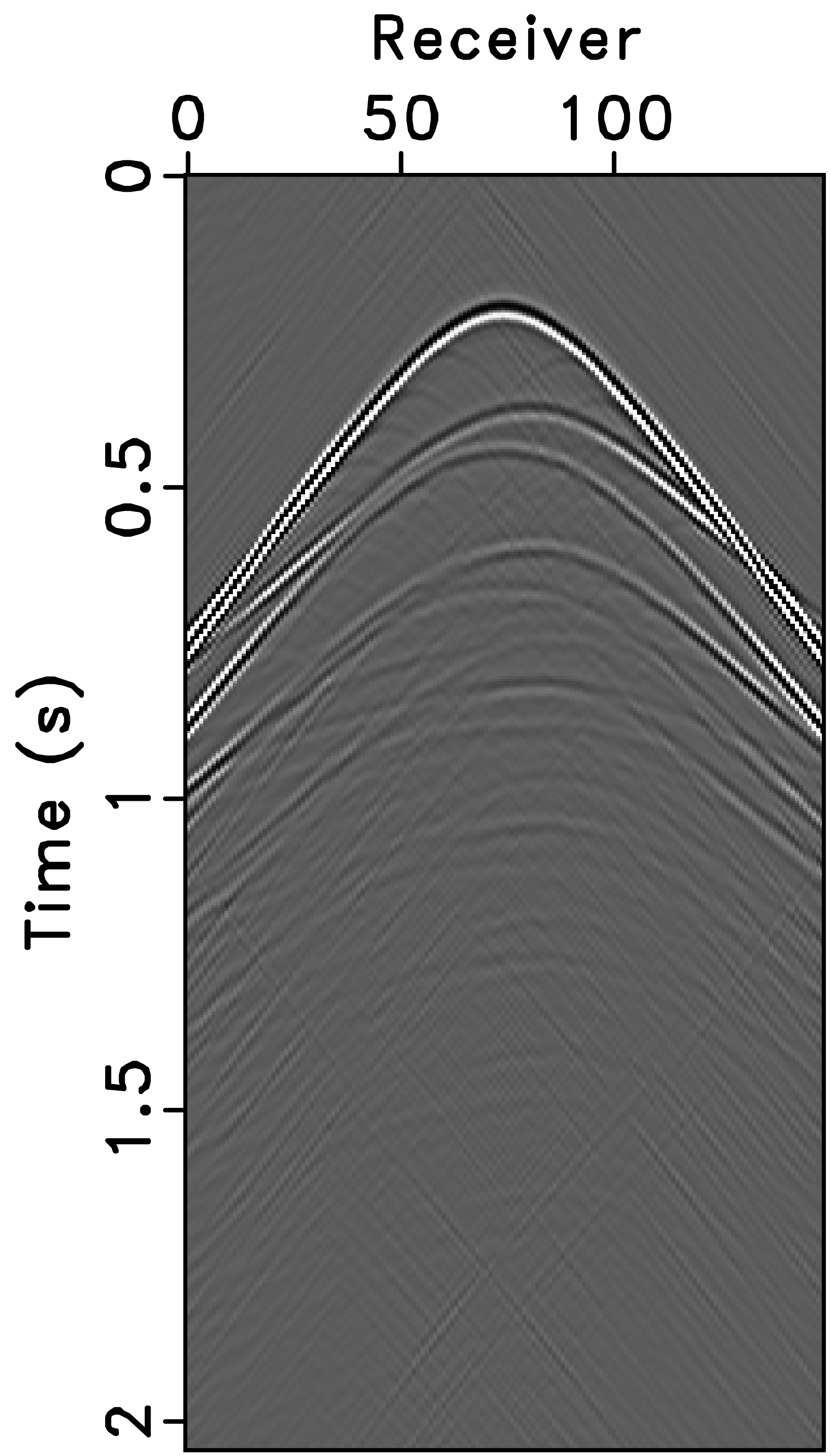
Background model



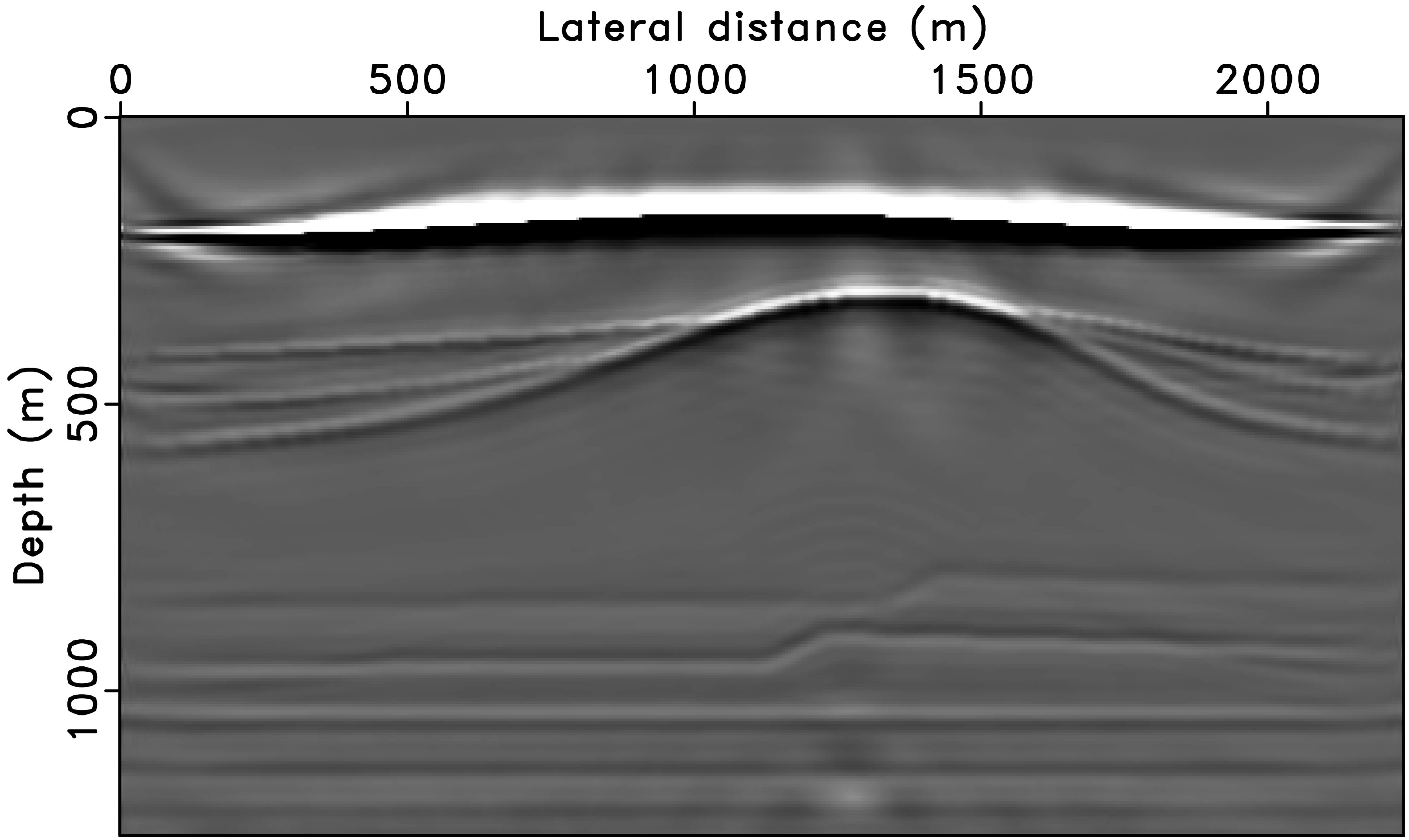
True perturbation



Linearized total data



Result



Inversion of the total up-going wavefield using all sequential sources and all frequencies
number of PDE solves: ~4.4 million (by calculation)

