Least-squares migration of full wavefield with source encoding

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Abstract

Multiples can provide valuable information that is missing in primaries, and there is a growing interest in using them for seismic imaging. In our earlier work, we proposed to combine primary estimation and migration to image from the total up-going wavefield. The method proves to be effective but computationally expensive. In this abstract, we propose to reduce the computational cost by removing the multi-dimensional convolution required by primary estimation, and reducing the number of PDE solves in migration by introducing simultaneous sources with source renewal. We gain great performance boost without compromising the quality of the image.

Introduction

Multiples are usually considered to be unwanted components in seismic data. Now there is a growing realization that they can actually provide valuable information rather than simply being temporally displaced duplicates of primary reflections. They provide wider illumination angles, and are more sensitive to velocity changes (Verschuur, 2006). Great efforts have been made to use multiples in seismic imaging, especially the class of multiples referred to as " surface-related" multiples (Verschuur, 2006). Some recent advances are described by Lin et al. (2010); Whitmore et al. (2010); Verschuur (2011); Liu et al. (2011). One particularly relevant category of these methods identifies surface-related multiples as the response of the down-going seismic reflections at ocean surface (Guitton, 2002; Whitmore et al., 2010). However, two key problems need to be solved for these methods to work. The first one is to separate multiples from primaries with high fidelity, which has been partially solved by surface-related multiple elimination (SRME) (Verschuur et al., 1992), and the second one is to reduce migration artifacts (Lu et al., 2011; Liu et al., 2011) when applying imaging condition to multiples. Additionally, it is also preferable to use primaries and multiples, i.e., the full wavefield, simultaneously rather than separately.

With the recent development of an inversion alternative to SRME, namely primary estimation by sparse inversion (EPSI) (van Groenestijn and Verschuur, 2009; Lin and Herrmann, 2010), full-wavefield migration has become viable (Lin et al., 2010; Verschuur, 2011). Instead of separating the primary and multiple wavefields, EPSI inverts the surface-free Green's function from the total up-going wavefield. By applying combined inversion of the EPSI modelling operator and the Born modelling operator using a sparsity promoting formulation (Lin et al., 2010; Tu et al., 2011b,a), imaging from the total up-going wavefield yields satisfactory results with very few artifacts. However, this combined inversion, which involves repeated multi-dimensional convolution and migration, is computationally expensive. In this abstract, we propose to reduce the computational cost by (i) using wave-equation solver to carry out the multi-dimensional convolution and (ii) reducing the number of PDE solves by employing simultaneous sources with source renewals (Herrmann and Li, 2011), which enables us to further reduce the number of simultaneous sources without degrading imaging quality.

Full-wavefield migration with sparsity promotion

The relation between the total up-going wavefield and the surface-free Green's function can be mathematically described by the SRME formulation (Verschuur et al., 1992):

$$\overbrace{\hat{\mathbf{P}}}^{\text{upgoing}} = \hat{\mathbf{G}}\,\overbrace{(\hat{\mathbf{Q}} - \hat{\mathbf{P}})}^{\text{downgoing}} \quad (1)$$

in a monochromatic manner. Here, the free-surface reflection is assumed to be -1. The matrix $\hat{\mathbf{G}}$ represents surface-free Green's function, $\hat{\mathbf{P}}$ the total up-going wavefield, and $\hat{\mathbf{Q}}$ the source signature. Hatted quantities represent monochromatic variables. Each bold upper-case variable is a matrix comprised of a single frequency slice of the wavefield. When written in the canonical form of a linear operator acting on a vector, equation (1) can be represented as:

$$\underbrace{\mathscr{F}_{\mathbf{t}}^{*}\text{Blockdiag}_{\mathbf{1}\cdots\mathbf{n_f}}[(\hat{\mathbf{Q}} - \hat{\mathbf{P}})^{*} \otimes \mathbf{I}]\mathscr{F}_{\mathbf{t}}}_{\mathbf{E}}\,\mathbf{g} = \mathbf{p} \quad (2)$$

(Lin and Herrmann, 2010), where lower-case quantities $\mathbf{g}$ and $\mathbf{p}$ represent vectorized wavefields; $\mathscr{F}_{\mathbf{t}}$ is the Fourier transform that operates along the time axis of the vectorized wavefield $\mathbf{g}$, and its adjoint operator $\mathscr{F}_{\mathbf{t}}^{*}$ brings the wavefield back to the time domain; n_f is the number of frequencies. The block-diagonal term varies over frequencies. The symbol $\otimes$ refers to the Kronecker product, which turns matrix multiplications into matrix-vector multiplications. And we refer to $\mathbf{E}$ as the EPSI (modelling) operator, which maps the entire up-going wavefield from the surface-free Green's function. Note that the Green's function in formulation (1) or (2) contains internal multiples.

