Recent Developments In Preconditioning the FWI Hessian - A Dimensionality Reduction Approach

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Abstract: Various techniques in performing least-squares migration revolve around the mathematical approximation of the so-called Gauss-Newton Hessian or normal operator (that is, demigration followed by migration) as a pseudo-differential operator. In this project, we extend the ideas presented originally in L. Demanet et al, which attempt to recover the action of the inverse Hessian by observing its action on a few random test vectors, to a dimensionality-reduction scenario by using simultaneous sources. Despite the noise introduced into the image by the simultaneous sources, the preconditioner is able to reasonably compensate for the amplitude distortion introduced by the normal operator.

To find the extended abstract corresponding to this work, follow this QR code.

Motivation and approach:

- Using the complete data in least-squares migration is far too costly (from both a memory and a time perspective)
- Even with a dimensionality reduction approach such as employing simultaneous sources, using iterative solvers such as LSQR for least-squares migration is still prohibitively expensive and we must employ a preconditioner
- Under certain assumptions, the action of the Hessian (and the inverse Hessian) can be approximated as a pseudo-differential operator

- If J is our demigration operator, then

$$Hm(x) = J^*Jm(x) \approx \int e^{2\pi i k\cdot x} a(x,k)\hat{m}(k)dk$$

- The symbol, $a(x,k)$, is a smooth function which satisfies

$$|\partial_k^\alpha \partial_x^\beta a(x,k)| \leq C_{\alpha\beta}(1+|k|^2)^{(e-|\alpha|)/2}$$

$$\alpha = (\alpha_1,\alpha_2), \partial_k^\alpha = \frac{\partial^{\alpha_1}}{\partial k_1^{\alpha_1}}\frac{\partial^{\alpha_2}}{\partial k_2^{\alpha_2}}$$

Operator Expansion

- We can approximately expand the symbol as a linear combination of known functions

$$a(x,k) \approx \sum_{\lambda,q_1,q_2} c_{\lambda,q_1,q_2} e^{2\pi i\lambda\cdot x} e^{iq_1\theta} TL_{q_2}(|k|)|k|^e$$

$TL_q(x)$ is the qth rational Chebyshev polynomial, e is the *order* of the operator

- This decomposition lets us compactly write

$$H \approx \sum_j c_j B_j$$

or alternatively

$$H^{-1} \approx \sum_j d_j B_j$$

for unknown coefficient vectors c_j or d_j and known operators B_j.

- We solve for these coefficients by the so-called method of *matrix-probing*, i.e. by enforcing

$$H^{-1}y_k \approx \sum_j d_j B_j y_k$$

for known suitable known vectors y_k and solving the resulting linear system for d_j

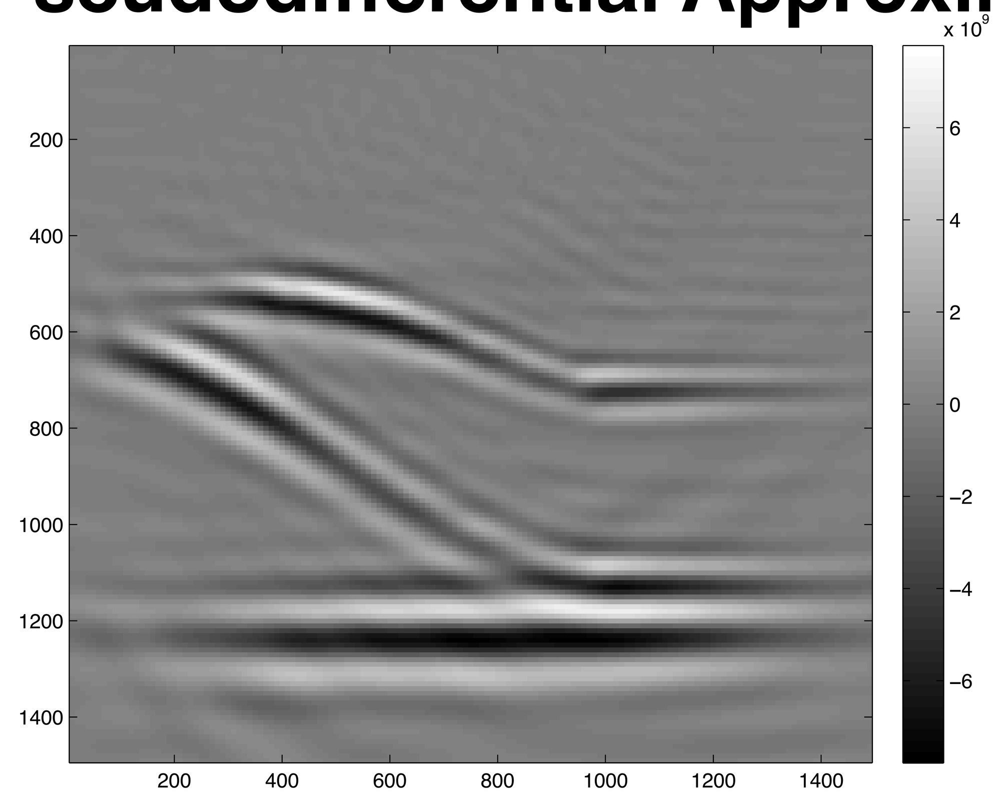
Dimensionality Reduction via Simultaneous Sources