Dimensionality reduction

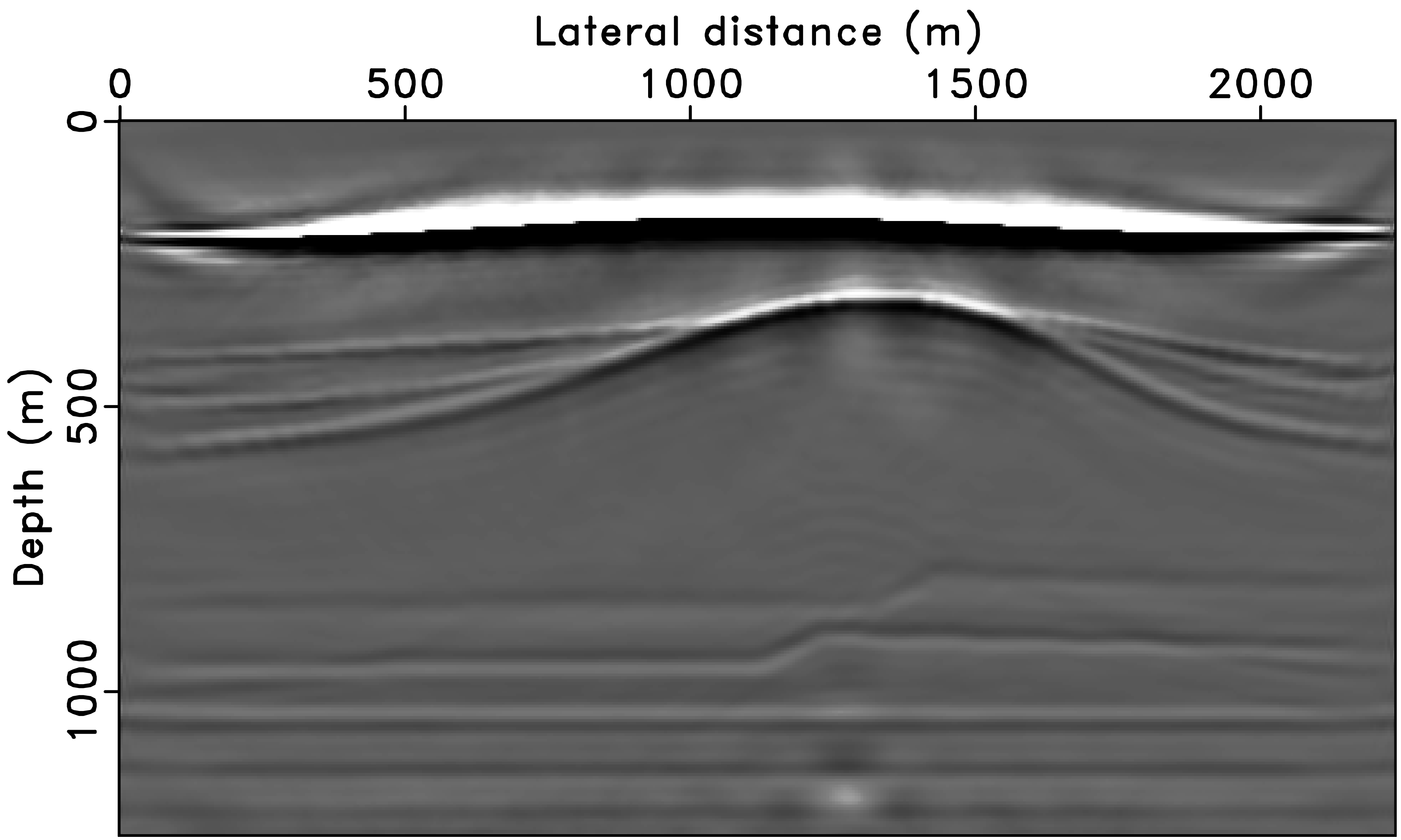
$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \operatorname*{argmin}_{\delta\mathbf{x}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

source: combine sources into a few simultaneous sources, using Gaussian distributed random weights

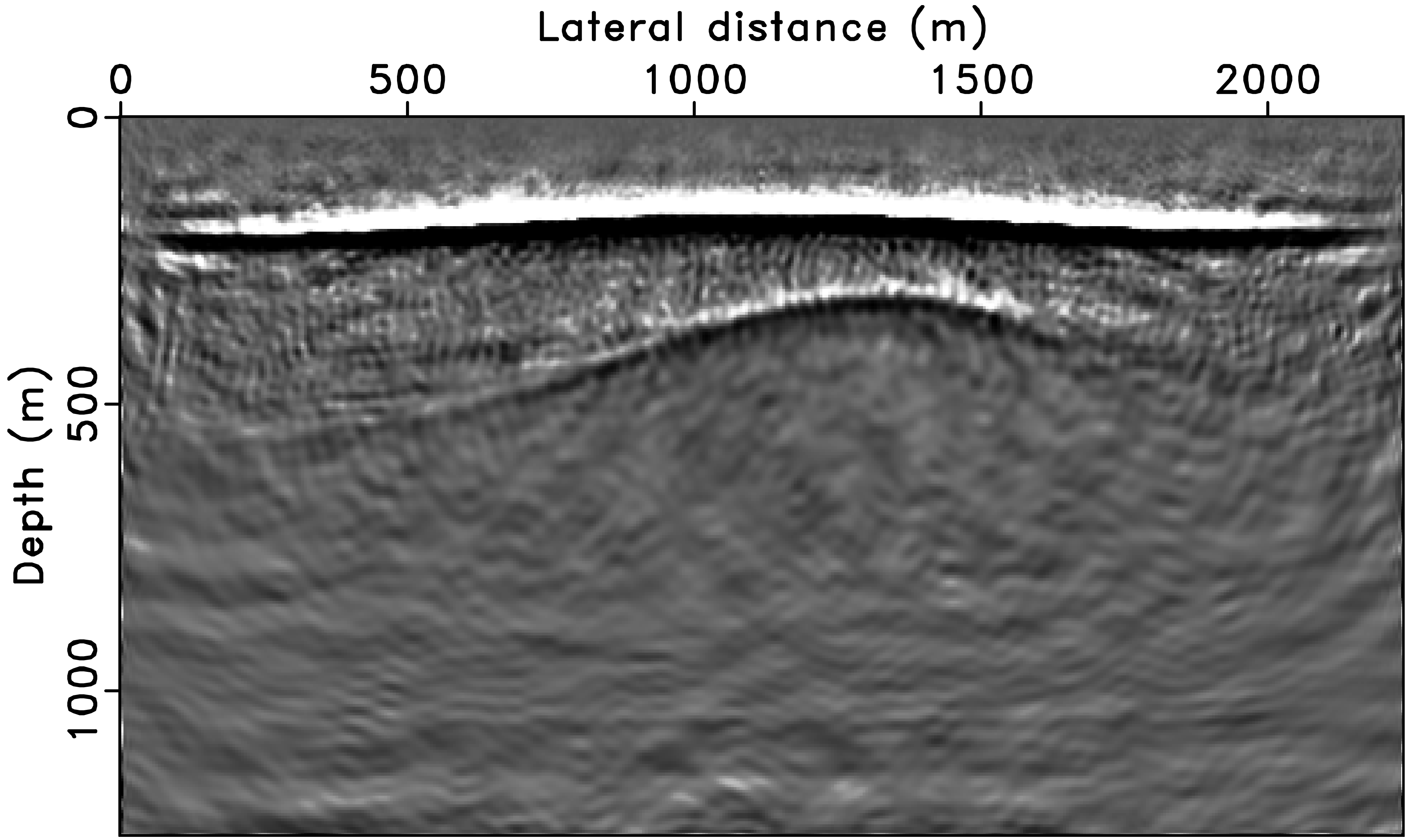
frequency: randomly choose a subset of frequencies

Result with 15x speed-up



Inversion of the total up-going wavefield using 10 simultaneous sources and all frequencies
number of PDE solves: ~0.3 million

Too much subsampling brings artifacts



Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand

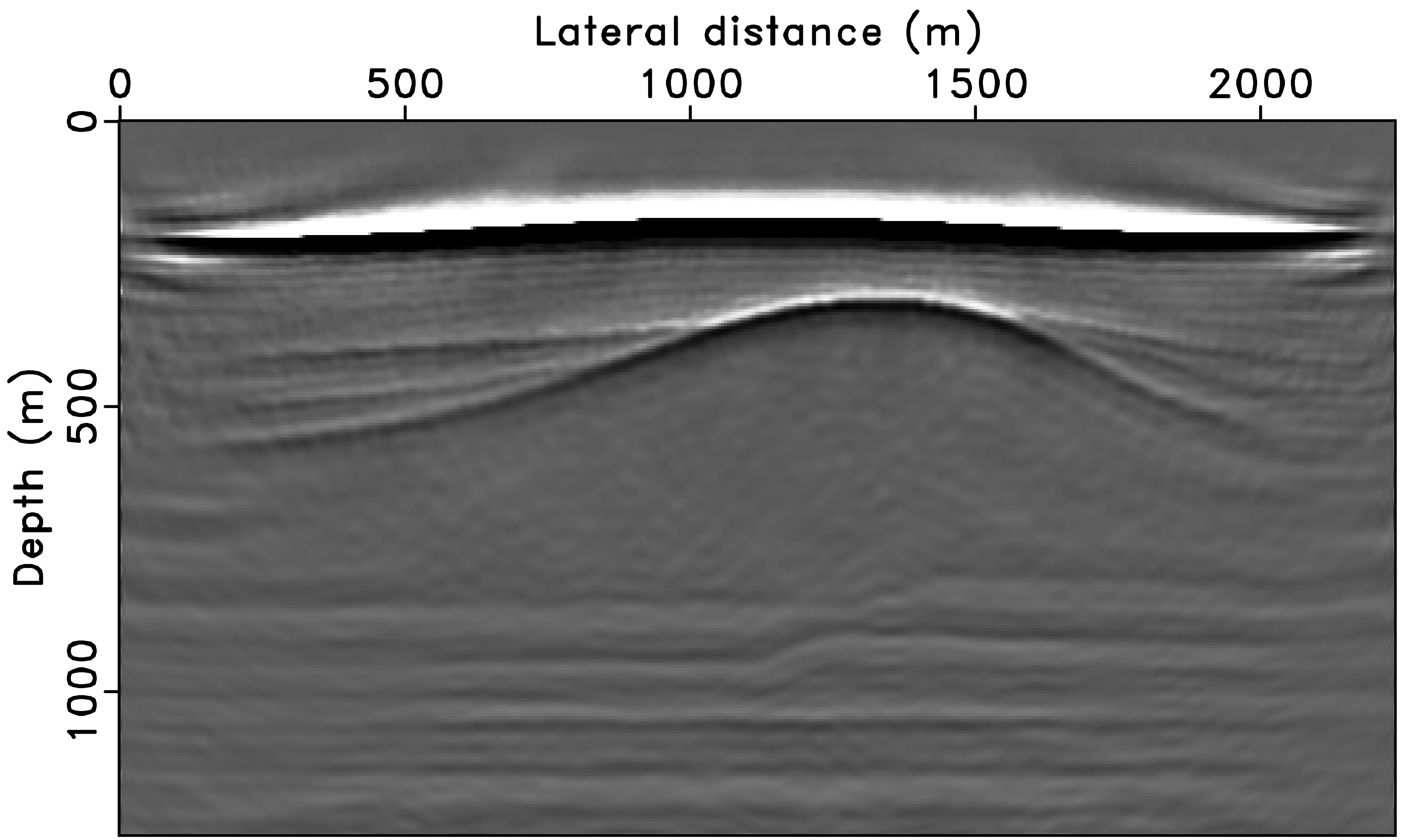
Draw new subsampling operator

- $\text{SPG}\ell_1$ solves a series of subproblems:

$$\underset{\delta\mathbf{x}}{\text{argmin}} \, ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H \delta\mathbf{x}||_2$$
$$\text{subject to } ||\delta\mathbf{x}||_1 \leq \tau$$