By ignoring internal multiples, we approximate the Green's function $\mathbf{g}$ in equation (2) with the linearized

Green's function $\delta \mathbf{g}$ modelled by the linearized Born (modelling) operator $\mathbf{K}$:

$$\delta \mathbf{g} = \mathbf{K}(\mathbf{m_0}, \mathbf{I})\delta \mathbf{m}. \tag{3}$$

Here, $\delta \mathbf{m}$ is model perturbations over a smooth background model $\mathbf{m_0}$, and the modelling involves solving wave equations for the impulsive source array $\mathbf{I}$. With this approximation, equation (2) can be represented as:

$$\mathbf{EK}(\mathbf{m_0}, \mathbf{I})\delta \mathbf{m} \approx \mathbf{p}. \tag{4}$$

We can then estimate $\delta \mathbf{m}$ from up-going wavefield $\mathbf{p}$ with a sparsity-promoting formulation that leverages sparsity in the curvelet domain (Lin et al., 2010):

$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta \mathbf{x}}{\operatorname{argmin}} ||\delta \mathbf{x}||_1 \ \text{ subject to } ||\mathbf{p} - \mathbf{EK}(\mathbf{m_0}, \mathbf{I})\mathbf{S}^* \delta \mathbf{x}||_2 \leq \sigma, \tag{5}$$

where $\mathbf{S}^*$ is curvelet synthesis operator, and σ is adjusted to allow for misfit from internal multiples, noise, or any modelling error.

This combined inversion is conceptually ideal in the sense that it exploits all the available subsurface illumination for imaging (caveat: non-surface-related multiples are ignored). We have demonstrated its effectiveness in our earlier work (Lin et al., 2010; Tu et al., 2011b,a). However, solving equation (5) is computationally expensive in the sense that: (i) the EPSI operator involves multi-dimensional convolution; (ii) the Born operator involves solving wave equations for all sequential sources since EPSI requires full sampling. For this method to be applicable to field seismic data, we address the high computational cost from these two aspects.

Embed multi-dimensional convolution in the Born operator

Looking in more detail at the SRME formulation, we can identify two source terms: point source $\mathbf{Q}$ that maps $\mathbf{G}$ to primaries, and the down-going surface reflections $-\mathbf{P}$ acting as an areal source that maps $\mathbf{G}$ to surface-related multiples. If we can inject $\mathbf{Q} - \mathbf{P}$ as the source term in the wavefield simulator, the computational cost of multi-dimensional convolution can be saved.

First, we write the Born operator explicitly in a monochromatic manner:

$$\begin{aligned} \hat{\mathbf{K}}(\mathbf{m_0}, \hat{\mathbf{I}})\delta \mathbf{m} &= \operatorname{vec}(\mathbf{D_r}\hat{\mathbf{H}}^{-1}(\mathbf{m_0})(\omega^2 \hat{\mathbf{H}}^{-1}(\mathbf{m_0})\operatorname{Diag}(\delta \mathbf{m})\mathbf{D_s^*}\hat{\mathbf{I}})) \\ &= \operatorname{vec}(\hat{\mathbf{K}}(\mathbf{m_0})\operatorname{Diag}(\delta \mathbf{m})(\mathbf{D_s^*}\hat{\mathbf{I}})), \end{aligned} \tag{6}$$

where $\hat{\mathbf{H}}$ is the time-harmonic Helmholtz operator, $\mathbf{D_r}$ is the detection operator that extracts data at receiver positions, and $\mathbf{D_s^*}$ injects sources at source locations. $\hat{\mathbf{K}}$ is overloaded to simplify the representation. Operation "vec" is to vectorize a wavefield.