- This is this approach outlined in L. Demanet et al. [1]

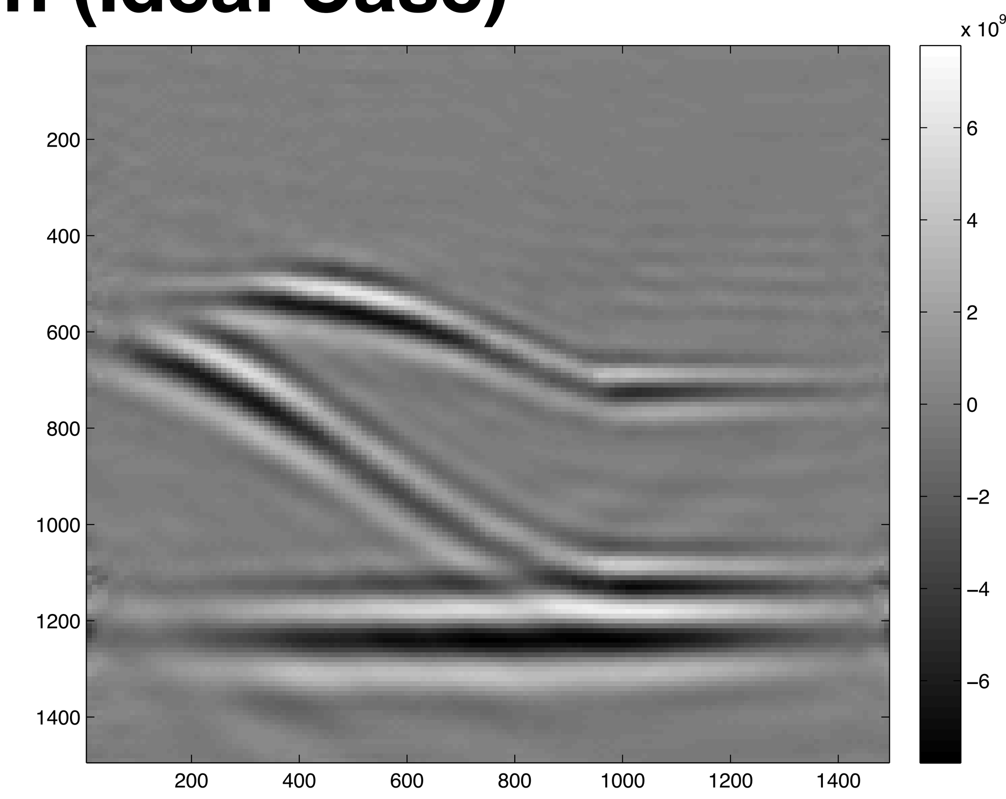
- We use these ideas in conjunction with simultaneous sources, to further enhance the convergence of our iterative solver and while also decreasing the overall work involved in least-squares migration.