- redraw subsampling operator for each new subproblem

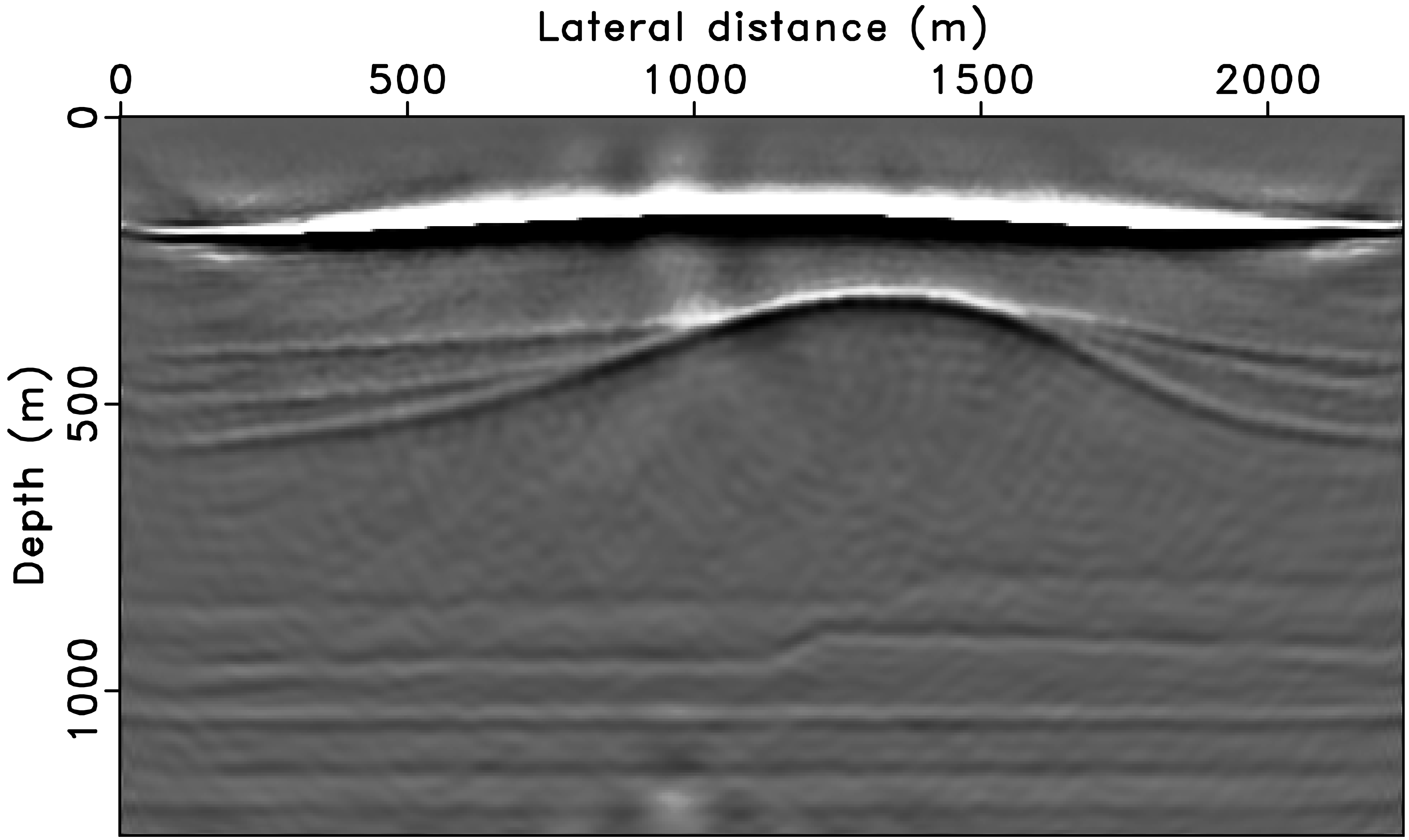
Draw new sim. sources



Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand (by calculation)