By the associativity of matrix multiplication, equation (4) can be monochromatically expressed as:

$$\begin{aligned} \hat{\mathbf{P}} &\approx \delta \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}}) \\ &= \hat{\mathbf{K}}(\mathbf{m_0})\operatorname{Diag}(\delta \mathbf{m})(\mathbf{D_s^*}\hat{\mathbf{I}})(\hat{\mathbf{Q}} - \hat{\mathbf{P}}) \\ &= \hat{\mathbf{K}}(\mathbf{m_0})\operatorname{Diag}(\delta \mathbf{m})(\mathbf{D_s^*}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})). \end{aligned} \tag{7}$$

Following the notation in equation (2) and (6), it reveals:

$$\mathbf{EK}(\mathbf{m_0}, \mathbf{I})\delta \mathbf{m} = \mathbf{K}(\mathbf{m_0}, \mathbf{Q} - \mathbf{P})\delta \mathbf{m}, \tag{8}$$

which means that the costly multi-dimensional convolution can be alternatively carried out by the the wave-equation solver by injecting the compound source $\mathbf{Q} - \mathbf{P}$ in the Born operator. Note that source-receiver reciprocity, imposed by the SRME formulation, is still required for equation (7) to hold. Otherwise matrix multiplication between term $\hat{\mathbf{I}}$ and term $\hat{\mathbf{Q}} - \hat{\mathbf{P}}$ is infeasible.

Reduce number of PDE solves by source encoding

By identifying $\mathbf{Q}-\mathbf{P}$ as a source term in the Born operator, not only can we remove the operation of multi-dimensional convolution, we can also reduce the number of PDE solves by using fewer number of sources. Due to source linearity, subsampling the wavefield is equivalent to subsampling the sources in the wavefield simulator, i.e., $\mathbf{RMK}(\mathbf{m_0},\mathbf{Q}-\mathbf{P})\delta\mathbf{m} = \mathbf{K}(\mathbf{m_0},\mathrm{vec}^{-1}\mathbf{RM}\mathrm{vec}(\mathbf{Q}-\mathbf{P}))\delta\mathbf{m}$, where $\mathbf{RM}$ is the subsampling operator and operation "vec^{-1}" reshapes vectors back to matrices. By combining equation (8), a dimensionality-reduced approach of equation (5) can be represented as:

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \operatorname*{argmin}_{\delta\mathbf{x}} ||\delta\mathbf{x}||_1 \quad \text{subject to } ||\mathbf{RMp}-\mathbf{K}(\mathbf{m_0},\mathrm{vec}^{-1}\mathbf{RM}\mathrm{vec}(\mathbf{Q}-\mathbf{P}))\mathbf{S}^*\delta\mathbf{x}||_2 \leq \sigma. \tag{9}$$

Following Herrmann and Li (2011), the subsampling operator can be factorized into a mixing operator $\mathbf{M}$ that in our case blends all sequential sources into simultaneous sources with random Gaussian-distributed weights, and a restriction operator $\mathbf{R}$ that uniform-randomly chooses a subset of simultaneous sources from all of them.

Mixing the sources will introduce cross-talks (Romero et al., 2000). When the number of simultaneous sources satisfies the empirical recovery condition of compressive sensing (CS), the cross-talks are accounted for by the sparsity-promoting formulation. However, as we decrease the number of simultaneous sources, more artifacts show up. By drawing new simultaneous sources (Herrmann and Li, 2011), these artifacts can be mitigated. Using SPGℓ_1 (van den Berg and Friedlander, 2008) as the solver, we draw a new subset of simultaneous sources once a LASSO subproblem is solved (Herrmann and Li, 2011). This removes correlations that build up between the iterate and the source weights, which in turn allows us to work with fewer simultaneous sources. We refer to this practice as source renewal.

Synthetic example

Linearized data is made according to the right hand side of equation (8). We trim and smooth the salt dome model (van Groenestijn and Verschuur, 2009) to get a smooth macro model $\mathbf{m_0}$, and get the model perturbation $\delta\mathbf{m}$ by subtraction. Model dimensions are Z=1250m, X=2235m with 5m interval; there are 150 co-located sources and receivers with 15m interval. Frequency band of the data is 0-60Hz.