- We consider a synthetic clay model with data generated from 101 sources (10 simultaneous sources, respectively) and a random sample of 26 frequencies from 1.5Hz to 19.5Hz.

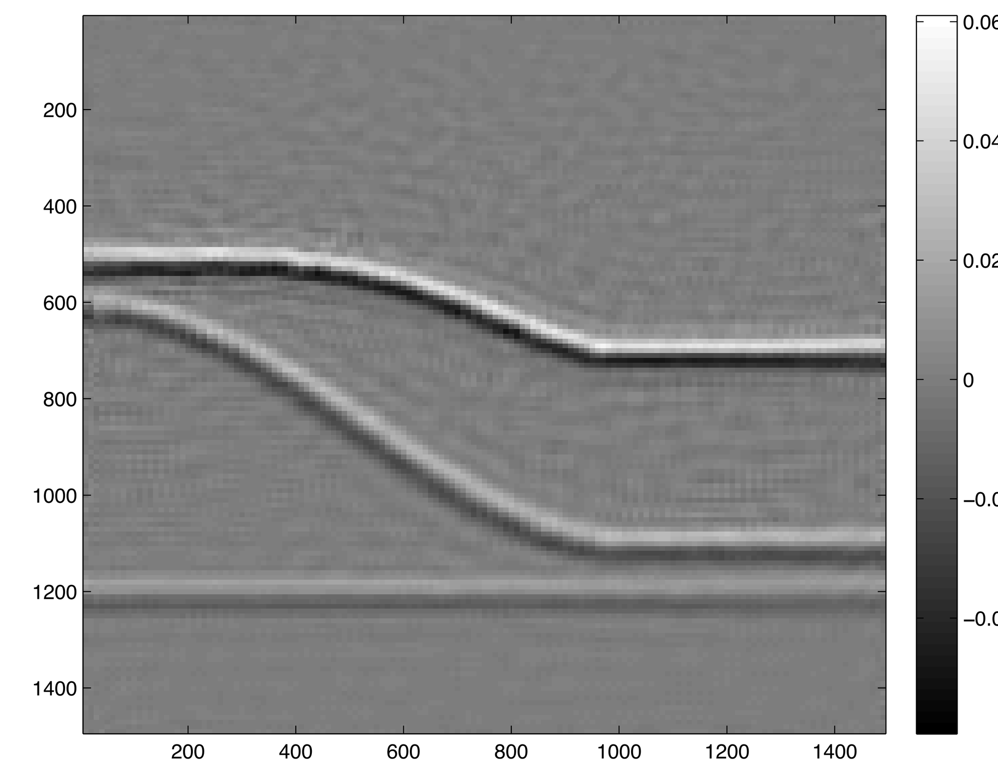
Pseudodifferential Approximation (Ideal Case)