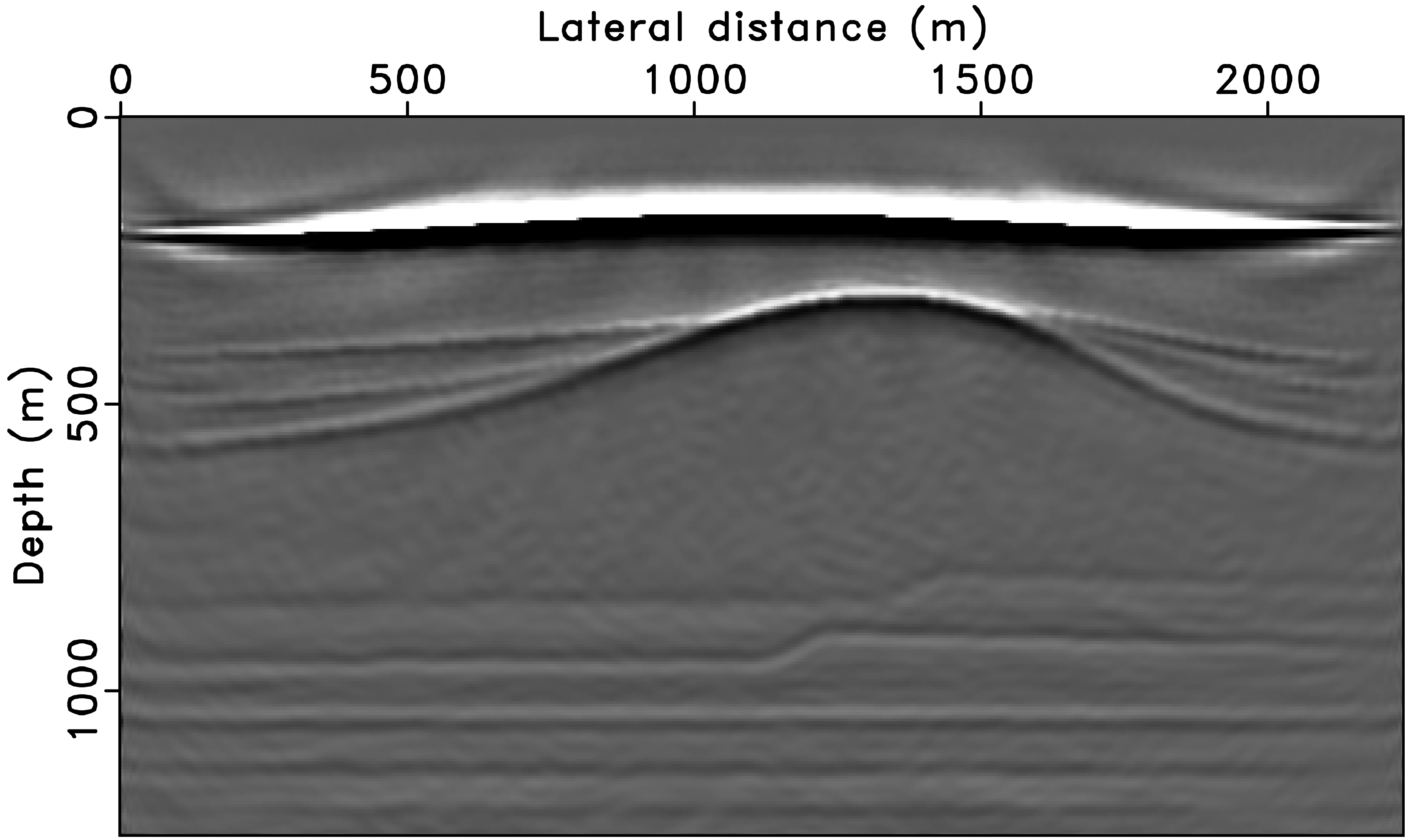
SLIM

Draw new frequencies



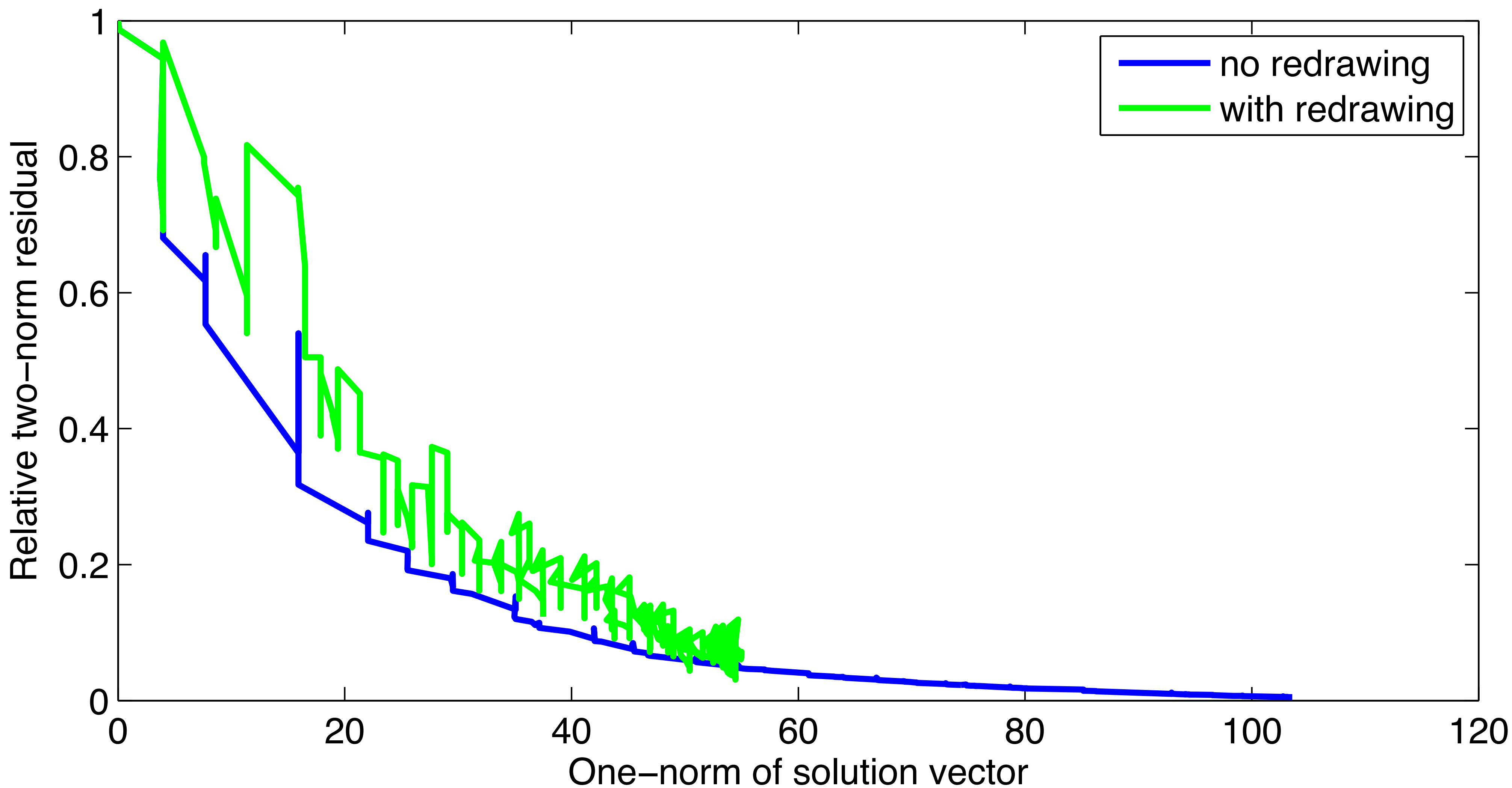
Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand (by calculation)

Draw new sim. sources and frequencies

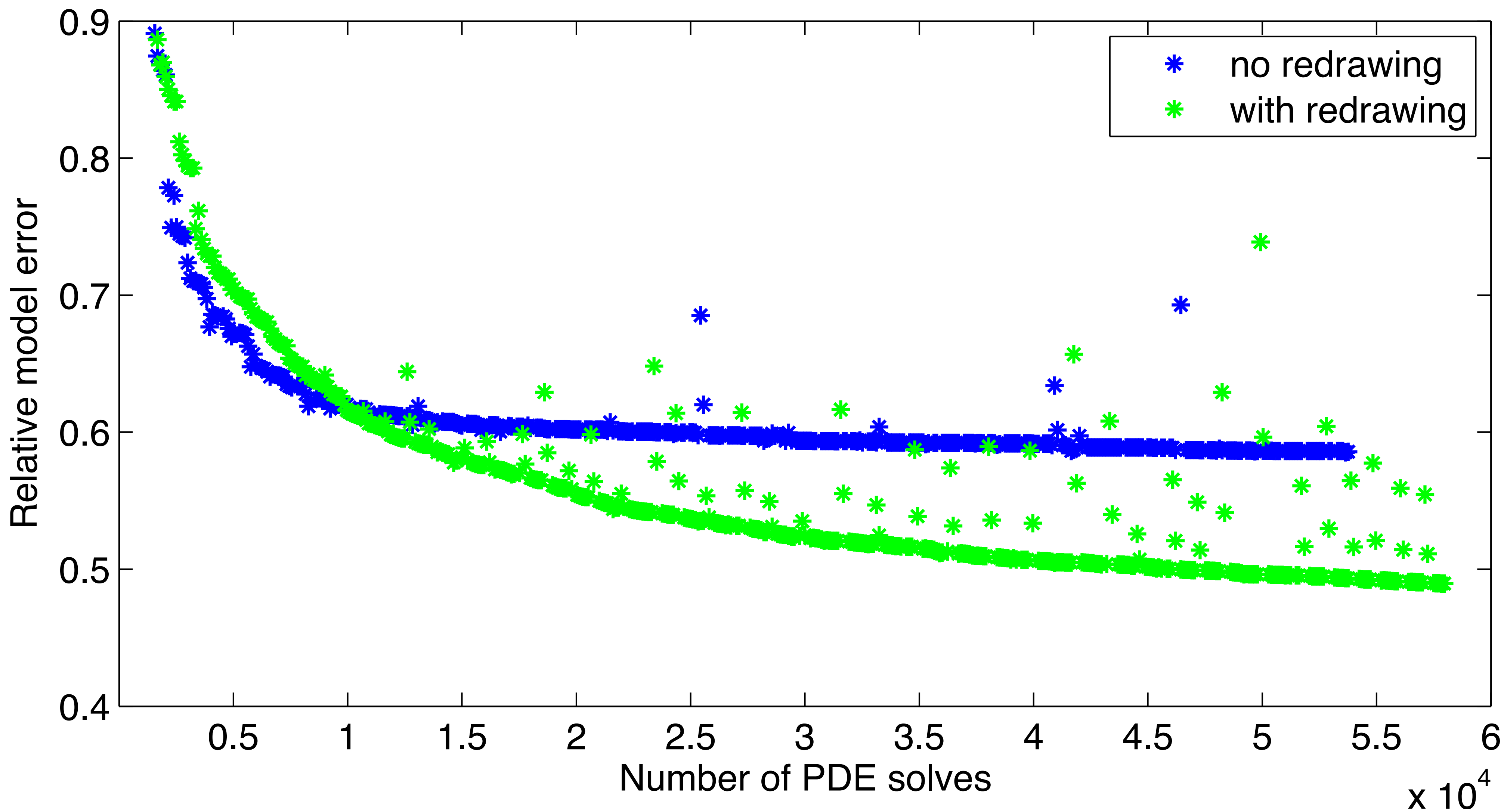


Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand (by calculation)