We compare four scenarios using different number of sources. The first one is according to equation (5), where we use all 150 sequential sources. The remaining three are according to equation (9), where we use 10, 2, and 2 simultaenous sources respectively. In the last scenario, source renewal is applied. All non-zero frequencies inside data bandwidth are used with ~0.5Hz increment. We can see from the result that with 10 simultaneous sources, we can already get an image almost as clear as when we use all sequential sources. When we reduce the number of simultaneous sources to 2, artifacts appear. However, after we apply source renewal, these artifacts are largely eliminated. This leads to a theoretical speed-up of 75X compared with using all sequential shots.

Conclusions

In our earlier work, we proposed to migrate from the total up-going wavefield by combined inversion of the EPSI modelling operator and the Born modelling operator with a sparsity-promoting formulation. In this paper, we proposed to reduce its computational cost by removing multi-dimensional convolutions, and using source encoding to reduce the number of PDE solves. By using source renewal, we further reduce the number of simultaneous sources without introducing significant artifacts.

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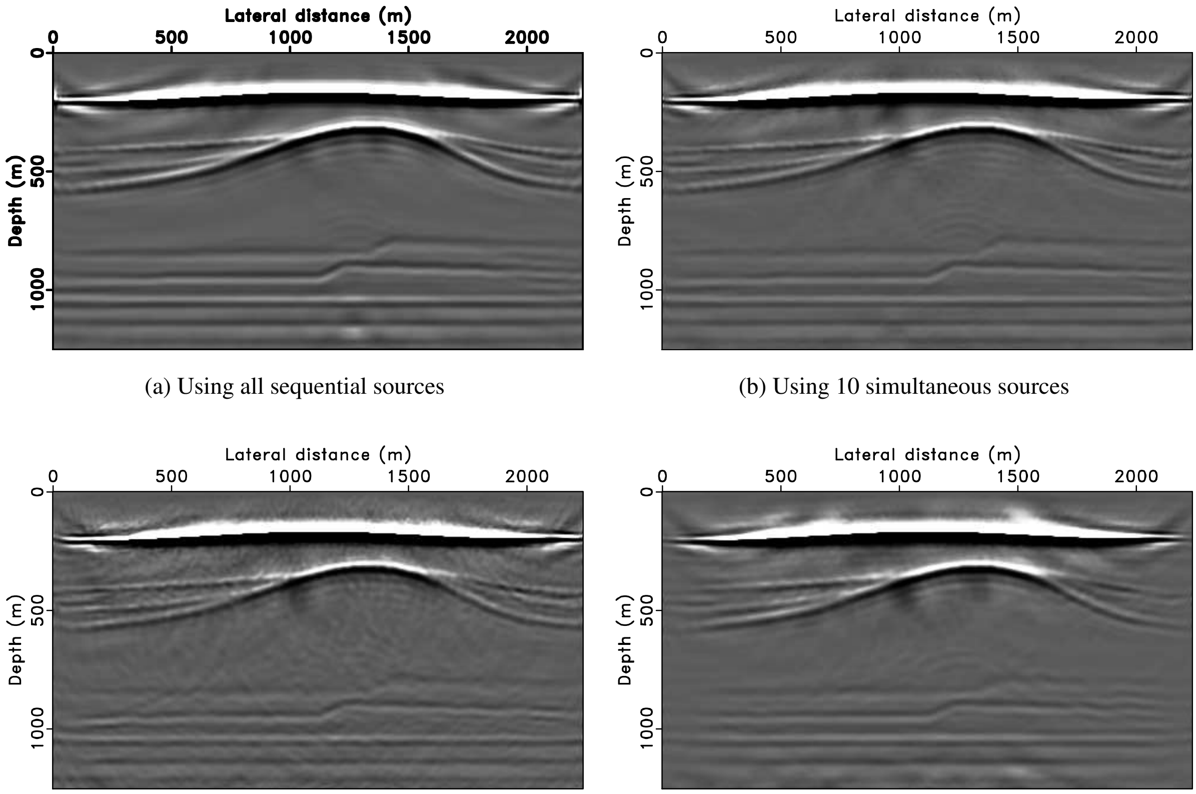

(a) Using all sequential sources

(b) Using 10 simultaneous sources

(c) Using 2 simultaneous sources

(d) Using 2 simultaneous sources with renewal

Figure 1: Full-wavefield migration using different sources

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