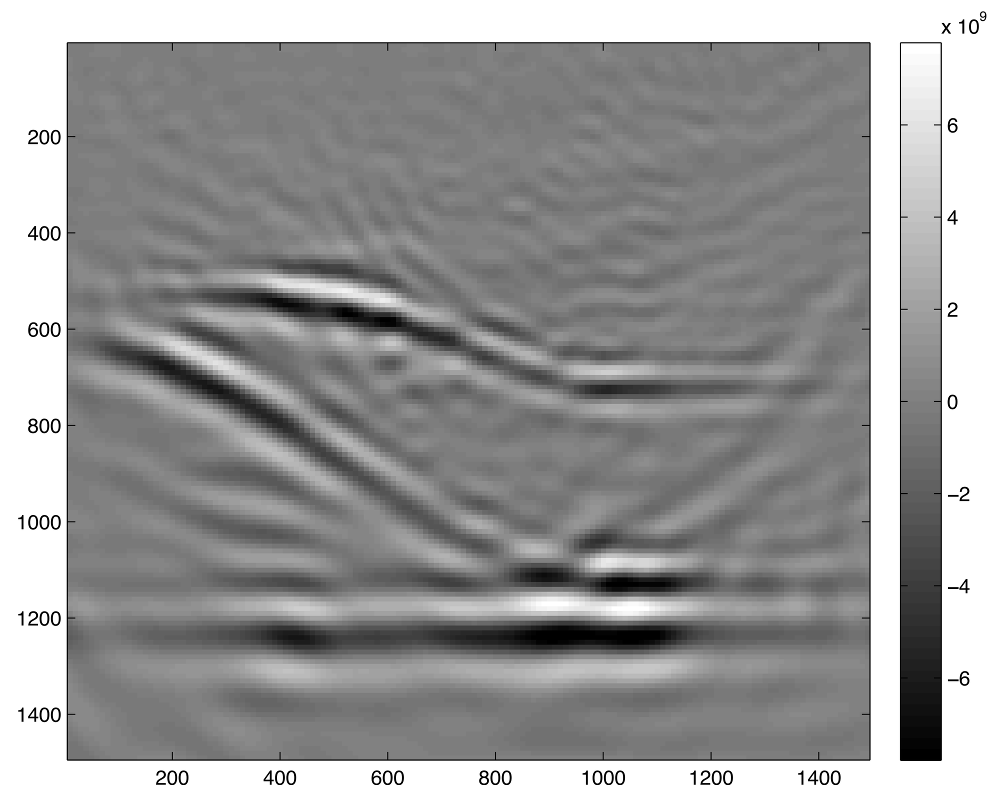
True Hessian - Full Sources



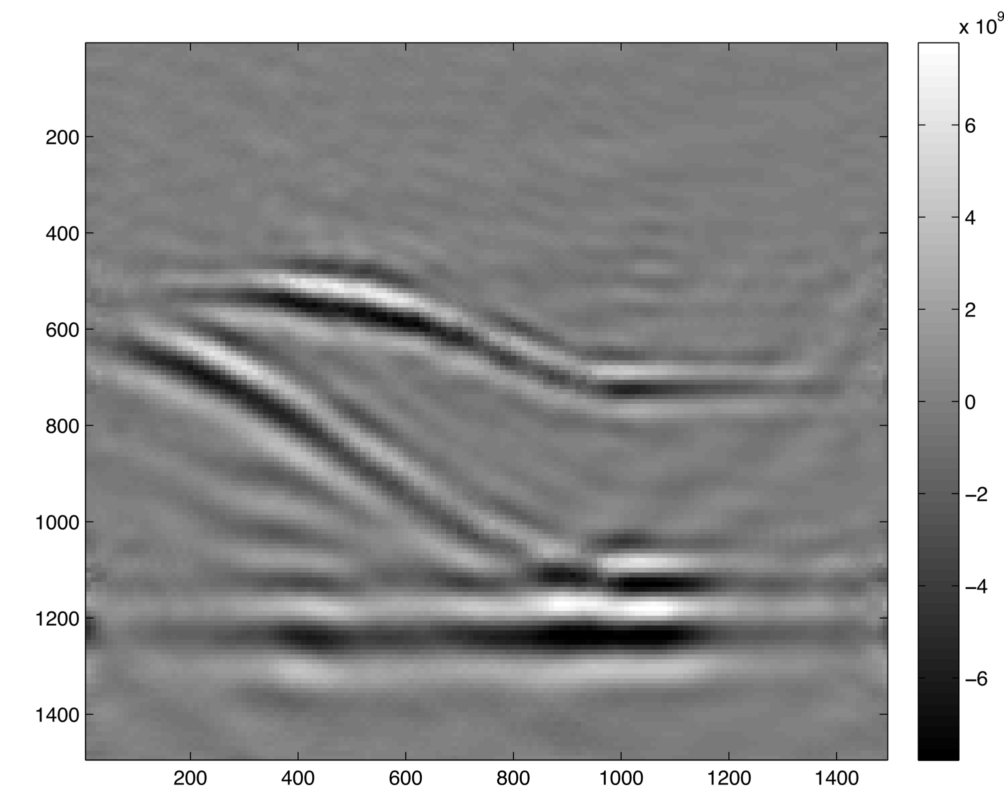
Approximate Hessian - true perturbation as a test function