Solution path

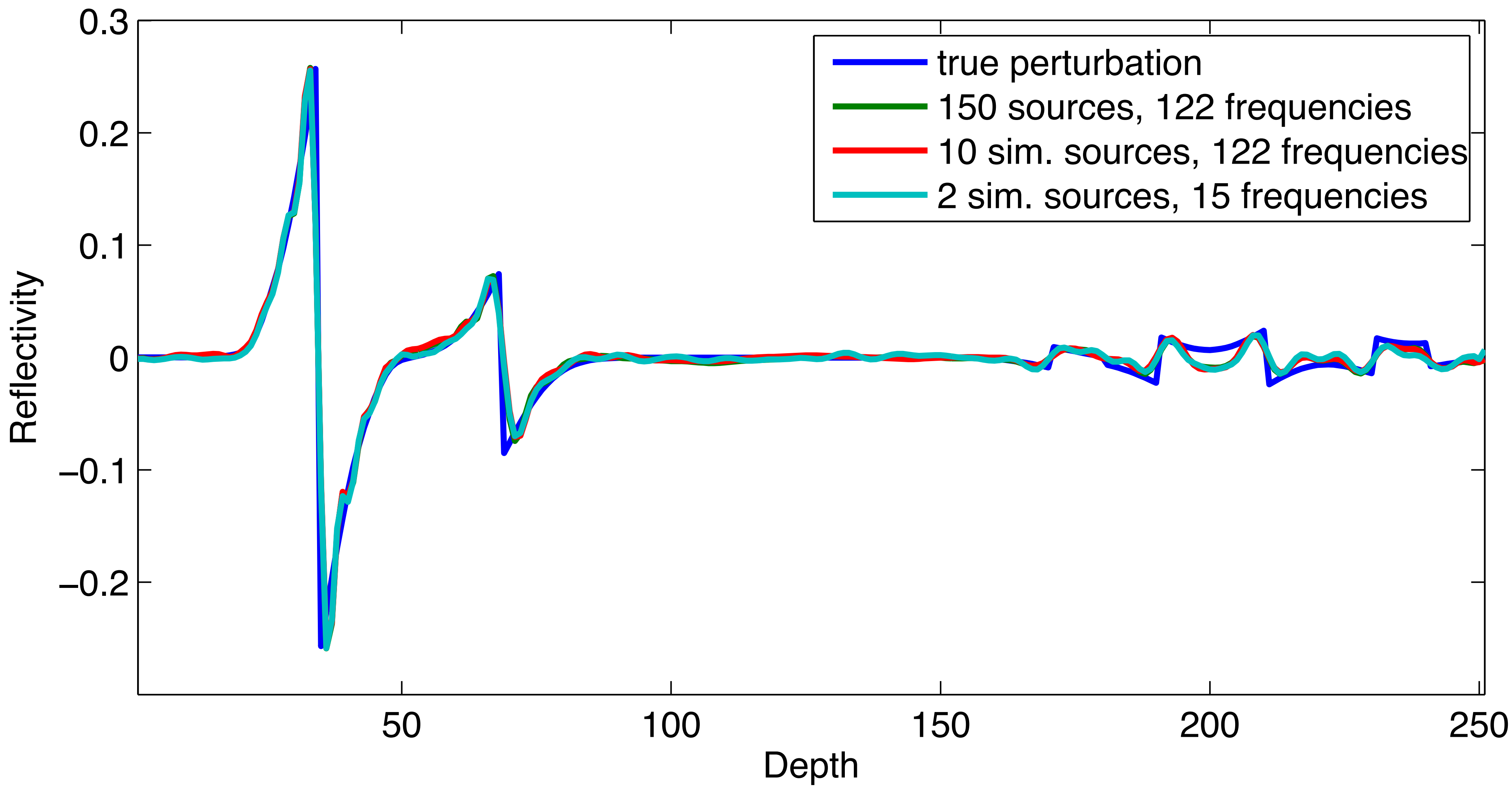


Model error decrease



Note: outliers are intermediate line-search results, not a concern; number of PDE solves in practice has ~50% overhead due to line search, etc.

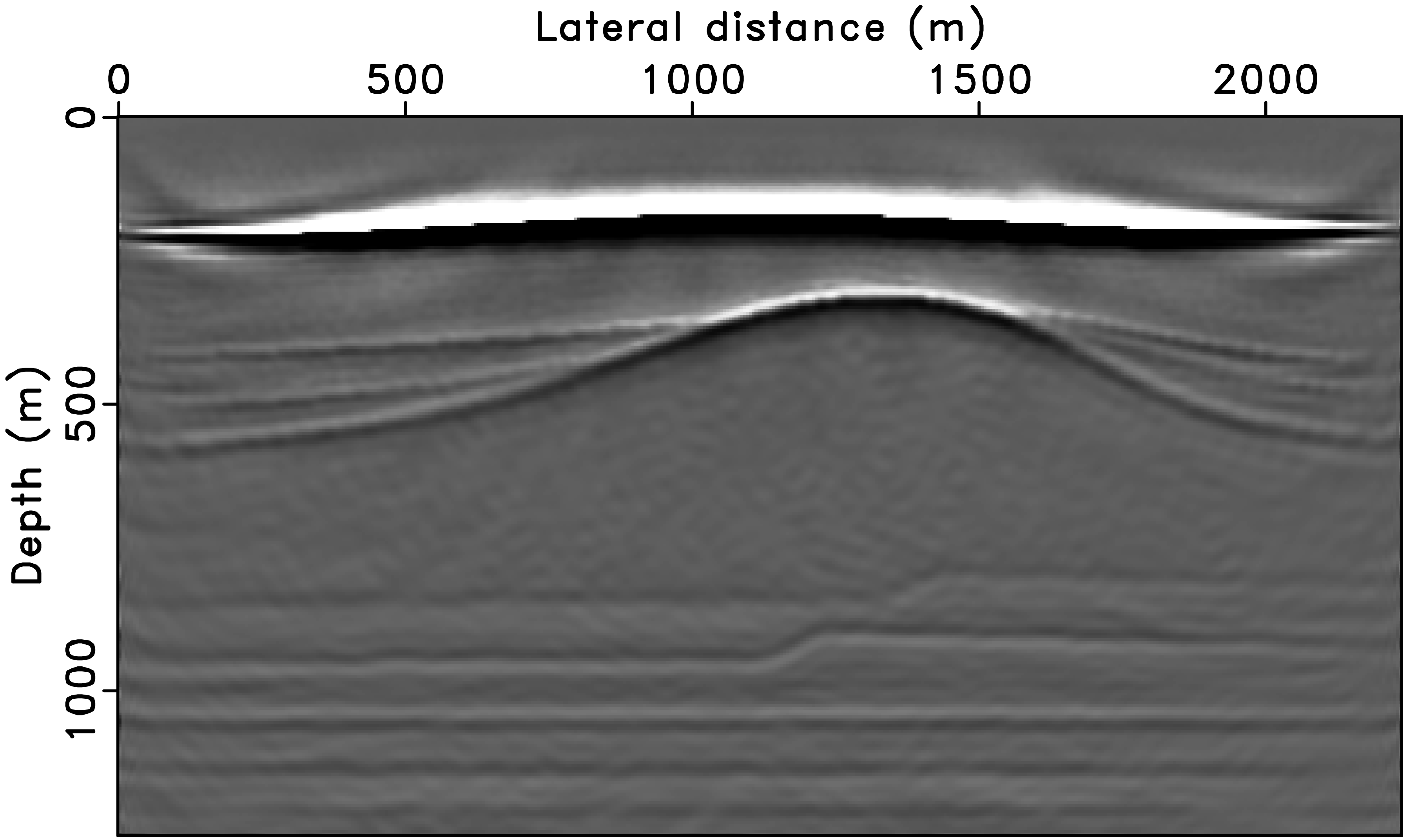
Inversion results



Trace to trace comparison: the 224th trace of model perturbation

Comparison: batch size

[same budget of PDE solves]



Fast imaging of total data

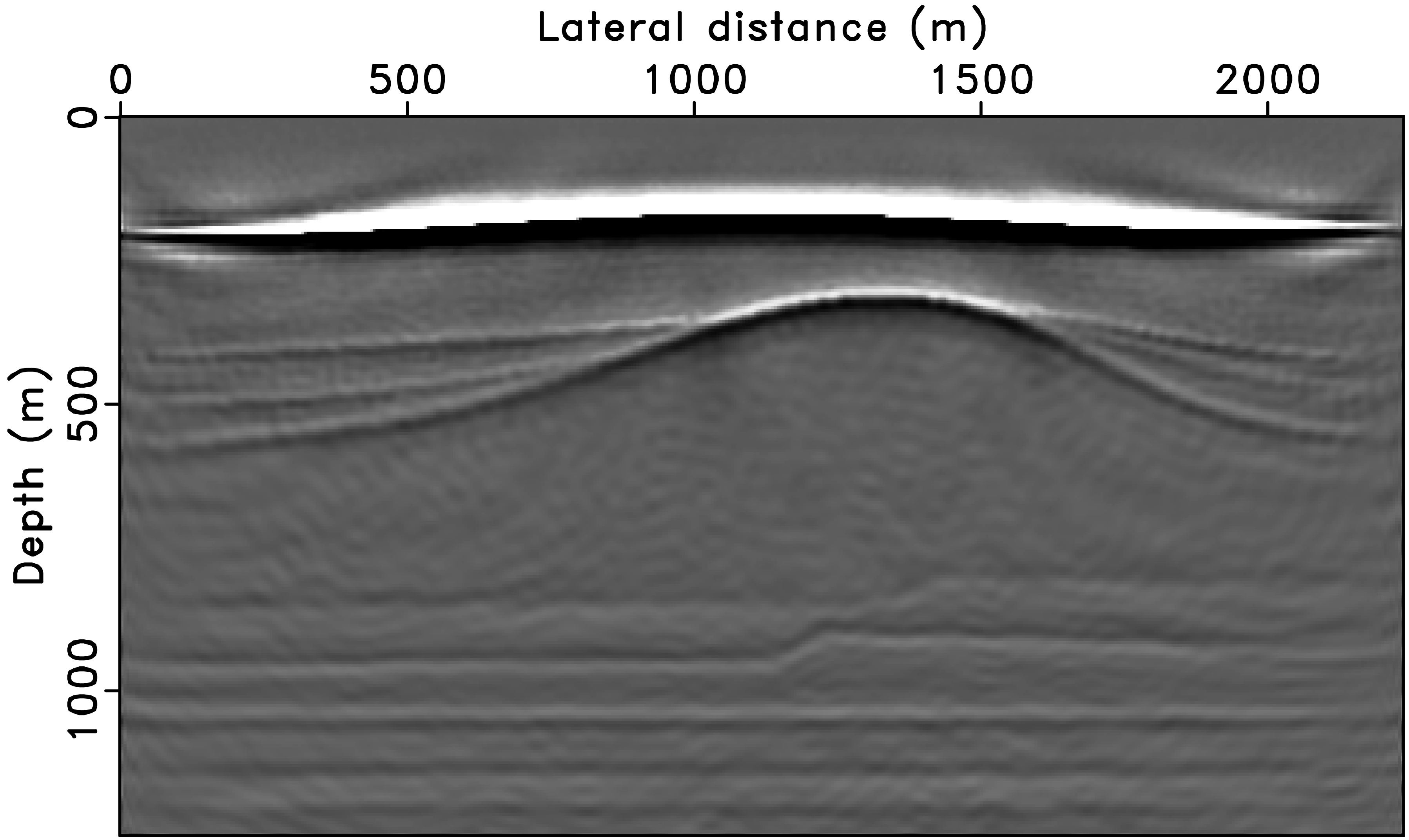
Batch size: 30 (2 simultaneous sources and 15 frequencies)

