Fast imaging with multiples by sparse inversion
Ning Tu and Felix J. Herrmann
Motivation

- making use of primaries and multiples *simultaneously*
- avoiding imaging *artifacts* from multiples
- looking for a computationally *efficient* approach
Primaries & multiples: not “OR” but “AND”

- primaries have a higher signal-to-noise ratio
- multiples provide extra illumination if correctly used
Artifacts from multiples

Whitmore et al., 2010; Liu et al., 2011

Reverse time migration of *multiples* using total data as “source”
Not there for primaries

Reverse time migration of *primaries*
When a free-surface presents artifacts

\[
(i+1)\text{-th order reflection} \xrightarrow{\text{backward propagation}} \text{cross-correlation} \xrightarrow{} \text{image}
\]

\[
i\text{-th order reflection} \xrightarrow{\text{forward propagation}} \text{cross-correlation} \xrightarrow{} \text{artifacts}
\]

\[
(i-1)\text{-th order reflection} \xrightarrow{\text{forward propagation}}
\]

\[
\{i\text{-th order reflection} \}
\]

\[
(i+2)\text{-th order reflection} \xrightarrow{\text{backward propagation}}
\]

\[
\ldots
\]

\[
\ldots
\]

Liu et al., 2011

Wednesday, June 20, 2012
Want to avoid them?

Imaging of multiples by *inversion*

Lin et. al., 2010; Tu and Herrmann, 2011a
Inversion? Sounds expensive...

- repeated evaluations of the Born scattering operator and its adjoint
- each evaluation requires solving $4^\ast\ (#source)^\ast\ (#frequencies)$ PDEs
Sneak peek of our result
[with a computational budget of a single RTM with all data]

Fast imaging of *total up-going wavefield* by sparse inversion
Method
Incorporating the free surface

Total data and the surface-free Green’s function can be related by the SRME formulation:

\[ P_{\omega_i} = G_{\omega_i} (Q_{\omega_i} + R_{\omega_i} P_{\omega_i}) \]

- \( P \): total up-going wavefield
- \( G \): surface-free Green’s function
- \( Q \): source wavelet
- \( R \): surface reflectivity
Expressed in model space

\[ P_{\omega_i} = \text{vec}^{-1}(F_{\omega_i}[m, I])(Q_{\omega_i} + R_{\omega_i}P_{\omega_i}) \]
\[ = D_r H_{\omega_i}^{-1}[m](D_s^* I)(Q_{\omega_i} + R_{\omega_i}P_{\omega_i}) \]
\[ = D_r H_{\omega_i}^{-1}[m](D_s^* (Q_{\omega_i} + R_{\omega_i}P_{\omega_i})) \]
\[ = \text{vec}^{-1}(F_{\omega_i}[m, Q_{\omega_i} + R_{\omega_i}P_{\omega_i}]) \]

\textbf{F} : modelling operator
\textbf{m} : true velocity/density model
\textbf{I} : impulsive source array
\textbf{D} : detection operator at receiver/source locations
**Linearization**

\[ p_{\omega_i} = \nabla F_{\omega_i} \left[ m_0, Q_{\omega_i} + R_{\omega_i} P_{\omega_i} \right] \delta m \]

- \( \nabla F \): Born scattering operator
- \( m_0 \): background model
- \( \delta m \): model perturbation
- \( P_{\omega_i} \): vectorized wavefield
Stacking over frequencies

\[
p = \left[ \nabla F_{\omega_1} (m_0, Q_{\omega_i} + R_{\omega_i} P_{\omega_i}) \right] \delta m \\
\vdots \\
\nabla F_{\omega_{nf}} (m_0, Q_{\omega_i} + R_{\omega_i} P_{\omega_i}) \\
\vdots \\
\nabla F [m_0, Q + RP] \delta m
\]

\[\delta m = \nabla F^\dagger [m_0, Q + RP] p\]
Sparse inversion

We use a sparsity-promotion formulation:

$$\delta \tilde{m} = C^H \arg\min_{\delta x} ||\delta x||_1$$

subject to $$||p - \nabla F[m_0, Q + RP]C^H \delta x||_2 \leq \sigma$$

$C$: curvelet transform
solver: SPG$\ell_1$
Demonstrative examples

- model grid spacing: 5 meters
- using linearized data:
  \[ \nabla F[m_0, Q + RP] \delta m \]
- 150 collocated sources/receivers
- 122 frequencies in 0-60Hz range
Background model
True perturbation
Linearized total data
Inversion of the total up-going wavefield using all sequential sources and all frequencies
number of PDE solves: ~4.4 million (by calculation)
Dimensionality reduction

\[ \delta \tilde{m} = C^H \arg\min_{\delta x} ||\delta x||_1 \]

subject to \[ ||p - \nabla F[m_0, Q + RP]C^H \delta x||_2 \leq \sigma \]

**source**: combine sources into a few simultaneous sources, using Gaussian distributed random weights

**frequency**: randomly choose a subset of frequencies
Result with 15x speed-up

Inversion of the total up-going wavefield using 10 simultaneous sources and all frequencies
number of PDE solves: ~0.3 million
Too much subsampling brings artifacts

Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand
Draw new subsampling operator

- $\text{SPG}\ell_1$ solves a series of subproblems:

$$\arg\min_{\delta x} \| p - \nabla F[m_0, Q + RP]C^H \delta x \|_2$$

subject to $\|\delta x\|_1 \leq \tau$

- redraw subsampling operator for each new subproblem
Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies

number of PDE solves: 36.6 thousand (by calculation)
Draw new frequencies

Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies

number of PDE solves: 36.6 thousand (by calculation)
Draw new sim. sources and frequencies

Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies

number of PDE solves: 36.6 thousand (by calculation)
Solution path

One−norm of solution vector

Relative two−norm residual

0

20

40

60

80

100

120

0

0.2

0.4

0.6

0.8

1

no redrawing

with redrawing
Model error decrease

Note: outliers are intermediate line-search results, not a concern; number of PDE solves in practice has ~50% overhead due to line search, etc.
Inversion results

Trace to trace comparison: the 224th trace of model perturbation
Comparison: batch size

[same budget of PDE solves]

Fast imaging of total data

Batch size: 30 (2 simultaneous sources and 15 frequencies)
Iteration: 305
Number of PDE solves: 36.6 thousand (by calculation)
Comparison: batch size

[same budget of PDE solves]

Fast imaging of total data

**Batch size:** 15 (1 simultaneous sources and 15 frequencies)
**Iteration:** 610
**Number of PDE solves:** 36.6 thousand (by calculation)
Comparison: batch size

[same budget of PDE solves]

Fast imaging of total data
Batch size: 60 (4 simultaneous sources and 15 frequencies)
Iteration: 152
Number of PDE solves: 36.6 thousand (by calculation)
Comparison: batch size

[same budget of PDE solves]
The Sigsbee2B model

- model grid spacing: 7.62m
- using linearized data
- 174 sequential sources
- 278 frequencies in 0-34Hz range
- using 8 simultaneous sources and 15 frequencies with redrawing
The Sigsbee2B model
Background model
True perturbation
Fast inversion of primaries
[with a computational budget of a single RTM with all data]
Fast inversion of total data

[with a computational budget of a single RTM with all data]
Conclusions

- An formulation is derived to image the total data based on the SRME formulation.
- Non-causal cross correlations when imaging multiples can be avoided by inversion.
- We greatly speed up the inversion by subsampling and redrawing.
Future work

- take source/receiver ghosts into consideration
- accurate estimation of source wavelet
This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, and WesternGeco.

Thanks for your attention!