Approximate Inverse Hessian - true perturbation as a test function



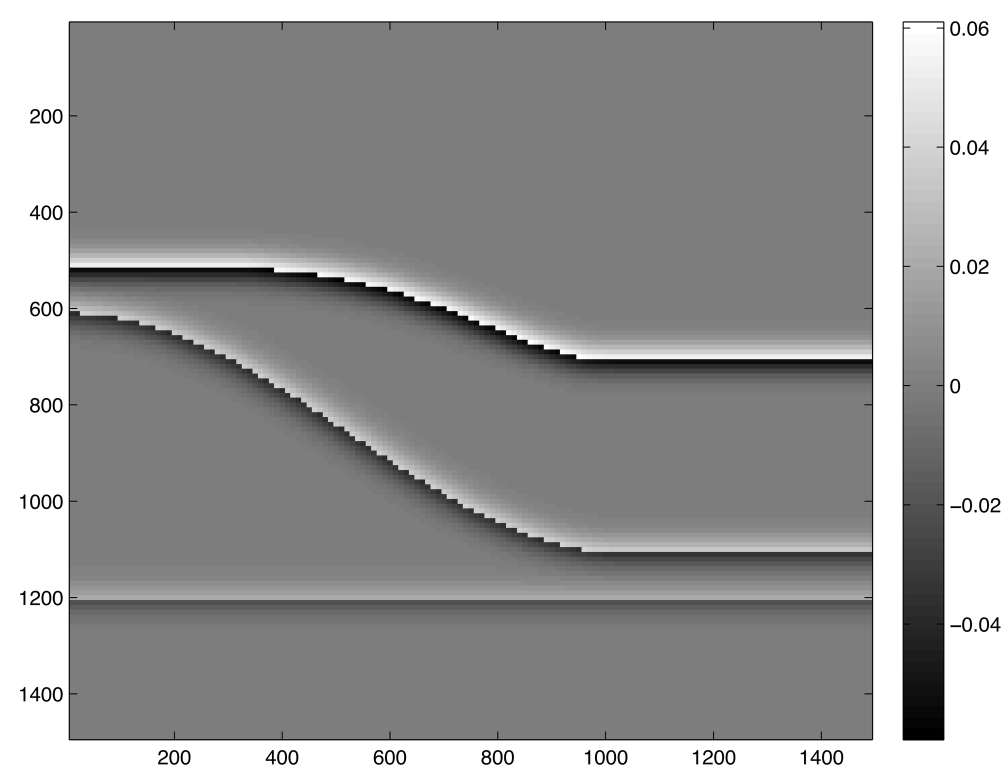
True Hessian - Simultaneous Sources



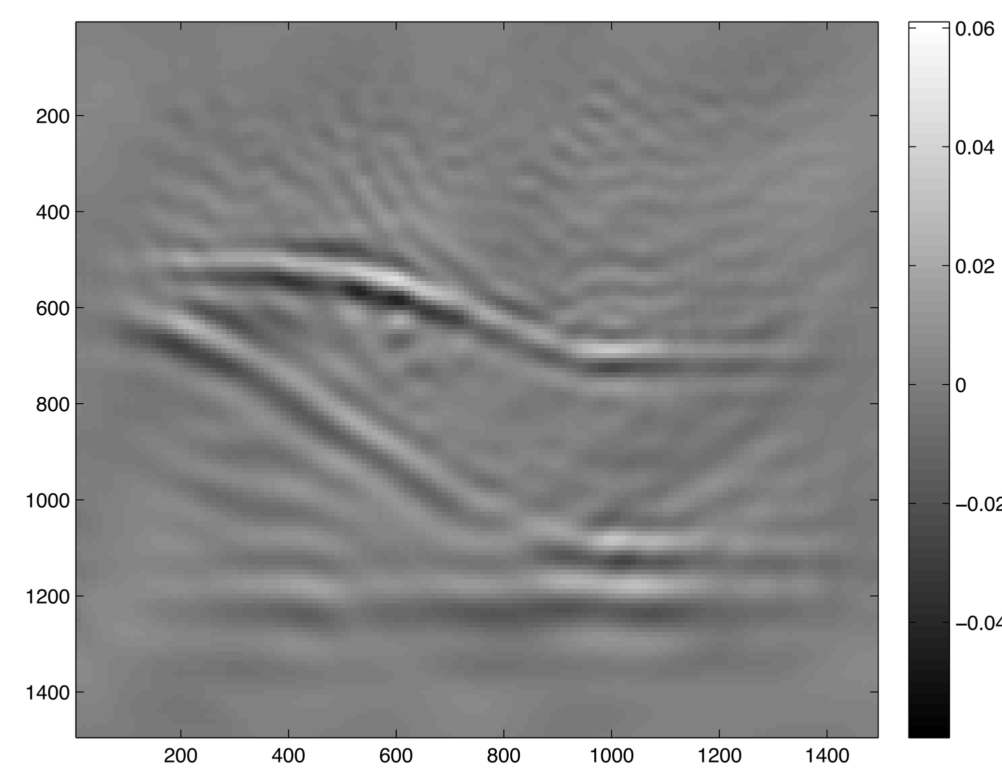
Approximate Hessian - true perturbation as a test function