Iteration: 305

Number of PDE solves: 36.6 thousand (by calculation)

Comparison: batch size

[same budget of PDE solves]



Fast imaging of total data

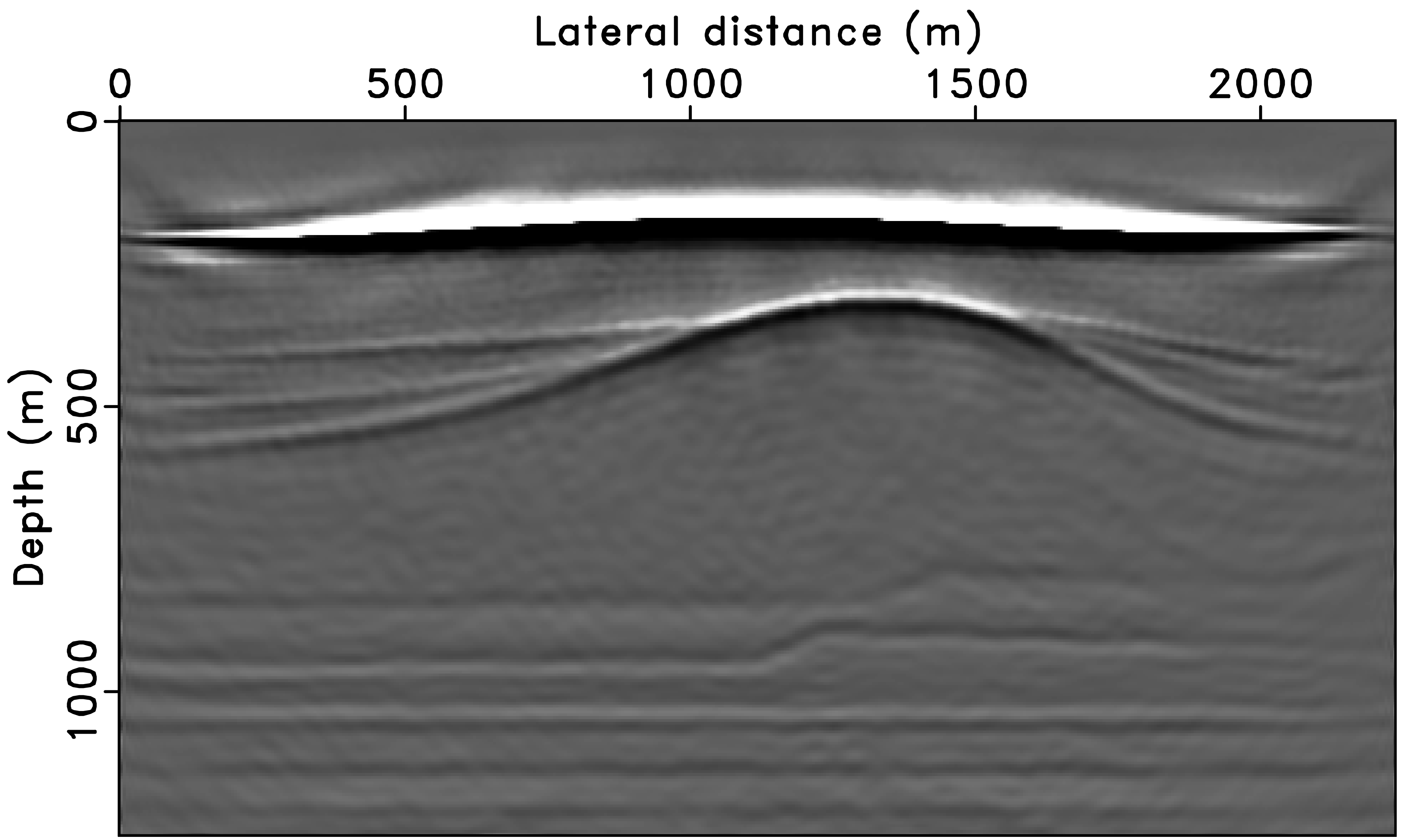
Batch size: 15 (1 simultaneous sources and 15 frequencies)

Iteration: 610

Number of PDE solves: 36.6 thousand (by calculation)

Comparison: batch size

[same budget of PDE solves]



Fast imaging of total data

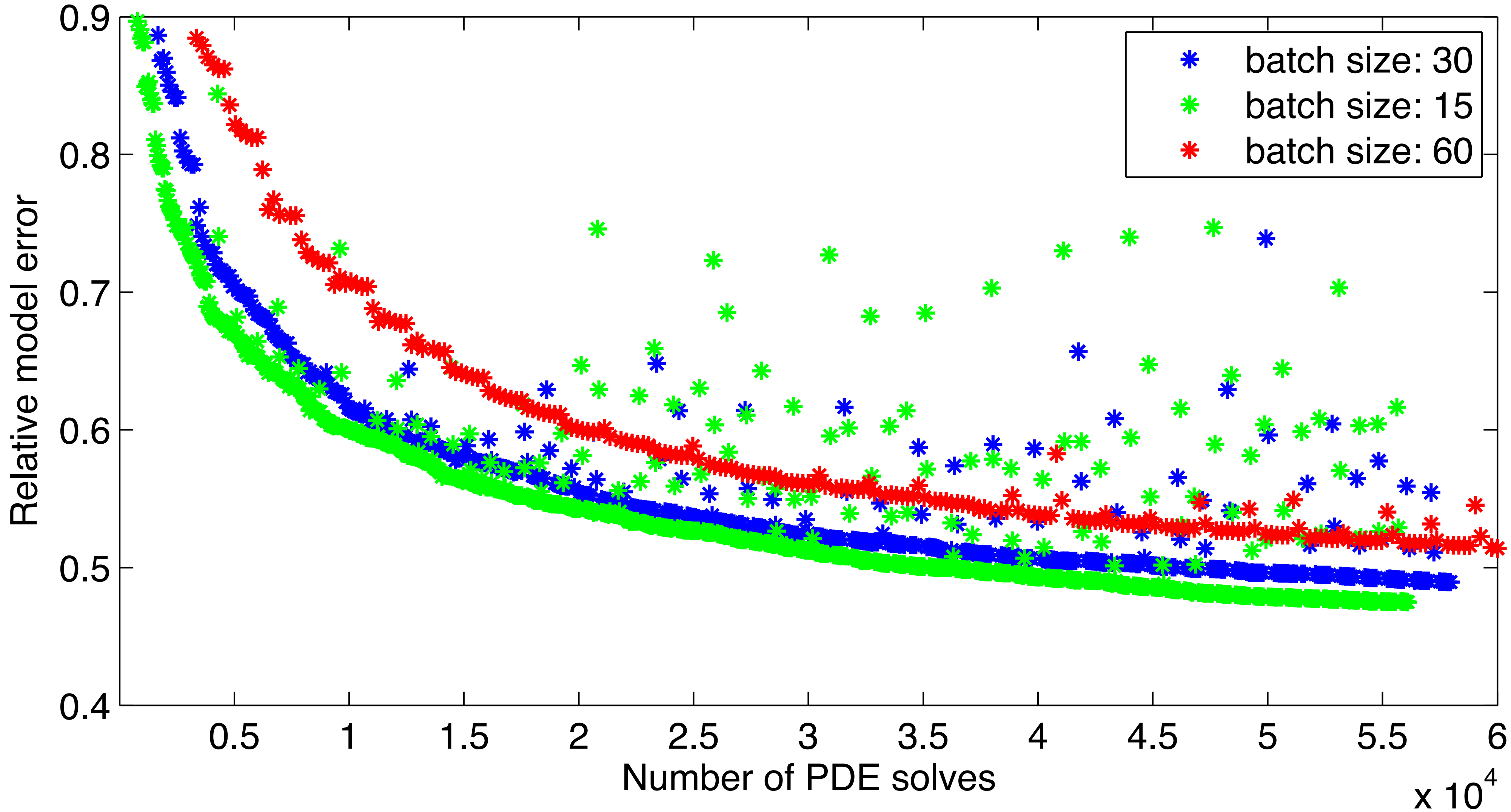
Batch size: 60 (4 simultaneous sources and 15 frequencies)

Iteration: 152

Number of PDE solves: 36.6 thousand (by calculation)