Pseudodifferential Approximation of the Inverse Hessian

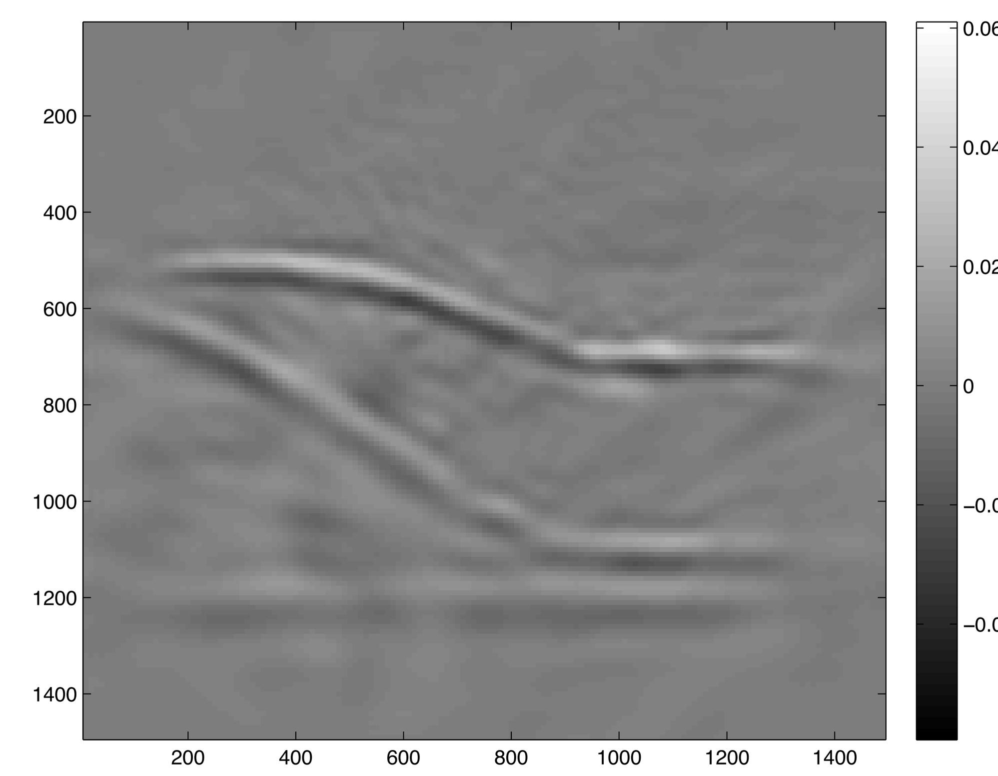
- Simultaneous sources, zero order Hessian



Model Perturbation



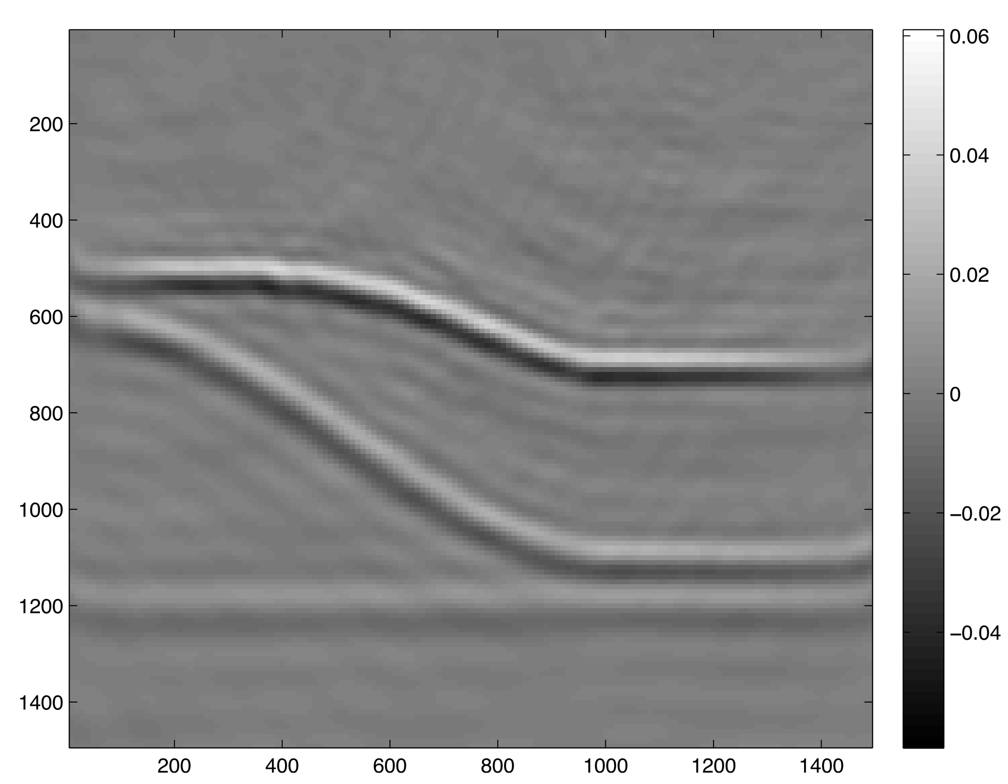
Approximate Inverse Hessian using noisy test functions



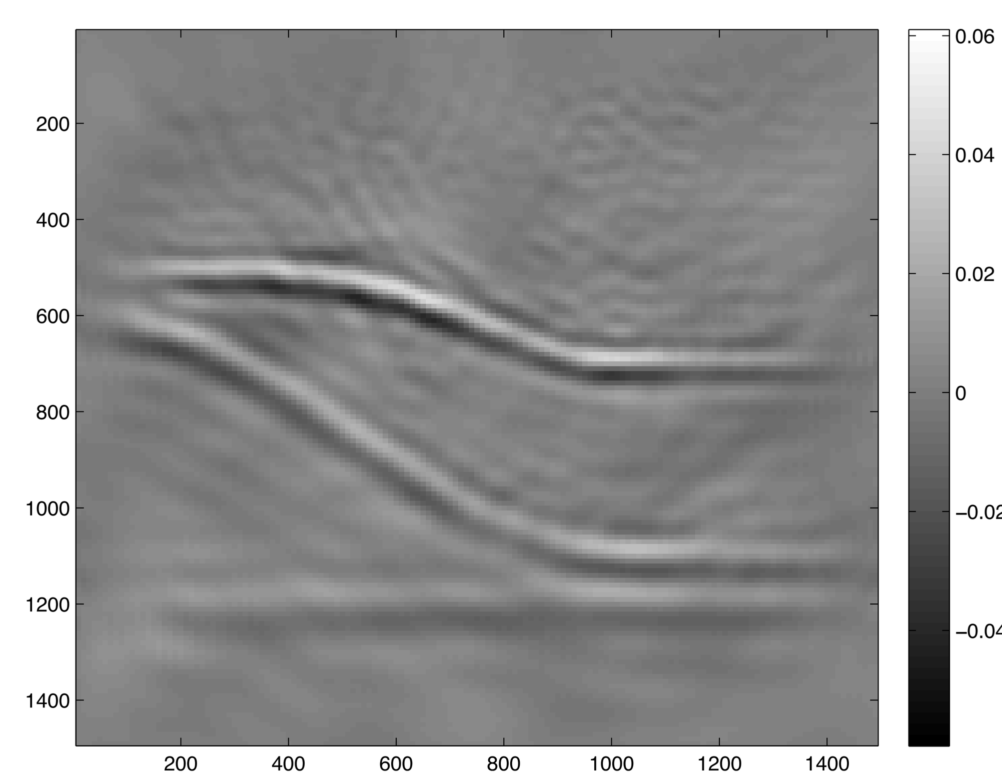
Approximate Inverse Hessian using a scaled, remigrated image test function