Comparison: batch size

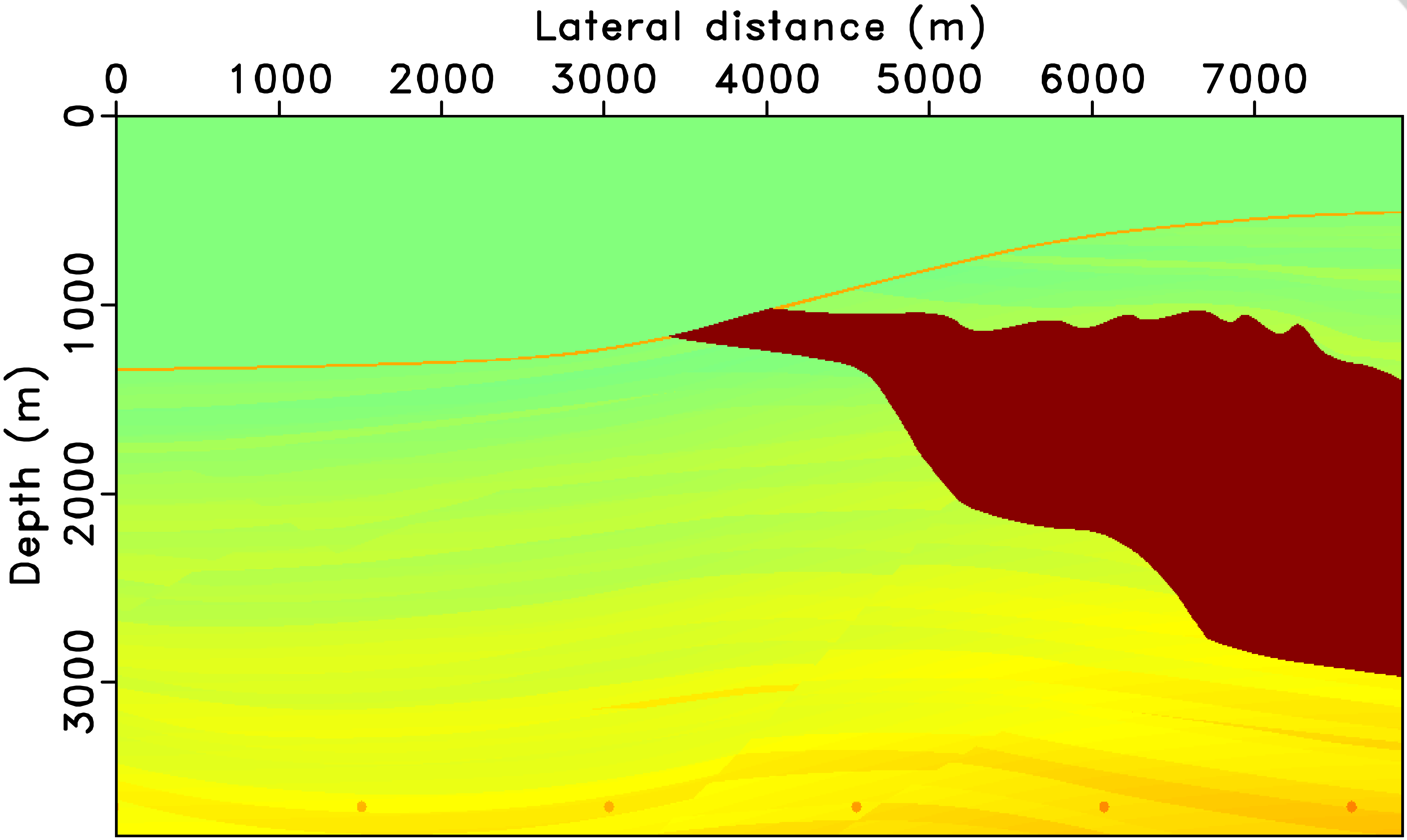
[same budget of PDE solves]



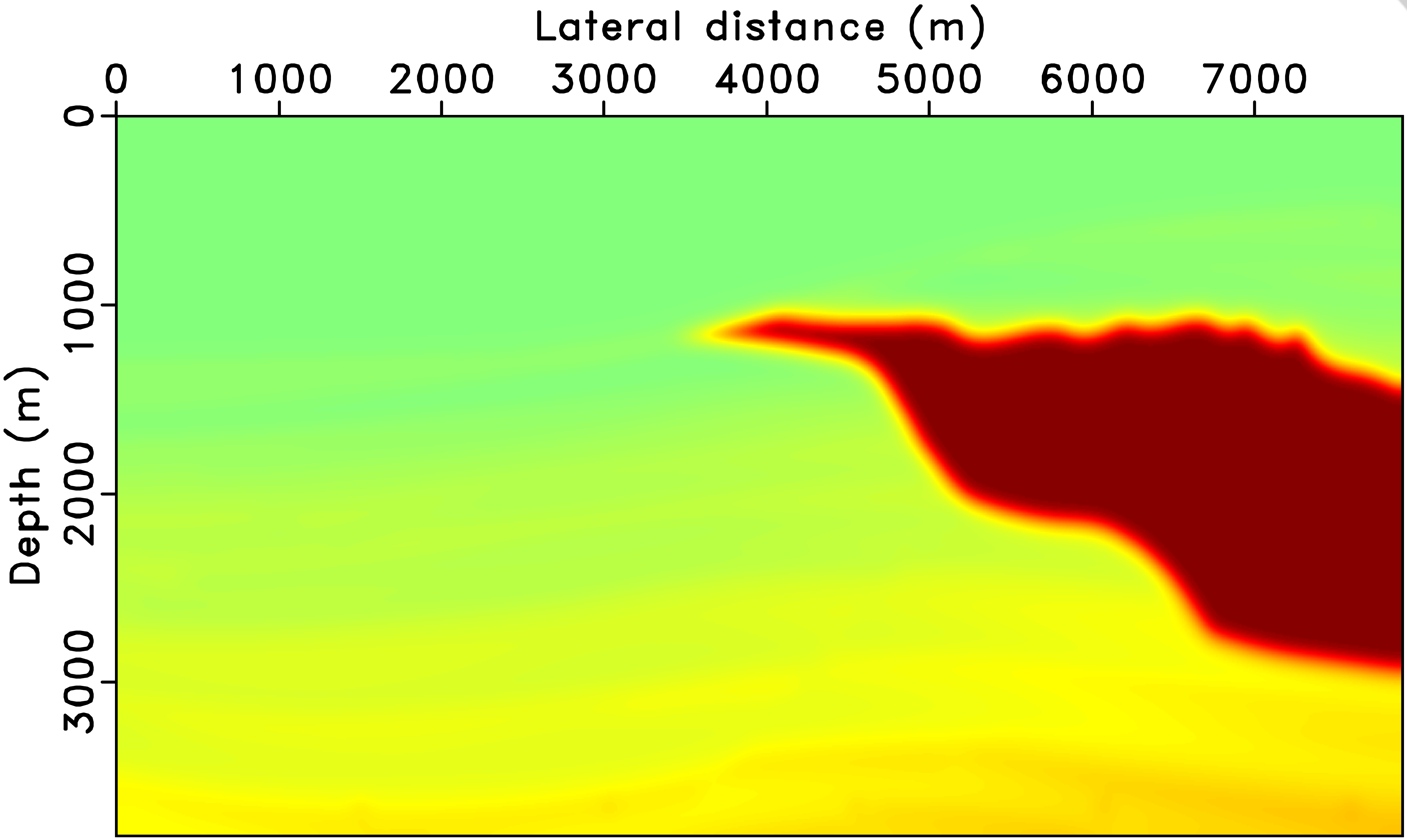
The Sigsbee2B model (cropped)

- model grid spacing: 7.62m
- using linearized data
- 174 sequential sources
- 278 frequencies in 0-34Hz range
- using 8 simultaneous sources and 15 frequencies with redrawing