Preconditioned Images

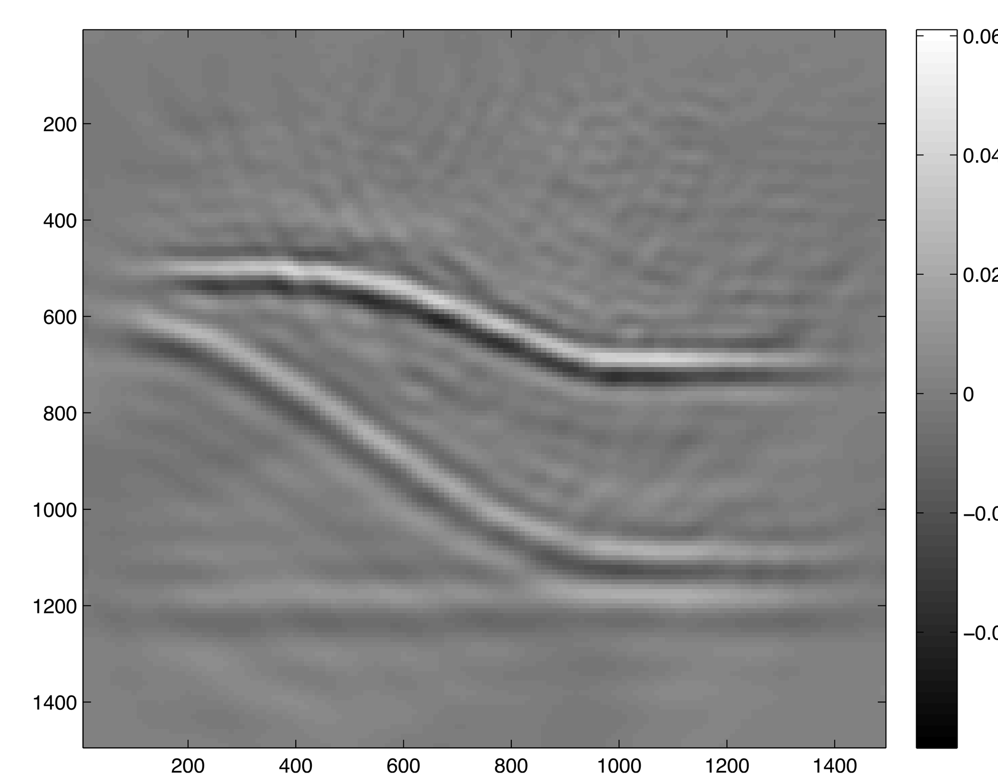
- Simultaneous sources, zero order Hessian



No preconditioning - 15 iterations of LSQR



Noisy test functions - 2 iterations of PCG



Noisy curvelet test functions - 3 iterations of PCG

Future Work

- Determine the role of each expansion parameter in the accuracy of the preconditioner
- Apply this preconditioner in the context of a sparsity promoting program
- Modify the preconditioner construction in order to preserve positive semidefiniteness (of the true Hessian and inverse Hessian)

References:

1. Laurent Demanet, et al. Matrix probing: a randomized preconditioner for the wave-equation hessian. *Applied and Computational Harmonic Analysis*, 32(2):155-168, March 2012.
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Acknowledgements