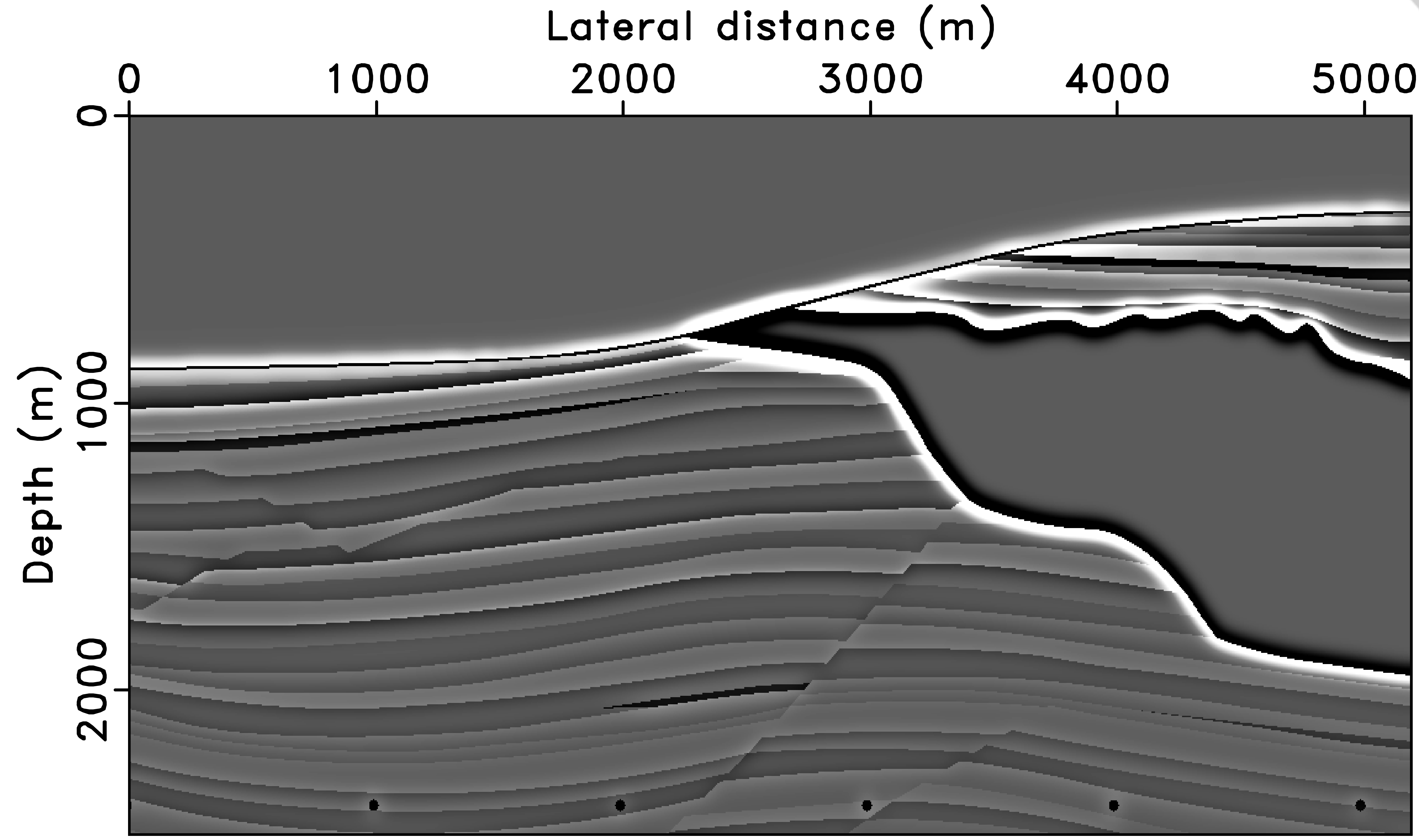
The Sigsbee2B model



Background model



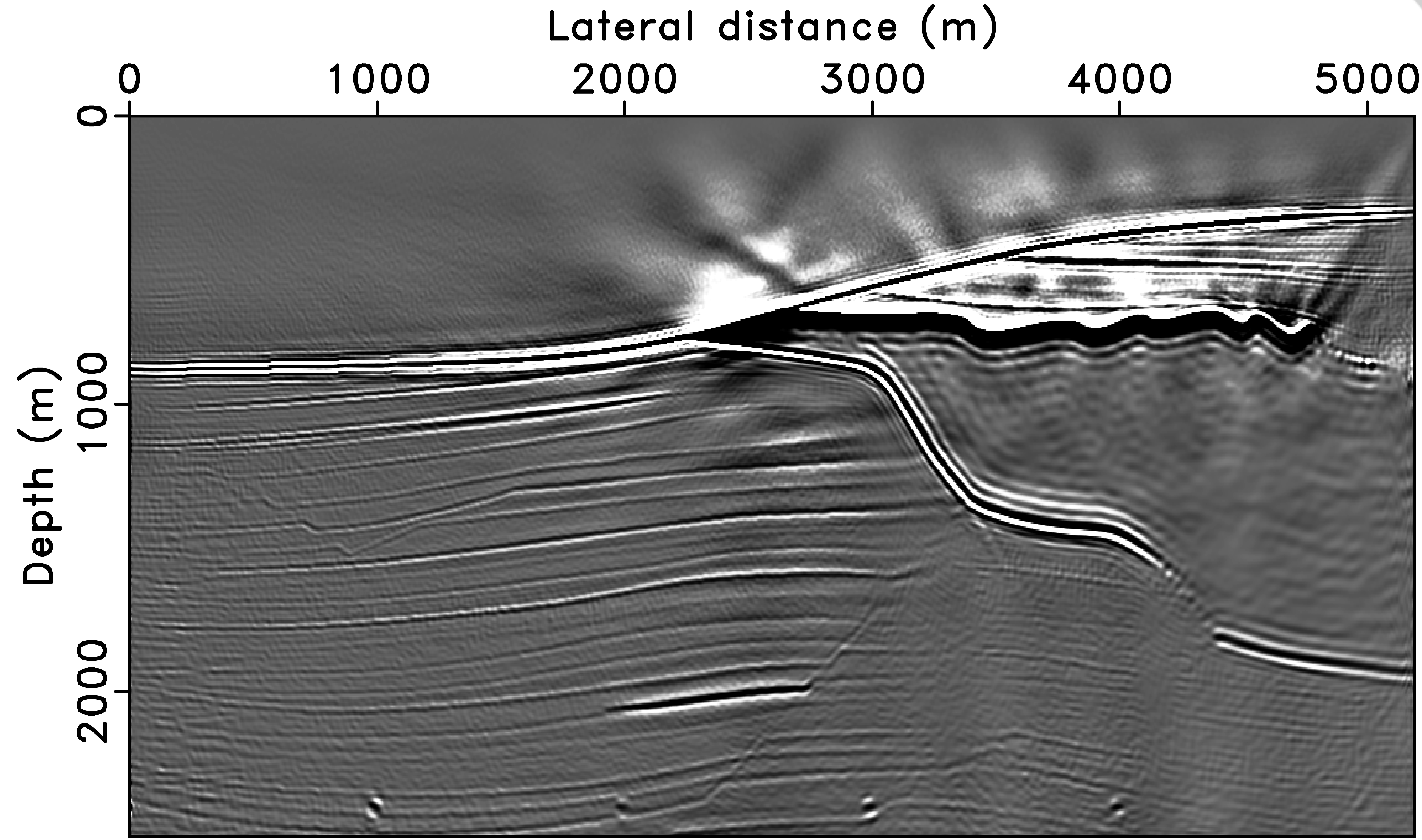
True perturbation



SLIM

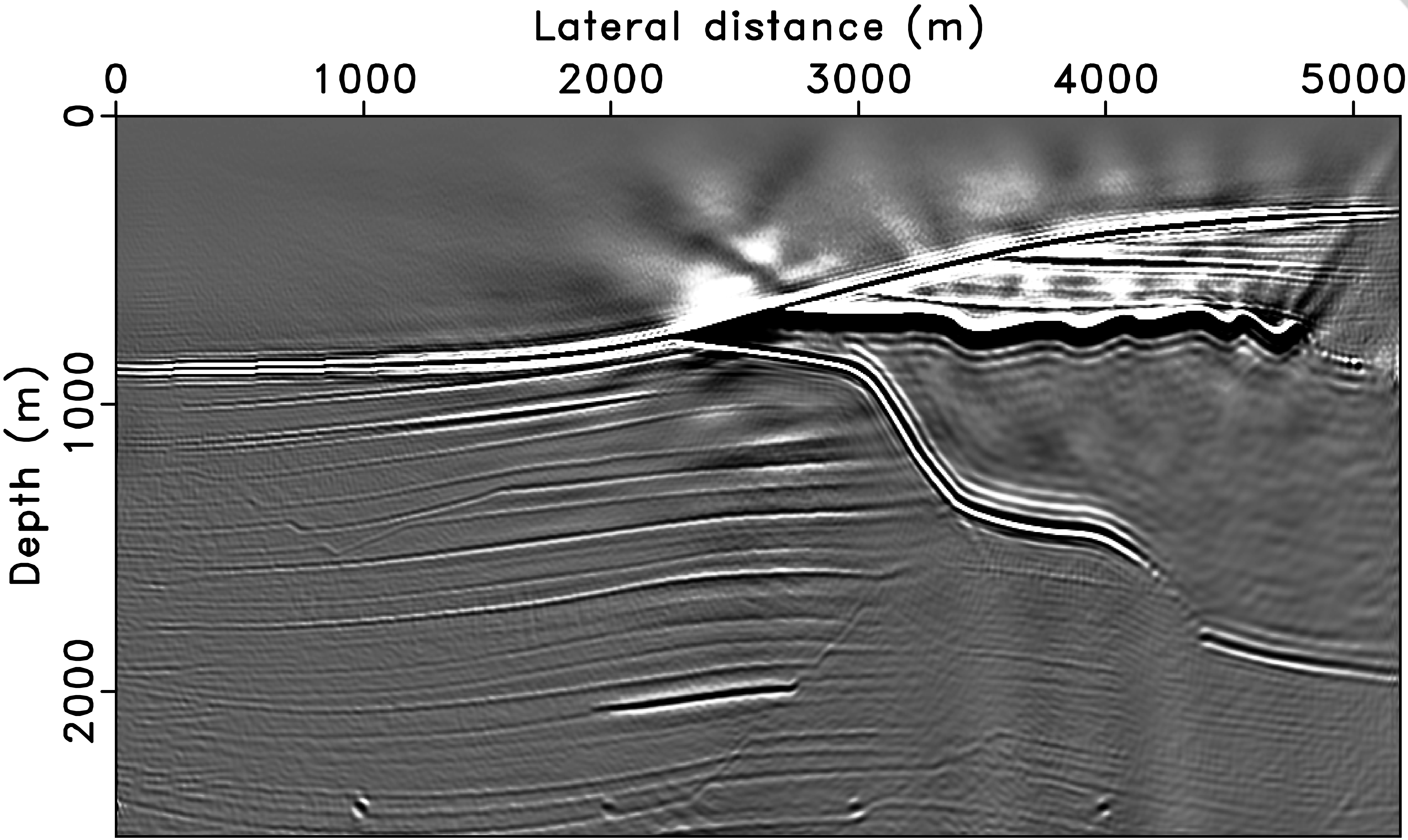
Fast inversion of primaries

[with a computational budget of a single RTM with all data]



Fast inversion of total data

[with a computational budget of a single RTM with all data]



SLIM

Conclusions

- An formulation is derived to image the total data based on the SRME formulation.
- Non-causal cross correlations when imaging multiples can be avoided by inversion.
- We greatly speed up the inversion by subsampling and redrawing.

Future work

- take source/receiver ghosts into consideration
- accurate estimation of source wavelet

Acknowledgements

Thanks for your attention!

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, and WesternGeco.